REALISTIC SHELL-MODEL STUDIES OF NUCLEAR STRUCTURE AND BETA DECAY

Women in Nuclear and Hadron Theoretical Physics: the last frontier

Genova, December 10-11 2018

Istituto Nazionale di Fisica Nucleare

A. Gargano, Sezione di Napoli

OUTLINE

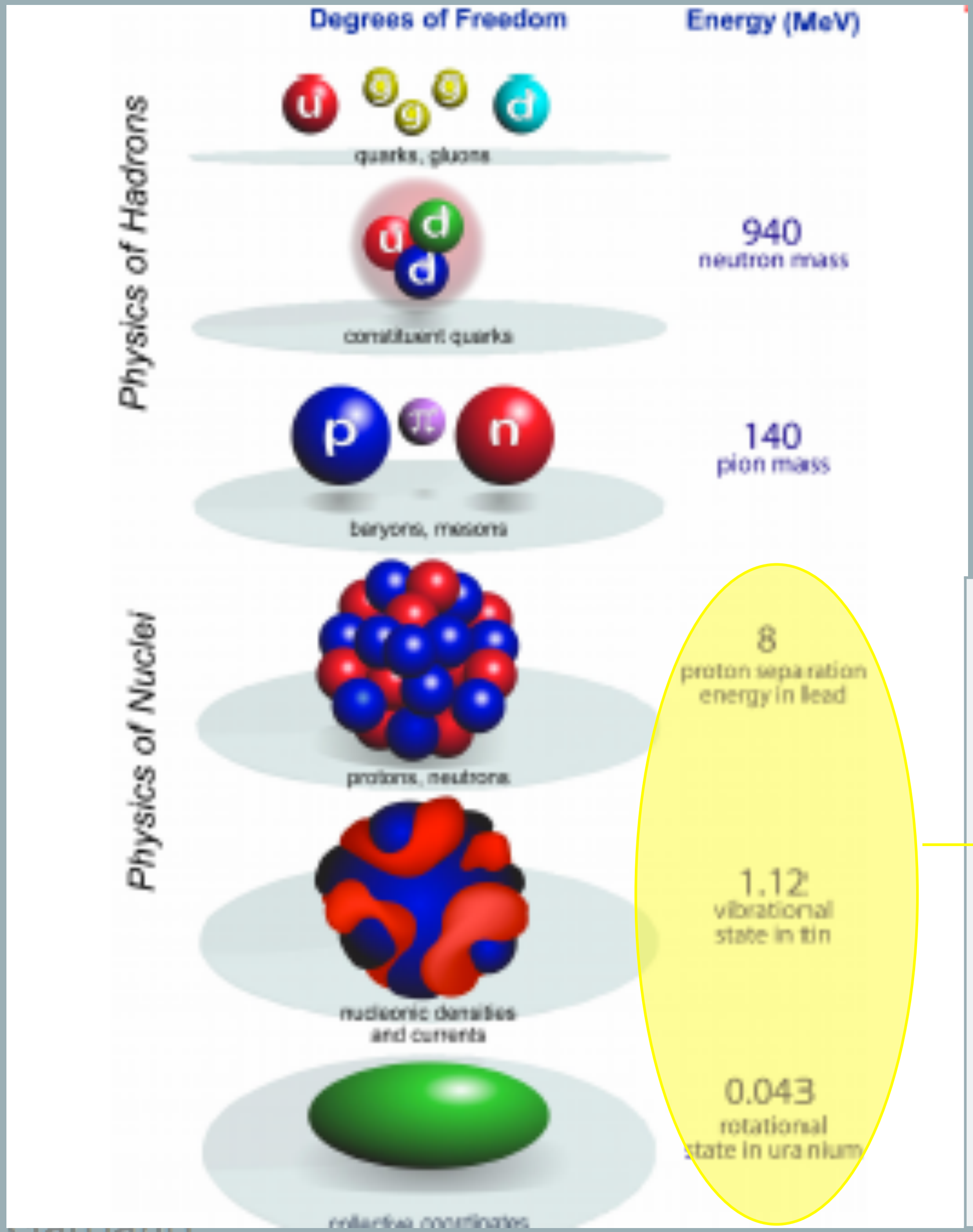
- What is nuclear structure and why carry out nuclear structure studies?

