

# *TREDI: a review*

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3<sup>rd</sup> workshop on μBI - Frascati Mar 24-26, 2010

# TREDI ...

*... is a multi-purpose macroparticle 3D Monte Carlo, devoted to the simulation of electron beams through*

- ✓ *Rf-guns, DC-guns*
- ✓ *Linacs (TW & SW)*
- ✓ *Solenoids*
- ✓ *Bendings*
- ✓ *Quadrupoles*
- ✓ *Undulators*
- ✓ *Laser Pulses*

*(Gauss-Hermite & Gauss-Laguerre modes)*

...

# History

- **1992-1995** - Start: EU Network on RF-Injectors<sup>(1)</sup>
  - Fortran / DOS
  - Procs: "VII J.D'Etude Sur la Photoem. a Fort Courant" Grenoble 20-22 Sept. 1995 (PC@20MHz ~10<sup>2</sup> particles)
- **1996-1997** - Covariant smoothing of SC Fields
  - Ported to C/Linux (PC@133MHz)
  - FEL 1996 - NIM A393, p.434 (1997)
- **1998** J. Rosenzweig & P. Musumeci, PRE 58 2737 (1998) Diamagnetic fields in und. Dyn.
- **1998-1999** - Simulation of bunching in low energy FEL<sup>(2)</sup>
  - Added Devices (SW Linac - Solenoid - UM) (PC@266MHz)

J.B. Rosenzweig & P. Musumeci, PRE 58, 27-37, (1998) - Diamagnetic fields due to finite dimensions of intense beams in high-gain FELs - FEL 1998 - NIM A436, p.443 (1999)
- **2001-2002** - Italian initiative for Short λ FEL
- **2002-2003** - CSR (DESY Zeuthen) & ICFA (Chia Laguna) Workshops
  - 1<sup>st</sup> parallel (MPI) implementation (IBM SP & Linux Cluster ~10<sup>3</sup> particles)
  - 1<sup>st</sup> serious attempt to account of CSR
  - L. Giannessi & M.Q., Phys. Rev. ST Accel. & Beams Vol. 6, 120101 (2003)
  - AA. VV. Code Comparison for Simulations of Photo-Injectors PAC03 (FPAG043 )
- **2004→Today** - SPARC+EUROFEL
  - SPARC Beam Dynamics Group (EUROFEL Cluster - 84 cores)
  - L. Giannessi, M.Q. & C. Ronsivalle EPAC04 (THPLT054) & FEL04 (TUPOS12)
- **Today→Future: Many upgrades:**
  - New algorithms to account for SC effects (fast 2D version, parallel PDE, FMPA...)
  - FEL dynamics, 1-to-1 simulations (parallel)

(1) Contributions from A. Marranca

(2) Contributions from P. Musumeci

# *Motivations*

- *Retarded effects*
- *3D effects* →
  - *Dishomogeneities of cathode's  $Q_E$*
  - *Misalignments (Laser, Devices)*
  - *Multipolar terms in accelerating fields*
  - *Rotation of devices*
- *Retarded effects*
  - *CSR emission in bendings, compressor*

# Features

## ■ SF effects accounted for in

- "Retarded" or "Instantaneous" mode
- "Point2point" or "Point2Grid" mode
  - Cartesian, Cylindrical & Elliptic 2D & 3D grids
- FMPA (Hyerarchical) algorithm
- lack (yet) a PDE approach

## ■ Devices

- Every device is both rotatable & displaceable either analytical or mapped
- Maps 1D → Off axis device's profile expansion
- Maps 2D → Assume axi-symmetry (+SuperFish)
- Maps 3D → Maps from HFSS, CST, Radia...

## ■ Dynamics

- Selectable accuracy, error estimation
- Adaptive 4° or 5° order Runge-Kutta

# Features cont'd

- *20000+ lines in ANSI C, Scalar & Parallel (MPI)*
- *Multi-platform*
  - Unix (Linux, AIX) & Windows (MPICh, multi-core)
  - LAM, MPICh, OpenMPI
- *Binary output in NCSA HDF5 format* →
  - *Self Documenting*
  - *Portable (no endianness/alignment problems)*
  - *Allows compression, ease piecewise simulations,*
- *Tcl/Tk Gui (layout definition)*
- *Mathematica, MathCad frontends (pre- & post-processing), Cern PAW*
- *Still lacks a decent documentation ...*
- *Free! (GPL)*

# Equations of motion...

$$\begin{cases} \vec{\chi} = \vec{r} \\ \vec{\pi} = \gamma \vec{\beta} = \frac{\vec{p}}{mc} \end{cases}$$

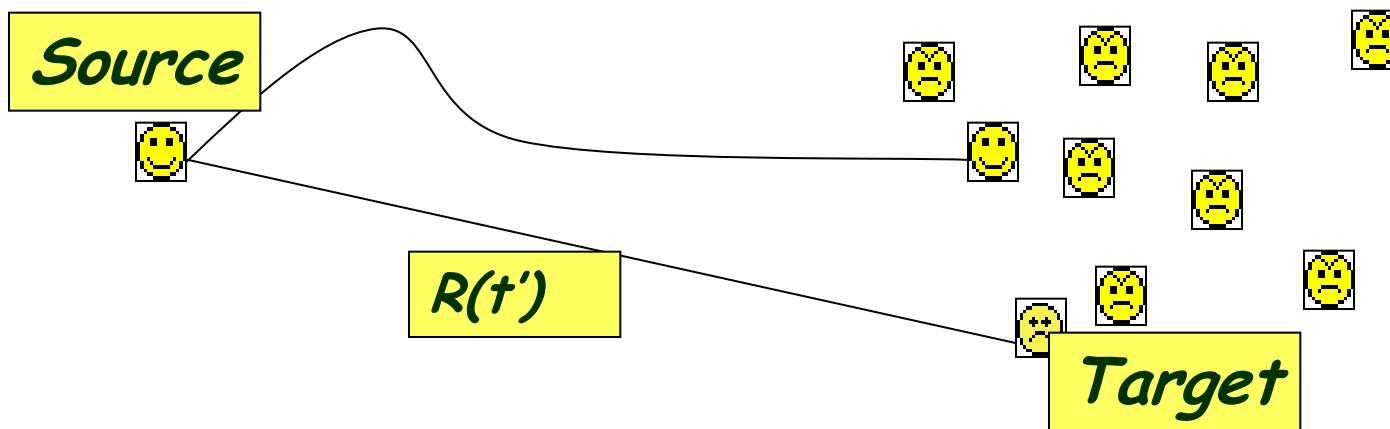
$$\begin{cases} \frac{d\vec{\chi}}{d\tau} = \frac{\vec{p}}{\gamma mc} \\ \frac{d\vec{\pi}}{d\tau} = \frac{e}{mc^2} \left( \vec{E} + \frac{\vec{\pi}}{\gamma} \times \vec{B} \right) \end{cases} = \frac{\vec{\pi}}{\gamma}$$

