Generalized unitarity & W/Z + jets @ NLO

Giulia Zanderighi

Oxford Theoretical Physics & STFC

Based on work done with Keith Ellis, Walter Giele, Zoltan Kunszt, Kirill Melnikov

Linear Collider meeting, Perugia, September 2009

International Linear Collider

- we all believe that no matter what will be discovered (or not) at the LHC, the ILC will provide complementary information
- given the high energy involved, the ILC can be a discovery machine, but thanks to the very clean e+e- environment the ILC will be mainly a precision machine

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From the high precision of the ILC we expect to

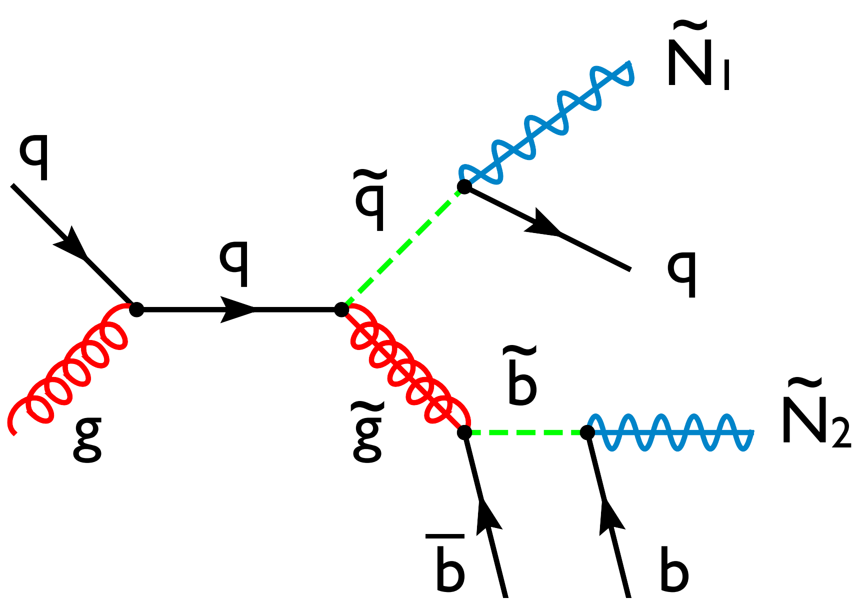
- identify the nature of new physics (discovered at the LHC?) by doing direct and indirect measurements of particle properties
- constrain new physics and model parameters (e.g. heavy masses, couplings)

Why NLO

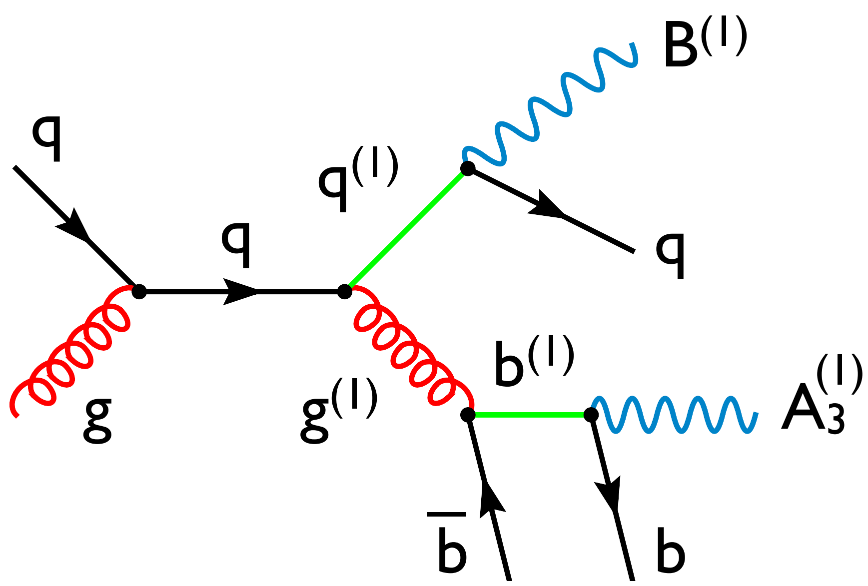
Accurate theoretical perturbative predictions desirable at hadron colliders and indispensable at e^+e^- linear collider in order to match the accuracy of experimental measurements. Processes with many particles in the final state are the most important backgrounds and have typically have much larger uncertainties at LO

☛ NLO predictions essential

SUSY:



UED:



This talk

- Brief reminder of main ideas of D-dimensional unitarity at NLO
- Recent new results for Tevatron/LHC for W + 3jet production
- Towards applications for LEP/ILC (➠ 5 jets ,V + multi-jets)

References:

- Ellis, Giele, Kunszt '07	*[Unitarity in D=4]*
- Giele, Kunszt, Melnikov '08	*[Unitarity in D≠4]*
- Giele & GZ '08	*[All one-loop N-gluon amplitudes]*
- Ellis, Giele, Melnikov, Kunszt '08	*[Massive fermions, ttggg amplitudes]*
- Ellis, Giele, Melnikov, Kunszt, GZ '08	*[W+5p one-loop amplitudes]*
- Ellis, Melnikov, GZ '09	*[W+3 jets]*

These papers heavily rely on previous work

- Bern, Dixon, Kosower '94	*[Unitarity, oneloop from trees]*
- Ossola, Pittau, Papadopoulos '06	*[OPP]*
- Britto, Cachazo, Feng '04	*[Generalized cuts]*
- *[....]*	

NLO: the traditional way

- draw all possible Feynman diagrams (use automated tools)
- write one-loop amplitudes as $\sum$ (coefficients × tensor integrals)
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Problem solved in principle, but brute force approaches plagued by worse than factorial growth ⇒ difficult to push methods beyond N=6 because of high demand on computer power, but N>5 if great interest at the LHC/ILC

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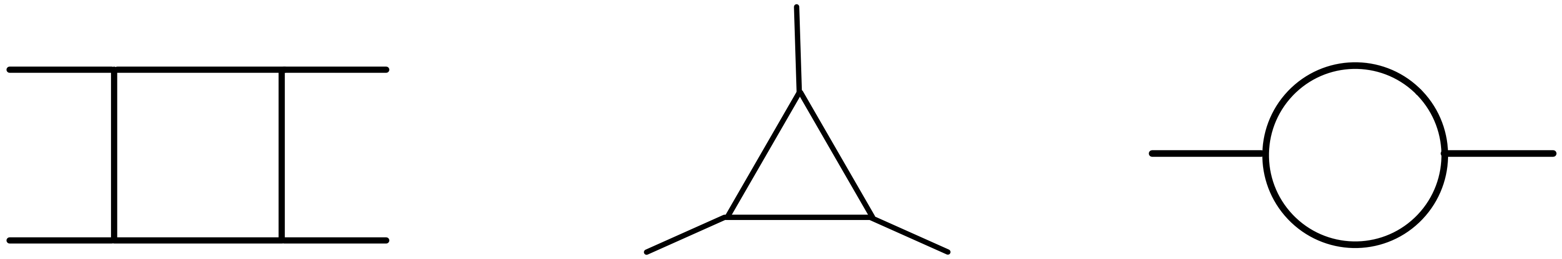
Many new ideas recently. I will talk about generalized unitarity and show its *simplicity, generality, efficiency, and thus suitability for automation*

Decomposition of the one-loop amplitude

$$\mathcal{A}_N^D = \sum_{[i_1|i_4]} d^D_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} \quad + \quad \sum_{[i_1|i_3]} c^D_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^D_{i_1 i_2} I^{(D)}_{i_1 i_2} \; ^{*}$$



Remarks:

- higher point function reduced to boxes + vanishing terms
- coefficients depend on D (i.e. on ϵ) $\Rightarrow$ rational part
- box, triangles and bubble integrals all known analytically

['t Hooft & Veltman '79; Bern, Dixon Kosower '93, Duplancic & Nizic '02; Ellis & GZ '08, public code $\Rightarrow$ http://www.qcdloop.fnal.gov]

* if non-vanishing masses: tadpole term;   notation: $[i_1|i_m] = 1 \leq i_1 < i_2 \ldots < i_m \leq N$

Cut-constructible part

Start from

$$\mathcal{A}_N^{\rm cut} = \sum_{[i_1|i_4]} d_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b_{i_1 i_2} I^{(D)}_{i_1 i_2} = \int \frac{d^D l}{i(\pi)^{D/2}} \mathcal{A}_N^{\rm cut}(l)$$

with

$$I^D_{i_1 \cdots i_M} = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{1}{d_{i_1} \cdots d_{i_M}}$$

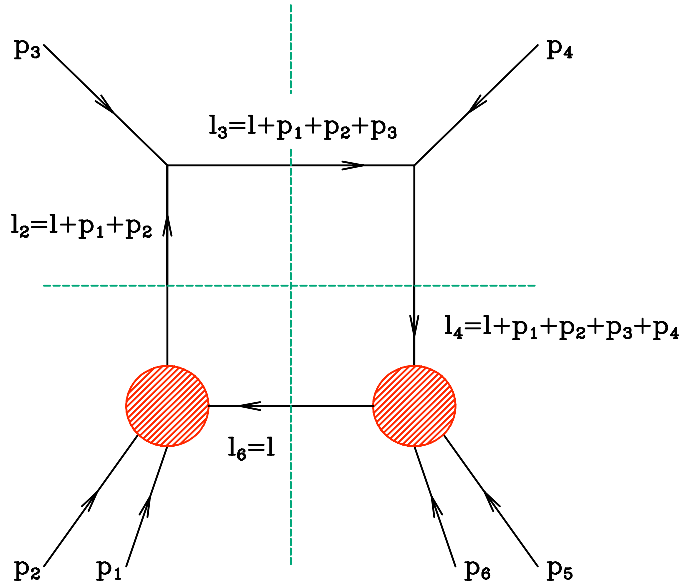
Look at the integrand

$$\mathcal{A}_N^{\rm cut}(l) = \sum_{[i_1|i_4]} \frac{\bar{d}_{i_1 i_2 i_3 i_4}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\bar{c}_{i_1 i_2 i_3}}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_1]} \frac{\bar{b}_{i_1 i_2}}{d_{i_1} d_{i_2}}$$

Get cut numerators by taking residues: i.e. set inverse propagator = 0
In D=4 up to 4 constraints on the loop momentum (4 onshell propagators) ⇒ get up to box integrals coefficients

