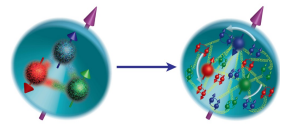


University of Pavia

TMDs at Jlab: present and future

December 19-20, 2018



Mapping the kinematic regions of SIDIS

Mariaelena Boglione



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DEGLI STUDI
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ALMA UNIVERSITAS
TAURINENSIS

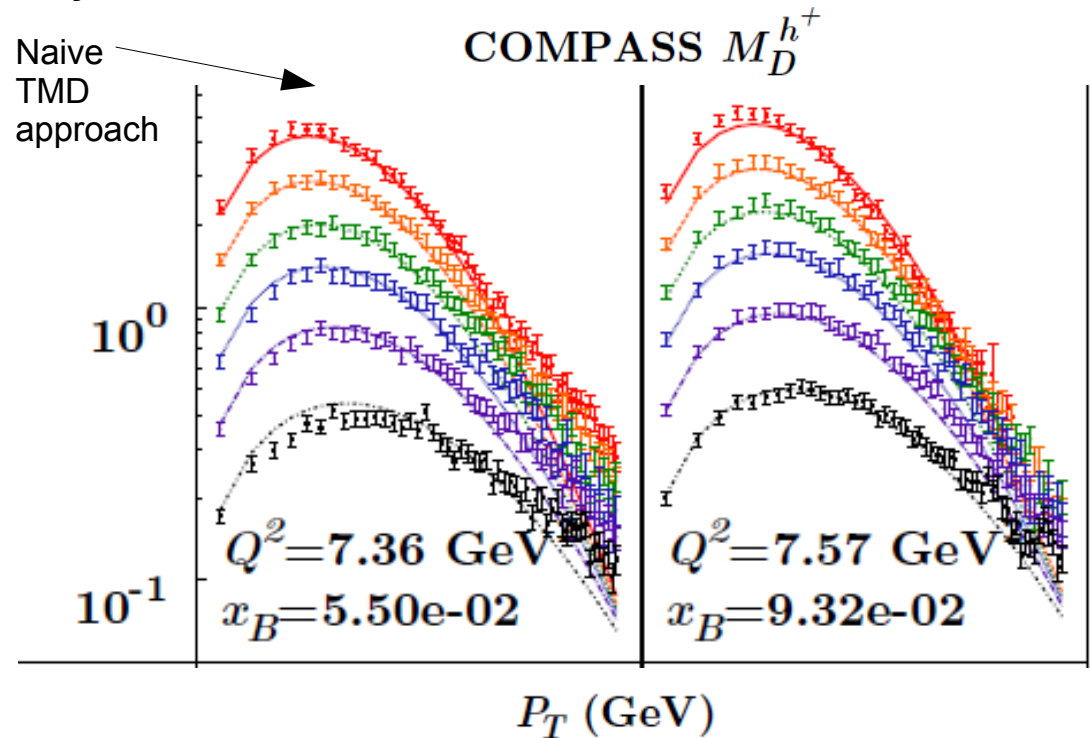
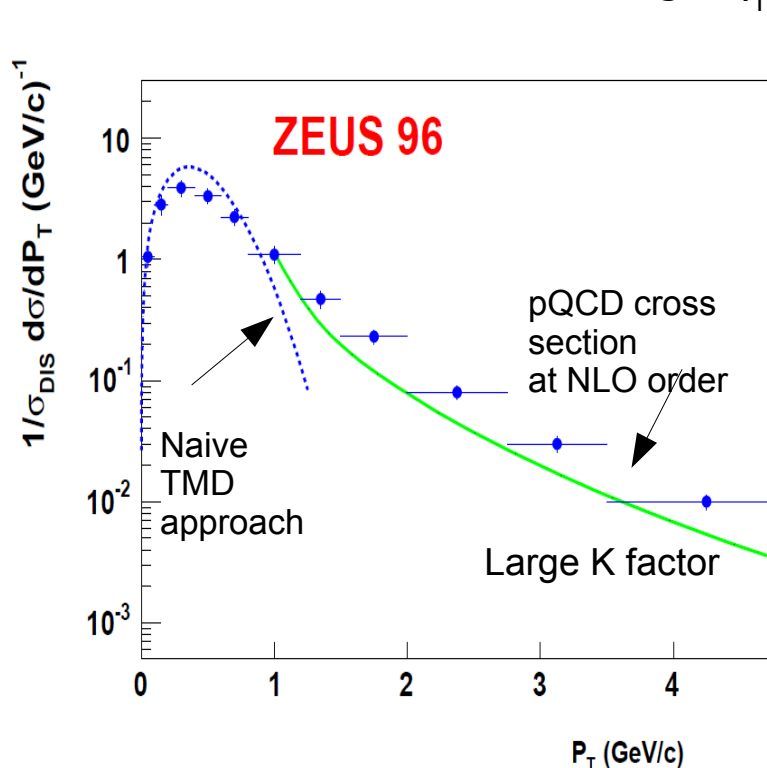


*In collaboration with J.O. Gonzalez Hernandez, S. Melis and A. Prokudin
and with J. Collins, L. Gamberg, T. Rogers, N. Sato, R. Taghavi*

TMD factorization in SIDIS

As mentioned above

- fixed order pQCD calculation fail to describe the SIDIS cross sections at small q_T ,
- the cross section tail at large q_T is clearly non-Gaussian.



Anselmino, Boglione, Prokudin, Turk, *Eur.Phys.J. A31* (2007) 373-381

ZEUS Collaboration (M. Derrick), *Z. Phys. C 70*, 1 (1996)

Anselmino, Boglione, Gonzalez, Melis, Prokudin, *JHEP* 1404 (2014) 005

COMPASS, Adolph et al., *Eur. Phys. J. C 73* (2013) 2531

Need resummation of large logs and matching perturbative to non-perturbative contributions

Naive TMD approach

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

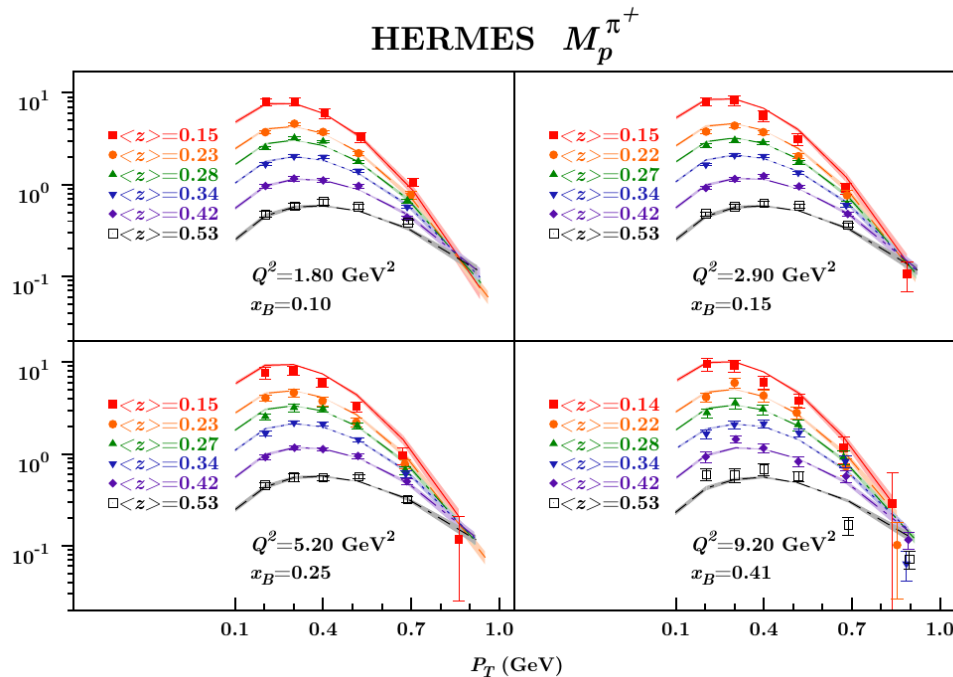
- Simple phenomenological ansatz can reproduce low q_T data

$$f_{q/p}(x, k_\perp) = f(x) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2 / \langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$



$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 1.69$$

Airapetian et al, Phys. Rev. D 87 (2013) 074029

Naive TMD approach

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, *JHEP* 1404 (2014) 005, ArXiv:1312.6261

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

$$\langle k_\perp^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

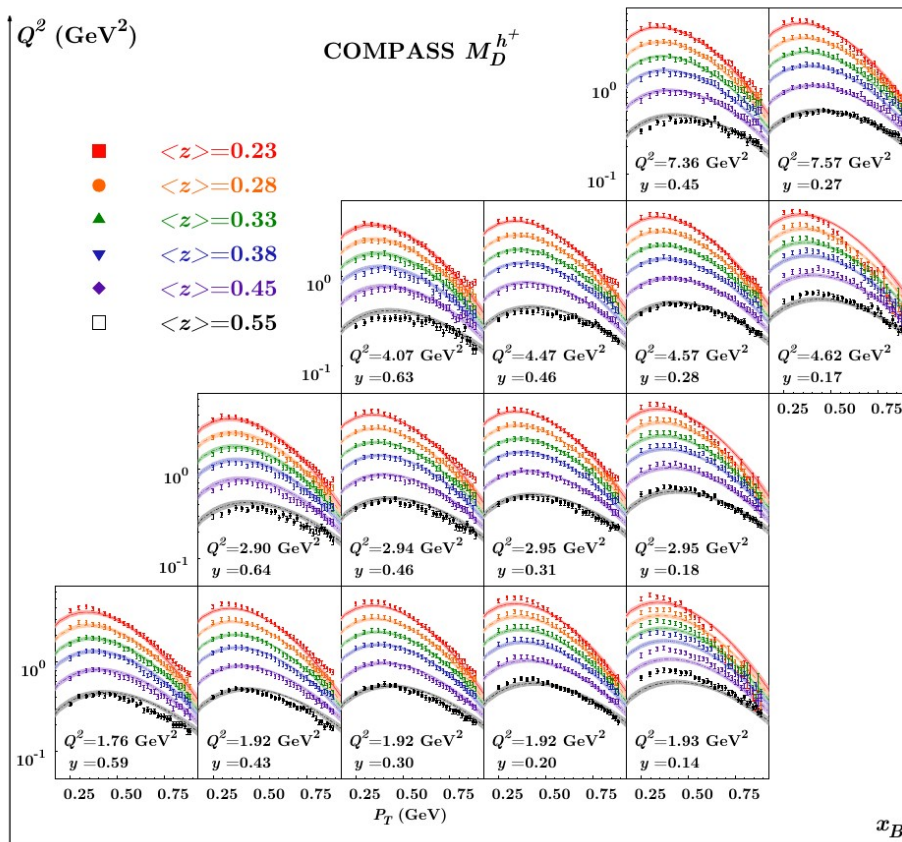
$$\chi_{\text{dof}}^2 = 3.42$$

Fit over 6000 data points with 2 free parameters

$$N_y = A + B y$$

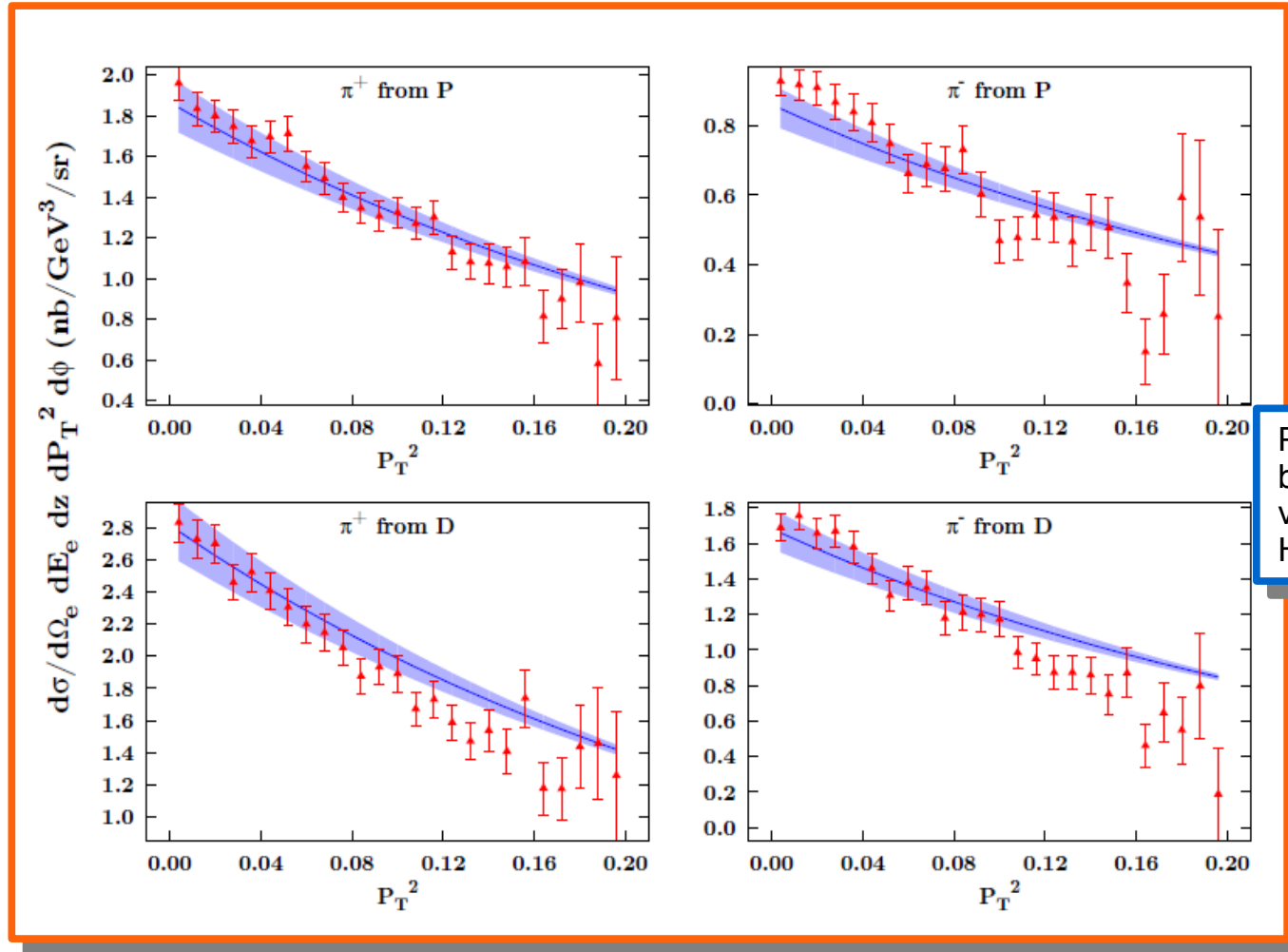
“The point-to-point systematic uncertainty in the measured multiplicities as a function of p_T^2 is estimated to be 5% of the measured value. The systematic uncertainty in the overall normalization of the p_T^2 -integrated multiplicities depends on z and y and can be as large as 40%”.

