RF Compression and Microbunching

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Why bother?

- Velocity bunching represents a possible and interesting complement to magnetic compression (energy chirped beam through a chicane)
  - ++ Avoids CSR effects that may plague magnetic compression
  - - - Enhances SC effects [beam manipulation at low energy]

- Proof of principle of RF compression established. Recent experiments at SPARC suggest that it can be done w/o undermining emittance compensation

- Bunch compression introduces potential for microbunching

- In principle both RF and Magnetic compression could support microbunching
  - The basic ingredients are present in both cases: collective forces + dispersion (i.e. energy dependent time of flight)
  - One could hope that RF compression, done at lower energy, should be more benign (plasma oscillations to wipe out density modulations).

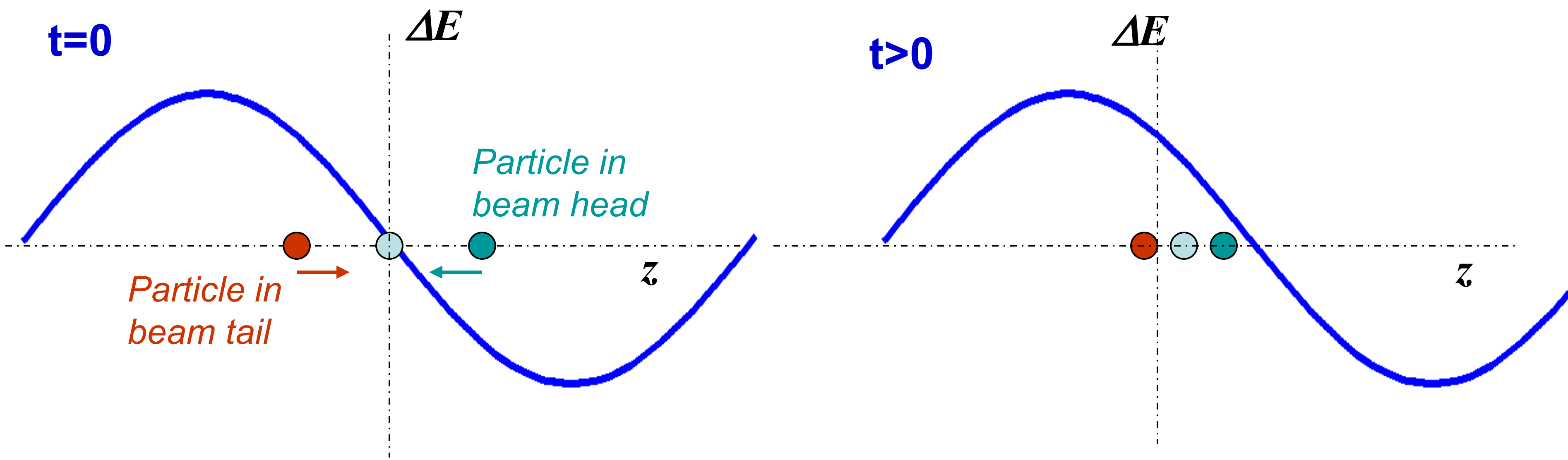
- Approach: develop a linear theory along the lines of the theory developed for microbunching instability through a chicane
  - A number of approximations have to be made, which may be questionable
  - Goal is to have a tool for comparative studies and understanding of basic scaling (if not a tool for extracting accurate absolute answers)

Outline of talk

- Motivations ✓
- Basics on velocity bunching
- Model of dynamics adopted for linear theory
- Validation of model against macroparticle simulations
- RF vs. Magnetic Compression (preliminary)

How RF compression works

- Compression in TW structure (slightly different from compression through RF cavity buncher).
- Exploits slow motion of mildly relativistic electrons in RF bucket and establishment of v_z/z correlation



$$\frac{dt}{ds} = \frac{\gamma}{\sqrt{\gamma^2 - 1}},$$

$$\frac{d\gamma}{ds} = \frac{eE_0}{mc^2}\sin\psi, \quad with \quad \psi = k_{rf}s - \omega_{rf}t + \psi_0$$

(Will assume RF TW has phase velocity c)

RF compression demonstrated experimentally

Piot et al. Phys. Rev. ST Accel. Beams 6, 033503 (2003)

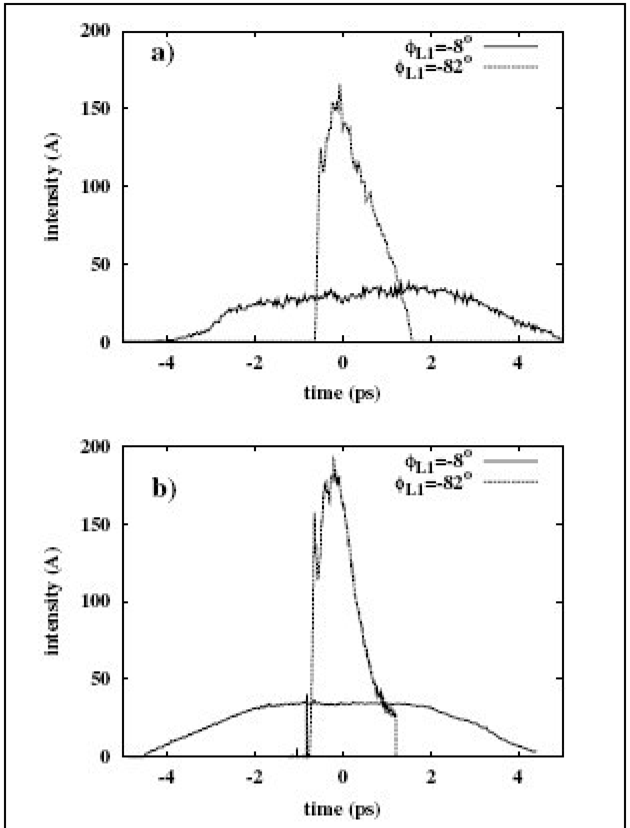
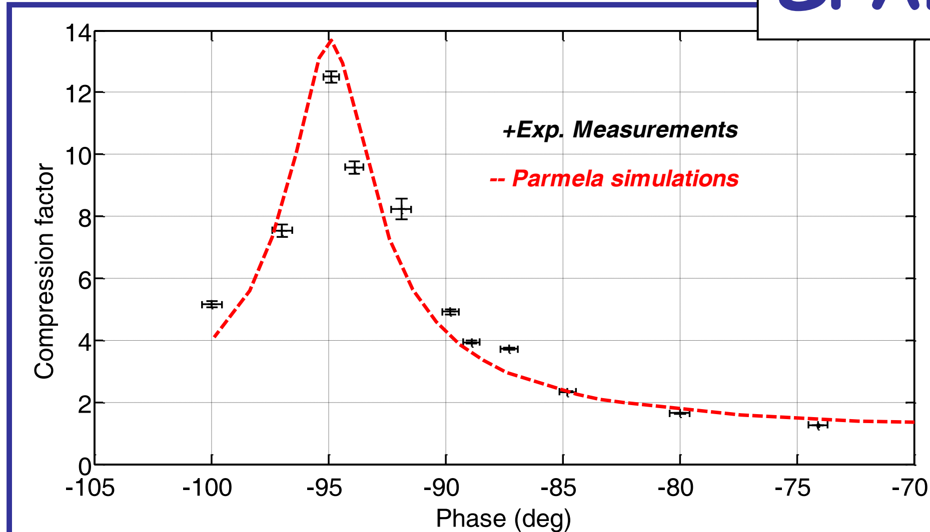
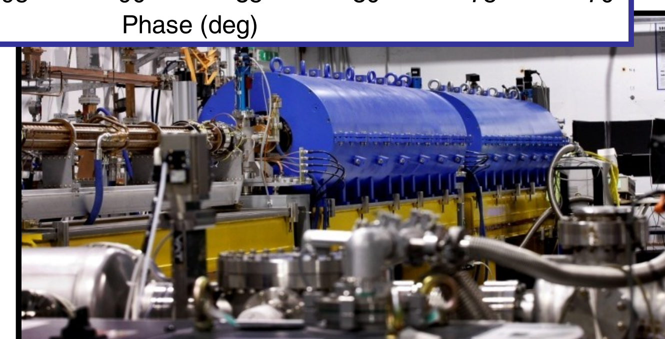


FIG. 6. Comparison of the bunch time profile for L1 −8° and −82° off crest. (a) was generated by tracking simulation; (b) is a direct measurement using the zero-phasing method. The time > 0 corresponds to the bunch tail

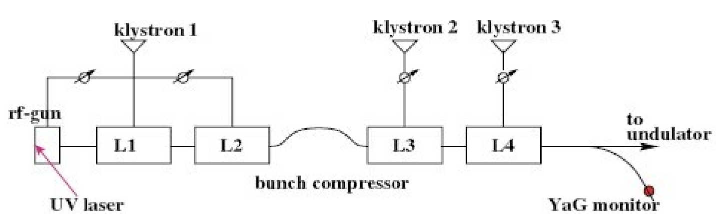
SPARC





Ferrario *et al.*
Phys. Rev. Lett. 104, 054801 (2010)

DUV-FEL



Linear approximation* of single-particle dynamics in RF structure

- Determine orbit for reference orbit by solving:
$$\frac{dt}{ds} = \frac{\gamma}{\sqrt{\gamma^2 - 1}},$$
$$\frac{d\gamma}{ds} = \frac{eE_0}{mc^2}\sin\psi,$$

- Linearize equations in terms of the variable $\boldsymbol{x} = (\Delta z, \Delta\gamma)$ expressing deviation from the reference orbit about the reference orbit. We use position $\Delta z \sim -c\beta\Delta t$ instead of time (and here assume $\beta \sim 1$).