Construction of the box residue

Four cut propagators are onshell
⇒ the amplitude factorizes into 4 tree-level amplitudes



$$
\begin{aligned}
\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^\pm)\Big) \quad &= \quad \mathcal{M}^{(0)}(l_i^\pm; p_{i+1}, \ldots, p_j; -l_j^\pm) \times \mathcal{M}^{(0)}(l_j^\pm; p_{j+1}, \ldots, p_k; -l_k^\pm) \\
&\times \quad \mathcal{M}^{(0)}(l_k^\pm; p_{k+1}, \ldots, p_l; -l_l^\pm) \times \mathcal{M}^{(0)}(l_l^\pm; p_{l+1}, \ldots, p_i; -l_i^\pm)
\end{aligned}
$$

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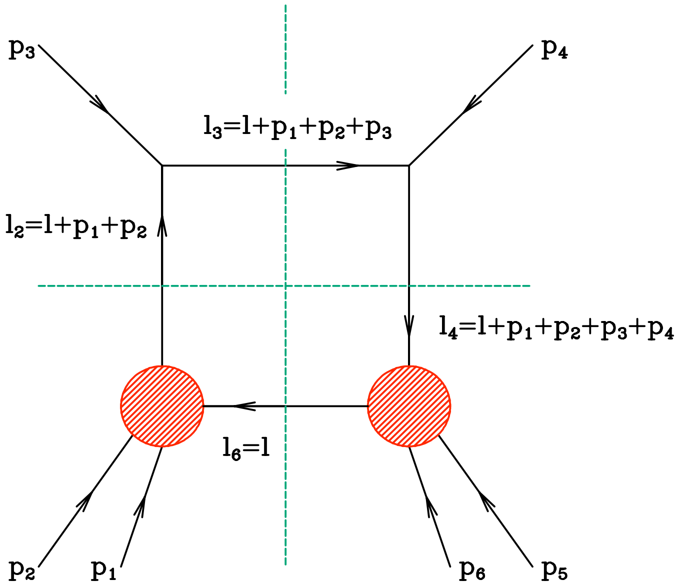


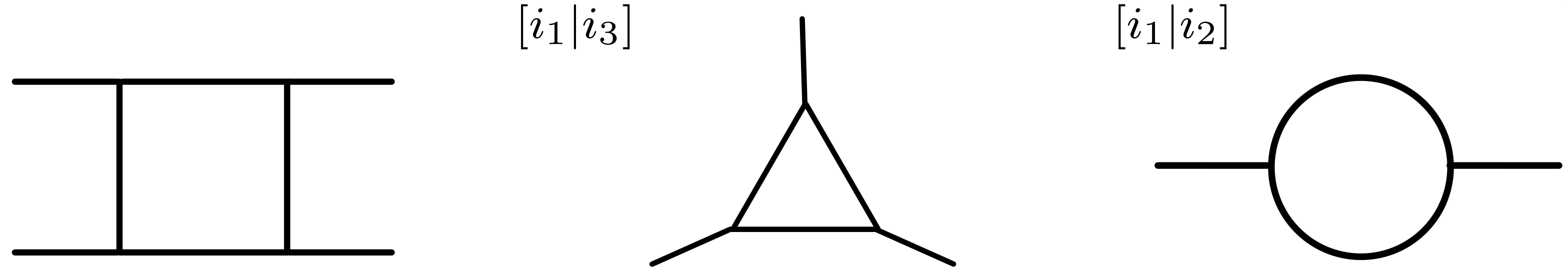
Remarks:

- implicit sum over two helicity states of the four cut gluons
- tree-level three-gluon amplitudes are non-zero because the cut gluons have complex momenta
- similarly, product of 3 (2) tree level amplitudes allows one to compute the coefficients triangles (bubbles) once box contributions are subtracted
- this procedure, in D=4, gives the cut-constructible part of the amplitude

One-loop virtual amplitudes

Cut constructible part can be obtained by taking residues in D=4

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



Rational part: can be obtained with D ≠ 4

Generic D dependence

Two sources of D dependence

dimensionality of loop momentum D

# of spin eigenstates/ polarization states D_s

Keep D and D_s distinct

$$\boxed{\mathcal{A}^D \Rightarrow \mathcal{A}^{(D,D_s)}}$$

Two key observations

1. External particles in D=4 ⇒ no preferred direction in the extra space

   ☛ in arbitrary D up to 5 constraints ⇒ get up to pentagon integrals

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2. Numerator $\mathcal{N}$ depends linearly on D_s ($2^{D_s/2}$ for fermion loops)

$$\mathcal{N}^{D_s}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

☛ evaluate at any D_{s1}, D_{s2} ⇒ get $\mathcal{N}_0$ and $\mathcal{N}_1$, i.e. , full $\mathcal{N}$

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Choose D_{s1}, D_{s2} integer ⇒ suitable for numerical implementation

[continue then to $D_s = 4 - 2\epsilon$ 't-Hooft-Veltman scheme, or $D_s = 4$ FDH scheme]

In practice

- Start from

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

- Use unitarity constraints to determine the coefficients, computed as products of tree-level amplitudes with complex momenta in higher dimensions

- Berends-Giele recursion relations are natural candidates to compute tree level amplitudes: they are very fast for large N and very general (spin, masses, complex momenta)

Berends, Giele '88

Virtual: final result

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1i_2i_3i_4i_5}\; I^{(D)}_{i_1i_2i_3i_4i_5}$$
$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1i_2i_3i_4}\; I^{(D)}_{i_1i_2i_3i_4} - \frac{D-4}{2}\, d^{(2)}_{i_1i_2i_3i_4}\; I^{(D+2)}_{i_1i_2i_3i_4} + \frac{(D-4)(D-2)}{4}\, d^{(4)}_{i_1i_2i_3i_4}\; I^{(D+4)}_{i_1i_2i_3i_4} \right)$$
$$+ \sum_{[i_1|i_3]} \left(c^{(0)}_{i_1i_2i_3}\; I^{(D)}_{i_1i_2i_3} - \frac{D-4}{2} c^{(9)}_{i_1i_2i_3}\; I^{(D+2)}_{i_1i_2i_3} \right) + \sum_{[i_1|i_2]} \left(b^{(0)}_{i_1i_2}\; I^{(D)}_{i_1i_2} - \frac{D-4}{2} b^{(9)}_{i_1i_2}\; I^{(D+2)}_{i_1i_2} \right)$$

Cut-constructable part:

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} d^{(0)}_{i_1i_2i_3i_4}\; I^{(4-2\epsilon)}_{i_1i_2i_3i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1i_2i_3}\; I^{(4-2\epsilon)}_{i_1i_2i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1i_2}\; I^{(4-2\epsilon)}_{i_1i_2}$$

Rational part:

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1i_2i_3i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1i_2i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1}-q_{i_2})^2}{6} - \frac{m_{i_1}^2 + m_{i_2}^2}{2} \right) b^{(9)}_{i_1i_2}$$

Vanishing contributions: $\mathcal{A} = \mathcal{O}(\epsilon)$

Rocket science!

Eruca sativa =Rocket=roquette=arugula=rucola

Recursive unitarity calculation of one-loop amplitudes

So far computed one-loop amplitudes:

- ✓ N-gluons
- ✓ qq + N-gluons
- ✓ qq + W + N-gluons
- ✓ qq + QQ + W
- ✓ tt + N-gluons
- ✓ tt + qq + N-gluons [Schulze]

NB: N is a parameter in Rocket

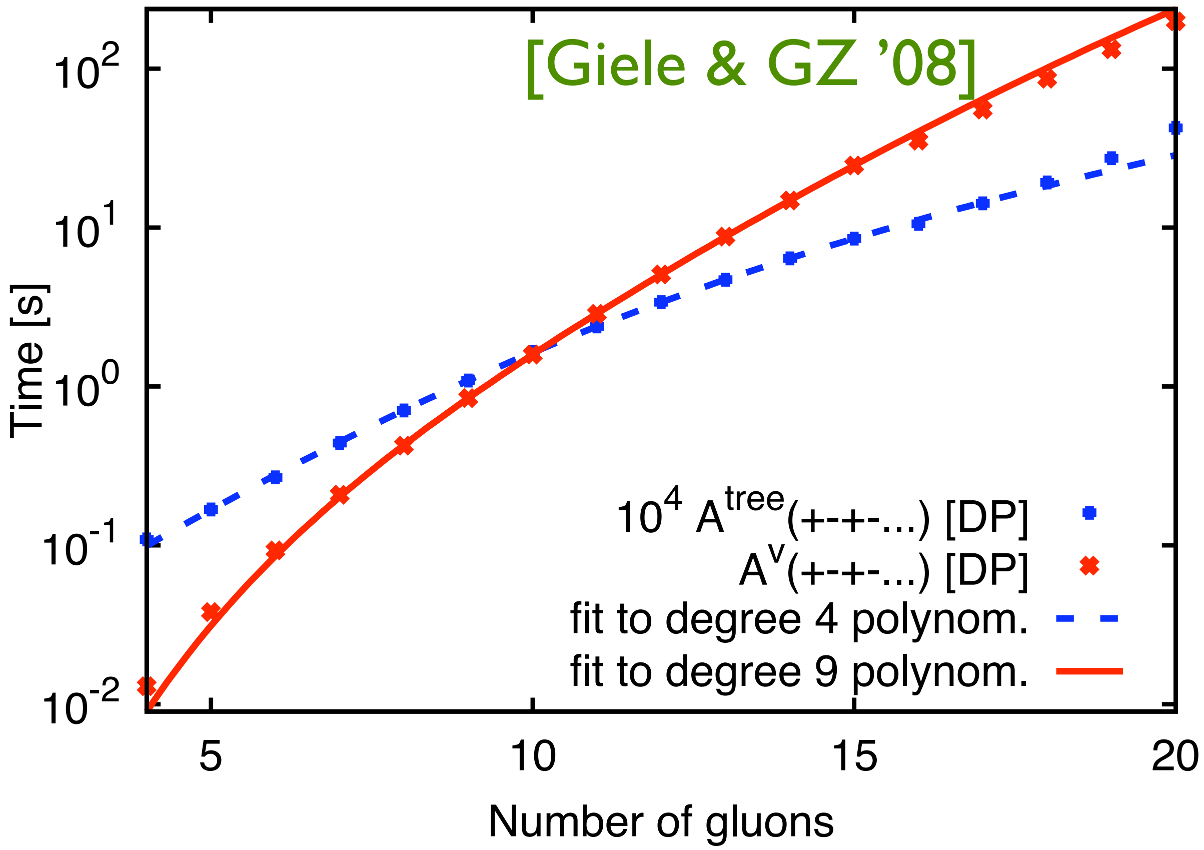
In perspective, for gluons:

$N = 6 \Rightarrow$ 10860 diags.

