Phase transitions in 2HDM during Universe expansion and present values of parameters.

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Minimal SM ($\equiv$ 1HDM)

- One Higgs doublet – single vacuum, conserving charge and CP.
- Vacuum expectations are given by minimization of Gibbs potential $V_G(T) = V(\phi) + aT^2\phi^\dagger\phi$
- Early Universe – high temperature T.
- Cooling $\langle\phi\rangle = 0 \to \langle\phi\rangle \neq 0$ – single EWSB phase transition.

Two Higgs doublet model (2HDM)

- Two Higgs doublets, up to 5 observable Higgses, $H^\pm$, h^0_{1-3}
- Vacua with different properties can exist.
- It is possible to have a sequence of phase transitions (between different vacua) during cooling of the Universe. [Ginzburg,Kanishev,2007]

Higgs potential

Two scalar weak isospinors ϕ_1, ϕ_2. Isoscalar combinations:

$$x_1 = \phi_1^\dagger \phi_1, \quad x_2 = \phi_2^\dagger \phi_2, \quad x_3 = \phi_1^\dagger \phi_2 \,.$$

$$V = \tfrac{1}{2}\lambda_1 x_1^2 + \tfrac{1}{2}\lambda_2 x_2^2 + \lambda_3 x_1 x_2 + \lambda_4 x_3^\dagger x_3 + \left[\tfrac{1}{2}\lambda_5 x_3^2 + h.c.\right]$$
$$-\tfrac{1}{2}\left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + h.c.\right)\right]$$

No terms $(\lambda_6 x_1 + \lambda_7 x_2)\, x_3 + h.c.$. (They hardly violate Z_2 symmetry $\phi_1\phi_2 \leftrightarrow -\phi_1\phi_2$. We have arguments against this violation.)

Particular case: Explicit CP: λ_5, m_{12}^2 are real.

This case is well representative and very transparent.

Useful parameterization

$$m_{11}^2 = m^2(1-\delta)\,, \quad m_{22}^2 = k^2 m^2(1+\delta)\,, \quad m_{12}^2 = \mu k m^2\,; \quad k \stackrel{def}{=} \sqrt[4]{\lambda_2/\lambda_1}\,.$$

$\boxed{\delta = 0}$: $k - symmetry$ of potential (NOT Lagrangian) $\phi_1 \leftrightarrow k\phi_2$.

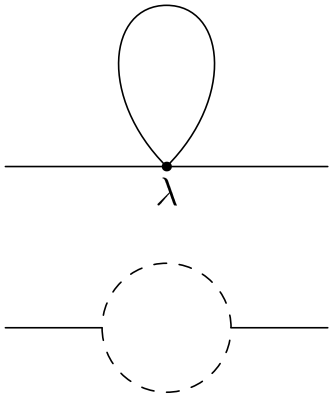
Useful abbreviations

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad \tilde{\lambda}_{345} = \lambda_3 + \lambda_4 - \lambda_5,$$
$$\Lambda_{345\pm} = \sqrt{\lambda_1\lambda_2} \pm \lambda_{345}, \; \tilde{\Lambda}_{345\pm} = \sqrt{\lambda_1\lambda_2} \pm \tilde{\lambda}_{345}, \; \Lambda_{3\pm} = \sqrt{\lambda_1\lambda_2} \pm \lambda_3.$$

Temperature dependence

At high temperature vacuum state is determined by minimum of the Gibbs potential $V_G = Tr\left(Ve^{-\hat{H}/T}\right)/Tr\left(e^{-\hat{H}/T}\right) \equiv V + \Delta V$. .

The first correction to potential is given by this diagrams. It is calculated with Matsubara diagram technic. At $T^2 \gg m_i^2$ scalar loop contributions ^{s} and gauge boson loop contributions ^{g} are



$$m_{11}^2(T) = m_{11}^2(0) - 2c_1 m^2 w, \quad m_{22}^2(T) = m_{22}^2(0) - 2k^2 c_2 m^2 w,$$

$$m_{12}^2(T) = m_{12}^2(0); \quad c_i = c_i^s + c_i^g + c_i^f, \quad w = \frac{T^2}{12m^2}$$

$$c_1^s = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{2}, \; c_2^s = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{2k^2}, \; c_1^g = c_2^g = (3g^2 + g'^2)B\,.$$

$(\mu,\, \delta)$ plane

Rewrite previous. $F(T)$ – value of F at temperature T,
$F \equiv F(0)$ – present value ($T = 0$).

$$m^2(T) = m^2\,(1 - (c_2 + c_1)w)\,, \quad \mu(T) = \mu\frac{m^2(0)}{m^2(T)},$$

$$\delta(T) = \frac{\mu(T)}{\mu}(\delta - P) + P \qquad \left(P = \frac{c_2 - c_1}{c_2 + c_1}\right)$$

Curve $(\mu(T), \delta(T))$ – is a straight ray in (μ, δ) plane.
Rays start from $(\mu(0), \delta(0))$. These rays cross in $\mathcal{P}_0 = (0, P)$.

- at $\mu(T)/\mu > 0$ – 1-st sheet of (μ, δ) plane
- at $\mu(T)/\mu < 0$ – 2-nd sheet

This variation of parameters result in change of vacuum states during cooling of Universe.