N.B. :  $\tau = c \cdot t$

*Devices+SelfFields*

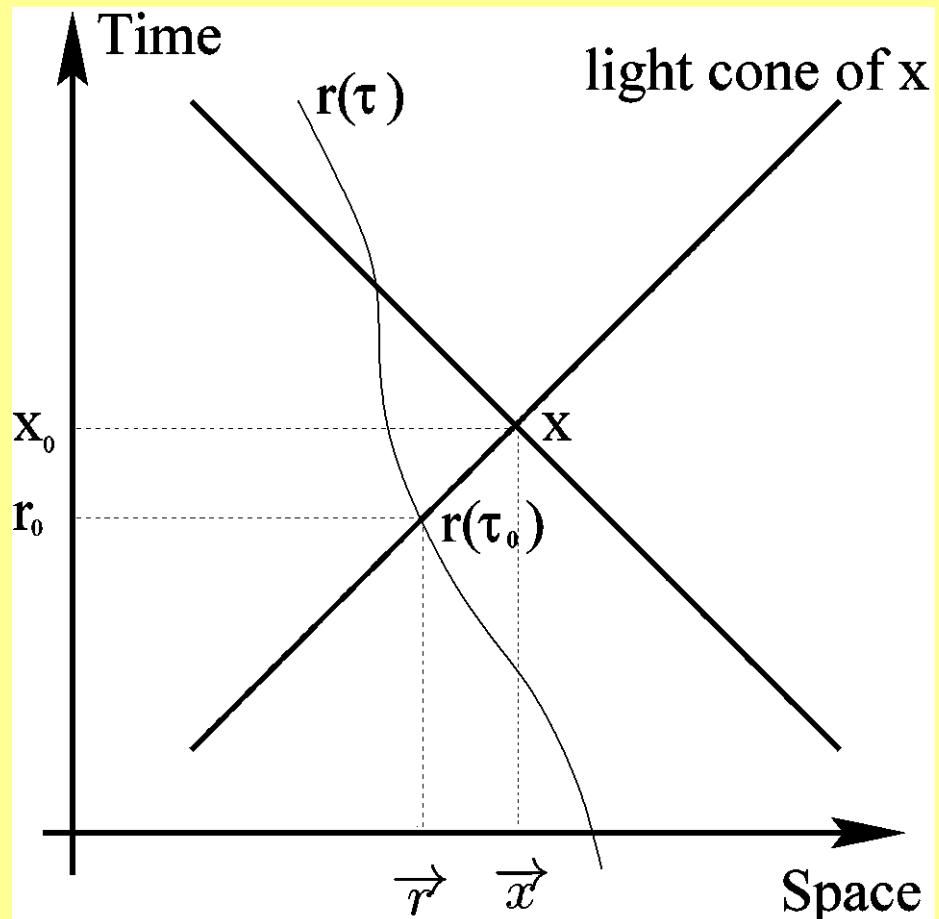
## *SELF FIELDS ...*

*... are accounted for by means of Lienard-Wiechert retarded potentials*

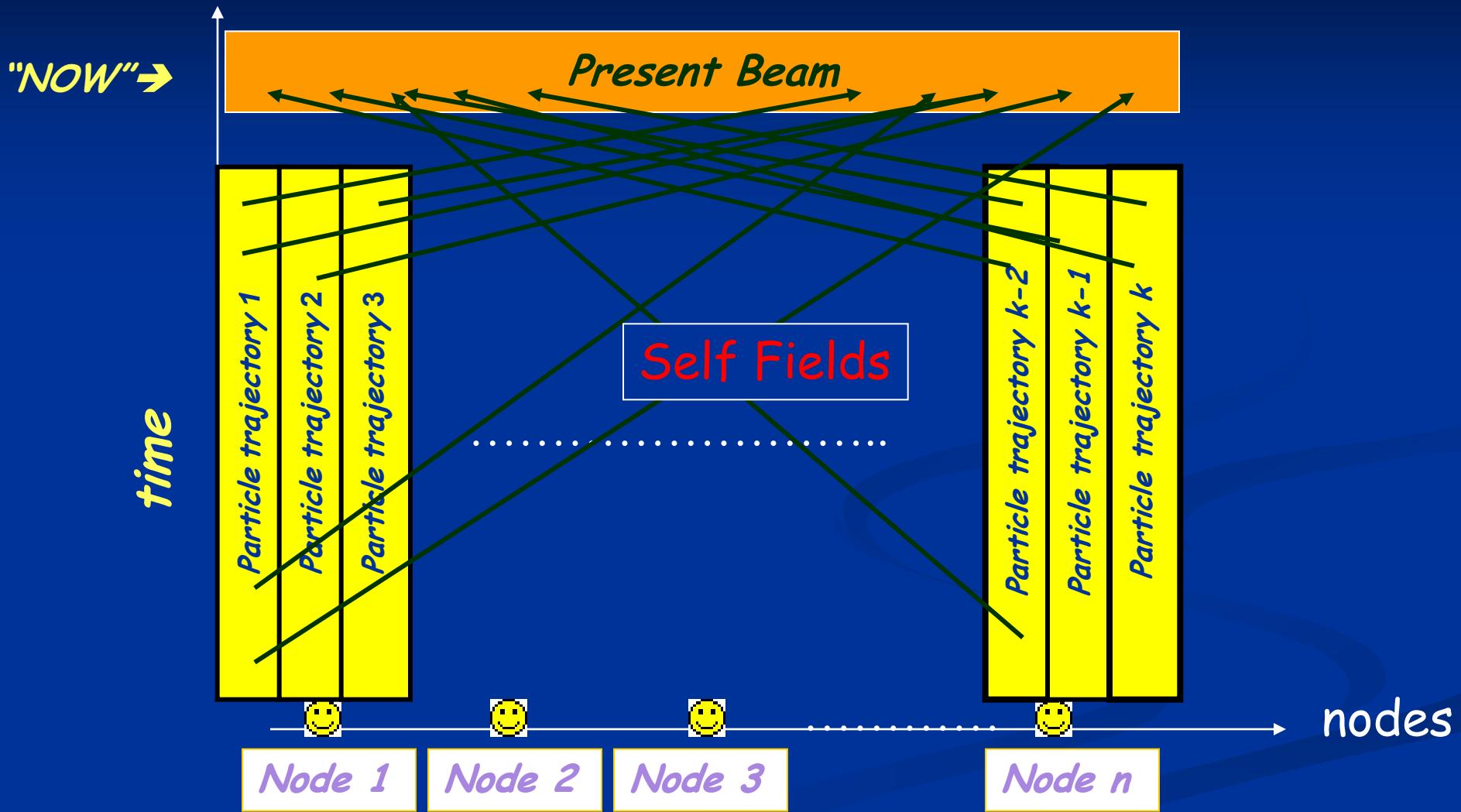


# *Retard Condition*

$$\tau_{ret} \rightarrow [x - r(\tau)]^2 = 0$$



# Parallelization of Self Fields (2002)



# SF fields:

$$\vec{E} = q \underbrace{\frac{(\hat{n} - \vec{\beta})}{\gamma^2 (1 - \hat{n} \cdot \vec{\beta})^3 R^2}}_{\text{Vel. field}} + \underbrace{\frac{q}{c} \frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \hat{n} \cdot \vec{\beta})^3 R}}_{\text{Accel. field}}$$

$$\vec{B} = \hat{n} \times \vec{E}$$

$$\hat{n} = \frac{\vec{x} - \vec{r}(t_{ret})}{|\vec{x} - \vec{r}(t_{ret})|} = \frac{\vec{R}(t_{ret})}{|\vec{R}(t_{ret})|}$$

$$\text{Retarded time} \quad t' = t - \frac{R(t')}{c}$$

(see e.g. Classical Electrodynamics, J. D. Jackson)

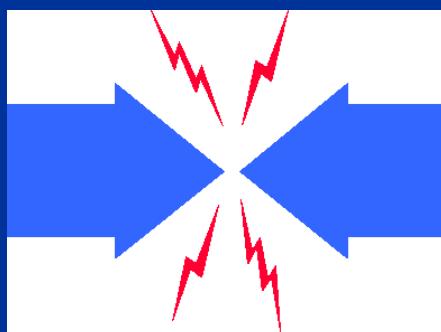
# Problem: fields regularization

- Real beam  $\sim 10^{10}$  particles

- Macroparticles  $\sim 10^3 - 10^6$

→ huge charges ( $> 10^6$  e)

Pseudo-collisional effects



# *Known therapies:*

- Share charge among vertices of a regular grid  
(require a fine mesh to reduce noise)
- Solve Poisson equation in the "rest frame"  
*(if any) → boost fields in the lab. frame*

## *Typical problems:*

- assume instantaneous interactions
- sensible for quasi-static Coulomb fields, low energy spread etc.
- non Lorentz invariant

# Alternative

- Get rid of 3-anything  
(i.e. quantities not possessing a definite Lorentz character)

$$\vec{E}, \vec{B}, \cos(\varphi), \int_0^R dR \dots$$

- Use 4-quantities instead

$$q, \tau$$

$$(x - r) \cdot v$$

$$v^\mu, w^\mu$$

$$(x - r)^\mu v^\nu$$

$$F^{\mu\nu}$$

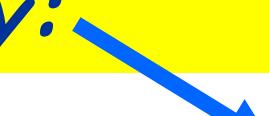


# Field strength produced by an accelerated charge (covariant form)

$$\begin{aligned}
 \vec{E} &= q \underbrace{\frac{(\hat{n} - \vec{\beta})}{\gamma^2 (1 - \hat{n} \cdot \vec{\beta})^3 R^2}}_{\text{Vel. field}} + \frac{q}{c} \underbrace{\frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \hat{n} \cdot \vec{\beta})^3 R}}_{\text{Accel. field}} \\
 \vec{B} &= \hat{n} \times \vec{E}
 \end{aligned}$$

$$F^{\mu\nu} = \frac{q}{(x - r) \cdot V} \frac{d}{d\tau} \left[ \frac{(x - r)^\mu V^\nu - (x - r)^\nu V^\mu}{(x - r) \cdot V} \right]_{ret}$$

$\tau$  is the (source) proper time and  
 $V$  is the (source) 4-velocity:


 $V = (\gamma, \gamma \vec{\beta})$

# Separation in vel.+accel. terms

$$F^{\mu\nu} = q \left[ \frac{T^{\mu\nu}(V)}{S^3(V)} + \frac{T^{\mu\nu}(W)}{S^2(V)} - S(W) \frac{T^{\mu\nu}(W)}{S^3(V)} \right]$$

$$\begin{aligned} \text{Velocity term} &= F_V^{\mu\nu} & \text{Acceleration terms} &= F_A^{\mu\nu} \\ &= \overbrace{\frac{q}{S^3(V)} T^{\mu\nu}(V)} &+ \overbrace{\frac{q}{S^2(V)} T^{\mu\nu}(W) - \frac{q}{S^3(V)} S(W) T^{\mu\nu}(W)} \end{aligned}$$

The natural decomposition of  
EM fields holds in covariant  
form!

