Analytical and Numerical Models of Longitudinal Space-Charge Induced Optical Microbunching

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Microbunching Instability III
Frascati,
March 24, 2010

The Problem

-Observed coherent *optical* transition radiation (COTR) from FEL injector beams

-Some structure formation in beam at microscopic (<μm) level; near the mean inter-particle distance. How?

-Phenomenon related to transversal of dispersive sections

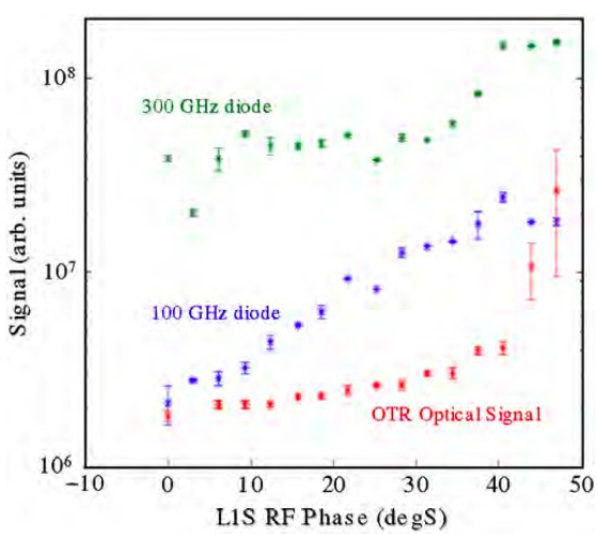
-Longitudinal and transverse spectra show stochastic behavior, 3D effects

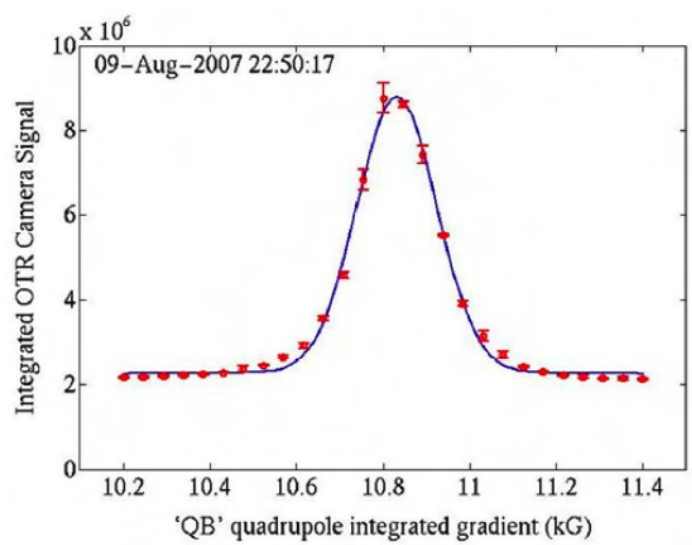
Observations

-Data from LCLS, DESY FLASH, ANL, etc…

-*No COTR* upstream of bends

-Large enhancement possible after 1st bends





D. Dowell, *et al. PRST-AB* 11, 030703 (2008)

Spectral information constrains micobunching models: FLASH example

- High resolution COTR exp'ts reported
  - FEL 2008, FEL 2009, by Schmidt, *et al.*
  - Multi-spike spectra
  - Transverse imaging indicates *not* simple 1D bunching

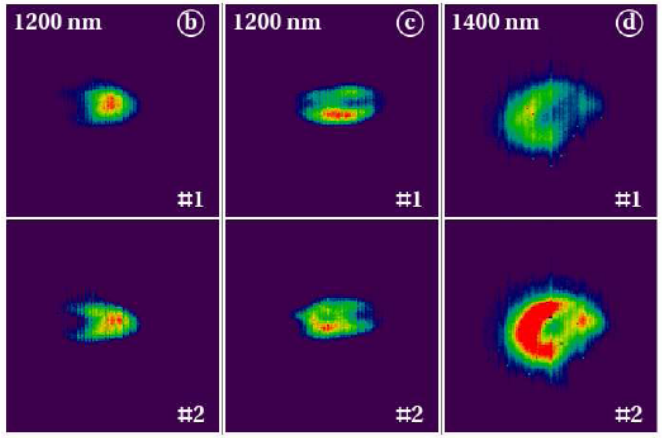


Figure 7: Selection of transverse profiles at different on-crest configurations and various spectral filters. The dimensions of the images are $2 \times 2\,\mathrm{mm}^2$

Schmidt, et al., FEL 2009 (WEPC50)

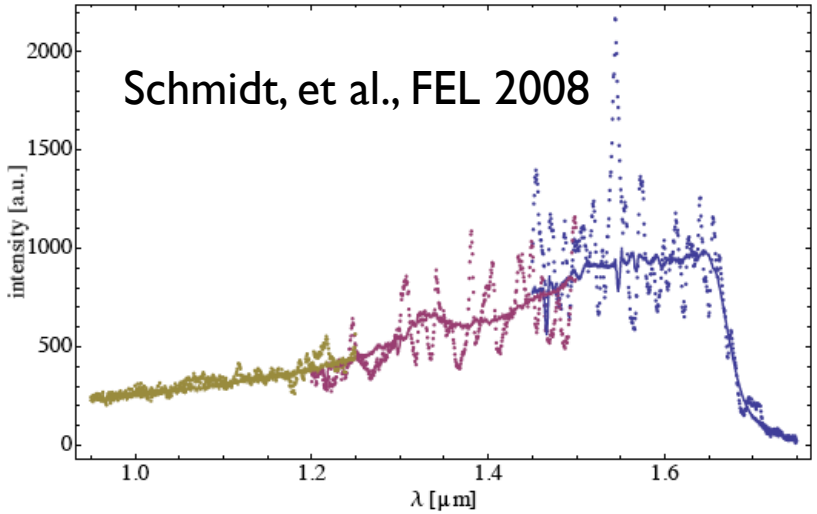


Figure 7: Single shot (dots) and averaged (line) CTR spectrum between 0.95 μm and 1.7 μm measured with a commercial InGaAs spectrograph. The response of the detector is basically flat between 1.0 μm and 1.7 μm.