Our problems

1. what are possible sequences of phase states allowed in 2HDM?
2. what present values of parameters correspond to each possible sequence of phase states?
3. How can vary physical parameters during cooling down?
4. What it mean for cosmology?

Extremes of potential

Extremum: point with $\partial V/\partial\phi_i|_{\langle\phi_j\rangle} = \partial V/\partial\phi_i^\dagger\Big|_{\langle\phi_j\rangle} = 0$

in the extremum $\langle x_i\rangle \overset{def}{=} y_i$, e.g. $y_3 = \langle\phi_1^\dagger\rangle\langle\phi_2\rangle$.

The extremum energy is $\mathcal{E}_N^{ext} = V(\langle\phi_i\rangle_N)$.

The minimum with lowest energy – vacuum.

(1) No more than two minima of potential can coexist.

(2) The minima having some symmetry and violating it can not coexist.

(1)$\Rightarrow$ if some minimum is degenerated, other extrema are not minima.

Choice of axis for $\langle\phi_1\rangle$:

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \;\; \langle\phi_2\rangle = \begin{pmatrix} u \\ v_2 e^{-i\xi} \end{pmatrix}, \;\; u, v_1, v_2 > 0, \;\; v^2 = v_1^2 + v_2^2 + u^2, \;\; \tan\beta = \frac{v_2}{v_1}$$

Types of extrema

Electroweak conserving point (EWc): $\langle\phi_1\rangle = \langle\phi_2\rangle = 0$

Electroweak symmetry is not broken. Gauge bosons and fermions are massless.

Neutral EWSB extrema: $u = 0$

5 physical Higgs particles (h_1, h_2, h_3 and $H^\pm$)

For explicitly CP conserving potential (all parameters are real)

- Spontaneously CP violating (sCPv)] $\xi \neq 0$ − two degenerate extrema
- CP conserving (CPc)] $\xi = 0$ − up to 4 such extrema

Charged extremum: $u \neq 0$

Electric charge is not conserved, photon is massive, Higgs fields have no definite electric charge. [A.Barroso, R.Santos]

EW symmetric phase and EWSB phase transition

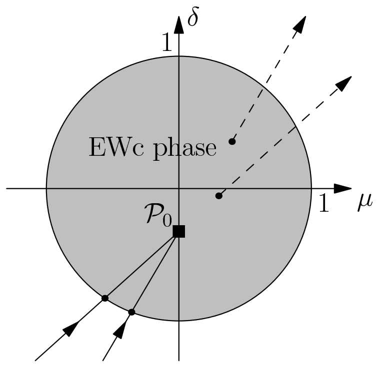
The EWs point $\langle\phi_1\rangle = \langle\phi_2\rangle = 0$ is minimum of potential – vacuum, if

$$m_{11}^2 < 0\,, \;\; m_{22}^2 < 0 \;\; and \;\; m_{11}^2 m_{22}^2 \geq |m_{12}^2|^2\,,$$

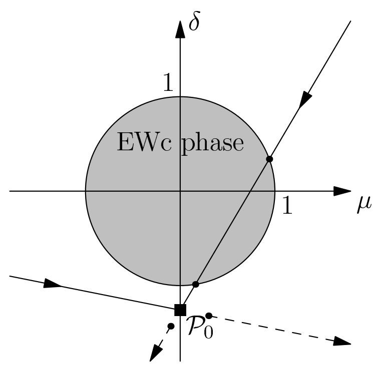
i. e. within the circle $\mathbb{C}: \; \mu^2 + \delta^2 = 1$ at the second sheet of (μ, δ) plane, i.e. at $\mu(T)/\mu < 0$.

EWSB phase transition takes place at the crossing of $\mathbb{C}$ with the ray.

- $c_1 > 0$ and $c_2 > 0 \Rightarrow \mathcal{P}_0$ within $\mathbb{C}$, one EWSB transition
- $c_1 c_2 < 0 \Rightarrow \mathcal{P}_0$ outside $\mathbb{C}$, none EWSB transition or crossing of EW phase



One EWSB transition



None or two EWSB transitions

1-st sheet of (μ, δ) plane

- **Separation of sectors in λ_i space**. Each sector ALLOW separate set of vacuum states, in addition to CPc.

- **In each sector — the study of different phase states in (μ, δ) plane**.

- **Possible phase transitions**

CPc extrema

The CPc extrema exist in entire space of parameters λ_i.

For the CPc extrema two cubic equations represented extremum condition are easily transformed into the relation for quantities $v^2 = 2(y_1 + y_2)$ and equation for $\tau = k \tan\beta = k\sqrt{y_2/y_1}$:

$$v^2 = m^2(k^2+\tau^2)\frac{1-\delta+\mu\tau}{\lambda_{345}\tau^2+\sqrt{\lambda_1\lambda_2}}\,,$$
$$\sqrt{\lambda_1\lambda_2}\mu\tau^4 + (\Lambda_{345-} - \Lambda_{345+}\delta)\tau^3 - (\Lambda_{345-} + \Lambda_{345+}\delta)\tau - \sqrt{\lambda_1\lambda_2}\mu = 0\,.$$

(no more than 4 real solutions with positive v^2).

$$\mathcal{E}_{CPc} = -\frac{m^4k^2}{8}\cdot\frac{(1-\delta+\mu\tau)[1-\delta+2\mu\tau+\tau^2(1+\delta)]}{\lambda_{345}\tau^2+\sqrt{\lambda_1\lambda_2}}\,.$$

Sectors in λ_i space

The space of all λ_i that satisfy positivity constraints, is subdivided into 4 non-overlapping sectors that allow for certain type of phases and phase transitions.