- The transfer map M yielding $\boldsymbol{x}_s = M(s_0 \to s)\boldsymbol{x}_{s_0}$ obeys $$\frac{dM}{ds} = AM$$

with $$A(s) = \begin{pmatrix} 0 & [\gamma_r^2(s) - 1]^{-3/2} \\ -\alpha k_{\rm rf}^2 \cos(k_{\rm rf} s + k_{\rm rf} z_r(s) + \psi_0) & 0 \end{pmatrix}$$

and initial condition $M(s_0) = 1$

**Caveat: In this talk "linear" is used with two different meanings:*
i) Linear with respect to RF dynamics
ii) Linear (in current) with respect to collective effects

In some cases one can write manageable approx. analytical expressions for linear motion

$$M_{11} \simeq \frac{1}{\cosh^2(\hat{k}s/\sqrt{2})},$$

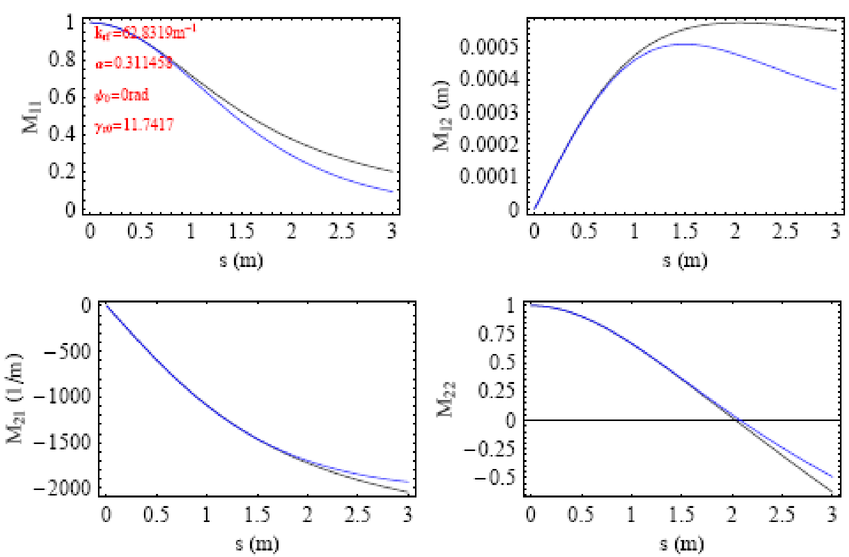
$$M_{12} \simeq \frac{3s}{4\gamma_{r0}^3 \cosh^2(\hat{k}s/\sqrt{2})} + \frac{\tanh(\hat{k}s/\sqrt{2})}{2\sqrt{2}\gamma_{r0}^3\hat{k}},$$

$$M_{21} \simeq \frac{8\hat{k}\gamma_{r0}^3}{(32+9s^2\hat{k}^2)^{1/2}}\left(\frac{B^{2/3}}{2^{5/3}} - \frac{2^{5/3}}{B^{2/3}}\right),$$

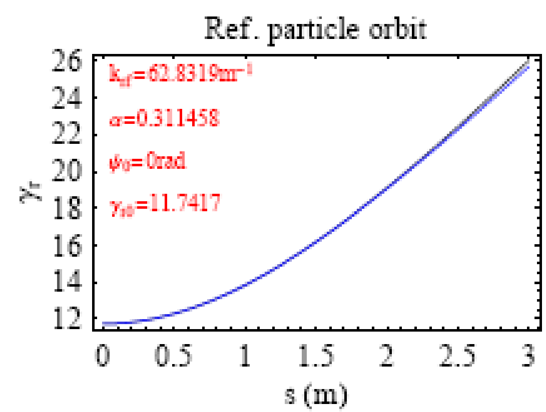
$$M_{22} \simeq \frac{8\sqrt{2}}{(32+9s^2\hat{k}^2)^{1/2}}\left(\frac{B^{1/3}}{2^{5/6}} + \frac{2^{5/6}}{B^{1/3}}\right) - 3.$$

$$B = (32+9s^2\hat{k}^2)^{1/2} -$$

$$\hat{k} = k_{\rm rf}\sqrt{\alpha/\overline{\gamma}^3}$$



- Formulas apply for zero-phase crossing, moderate compression
- For more general case solve for M numerically

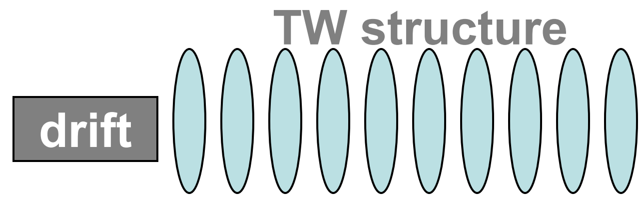


$$\gamma(s) \simeq \overline{\gamma}\left(4A^{-2/3} + A^{2/3} - 3\right)$$

$$A = (\sqrt{32+a^2} - a)/2$$

$$a = 3s\hat{k} - [6(x-1)^{1/2} + (x-1)^{3/2}]_{x=\gamma_{r0}/\overline{\gamma}},$$

Entries of transfer matrix, initial chirp, determine compression factor

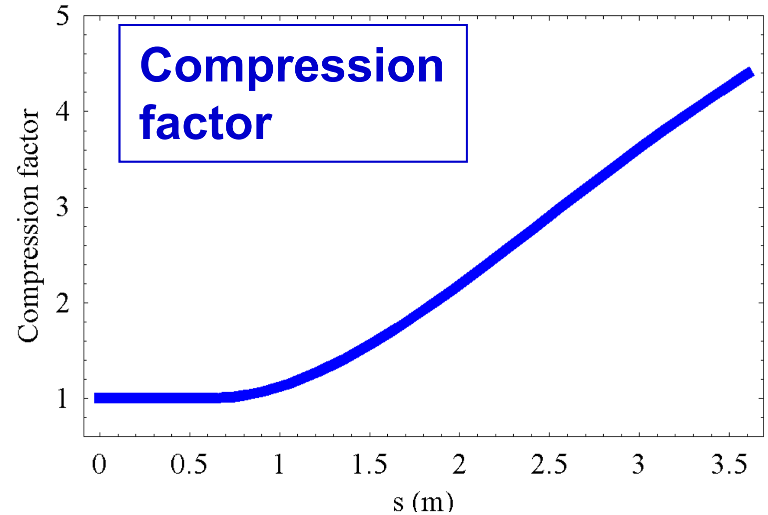


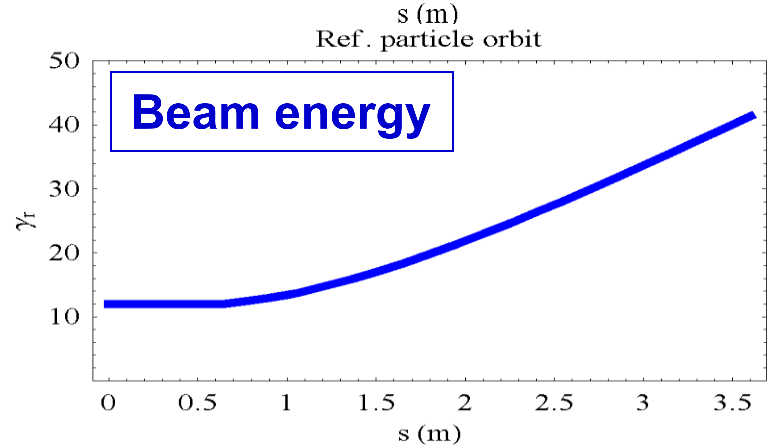
$$C = \frac{1}{M_{11}(s) + hM_{12}(s)}$$

=1 for magnetic chicane

Initial chirp

This is ~R_{56}

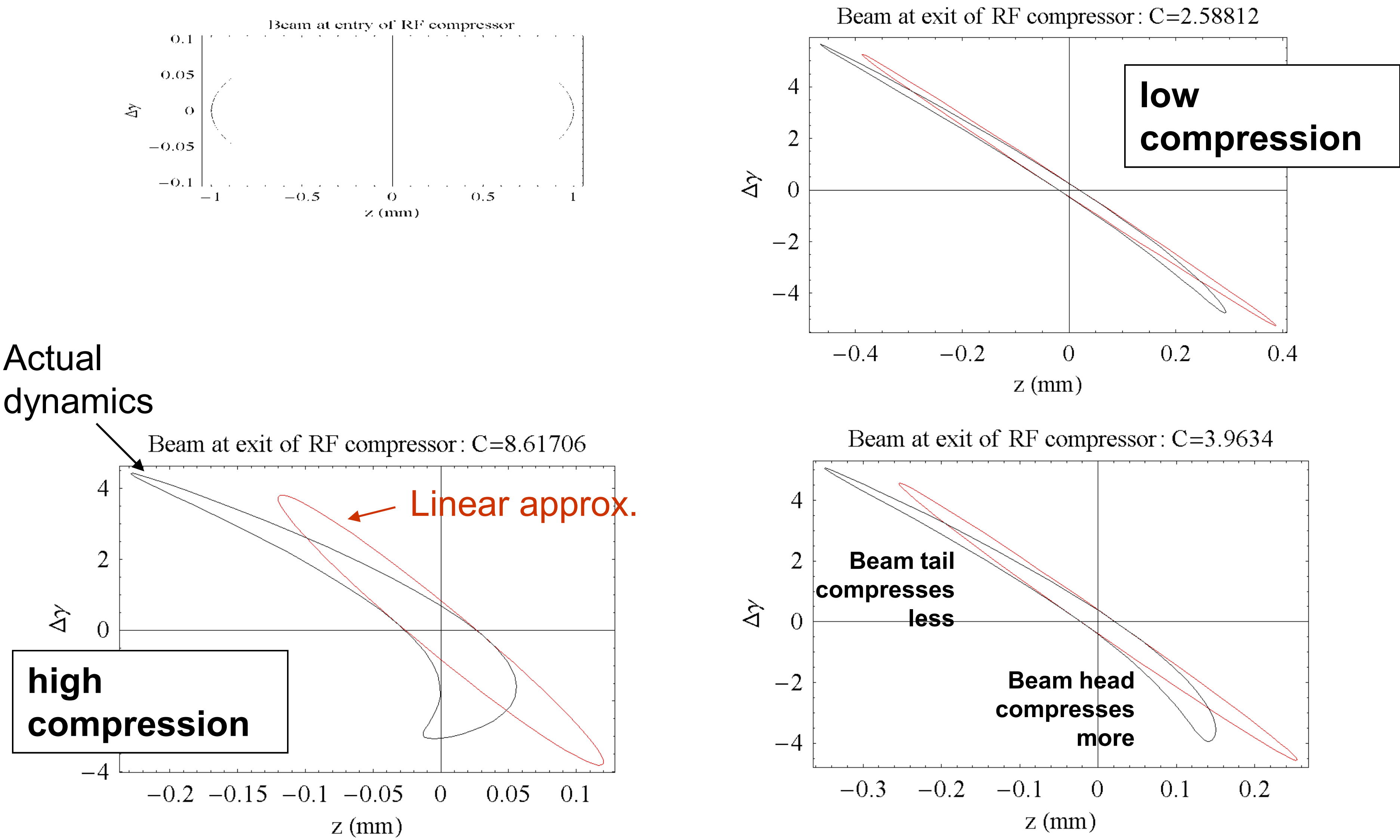




~SPARC parms

f_{rf}	2.8GHz
ψ_0	-2 deg
E_0	5.6MeV
E_z	25 MeV/m

Linear approx. of single-particle dynamics is good only for low/moderate compression



Space charge is treated using 1D LSC impedance model

$$Z_{\mathrm{avg}}(k) = \frac{iZ_0}{\pi\gamma r_b}\frac{1-2I_1(\xi)K_1(\xi)}{\xi}\bigg|_{\xi=kr_b/\gamma}.$$

- For the space-charge impedance assume 1D model obtained from taking average of E_z electric field generated by beam with circular cross-section of radius r_b and uniform transverse density.
  - Somewhat better than taking Z corresponding to the on-axis electric field of same beam density
  - To some degree model can be used to represent the impedance from beam with a more general transverse density profile by adjusting r_b to some effective value.