Previous Theoretical Work

September 2008
SLAC-PUB-13392

THREE-DIMENSIONAL ANALYSIS OF LONGITUDINAL SPACE CHARGE MICROBUNCHING STARTING FROM SHOT NOISE*

D. Ratner, A. Chao, Z. Huang†
Stanford Linear Accelerator Center, Stanford, CA 94309, USA

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **11**, 034401 (2008)

Models of longitudinal space-charge impedance for microbunching instability

Marco Venturini*
Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720, USA
(Received 20 August 2007; published 25 March 2008)

"Collective-Interaction Control and Reduction of Optical Frequency Shot Noise in Charged-Particle Beams"
A. Gover and E. Dyunin
Phys. Rev. Lett. 102, 154801 (2009)

The High Frequency Limit of the Space-Charge Fields

In microbunching instability theory, a relevant quantity is the longitudinal fourier transform of the longitudinal electric field

$$E(k,\vec{r}) = \frac{-eik}{2\pi\gamma^2\varepsilon_0}\sum e^{-ikz_i} K_0\left(\frac{k\left|\vec{r}-\vec{r}_i\right|}{\gamma}\right)$$

The modified bessel function falls off exponentially for values of the argument bigger than 1.
We can identify two limits:

High Frequency Limit

Relevant Limit for Optical Microbunching

The High Frequency Limit of the Space-Charge Fields

In the High Frequency Limit, the Fourier transform of the field is composed of several coherent spots uncorrelated with respect to each other.

$$L_c \approx \frac{\gamma\lambda}{4}$$

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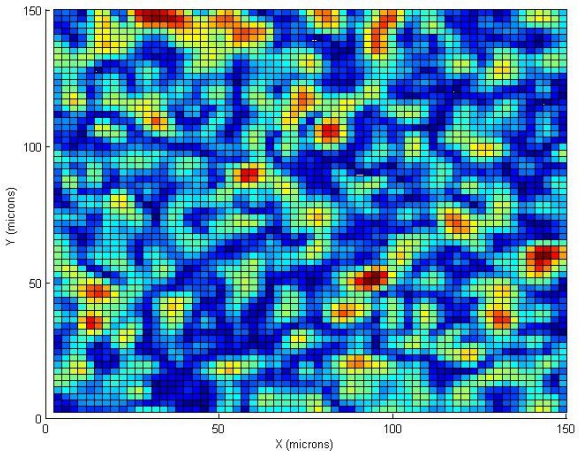
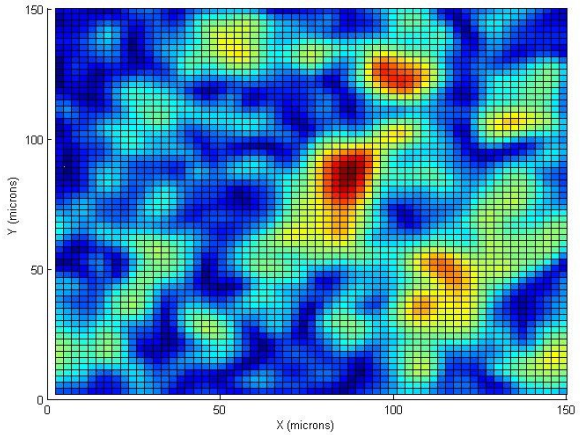
Examples for γ = 100

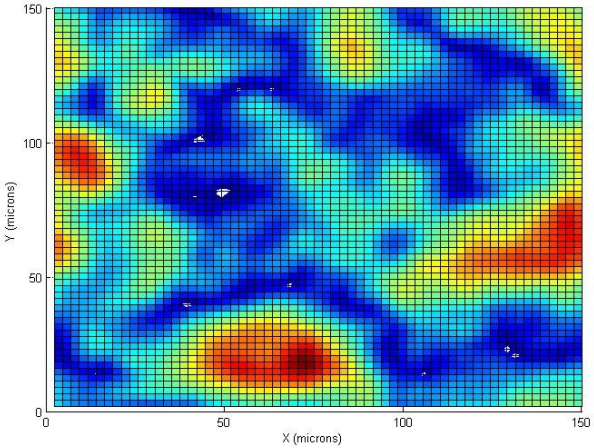
λ = 0.12 μm



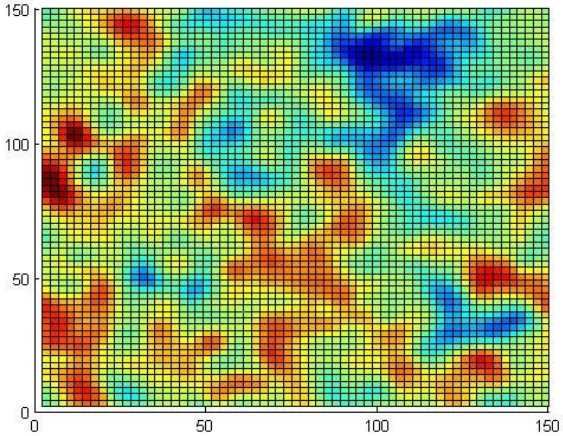
λ = 0.6 μm



Electric Field Amplitude in X-Y plane



λ = 1.2 μm



Space Domain

Micro-Bunching Instability in the High Frequency Limit

Full three dimensional problem: angular dependence of the micro-bunching needs to be understood
(IMPORTANT FOR COTR MEASUREMENTS!)