Sector I:	$\Lambda_{345-} > 0$	$\lambda_5 < 0$	$\lambda_4 + \lambda_5 < 0$
Sector II:	$\Lambda_{345-} < 0$	$\Lambda_{3-} < 0$	$\tilde{\Lambda}_{345-} < 0$
Sector III:	$\lambda_5 > \lambda_4$	$\lambda_5 > 0$	$\tilde{\Lambda}_{345-} > 0$
Sector IV:	$\lambda_5 < \lambda_4$	$\Lambda_{3-} > 0$	$\lambda_4 + \lambda_5 > 0$

Sect. I: $\boxed{\Lambda_{345-} < 0,\ \Lambda_{3-} < 0,\ \tilde{\Lambda}_{345-} < 0}$.

Only CPc states are possible.

First order phase transition is possible

- Let $\delta = 0$ at certain temperature T_{tr}. At this temperature potential is k-symmetric $\Rightarrow$ two degenerate extrema.
- At other temperatures potential is not k-symmetric $\Rightarrow$ extrema are non-degenerate.
- Possibility to have first order phase transition between such extrema.

At $\delta = 0$ one can solve the equation on τ:

$$(\tau^2 - 1)\left[\sqrt{\lambda_1\lambda_2}\,\mu(\tau^2+1) + \tau\Lambda_{345-}\right] = 0$$

(Solutions $\boldsymbol{A}_\pm$): $\tau_A = \pm 1$, $\mathcal{E}_{CPcA} = -\frac{m^4k^2}{4} \cdot \frac{(1\pm\mu)^2}{\Lambda_{345+}}$.

(Solutions $B_\pm$): $\tau_{B_\pm} = \frac{-\Lambda_{345-}}{2\mu\sqrt{\lambda_1\lambda_2}}(1 \pm \sqrt{1 - \frac{4\mu^2\lambda_1\lambda_2}{\Lambda^2_{345-}}})$.

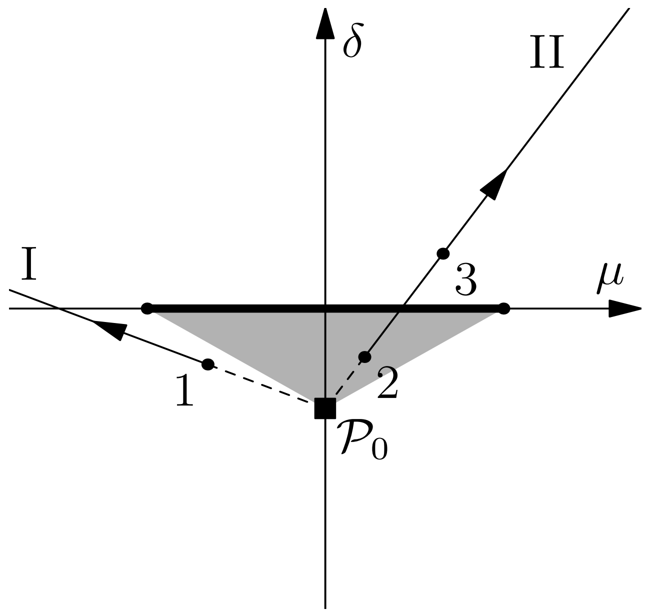
$\mathcal{E}_{CPcB\pm} = -\frac{m^4k^2}{4}\left(\frac{1}{2\sqrt{\lambda_1\lambda_2}} - \frac{\mu^2}{\Lambda_{345-}}\right)$ – degenerate in energy.

Solutions $(B_\pm)$ can coexist only at $\delta = 0$ and:

$|\mu| \le |\Lambda_{345-}|/(2\sqrt{\lambda_1\lambda_2})$, – the interval on (μ,δ) plane.

The (μ, δ) plane. Sector I.

First order transition segment — very thick line. The possible evolution of physical states — rays, directed to the side of growth of temperature. Small dark circles — variants of present values of parameters. The shaded area cover all present values of (μ, δ) in which in the past first order phase transition occurs (like p. 2).



Possible sequences of phase transitions

For present point 2 — EWs → CPc1 → CPc2

For present points 1 or 3 — EWs → CPc.

Solutions $(B_\pm)$. Case $\delta\neq 0$.