*Where:*

$$\begin{cases} w &= \frac{dV}{d\tau} && \text{is the 4 - acceleration} \\ s(U) &= U \cdot (x - r) && \text{is a Lorentz scalar} \\ T^{\mu\nu}(U) &= (x - r)^\mu U^\nu - (x - r)^\nu U^\mu && \text{is an antisymmetric tensor} \end{cases}$$

$$s(U) \rightarrow s(U, R) \quad e.g. \quad s(V) = \gamma R (1 - \hat{n} \cdot \vec{\beta})$$

*Note:*

$$T^{\mu\nu}(U) \rightarrow T^{\mu\nu}(U, R)$$

# *Velocity term:*

*Lorentz scalar*

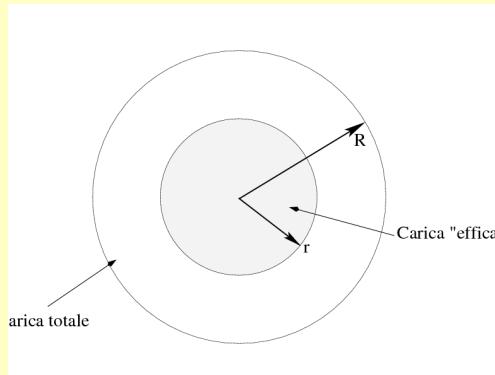
*Tensorial structure*

$$F_V^{\mu\nu} = \frac{q}{S^3(V)} T^{\mu\nu}(V)$$

*Lorentz scalar*

$$R \rightarrow 0 \Rightarrow S(V) \rightarrow 0$$

- Solution: source macro particles are given a form factor (i.e. a finite extension in space: Q.E.D. Dirac/Pauli form factors corrections to current scattering amplitudes)



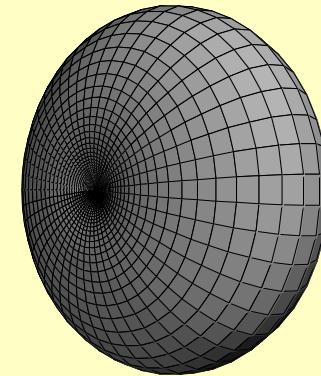
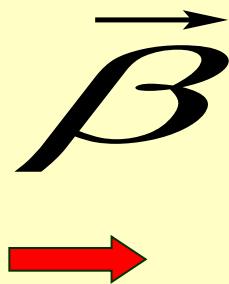
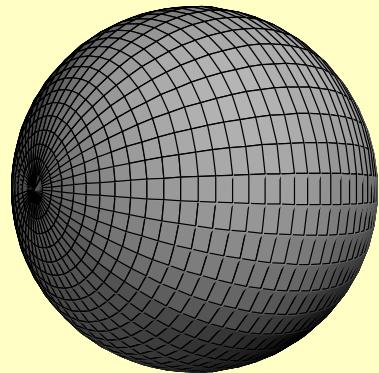
- Effective charge**

$$\lim_{R \rightarrow 0} \frac{Q_{\text{eff}}(x - r)}{S^3(V)} \rightarrow 0$$

- A scaled replica of the (retarded) beam:
- Same aspect ratio

$$\sigma_{mp} \approx \frac{\sigma_{beam}}{\sqrt[3]{N}}$$

- Velocity (static, Coulomb) fields travel at speed of light, too!  $\rightarrow$  introduce a covariant (4D) generalization of purely geometric form factor:



$$\hat{\Sigma}^{-1} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \hat{\sigma}^{-1} & \\ 0 & & & \end{pmatrix}}_{\text{Rest frame}}$$

$$\Rightarrow \quad \hat{\Sigma}'^{-1} = \overbrace{\begin{pmatrix} \overbrace{\vec{\beta}^T \hat{\sigma}'^{-1} \vec{\beta}}^{1 \times 1} & \overbrace{- \vec{\beta}^T \hat{\sigma}'^{-1}}^{1 \times 3} \\ \overbrace{- \hat{\sigma}'^{-1} \vec{\beta}}^{3 \times 1} & \overbrace{\hat{\sigma}'^{-1}}^{3 \times 3} \end{pmatrix}}^{\text{Lab frame (primed)}}$$

# Introduce a 1D shell parameter

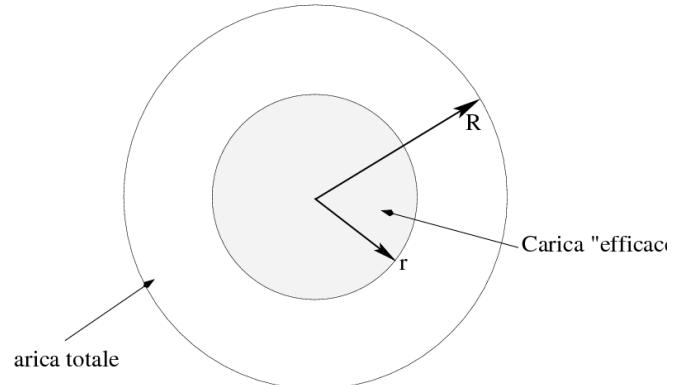
$$R_0^2 = \overbrace{\Delta \vec{x}^T \cdot \hat{\sigma}^{-1} \cdot \Delta \vec{x}}^{\text{Rest Frame (3-vectors)}} = \overbrace{\Delta x^T \cdot \hat{\Sigma}^{-1} \cdot \Delta x}^{\text{Lab Frame (4-vectors)}}$$

e.g.: gaussian shape:

$$\rho(\vec{r}) = \overbrace{\frac{q}{\sqrt{(2\pi)^3 \det \hat{\sigma}}} \exp\left(-\frac{\Delta \vec{x}^T \cdot \hat{\sigma}^{-1} \cdot \Delta \vec{x}}{2}\right)}^{\text{Rest Frame (3-vectors)}} \downarrow$$

$$\rho'(r) = \overbrace{\frac{q}{\sqrt{(2\pi)^3 \det \hat{\sigma}'}} \gamma \exp\left(-\frac{\Delta x'^T \cdot \hat{\Sigma}'^{-1} \cdot \Delta x'}{2}\right)}^{\text{Lab Frame (4-vectors)}}$$

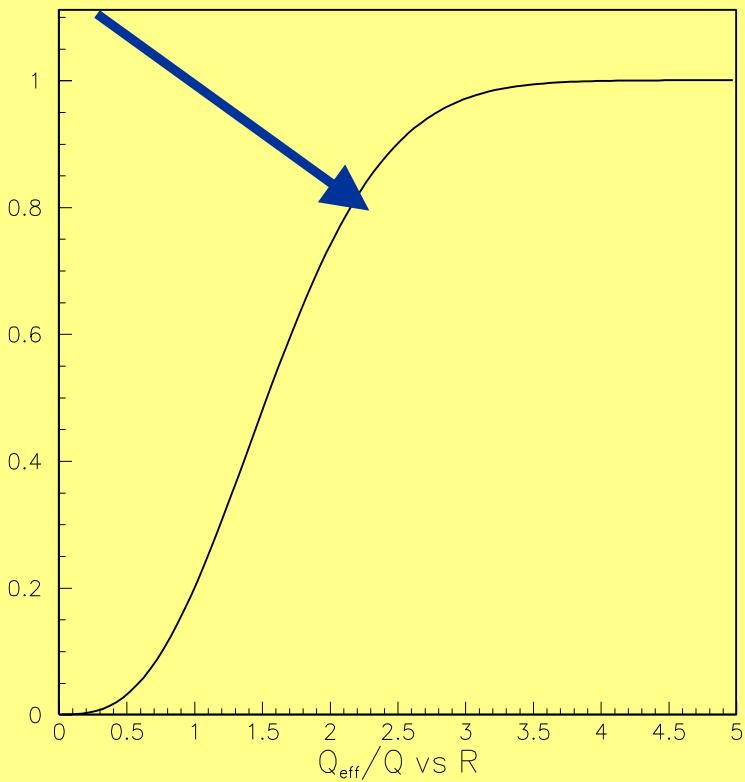
■ Effective charge total charge included by the iso-density surface associated to the value of  $\rho$  ( $\rho'$ ) at observer point: ➔