$$b = \sum e^{i(kz_n + k\theta x_n)}$$

Physical Picture:

beam strongly focused through a drift

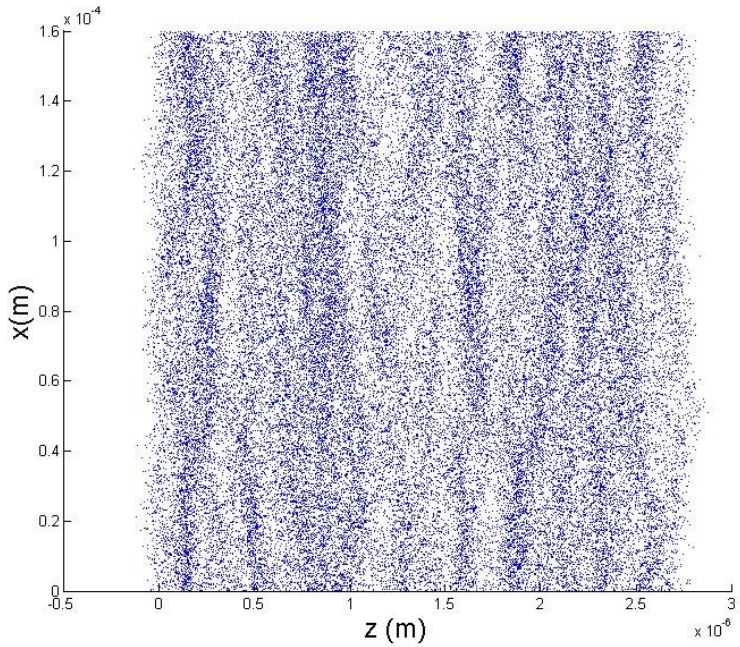
↓

Space-charge forces generated by shot-noise induce energy modulation

↓

Longitudinal and transverse position rearranged by optical elements: $R_{i,j}$

Laminar Beam Theory



In the laminar beam approximation, the transverse structure of the energy modulation matches that of the electric field

$$G \propto \left(\frac{1}{+\ \gamma\vartheta^{2}} \right)$$

R12 and R34 terms after the drift reduce angular width due to transverse phase mixing but the intrinsic angular width of the energy modulation is $1/\gamma$

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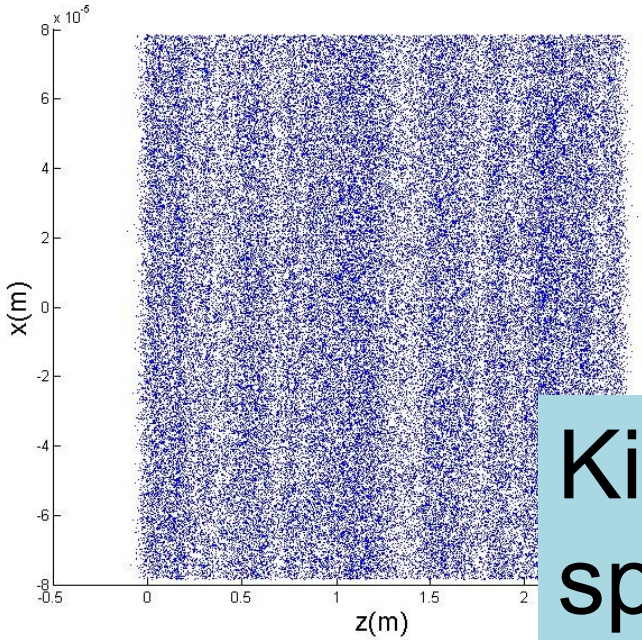
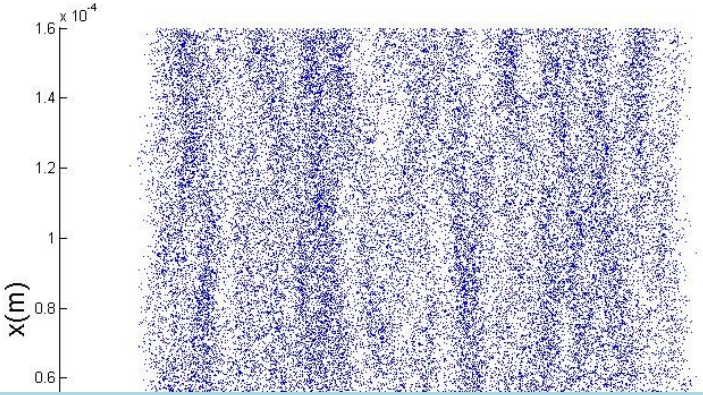
Thermal Effects in the Drift

Transverse thermal motion during the drift smoothes the transverse distribution of the microbunching: narrower angular width of energy modulation and microbunching gain





With Emittance

Kinetic treatment of space-charge interactions needed

The Landau Problem for Ultra-Relativistic Electrons

-Linearized Collisionless Vlasov's Equation

$$\frac{\partial f_1}{\partial \tau} + \vec{\beta}_\perp \cdot \nabla_\perp f_1 + \frac{p}{\gamma^2}\frac{\partial f_1}{\partial z} - \frac{eE_z}{\gamma m c^2} n_0 \frac{\partial f_v}{\partial p} = 0$$

-Poisson's equation in rest frame

$$\left[\nabla_\perp{}^2 + \frac{1}{\gamma^2}\frac{\partial^2}{\partial z^2}\right]\phi = \frac{e}{\gamma \epsilon_0}\int f_1 dp d^2\vec{\beta}$$

$$E_z = -\frac{1}{\gamma}\frac{\partial \phi}{\partial z}$$

Lorentz invariance of longitudinal Electric field

The Landau Problem for Ultra-Relativistic Electrons

Assumptions:

-3D limit of space-charge forces
-System isotropic in the transverse plane
-Paraxial approximation
-Neglect transverse forces

$$\hat{f}_1 = \int f_1 e^{-i\left(k_z z + \vec{k}_\perp \cdot \vec{x}\right)} dz d^2\vec{x}$$

$$\tilde{\hat{f}}_1 = \int_0^\infty \hat{f}_1 e^{-s\tau} d\tau$$

$$k_z = k \quad k_x = \theta k$$

$$s\tilde{\hat{f}}_1 - \hat{f}_1\Big|_{\tau=0} + i(k\theta\beta_x + k\frac{p}{\gamma^2})\tilde{\hat{f}}_1 - \frac{e\tilde{\hat{E}}_z}{\gamma m c^2} n_0 \frac{\partial f_v}{\partial p} = 0$$

$$\tilde{\hat{E}}_z = -\frac{i}{k}\frac{e}{\epsilon_0}\frac{1}{1+(\gamma\theta)^2}\int \tilde{\hat{f}}_1 dp d^2\vec{\beta}$$

The Landau Problem for Ultra-Relativistic Electrons

$$\tilde{\omega}_p^2 = \frac{e^2 n_0}{\epsilon_0 m \gamma^3} \frac{1}{1+(\gamma\theta)^2}$$

$$\tilde{\hat{f}}_1 = \frac{1}{s + ik(\theta\beta + \frac{p}{\gamma^2})} \left(\hat{f}_1 \Big|_{\tau=0} - \frac{1}{c^2} \frac{\tilde{\omega}_p^2}{} \frac{\partial f_v}{\partial p} \frac{\gamma^2}{ik} \int \frac{\hat{f}_1 \Big|_{\tau=0}}{s + ik(\theta\beta + \frac{p}{\gamma^2})} dp d^2\vec{\beta} \right)$$

$$\epsilon_p = 1 + \frac{\tilde{\omega}_p^2}{c^2} \frac{\gamma^2}{\mathrm{ik}} \int \frac{\frac{\partial f_v}{\partial p}}{s + \mathrm{ik}\left(\theta\beta + \frac{p}{\gamma^2}\right)} dp d^2\vec{\beta}$$

Plasma Dielectric Function with Longitudinal Laminarity

$$k\sigma_p/\gamma^2 << \tilde{\omega}_p/c$$

$$\epsilon_p = 1 - \frac{1}{K^2}\int_{\tilde{c}} \frac{\frac{\partial F}{\partial B}}{\frac{\Omega}{K} + B} dB$$

$$\tilde{k}_D = \tilde{\omega}_p/c\sigma_\beta$$

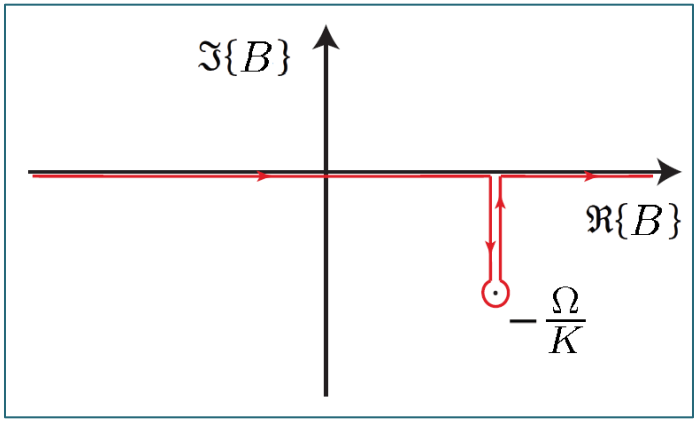
$$K = k\theta/\tilde{k}_D$$

$$\Omega = cs/i\tilde{\omega}_p$$

$$F = \frac{1}{(2\pi)^{1/2}} e^{-\frac{B^2}{2}}$$

The dielectric function has an infinite number of zeros but we will only take into account the two dominant ones (smallest damping coefficient)