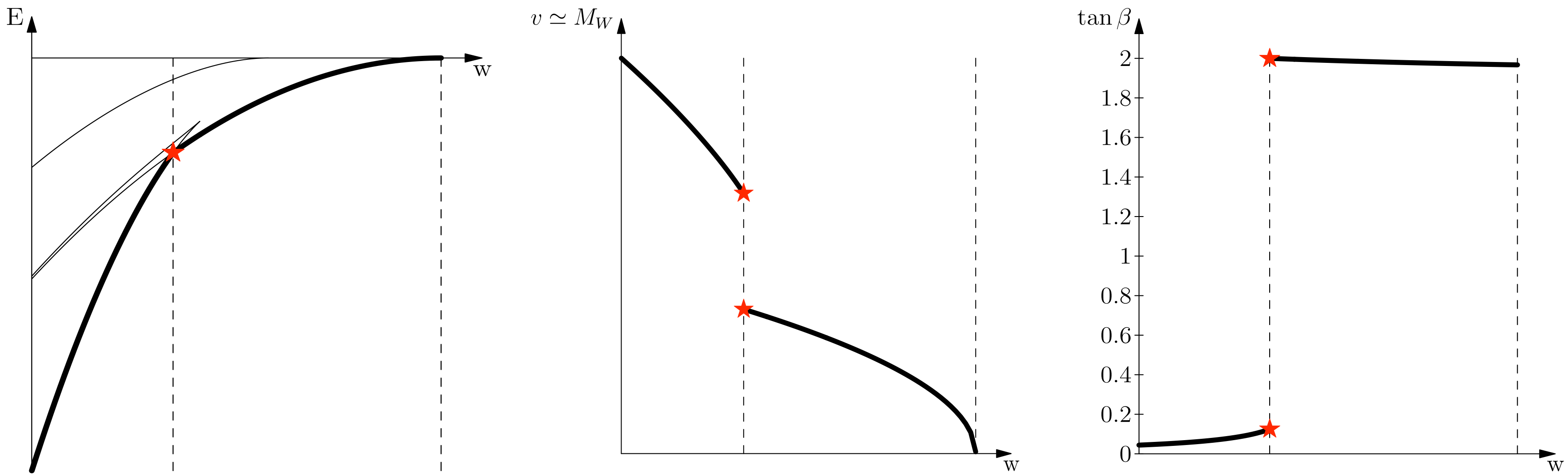
With variation of temperature quantity δ passes through 0.
At $\delta \ll 1$ all calculations are easy. We see:

At $\delta\neq 0$ the degeneracy splits -v.e.v.'s vary stepwise —
— 1-st order phase transition.

The latent heat

$$Q_{-\to +}=T\frac{\partial\mathcal{E}_+}{\partial T}-T\frac{\partial\mathcal{E}_-}{\partial T}\bigg|_{\delta\to 0}=\frac{m^4k^2}{2}\left(\frac{2\mu^2\sqrt{\lambda_1\lambda_2}-\Lambda_{345-}}{\mu^2\sqrt{\lambda_1\lambda_2}+\lambda_{345}}\right)\frac{R}{\sqrt{\lambda_1\lambda_2}}(c_1-c_2)w.$$

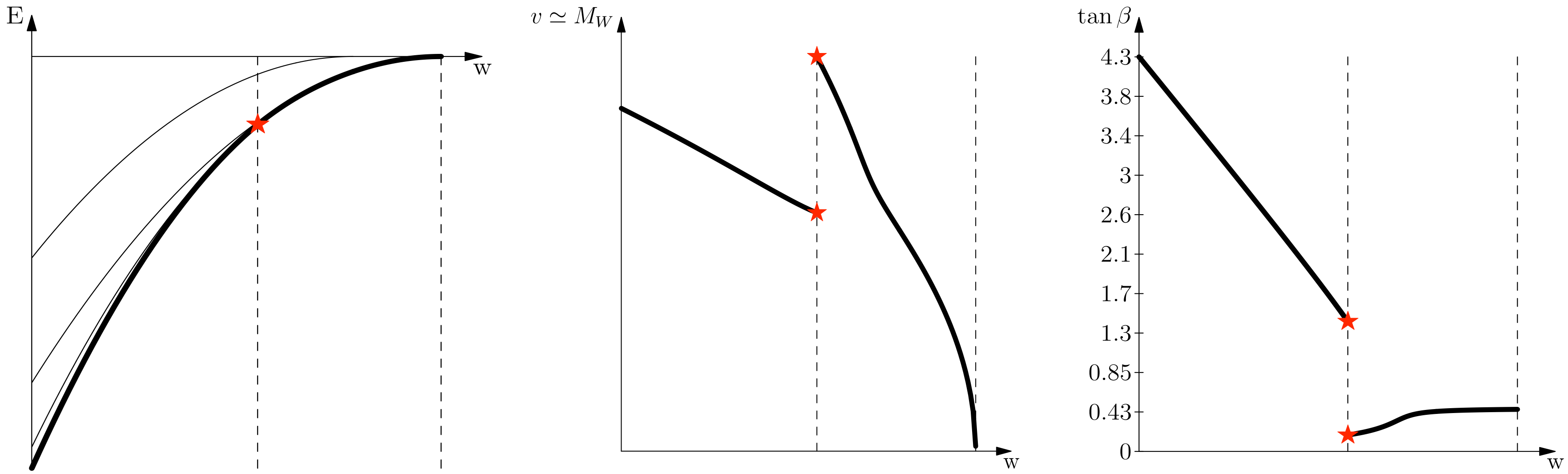
Sector I. Case with 1-st order transition CPc1→CPc2



One can see discontinuity in v^2 and $\tan\beta$ evolution.

In the left plot energies of ALL extremum states are presented.

Sector I. Another set of parameters



Before phase transition v^2 is bigger than after phase transition.

Sector II: $\boxed{\lambda_5 > 0, \;\; \lambda_5 > \lambda_4, \;\; \tilde{\Lambda}_{345-} > 0}$.