$N = 7 \Rightarrow$ 168925 diags.

Successfully computed up to N=20

Time for oneloop N-gluon loop amplitudes



- ☛ time $\propto N^9$ as expected
- ☛ independent of the helicity configuration

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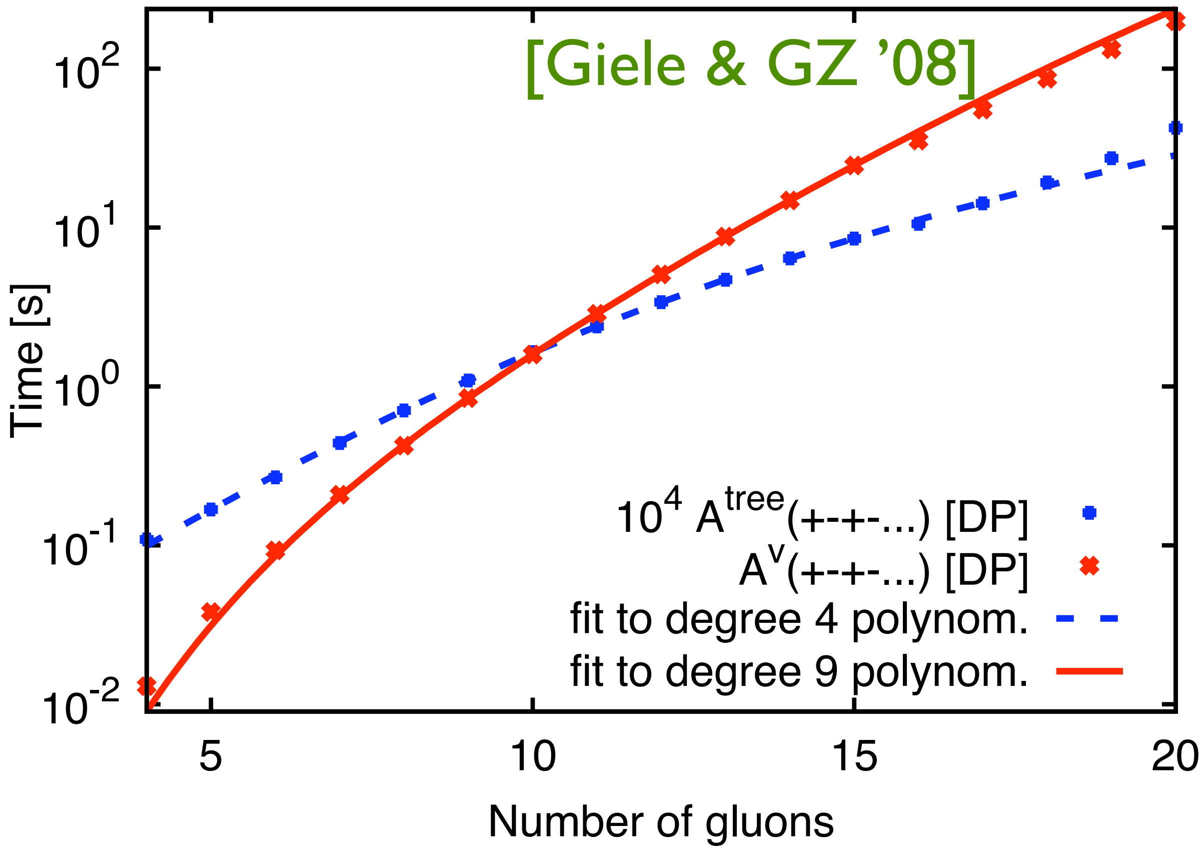

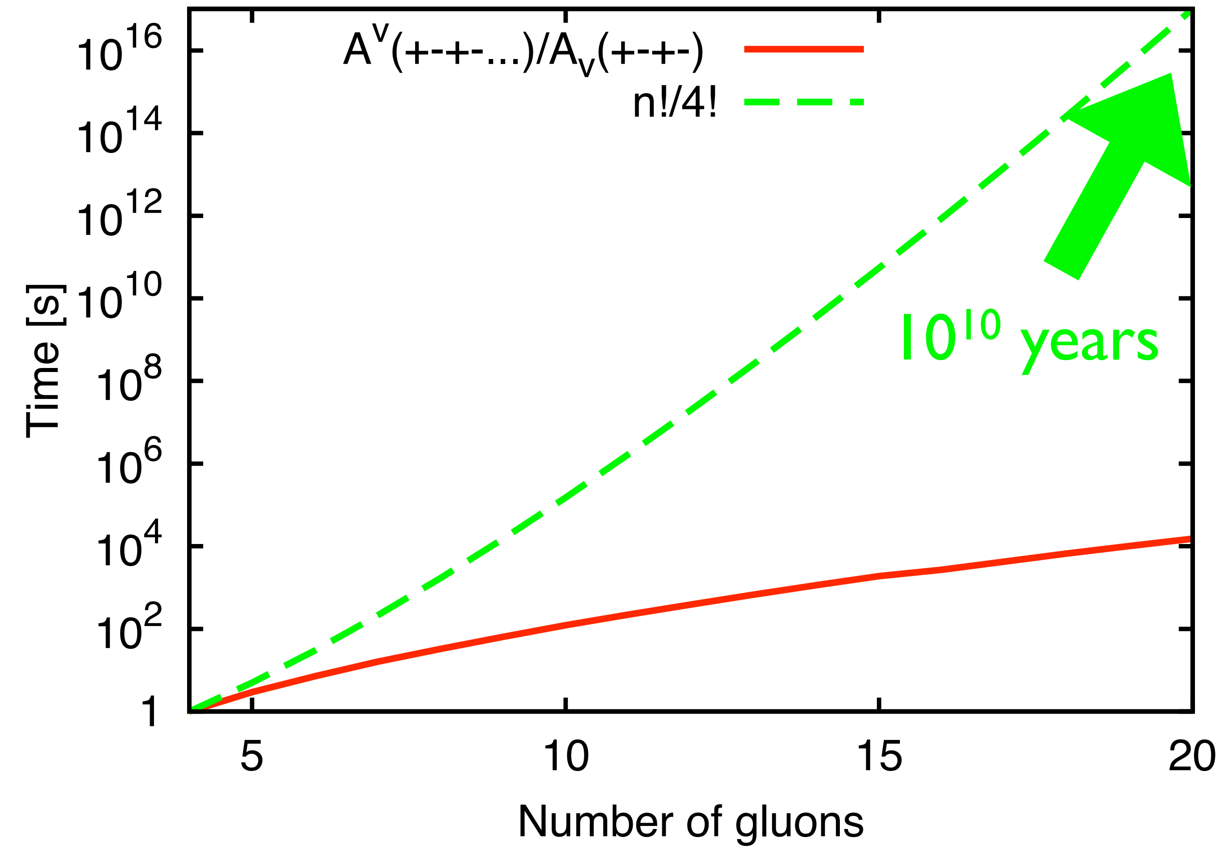

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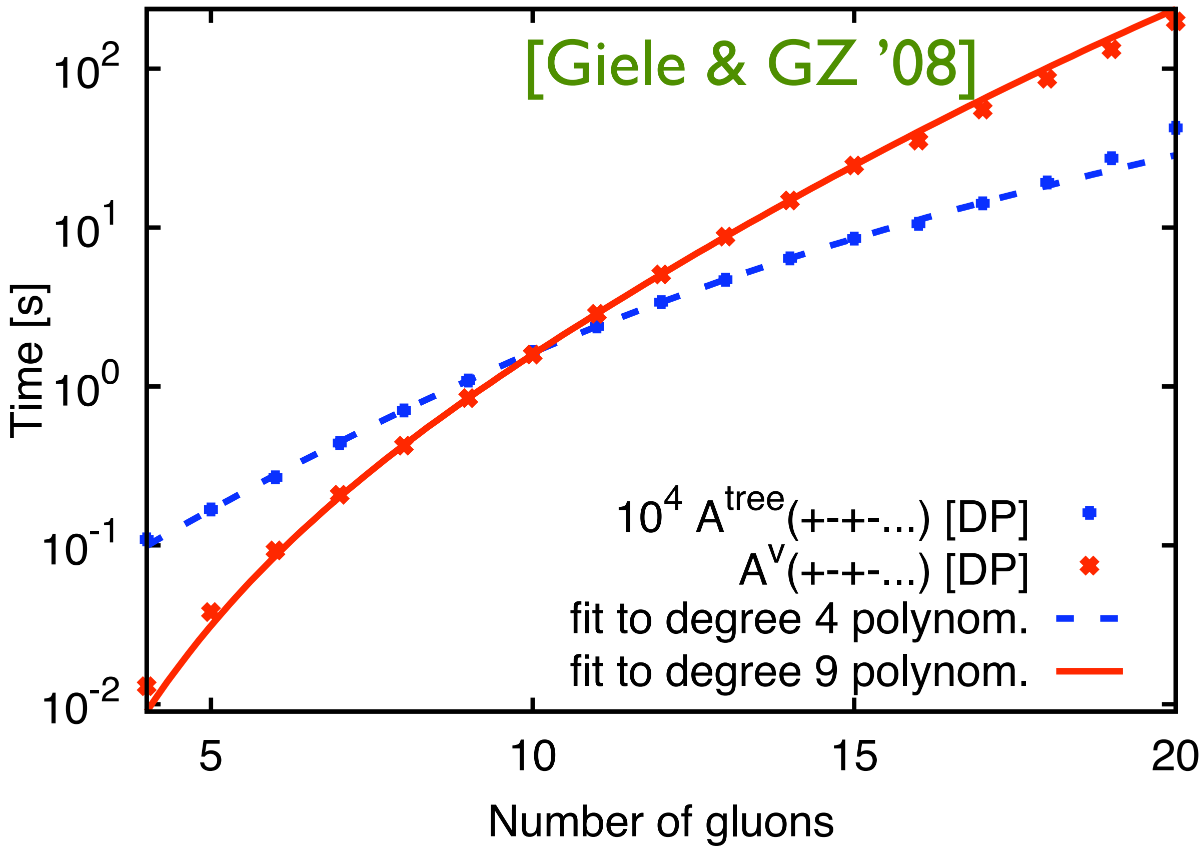