Erratum Eur.Phys.J. C75 (2015) 2, 94



Comparison with Jlab6 data - HALL C

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261



Predictions obtained by using the parameter values extracted from HERMES multiplicities

R. Asaturyan et al., Phys. Rev. C85, 015202 (2012)

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities: flavour dependence

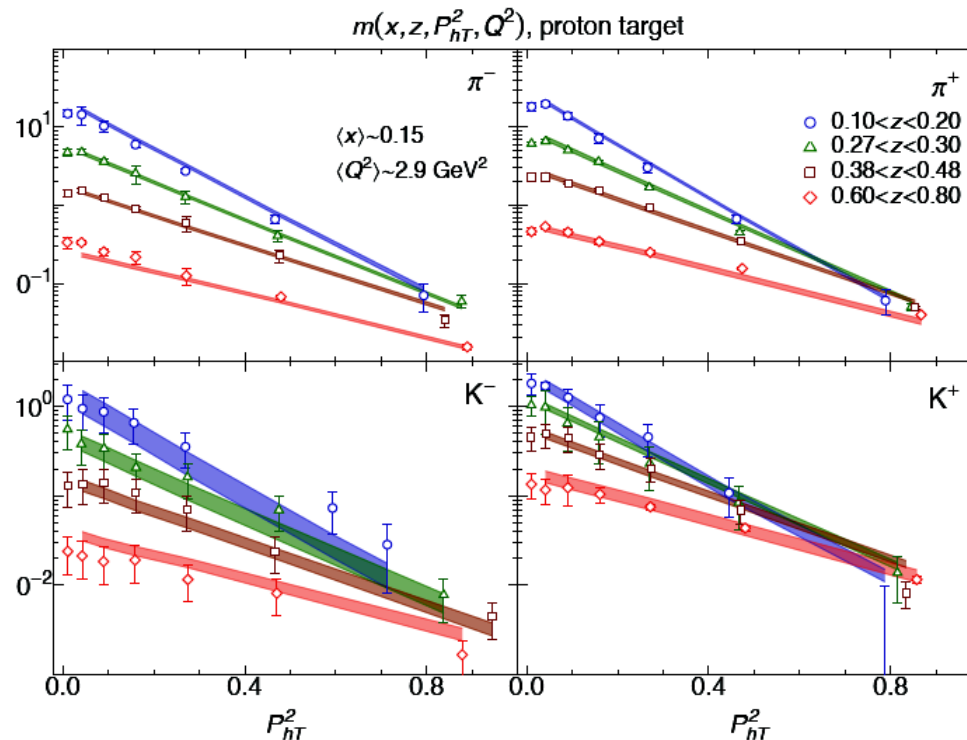
A. Signori, A. Bacchetta, M. Radici, G. Schnell, JHEP 1311 (2013) 194

proton target global $\chi^2 / \text{d.o.f.} = 1.63 \pm 0.12$
 no flavor dep. 1.72 ± 0.11

5/7 parameters
 Much more complex
 parametrization of x
 and z dependence

π^-
 1.80 ± 0.27
 1.83 ± 0.25

K^-
 0.78 ± 0.15
 0.87 ± 0.16



π^+
 2.64 ± 0.21
 2.89 ± 0.23

K^+
 0.46 ± 0.07
 0.43 ± 0.07

Resummation of large logarithms

- To ensure momentum conservation, write the cross section in the Fourier conjugate space

$$\delta^2(\mathbf{q}_T - \mathbf{k}_{1T} - \mathbf{k}_{2T} - \dots - \mathbf{k}_{nT} + \dots) = \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{b}_T \cdot (\mathbf{q}_T - \mathbf{k}_{1T} - \mathbf{k}_{2T} - \dots - \mathbf{k}_{nT} + \dots)}$$

$$\frac{1}{\sigma_0} \frac{d\sigma}{dQ^2 dy dq_T^2} = \left[\int \frac{d^2\mathbf{b}_T e^{i\mathbf{q}_T \cdot \mathbf{b}_T}}{(2\pi)^2} X_{div}(b_T) \right] + Y_{reg}(q_T)$$

$$X_{div}(b_T) \longrightarrow W(b_T) = \exp[S(b_T)] \times (\text{PDFs and Hard coefficients})$$

$$\frac{d\sigma^{total}}{dx dy dz dq_T^2} = \pi\sigma_0^{DIS} \int \frac{d^2\mathbf{b}_T e^{i\mathbf{q}_T \cdot \mathbf{b}_T}}{(2\pi)^2} W^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}(x, z, q_T, Q)$$

Resummed part

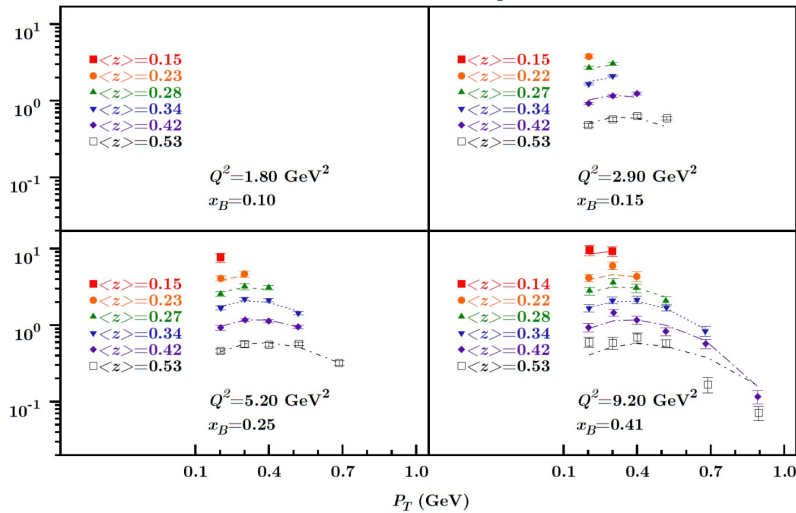
Regular part

Fit of HERMES and COMPASS data Attempting "Resummation" in SIDIS ...

J. Osvaldo Gonzalez Hernandez, work in progress

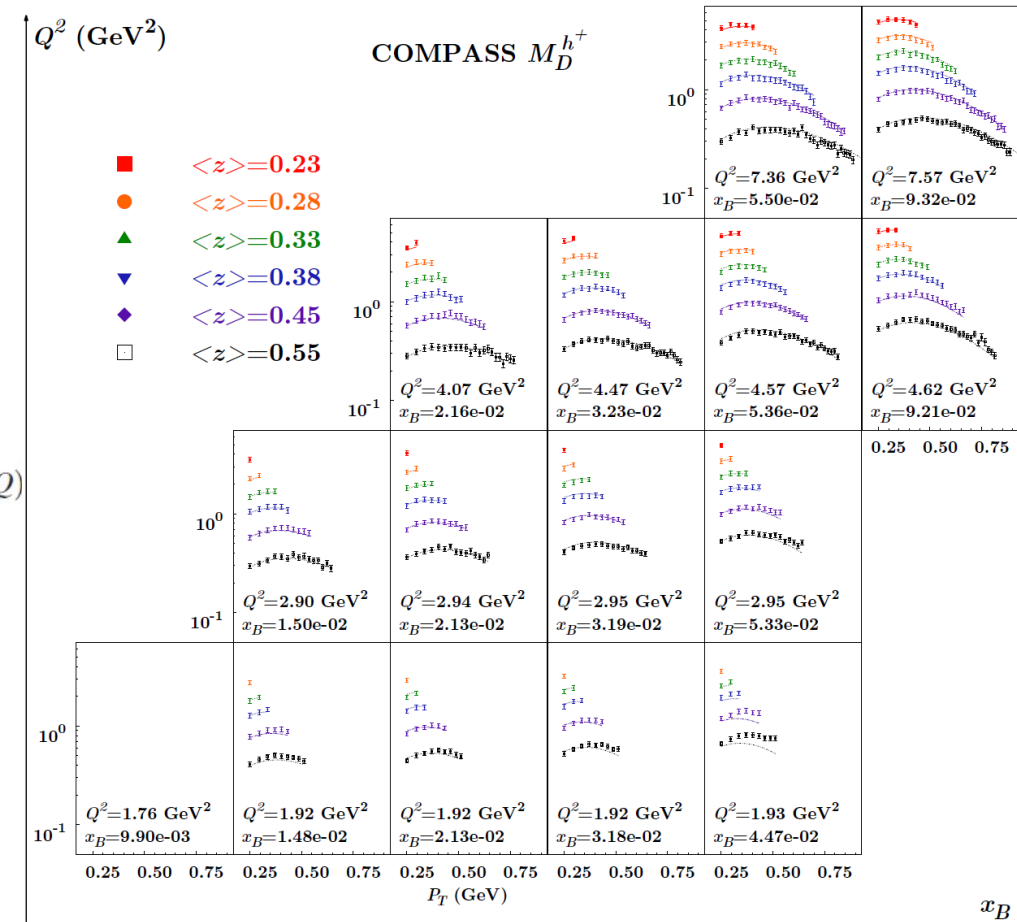
$$\chi^2_{\text{HERMES}} = 1.32$$

HERMES $M_p^{\pi^+}$



$$\chi^2_{\text{tot}} = 1.17$$

$$\chi^2_{\text{COMPASS}} = 1.12$$



$$\frac{d\sigma}{dx dy dz dq_T^2} = \pi\sigma_0^{DIS} \left\{ \int \frac{d^2\mathbf{b}_T e^{i\mathbf{q}_T \cdot \mathbf{b}_T}}{(2\pi)^2} W^{SIDIS}(x, z, b_*, Q, C_1, C_2, C_3) F_{NP}^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}(x, z, q_T, Q, C_4) \right\}$$

$$F_{NP}^{SIDIS}(x, z, Q) = \exp \left\{ \left[-\frac{g_1 + g_1 f/z^2}{2} - g_2 \ln(Q/(2Q_0)) - g_1 g_3 \ln(10x) \right] b_T^2 \right\}$$

- N ~ 2 (One overall normalization parameter is required)
- g1 ~ 0.5 (too large compared to the value extracted from DY data)
- g2 ~ 0.5
- g3 ~ -0.03

Global fits

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, JHEP06 (2017) 081

Extraction of partonic transverse momentum distributions from semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production

Alessandro Bacchetta,^{a,b} Filippo Delcarro,^{a,b} Cristian Pisano,^{a,b} Marco Radici^b and Andrea Signori^c

^aDipartimento di Fisica, Università di Pavia, via Bassi 6, I-27100 Pavia, Italy

^bINFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy

^cTheory Center, Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, U.S.A.

E-mail: alessandro.bacchetta@unipv.it, filippo.delcarro@pv.infn.it, cristian.pisano@unipv.it, marco.radici@pv.infn.it, asignori@jlab.org

ABSTRACT: We present an extraction of unpolarized partonic transverse momentum distributions (TMDs) from a simultaneous fit of available data measured in semi-inclusive deep-inelastic scattering, Drell-Yan and Z boson production. To connect data at different scales, we use TMD evolution at next-to-leading logarithmic accuracy. The analysis is restricted to the low-transverse-momentum region, with no matching to fixed-order calculations at high transverse momentum. We introduce specific choices to deal with TMD evolution at low scales, of the order of 1 GeV². This could be considered as a first attempt at a global fit of TMDs.

JHEP06 (2017) 081

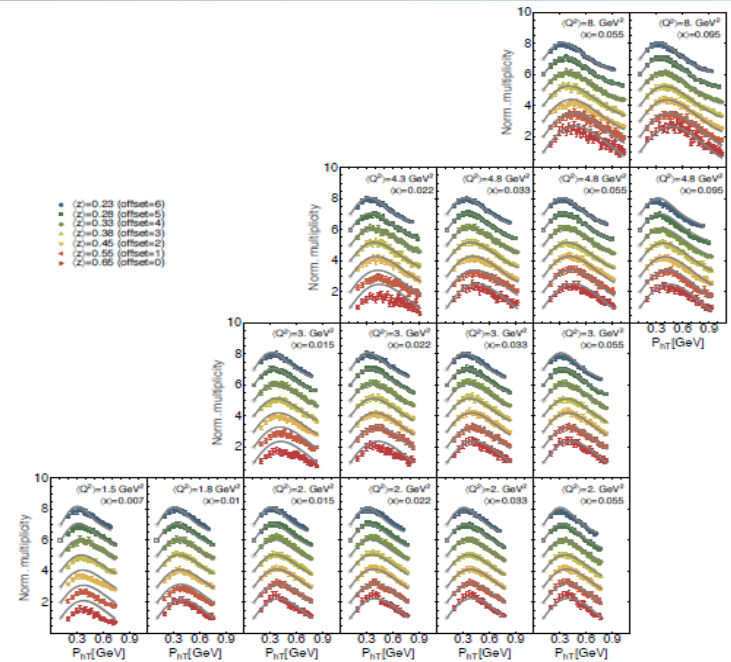


Figure 5. COMPASS multiplicities for production of negative hadrons (π^-) off a deuteron for different (x) , (z) , and (Q^2) bins as a function of the transverse momentum of the detected hadron P_{hT} . Multiplicities are normalized to the first bin in P_{hT} for each (z) value (see (3.1)). For clarity, each (z) bin has been shifted by an offset indicated in the legend.

$$\chi^2_{\text{tot}} = 1.55$$

- Y-term is neglected
- Sum of two Gaussian k_T distributions is introduced

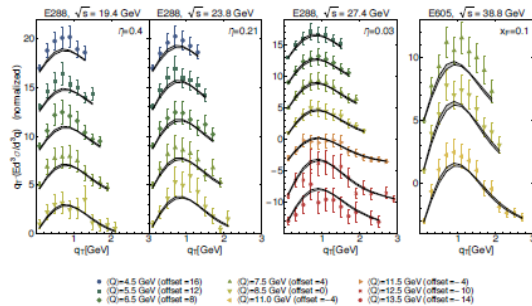


Figure 7. Drell-Yan differential cross section for different experiments and different values of \sqrt{s} and for different (Q) bins. For clarity, each (Q) bin has been normalized (the first data point has been set always equal to 1) and then shifted by an offset indicated in the legend.

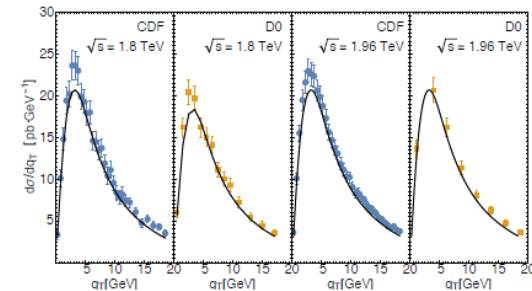


Figure 8. Cross section differential with respect to the transverse momentum q_T of a Z boson produced from pp collisions at Tevatron. The four panels refer to different experiments (CDF and D0) with two different values for the center-of-mass energy ($\sqrt{s} = 1.8$ TeV and $\sqrt{s} = 1.96$ TeV). In this case the band is narrow due to the narrow range for the best-fit values of g_2 .