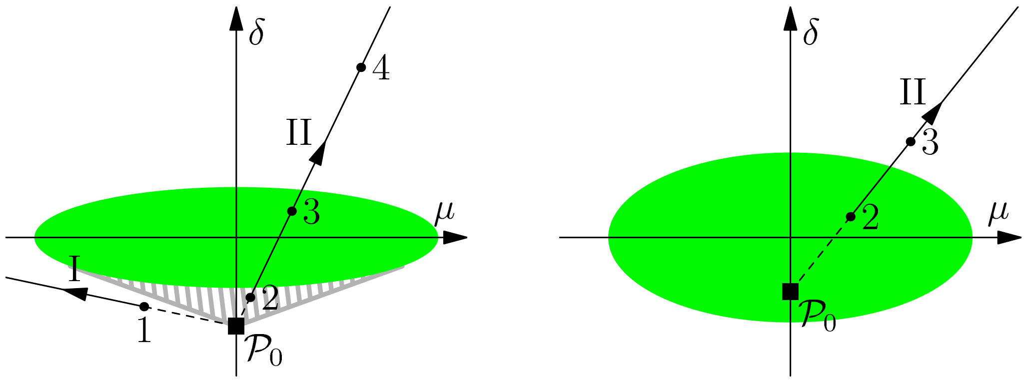
The sCPv states are possible.

Up to 3 second order phase transitions

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \;\; \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi}/\sqrt{2} \end{pmatrix}, \; \xi \neq 0$$

Potential is CP symmetric $\Rightarrow$ the sCPv extremum is doubly degenerated (in the sign of ξ).

If sCPv extremum is minimum, it is global one – vacuum.



In this figure we present **(μ, δ)** plane for different P_0. Green — sCPv states. The possible evolution of physical states — rays, directed to the side of growth of temperature. Small dark circles at these rays correspond possible present values of parameters. The shaded area cover all present values of (μ, δ) in which in the past the sCPv phase was crossed (like point 2).

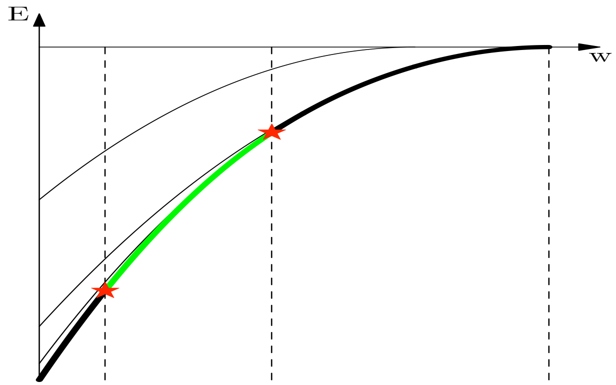
Possible sequences of phase transitions (all — 2-nd order):

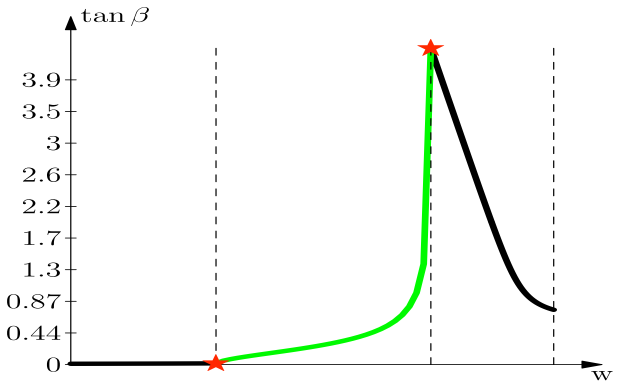
For present point 2 — EWs → CPc1 → scPv→ CPc2

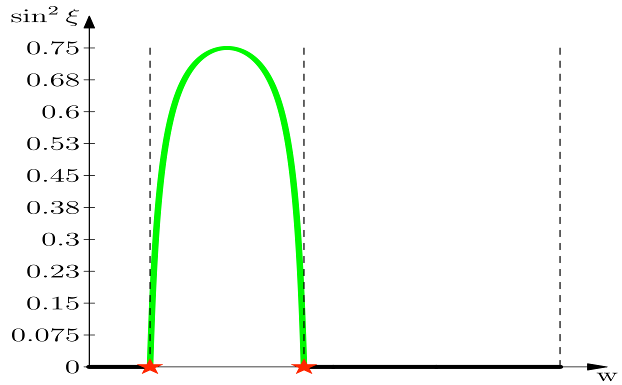
For present point 3 — EWs → CPc1 → scPv

For present points 1 or 4 — EWs → CPc.

Evolution of physical states in Sector II.