Landau Contour

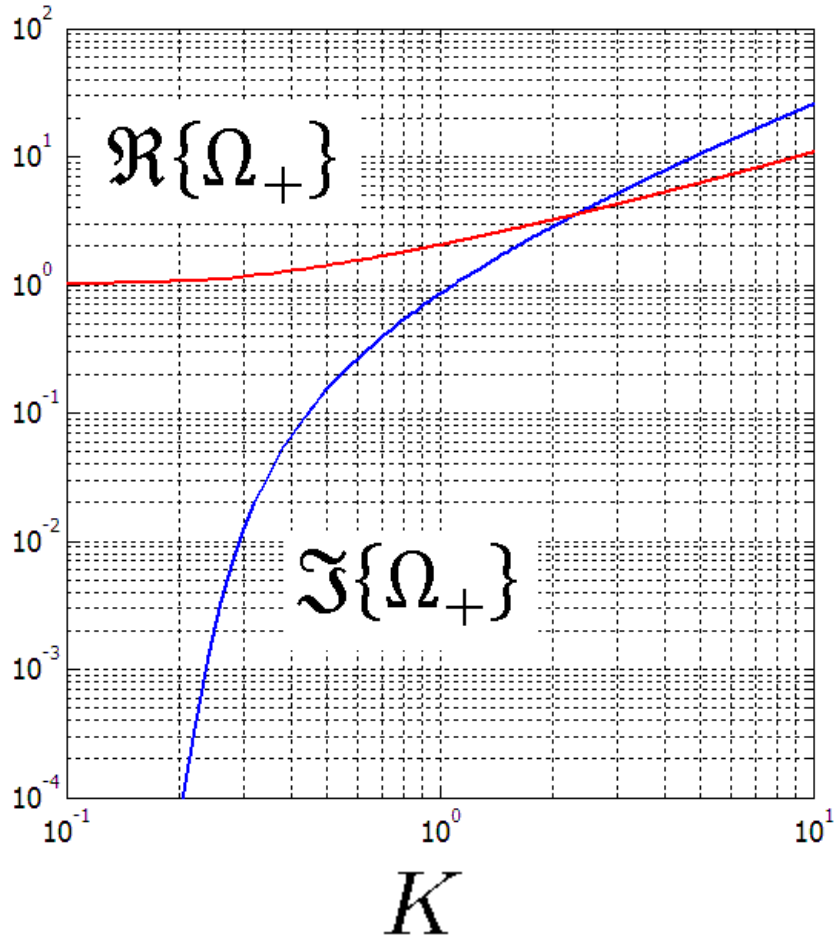


Roots have to be found numerically

$$\Omega_+ = \Omega_R + i\Omega_I$$

$$\Omega_- = -\Omega_R + i\Omega_I$$

Roots of the Plasma Dielectric Function



$$\Omega = cs/i\tilde{\omega}_p$$

For ***K*** > 1 response is

STRONGLY DAMPED !

J. D. Jackson,

"Longitudinal Plasma Oscillations"

Journal of Nuclear Energy C
Vol 1, number 4, 1960

Physical Interpretation

For ***K > 1*** damping term is important even for drift lengths that are much smaller than the plasma oscillation period (as usually assumed in Micro-Bunching Instability Problems).

K > 1 means $k\theta > k_d$

Loosely Speaking:

Transverse thermal motion limits energy modulation to transverse scales bigger than:

$$\lambda_d / 2\pi = c\sigma_\beta / \omega_p$$

Due to the intrinsic cut-off angle of single electron longitudinal field, thermal effects are important only if:

$$k\sigma_\beta > \gamma\omega_p / c \quad (\text{or } \lambda_d > \gamma\lambda)$$

Micro-Bunching After Longitudinal Dispersion

For the moment we only keep the R56 term

$$b_{R_{56}} = i\frac{\tilde{\omega}_p}{c}\gamma^2 R_{56} e^{-\frac{(k\sigma_p R_{56})^2}{2}}$$

$$\frac{1}{N}$$

$$\frac{K}{1-\frac{\Omega_j}{1+K^2}}$$

Summation over two dominant roots

Coupling between shot-noise and plasma oscillation modes:

needs a statistical treatment (compute square and perform statistical averaging)

$$< |b_{R_{56}}|^2 > = \frac{1}{N}\left(\frac{\tilde{\omega}_p}{c}\gamma^2 R_{56}\right)^2 e^{-(k\sigma_p R_{56})^2} \int dB F(B)$$

$$\left(\sum_j e^{i\Omega_j \frac{\tilde{\omega}_p}{c} L_d} \frac{K}{1-\frac{\Omega_j^2}{1+K^2}} \frac{1}{\frac{\Omega_j}{K}+B}\right)$$

$$\left(\sum_{j'} e^{i\Omega_{j'} \frac{\tilde{\omega}_p}{c} L_d} \frac{K}{1-\frac{\Omega_{j'}^2}{1+K^2}} \frac{1}{\frac{\Omega_{j'}}{K}+B}\right)^*$$

Laminar Beam Limit (or Small Observation Angle Limit)

For ***K << 1*** we recover a known result
(up to a geometric factor of 1/3 due to the assumptions on the 0-th order charge distribution)

$$g = \left(\left(\frac{\gamma \omega_p}{c} \right)^2 \frac{1}{1 + (\gamma\theta)^2} R_{56} L_d \right)^2 e^{-(k\sigma_p R_{56})^2}$$

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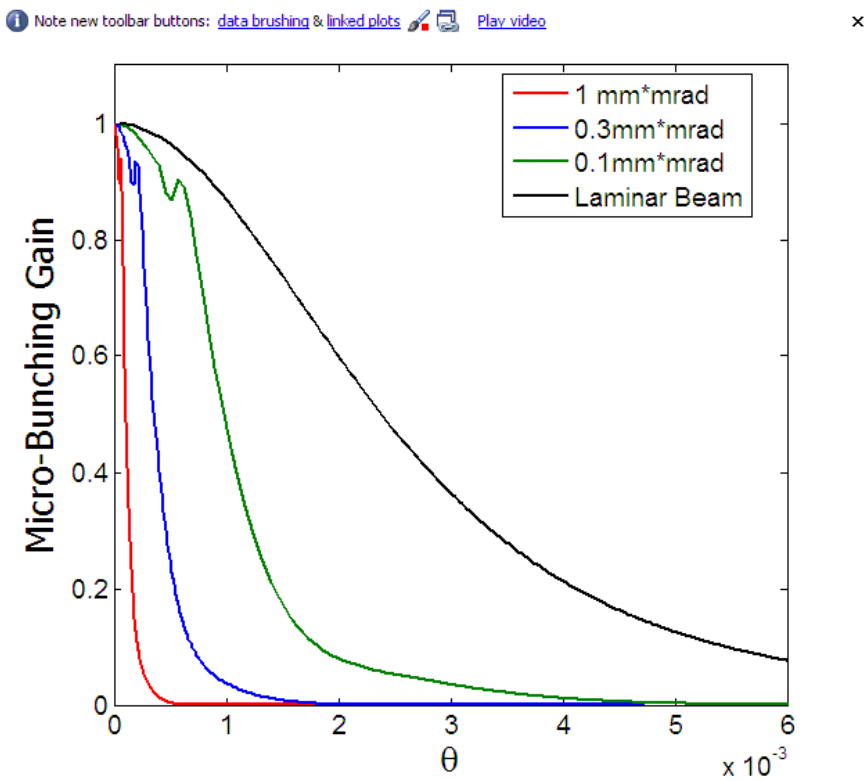
Numerical Example