- Cons: We lose 3D effects that at low energy can be important (particularly at short wavelengths).
  - Model gets better as beam is accelerated along structure
  - Comparison with macroparticle simulations (see later slides) ~OK.
  - Model could be improved by making assumption of laminar beam and retaining *r*-dependence of E_z field (see J. Wu *et al*., PRST-AB **11** 040701, 2008).
- Pros: Reduced dimensionality makes model more handy.

Linear theory for gain function

- We are interested in determining the gain function initial density perturbations with wavelength much smaller than bunch length
- Adopt the coasting beam approximation (gauss energy distribution) for unperturbed beam density
  - Neglect space-charge induced chirping
  - A theory applicable to bunched beam could be developed
- 2D Beam density in phase space in the form $f=f_0+f_1$
- Assume initial perturbation consisting of a sinusoidal density perturbation with wavenumber k_0

$$f_1(x_0; s_0) = Ae^{ik_0 z_0} \frac{e^{-(p_0 - hz_0)^2/2\sigma_p^2}}{\sqrt{2\pi}\sigma_p} + c.c.,$$

- Collective effects expressed in terms of Impedance

$$\frac{dp}{ds} \equiv F(\tilde{\rho}, z_s; s) = -\frac{e^2 n_0}{mc} \int_{-\infty}^{\infty} dk e^{ikz_s} Z(k;s)\tilde{\rho}(k;s)$$

- Solve the linearized Vlasov equation. Determine the instability gainfunction *i.e.* ratio between *relative* amplitudes of density perturbation at exit and entry of RF compresson

Expression of gain obeys integral equation formally identical to that of magnetic compressor

- FT of perturbation at s>0 $\tilde{\rho}_1(k;s) = b(k;s)\delta(k_0 - \frac{k}{C(s)})$
- Integral Eq. for bunching function

$$b(C(s)k_0; s) = Ae^{-[Ck_0 M_{12}\sigma_p]^2/2} + \int_{s_0}^{s} ds' K(s', s) b(C(s')k_0; s')$$

- Kernel of integral equation

$$K(s', s) = 4\pi i \frac{I(s')}{I_A} C(s) k_0 M_{12}(s' \to s) \frac{1}{Z_0} Z\left(C(s')k_0, s'\right) \exp\left(-\frac{\sigma_p^2 k_0^2}{2}[C(s')M_{12}(s') - C(s)M_{12}(s)]^2\right)$$

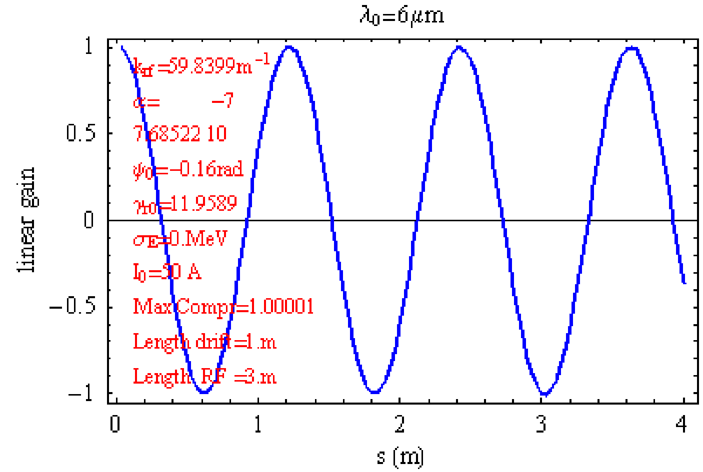
- In comparison with bunching in chicane:
  - Longitudinal dynamics is decoupled from transverse (1D theory suffices); a priori knowledge of transverse beam size along s is assumed (for determining impedance)
  - Compression factor has slightly different analytical form

$$C_{RF}(s) = \frac{1}{M_{11}(s) + hM_{12}(s)} \quad vs. \quad C_{BC}(s) = \frac{1}{1 + hR_{56}(s)}$$

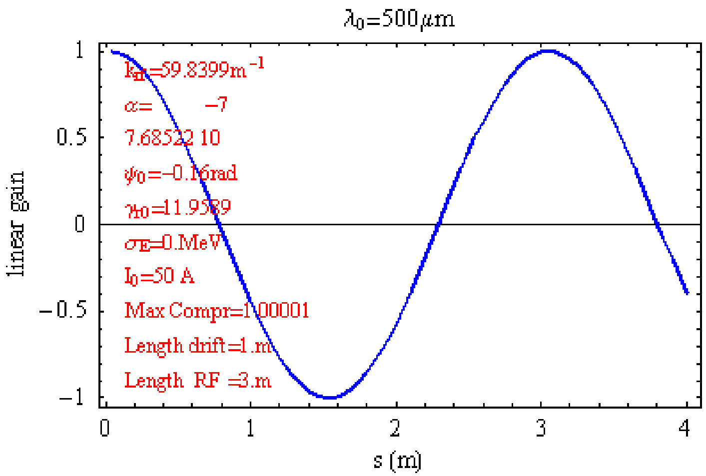
In the absence of RF forces we recover dynamics of long. plasma oscillations

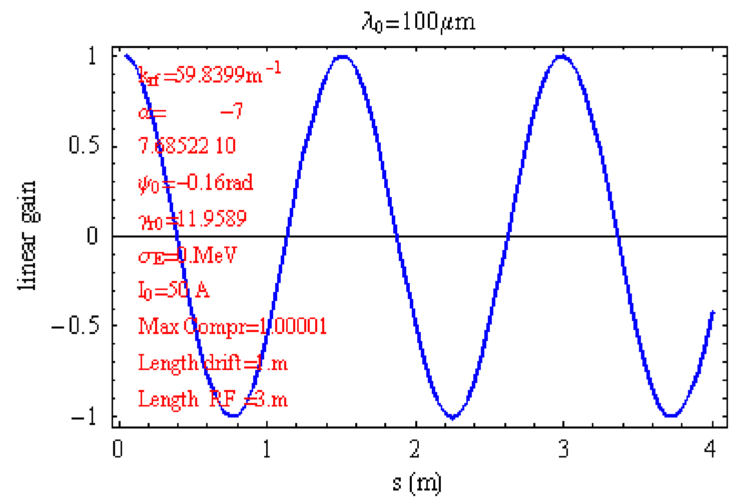
- Amplitude of plasma oscillations for cold beams

$$b(s) = A\cos\left(s\sqrt{4\pi k \frac{I}{\gamma^3 I_A}\frac{Z(k)}{Z_0}} \right)$$



E=5.6MeV, I=50 A



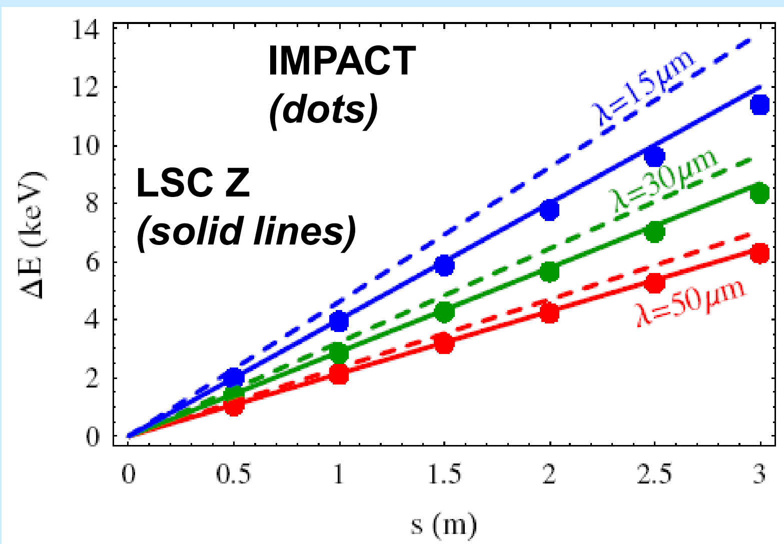


Aside: gain along a magnetic bunch compressor; Theory vs. Macroparticle simulations

- Application of linear theory to microbunching through magnetic compressors has generally been satisfactory

Check impedance model:

Evolution of energy modulation Seeded by sinusoidal charge mod

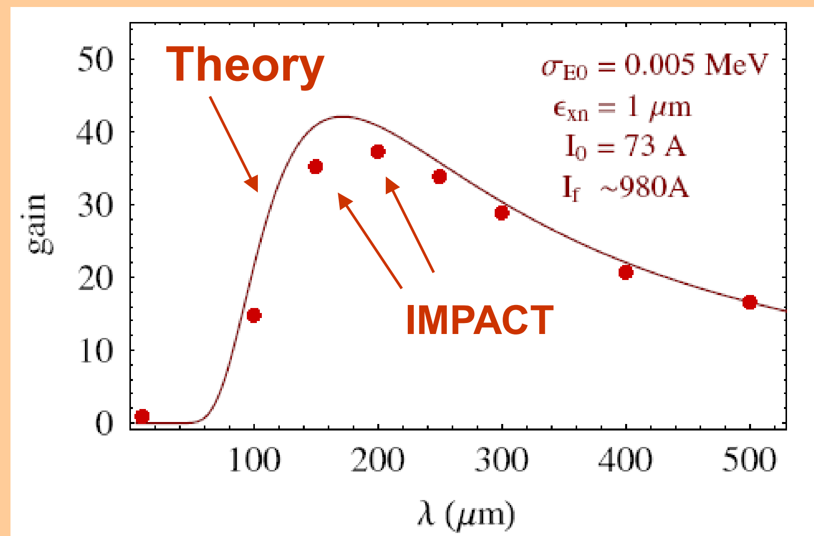


$$Z_{\text{avg}}(k) = \frac{iZ_0}{\pi\gamma r_b}\frac{1-2I_1(\xi)K_1(\xi)}{\xi}\bigg|_{\xi=kr_b/\gamma}$$