Global fits

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, JHEP06 (2017) 081

Extraction of partonic transverse momentum distributions from semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production

Alessandro Bacchetta,^{a,b} Filippo Delcarro,^{a,b} Cristian Pisano and Andrea Signori^c

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^cTheory Center, Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, U.S.A.

E-mail: alessandro.bacchetta@unipv.it, filippo.delcarro@pv.infn.it, cristian.pisano@unipv.it, marco.radici@pv.infn.it, asignori@jlab.org

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Although the shape in transverse momentum space is well described, **normalization** is very problematic

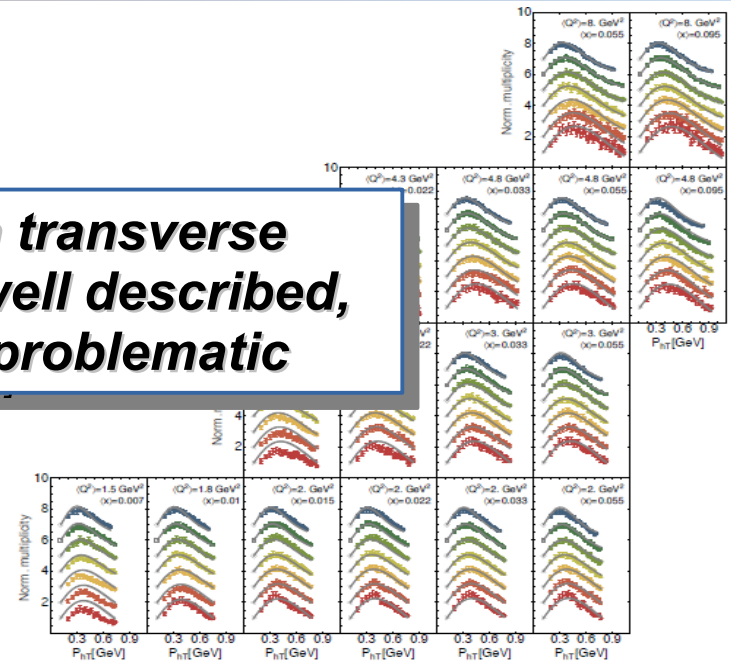


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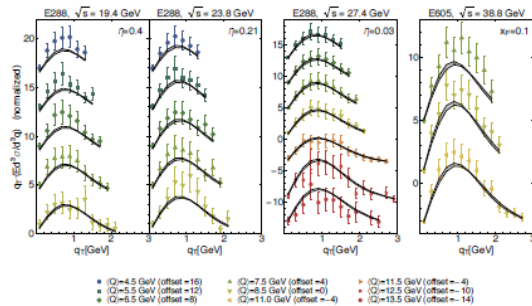


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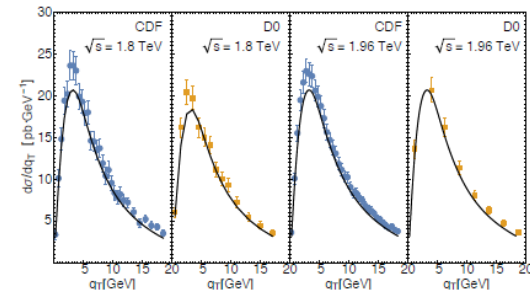
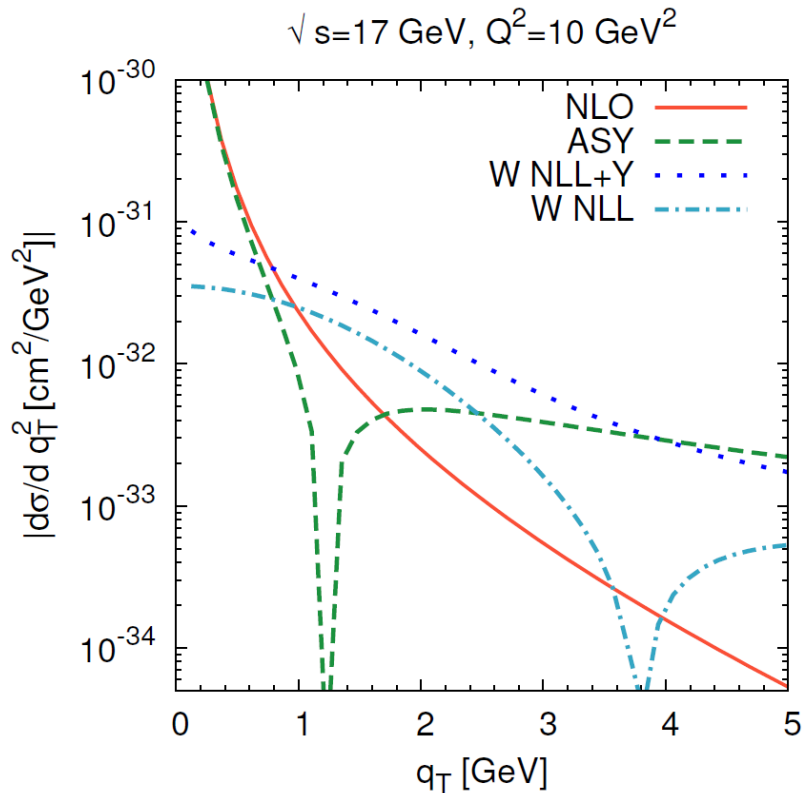


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SIDIS - Y factor



- The Y factor is very large (even at low q_T)
- However, it could be affected by **large** theoretical uncertainties

Boglione, Gonzalez, Melis, Prokudin, JHEP 02 (2015) 095

The Y factor cannot be neglected !!!

- New prescription for Y factor, b^* and W

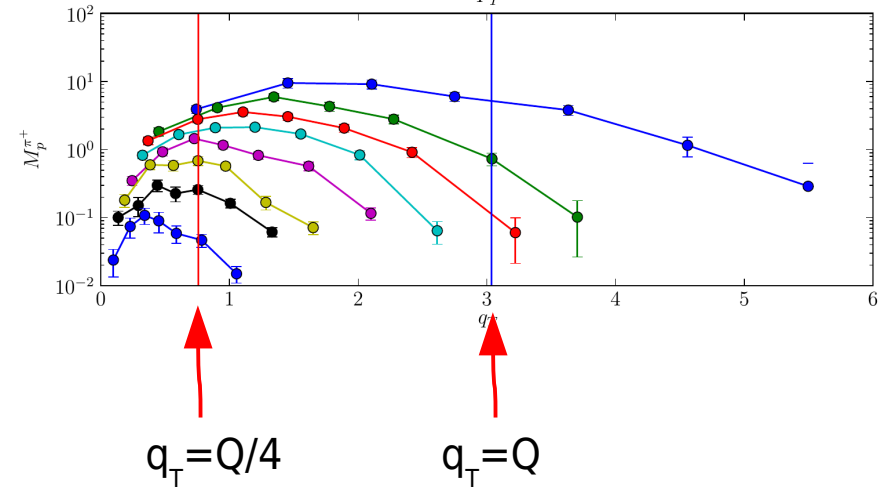
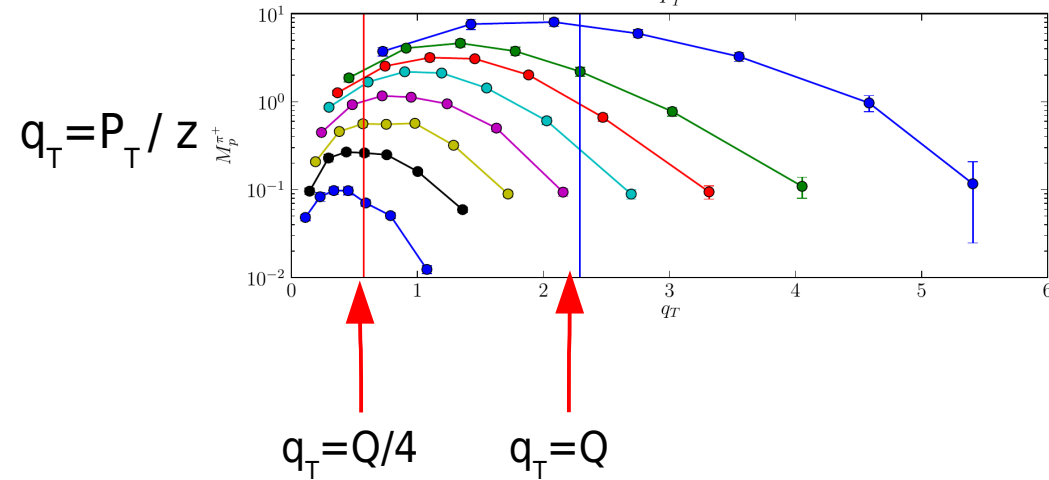
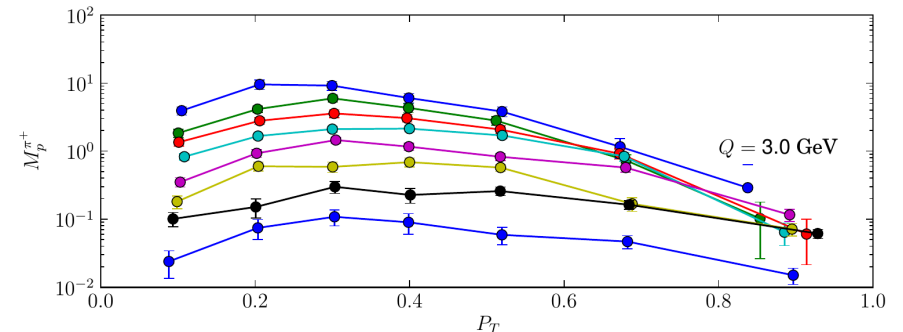
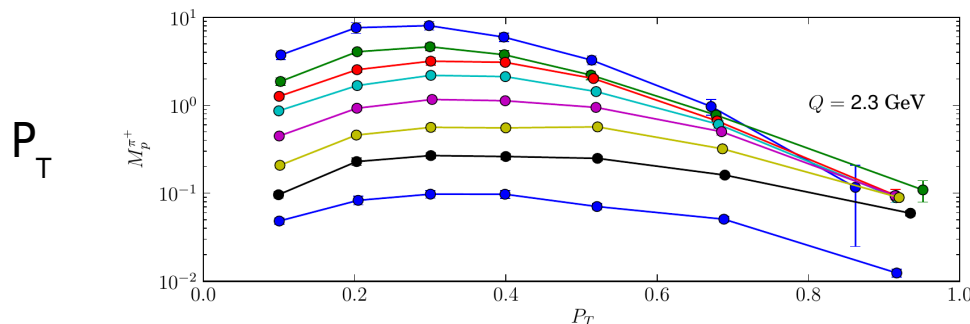
Collins, Gamberg, Prokudin, Rogers, Sato, Wang, Phys. Rev. D 94 (2016) 034014

$$\frac{d\sigma^{\text{NLO}}}{dx dy dz dq_T^2} = \frac{d\sigma^{\text{ASY}}}{dx dy dz dq_T^2} + \textcircled{Y}$$

$$\sigma^{\text{ASY}} = Q^2/q_T^2 [A \text{Ln}(Q^2/q_T^2) + B + C]$$

Other issues related to TMD regions ...

- TMD regions are defined in terms of q_T and not in terms of P_T



Possible issues ...

- This fit gives a very high quality description of a wide amount of data points
- However, there are a few issues that are worth mentioning:
 - ★ The NLO SIDIS cross section is not correctly normalized $\rightarrow N \sim 2$
 - ★ The Y factor has been neglected
 - ★ Difficult to reconcile Drell-Yan and SIDIS data into the fit

Large transverse momentum behaviour in SIDIS

J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang, *Phys.Rev. D98* (2018) no.11, 114005

Challenges with Large Transverse Momentum in Semi-Inclusive Deeply Inelastic Scattering

J. O. Gonzalez-Hernandez,^{1,2,3,*} T. C. Rogers,^{1,4,†} N. Sato,^{4,‡} and B. Wang^{1,4,5,§}

¹Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

²Dipartimento di Fisica, Università di Torino, Via P. Giuria 1, 10125 Torino, Italy

³INFN-Sezione Torino, Via P. Giuria 1, 10125 Torino, Italy

⁴Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

⁵Zhejiang Institute of Modern Physics, Department of Physics, Zhejiang University, Hangzhou 310027, China

(Dated: 13 August 2018)

We survey the current phenomenological status of semi-inclusive deep inelastic scattering at moderate hard scales and in the limit of very large transverse momentum. As the transverse momentum becomes comparable to or larger than the overall hard scale, the differential cross sections should be calculable with fixed order pQCD methods, while small transverse momentum (TMD factorization) approximations should eventually break down. We find large disagreement between HERMES and COMPASS data and fixed order calculations done with modern parton densities, even in regions of kinematics where such calculations should be expected to be very accurate. Possible interpretations are suggested.

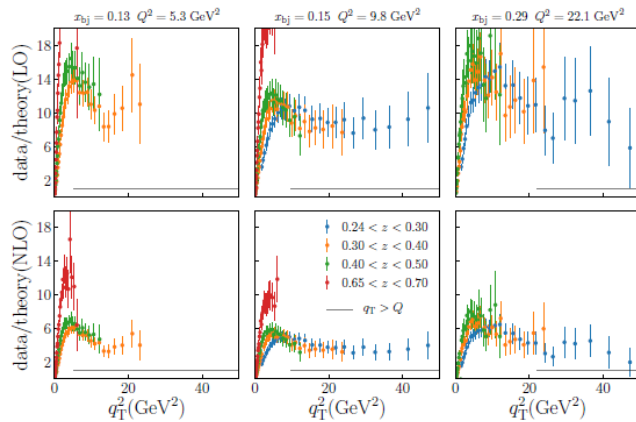


FIG. 5. Ratio of data to theory for several near-valence region panels in Fig. 4. The grey bar at the bottom is at 1 on the vertical axis and marks the region where $q_T > Q$.

