

Light and heavy clusters in warm stellar matter

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the last frontier - WTPLF 2018**

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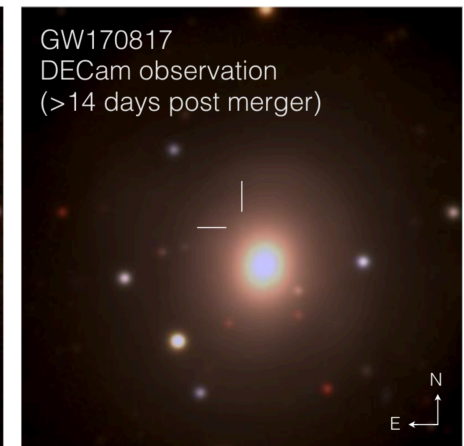
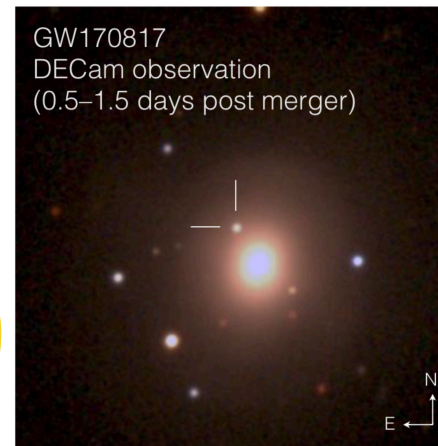
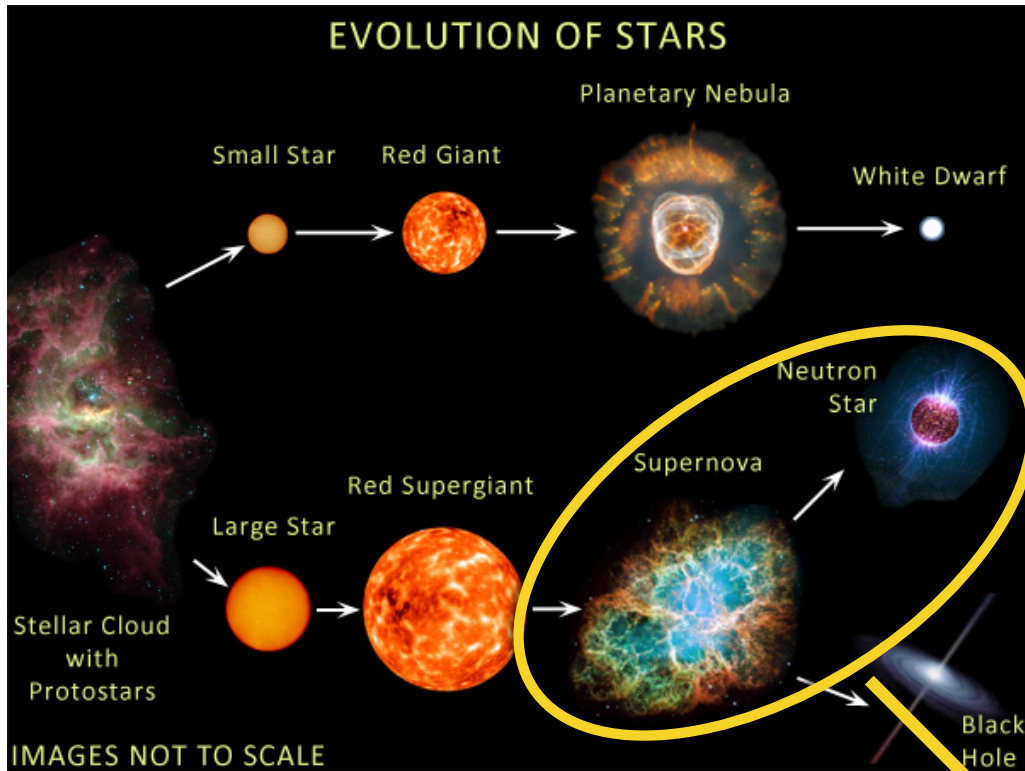


Where do these clusters form?

in <http://essayweb.net/astronomy/blackhole.shtml>

in <https://www.ligo.org/detections/GW170817.php>

Credit: Soares-Santos et al. and DES Collab



NS mergers

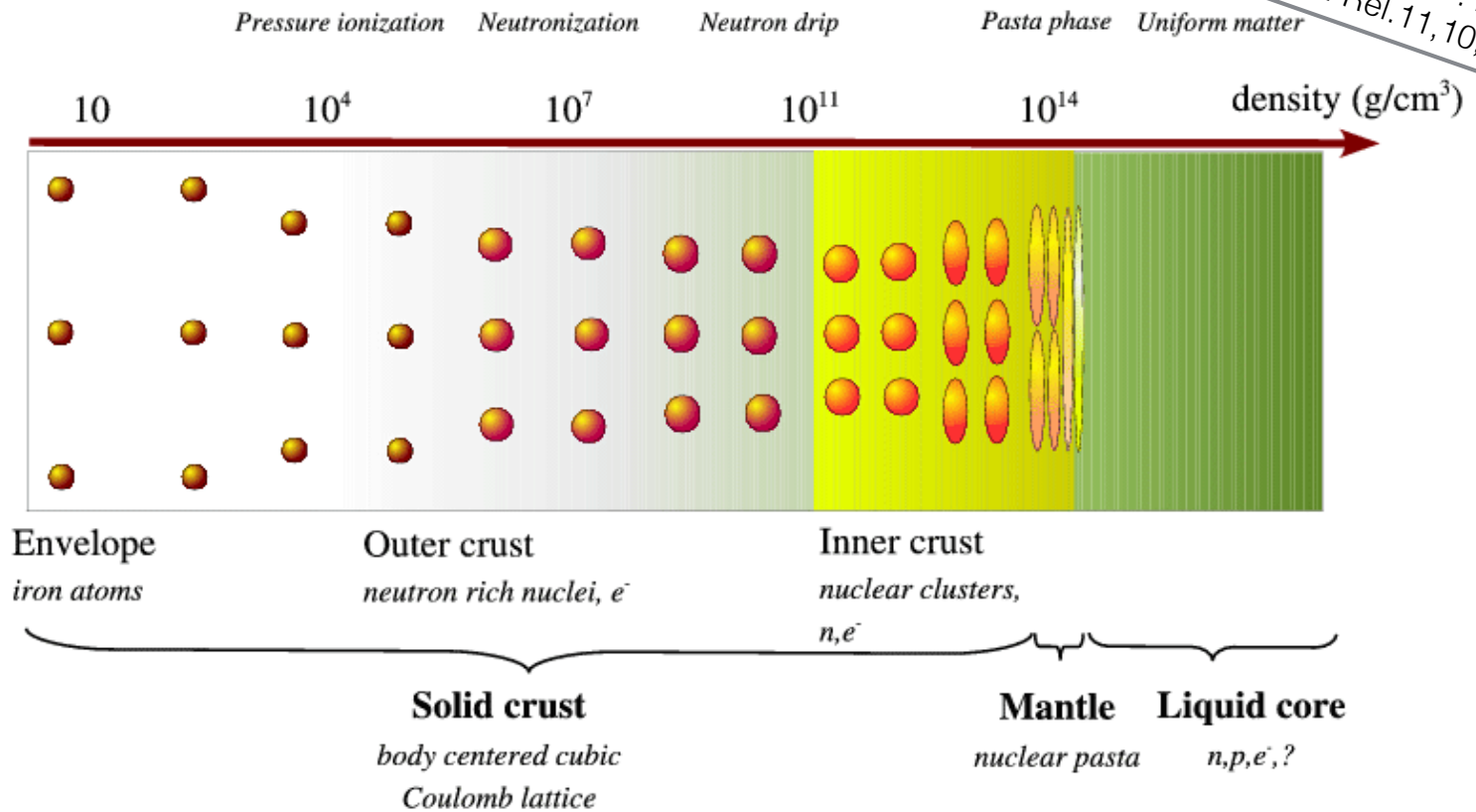
scenarios where light and heavy clusters are important