Cold beam with circular cross-section along drift

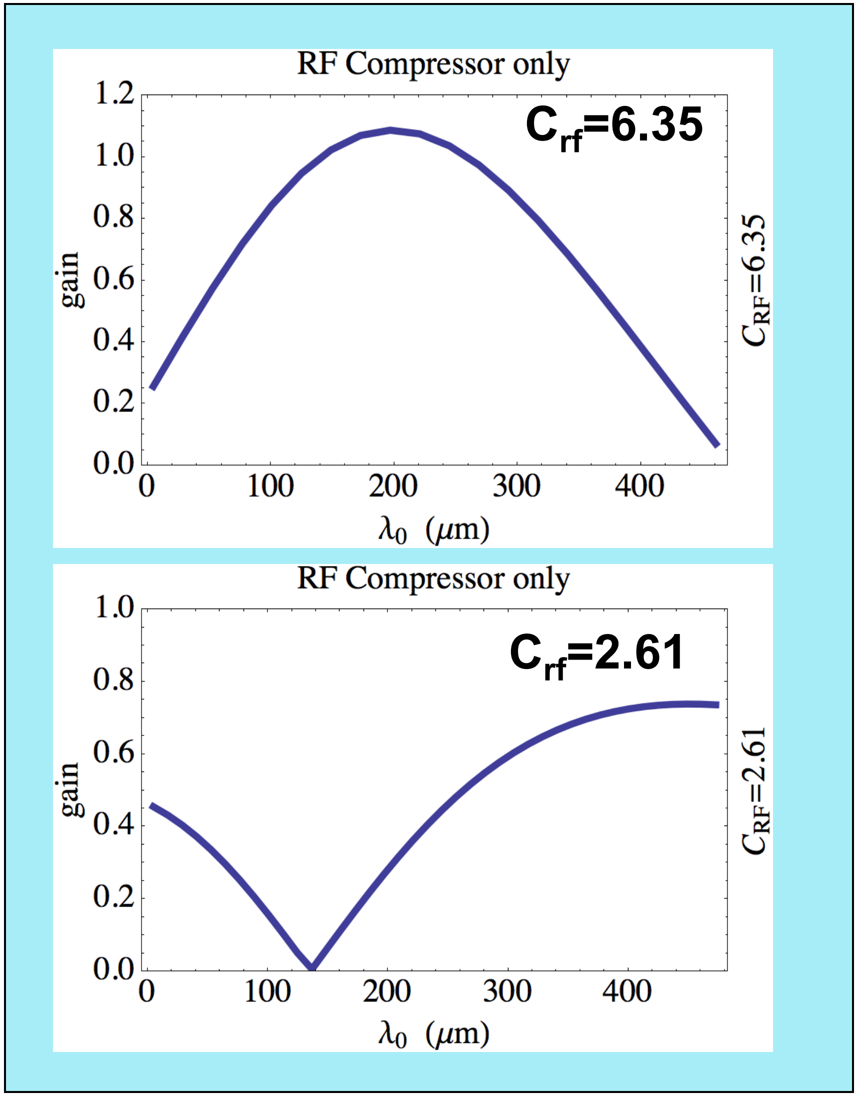
J. Qiang et al., PRST-AB 12 100702 (2009)

Gain function through L1 +chicane (LSC + CSR)



39m+11m section
E=40 to 240 MeV
R_{56}=12.3cm
I_0=73A (initial peak current)
Compression ~ 14

Examples of gain curve through RF compressor



f_{rf}	2.8GHz
I_0	50 A
E_0	5.6MeV
E_z	25 MeV/m

0.6 m drift
+3 m structure

gain= (initial rel. amplitude)/(initial rel. amplitude)

Validating 1D theory against macroparticle Simulations

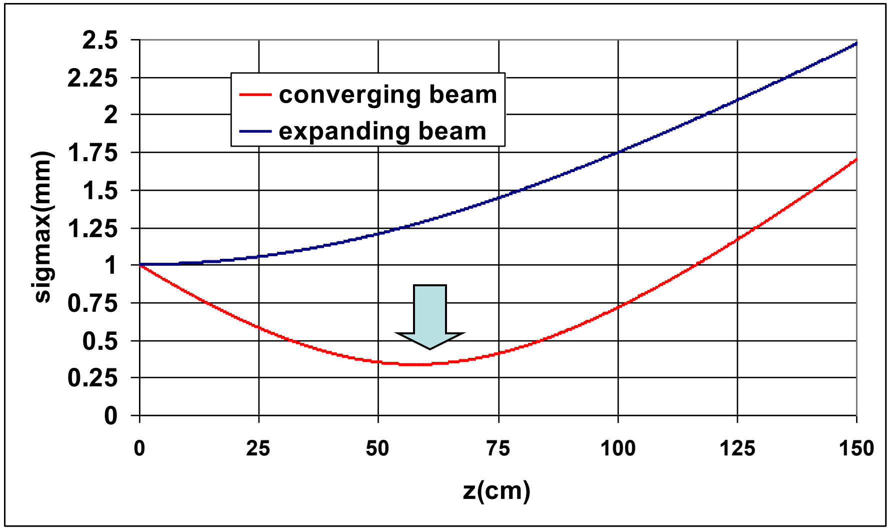
NUMERICAL MODEL: the beam

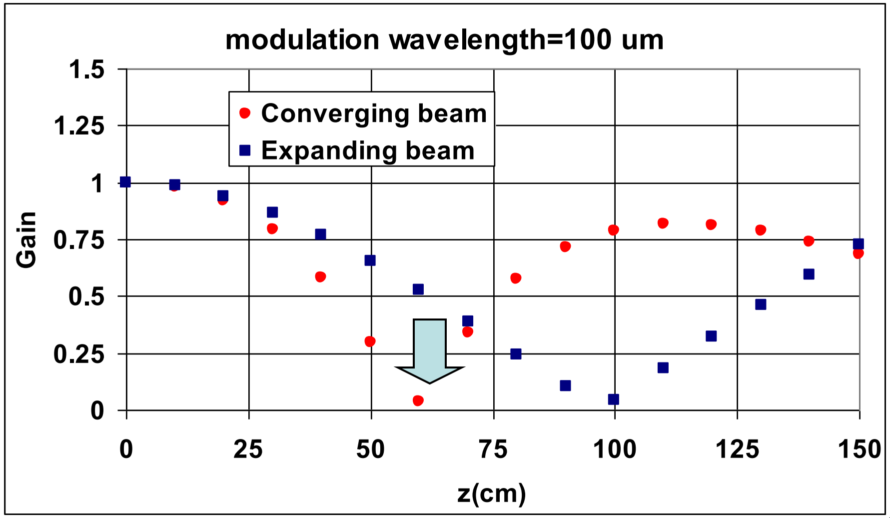
- Used code: TSTEP (a derivative of PARMELA code), a serial code limited to 10M of max. number of macro-particles
- Beam parameters: Q=1 nC, flat temporal distribution (10° at 2856 MHz), uniform transverse distribution,Energy=5.6 MeV,no energy spread
- Range of initial modulation wavelengths λ_m: 50-300 μm
- Initial modulation amplitude: 10%
- Max. number of macroparticles used in simulations: 4,5M
- Radial mesh automatical adjusted by the code, longitudinal mesh length Δzsc=5 mm, Nz=1200 or 2400 depending on the bunch compression
- Computation of microbunching gain on beam core to minimize edge effects

Macroparticle simulations

NUMERICAL MODEL: the beamline

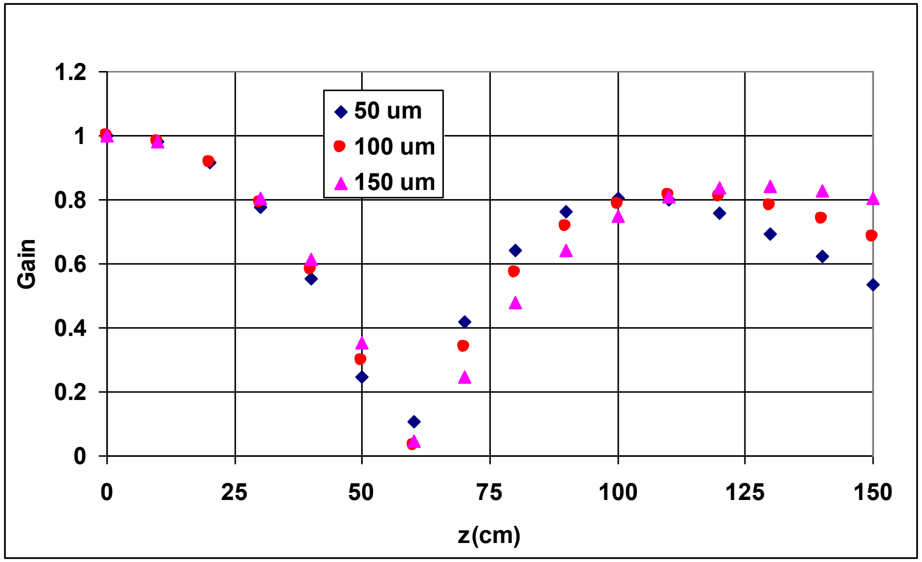
- Beam line (RF gun not included): drift+RF compressor (3m TW linac, E=25 MV/m, no emittance compensation solenoid)
- Choice of the length of the leading drift: equal to the distance in which the first minimum of density modulation occours with a complete transfer from density to energy modulation.

