

Electron-Beam Noise and Spontaneous Emission Suppression and the Fundamental Coherence Limits of Free Electron Radiators

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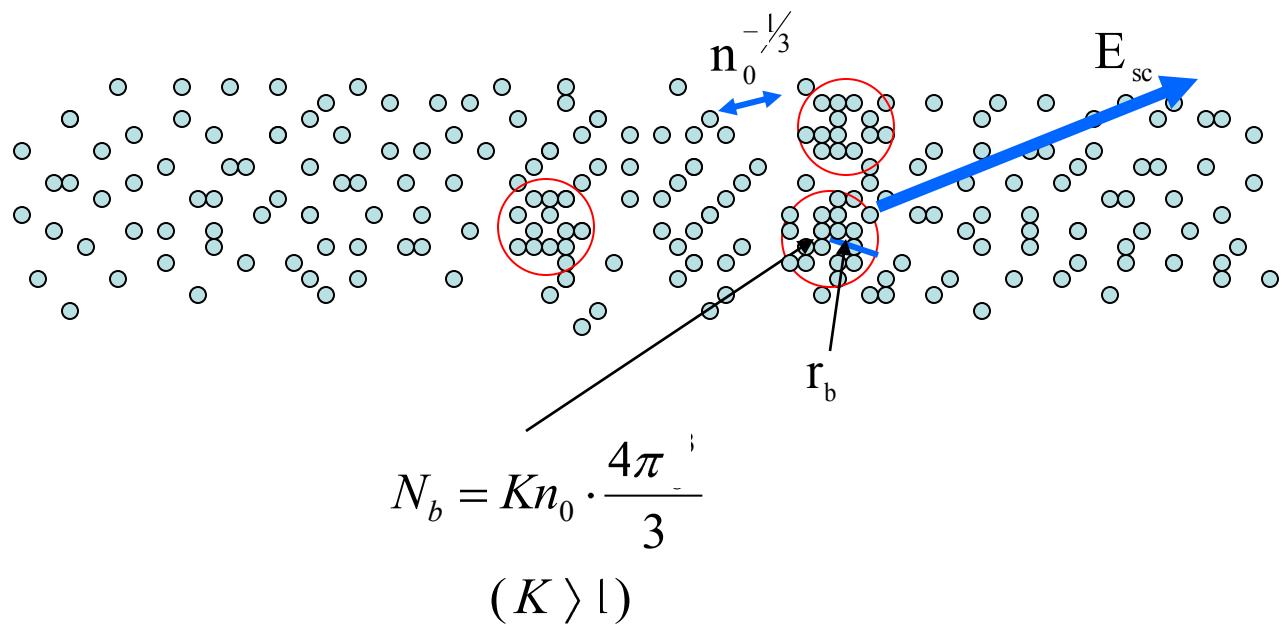
Tel-Aviv, Israel

**Micro-bunch – March 24-26, 2010
Frascati, Italy**



New Physics of Collective Micro-Dynamics in a Charged Particle Beam:

- Spatially coherent Coulomb interaction micro-dynamics.
- Yet unobserved effects of particle self-ordering and current shot noise suppression at optical frequencies.



3-D Homogenization Trend

A simple physical argument:

Inter-particle Coulomb force:

$$F_{coul} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_b^{1/3}}$$

Space-charge force:

$$F_{sc} = \frac{1}{4\pi\epsilon_0} \frac{eN_b}{r_b^2} = \frac{e^2}{4\pi\epsilon_0} \frac{Kn_0(4\pi r_b^3/3)}{r_b^2} = K \frac{e^2 n_0 r_b}{3\epsilon_0}$$

When $F_{sc} > F_{coul}$?

$$F_{sc}/F_{coul} \approx K4(r_b / n_0^{-1/3}) \rangle 1$$

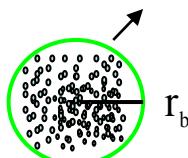
Answer: Always! (if $K > 1$ in a sphere of radius $r_b > n_0^{-1/3}$)

Note: Process leads to velocity spread growth

Bunch Expansion Time

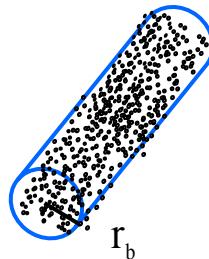
$$t_{\pi/2} = \frac{\pi}{\omega_r} :$$

3D - sphere



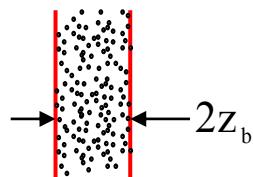
$$r_b(\pi/2) = .21r_b$$

2D - pencil



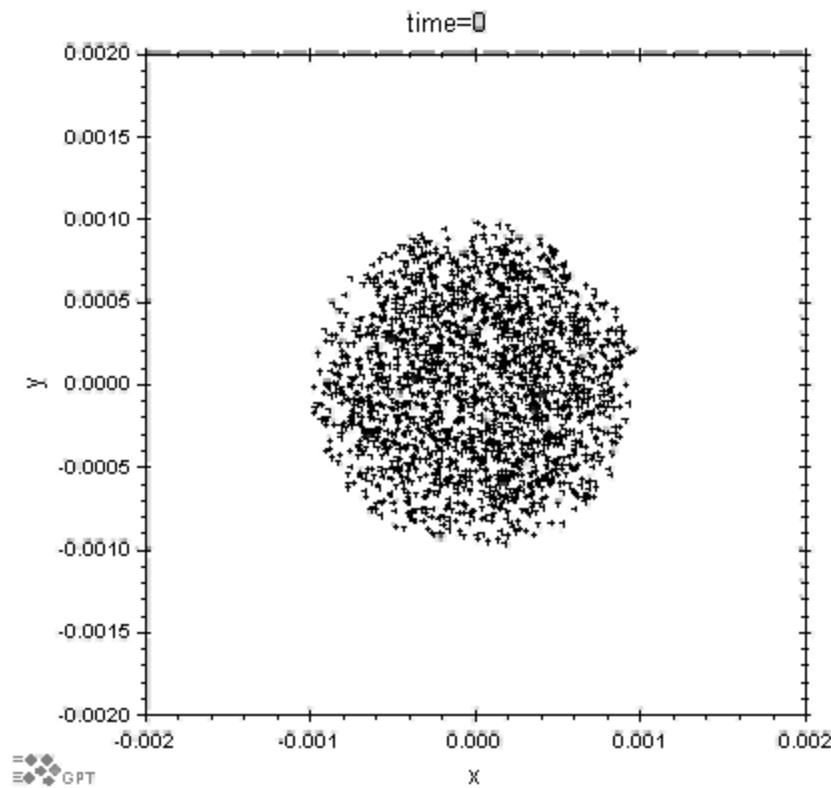
$$r_b(\pi/2) = .62r_b$$

1D - layer



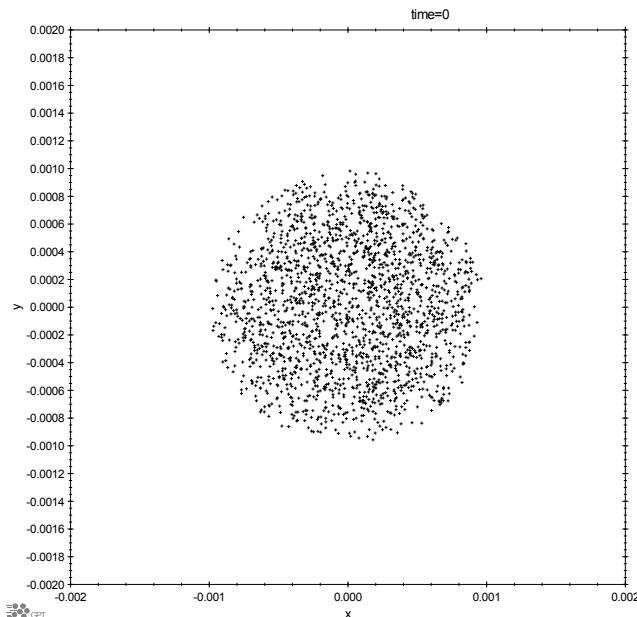
$$z_b(\pi/2) = .83z_b$$

Expansion of a Sphere Shaped Bunch of Uniformly Distributed Charges in Time Period $t = \tau / 2\omega_r$

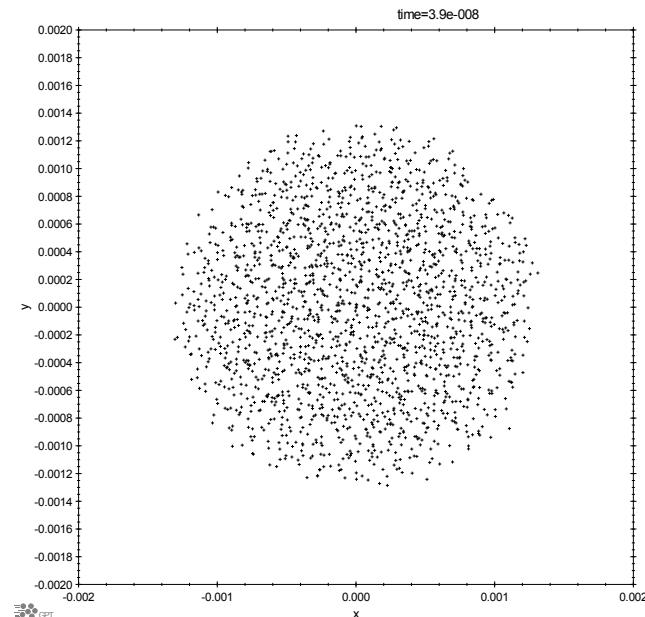


t=0[Sec], R=1[mm]

Expansion of a Sphere Shaped Bunch of Uniformly Distributed Charges in Time Period $t = \tau / 2\omega_r$