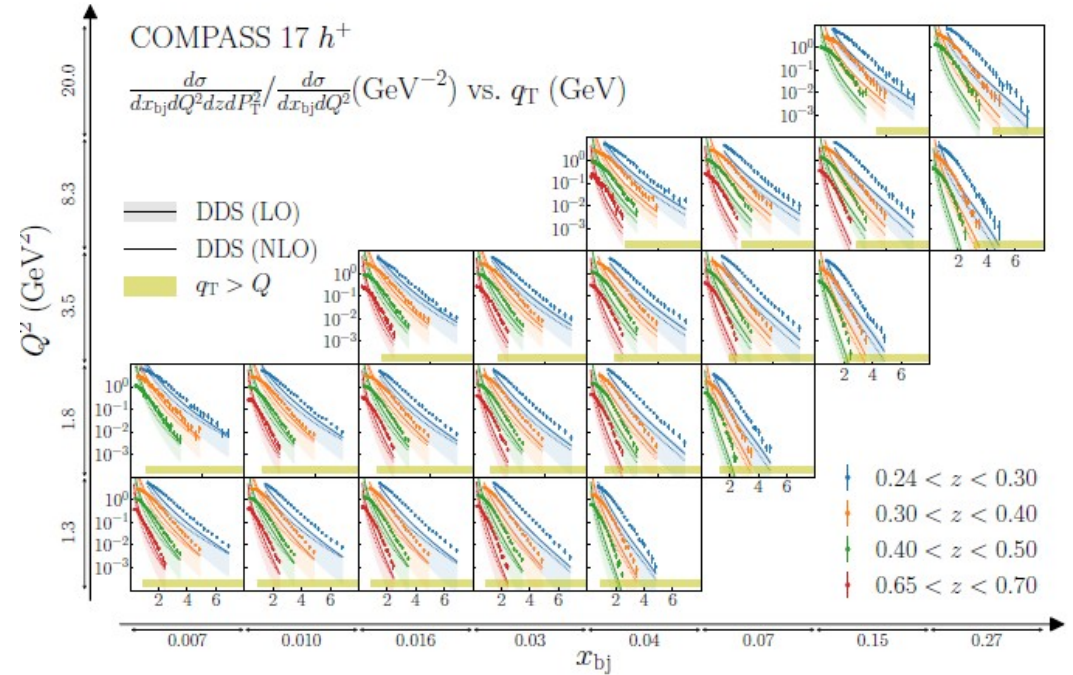
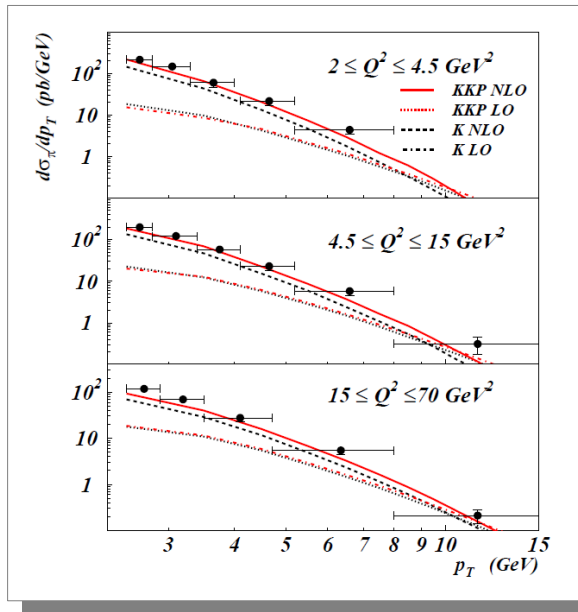


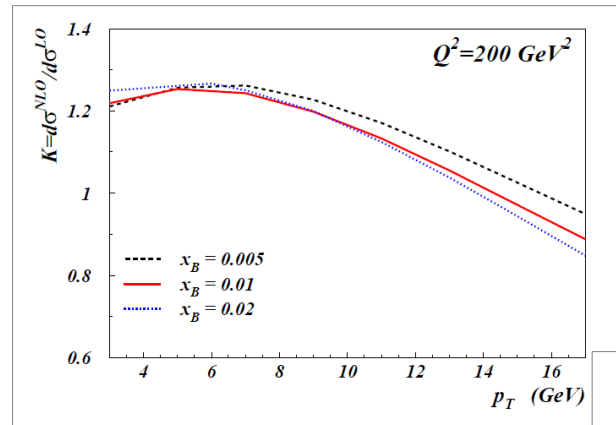
FIG. 4. Calculation of $O(\alpha_s)$ and $O(\alpha_s^2)$ transversely differential multiplicity using code from [22], shown as the curves labeled DDS. The bar at the bottom marks the region where $q_T > Q$. The PDF set used is CJNLO [25] and the FFs are from [26]. Scale dependence is estimated using $\mu = ((\zeta_Q Q)^2 + (\zeta_{q_T} q_T)^2)^{1/2}$ where the band is constructed point-by-point in q_T by taking the min and max of the cross section evaluated across the grid $\zeta_Q \times \zeta_{q_T} = [1/2, 1, 3/2, 2] \times [0, 1/2, 1, 3/2, 2]$ except $\zeta_Q = \zeta_{q_T} = 0$. The red band is generated with $\zeta_Q = 1$ and $\zeta_{q_T} = 0$. A lower bound of 1 GeV is place on μ when $Q/2$ would be less than 1 GeV.

There are large discrepancies between data and fixed order calculations. They seem to be generated by collinear PDFs and FFs

Normalization and K factor



How can we address the normalization problem ???



■ K factor depends on p_T

■ Kinematics cuts can affect the size of K factors ... up to a factor 10 !

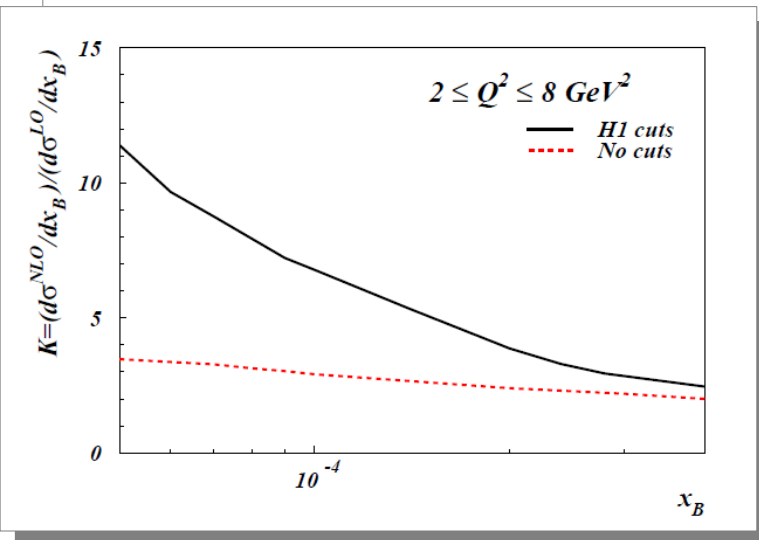
Stringent cuts on the pion production angle in H1 data suppresses LO and NLO contributions in a different way

Daleo, De Florian, Sassot, *Phys.Rev. D71* (2005) 034013

Daleo, De Florian, Sassot, *Braz.J.Phys.* 37 (2007) 585-590

Aktas et al., H1 Collaboration, *Eur. Phys. J. C36* (2004) 441

“The rather large size of the K-factor can be understood as a consequence of the opening of a new dominant (‘leading-order’) channel, and not to the ‘genuine’ increase in the partonic cross section [...]. The dominance of the new channel is due to the size of the gluon distribution at small x_B and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \rightarrow g + q + \bar{q}$ partonic process. In particular, without the experimental cuts for the final state hadrons, the gg component represents less than 25% of the total NLO contribution at small x_B .”



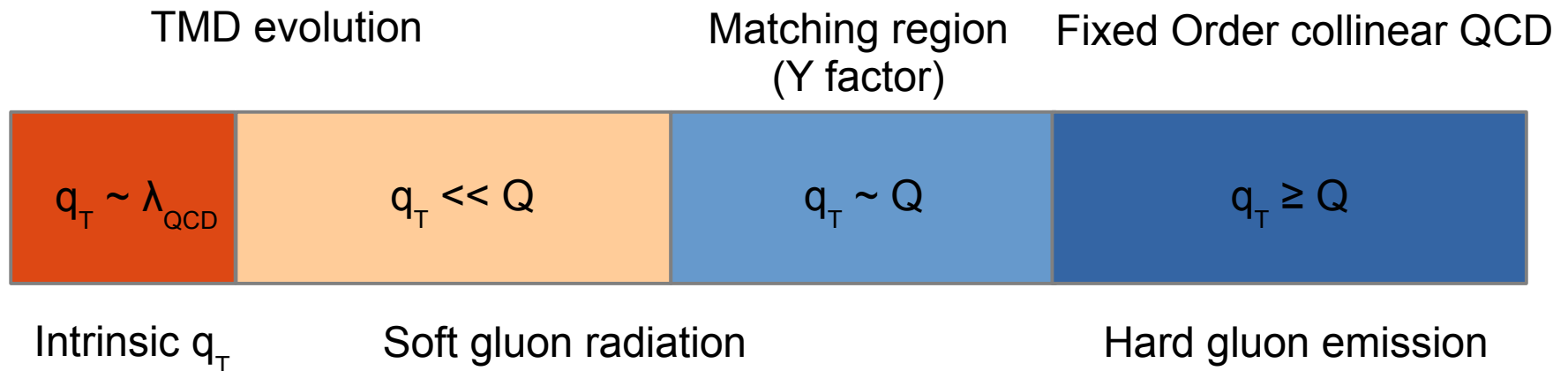
Daleo, De Florian, Sassot, *Phys.Rev. D71* (2005) 034013

Daleo, De Florian, Sassot, *Braz.J.Phys.* 37 (2007) 585-590

What's wrong ???

TMD regions

- The TMD factorization scheme works when 4 distinct kinematic regions can be clearly be identified
- They should be large enough and well separated

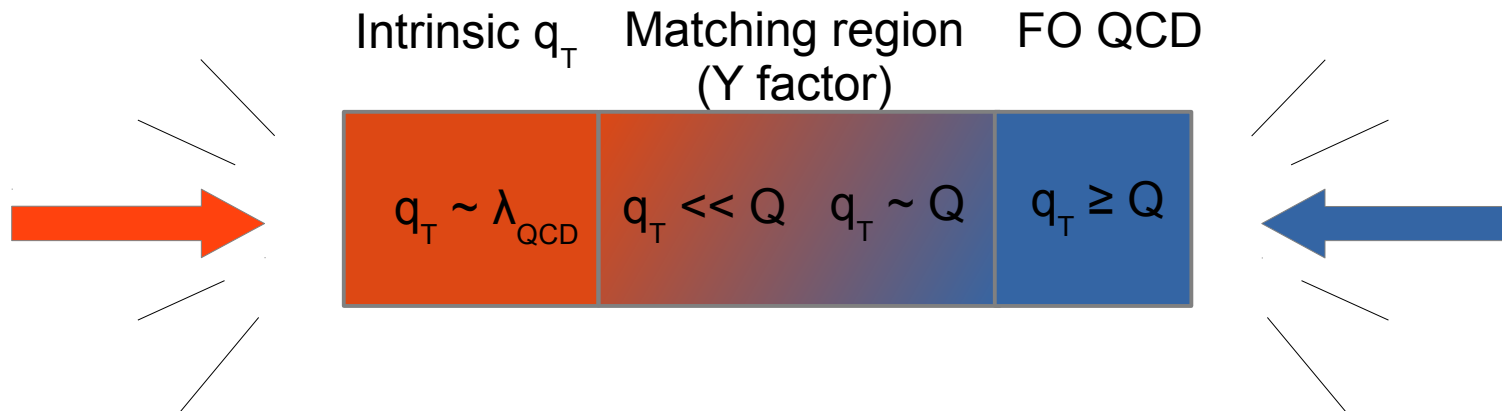


TMD regions

- For this scheme to work, 4 distinct kinematic regions have to be identified
- They should be large and well separated

Does not work in SIDIS !