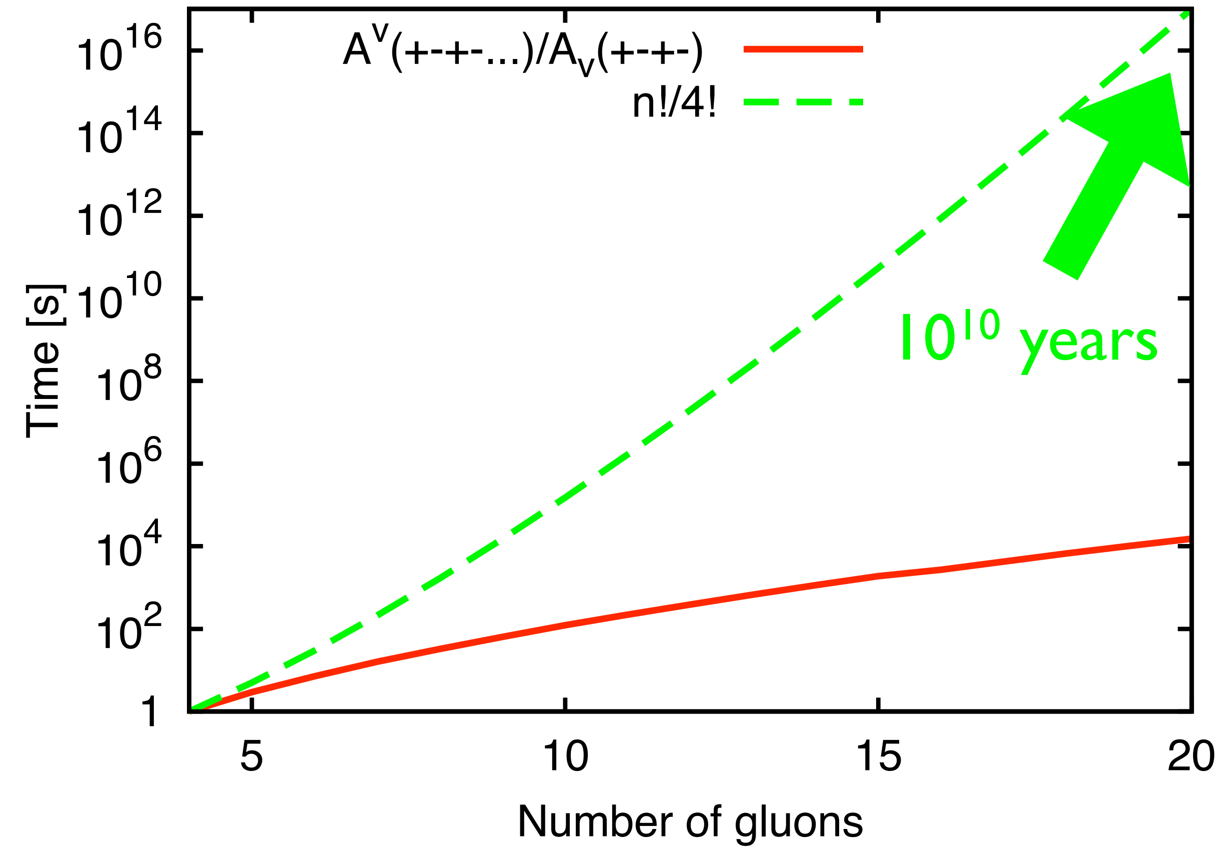

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Comparison with other methods: time roughly comparable

Berger, Bern, Cordero, Dixon, Forde, Ita, Kosower, Maitre '08
Giele & Winter '09
Lazopoulos '09

First physics application: W + 3 jets

1. W + 3 jets measured at the Tevaton, but LO varies by more than a factor 2 for reasonable changes in scales

	$W^{\pm}$, TeV	W^{+}, LHC	W^{-}, LHC
σ [pb], $\mu = 40$ GeV	74.0 ± 0.2	783.1 ± 2.7	481.6 ± 1.4
σ [pb], $\mu = 80$ GeV	45.5 ± 0.1	515.1 ± 1.1	316.7 ± 0.7
σ [pb], $\mu = 160$ GeV	29.5 ± 0.1	353.5 ± 0.8	217.5 ± 0.5

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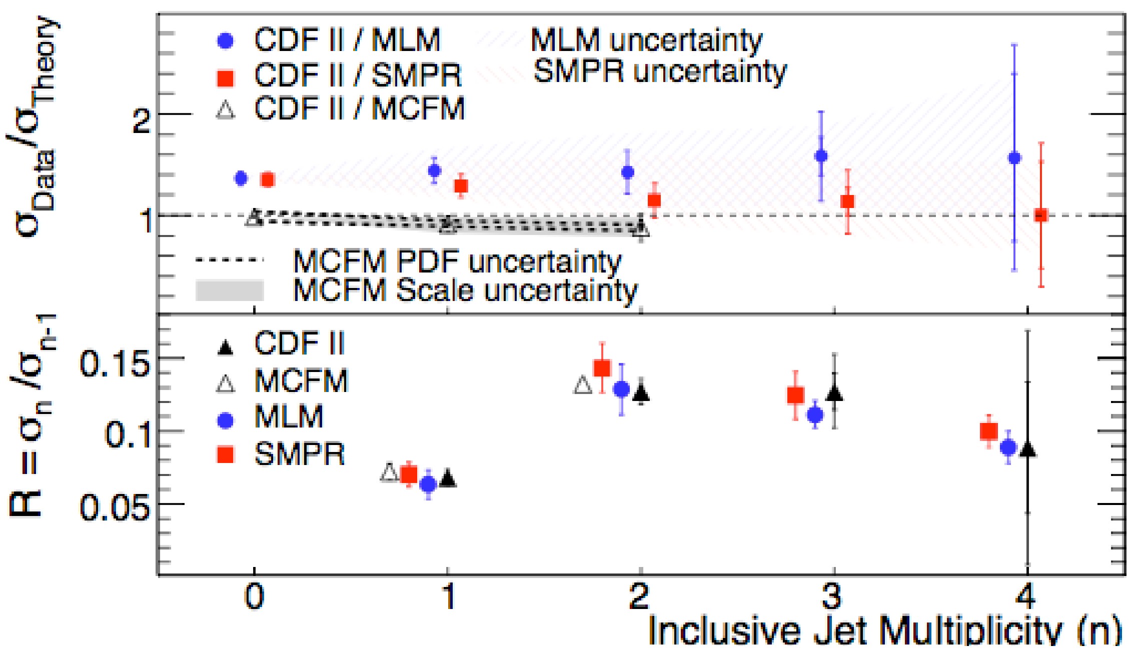
II. Measurements at the Tevaton: for W + n jets with n=1,2 data is described well by NLO QCD

$\Rightarrow$ verify this for 3 and more jets



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IV. Calculation highly non-trivial optimal testing ground

$0 \to \bar{u}\, d\, g\, g\, g\, W^+$ ⇒ 1203 +104 Feynman diagrams

$0 \to \bar{u}\, d\, \bar{Q}\, Q\, g\, W^+$ ⇒ 258 +18 Feynman diagrams

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258 +18 Feynman diagrams

V. Crossing of Z + 3 jets at proton colliders gives immediately 5 jet production at LEP/ILC. NB: data already available for 5,6 jets at LEP

Cross-section calculation

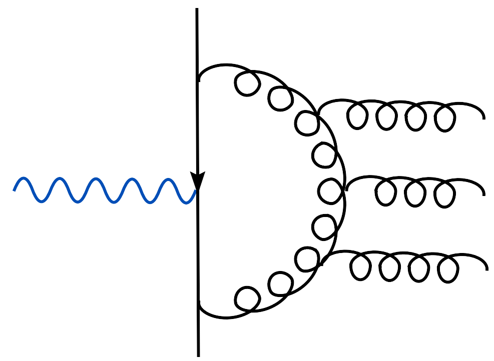
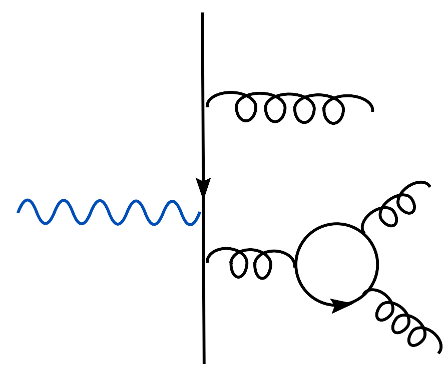
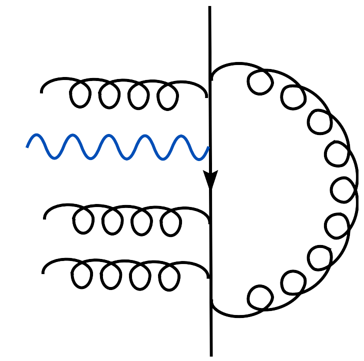
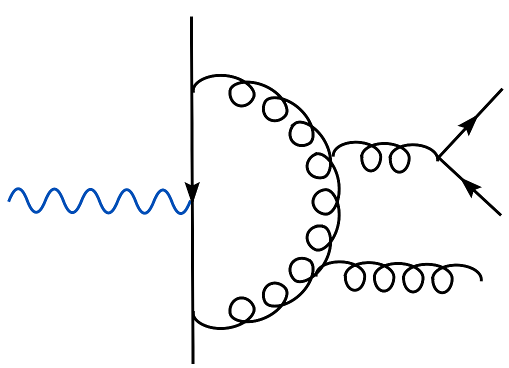
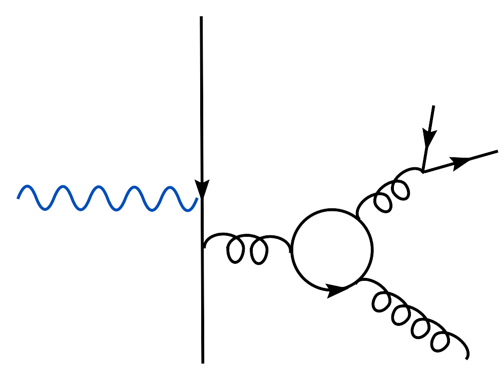
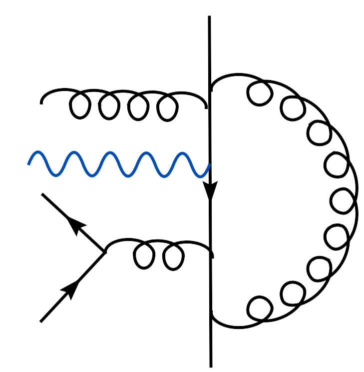
- Consider the NLO leading color approximation, keep n_f dependence exact (important for beta function) but neglect $1/N_c^2$ terms