$t=0[\text{Sec}], R=1[\text{mm}]$



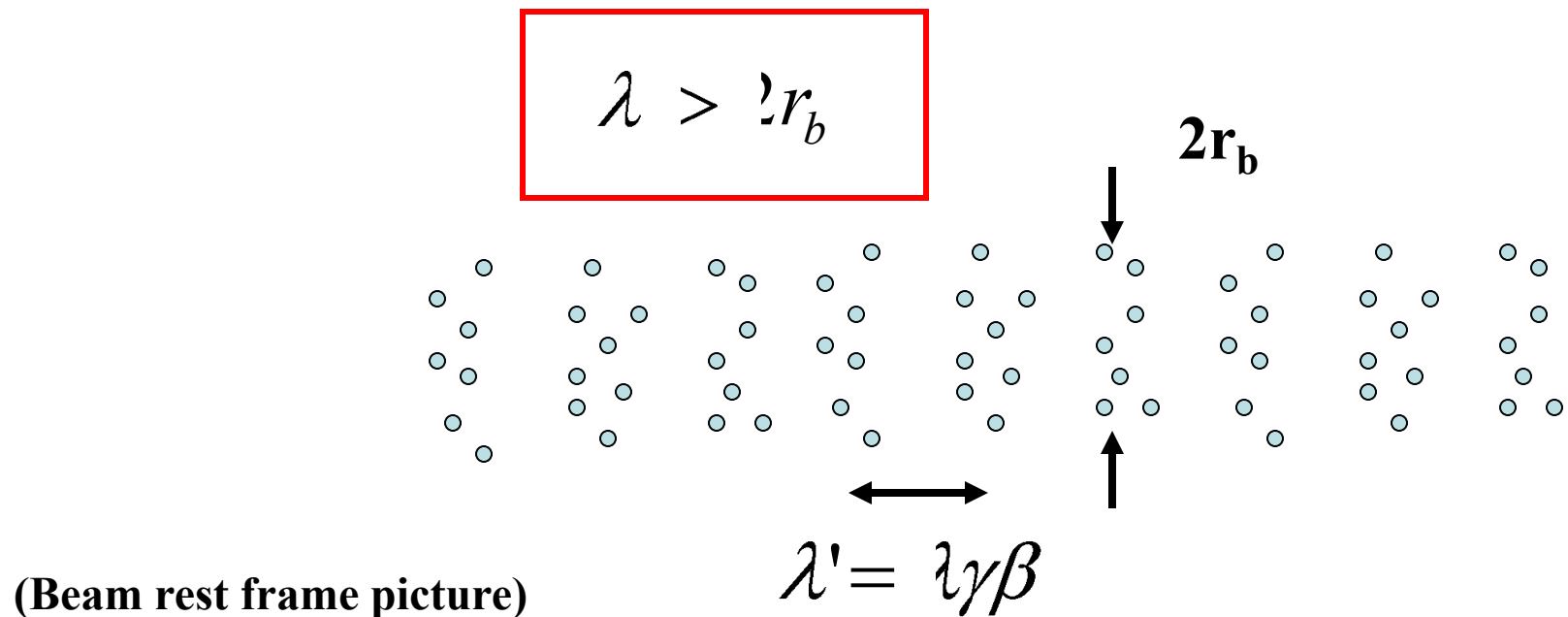
$t=9e-8[\text{Sec}], R=1.3[\text{mm}]$

The expansion ratio observed in GPT simulations was independent of the Initial radius or the number of sample charges.

Conditions for a Charged-Particle Beam to Exhibit Spatially-Coherent Current Shot-Noise Suppression:

$$\left| \overline{\tilde{i}(\omega)} \right|_{t \approx \pi/2} < \left| \overline{\tilde{i}(\omega)} \right|_{t=0} = I_b$$

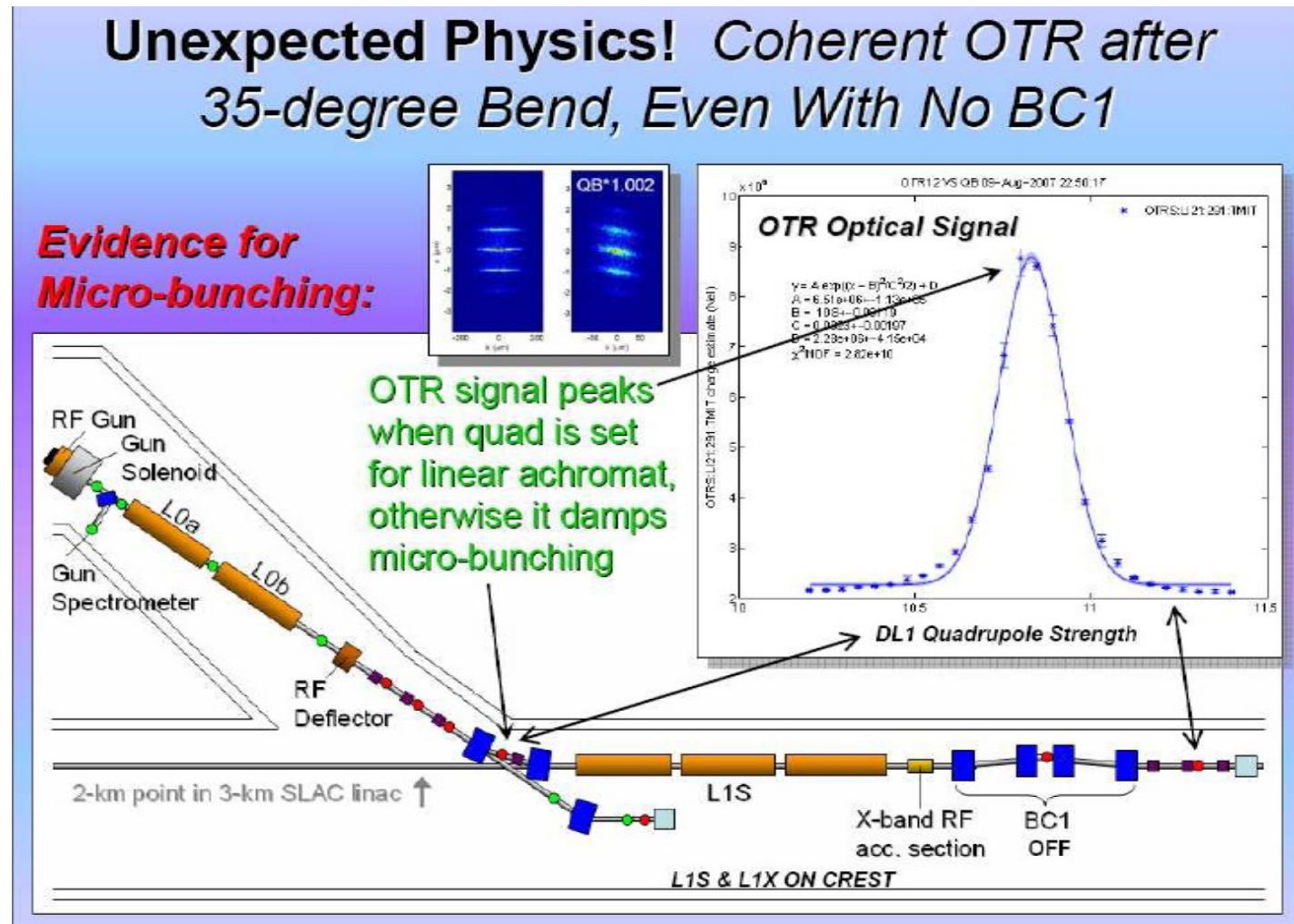
1. Cold beam: current shot-noise dominated (non-equilibrium plasma !)
2. Longitudinal interaction (single Langumir mode)



Coherent Optical Transition Radiation in LCLS/SLAC

D. Dowell, FEL Frontiers conference (Italy, Sept. 9-13, 2007)

R. Akre et al, Phys. Rev. ST-AB, 11, 030703 (2008)



ANALYTICAL FLUID-PLASMA LINEAR MODEL

[A. Gover, E. Dyunin, PRL 102, 154801 (2009)]

[H. Haus and F. N. H. Robinson, Proc. IRE 43, 981 (1955)]

Coherent Plasma Oscillation in an e-Beam Drift Section

$$\begin{aligned}\check{i} \mathbf{E}_d, \omega_r &= [\mathbf{Q}, \omega_r \cos \phi_r - \check{V} \mathbf{Q}, \omega_r \sin \phi_r / W_d] e^{i\phi_r} \mathbf{E}_d \\ \check{V} \mathbf{E}_d, \omega_r &= [-\check{i} \mathbf{Q}, \omega_r V_d \sin \phi_r + \check{\gamma} \mathbf{Q}, \omega_r \cos \phi_r] e^{i\phi_r} \mathbf{E}_d\end{aligned}$$

$$\check{V} \mathbf{E}, \omega_r = -\hbar c^2/e \check{\gamma} \mathbf{E}, \omega_r = -\hbar c^2/e \check{\gamma}_0^3 v_0 \check{V} \omega_r$$

(Chu's Relativistic Kinetic Voltage)

$$\phi_r = \frac{\vartheta}{v_z} L_d \quad \phi_r = \vartheta_r L_d \quad \theta_r = \dot{\vartheta}_p \frac{\omega_r}{v_0}$$

$$\omega_p = \left(\frac{e^2 n_0}{m \epsilon_r \gamma} \right)^{1/2} \quad W_d = \dot{\vartheta}_p^2 \sqrt{\mu_0 / \epsilon_0} / k \theta_{r,rd} A_e$$

Current Noise Transformation in a Drift Section

If $\overline{\text{Im}(\mathbf{Q}\tilde{\mathbf{y}}^*\mathbf{Q})} = 0$:

$$\overline{|i\mathbf{C}, \omega|^2} = \overline{|i\mathbf{Q}, \omega|^2} \cos^2 \phi_r + \frac{1}{W_d^2} |\tilde{V}\mathbf{Q}|^2 \sin^2 \phi_r$$

$$\overline{|\tilde{V}\mathbf{C}_d, \omega|^2} = V_d^2 \overline{|i\mathbf{Q}, \omega|^2} \sin^2 \phi_r + |\tilde{V}\mathbf{Q}|^2 \cos^2 \phi_r$$

At $\phi_p = \vartheta_{pr} L_d = \tau/2$:

$$\overline{|i\mathbf{C}_d, \omega|^2} = \overline{|\tilde{V}\mathbf{Q}, \omega|^2} / W_d^2 \quad (\approx 0)$$

$$\overline{|\tilde{V}\mathbf{C}_d, \omega|^2} = \overline{|\tilde{V}\mathbf{Q}, \omega|^2} W_d^2$$

Current Shot-Noise Reduction

$$\phi_r = \frac{J_r}{L_d} = \tau^2$$

$$\Rightarrow \frac{\overline{|i \mathbf{C}_d, \omega_\perp|^2}}{\overline{|V \mathbf{C}_d, \omega_\perp|^2}} = \frac{\overline{|V \mathbf{Q}, \omega_\perp|^2}}{\overline{|i \mathbf{Q}, \omega_\perp|^2}} / W_d^2$$

$$f_i = \frac{\overline{|i \mathbf{C}_d, \omega_\perp|^2}}{\overline{|i \mathbf{Q}, \omega_\perp|^2}} = \frac{\overline{|V \mathbf{Q}, \omega_\perp|^2}}{\overline{|i \mathbf{Q}, \omega_\perp|^2}} / W_d^2$$