TMD evolution



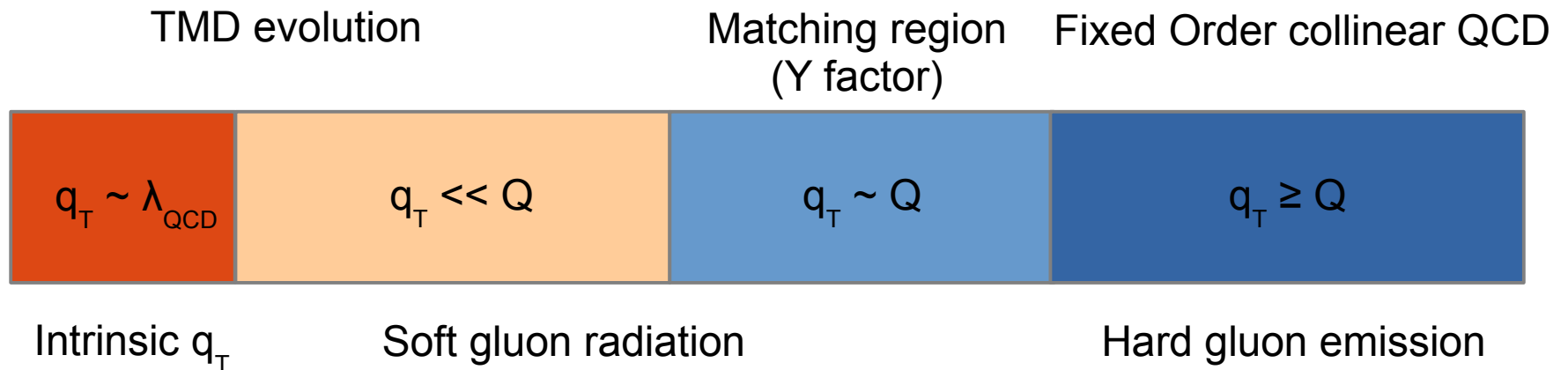
Mapping the kinematic regions of SIDIS

TMDs in SIDIS

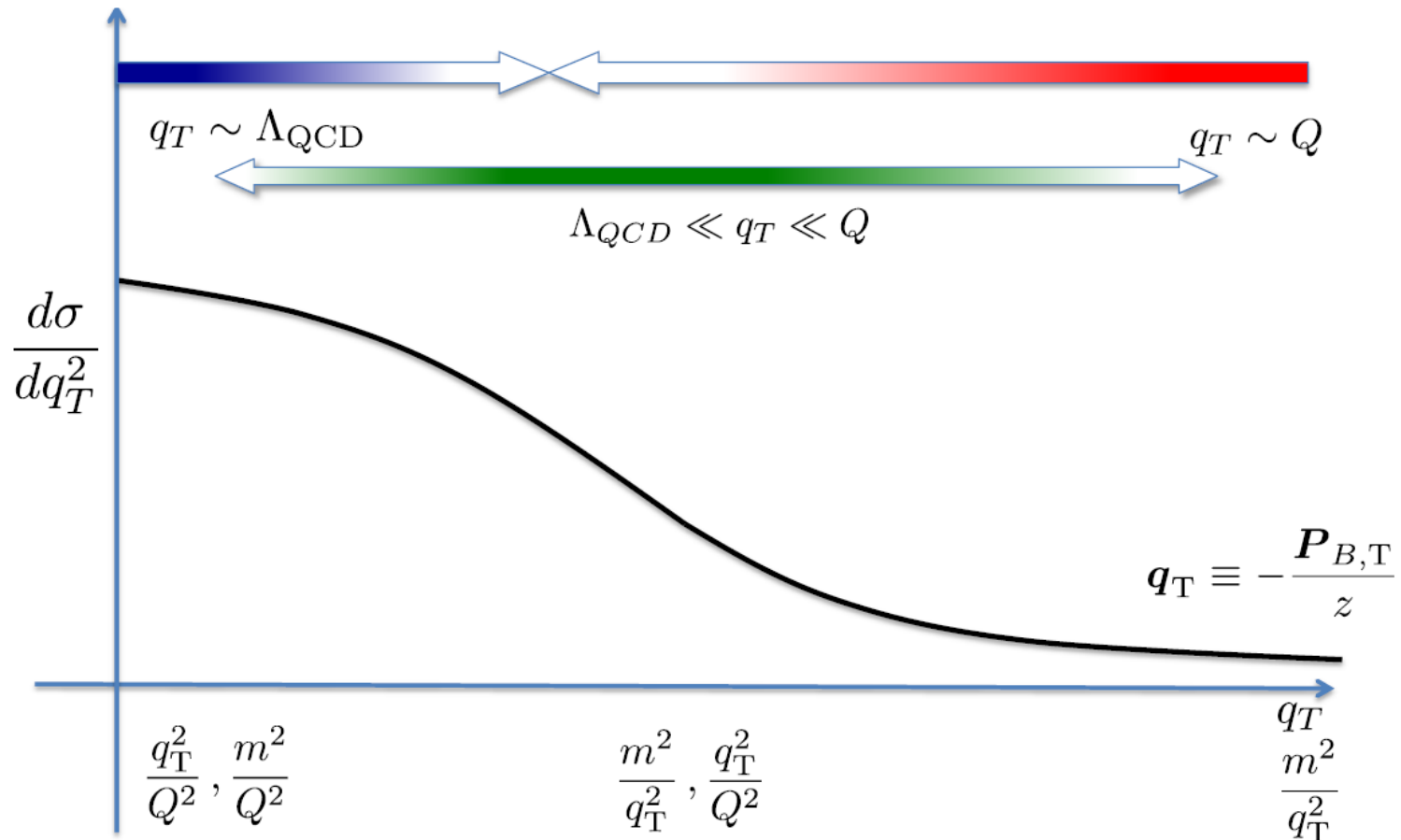
- Well-established collinear factorization theorems for SIDIS allow to study the hadron structure in terms of elementary constituents, and give access to the corresponding flavor dependence of PDFs and FFs.
- Beyond collinear factorization, transversely differential SIDIS at low transverse momentum is sensitive to the properties of TMDs.
- SIDIS experiments at moderate values of Q (1-3 GeV) are highly sensitive to intrinsic properties of hadron structure.
- Novel aspects of QCD might be exposed by studying this interesting but still poorly understood regime of SIDIS. However, there are also unique challenges in interpreting the experimental data.
- QCD factorization is necessary to describe the underlying physical mechanisms in terms of partonic degrees of freedom. However, it requires specific kinematic assumptions (e.g., very large or very small transverse momentum, or very large or very small rapidity).
- The interface between different physical regimes remains unclear in practice, especially when the hard scales involved are not that large.
- Estimating the kinematic boundaries of any specific QCD approach, approximation, or partonic picture requires at least some model assumptions, e.g., about the role of parton virtuality and the onset of non-perturbative or hadronic mechanisms.

TMD regions

- The TMD factorization scheme works when 4 distinct kinematic regions can be clearly be identified
- They should be large enough and well separated

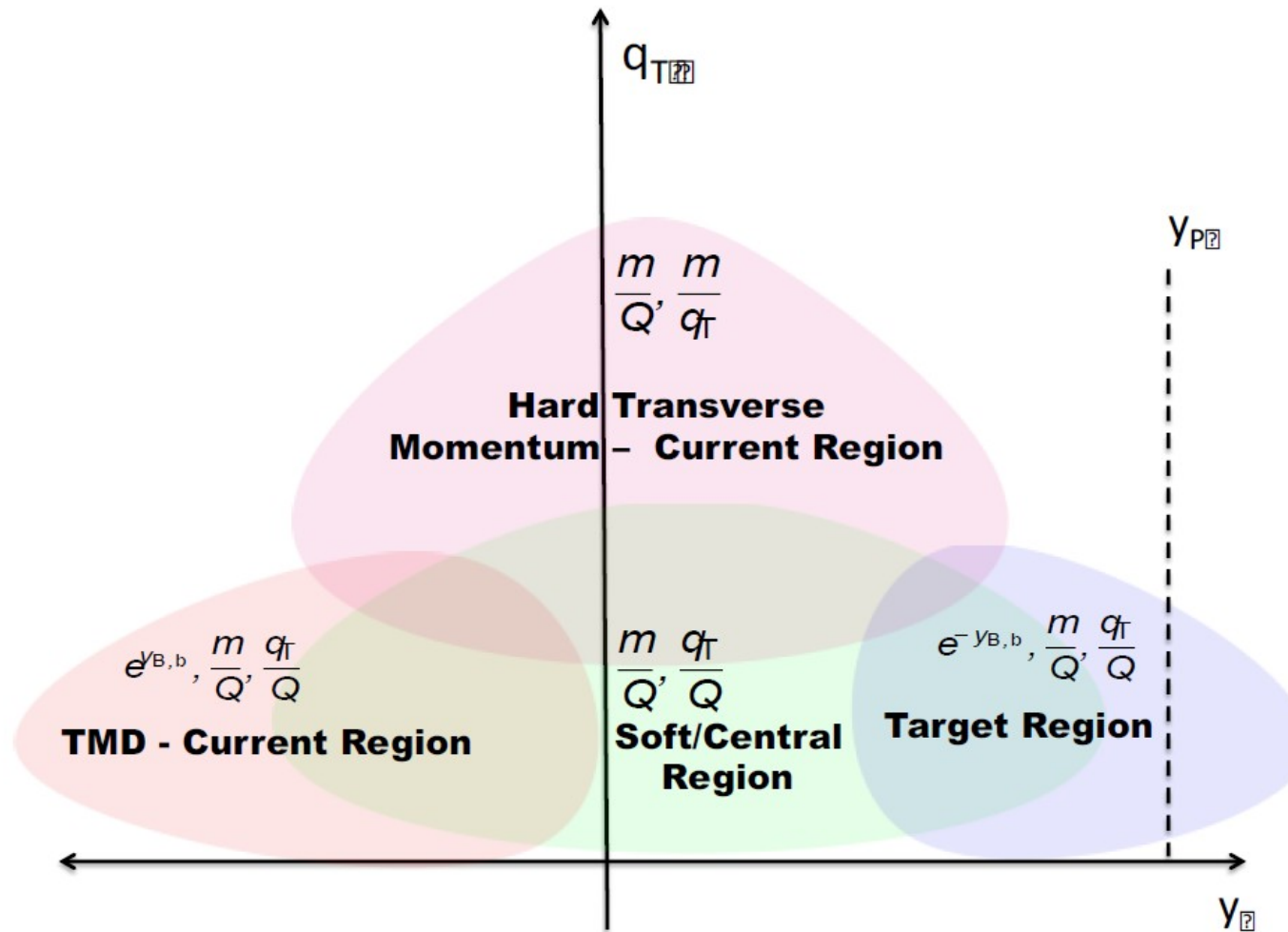


TMD regions



Courtesy of Ted Rogers

TMD regions



Courtesy of Ted Rogers

Kinematic variables

FACTORIZATION

Lightcone fractions

$$x_N = -\frac{q^+}{P^+}$$

$$z_N = \frac{P_B^-}{q^-}$$

EXPERIMENT

Momentum fraction

$$x_{Bj} = \frac{Q^2}{2P \cdot q}$$

$$z_h = \frac{P \cdot P_B}{P \cdot q} = 2x_{Bj} \frac{P \cdot P_B}{Q^2}$$

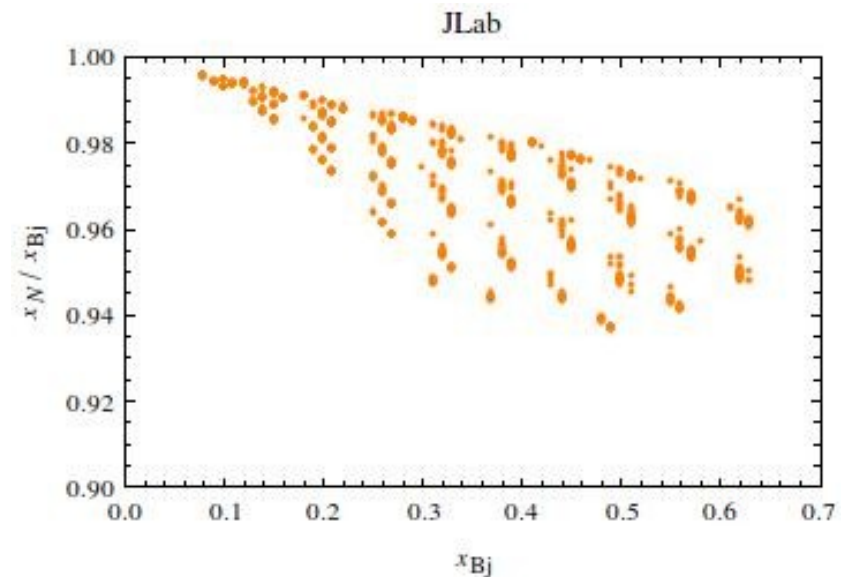
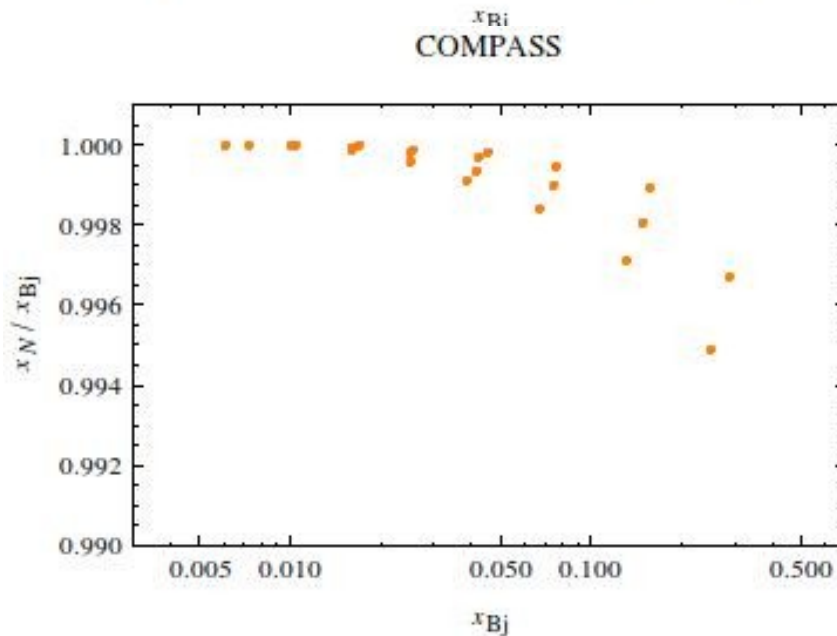
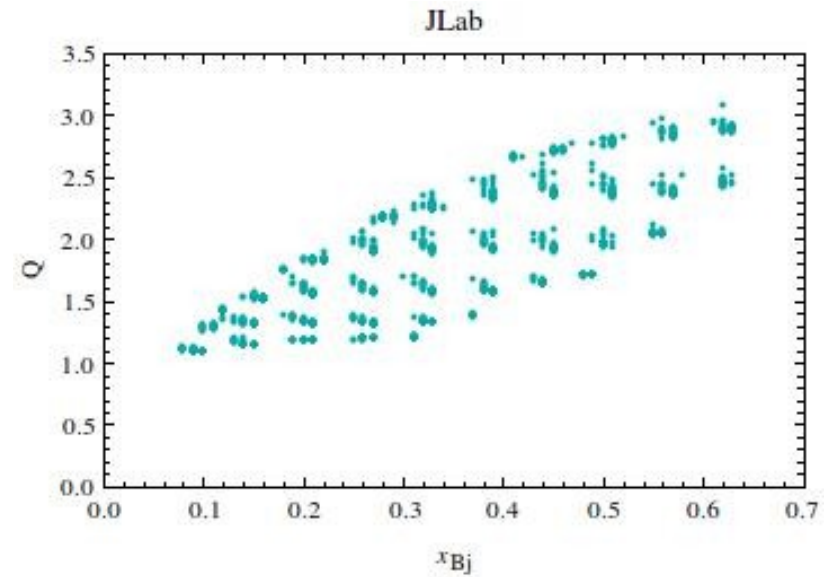
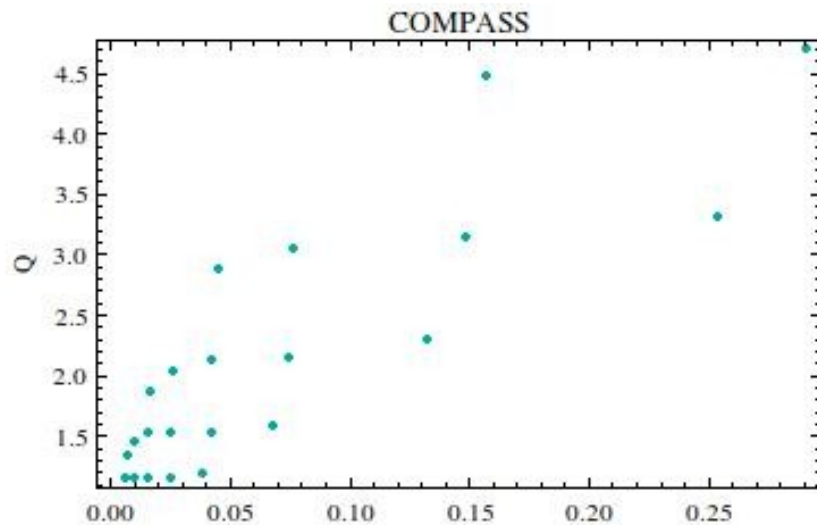
$$x_N = \frac{2x_{Bj}}{1 + \sqrt{1 + \frac{4x_{Bj}^2 M^2}{Q^2}}}$$

$$z_N = \frac{x_N z_h}{2x_{Bj}} \left(1 + \sqrt{1 - \frac{4M^2 M_{B,T}^2 x_{Bj}^2}{Q^4 z_h^2}} \right)$$