- Real radiation part:
  - leading color tree level W+6 parton amplitudes computed recursively
  - we use Catani-Seymour subtraction terms modified to deal with the minimal set of color structures needed at leading color

- Real + virtual implemented in the MCFM parton level integrator

Full-color NLO calculation also done by Berger et al. '09

Primitive amplitudes: color structures

	Leading color	Fermion loops	Subleading color
2-quark 3-gluon	 $\mathrm{LC} \equiv (N_c^2 - 1)N_c^3$	 $\mathrm{LC} \cdot \frac{n_f}{N_c}$	 $\cdots$
4-quark 1-gluon	 $\mathrm{LC} \cdot \frac{n_f}{N_c}$	 $\cdots$	 $\cdots$

Leading color adjustment

Define

$$\boxed{\mathcal{R}_{\mathcal{O}} = \frac{\int \mathcal{O}(p)\mathrm{d}\sigma^{\mathrm{FC}}_{LO}(\mu,p)}{\int \mathcal{O}(p)\mathrm{d}\sigma^{\mathrm{LC}}_{LO}(\mu,p)}}$$

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This turns out to be independent of factorization/renormalizaion and on the observable (e.g. bin of distribution)

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Define our best approximation to the NLO result as

$$\mathcal{O}^{\mathrm{NLO}} = r \cdot \mathcal{O}^{\mathrm{NLO,LC}}$$

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Leading color adjustment tested in W+1 and W+2jets: OK to 3 %

CDF cuts

$$p_{\perp,j} > 20\text{GeV} \qquad p_{\perp,e} > 20\text{GeV} \qquad E_{\perp,\text{miss}} > 30\text{GeV}$$

$$|\eta_e| < 1.1 \qquad M_{\perp,W} > 20\text{GeV}$$

$$\mu_0 = \sqrt{p_{\perp,W}^2 + M_W^2} \qquad \mu = \mu_R = \mu_F = [\mu_0/2, 2\mu_0]$$

- PDFs: cteq6l1 and cteq6m

- CDF applies lepton-isolation cuts. This is a O(10%) effect. Lepton-isolation and detector acceptance cuts are believed to cancel out
  No lepton isolation applied

- CDF uses JETCLU with R = 0.4, but this is not infrared safe, use a different jet-algorithm. Argument: difference small in inclusive cross-section [possibly larger in distributions]

Jet-algorithm choice

Leading order comparison:

Algorithm	R	$E_\perp^{\rm jet} > 20$ GeV	$E_\perp^{\rm 3rdjet} > 25$ GeV
JETCLU	0.4	$1.845(2)^{+1.101(3)}_{-0.634(2)}$	$1.008(1)^{+0.614(2)}_{-0.352(1)}$
SIScone	0.4	$1.470(1)^{+0.765(1)}_{-0.560(1)}$	$0.805(1)^{+0.493(1)}_{-0.281(1)}$
anti-$k_\perp$	0.4	$1.850(1)^{+1.105(1)}_{-0.638(1)}$	$1.010(1)^{+0.619(1)}_{-0.351(1)}$

[SIScone ⇒ Salam & Soyez '07; anti-kt ⇒ Cacciari, Salam, Soyez '08]

- anti-k_t is closest to JETCLU at LO, SIScone does not do well
  However, the meaning of LO for JETCLU is questionable (NLO infinite)

- do NLO calculation both with SISCone and anti-k_t
  - ➡ SISCone will allow us to compare with Berger et al.
  - ➡ anti-kt (if we had perfect data) would tell us is the LO agreement with JETCLU is fortuitous

Cross-section at the Tevatron

$$\sigma_{W+3j}(p_{\perp,j} > 25\,\mathrm{GeV}) = (0.84 \pm 0.24)\,\mathrm{pb}$$

CDF

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	$\mathrm{LO}^{\mathrm{LC}}$						
SIS.	$0.89^{+0.55}_{-0.31}$						
a-k_t	$1.12^{+0.68}_{-0.39}$						

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a-k_t	$1.12^{+0.68}_{-0.39}$	$1.01^{+0.62}_{-0.35}$					

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	LO^{LC}	LO^{FC}	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$				
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a-k$_t$	$1.12^{+0.68}_{-0.39}$	$1.01^{+0.62}_{-0.35}$	0.91				

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SIS.	$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91	$1.01^{+0.05}_{-0.17}$	$0.91^{+0.05}_{-0.12}$	$0.908^{+0.044}_{-0.142}$	
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$\Rightarrow$ agreement between independent calculations to within 3%

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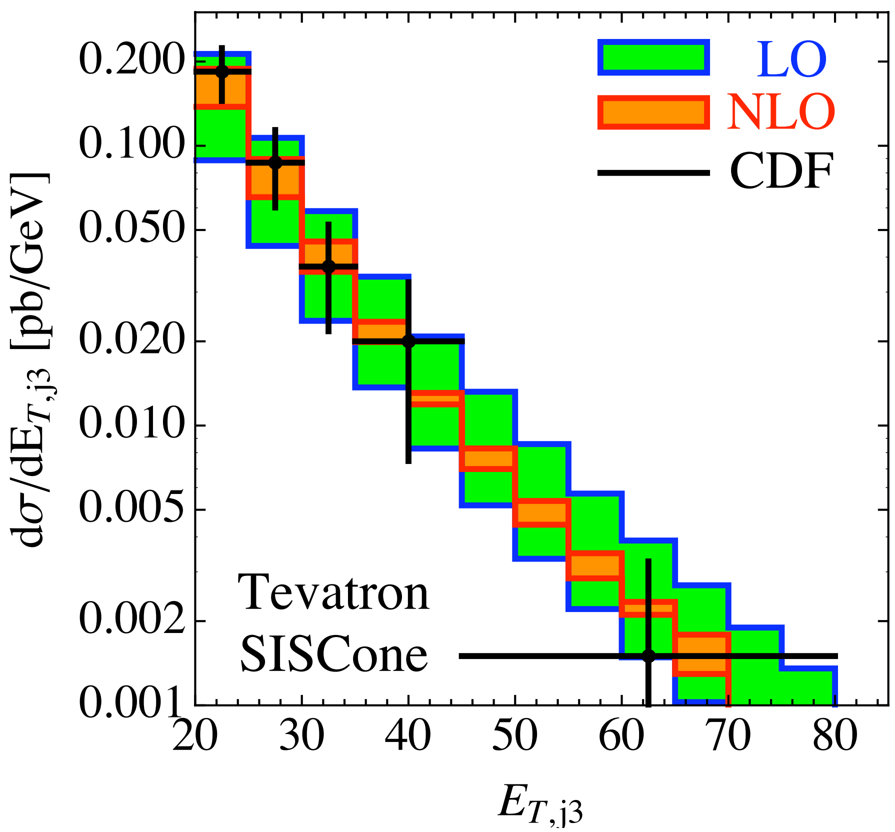
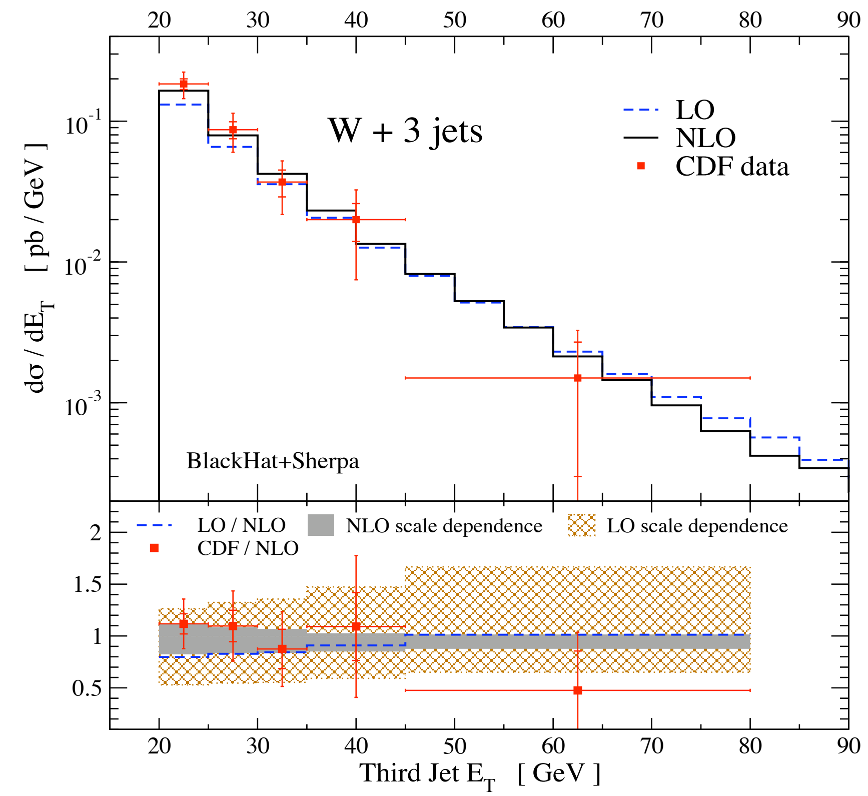
$\Rightarrow$ leading color approximation works very well. After leading color adjustment procedure it is good to 3%

$\Rightarrow$ important (10% or more) differences due to different jet-algorithms.
High precision comparison impossible if using different algorithms