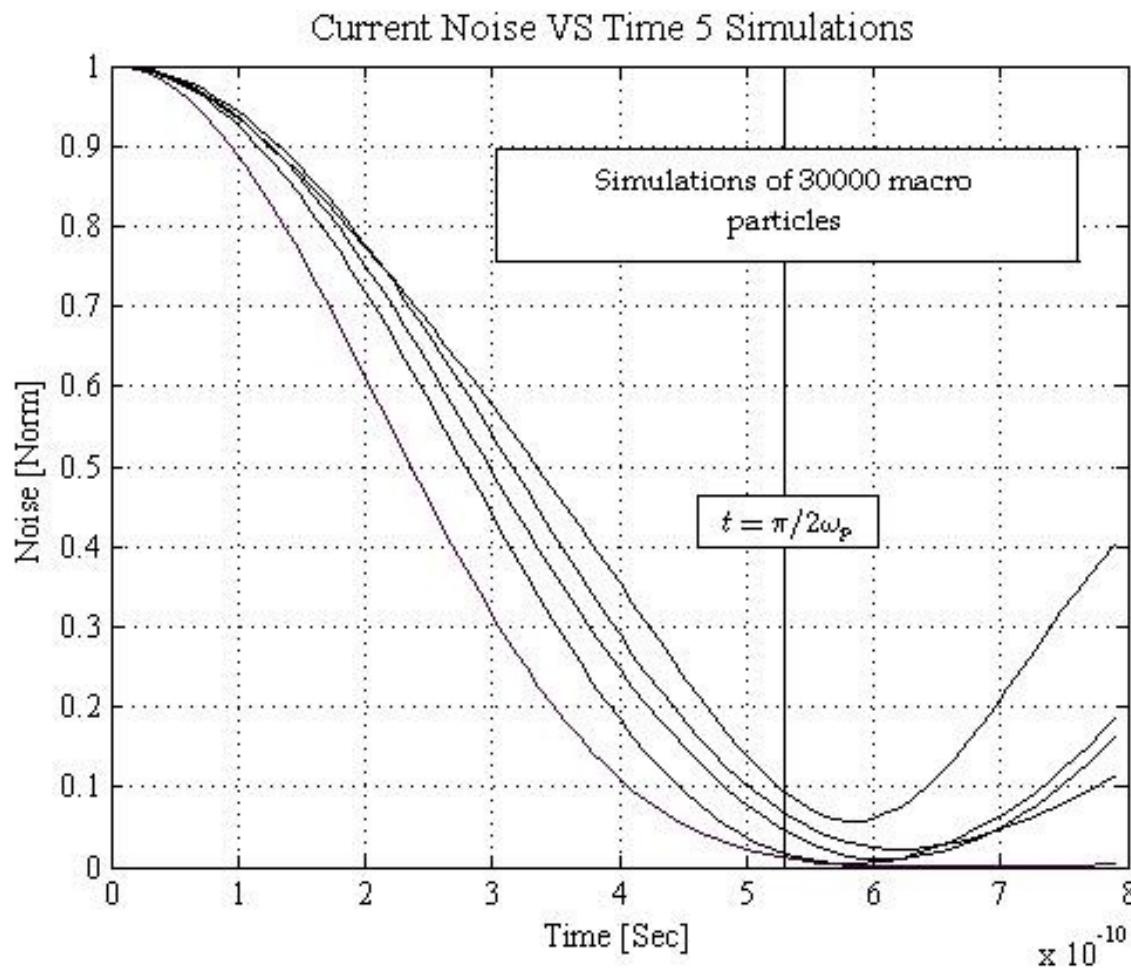
$\ll 1$ For current noise dominated beam

For LCLS parameters: $f_i = 2.5 \times 10^{-7}$

3-D Numerical Simulations

**A. Nause, E. Dyunin, A. Gover,
to be published JAP (2010)**

3D GPT Results of Simulations

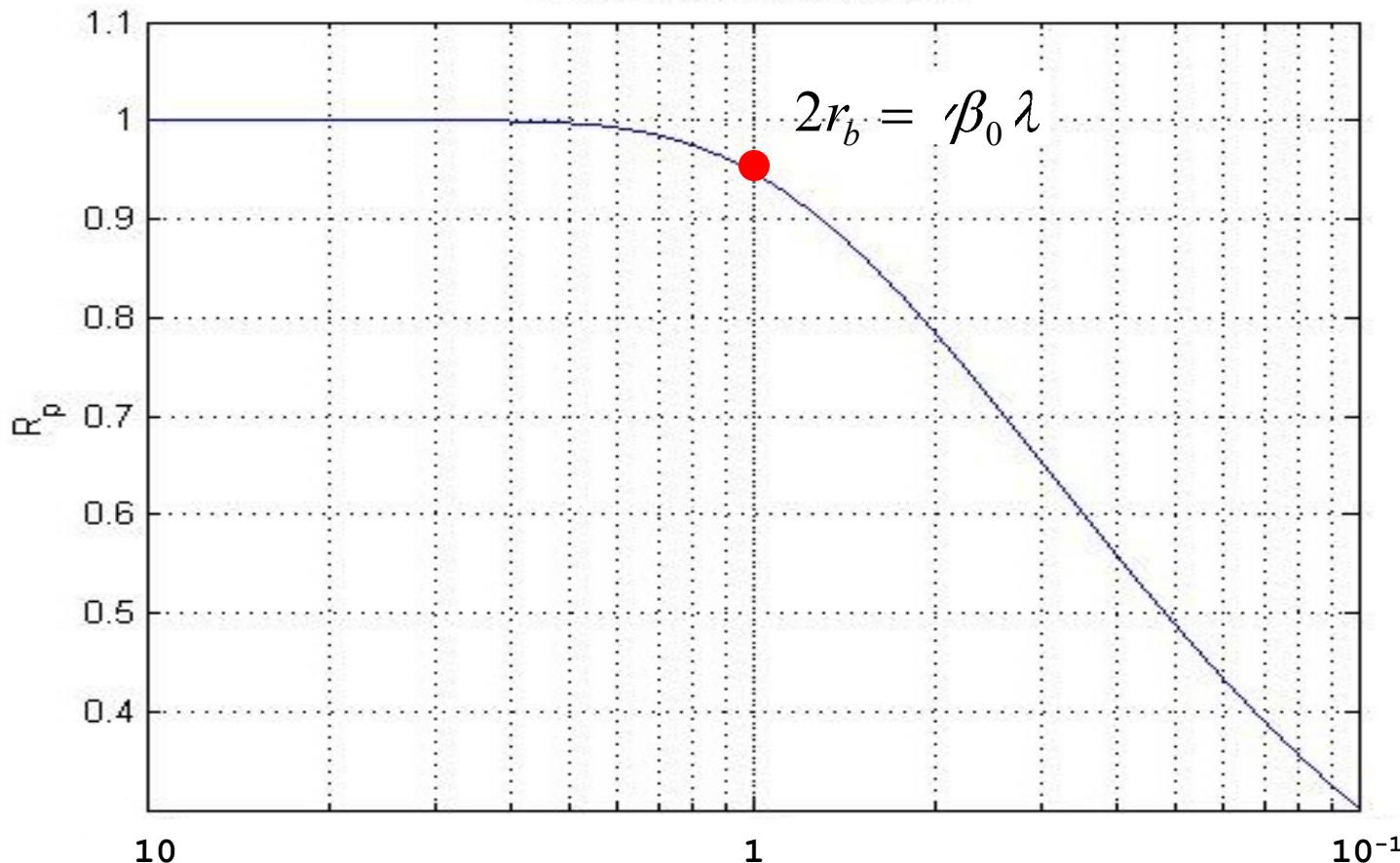


Plasma Reduction Factor – Fundamental Langmuir Plasma Wave Mode

$$r_p = \frac{1}{\tau} K_1(\pi)$$

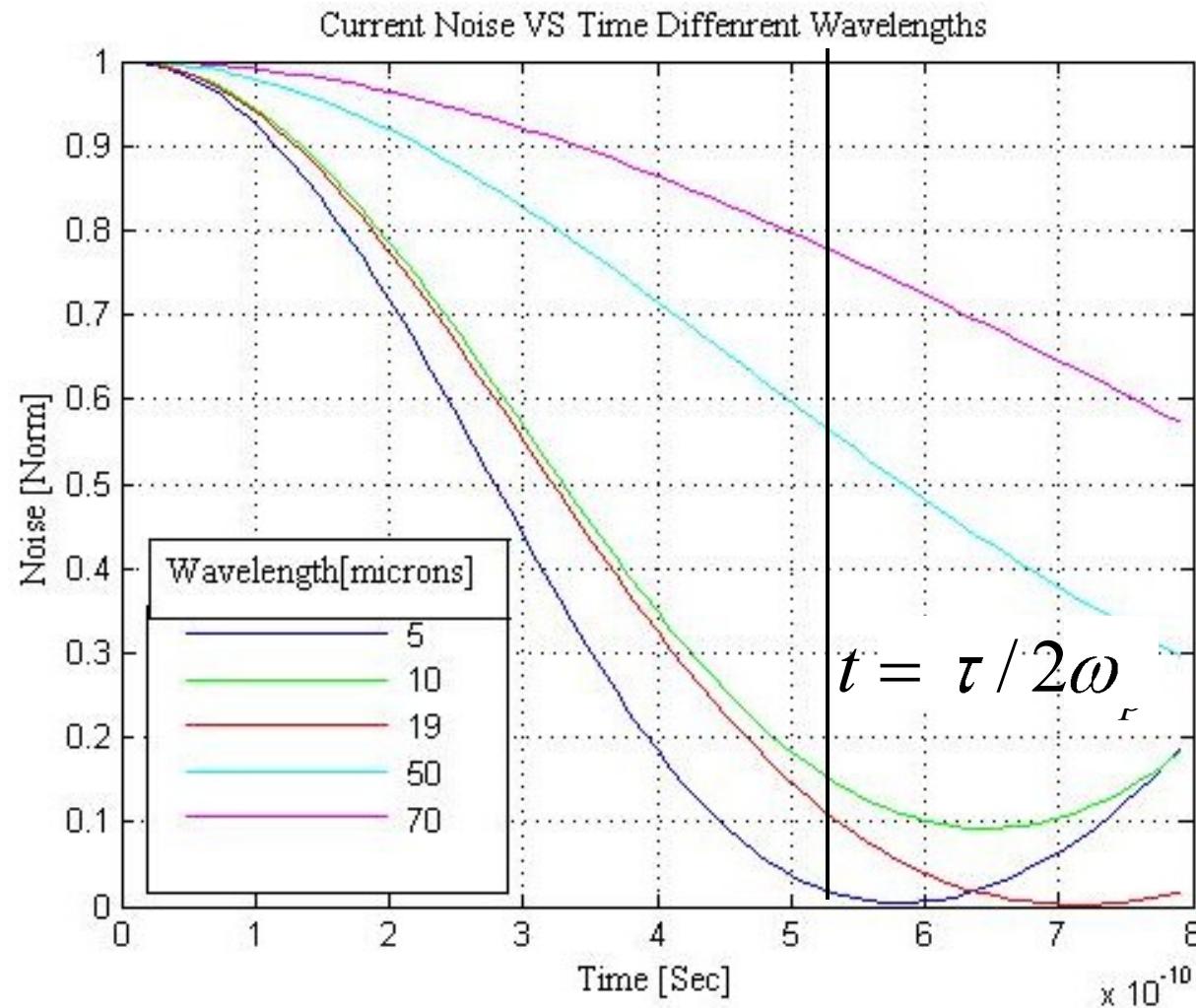
[M. Venturini, NIM-PR-A 599 (2009) 140]