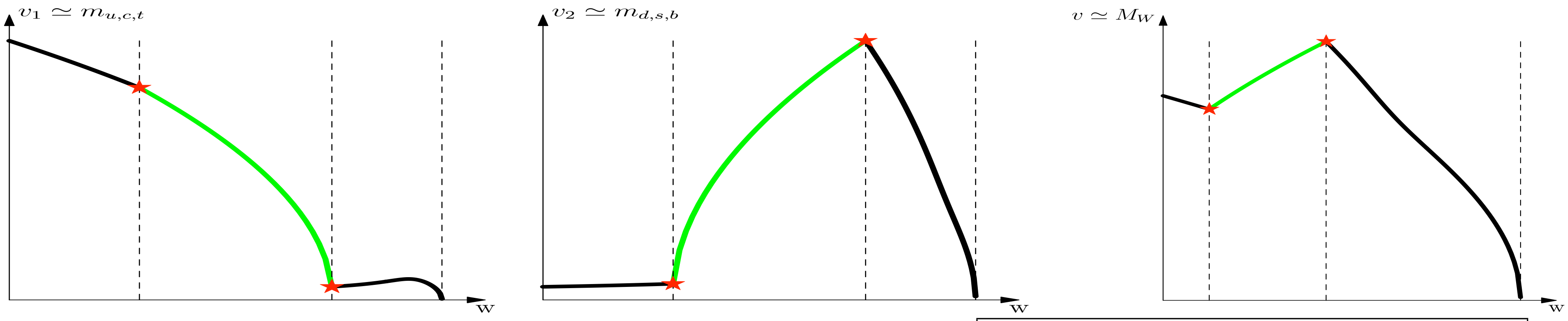




Left plot — evolution of the energy of all extrema (thin lines). The vacuum state – thick line. The phase transition points — red stars. Note small energy gap between sCPv and CPc extrema.

Central plot — evolution of $\tan\beta \equiv v_2/v_1$. The slope of curves changes stepwise in the transition points.

Right plot — evolution of order parameter $\sin^2\xi$.

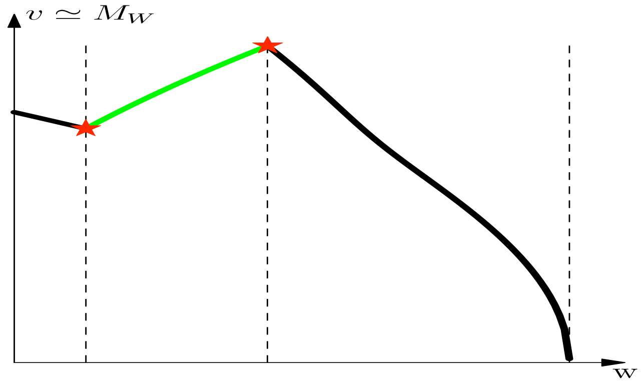


In these curves — v.e.v.'s.

Left – $v_1 \propto m_{u,c,t}$.

Center – $v_2 \propto m_{d,s,b}$.

Right – $v \propto M_W$. The evolution of v for another set of λ_i is also shown



Another set of λ_i

Sector III: $\boxed{\lambda_4 \pm \lambda_5 > 0\,, \quad \Lambda_{3-} > 0}$.

Charged vacuum states are possible.
Up to 3 second order phase transitions

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix}, \; Z = y_1 y_2 - y_3 y_{3^*} > 0$$

- Electric charge is not conserved.
- 4 (not 5) massive Higgs bosons and 4 (not 3) gauge bosons without definite charge.

Not modern state - BUT IN the PAST?

If charged extremum is minimum, it is global one – vacuum

For charged extremum all y_i are independent, so that minimum conditions describe system of linear equations with unambiguous solution, which define independently all y_i.

$$y_1 = \frac{m^2k^2}{2}\left(\frac{1}{\Lambda_{3+}} - \frac{\delta}{\Lambda_{3-}}\right), \;\; y_2 = \frac{m^2}{2}\left(\frac{1}{\Lambda_{3+}} + \frac{\delta}{\Lambda_{3-}}\right), \;\; y_3 = \frac{m^2k\mu}{2(\lambda_4+\lambda_5)};$$

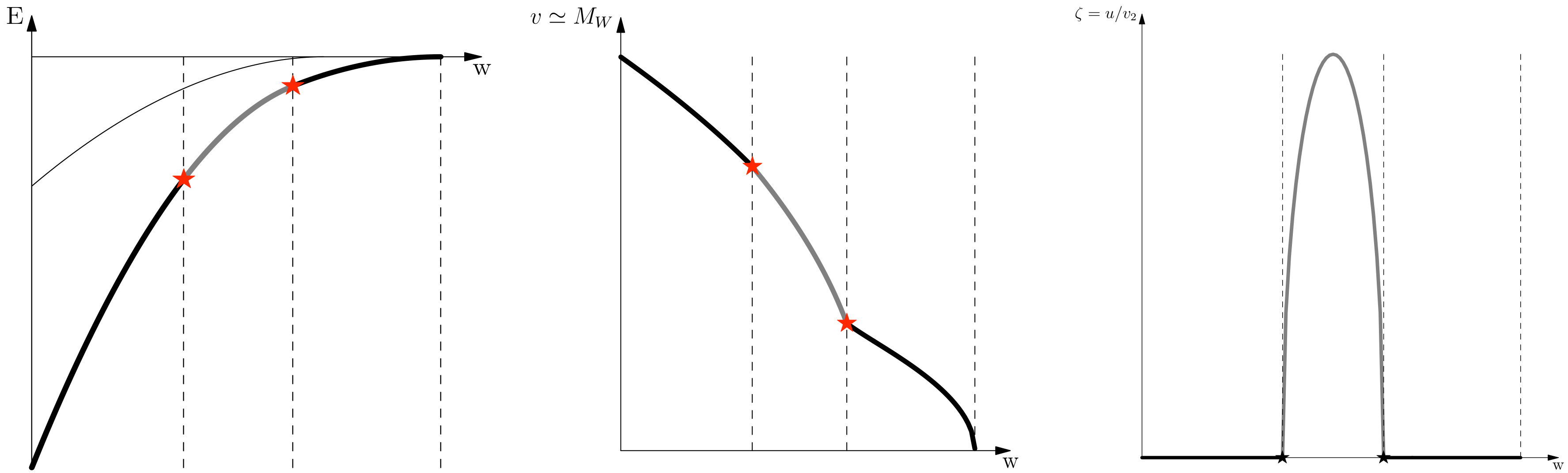
$$\mathcal{E}_{ch}^{ext} = -\frac{m^4k^2}{4}\left[\frac{1}{\Lambda_{3+}} + \frac{\delta^2}{\Lambda_{3-}} + \frac{\mu^2}{\lambda_4+\lambda_5}\right].$$