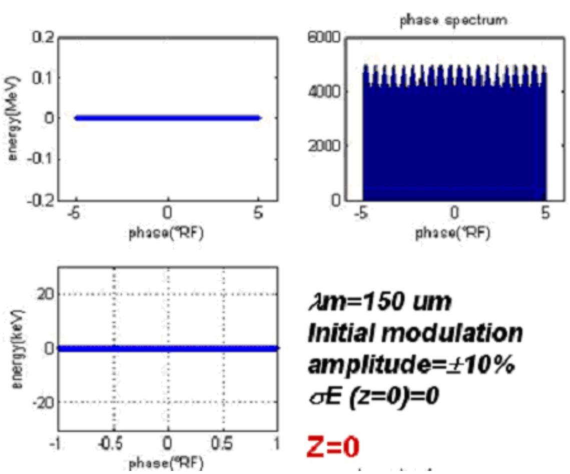




Macroparticle simulations

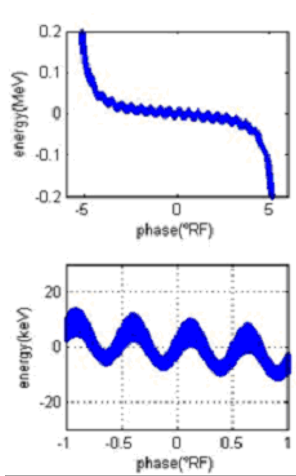
COMPUTED MICROBUNCHING GAIN vs. Z IN THE DRIFT





λm=150 um
Initial modulation amplitude=±10%
σE (z=0)=0

Z=0

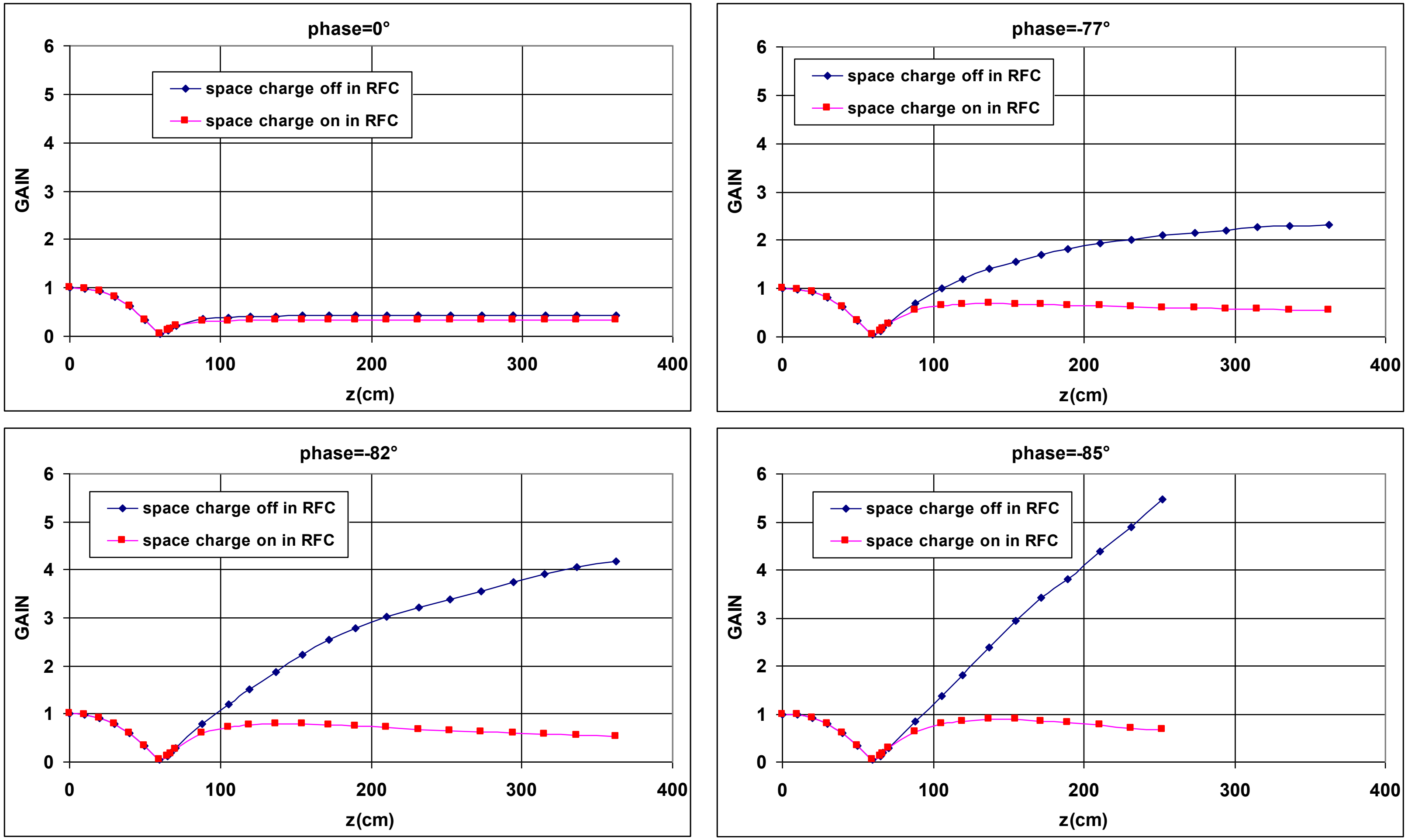


λm=150 um
Initial modulation amplitude=±10%
σE (z=0)=0

Z=60 cm

Macroparticle simulations

MICROBUNCHING GAIN VS Z IN THE DRIFT+RFC LINE: SPACE CHARGE EFFECT (GAIN DAMPING)



Macroparticle simulations: comparison with the theoretical model

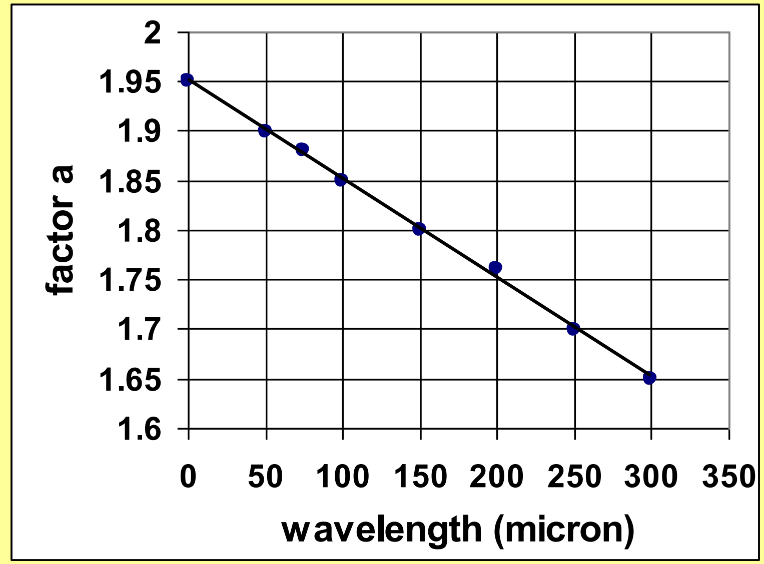
The computed radius variation has been incorporated in the theoretical model as

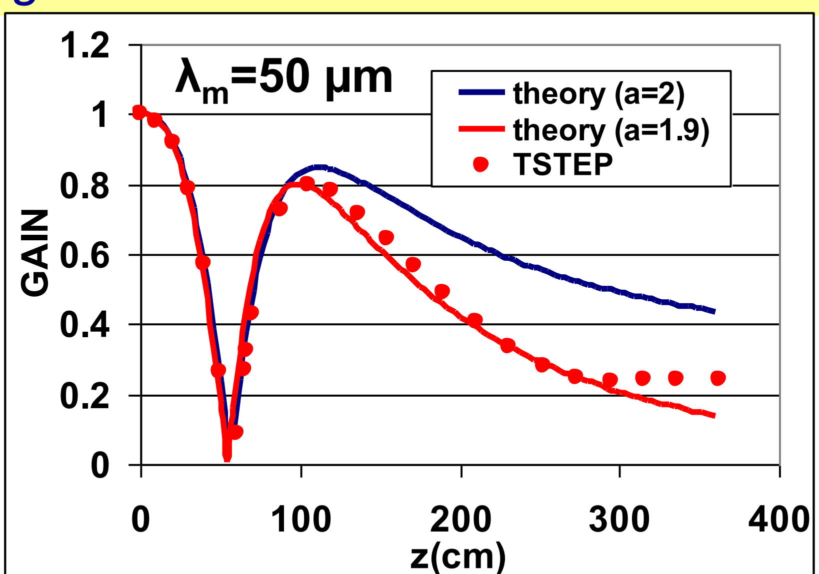
$r_b(z)=a\cdot\sigma_x(z)$

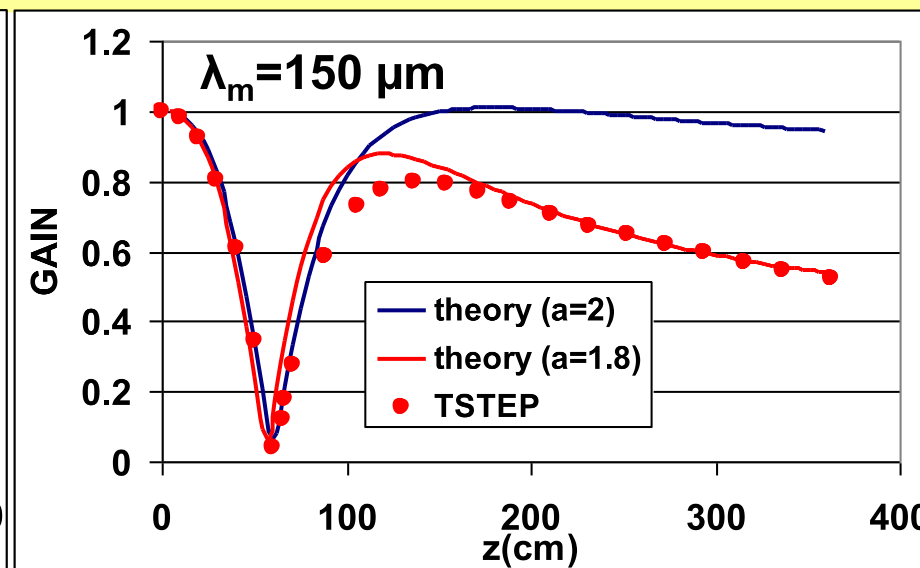
with a=1.95-0.001·λ(μm) and

σx=computed rms envelope

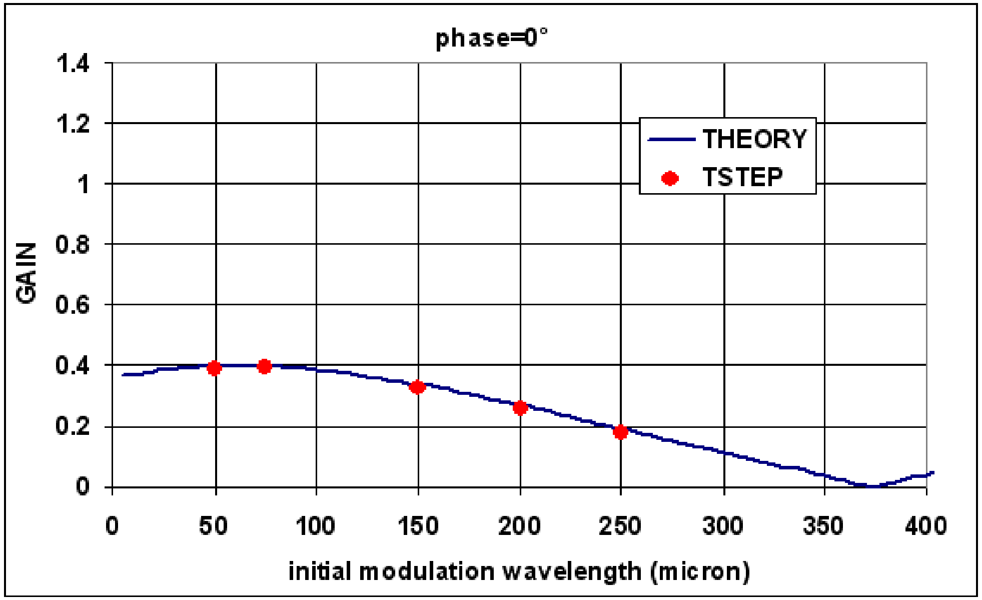
This condition gives the best agreement model-simulations