Plasma Reduction Factor



$$u = \frac{2r_b}{\gamma\beta\lambda}$$

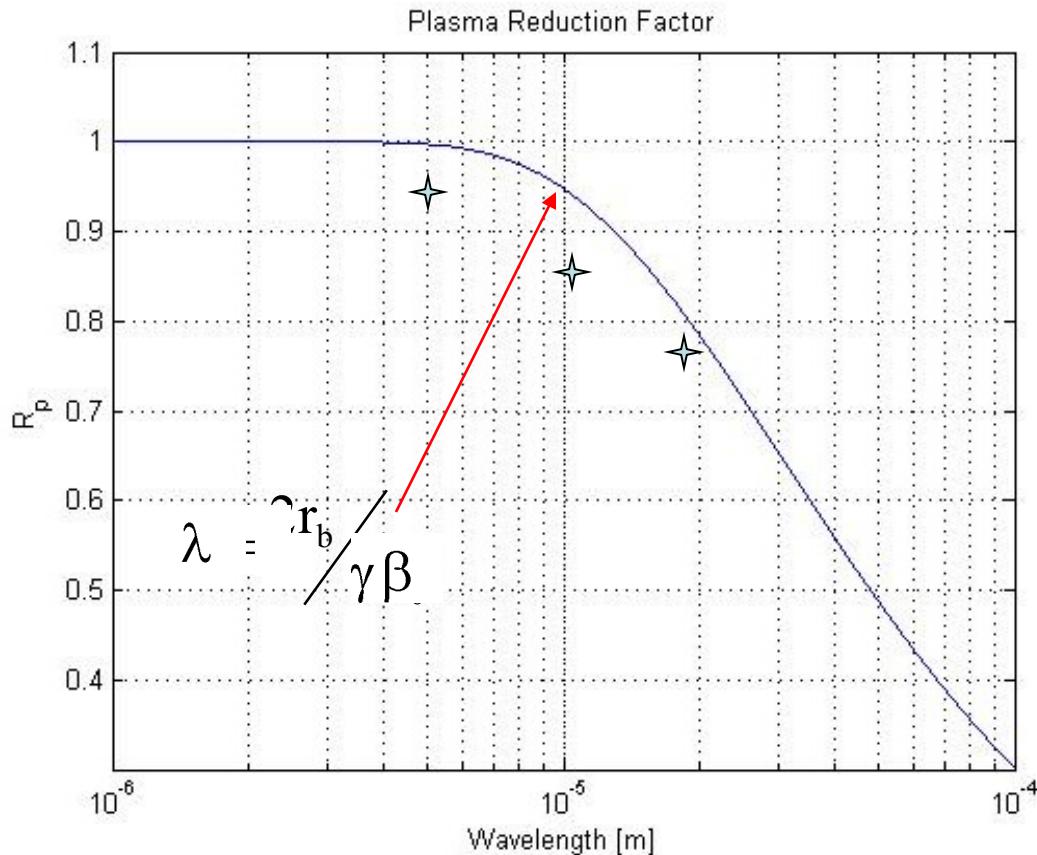
Minimal Current Noise Dependence on Plasma Reduction Factor



Minimal Current Noise Dependence on Plasma Reduction Factor

$$r_p = \left(-\frac{kr_b}{\gamma} K_1\left(\frac{kr_b}{\gamma}\right) \right)^{-2}$$

Where $K_1(x)$ is the modified Bessel function

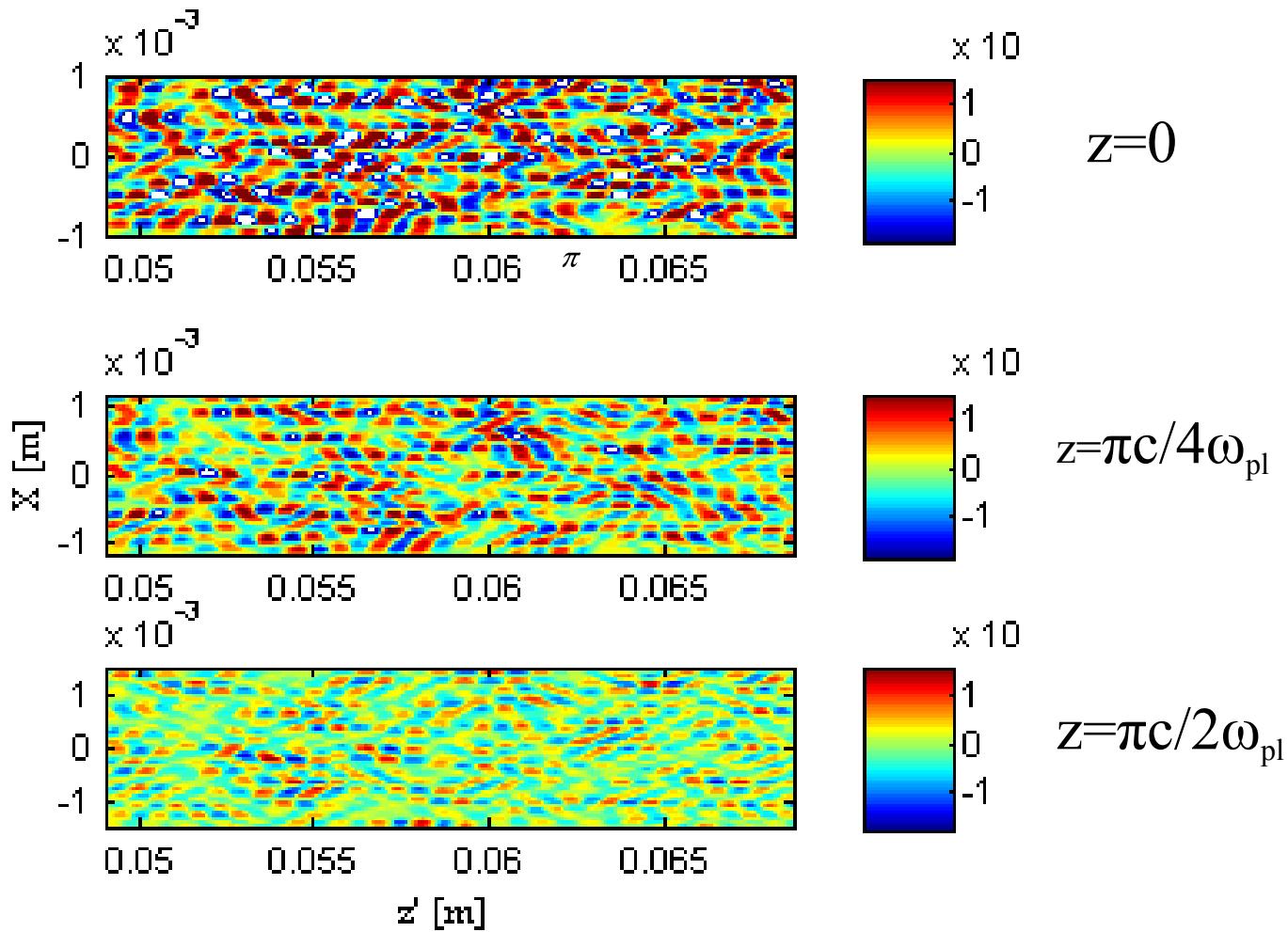


$\lambda[\mu\text{m}]$	Theoretical r_p	Simulation r_p
5	1	0.93
10	0.95	0.85
19	0.8	0.76
50	0.40	
70	0.38	

Charge Density Homogenization – Axially Filtered 5-10 [μm]

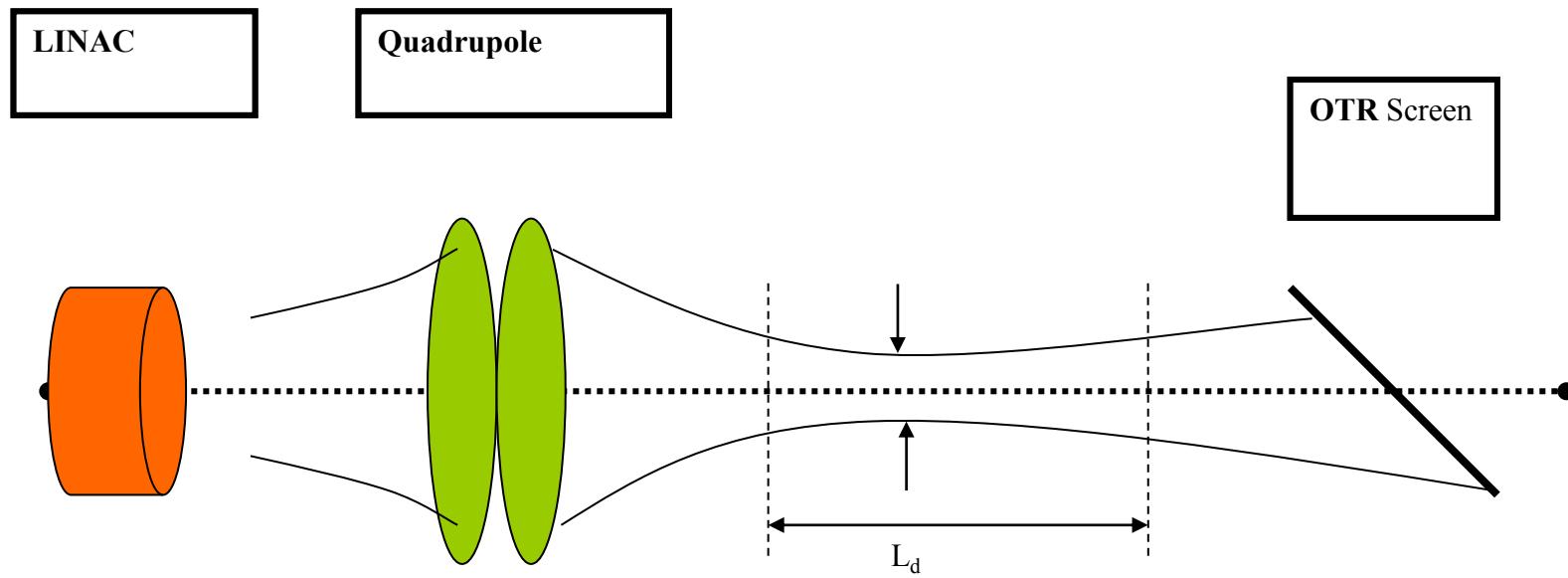
Simulation Parameters (FERMI 60k m.p):