σ_x=85μm
I = 40 A
γ= 270
λ_p = 34 m
L_d = 4 m



Molecular dynamics simulations

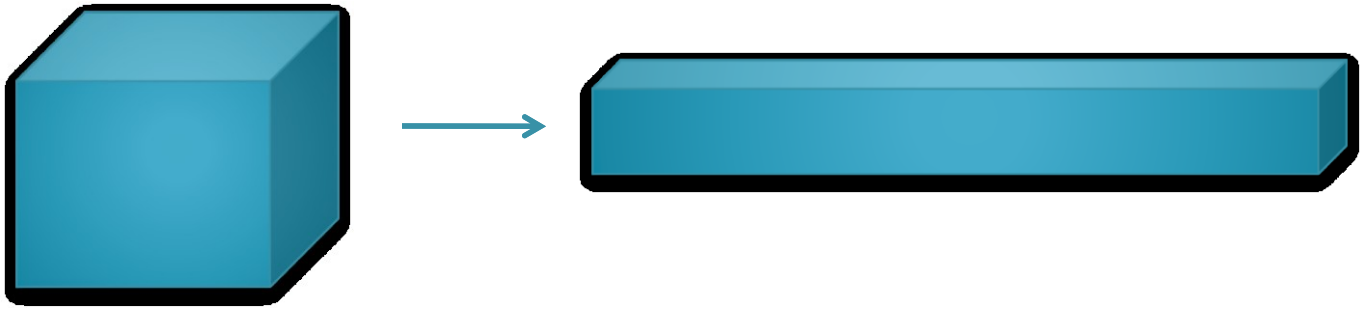
-Resolution well below λ needed

-Particle to Particle simulation for broad-band shot noise statistics

-Like to have full beam 3D geometry

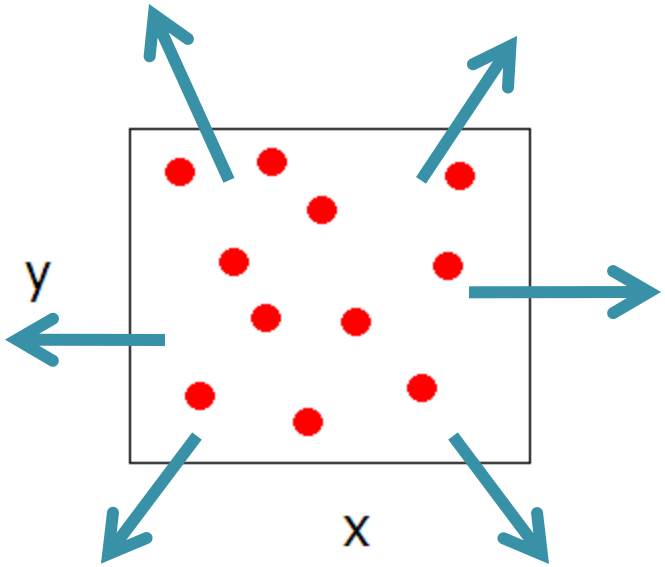
(Computationally intensive!)

-1st pass: simulate a small deformable box with periodic boundary conditions



Periodicity in 3D for High Resolution Simulations

Particle to particle simulation and high resolution needed for phenomena on the scale of the mean int. dist.



An unphysical outward pressure would cause the beam to expand if only a small fraction of the beam was simulated

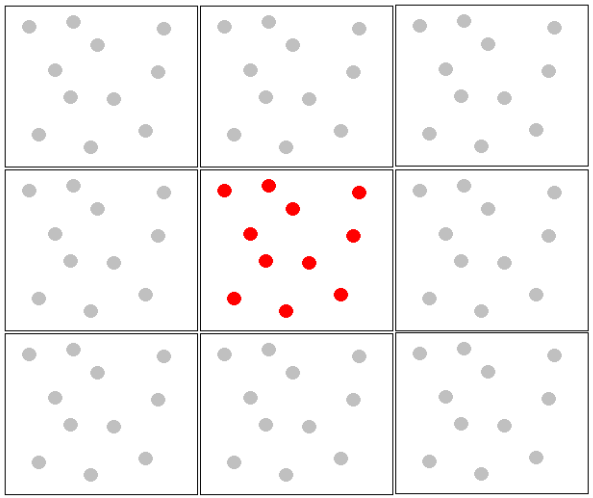
Interested in simulating a small fraction of the beam for high resolution.

Pressure from the rest of the beam needs to be included…

Periodicity in 3D for High Resolution Simulations

Periodicity in 3D: avoid beam expansion in transverse dimension.

Electrons effectively in the center of the beam!