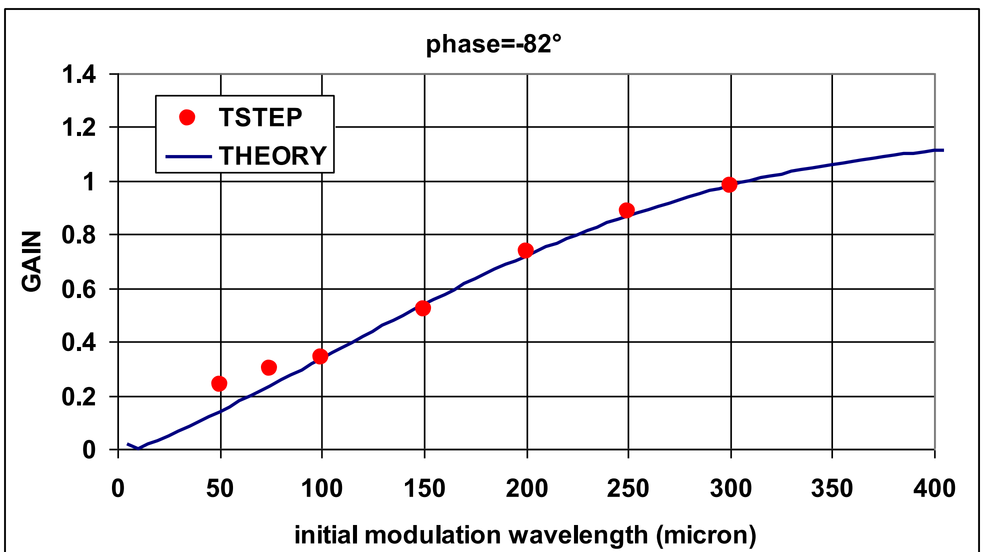






Macroparticle simulations: comparison with the theoretical model

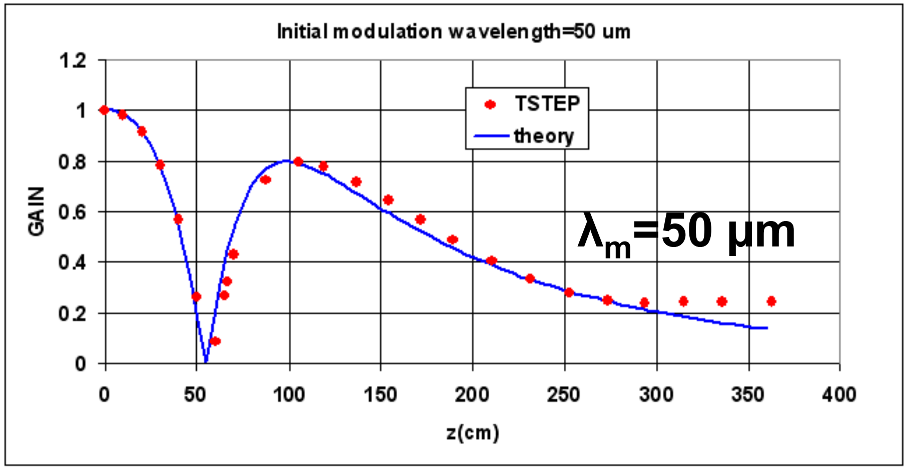


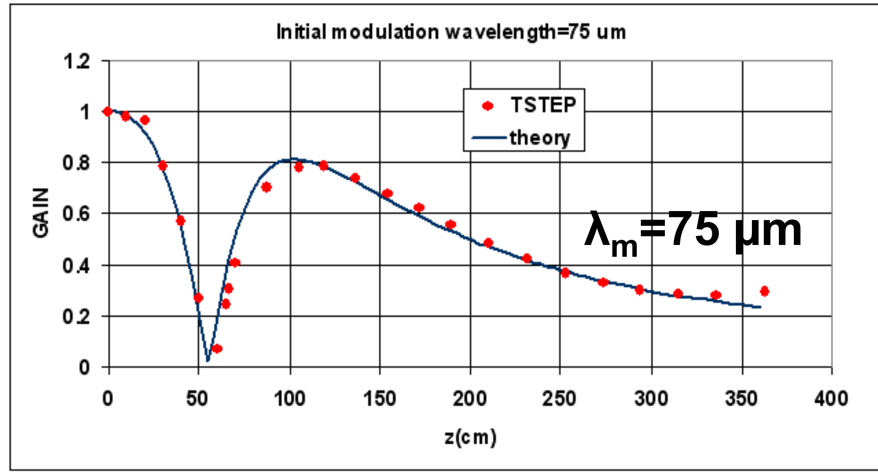


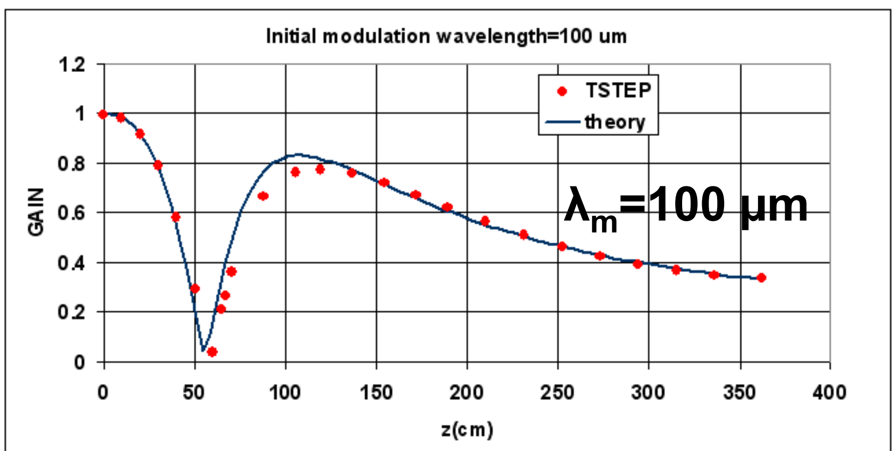
GAIN VS INITIAL MODULATION WAVELENGTH

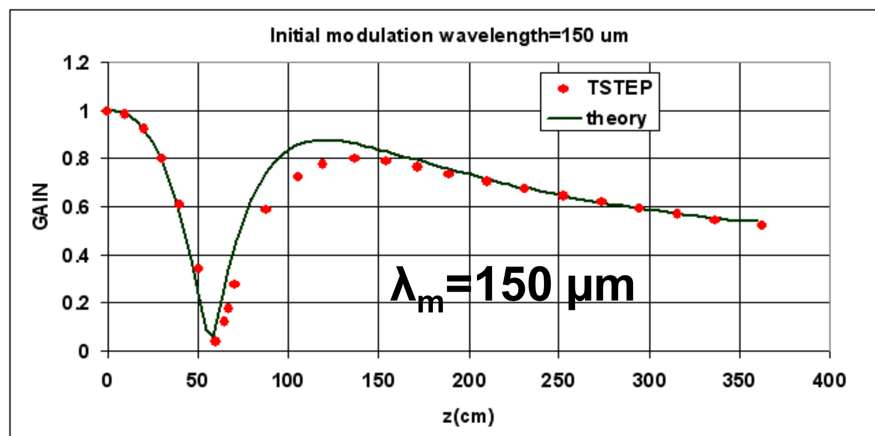
Macroparticle simulations: comparison with the theoretical model

GAIN VS Z FOR DIFFERENT INITIAL MODULATION WAVELENGTHS AND φ(RFC)=-82° (bunch compression~2)