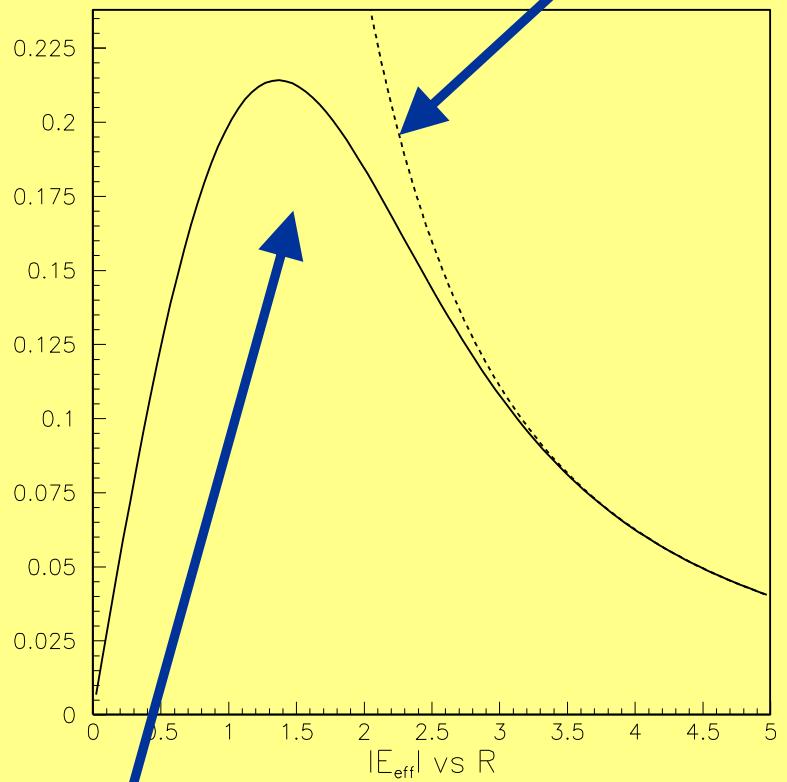
$$Q_{\text{eff}} \left( R = \sqrt{\Delta \mathbf{x}'^T \cdot \hat{\Sigma}'^{-1} \cdot \Delta \mathbf{x}'} \right) = q \left[ \operatorname{erf} \left( \frac{R}{2} \right) - \sqrt{\frac{2}{\pi}} R \cdot \exp \left( -\frac{R^2}{2} \right) \right]$$

$$E_{\text{eff}}(R) = \frac{Q_{\text{eff}} \left( R = \sqrt{\Delta \mathbf{x}'^T \cdot \hat{\Sigma}'^{-1} \cdot \Delta \mathbf{x}'} \right)}{|\Delta \vec{\mathbf{x}}'|^2}$$

*Eff. Charge*



*Un-smoothed ( $1/R^2$ ) fields*



*Effective vel. field*

# *Acceleration term*

*Lorentz scalars*

$$F_A^{\mu\nu} = \frac{q}{S^2(V)} T^{\mu\nu}(W) - \frac{q}{S^3(V)} s(W) T^{\mu\nu}(W)$$

*Lorentz scalars*

- Fields blow up when  $\vec{\beta} \approx \hat{n}$  ("collinear divergencies)



- Solution: target macro particles are given a finite extension in space:

- Let be  $S_0 = \Delta \mathbf{x}' \cdot \mathbf{V} = \gamma |\Delta \vec{\mathbf{x}}'| (1 - \hat{n} \cdot \vec{\beta})$

# *Smoothing of accel. fields*

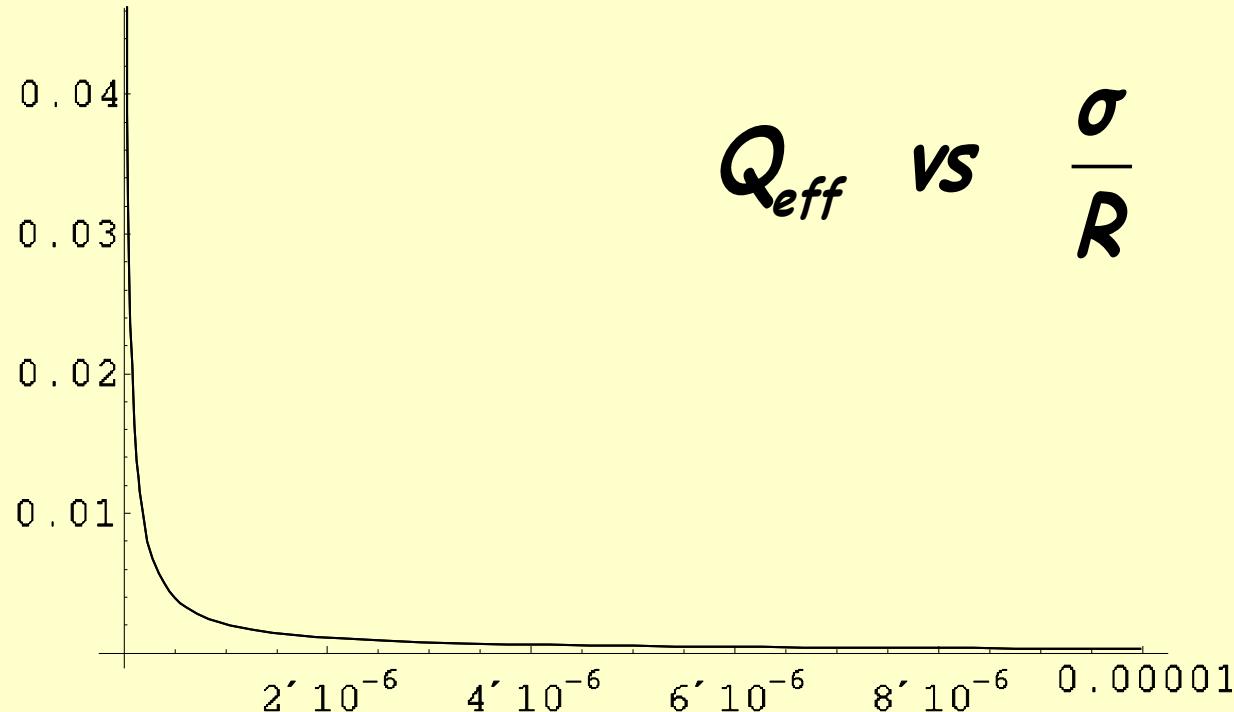
$$\begin{cases} \frac{1}{S^2(V)} \approx \frac{S_0^2}{\sqrt{2\pi}\sigma^3} \int_0^\infty d\rho \cdot \exp\left[-\frac{\rho^2}{2\sigma^2}\right] \rho^2 G_2(S_0) \\ \\ \frac{1}{S^3(V)} \approx \frac{S_0^3}{\sqrt{2\pi}\sigma^3} \int_0^\infty d\rho \cdot \exp\left[-\frac{\rho^2}{2\sigma^2}\right] \rho^2 G_3(S_0) \end{cases}$$

$$\text{Where: } G_2(S_0) = 2 \frac{S_0^2 - 2S_0\gamma\rho + \rho^2}{S_0^4 - 2S_0^2[\gamma^2(1 + \beta^2)]\rho^2 + \rho^4}$$

*while  $G_3$  is a too complicated to be worth showing*

# *Effectiveness of smoothing*

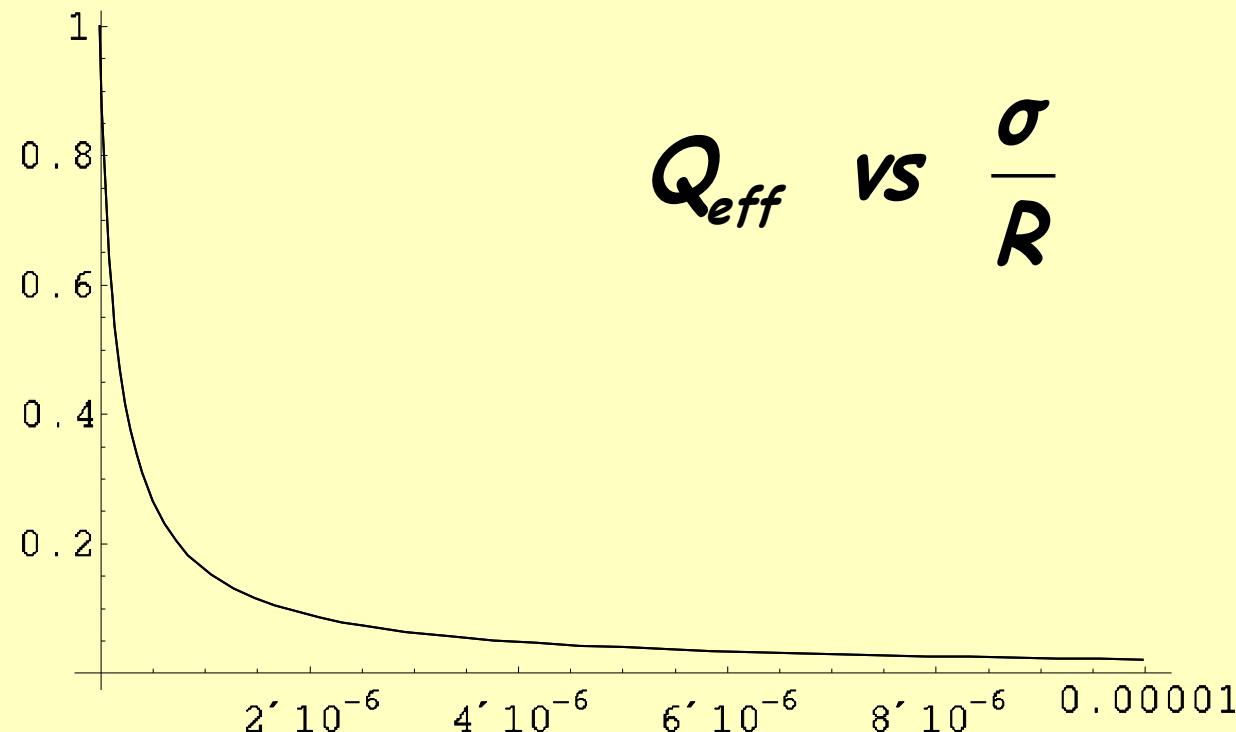
- $S_0 \sim 10^{-5}$  ("collinear")