Neutron stars

1. Outer crust
2. Inner crust
3. Core

- Divided in 3 main layers:
- NS: catalyzed cold stellar matter:

N. Chamel and P. Hansel,
Liv. Rev. Rel. 11, 10, 2008



- The clusters, light and heavy, also appear in CCSN (fixed y_p and finite T)
- In CCSN, the clusters can modify the neutrino transport, affecting the cooling of the PNS.

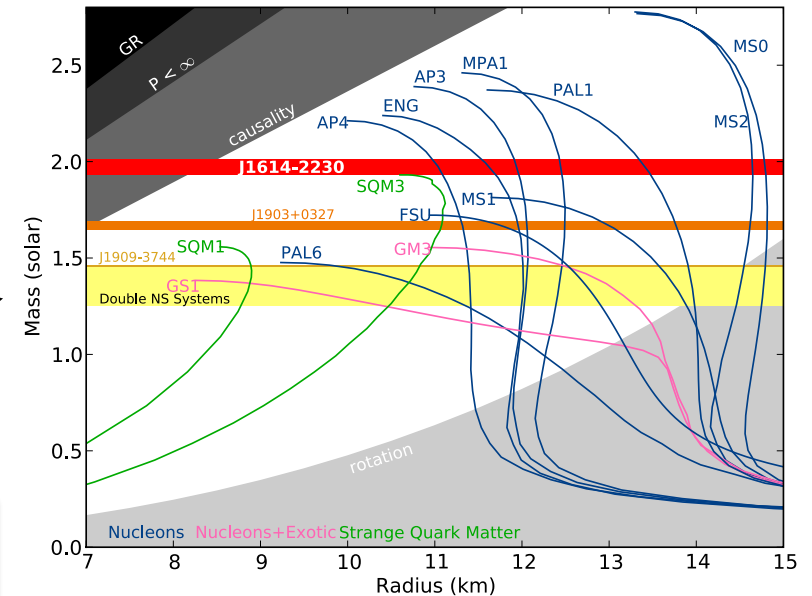
Describing neutron stars

P.B. Demorest *et al*,
Nature 467, 1081, 2010

Prescription:

1. EoS: $P(E)$ for a system at given ρ and T
2. Compute TOV equations
3. Get star $M(R)$ relation

Problem: Which phenomenological EoS to choose?

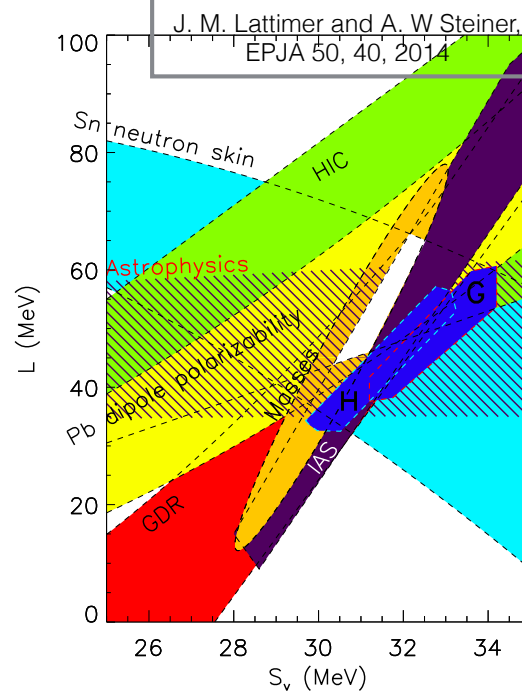
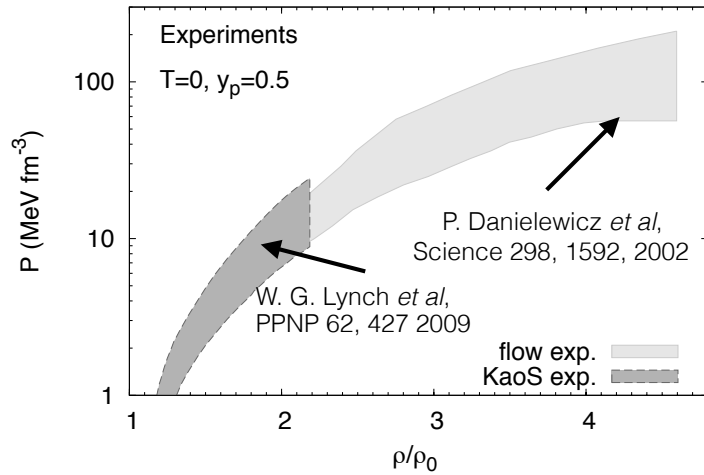


Many EoS models in literature: Phenomenological models (parameters are fitted to nuclei properties): **RMF, Skyrme...**

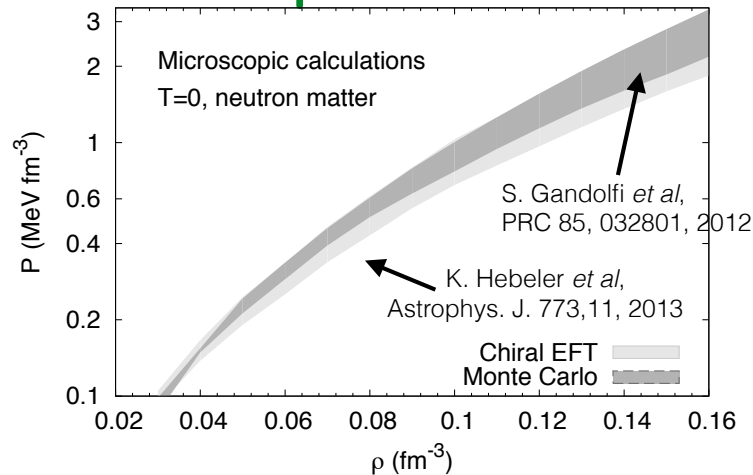
Solution: Need Constraints (Experiments, Microscopic calculations, Observations)