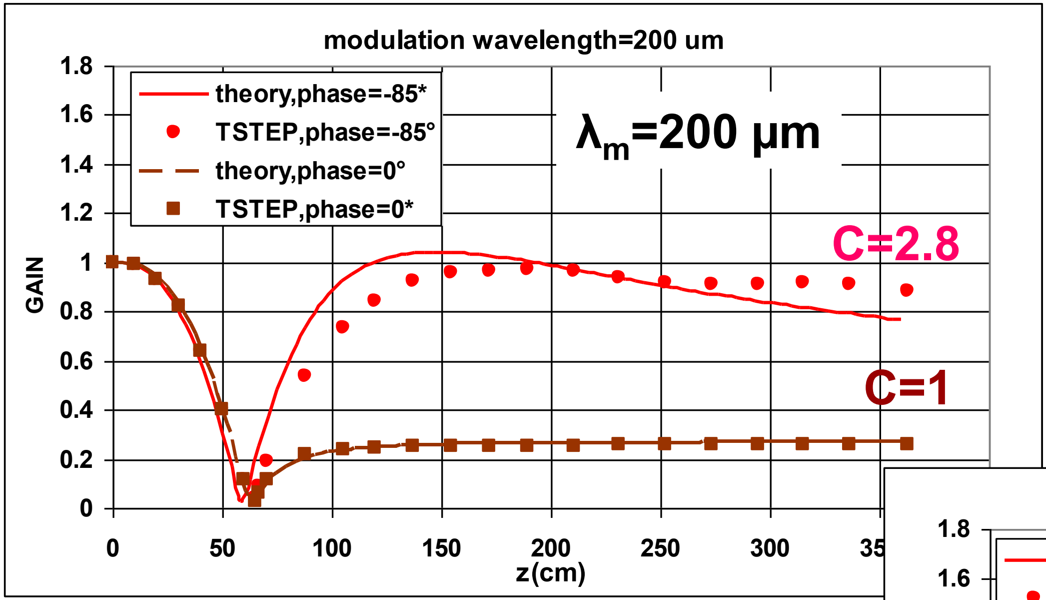


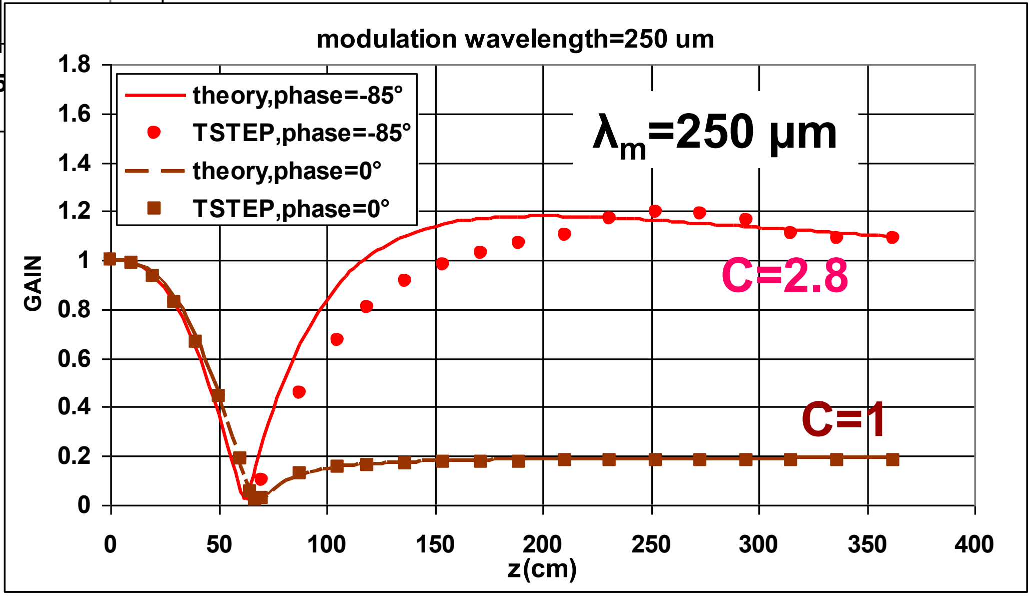




Macroparticle simulations: comparison with the theoretical model

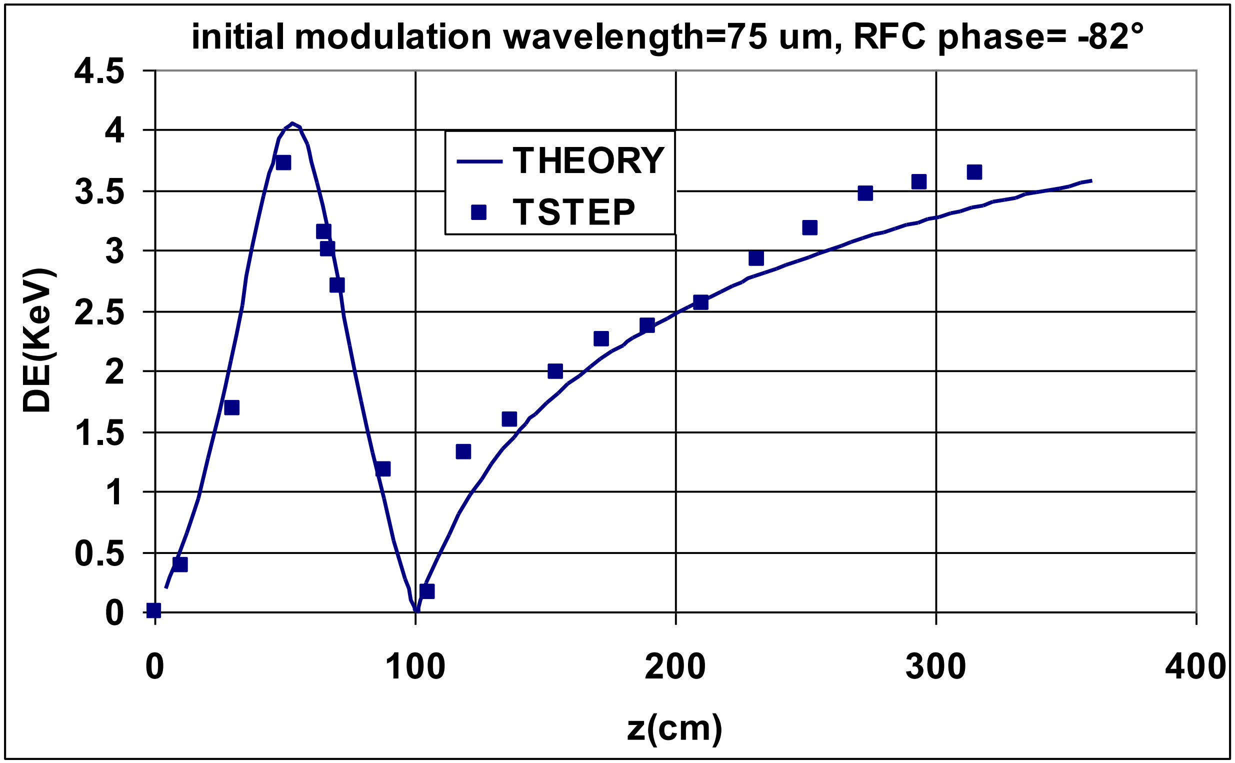
GAIN VS Z FOR TWO DIFFERENT LEVELS OF COMPRESSION



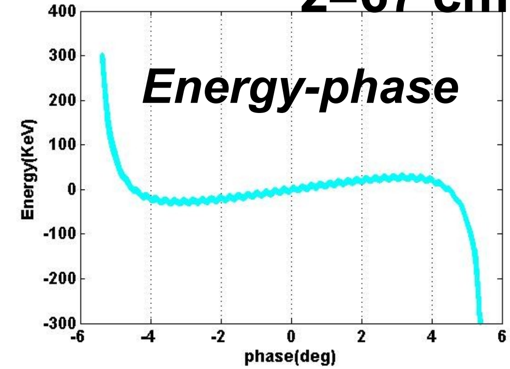


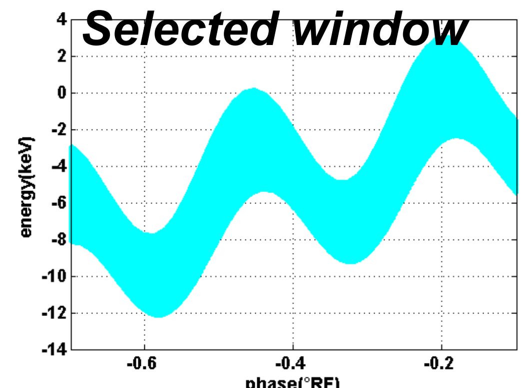
Macroparticle simulations: comparison with the theoretical model

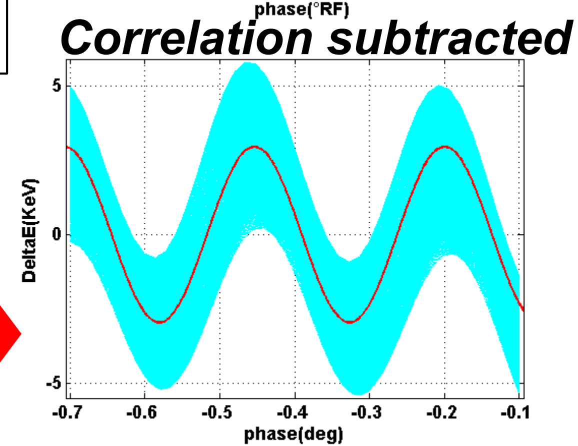
ENERGY MODULATION



z=67 cm



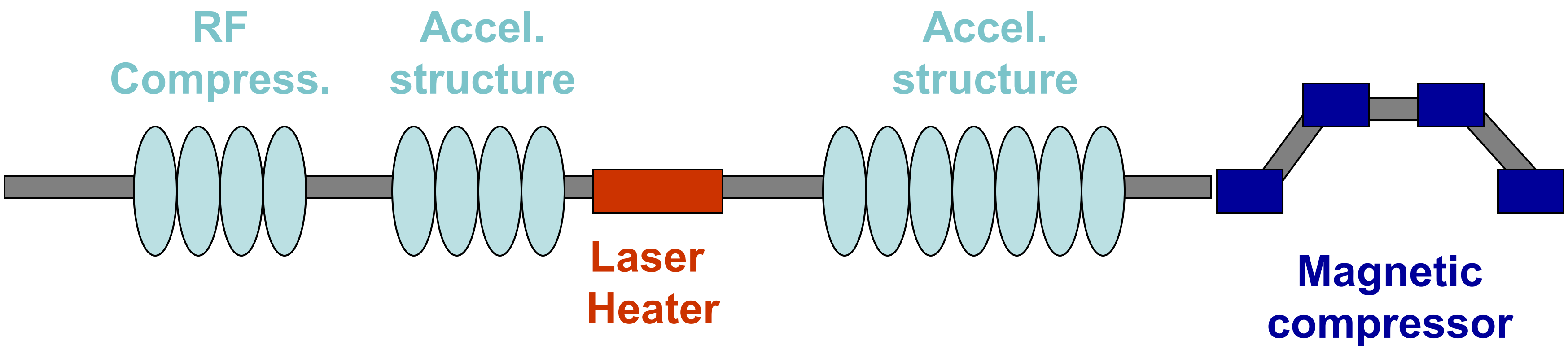




Main difference respect to the theoretical model: the growth of the uncorrelated energy spread

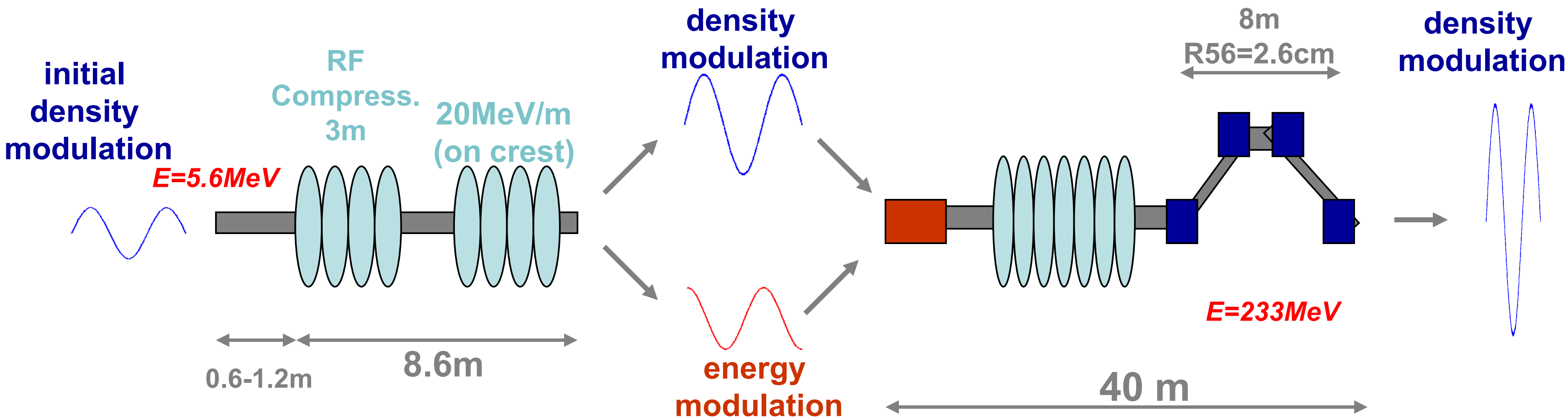
What is compared is the amplitude of the red curve (sinusoidal fit)

RF compression at low energy followed by magnetic compression



- In a conceivable practical scenario RF compression is to be supplemented by magnetic compression (one, perhaps more chicanes)
- Is there an optimal may to partition compression between RF and magnetic compressors in order to minimize the gain for the microbunching instability?