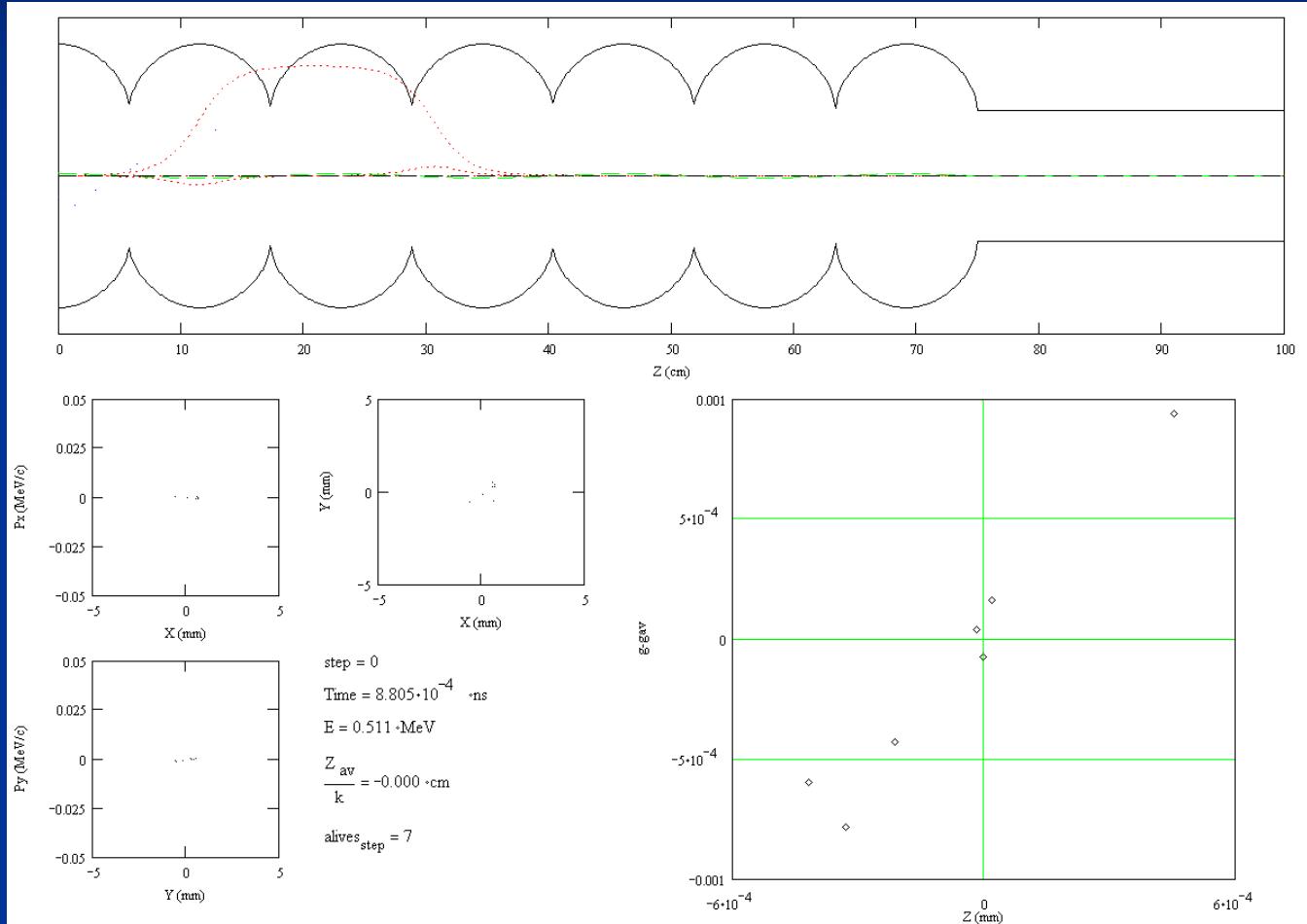
# Effectiveness of smoothing

(cont'd)

- $S_0 \sim 10^{-3}$  ("non collinear")

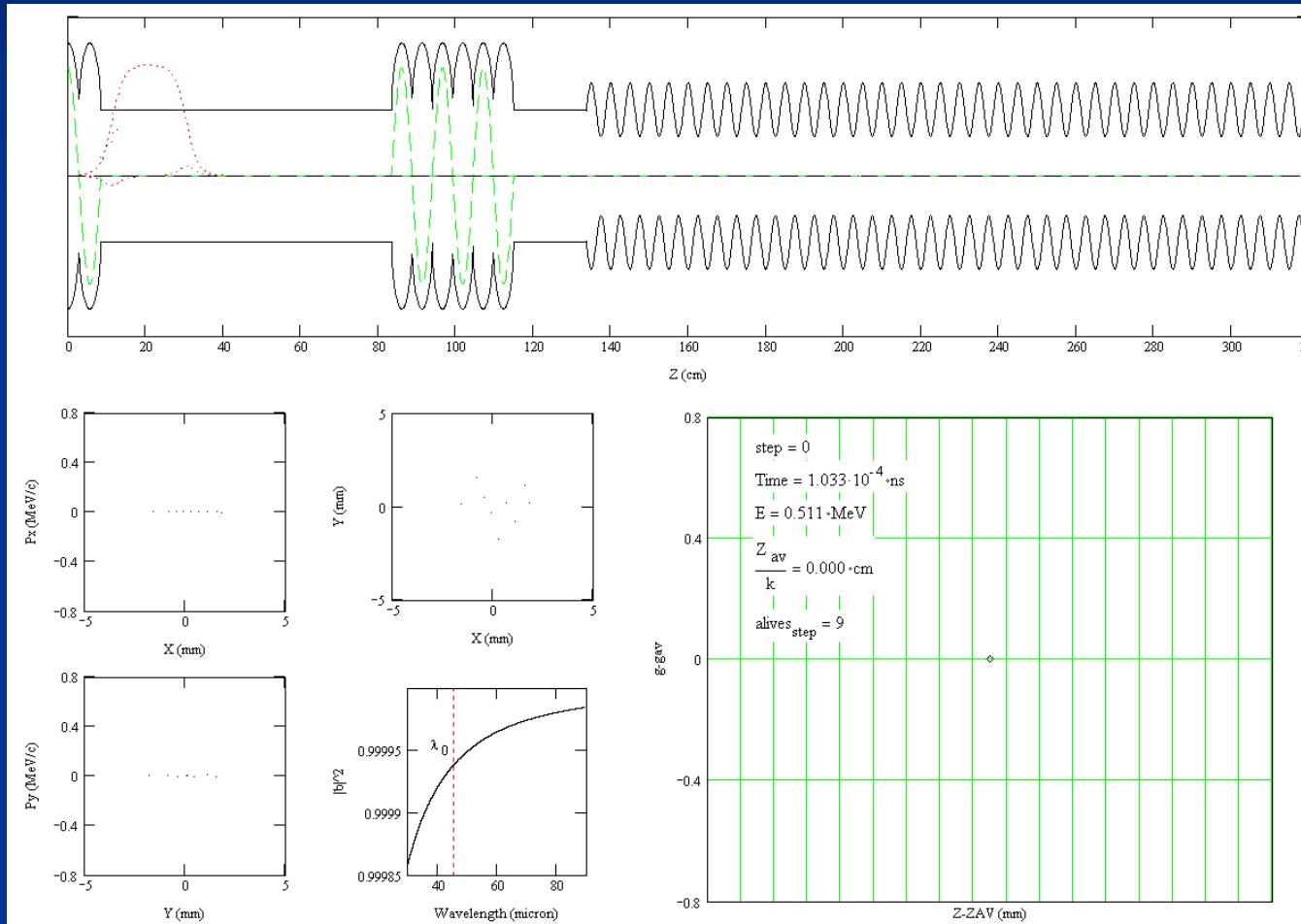


# *A $6+\frac{1}{2}$ cell RF Gun (1996)*



# FEL simulation

(L. Giannessi, P. Musumeci & M.Q., 1997)