EoS Constraints

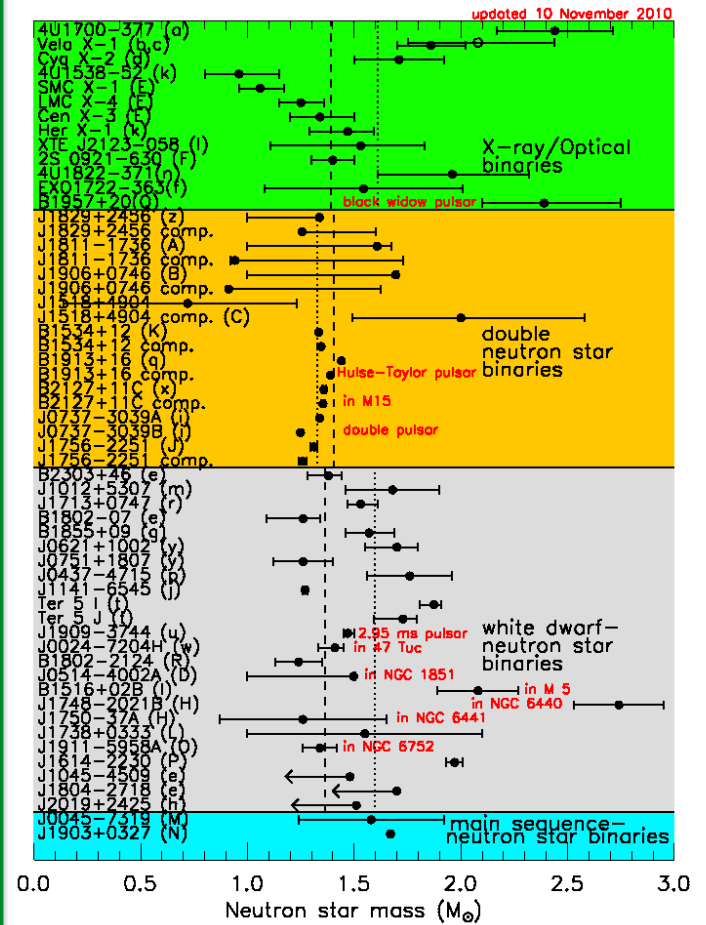
Experiments



Microscopic calculations



Observations



Choosing the EoS(s)

Problem: How to build the EoS for different star regions, Ts?

Solution: Choose 1 EoS for each NS layer:

- Outer crust EoS (BPS, HP, or RHS, ...) \rightarrow $M(R)$ not affected
- *Inner crust EoS* \rightarrow *pasta phases ? unified core EoS ?*
- Core EoS \rightarrow homogeneous matter
and then
 - Match OC EoS at the neutron drip with IC EoS
 - Match IC EoS at *crust-core transition* with Core EoS

check e.g. PRC 94, 035804 2016

Non-linear Walecka Model

mesons: mediation of nuclear force

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho}$$

nucleons

electrons

mesons

non-linear mixing coupling

$$\mathcal{L}_i = \bar{\psi}_i [\gamma_\mu i D^\mu - M^*] \psi_i$$

$$\mathcal{L}_e = \bar{\psi}_e [\gamma_\mu (i \partial^\mu + e A^\mu) - m_e] \psi_e$$

$$\mathcal{L}_\sigma = \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 - \frac{1}{3} \kappa \phi^3 - \frac{1}{12} \lambda \phi^4 \right)$$

$$\mathcal{L}_\omega = -\frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu + \frac{1}{4!} \xi g_v^4 (V_\mu V^\mu)^2$$

$$\mathcal{L}_\rho = -\frac{1}{4} \mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu$$

non-linear mixing coupling term:
responsible for density dependence of

E_{sym}

$$\mathcal{L}_{\omega\rho} = g_{\omega\rho} g_\rho^2 g_v^2 V_\mu V^\mu \mathbf{b}_\nu \cdot \mathbf{b}^\nu$$

Light clusters

- New degrees of freedom of the system.
- Interact with the medium via the meson couplings.

$$\mathcal{L} = \sum_{j=t,h} \mathcal{L}_j + \mathcal{L}_\alpha + \mathcal{L}_d$$

the vector cluster-meson coupling

$$g_{vj} = A_j g_v$$

with

$$\mathcal{L}_j = \bar{\psi} [\gamma_\mu iD_j^\mu - M_j^*] \psi, \quad iD_j^\mu = i\partial^\mu - g_{vj}\omega^\mu - \frac{g_\rho}{2} \tau_j \cdot \mathbf{b}^\mu$$

for the fermions tritons and helions,
and for the bosons alphas and deuterons, we have:

$$\mathcal{L}_\alpha = \frac{1}{2} (iD_\alpha^\mu \phi_\alpha)^* (iD_{\mu\alpha} \phi_\alpha) - \frac{1}{2} \phi_\alpha^* (M_\alpha^*)^2 \phi_\alpha,$$

$$\mathcal{L}_d = \frac{1}{4} (iD_d^\mu \phi_d^\nu - iD_d^\nu \phi_d^\mu)^* (iD_{d\mu} \phi_{d\nu} - iD_{d\nu} \phi_{d\mu}) - \frac{1}{2} \phi_d^{\mu*} (M_d^*)^2 \phi_{d\mu}, \quad iD_j^\mu = i\partial^\mu - g_{vj}\omega^\mu$$

In-medium effects - g_{sj}

PRC 97, 045805 2018

- Binding energy of each cluster: $B_j = A_j m^* - M_j^*$, $j = d, t, h, \alpha$

with $m^* = m - g_s \phi_0$ the nucleon effective mass and

$$M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j) \text{ the cluster effective mass.}$$

the scalar cluster-meson coupling

$$g_{sj} = x_{sj} A_j g_s$$

needs to be determined from exp. constraints

In-medium effects - δB_j

PRC 97, 045805 2018

- Binding energy of each cluster: $B_j = A_j m^* - M_j^*$, $j = d, t, h, \alpha$
with $m^* = m - g_s \phi_0$ the nucleon effective mass and

$$M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j)$$

the cluster effective mass.

binding energy shift

$$\delta B_j = \frac{Z_j}{\rho_0} (\epsilon_p^* - m \rho_p^*) + \frac{N_j}{\rho_0} (\epsilon_n^* - m \rho_n^*)$$

energetic counterpart of classical ExV mechanism

the energy states occupied by the gas are excluded:
double counting avoided!

associated with the gas lowest energy levels

$$\epsilon_j^* = \frac{1}{\pi^2} \int_0^{p_{F_j}(\text{gas})} p^2 e_j(p) (f_{j+}(p) + f_{j-}(p)) dp$$

$$\rho_j^* = \frac{1}{\pi^2} \int_0^{p_{F_j}(\text{gas})} p^2 (f_{j+}(p) + f_{j-}(p)) dp,$$