$$E = 100 \text{ [MeV]}, R = 1 \text{ [mm]}, I = 80 \text{ [A]}$$

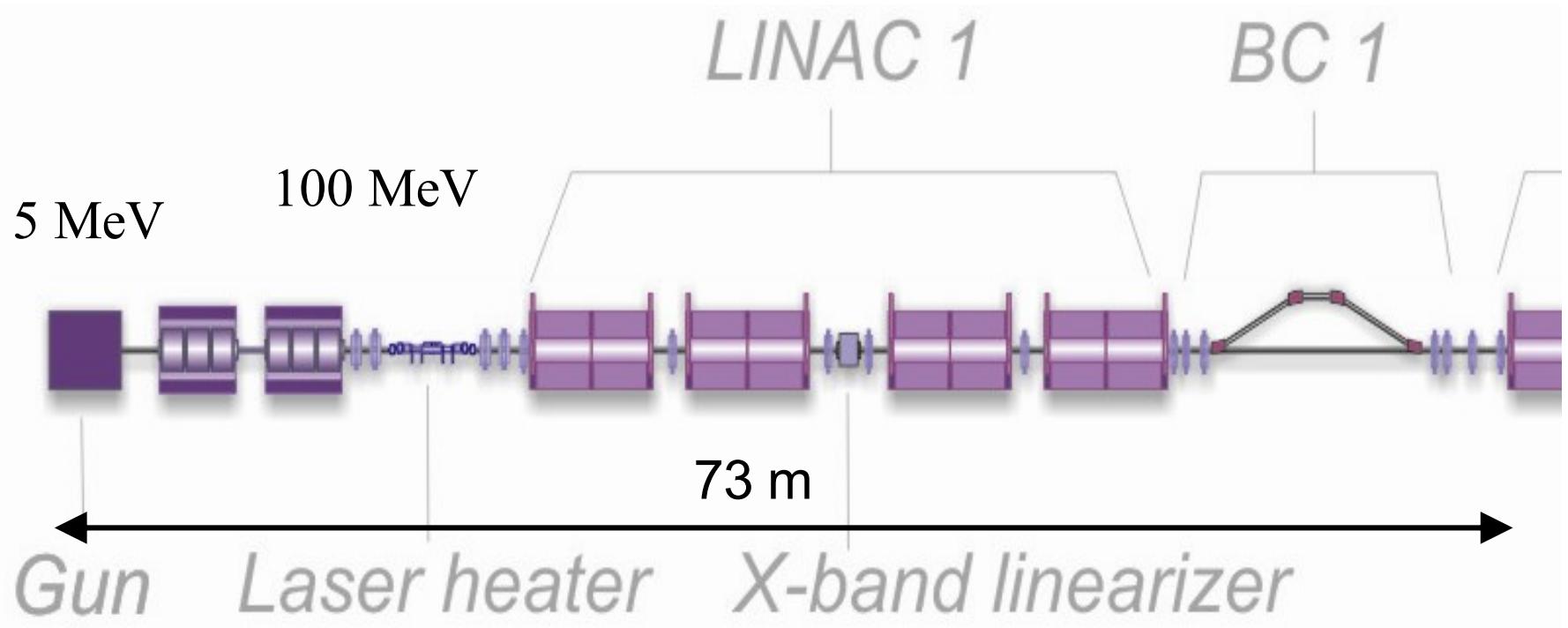


POSSIBLE EXPERIMENTAL VERIFICATION

Collective Interaction Drift Section in the Waist of a Free Propagation Beam



FERMI'S Gun – Injector – Laser-heater section (5-100 MeV)



3D Simulations Proof Shot Noise Control and Reduction by Collective Coulomb Interaction

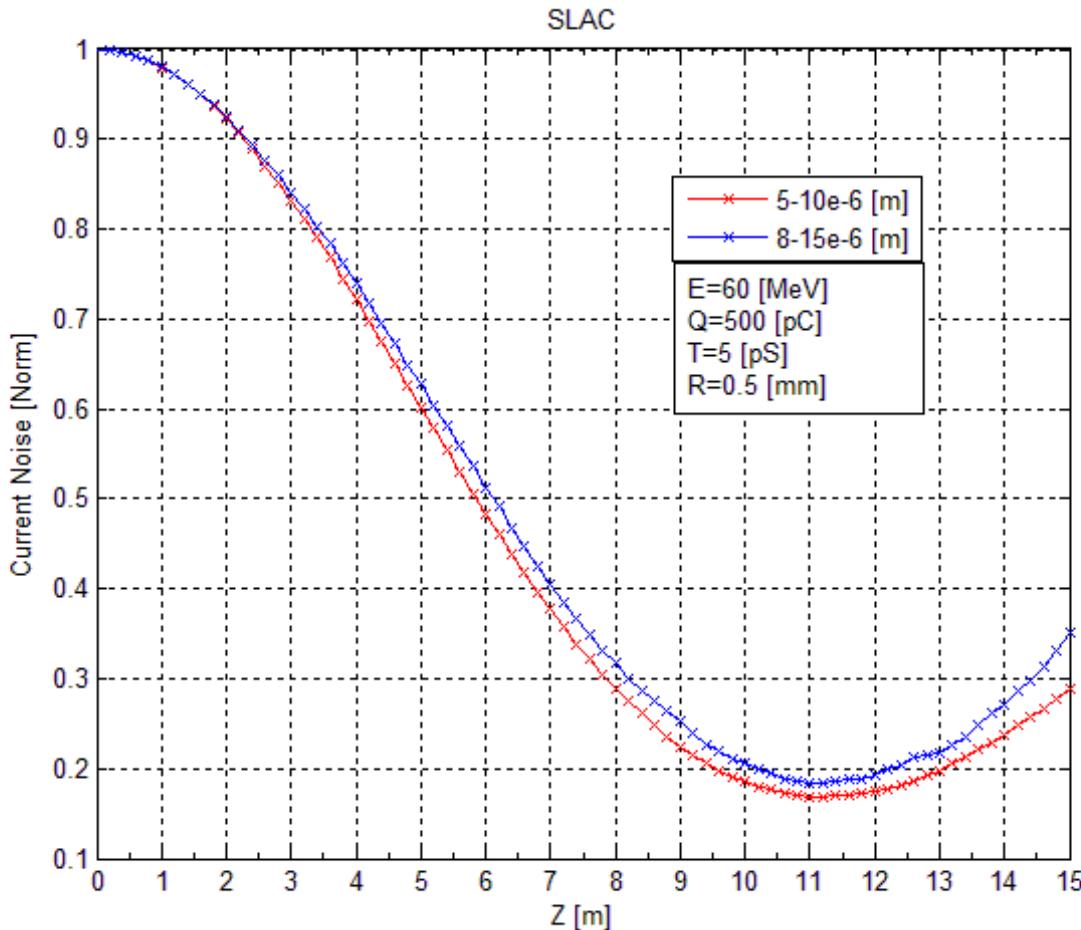
Beam Parameters – FERMI

Energy	100[MeV]
Current, Pulse Length	80[A], 9[pS]
Beam Radius	1[mm]
Drift Length	31[m]
Drift Time $t = \sqrt{\frac{2r_b}{\lambda\omega_r}}$	5.3e-10[S]

(Verifying single mode interaction condition) $\lambda\gamma\beta_c = 0.98 \text{ nm}$ ($\lambda=5\mu$)

$$2r_b = 1 \text{ nm}$$

Planned Experiments - LCLS

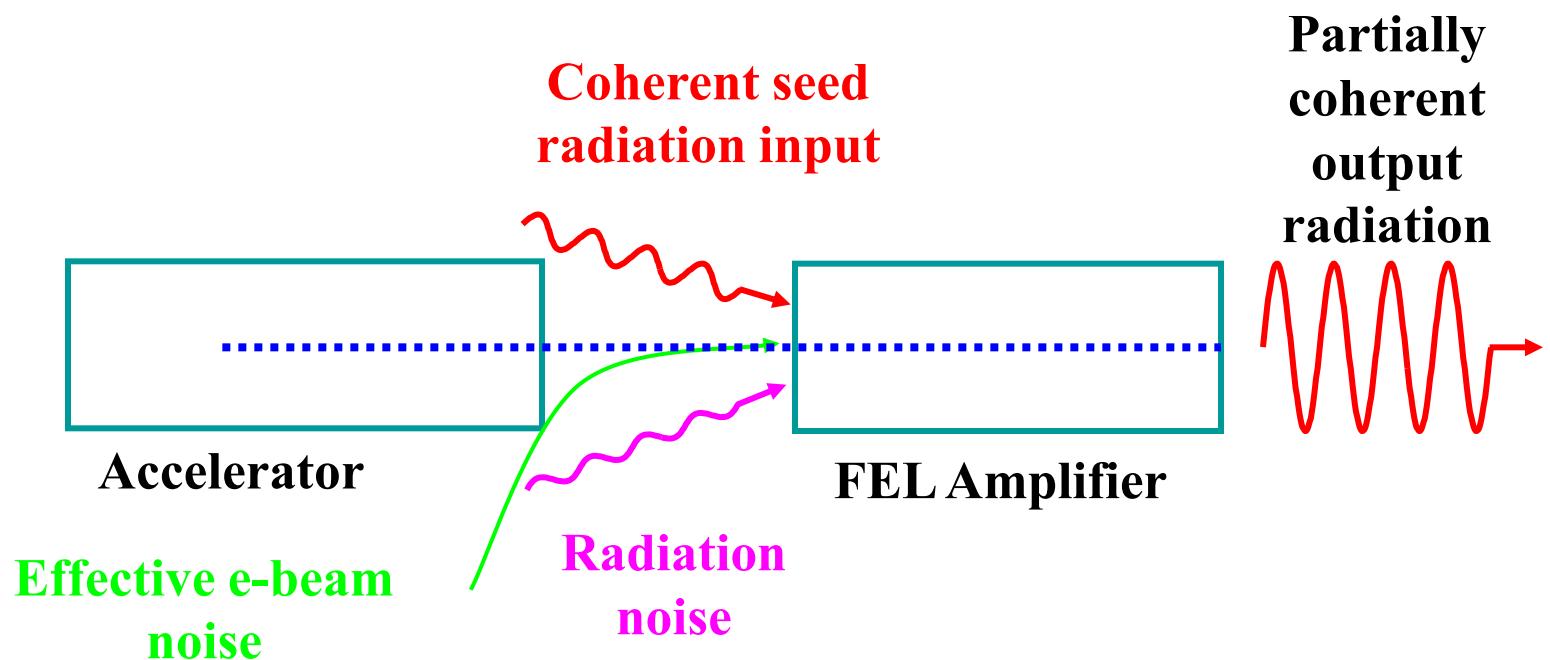


Implications:

- Spontaneous Emission Suppression
(Dicke's Sub-radiance).
- Radiation Coherence enhancement.
- Fundamental Coherence limits of
e-beam Radiators.