# SPARC



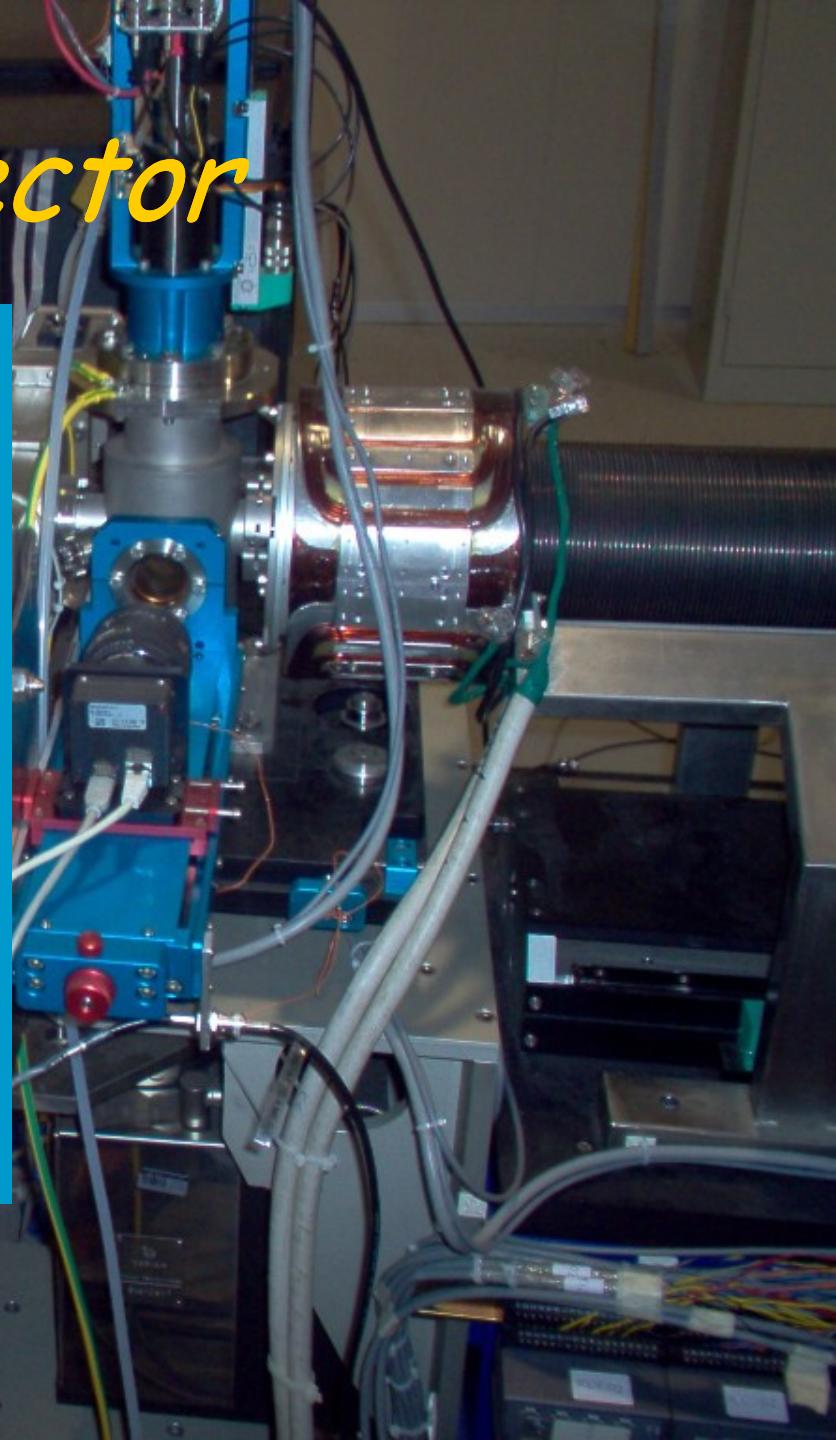
- Injector commissioning concluded Dec. 2006
- Linac installed & in vacuum July 2007
- Undulators (Accel) delivered May. 2007 magnetic characterization & alignment in process
- Linac conditioning starts Sept. 2007
- Beam transport through UM by July 2008



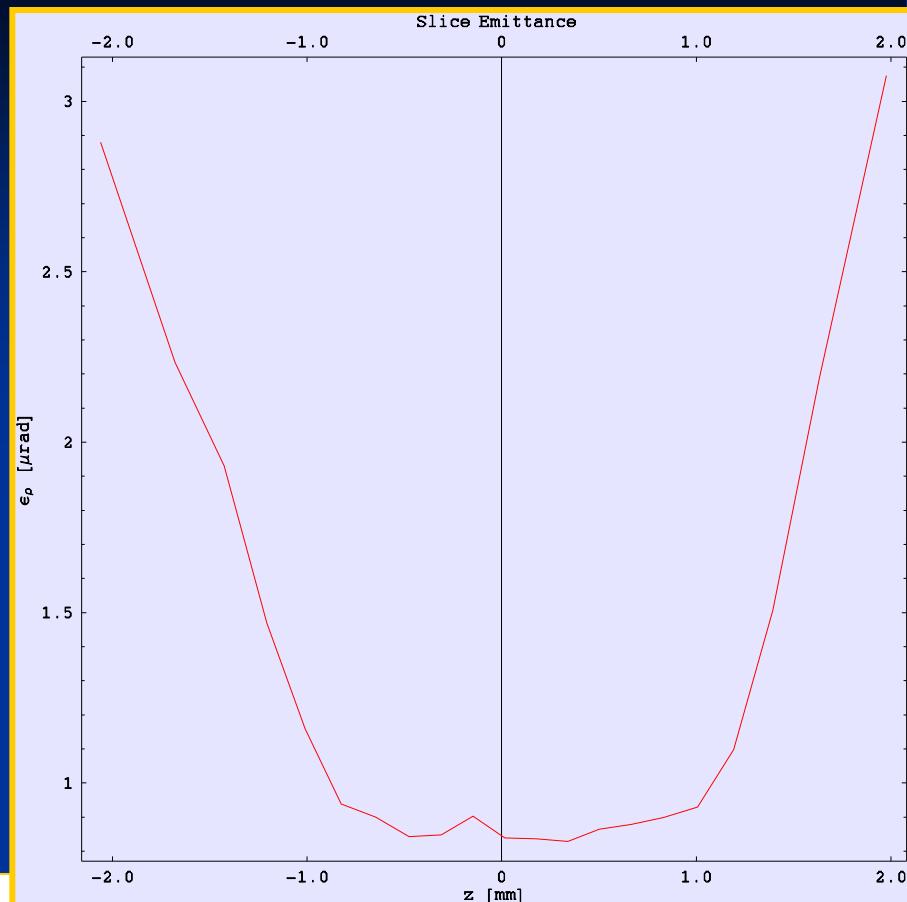
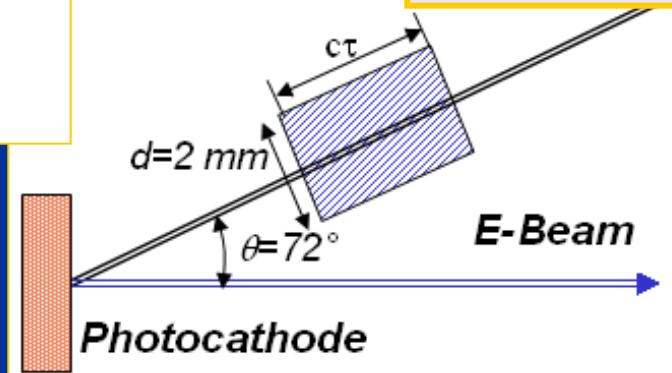
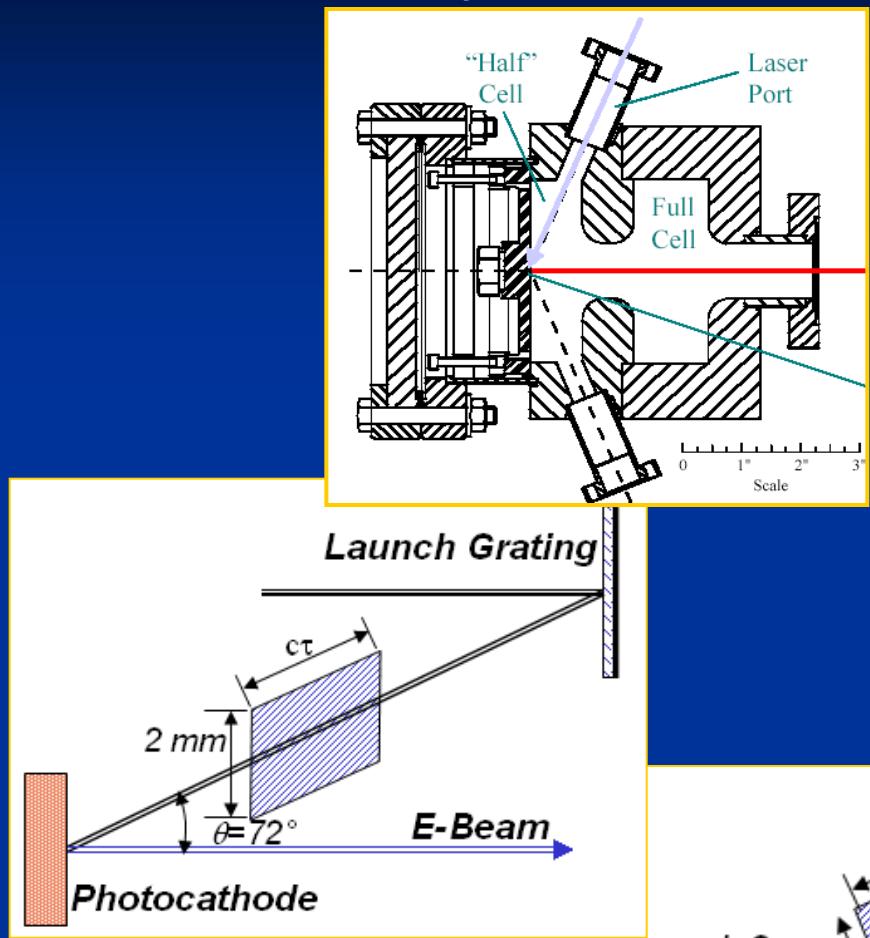
July 13 2007 SPARC Hall