EoS for HM with light clusters

- The total baryonic density is defined as:

$$\rho = \rho_p + \rho_n + 4\rho_\alpha + 2\rho_d + 3\rho_h + 3\rho_t$$

- The global proton fraction as

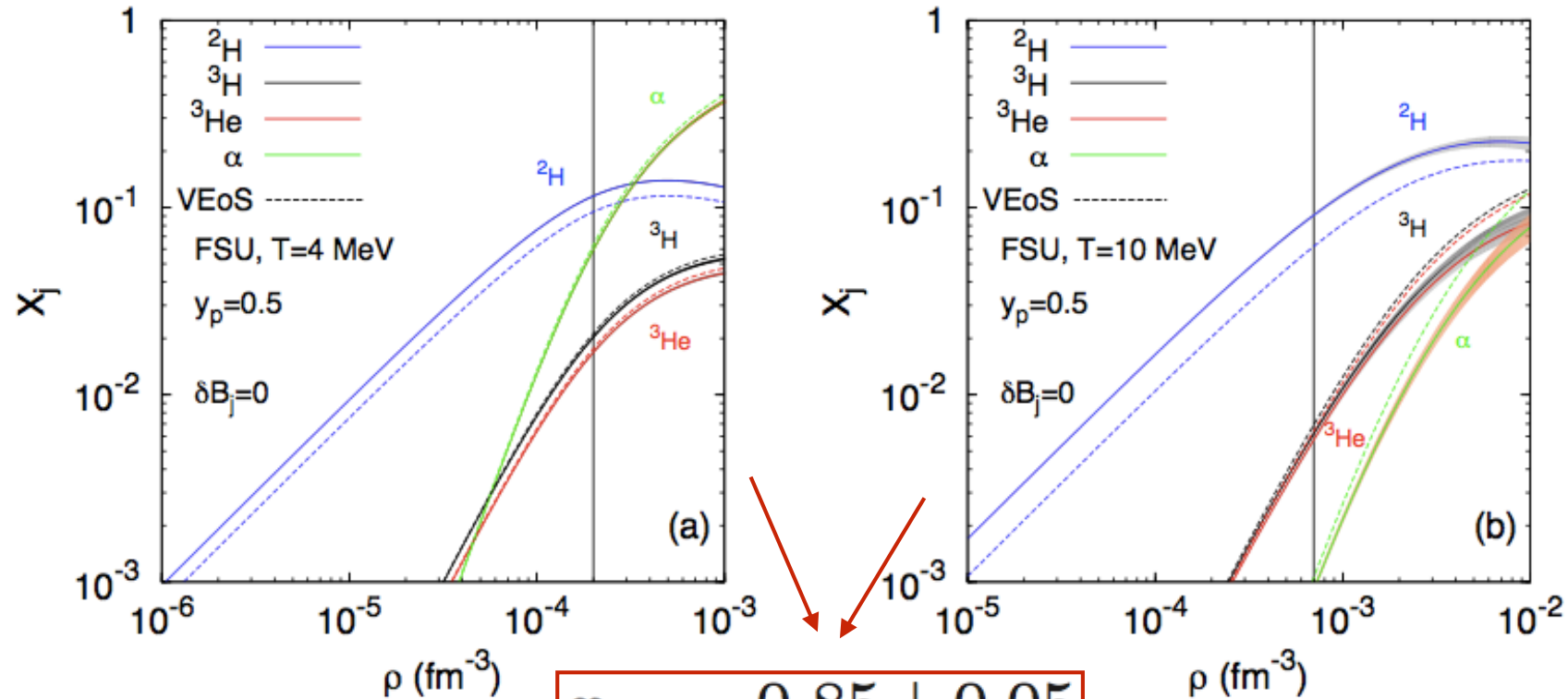
$$Y_p = y_p + \frac{1}{2}y_\alpha + \frac{1}{2}y_d + \frac{2}{3}y_h + \frac{1}{3}y_t$$

with $y_i = A_i(\rho_i/\rho)$ the mass fraction of cluster i .

- Charge neutrality must be imposed: $\rho_e = Y_p \rho$
- The light clusters are in chemical equilibrium, with the chemical potential of each cluster i defined as