[E. Dyunin, A. Gover, to be published JQE (2010)]

NOISE-EQUIVALENT RADIATION INPUT POWER

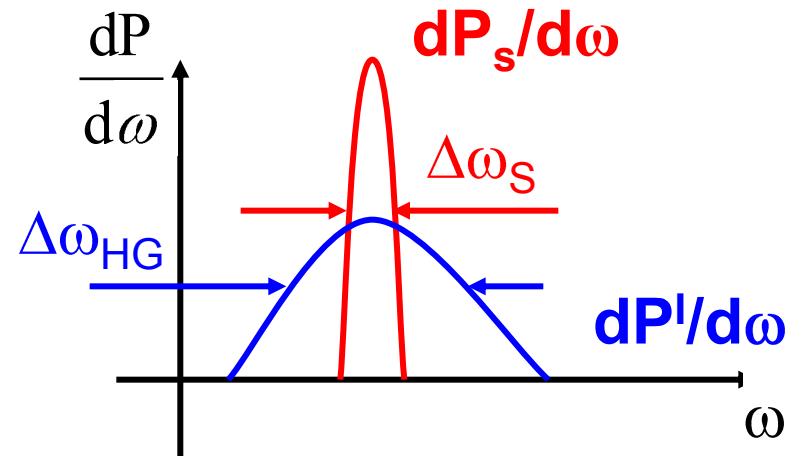


Effective Input noise

$$\left(\frac{P_{in}^{noise}}{d\omega} \right)_{eff} = \left(\frac{P_{in}^{coh}}{d\omega} \right)_{incoh} / G(\omega)$$

Coherence condition

$$P_s \Phi_{coh} \gg \left(\frac{P_{in}^{noise}}{d\omega} \right)_{eff} \Delta \omega$$



Dominating Current Shot-Noise:

$$P_s \Phi_{coh} \gg \frac{eI_b Z_0}{16\pi A_{em}} \left(\frac{a_w}{\gamma \beta_z} \right)^2 \Delta \omega$$

(seed radiation injection)

$$|\tilde{i}_s \Phi|^2 \gg eI_b \Delta \omega$$

(pre-bunching)

SASE Power Control

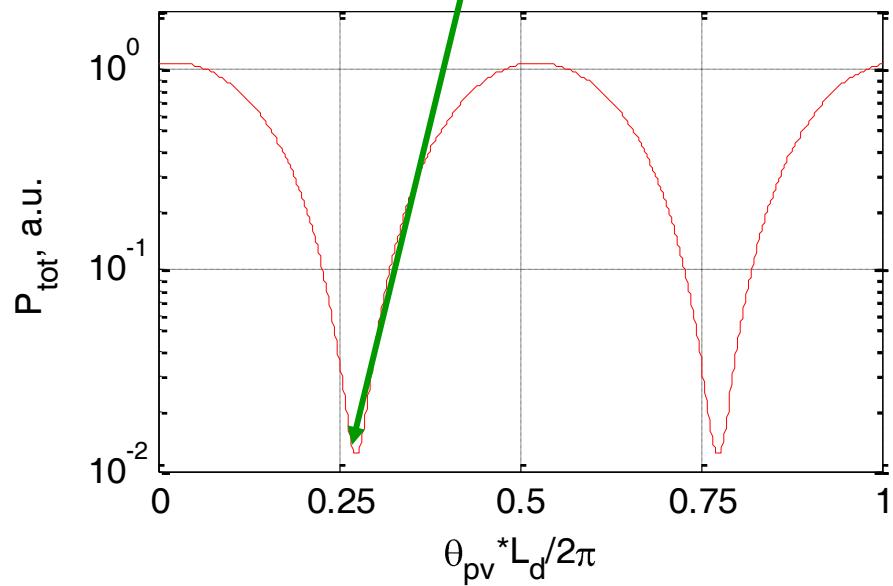


$$P_{tot} = \int \frac{dP}{d\omega} d\omega = \frac{2}{\pi} e I_b \int H_{tot}^{Ei}(\omega)^2 d\omega$$

$$L_d \sim \lambda_p / 4 = \pi / 2 \theta_p$$

(starting from current shot-noise at $z=0$, $\Delta E=0$)

E. Dyunin, A. Gover,
NIM A **593**, 49 (2008)



Fundamental Coherence Limits (Beyond Shot Noise Limit)

A Conservative of motion in a non-dissipative e-beam transport section:

$$\overline{|\tilde{i}_c|^2} \overline{|\tilde{V}_c|^2} = \delta \cdot c^2$$

Minimum input noise:

$$\left(\frac{dP_{in}}{d\omega} \right)_{min} = \frac{\delta c}{\pi} + \frac{\hbar\omega}{1 + e^{-\omega/T}}$$

Microwave/THz regime:

$$\left(\frac{dP_{in}}{d\omega} \right)_{min} = \frac{\delta E_c}{\pi} + \cancel{k_B T} \quad (\approx \frac{\delta E_c}{\pi})$$

(Cathode temperature limited)

Optical regime:

$$\left(\frac{dP_{in}}{d\omega} \right)_{min} = \cancel{\frac{\delta E_c}{\pi}} + i\omega \quad (\approx i\omega)$$

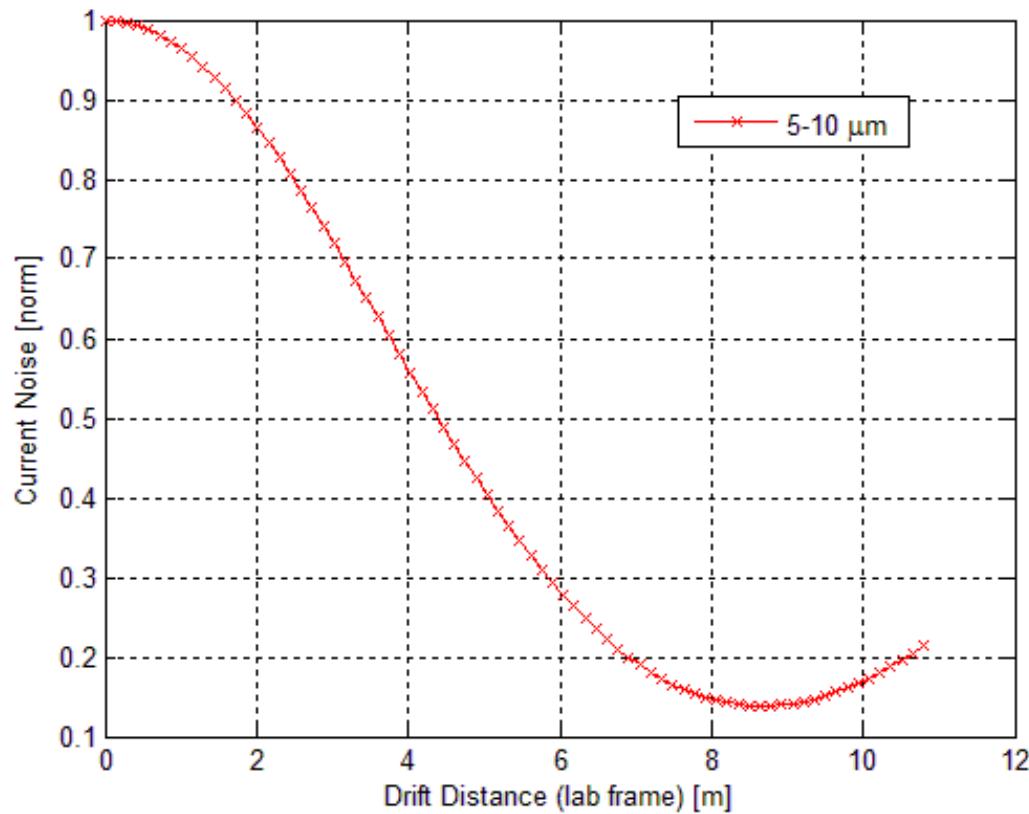
(Quantum limit)

Conclusions

- It is possible to adjust the e-beam current shot- noise level by controlling the longitudinal plasma oscillation dynamics.
- This can be used to enhance FEL coherence and relax seeding power requirement.
- After elimination of shot noise, IR/XUV FEL coherence is ultimately limited by the quantum input noise $dP/d\omega = i\omega$