Target mass corrections

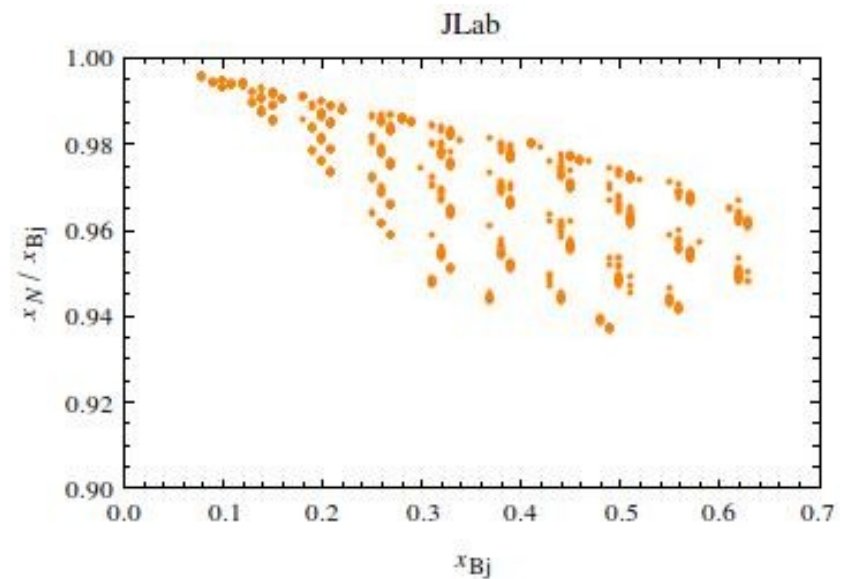
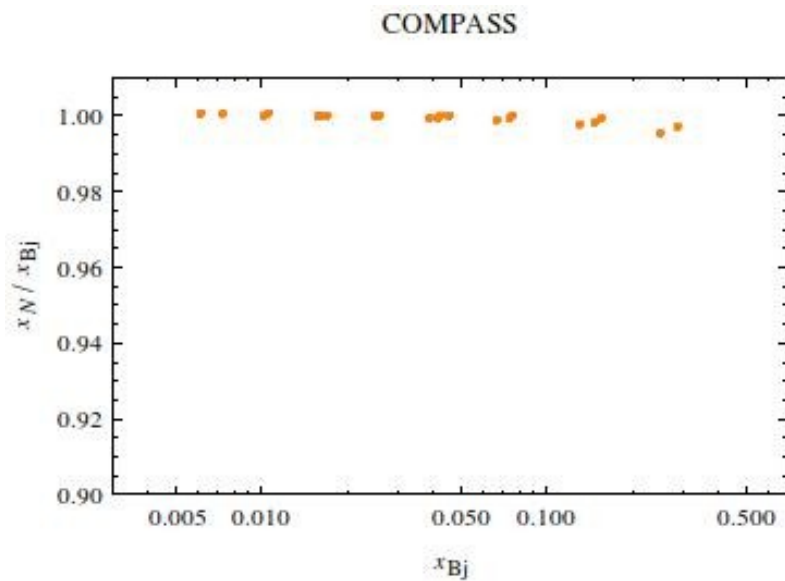
Sensitivity to final hadron mass

Kinematic variables

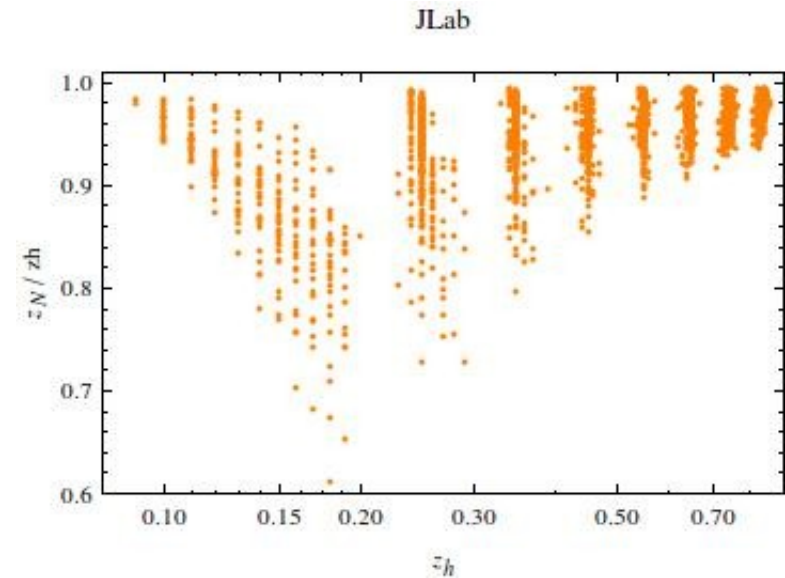
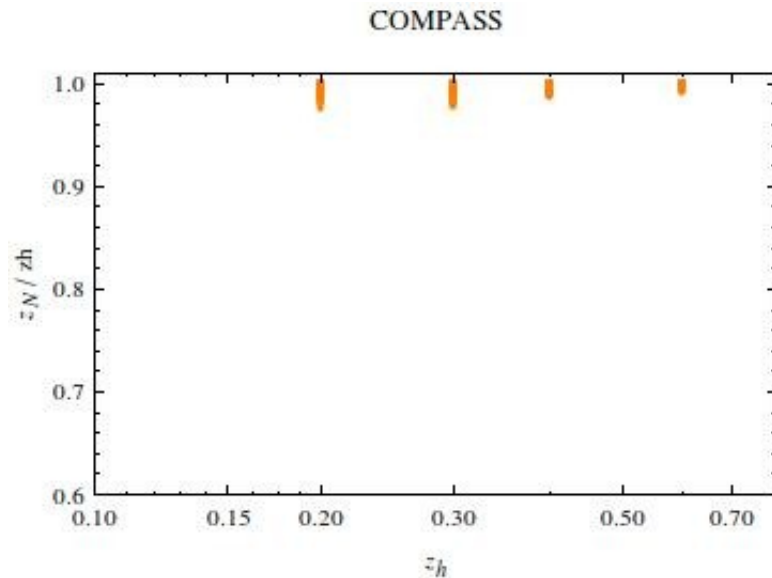


Kinematic variables

- x_N / x_{Bj} is a measure of the quality of the massless target approximation
- x_N / x_{Bj} is very sensitive to both target mass corrections

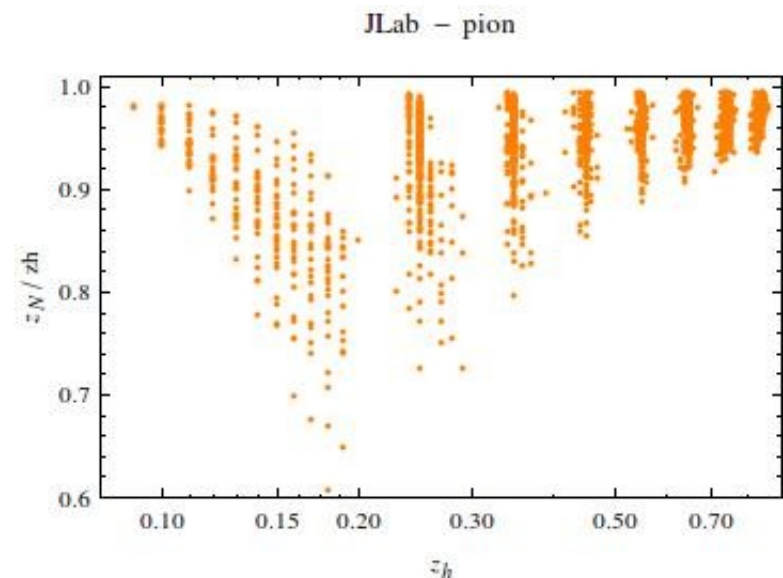
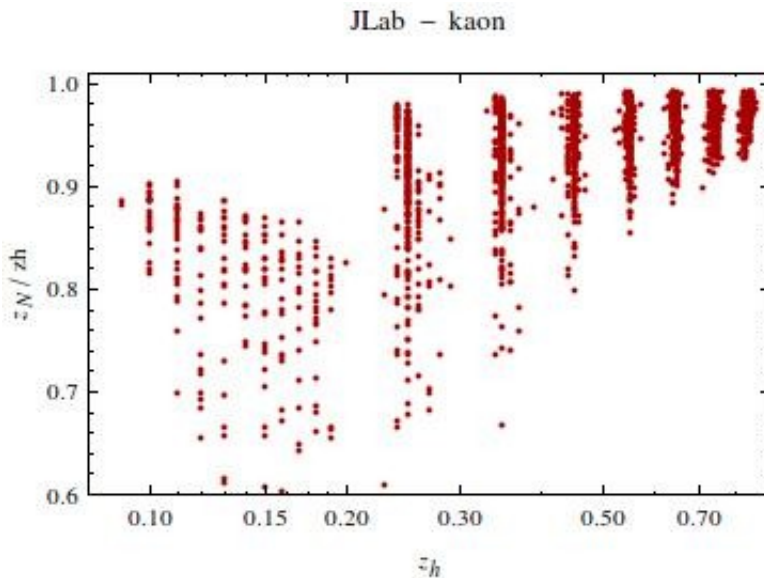


Kinematic variables



- z_N / z_h is a measure of the quality of the massless hadron approximations
- z_N / z_h is very sensitive to both target mass and hadron mass corrections

Kinematic variables

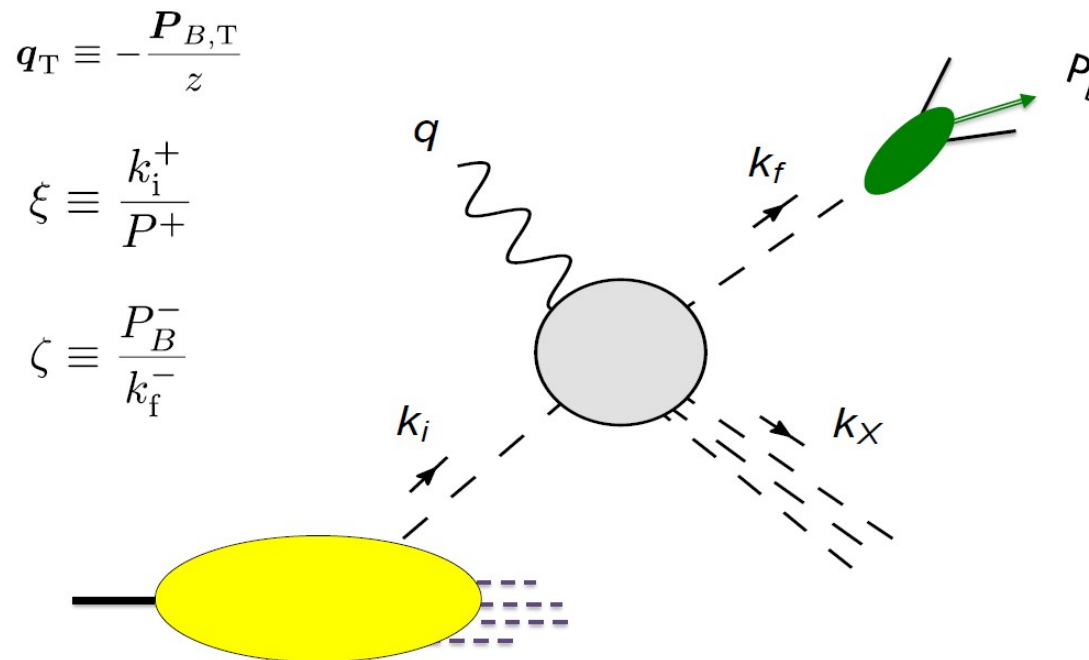


- z_N / z_h is a measure of the quality of the massless hadron approximations
- z_N / z_h is very sensitive to both target mass and hadron mass corrections
- Deviations from $z_N / z_h \sim 1$ become larger with increasing hadron masses

Partonic variables

- Assume that initial and final hadrons are the result of scattering and fragmentation by small-mass constituents. What are the possible kinematic configurations of those partons given a set of assumptions about their intrinsic properties? To approach this question one needs to apply some kind of factorization and, in turn, to deal with **partonic variables**.

Current fragmentation

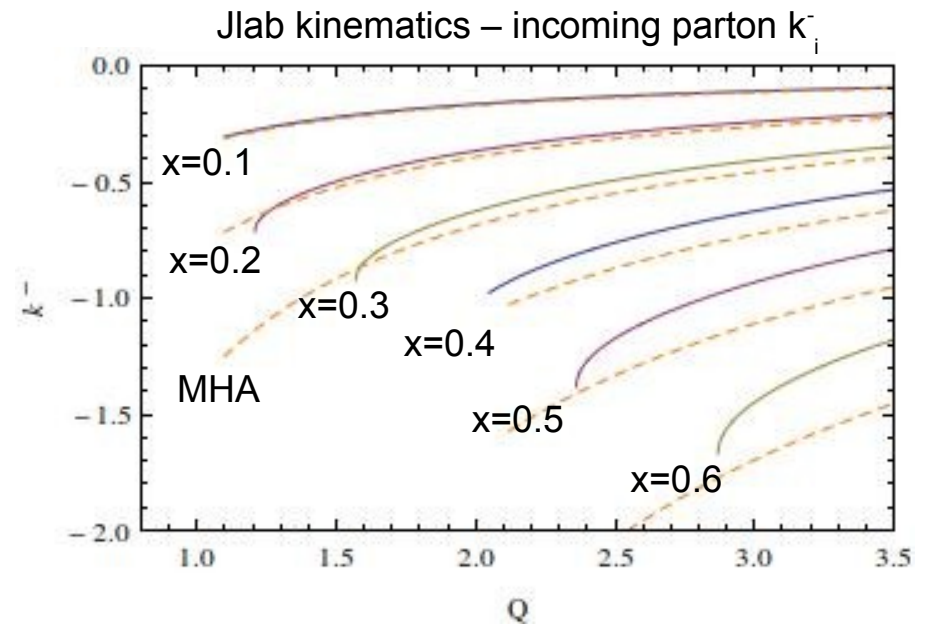
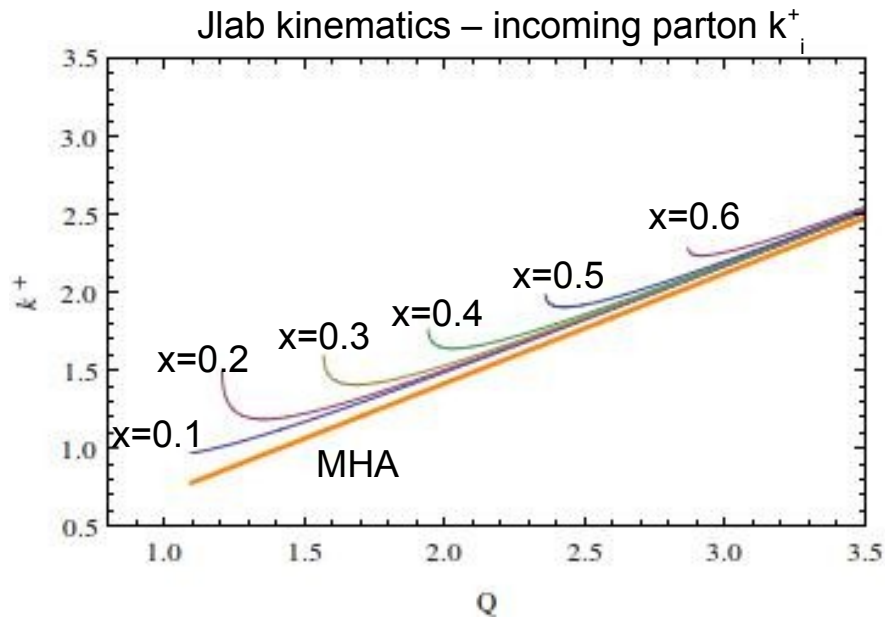


Courtesy of Ted Rogers

Partonic variables

$$\begin{aligned}\xi &\equiv \frac{k_i^+}{P^+} = x_N + O\left(\frac{m^2}{Q^2}\right) \\ &= x_{Bj} + O\left(\frac{x_{Bj}^2 M^2}{Q^2}\right) + O\left(\frac{m^2}{Q^2}\right)\end{aligned}$$