$$\mu_i = N_i\mu_n + Z_i\mu_p$$

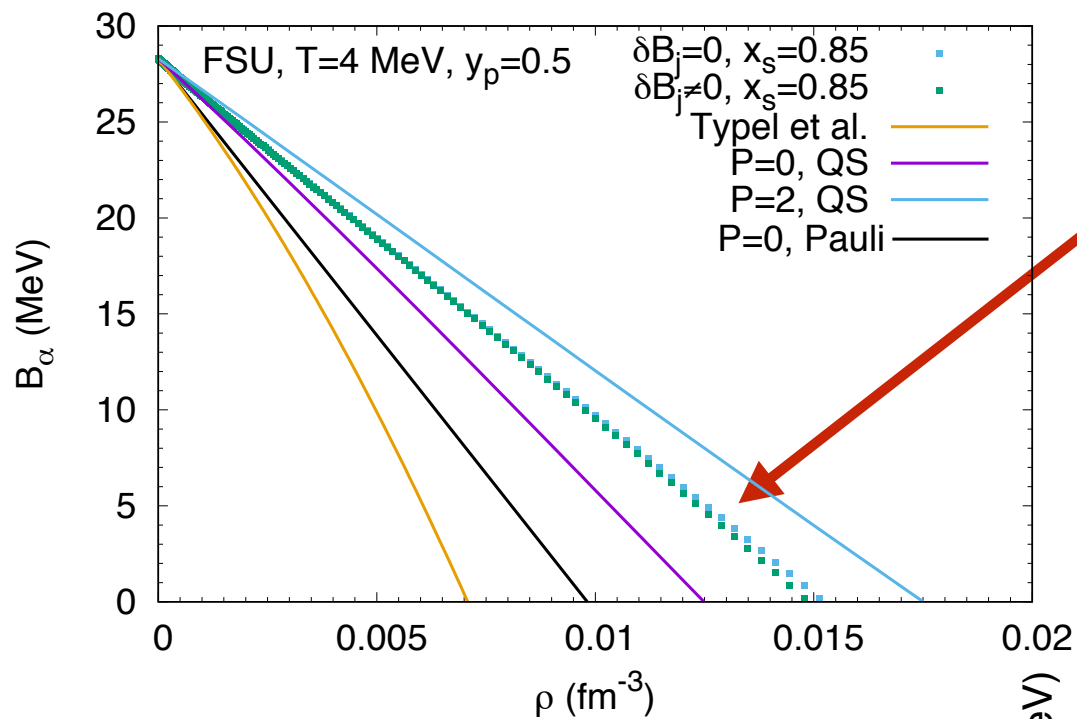
Determination of x_s : Virial EoS



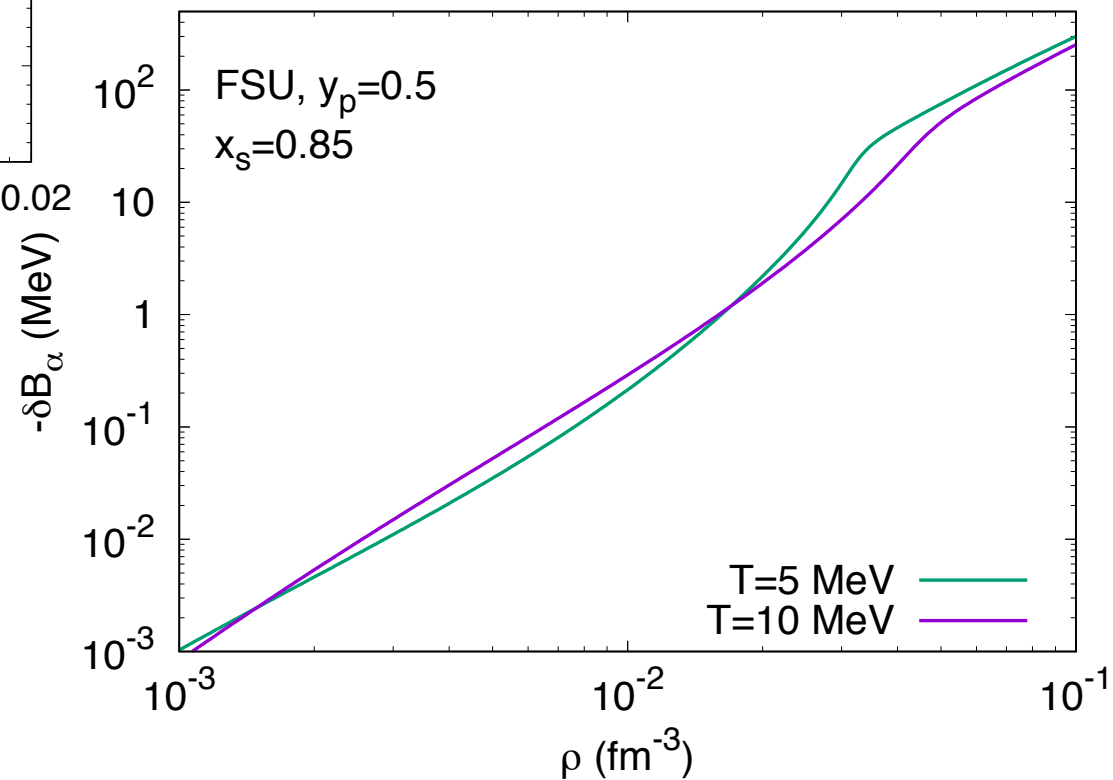
- **VEoS**: model-independent constraint, only depends on experimentally binding energies and scattering phase shifts.
- Provides correct zero-density limit for finite T EoS.
- Breaks down when interaction with particles becomes stronger:

δB_j takes action!!

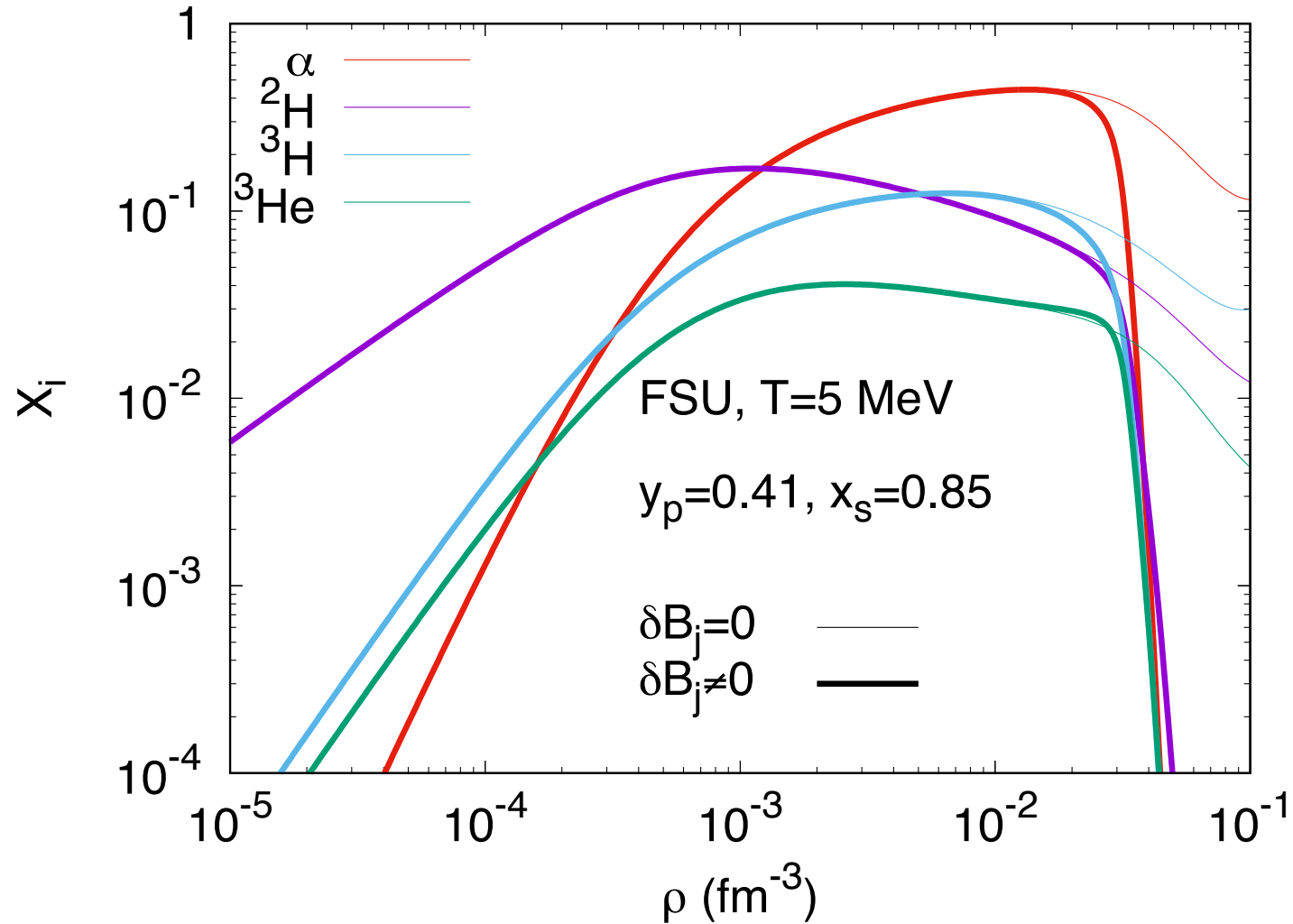
Contribution of δB_j



- δB_j completely negligible in the VEOs range of densities
- but rises fast for larger densities