Beam Line: ATF

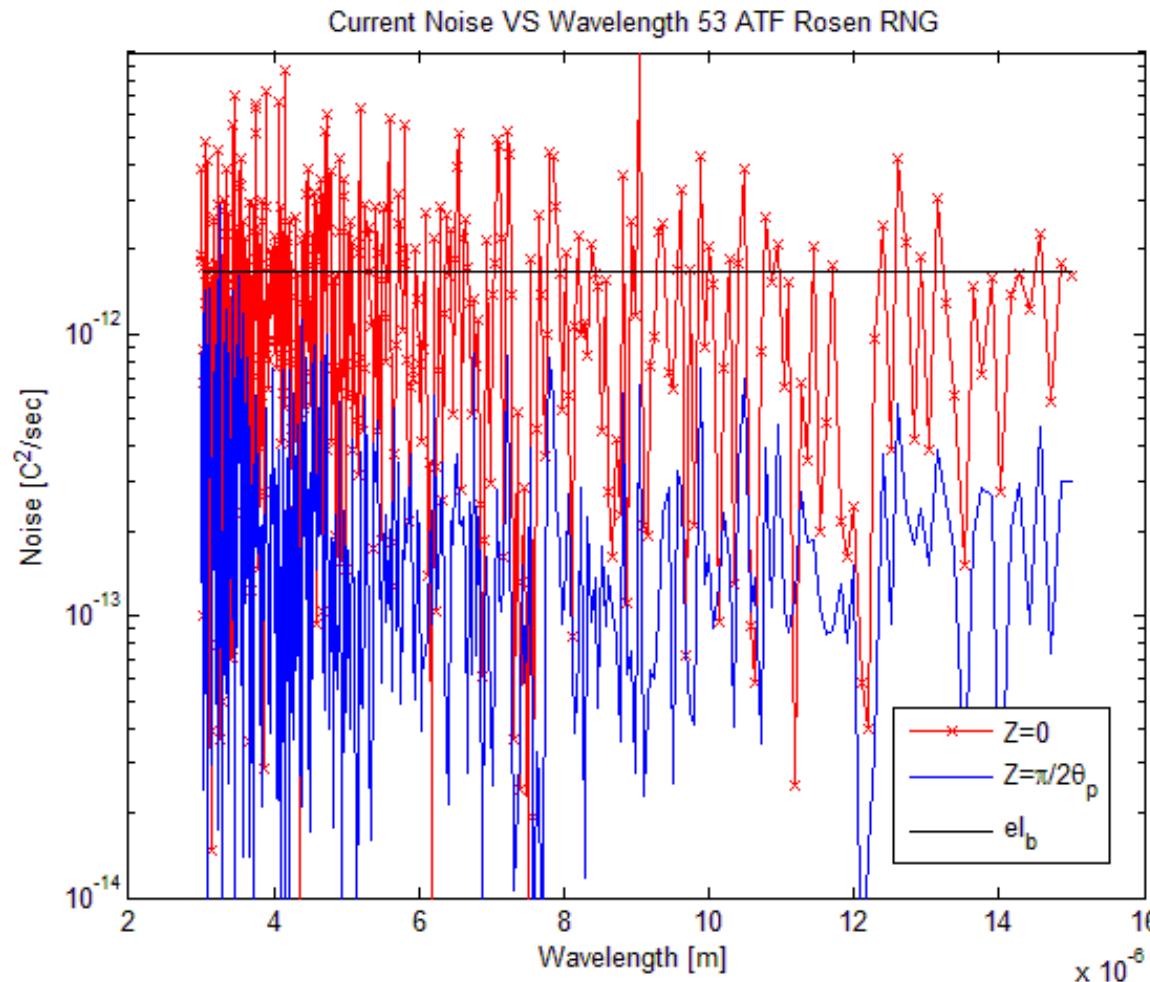
Planned Experiments - ATF



Current Noise Suppression – 3-15 [μm]

Simulation Parameters (ATF 30k m.p):

$E = 60$ [MeV], $R = 0.5$ [mm], $I = 100$ [A]



Beam Parameters Conditions

$$n_0 A_c \lambda \beta \gg$$

$$\Delta \beta_b = k L_d \Delta \beta / \beta_z \ll \pi$$

$$\Rightarrow \begin{cases} \gamma \ll \beta \gamma \lambda / 2L_d \\ \beta \gamma \sigma_{\phi} \lambda / L_d \gg \end{cases}$$

In free space (sufficient condition):

$$\varepsilon \ll 4I_0 / I_A \gamma \beta \gg r_{b0} = \left(\frac{\pi^2}{2} / \gamma \right) \left(I_0 / I_A \beta \gamma \right) L_d$$

Plasma Reduction Factor

1D model solution of Poisson equation (short length):

$$\tilde{E}_{sc} = - \frac{\sqrt{\mu/\epsilon}}{\pi} \frac{1}{\beta} \tilde{I} = - Z \epsilon \tilde{I} \quad \Rightarrow$$

$$Z \epsilon = \frac{\sqrt{\mu/\epsilon}}{\pi} \frac{1}{\beta}$$

Cylindrical beam model solution of Poisson equation:

[M. Venturini, NIM-PR-A 599 (2009) 140–145]

$$Z \epsilon = \frac{\sqrt{\mu_0/\epsilon_0}}{\pi \gamma} \frac{1 - K_1(\xi)}{\xi} \quad \text{Where} \quad \xi_b = \frac{\kappa r_b}{\gamma}$$

$$\Rightarrow r_p^2 = [- K_1(\xi)]^2$$

Random Signal Spectral Domain

$$\check{v}(\omega) = \int_{-\pi/2}^{\pi/2} v(t) e^{i\omega t} dt$$

$$\check{v}_z(\omega) = \int_{-\pi/2}^{\pi/2} v_z(t) e^{i\omega t} dt$$

$$\check{V}(\omega) = - \frac{\hbar c^2}{e} \gamma_0^3 v_0 \check{v}_z(\omega)$$

At the cathode (“virtual” cathode)

Current and velocity noise – uncorrelated:

$$\overline{|\tilde{i}(\omega_{\tau})|^2} = \frac{1}{\Gamma} \left\langle |\tilde{i}(\omega_{\tau})|^2 \right\rangle_{N_T} = \frac{eI_b}{c}$$

$$\overline{|\tilde{v}(\omega_{\tau})|^2} = \frac{1}{\Gamma} \left\langle |\tilde{v}(\omega_{\tau})|^2 \right\rangle_{N_T} = \frac{\mathcal{S}_c}{eI_b}$$

$$\left(\overline{|\tilde{i}(\omega_{\tau})|^2} \right)^{1/2} \left(\overline{|\tilde{v}(\omega_{\tau})|^2} \right)^{1/2} = \mathcal{S}_c$$

FEL Amplifier (seeded FEL)

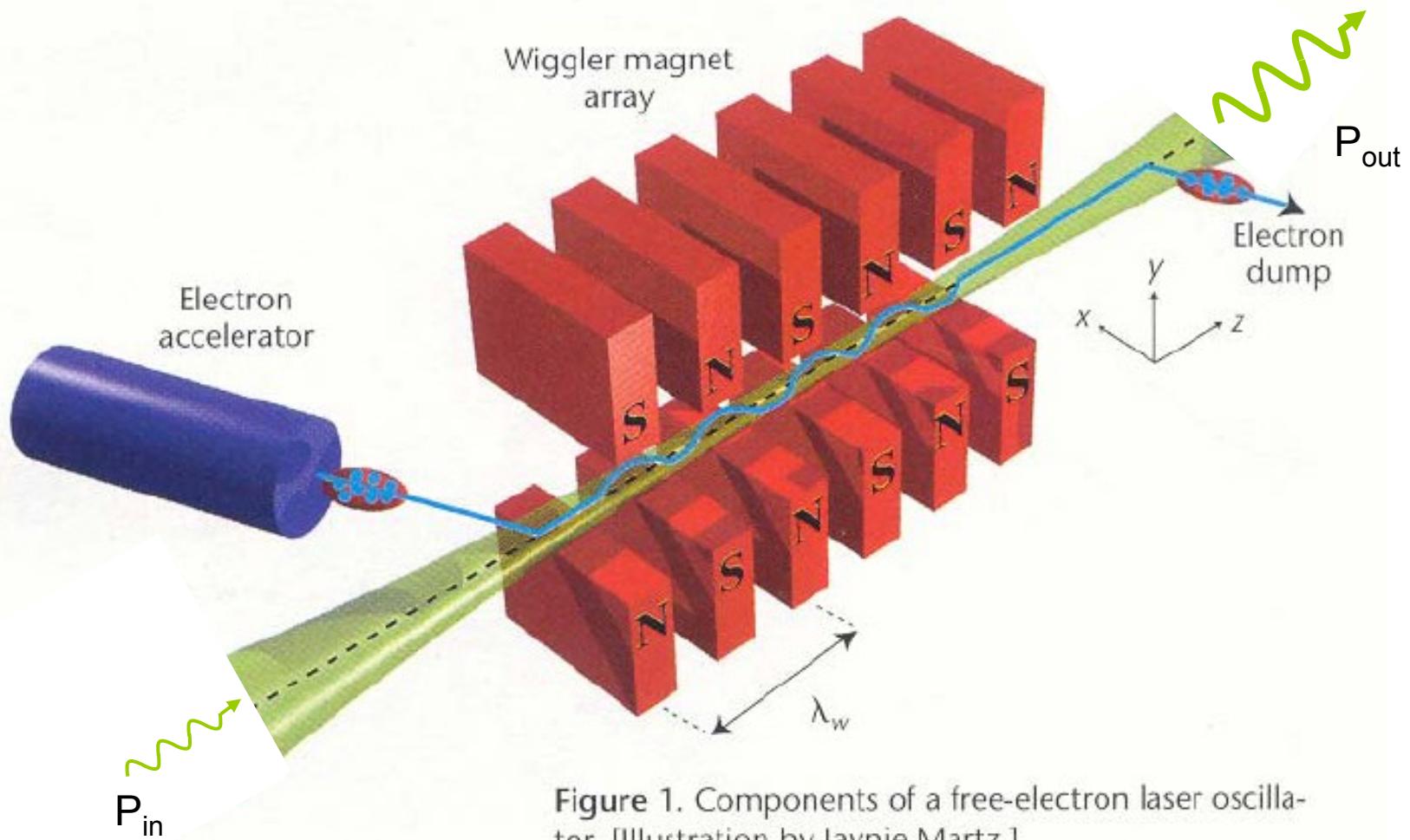
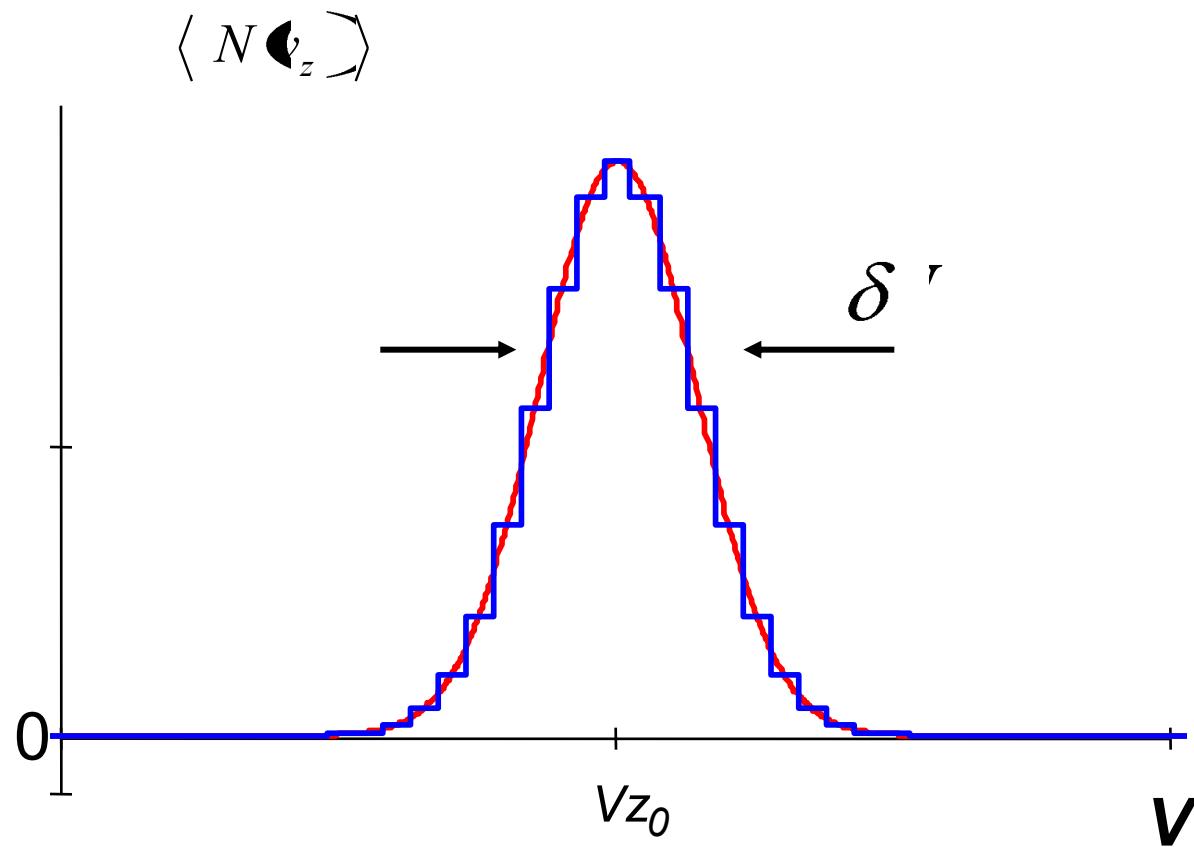
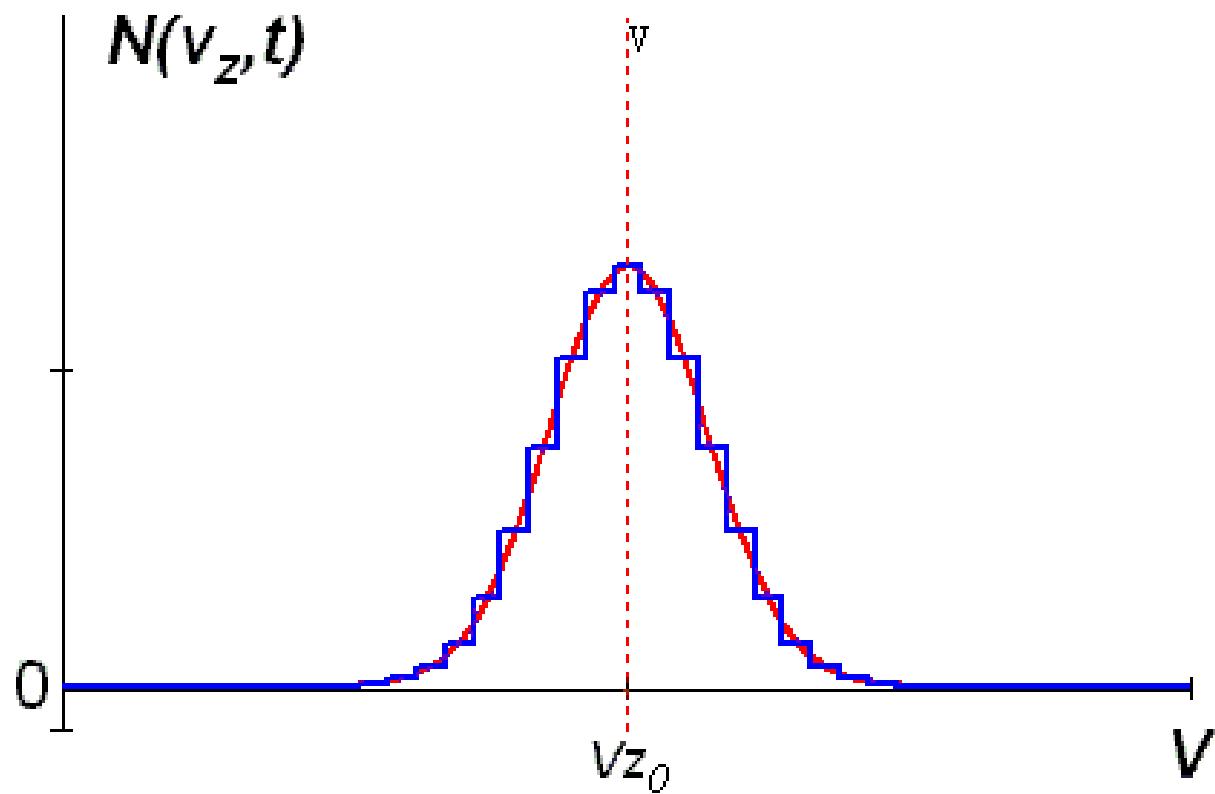