The condition $Z \equiv y_1y_2 - y_3^2 \equiv v_1^2u^2/4 > 0$ define the interior of ellipse

$$\frac{\mu^2}{a_1^2} + \frac{\delta^2}{a_2^2} < 1\,, \quad where\;\; a_1 = \frac{\lambda_4+\lambda_5}{\Lambda_{3+}}, \quad a_2 = \frac{\Lambda_{3-}}{\Lambda_{3+}}.$$

The picture in (μ, δ) plane and evolution of physical parameters are similar to those for sector II case (except physical properties within ellipse).

Sector III. Transition through charged extremum.



Order parameter: $\zeta = \frac{u}{\sqrt{v_1^2+v_2^2}}$.

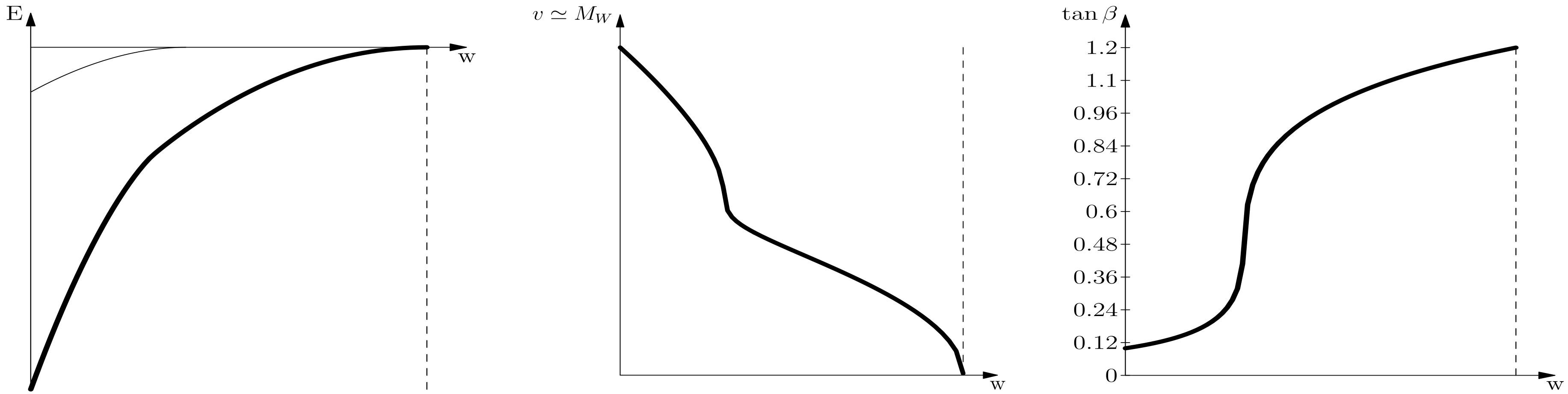
In the left plot energies of ALL extremum states are presented.

Sector IV: $\boxed{\lambda_5 < 0,\ \lambda_4 + \lambda_5 < 0,\ \Lambda_{345-} > 0}$.

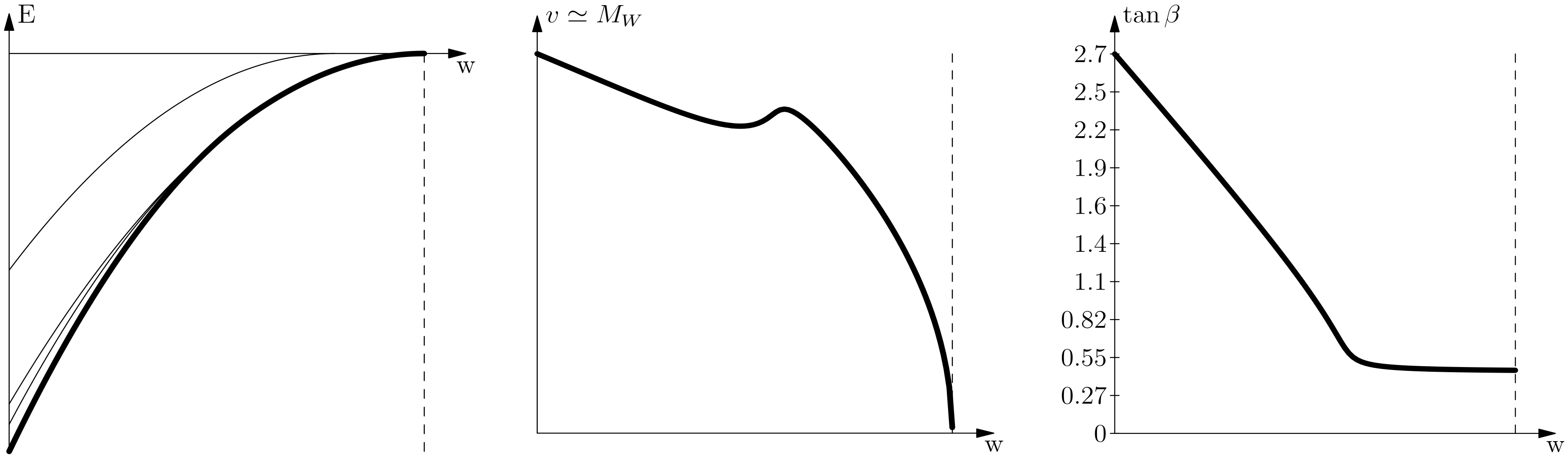
CPc states only.