Cluster fractions - effect of δB_j



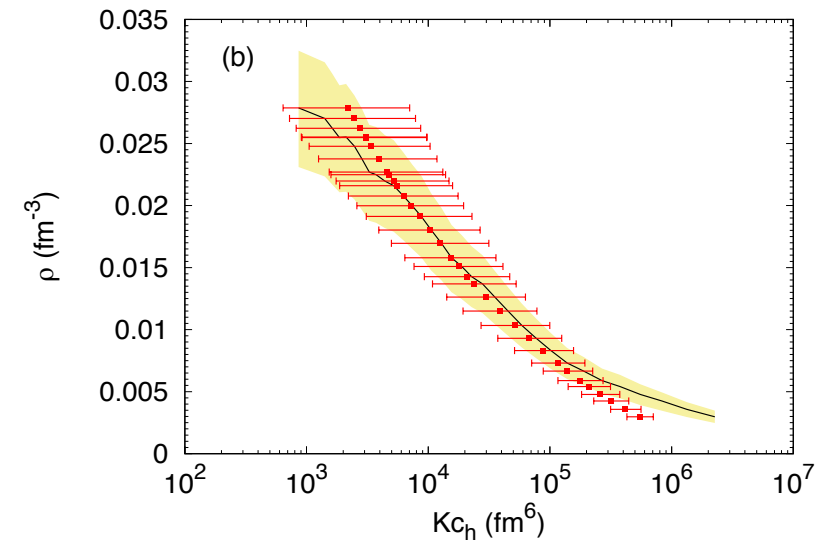
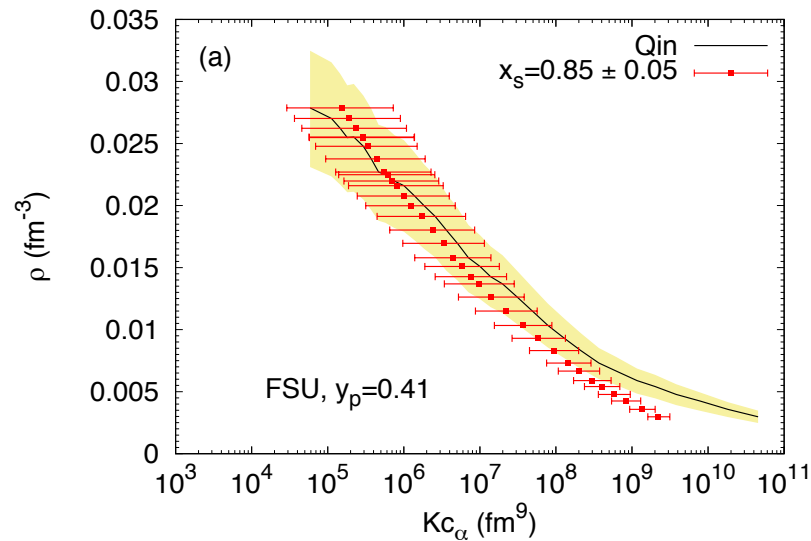
- δB_j important for dissolution of clusters!

Equilibrium constants

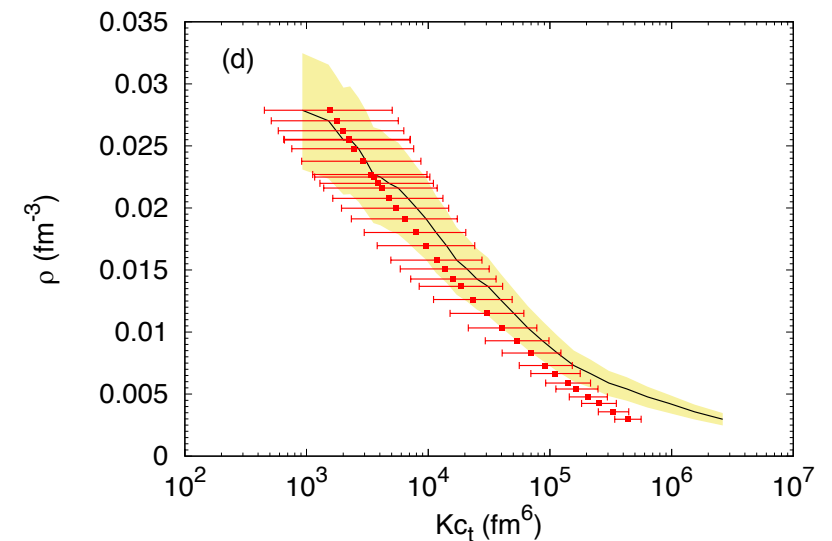
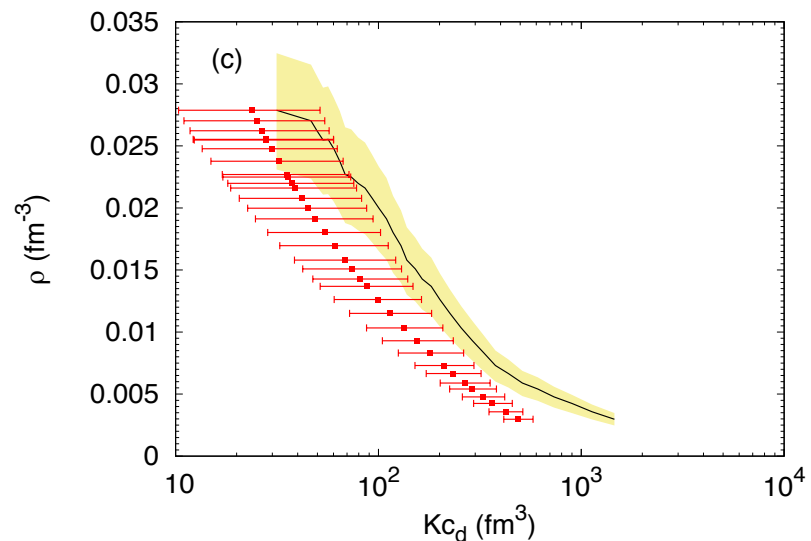
Qin et al, PRL 108, 172701 2012

- K_c calculated with data from HIC:

$$K_c[j] = \frac{\rho_j}{\rho_n^{N_j} \rho_p^{Z_j}}$$



- Unique existing constraint on in-medium modifications of light clusters at finite T