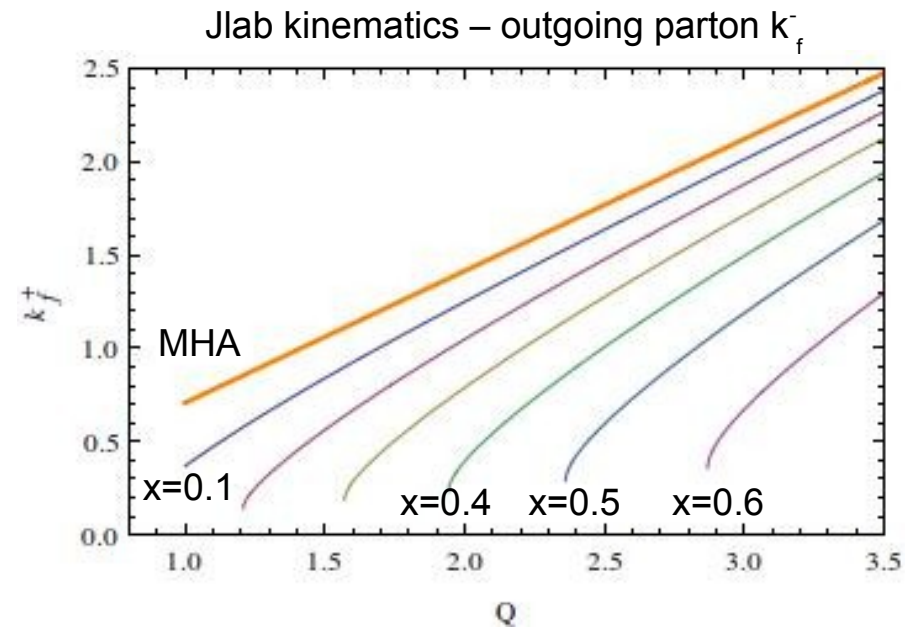
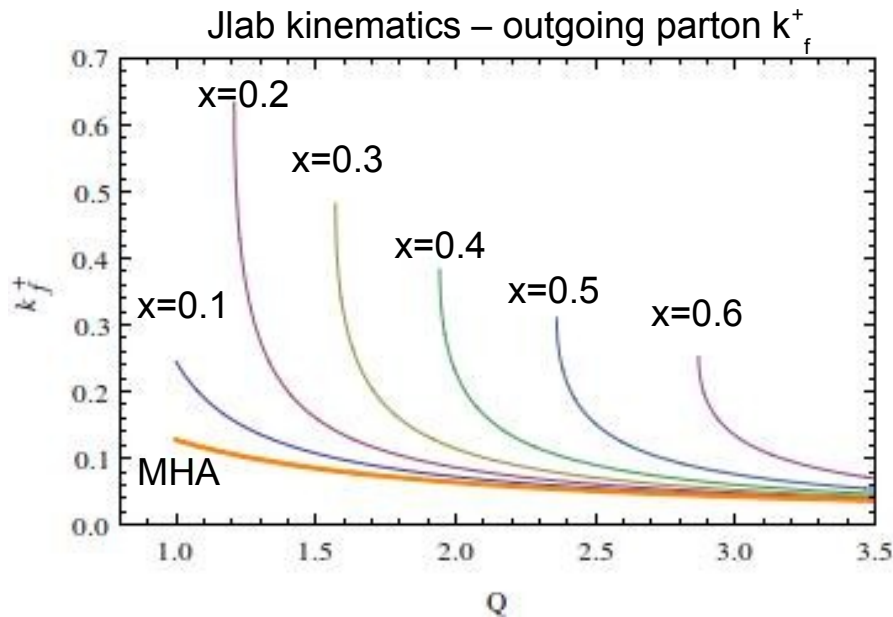
- Need to estimate some non perturbative quantities ...
- Important to ensure that observables do not depend strongly on those quantities



- Partonic variables strongly depend on hadron mass corrections, $O(M/Q)$
- Massless hadron approximation worsen with growing x_{Bj}

Partonic variables

- Assume that initial and final hadrons are the result of scattering and fragmentation by small-mass constituents. What are the possible kinematic configurations of those partons given a set of assumptions about their intrinsic properties? To approach this question one needs to apply some kind of factorization and, in turn, to deal with **partonic variables**.

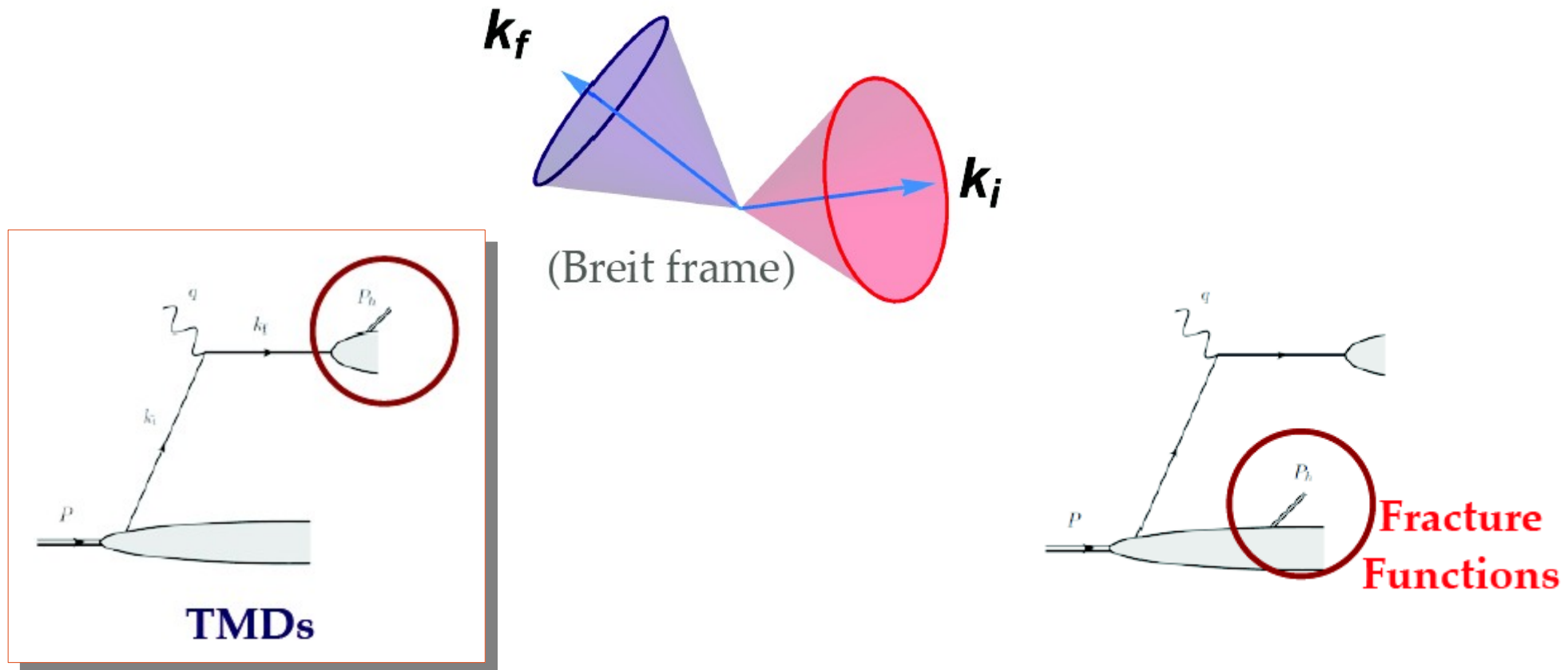


- Partonic variables strongly depend on hadron mass corrections, $O(M/Q)$
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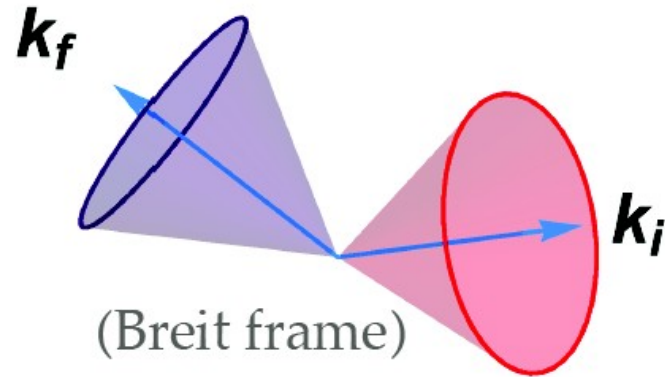
Kinematics of current region

Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato
Phys. Lett. B766 (2017) 245

Need a quantitative way to identify the region of validity of TMD factorization (**current region**)

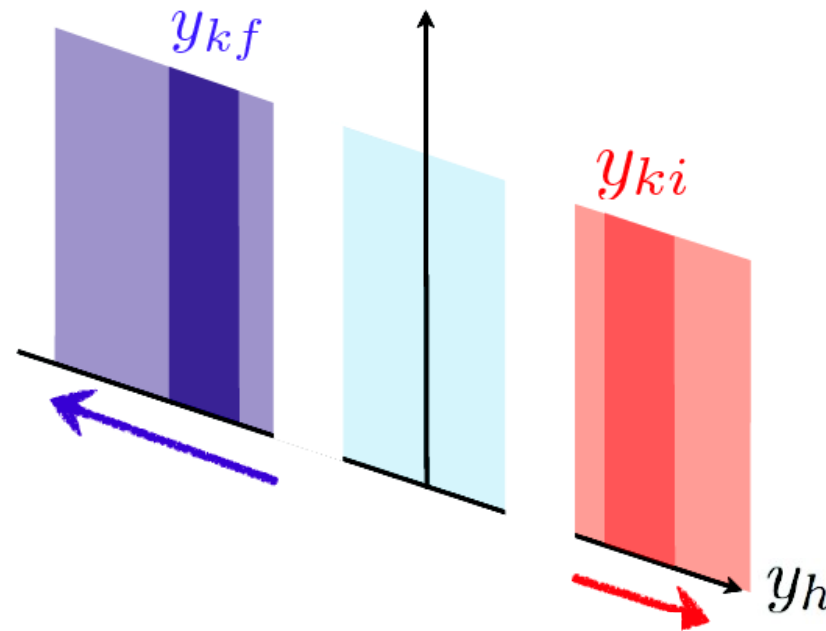


Kinematics of current region



Hadron rapidity

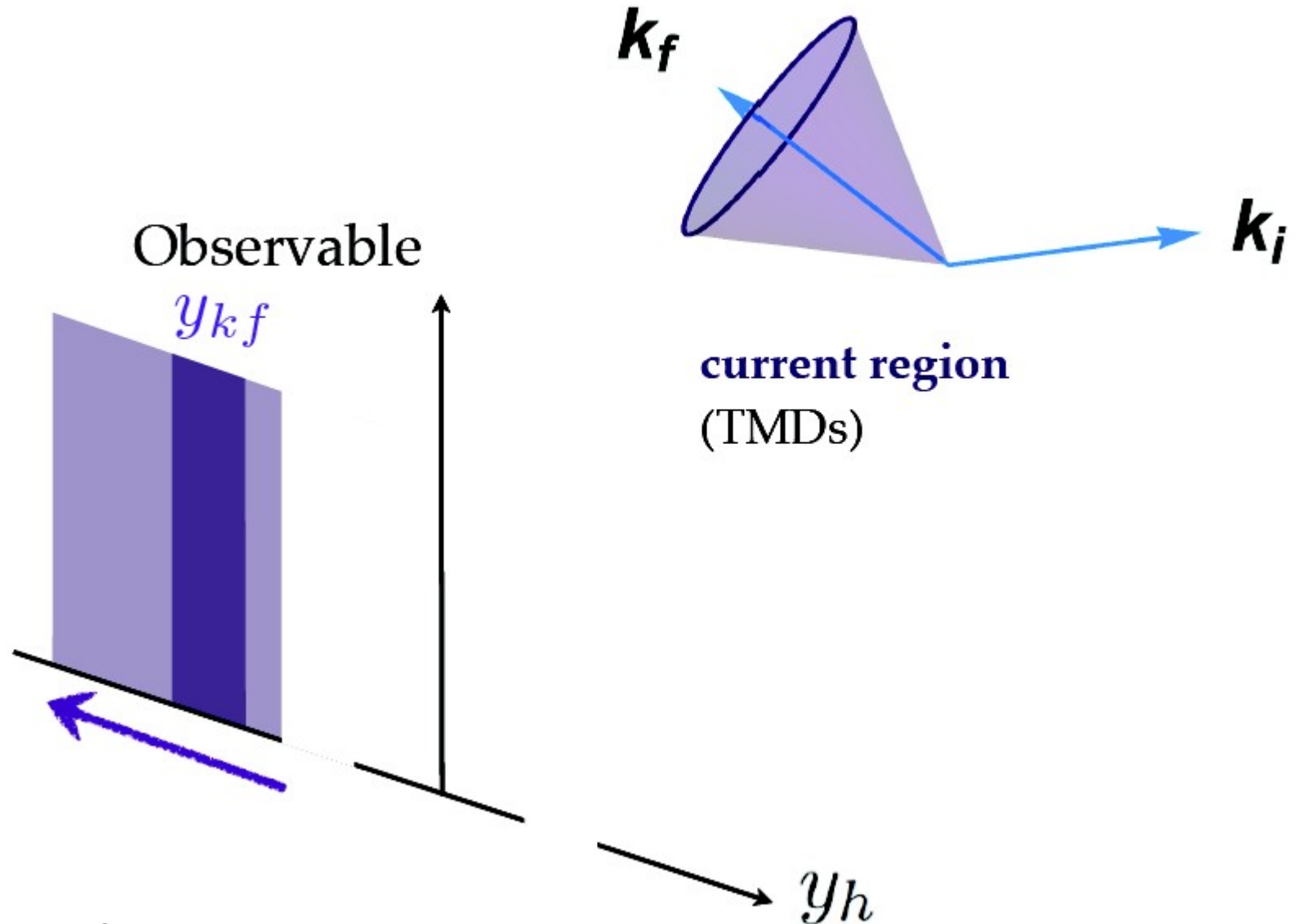
$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$



Current and fragmentation regions should be well separated in the observed hadron rapidity