# The Injector

- *BNL-SLAC-UCLA design 1.6 cells S-band rf-gun*
- *Realized @ UCLA Particle Beam Physics Laboratory*
- *Commissioning ended Dec. 2006*
- *Detailed beam characterization*
  - *1.5 mm mrad @ 0.8 nC - 92 A*
  - *Gaussian vs. flat top comparison*
  - *Emittance meter - emittance vs.z & double minimum observed*

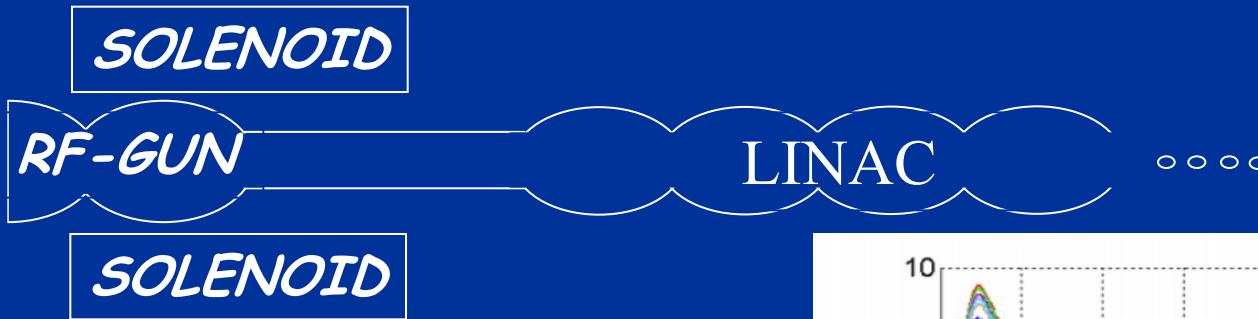


# "Slew" injection



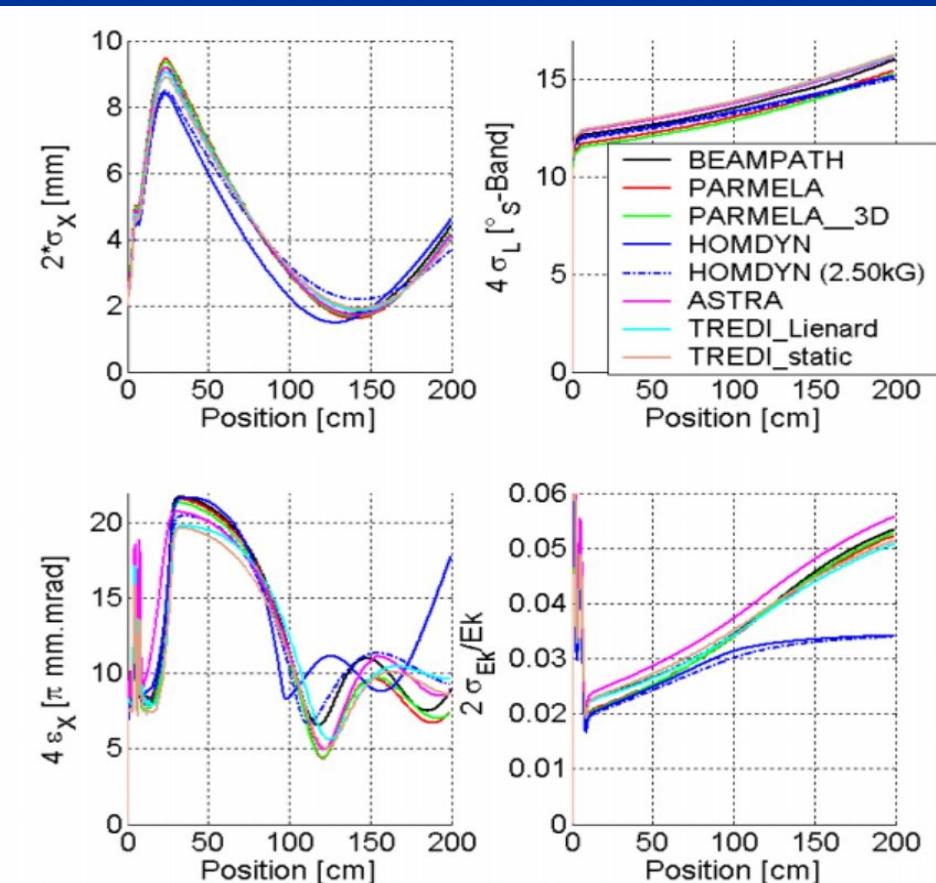
# RF-Gun benchmark

C. Limborg et al. Procs. PAC03

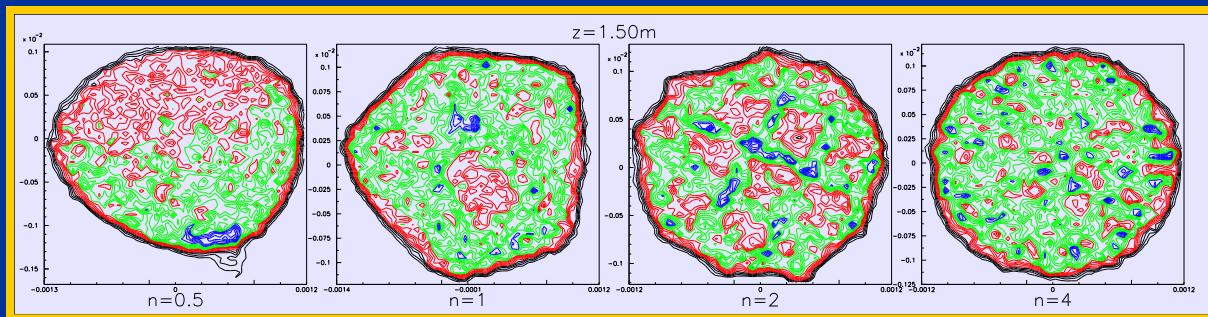
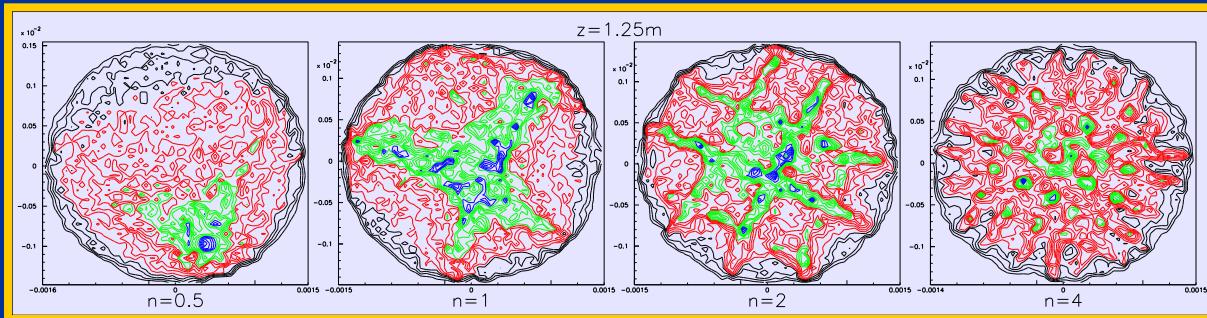
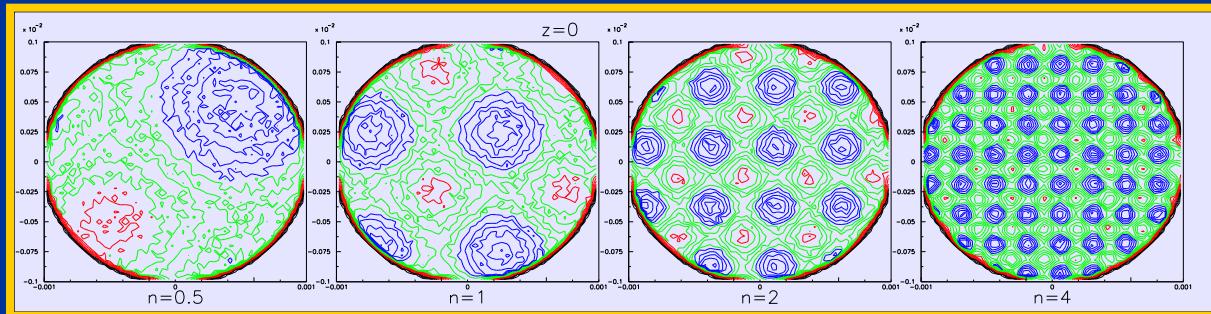


(BNL RfGun+Solenoid+Drift)

<i>Gradient</i>	>100 MV/m
<i>Charge</i>	1nC
<i>Pulse length</i>	10 ps (flat top)
<i>Spot radius</i>	1 mm
$\phi_e$	~32° (center)
<i>Solenoid field</i>	~0.2541 T