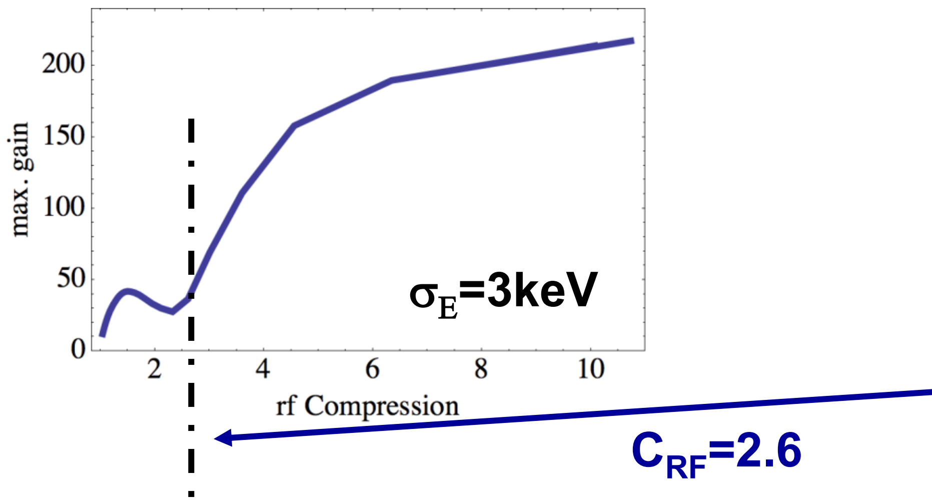
Model for analysis

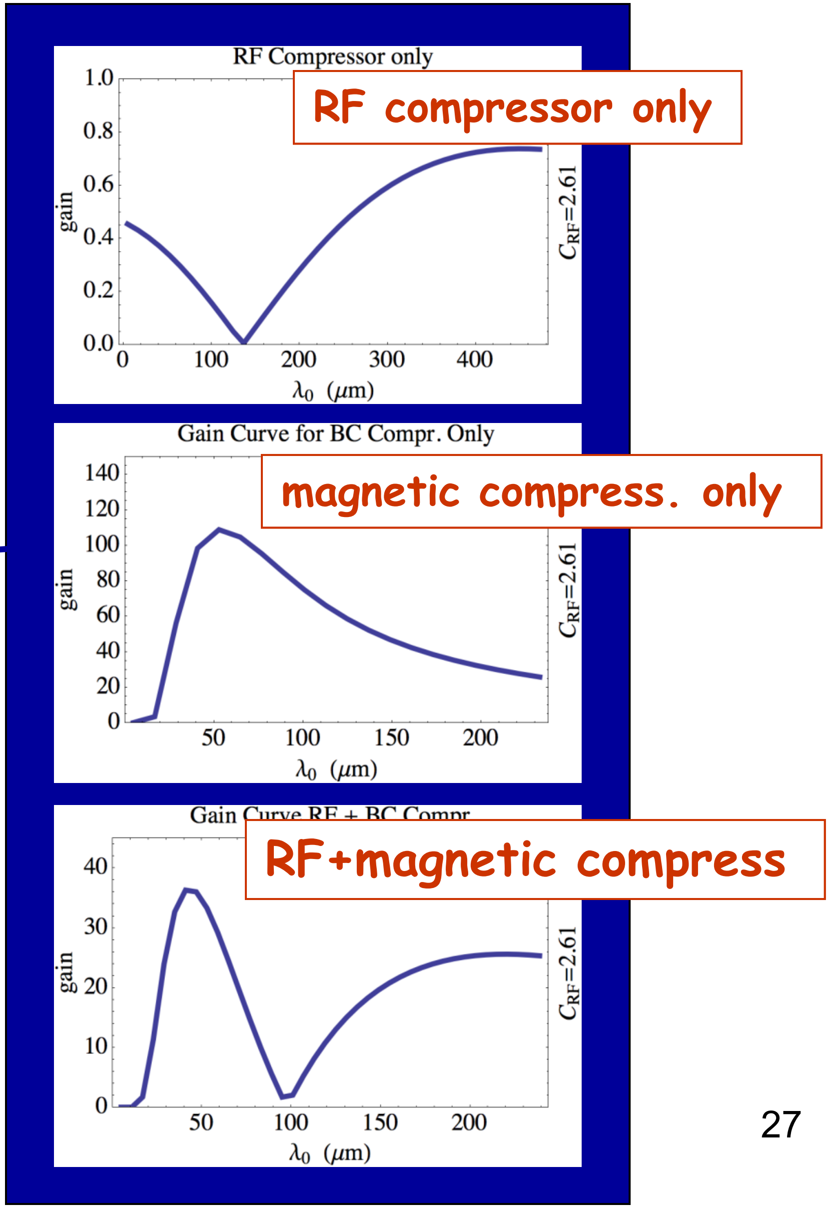


- Assume initial cold beam. RF compression done in first RF structure (3m). Second RF structure operated on crest. Third SR structure accelerates to 233MeV
- Laser heater introduces finite energy spread
- Fixed chicane with R_{56}=2.6cm (inspired to FERMI first BC); $\varepsilon_x=10^{-6}$ m
- Assume uniform transverse beam size throughout
- Fix initial peak current (50A) and overall compression $C=C_{RF}C_{BC}=22$.
- Vary RF compression factor (adjust chirp “by hand” to obtain $C_{BC}=C/C_{RF}$), monitor overall gain function (and its maximum)

Large RF compression generally boosts microbunching

Peak of overall gain function vs. RF compression factor

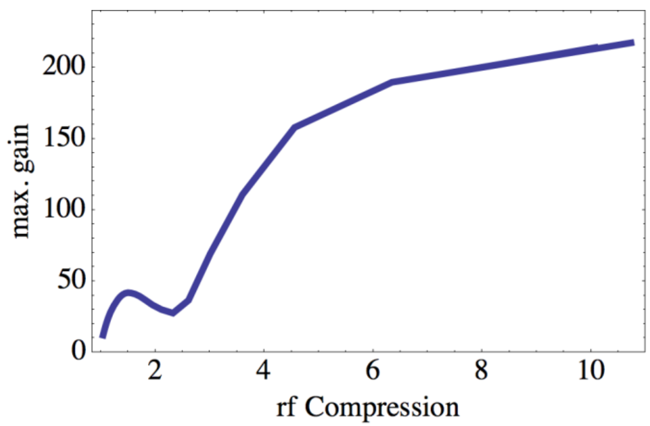




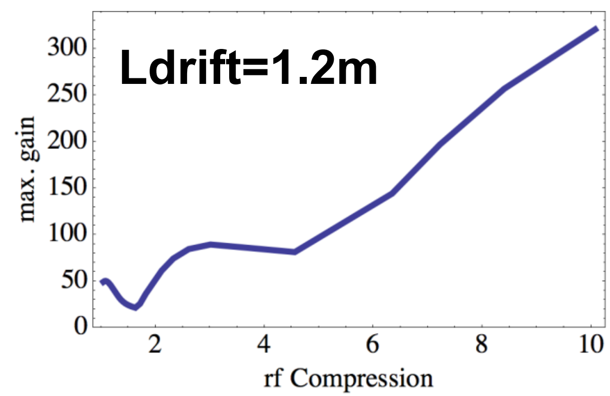
- Local minimum appears for moderate C_{RF}
- Minumum (zero) of gain through RF compressor ~overlaps to peak of gain for magnetic compressor.
- Profile of gain function through RF compressor depends on length of trailing dirft (next slide)

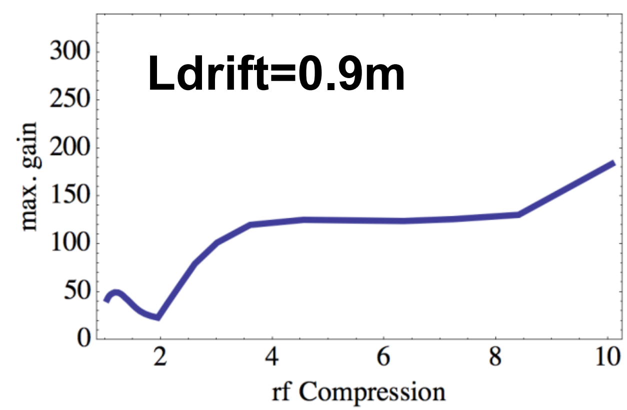
Overall gain sensitive to drift trailing RF compressor

- Initial perturbations considered for this study have all the same phase (purely density perturbations).
- Phase of perturbation at entrance of RF compressor is affected by length of trailing drift



Ldrift=0.6m





Conclusions

- As velocity bunching is being seriously considered for beam compression it is worhtwhile to investigate how it may affect the evolution of small beam perturbations.

- We have presented a "double-linear" theory for the evolution of small sinusoidal perturbations
  - Linear in terms of rf forces (limits applicability to low/moredate compression)
  - Linear in terms of amplitude of perturbations

- Comparison against macroparticle simulations not unsatisfactory. However:
  - It was limited to relatively long wavelengths, low compression
  - Good agreement obtained by adjusting empirically the ratio betwen parameter r_b appearing in theory and actual transverse rms sizes from simulations (hint that 3D effects are creeping in).
  - Model misses development of z/r correlated energy spread

- We used the theory to analyze gain through model of RF compressor + magnetic compressor with constant overall compression factor. Tentative (and preliminary) results:
  - Excessive RF compression is unfavorable
  - Low RF compression may not significantly enhance instability
  - Outcome sensitive to details of initial conditions (phase) of noise

Energy at exit of RF compressor vs. RF compression factor