Figure 1. Components of a free-electron laser oscillator. [Illustration by Jaynie Martz.]

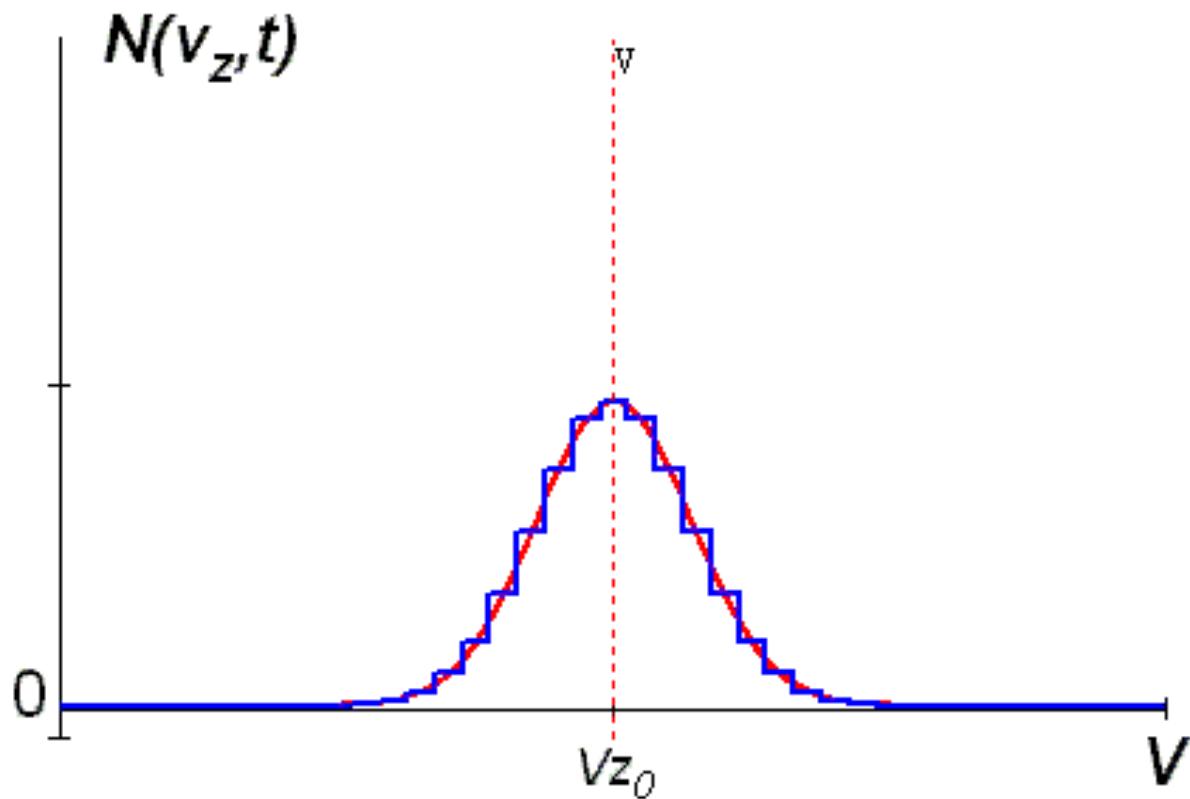
Axial Velocity Distribution of the E-beam Density Averaged over Time



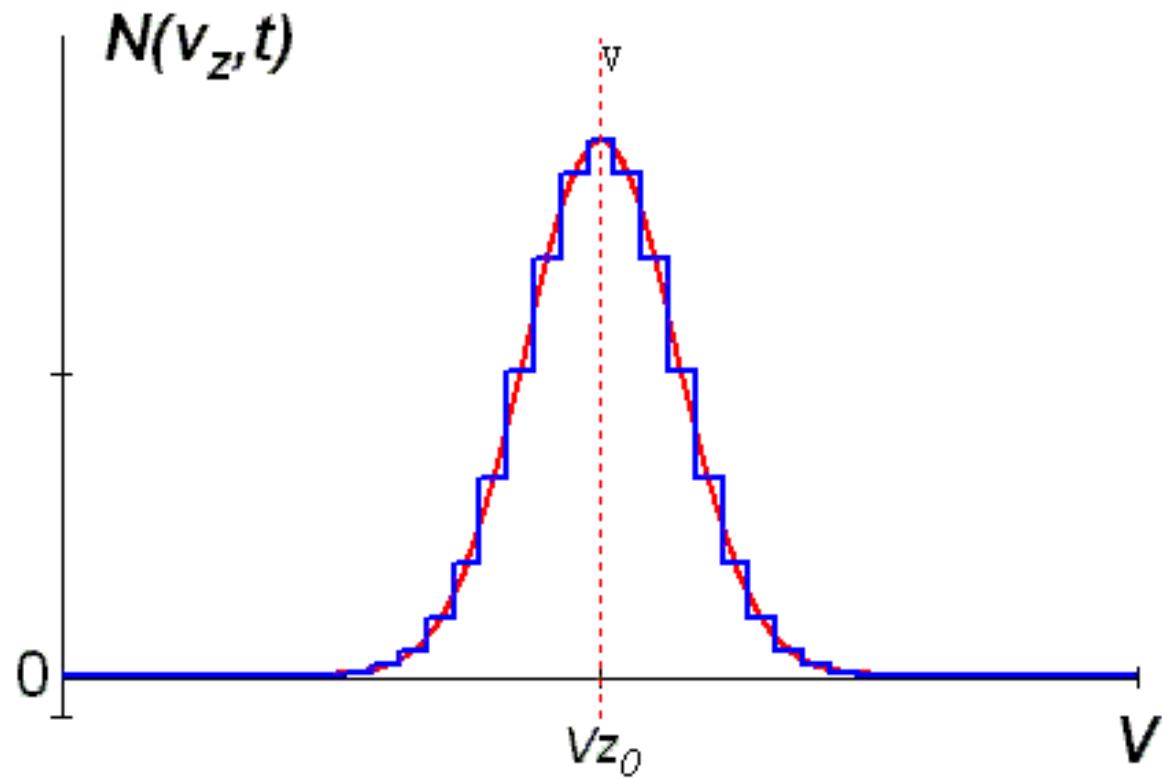
Current modulation



Velocity modulation



Current Fluctuations (Shot Noise)



Velocity Fluctuations