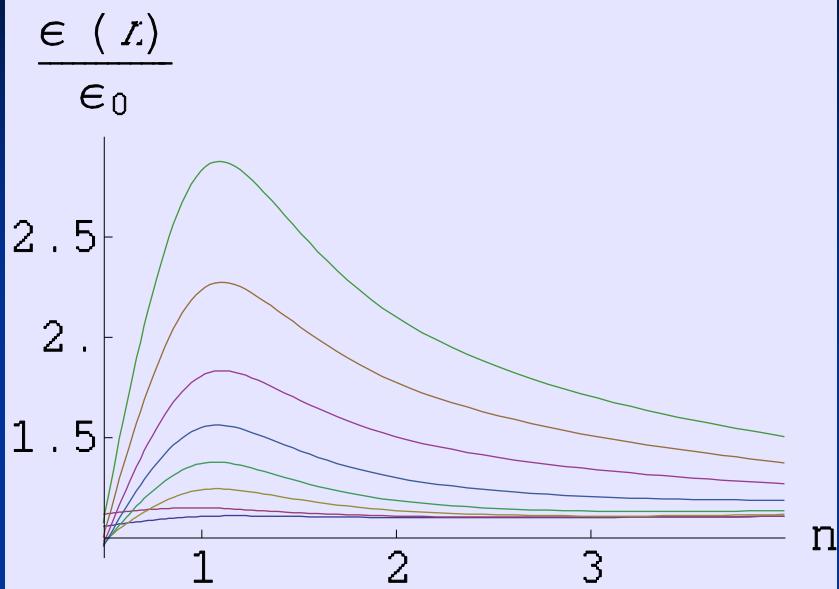


# L. G., M.Q. & C. Ronsivalle - procs. EPAC 04

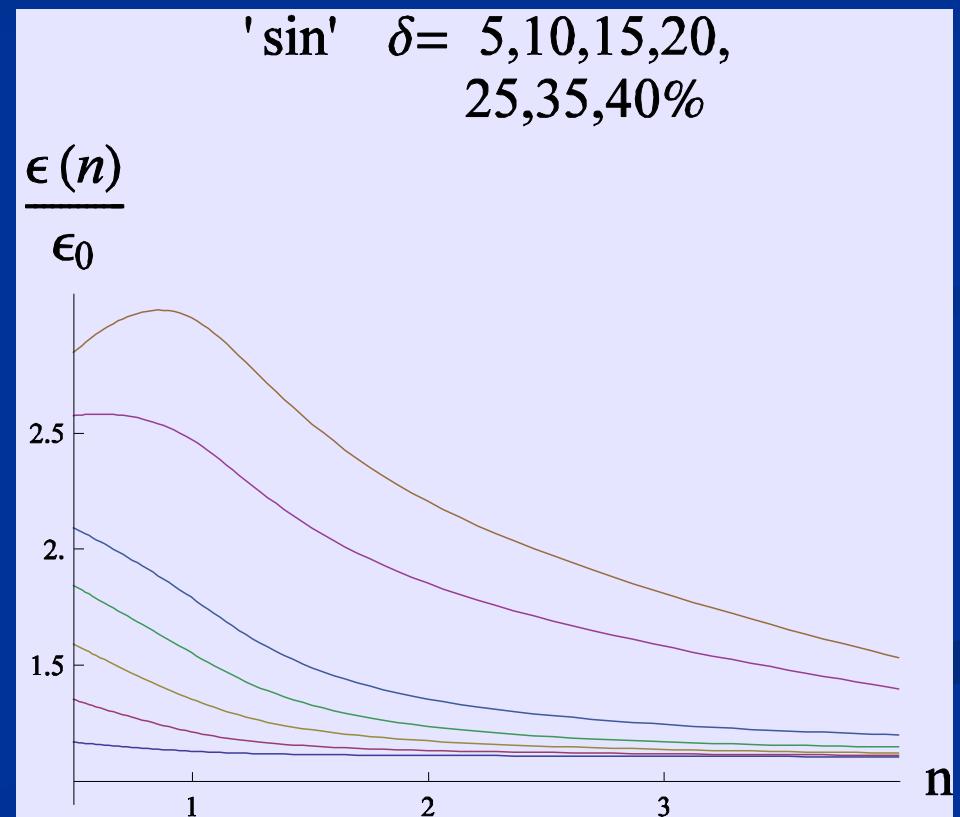


$$\rho(x, y) = \rho_0 [1 + \delta \cdot \sin(k_n x)] [1 + \delta \cdot \sin(k_n y)]$$

'cos'       $\delta = 5, 10, 15, 20,$   
 $25, 30, 35, 40\%$

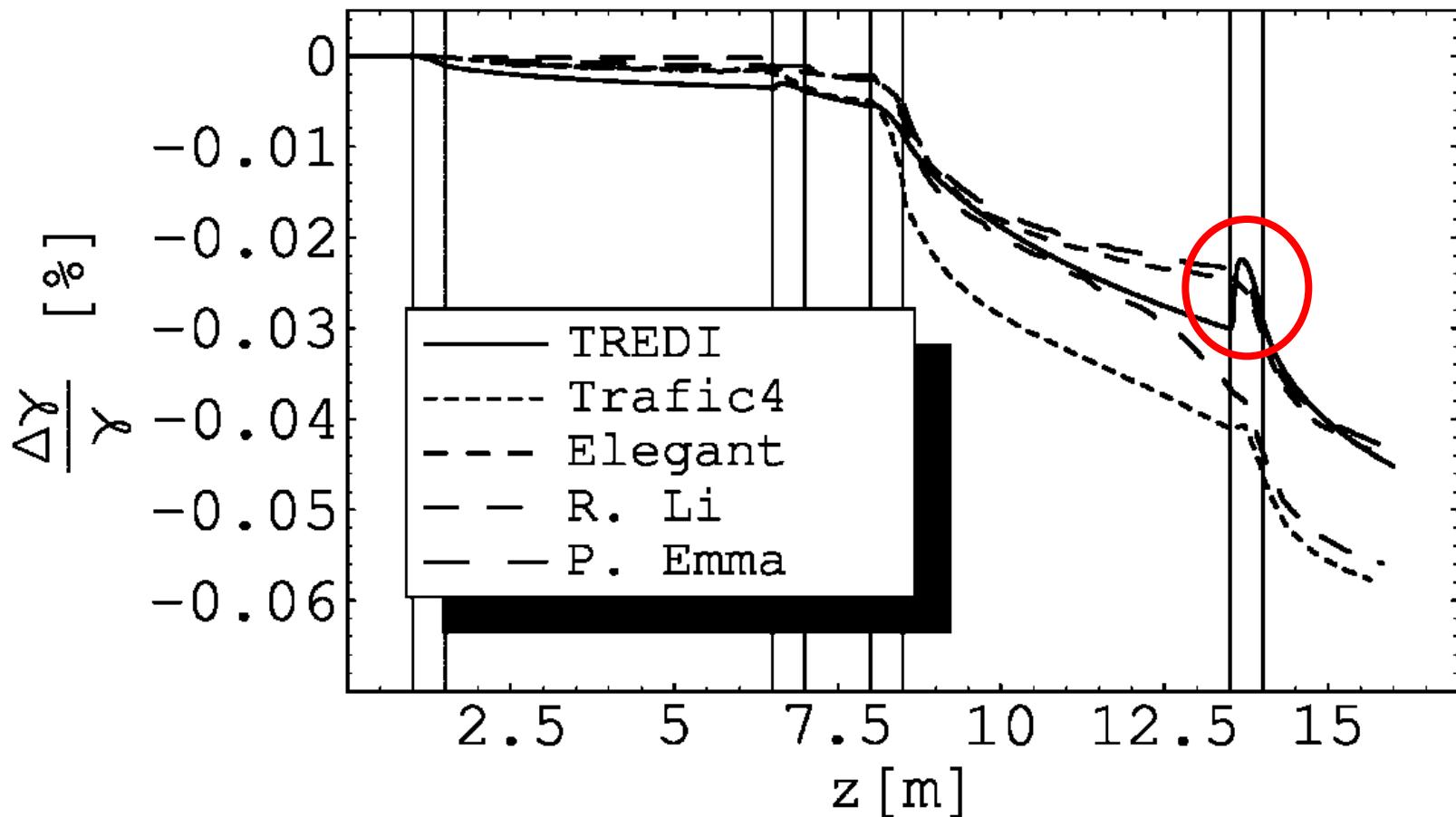


'sin'     $\delta = 5, 10, 15, 20,$   
 $25, 35, 40\%$



# Zeuthen 2002 workshop

Phys. Rev. STAB - Vol. 6, 120101 (2003)



## *Final remark*

- The single most fundamental issue deciding whether a multiparticle TREDI-like tracking code is a viable approach to CSR/ $\mu$ BI is the electrodynamics of extended charges
- →Need of a covariant, physically cogent smoothing mechanism for acceleration fields...

## ■ ...relevant literature:

*A Covariant Formulation of Classical  
Electrodynamics of Charges of Finite Extension*

*J. S. Nodvik, Ann. Of Phys. : 28, 225-319 (1964);*

*Synchrotron Radiation of a charge evenly  
distributed throughout a ball volume* (Non  
manifestly covariant treatment, smoothing in the  
frequency domain)

*R.A. Sedankov and G.I. Flesher, Russ. Phys. J., Vol.  
34, No. 9, Sept. 1991 + refs therein*

*Acknowledgements → C. Vaccarezza for urging us  
to revive TREDI*

*Apologies → C. Vaccarezza for our (apparent)  
laziness to revive TREDI*