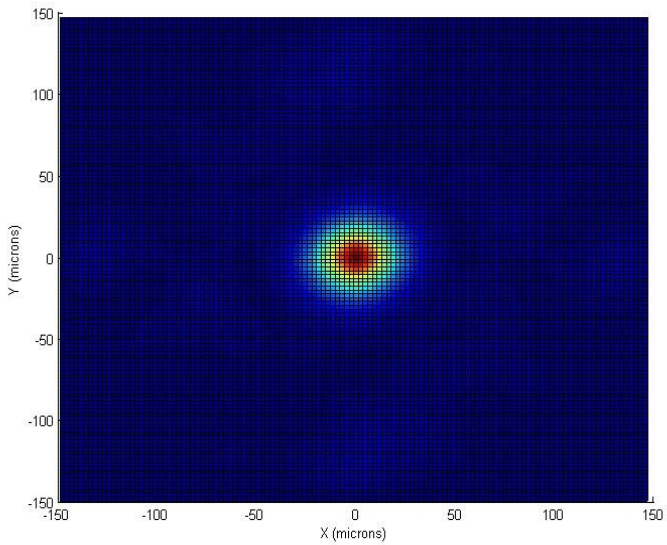


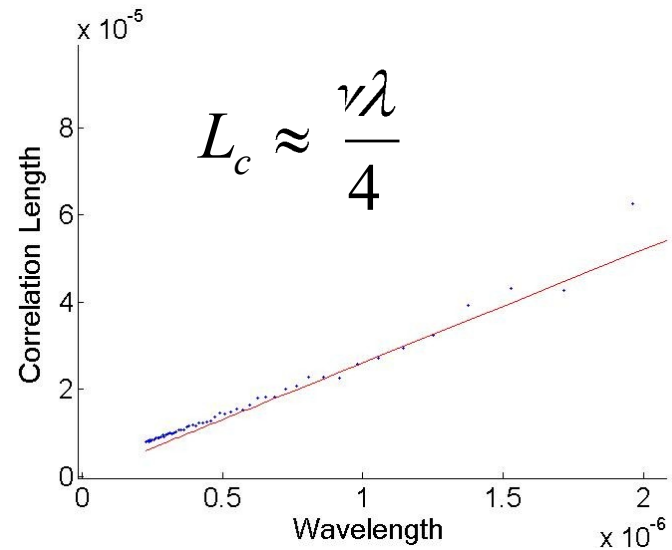
Periodicity enforced using Fourier methods for space-charge force calculation.

Field calculated in the rest frame and then Lorentz-transformed back to the Lab Frame

$$\rho(\vec{x}) \xrightarrow[\text{FFT}]{} \tilde{\rho}(\vec{k}) \longrightarrow \tilde{E}(\vec{k}) = \frac{i\vec{k}\,\tilde{\rho}(\vec{k})}{k^2} \xrightarrow[\text{IFFT}]{} \vec{E}(\vec{x})$$

Correlation Length





$$L_c \approx \frac{\gamma\lambda}{4}$$

Example: Self Correlation at 0,6μm for γ = 100

Computed Correlation Length vs Theoretical Curve

Beam Coordinate System and Forces

Electron dynamics studied in the electron beam coordinate system:

$$x' = x$$

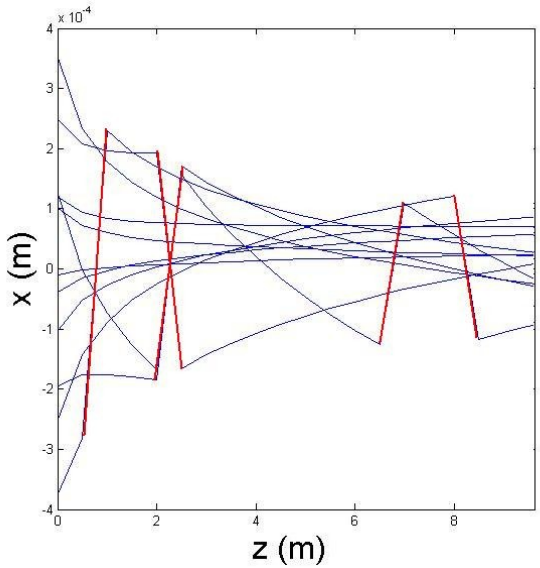
$$y' = y$$

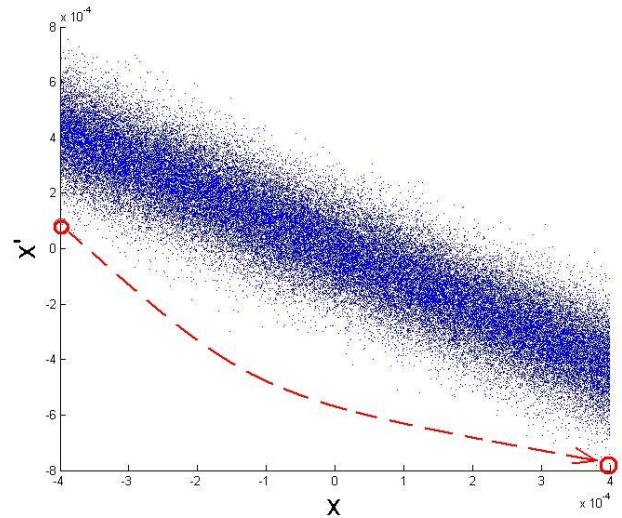
$$z' = z - vt$$

Equations of motion include:

-Microscopic space-charge forces (from FFT)
-Solenoid and RF focusing
-Macroscopic transverse space-charge
-Acceleration