Courtesy of Osvaldo Gonzalez

Kinematics of current region



Courtesy of Osvaldo Gonzalez

Current fragmentation region

- In the current region standard approximations hold:

$$k_i^2/Q^2 \rightarrow 0 \quad k_f^2/Q^2 \rightarrow 0$$

- Moreover, current hadrons are produced in such a way that the final hadron is exactly aligned with the fragmenting parton

$$k_f \cdot P_B \rightarrow 0$$

- Therefore, one can define the ratio

$$R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i}$$

which is small in the current region

We work in
the Breit
(brick-wall)
frame

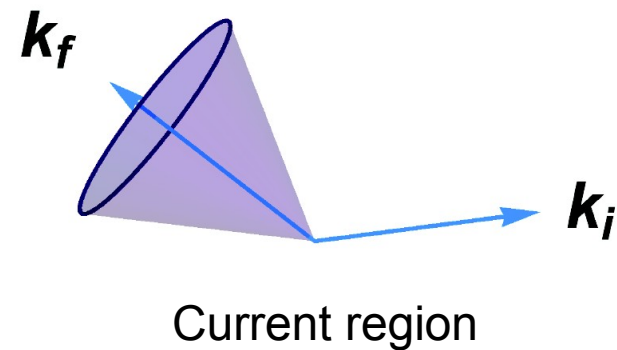
Kinematics of current region

Factorization implies power counting for the momenta

Small mass

$$P_h \cdot k_f = O(m^2)$$
$$P_h \cdot k_i = O(Q^2)$$

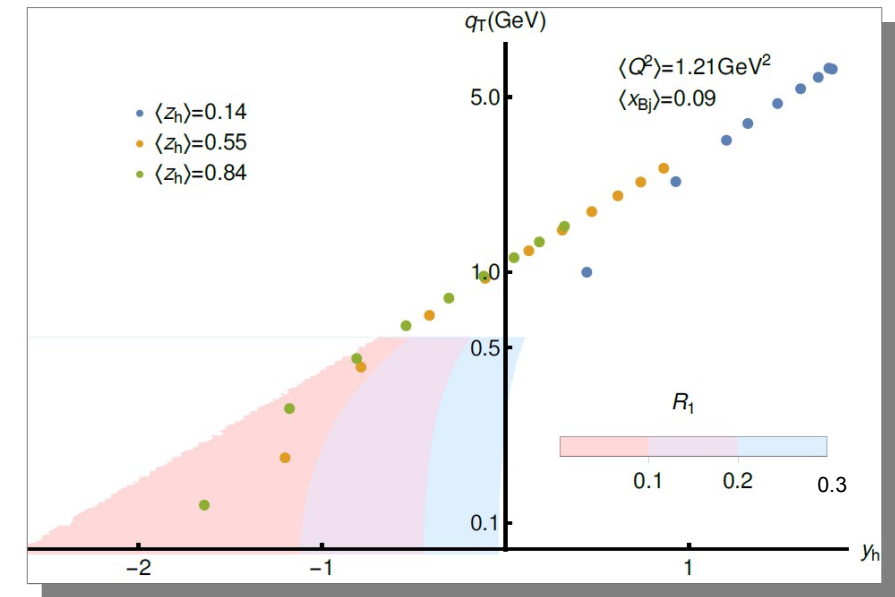
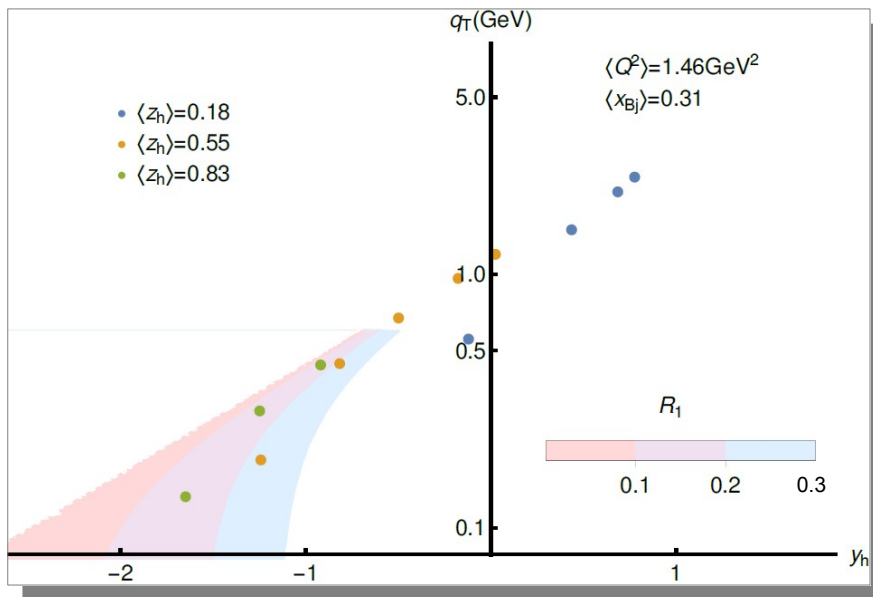
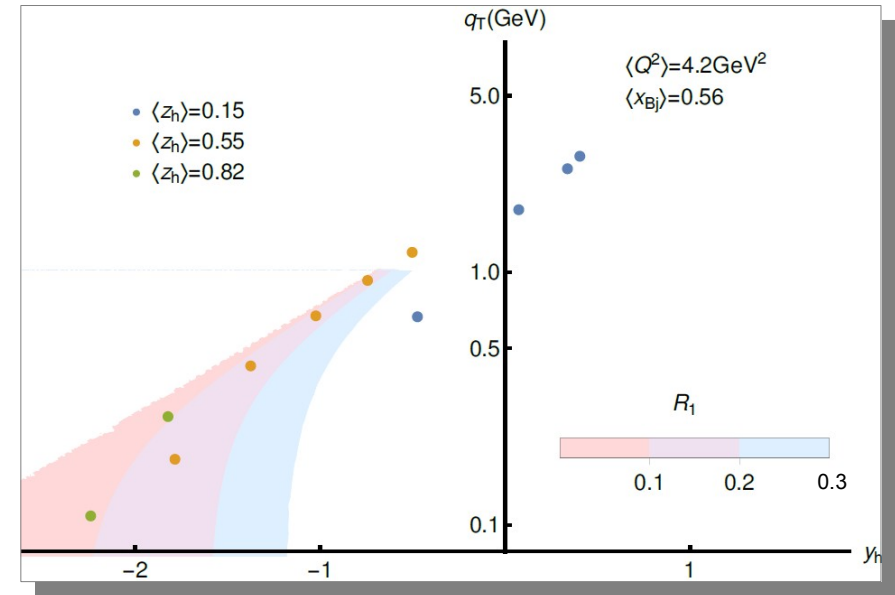
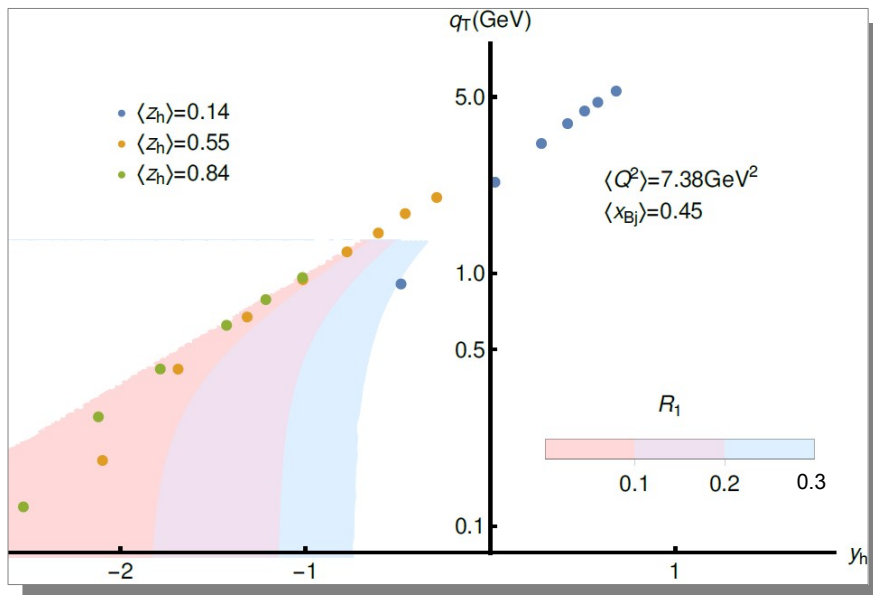
Hard scale



$$R_1(y_h, z_h, x_{bj}, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

Collinearity must be small in the current region

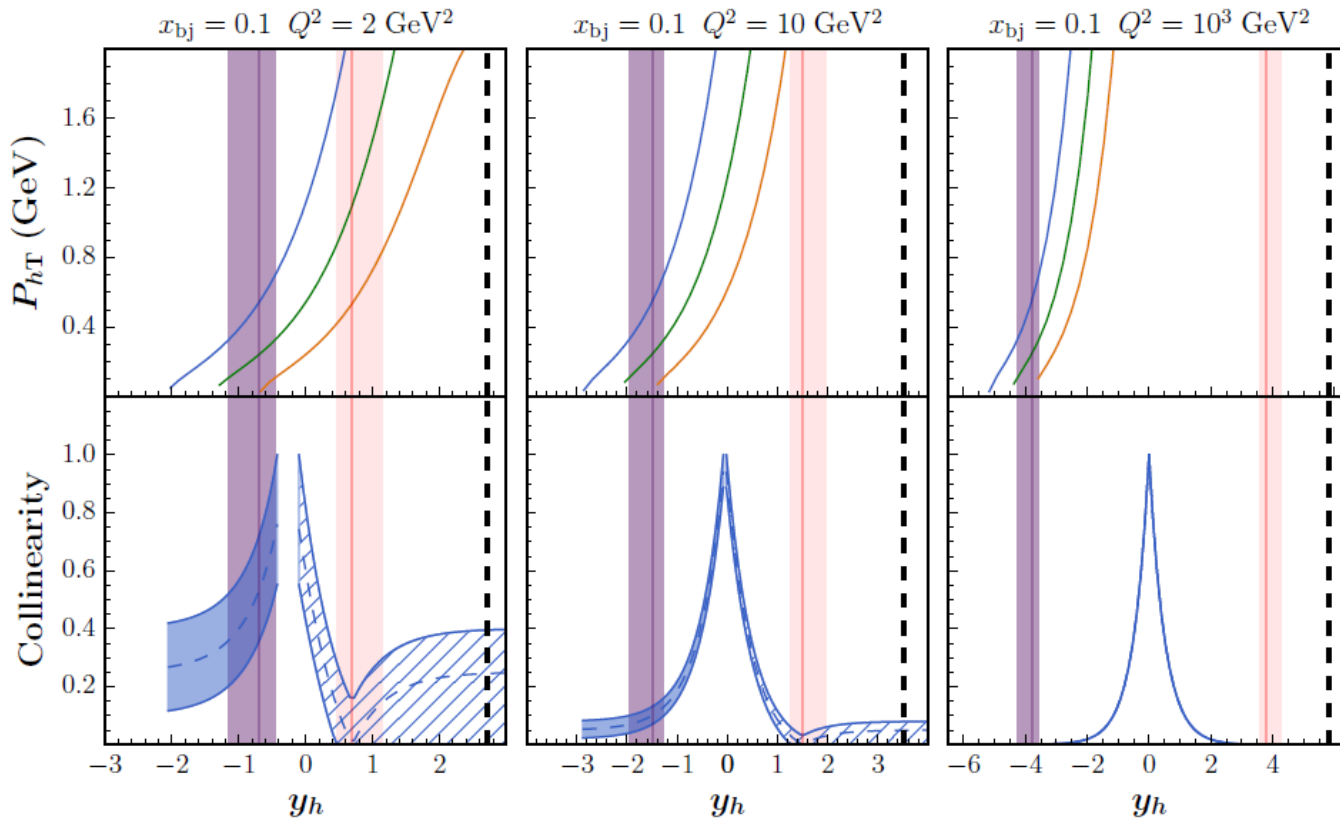
Current fragmentation region



Kinematics of current region

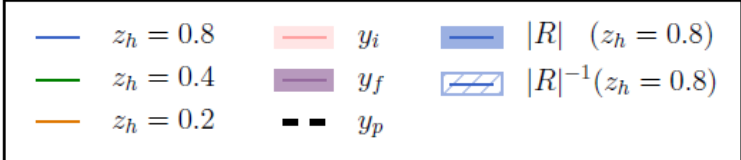
$R(y_h, z_h, x_{bj}, Q) \ll 1$: collinear to outgoing quark,
 $R(y_h, z_h, x_{bj}, Q)^{-1} \ll 1$: collinear to incoming quark.

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

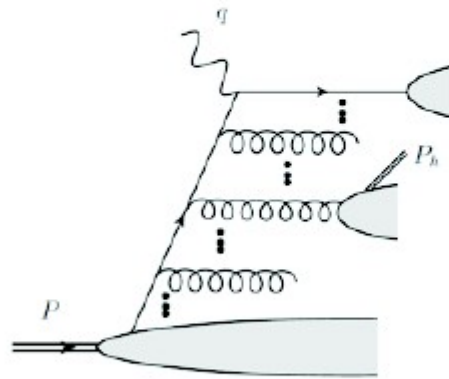
$$y_f = -\ln \frac{Q}{M_{fT}}$$



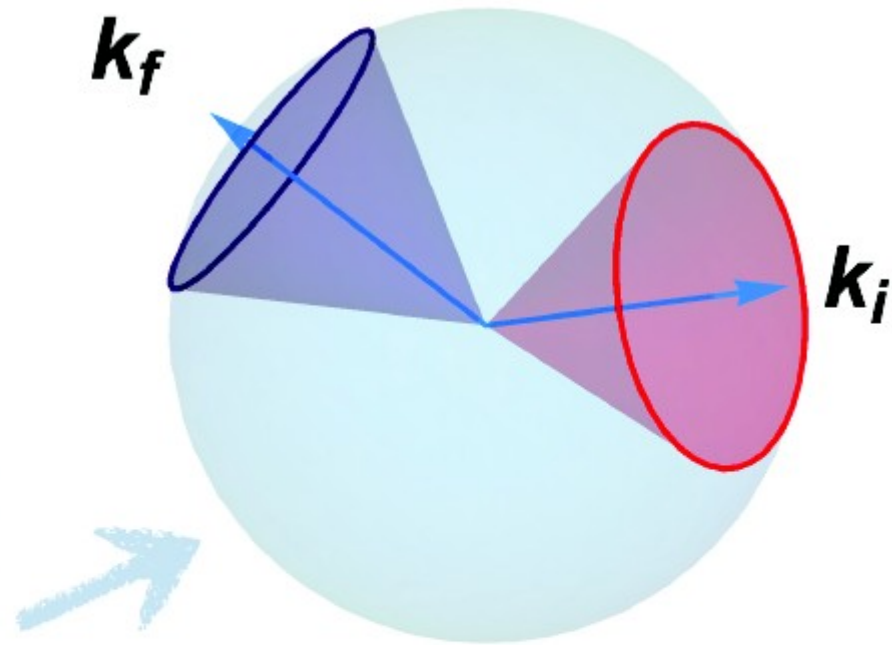
$$y_i = \ln \frac{Q}{M_{iT}}$$

Kinematics of soft region

$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$



soft



However, this neglects the soft fragmentation region

(No factorization theorem for this region)

Conclusions

- Phenomenological studies of TMD factorization and evolution have come a long way.
Many aspects of the interplay between perturbative and non-perturbative contributions are now better understood, but many others need further investigation.
- Special care has to be taken when dealing with moderate-to-low Q kinematic ranges, where power corrections from hadron masses can become relevant.
- Data selection is crucial in global fitting:
 - not too many
(only data within the ranges where the TMD factorization schemes work should be considered)
 - not too few
(too strict a selection can bias the fit results and neglect important information from experimental data)