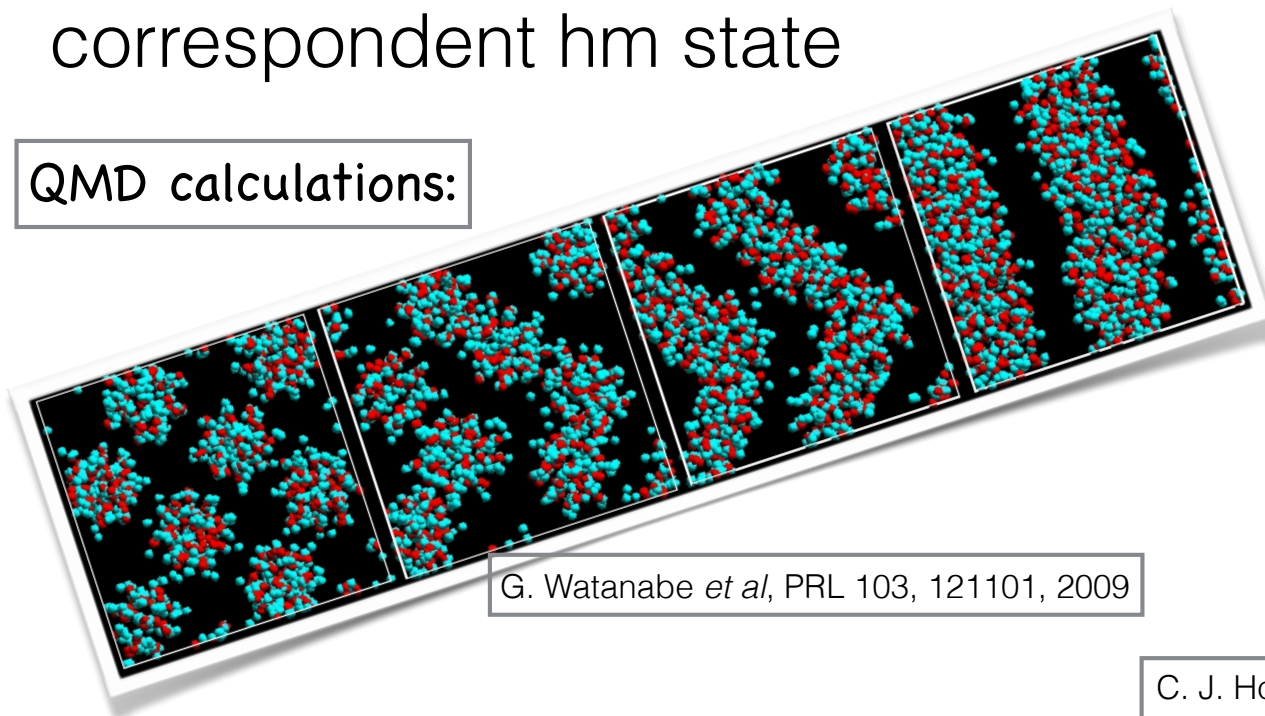


- Our model describes quite well exp data!

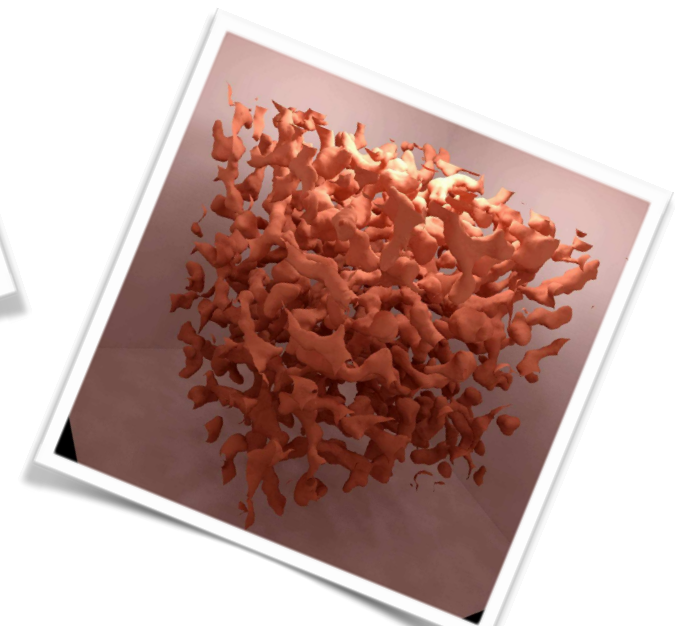
The pasta phases

- Competition between Coulomb and nuclear forces leads to frustrated system
- Geometrical structures, the **pasta phases**, evolve with density until they melt → **crust-core transition**
- Criterium: pasta free energy must be lower than the correspondent hm state

QMD calculations:



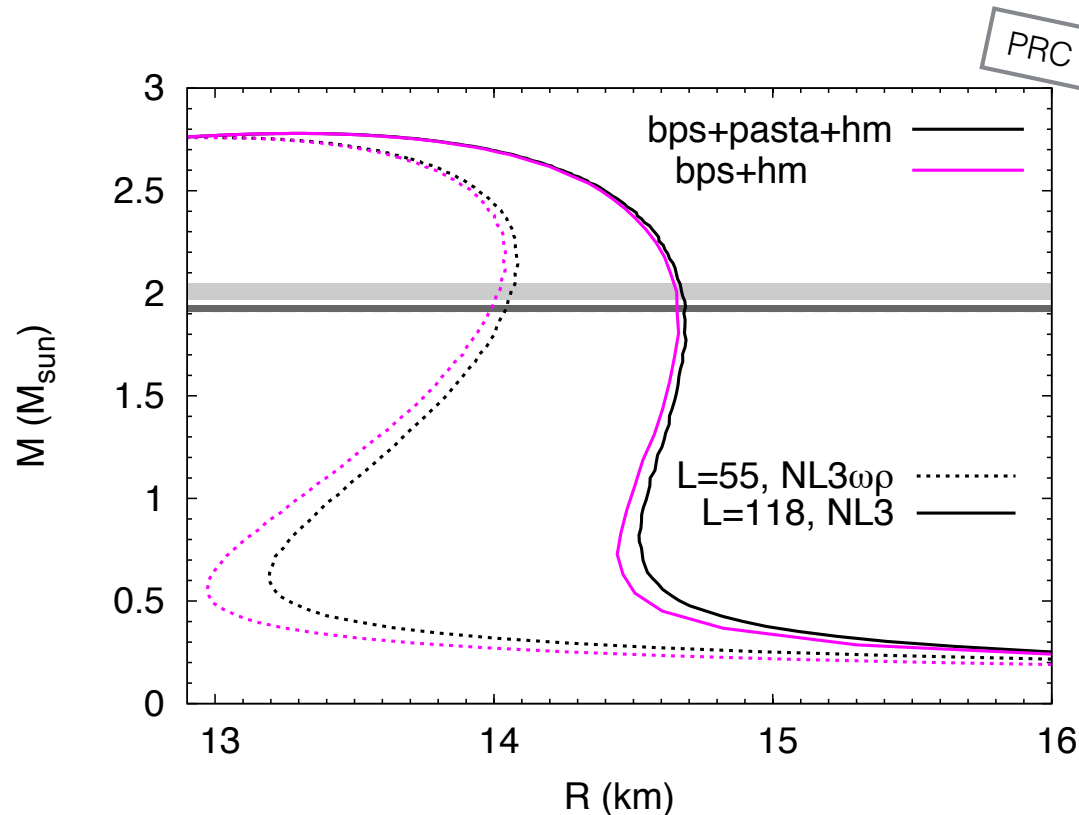
G. Watanabe *et al*, PRL 103, 121101, 2009



C. J. Horowitz *et al*, PRC 70, 065806, 2004

Why are these phases important?