Emittance Effects: Crossing The Box

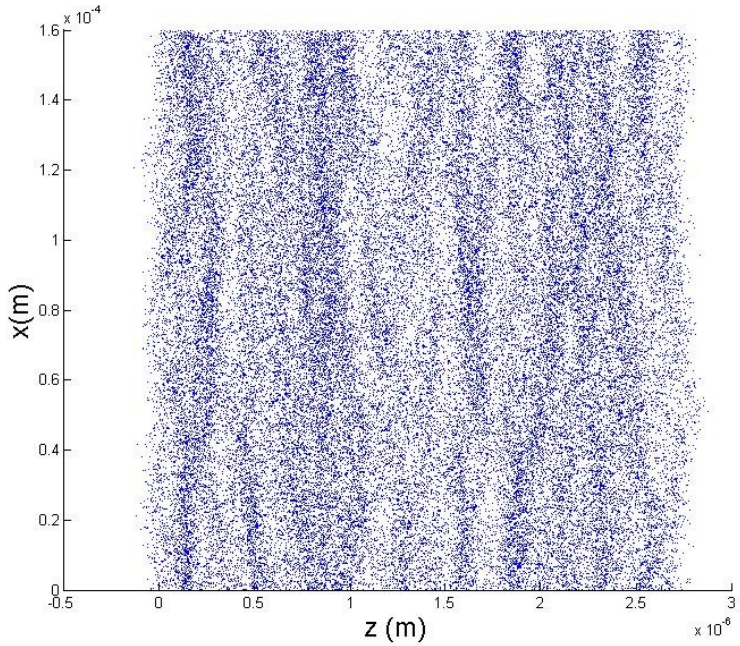




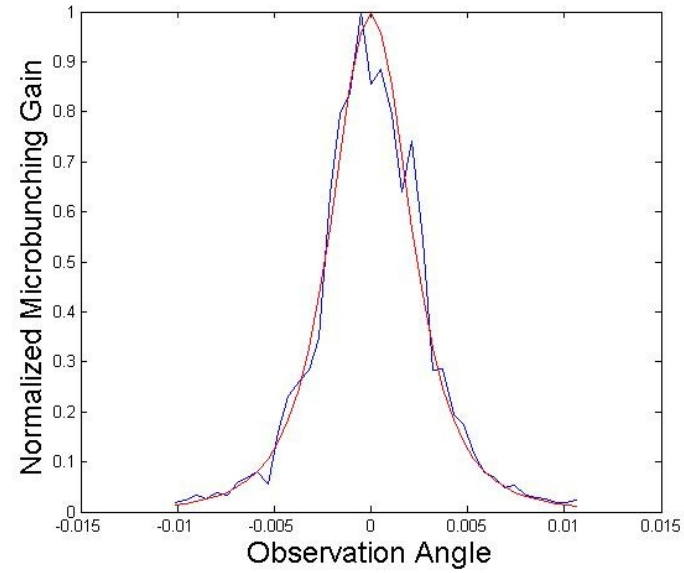
Particles hitting the box reappear on the opposite side consistently with periodic boundary conditions.

Correlated velocity spread is subtracted to velocity to preserve emittance.

Beam Passing Through a Waist



Example with no emittance

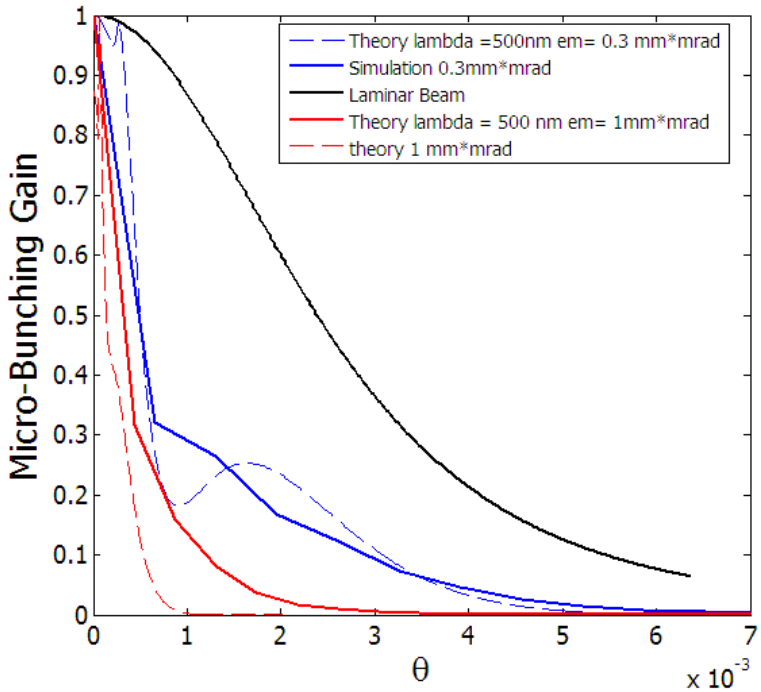


Angular dependence of the microbunching vs theory

$$G = \left(\frac{1}{+ \gamma \vartheta^{2}} \right)^{2}$$

Non-Laminar Beam

σ_x=85 μm
λ = 0.5 μm
I = 40 A
γ= 270
λ_p = 34 m
L_d = 2.5 m



Extension of the Code

Periodicity in three dimensions limits the use of the code only to high frequencies.

$$\sigma_x >> \lambda\gamma$$

A new code with periodicity only in the Z direction is currently under consideration:

-to extend the study of microbunching instabilitiy to lower frequencies.

-To properly treat the effect of transverse thermal motion on the angular distribution of microbunching.

Extension of the Code

$$(\nabla_{\perp} - k_z^2)\varphi(\vec{r}, k_z) = -\rho(\vec{r}, k_z)$$

Several options are considered for the field solver:

-Orthogonal Function Expansion
(Laguerre Gaussian Modes)

-Finite Differences

-Finite Elements

Possibility of exploiting the geometry of the Coulomb field in the frequency domain? (lower frequencies require a lower transverse resolution!).

Conclusions

- A microscopic approach to the theory of space-charge induced optical micro-bunching based on the plasma dielectric function has been developed.
- The model illuminates the role of transverse thermal motion in the formation of micro-bunching.
- Though an exact solution of the problem requires some numerical computational effort, the physics behind this phenomenon can be easily understood in terms of the scaling of the Landau damping coefficient with the dimensionless parameter K.
- A molecular dynamics numerical code for the styudy of high frequency space-charge related phenomena has been developed.
- The code has been matched to previous theories and used to confirm those of the non-laminar beam theory.

Future Work

- The present theory can be easily extended to include more matrix elements than R56 (for example R12 and R34 responsible for phase mixing and R11 andR33 that account for transverse magnification).
- Application of theory to noise-reduction schemes.
- The development of a theory based on orthogonal function expansion to account for finite beam size effects will be the subject of future investigation.
- To overcome the limitations of the current implementation, development of a new code with periodicity in one dimension is foreseen.

Acknowledgements

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