Sample distribution: $E_{t,j3}$



Ellis, Melnikov, GZ '09



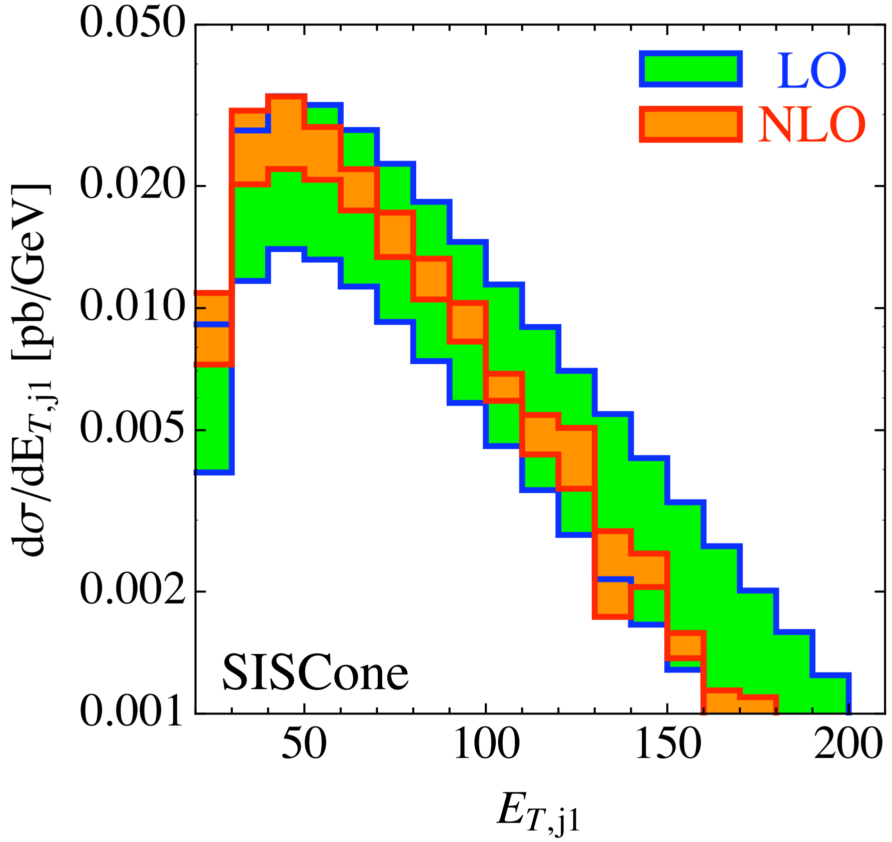
Berger et al '09

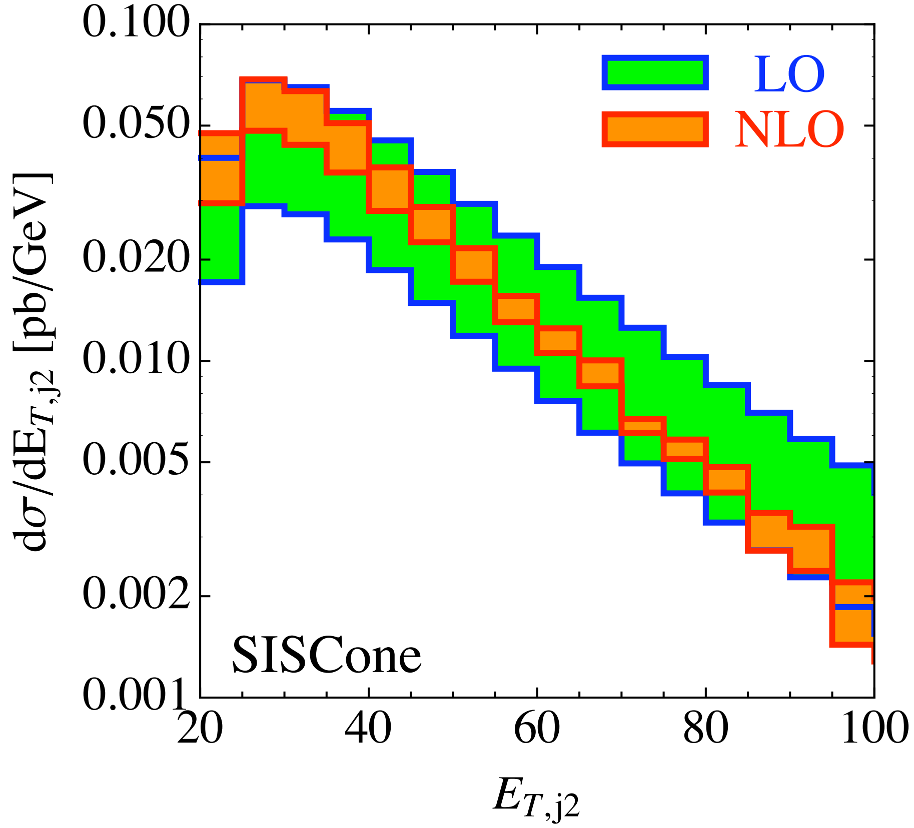
Comparison to data

- OK within large experimental errors
- even with reduced exp. errors, accurate comparison not possible because of different jet-algorithm used

Plots done by running 4 days (or less) all sub-processes in parallel

Sample distribution: $E_{t,j1}$ and $E_{t,j2}$





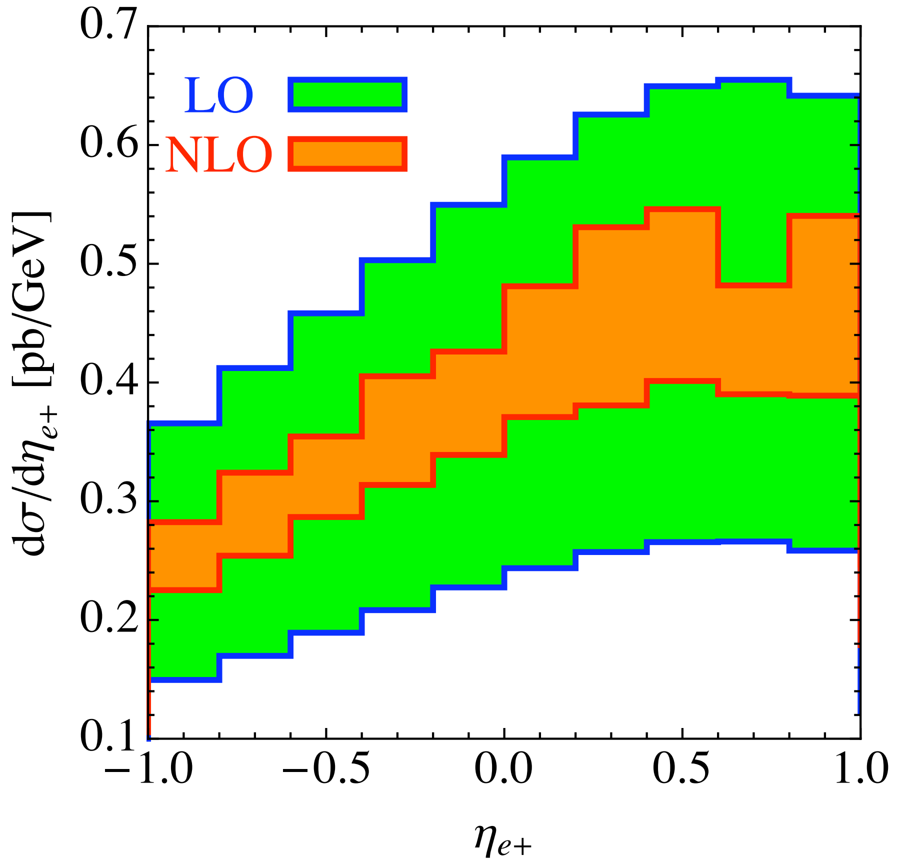
Ellis, Melnikov, GZ '09

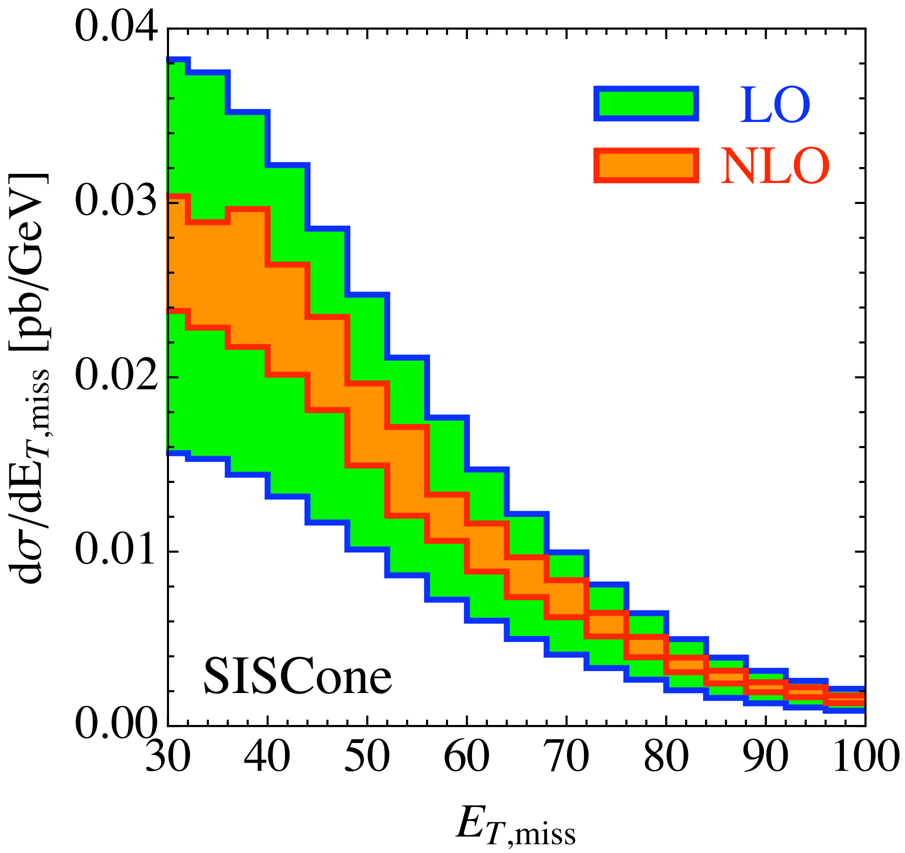
Hadronic observables:

- scale reduction (factor 4)
- change in shape

Plots done by running 4 days (or less) all sub-processes in parallel

Sample distribution: $E_{t,j1}$ and $E_{t,j2}$





Ellis, Melnikov, GZ '09

Leptonic observables:

- scale reduction (factor 4)
- inclusive K-factor works very well

Plots done by running 4 days (or less) all sub-processes in parallel

LHC cuts

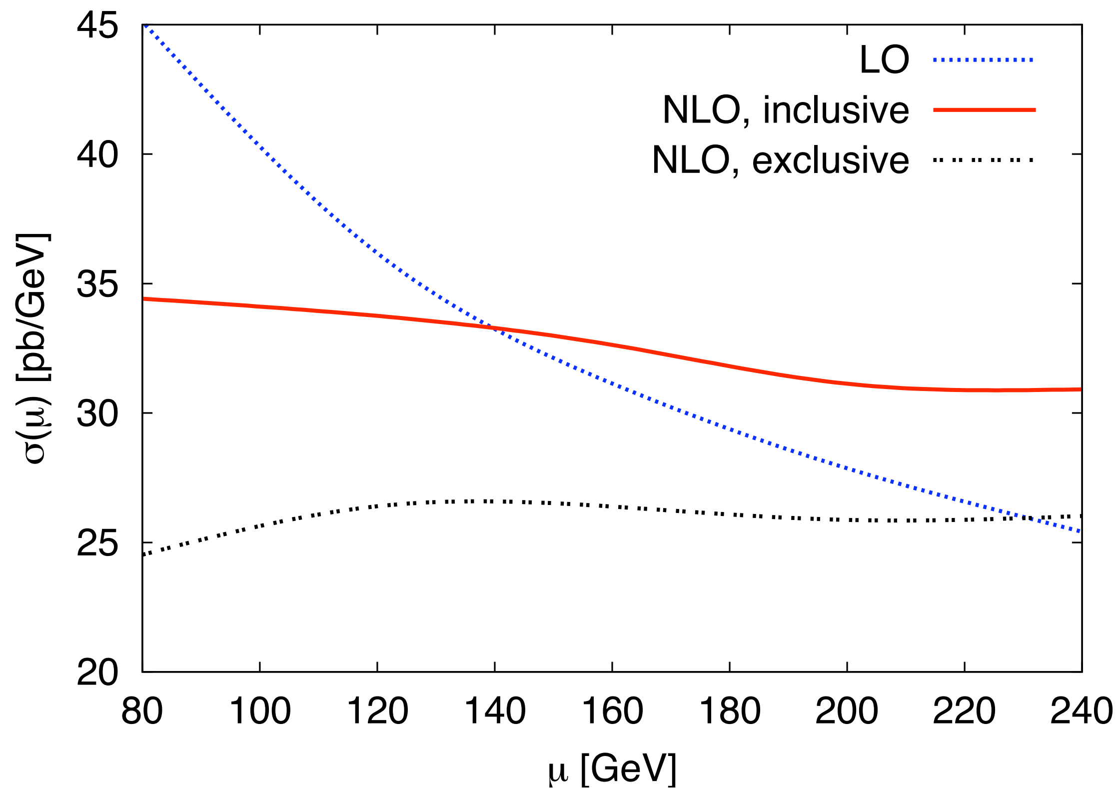
$$E_{\mathrm{CM}} = 10\,\mathrm{TeV} \qquad E_{\perp,\mathrm{jet}} = 30\,\mathrm{GeV} \qquad E_{\perp,e} = 20\,\mathrm{GeV}$$

$$E_{\perp,\mathrm{miss}} = 15\,\mathrm{GeV} \quad M_{\perp,W} = 30\,\mathrm{GeV} \quad |\eta_e| < 2.4 \quad |\eta_{\mathrm{jet}}| < 3$$

$$\mu_0 = \sqrt{p_{\perp,W}^2 + M_W^2} \qquad \mu = \mu_R = \mu_F = [\mu_0/2, 2\mu_0]$$

- Jet definition: SIScone with R = 0.5
- PDFs: cteq6l1 and cteq6m
- Other input parameters as before

LHC: W^+ +3 jet cross-section

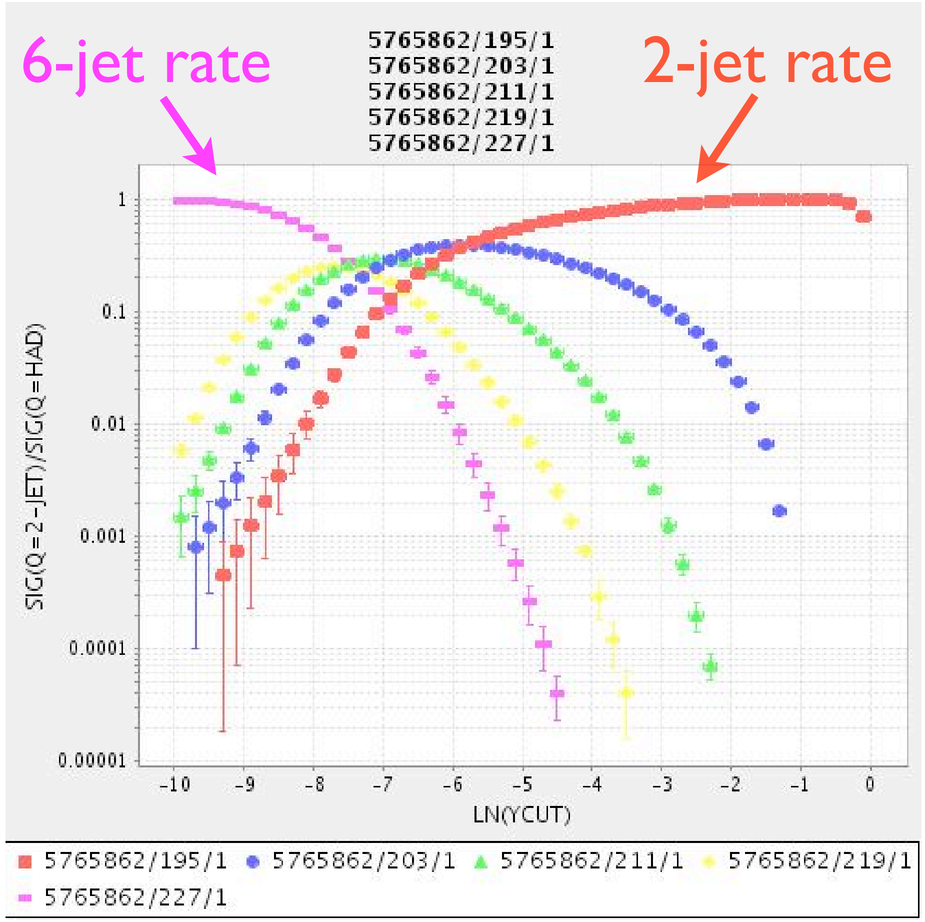


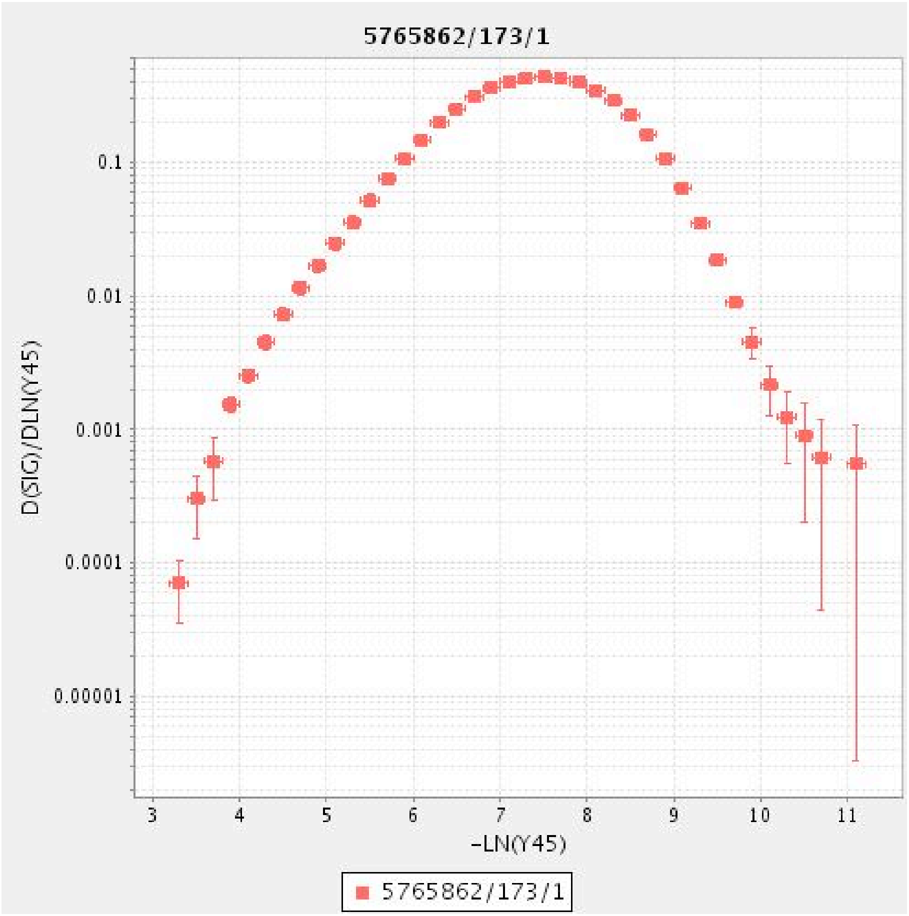
Ellis, Melnikov, GZ preliminary

- scale dependence considerably reduced at NLO
- NLO tends to reduce cross-section
- because of very large scale dependence of LO, quoting a K-factor not very meaningful

Z + 5 jets at e^+e^-

- ILC need precise predictions for high-multiplicity final states
- Jet rates measured very accurately already at LEP for up to 6 jets
- One of the most accurate extraction of α_s from 4-jet rates. 5-jet rates even more sensitive to the coupling but not yet known at NLO