- They may have an effect in the cooling of the star.
- They do have an effect in the radius of the stars, but not in the maximum mass:



For $1.4M_{\odot}$ stars, the RMF models that passed the constraints predict $R=13.6 \pm 0.3$ km and a crust thickness of $\Delta R=1.36 \pm 0.06$ km.

Pasta phases - calculation (I)

check PRC 91, 055801 2015

- Coexistence Phase (CP) approximation:
 - Separated regions of higher (pasta phases) and lower density (background nucleon gas).
 - Gibbs equilibrium conditions: for $T = T^I = T^{II}$:

$$\begin{aligned}\mu_p^I &= \mu_p^{II} \\ \mu_n^I &= \mu_n^{II} \\ P^I &= P^{II}\end{aligned}$$

- Finite size effects are taken into account by a surface and a Coulomb terms in the energy density, **after the coexisting phases are achieved**.
- Total \mathcal{F} and total ρ_p of the system:

$$\mathcal{F} = f\mathcal{F}^I + (1 - f)\mathcal{F}^{II} + \mathcal{F}_e + \epsilon_{surf} + \epsilon_{coul}$$

$$\rho_p = \rho_e = y_p\rho = f\rho_p^I + (1 - f)\rho_p^{II}$$

and

$$\epsilon_{surf} = 2\epsilon_{Coul}$$

Pasta phases - calculation (II)

check PRC 91, 055801 2015

- Compressible Liquid Drop (CLD) approximation:

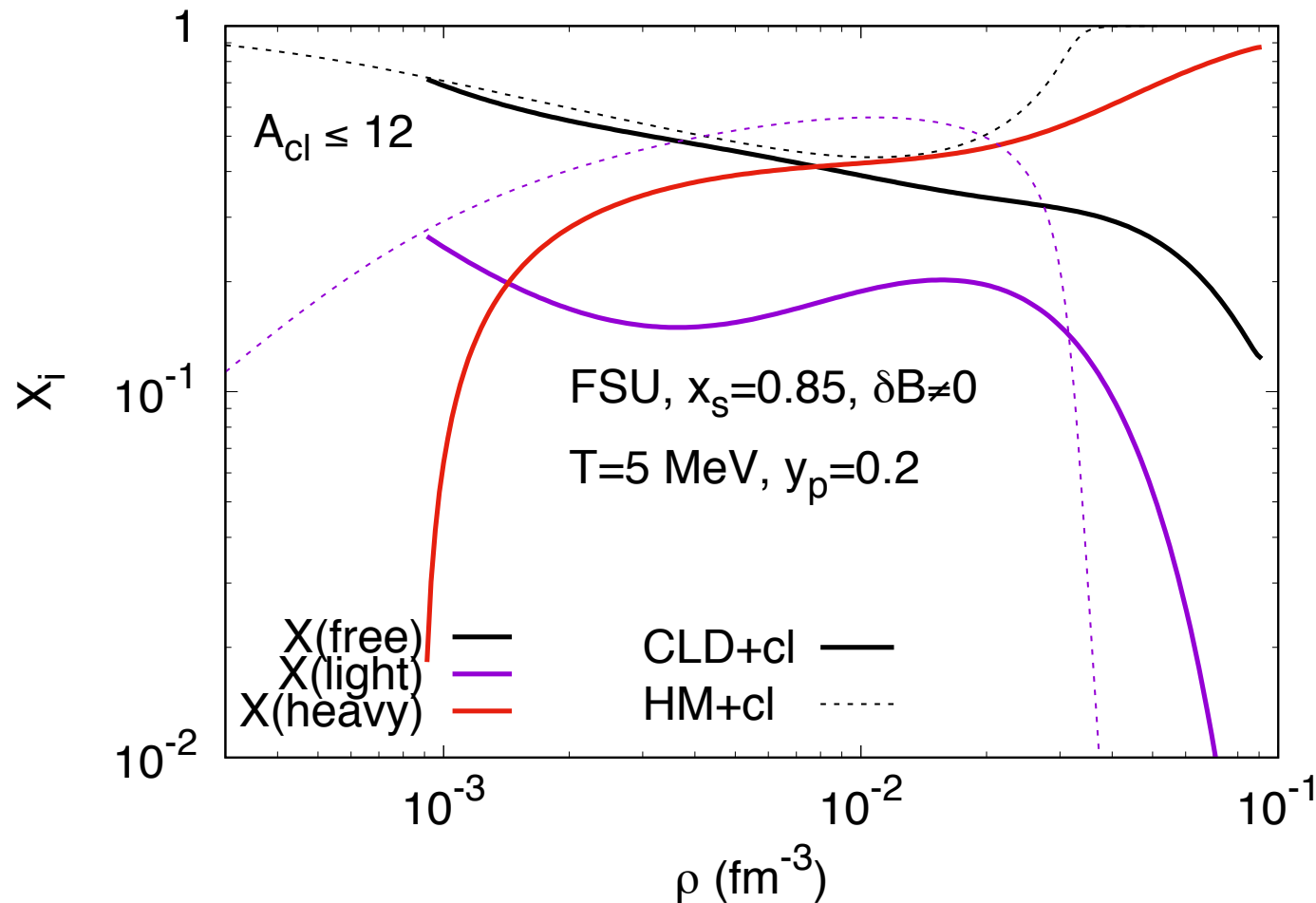
The total free energy density is minimized, **including the surface and Coulomb terms.**

The Gibbs equilibrium conditions become:

$$\begin{aligned}\mu_n^I &= \mu_n^{II}, \\ \mu_p^I &= \mu_p^{II} - \frac{\epsilon_{surf}}{f(1-f)(\rho_p^I - \rho_p^{II})}, \\ P^I &= P^{II} - \epsilon_{surf} \left(\frac{1}{2\alpha} + \frac{1}{2\phi} \frac{\partial \phi}{\partial f} - \frac{\rho_p^{II}}{f(1-f)(\rho_p^I - \rho_p^{II})} \right)\end{aligned}$$

Cluster fractions - CLD vs HM

- Heavy cluster with light clusters (CLD+cl) **VS.** homogeneous matter with light clusters (HM+cl).
- Light clusters with $A \leq 12$.



- The heavy cluster makes the light clusters less abundant but increases their melting density, as compared with the HM+cl calculation.
- The background of free nucleons also decreases in the presence of the heavy cluster.

Summary

- A simple parametrisation of in-medium effects acting on light clusters is proposed in a RMF framework.
- Interactions of clusters with medium described by modification of sigma-meson coupling constant.
- Clusters dissolution obtained by the density-dependent extra term on the binding energy.
- $x_{sj} = 0.85 \pm 0.05$ reproduces both virial limit and K_c from HIC.
- Light clusters and pasta structures are relevant and should be explicitly included in EoS for CCSN simulations and NS mergers.
- Extra constraints from experimental data are needed!!

Grazie mille!

Thank you!