Only EWSB phase transition.

The case with only EWSB transition, it is similar for all sectors



Non-monotonic evolution of $v \simeq M_W$ is possible.



GENERAL 1. Sequences of phase states

Start: EWSB phase transition from EWs phase to CPc phase at $T \sim 10 M_W$. Modern state — either CPc or CPv.
All possible sequences of phase states:

- (I) EW $\xrightarrow{II}$ CPc
- (II) EW $\xrightarrow{II}$ CPc $\xrightarrow{II}$ charged $\xrightarrow{II}$ CPc
- (III) EW $\xrightarrow{II}$ CPc $\xrightarrow{I}$ CPc
- (IV) EW $\xrightarrow{II}$ CPc $\xrightarrow{II}$ sCPv
- (V) EW $\xrightarrow{II}$ CPc $\xrightarrow{II}$ sCPv $\xrightarrow{II}$ CPc

Fields of parameters, allowing different ways of phase evolution were shown in our slides.
Transformation of these fields into the field of measurable parameters (like masses and some couplings) is natural task.

GENERAL 2. Properties of phase states
Rearrangement of particle mass spectrum

During evolution $v = v(T)$, $v_1 = v_1(T)$, $v_2 = v_2(T)$.
Masses of particles evolve (Model II for Yukawa, as in MSSM)

$$M_W(T) = M_W \frac{v(T)}{v}, \quad m_d(T) = m_d \frac{v_1(T)}{v_1}, \quad m_u(T) = m_u \frac{v_2(T)}{v_2}$$

Ratio of masses of quarks of the same charge remains unchanged:

$$m_t(T) : m_c(T) : m_u(T) = m_t : m_c : m_u$$

while within one generation:

$$\frac{m_u(T)}{m_d(T)} = \frac{m_u}{m_d} \frac{tan\beta(T)}{tan\beta}, \quad \frac{m_u(T)}{M_W(T)} = \frac{m_u}{M_W} \frac{cos\beta(T)}{cos\beta}, \quad \frac{m_d(T)}{M_W(T)} = \frac{m_d}{M_W} \frac{sin\beta(T)}{sin\beta}$$

In the most of considered examples values of $\tan\beta$ vary strong, for the first order phase transition (sequence (III)) – even stepwise. Therefore in the past quark mass spectrum can be rearranged, i.e. either $m_u > m_d \Rightarrow M_n < M_p$ or $m_c < m_s$ or even $m_t < m_b$,...

Certainly, this effect is smoothed in the hot Universe and during very small time. However, this rearrangement prepare very unusual initial state for baryogenesis and galaxies formation. In particular, the islands of normal matter (proto-galaxies) will be neutral without additional electrons.

This rearrangement of fermion masses looks like

one more phase transition in fermion subsystem

with own fluctuations, etc. The study of this possible phase transition is beyond reported approach.

GENERAL 3. First relations to cosmology ?

1. In the standard approach the temperature of phase transition is given by electroweak scale, it is very high. In our model the same is valid for the first EWSB transition. However, the temperature of last phase transition can be low enough (the point, representing modern state can be close to phase separation line).

2. This picture can influence for Microwave Background radiation and structure of proto-galaxies.

- New phase transitions $\Rightarrow$ new critical fluctuations.
- Small gaps between vacuum and nearest CPc extrema in sCPv and charged phases $\Rightarrow$ small energy barrier $\Rightarrow$ big fluctuations, bubbles.

3. If the charged phase was the intermediate phase, then electroneutrality was strongly violated $\Rightarrow$ strong relative motions after transition to modern neutral phase – ? relation to bariogenesis?.

Additional comments

- The application of this approach to the model with $m_{12} = 0$ allow to describe modern dark matter as specific phase of 2HDM (inert model) with specific evolution of phase states – M. Krawczyk report

- The application of this approach to 3HDM with $m_{13} = m_{23} = 0$ allows to have different modern phase states like in usual 2HDM plus dark Higgs component like that in inert model (see B. Grzadkowski, O.M. Ogreid, P. Osland hep-ph/0904.2173) The phase history of Universe in this case is reacher than in 2HDM.

THE END