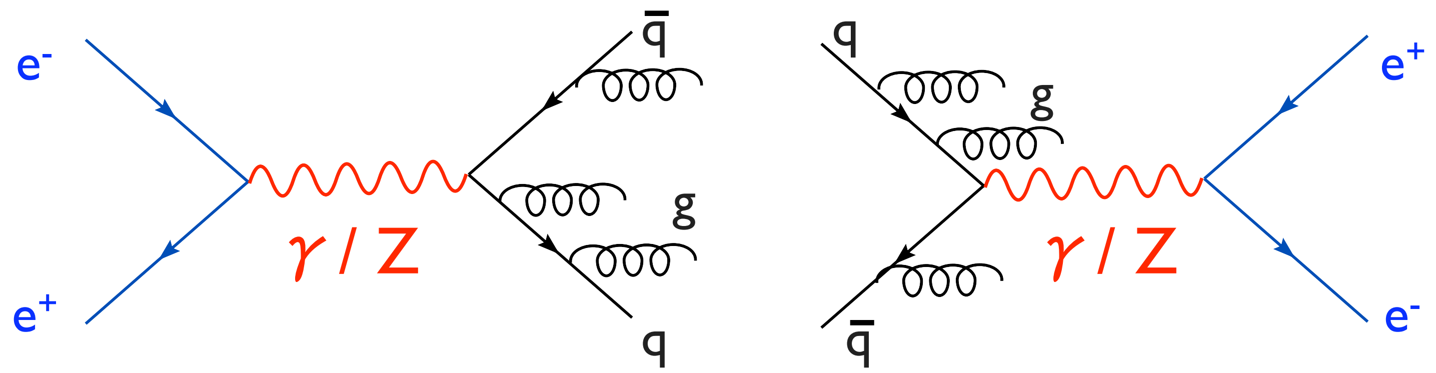




Aleph, similar plots available from Delphi, Opal, L3

Z+ 5 jets at e^+e^-

- Processes $e^+e^- \rightarrow X + 2$ jets can be obtained via a crossing of $PP \longrightarrow Z+ X$



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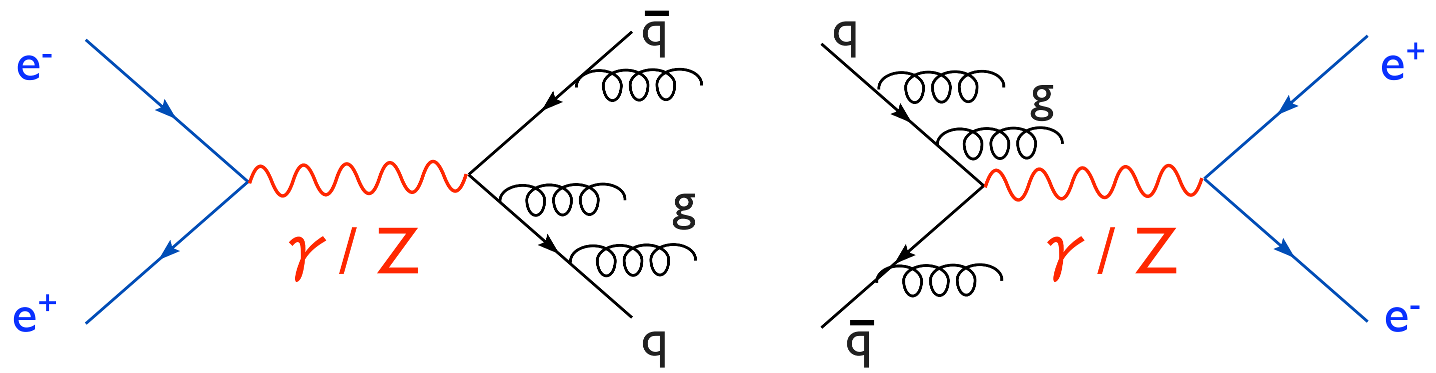
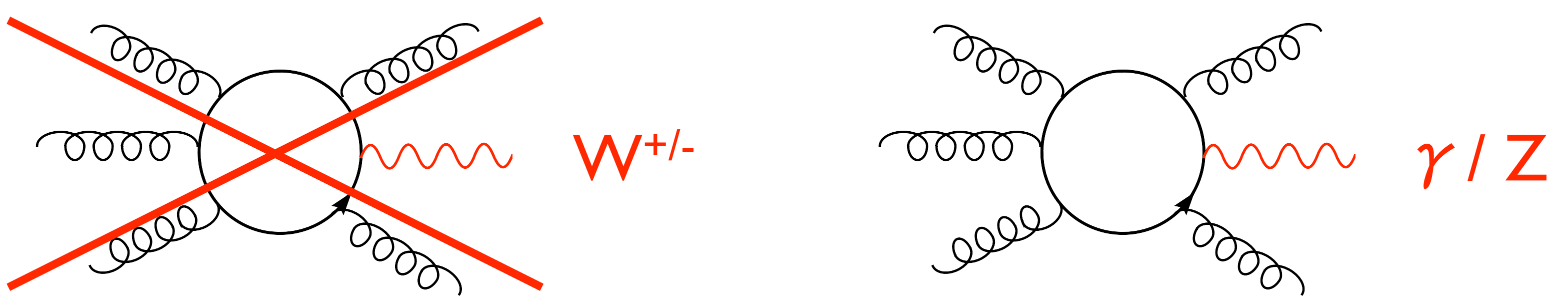


- Z+ X and W + X are very similar, however Z+X at NLO involves new diagrams with the Z radiated from a fermion loop



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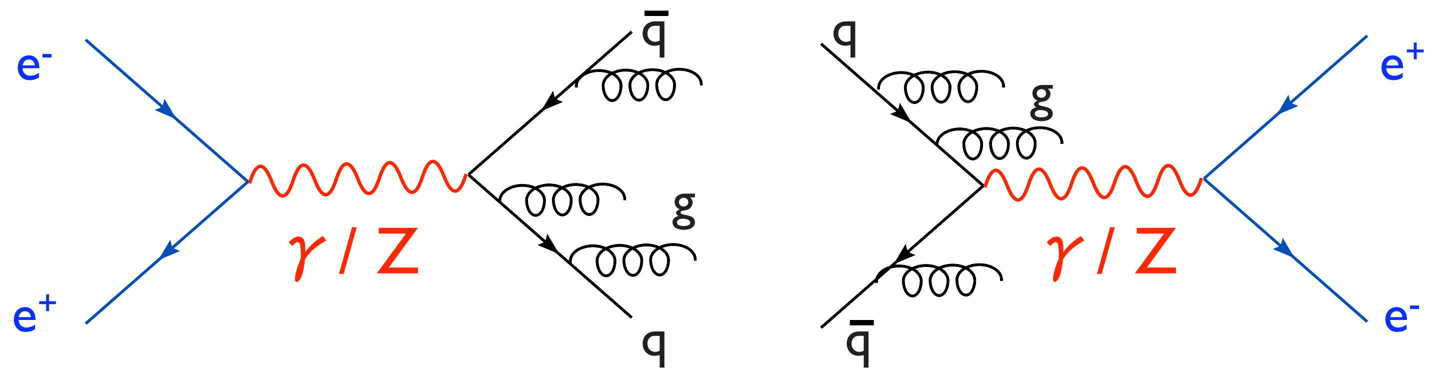

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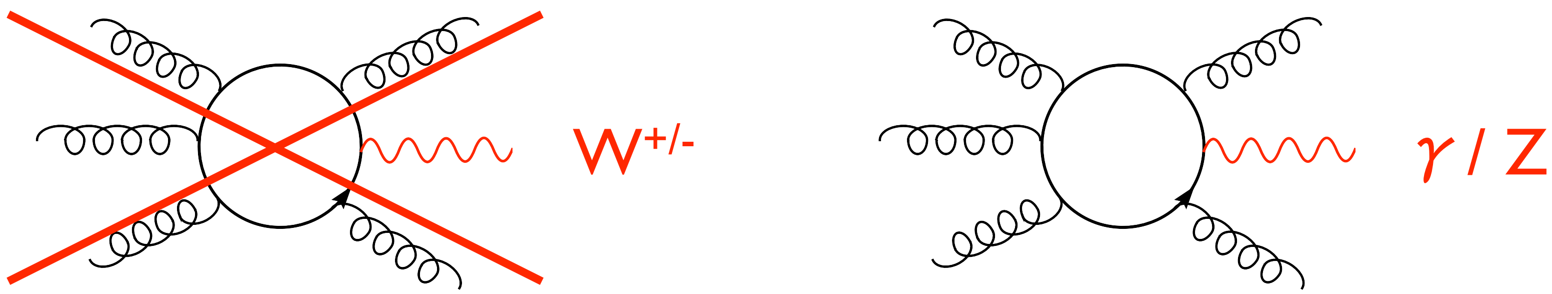
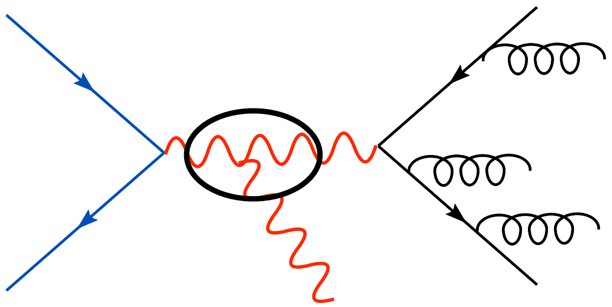


- Final states with additional vector bosons allow measurements of anomalous couplings sensitive to generic NP



Final remarks

In the last (5?) years there has been a breakthrough in NLO techniques. Generalized D-dimensional unitarity is one new method:

- ✗ general Berends-Giele recursion for tree level amplitudes: numerically efficient (large N), general (D, spins, masses)
- ✗ simple method, suitable for automation
- ✗ universal method (general masses, spins) and unified approach, no 'special' cases, no exceptions
- ✗ speed: numerical performance as expected (polynomial)
- ✗ transparent: full control on all parts
- ✗ maturity reached for cross-section calculations? Demonstrated by explicit calculations of W + 3 jets (but still room for improvements)

Other remarks

✗ despite new advanced techniques full-color calculations are hard. Leading color seems to be an excellent, cheaper approximation

✗ for precision comparisons measurements & theory must use the same jet-algorithm. Infrared unsafe cones can not be used at NLO and should be abandoned in future measurements

✗ current work on NLO calculations focuses on LHC processes but

- $e^+e^- \to X + 2$ jets can be obtained via a crossing of $PP \longrightarrow Z + X$. We are on the process of computing 5 jets final state for the ILC
- techniques developed for the LHC do provide solid ground for accurate predictions of generic ILC processes.