- Which are the available tools? → Nuclear structure models → Shell model → Realistic shell model

- Applications of realistic shell-model to nuclear structure:
  ✓ Results for GT strengths and nuclear matrix elements of the $2\nu\beta\beta$ decay for nuclei that are candidates for the observation of the $0\nu\beta\beta$ decay
  ✓ Very preliminary results for some $0\nu\beta\beta$ nuclear matrix elements

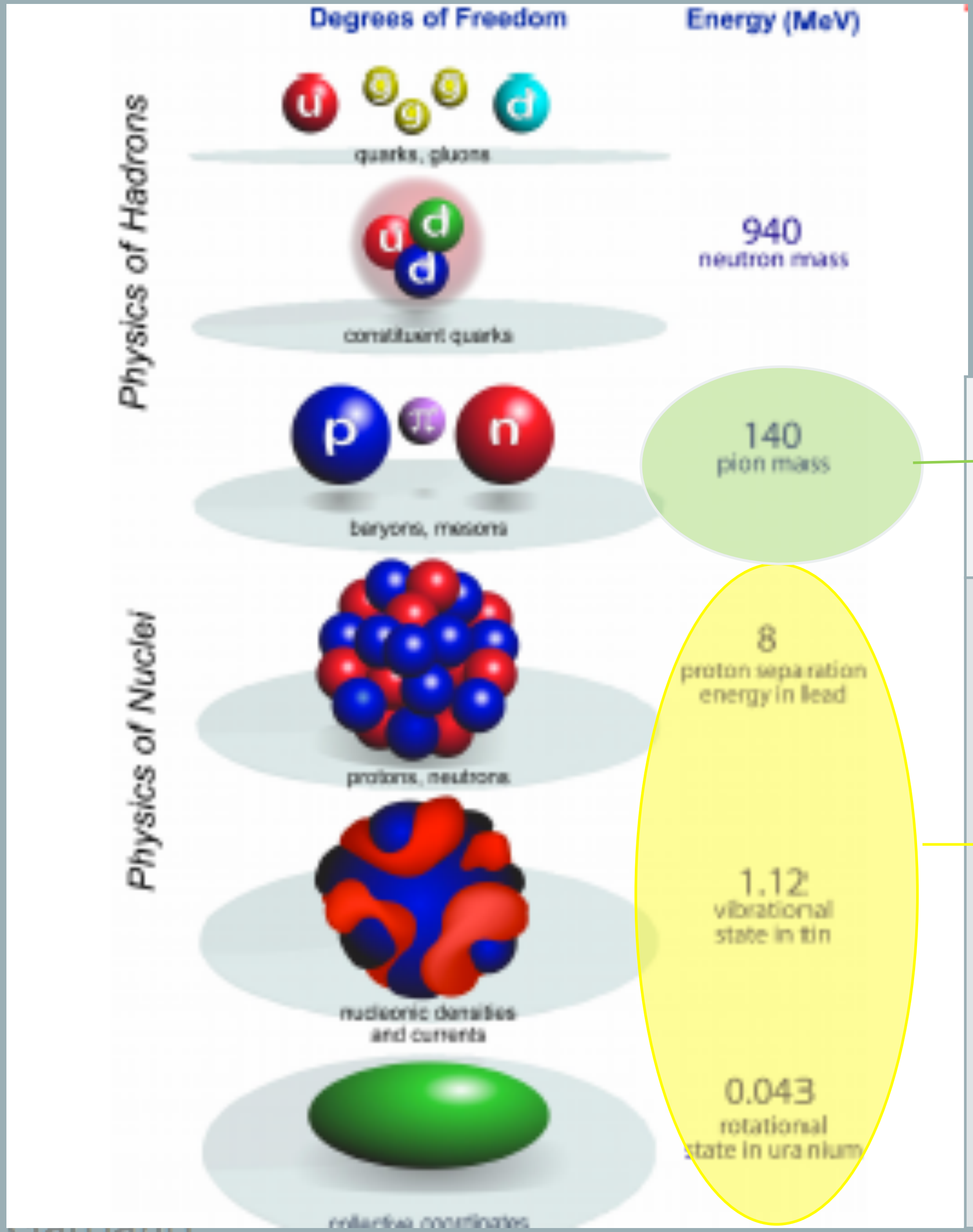
- Concluding remarks

DEGREES OF FREEDOM OF ATOMIC NUCLEI



Nuclear structure and reactions

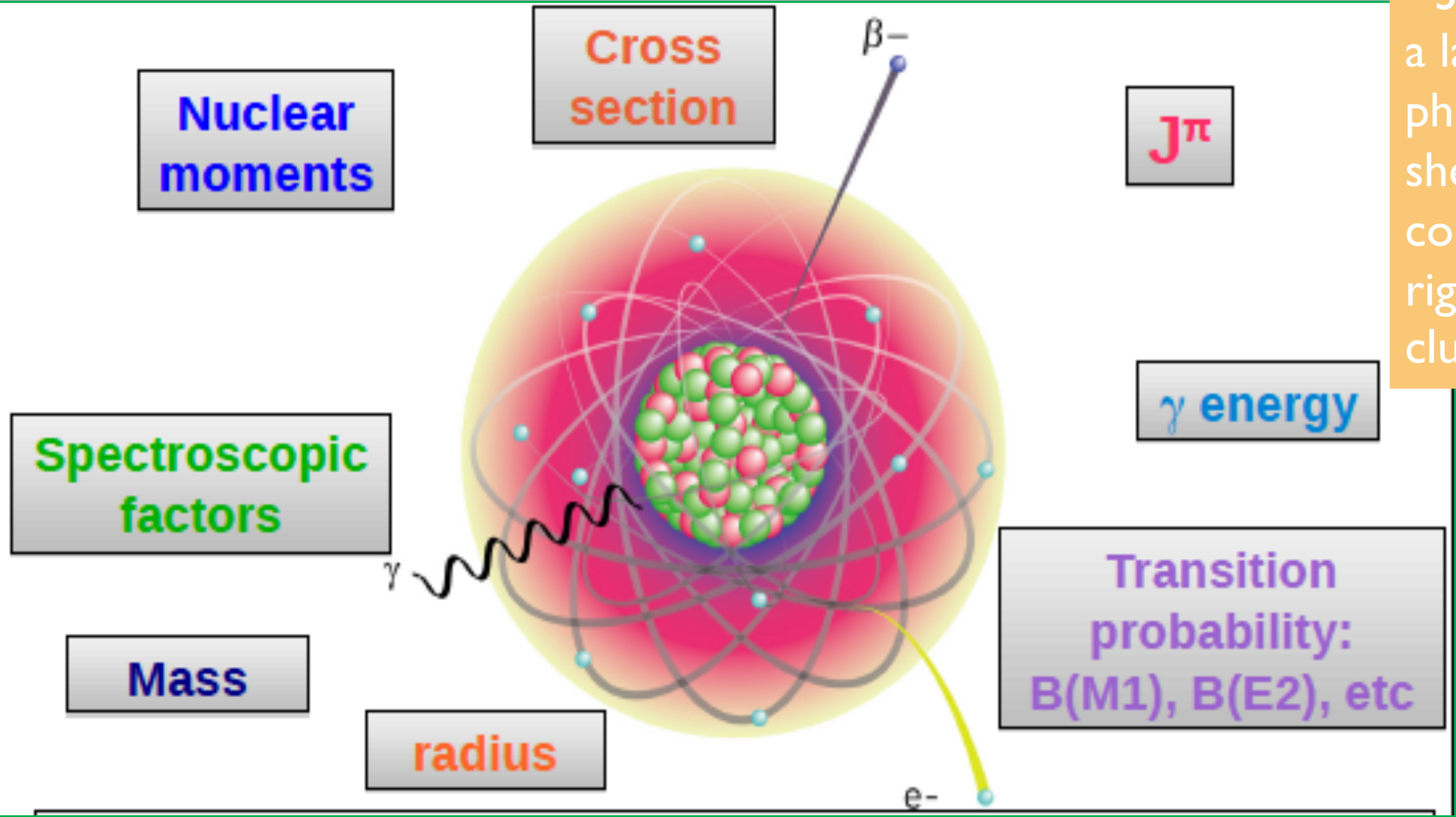
DEGREES OF FREEDOM OF ATOMIC NUCLEI



Hadron nuclear interface

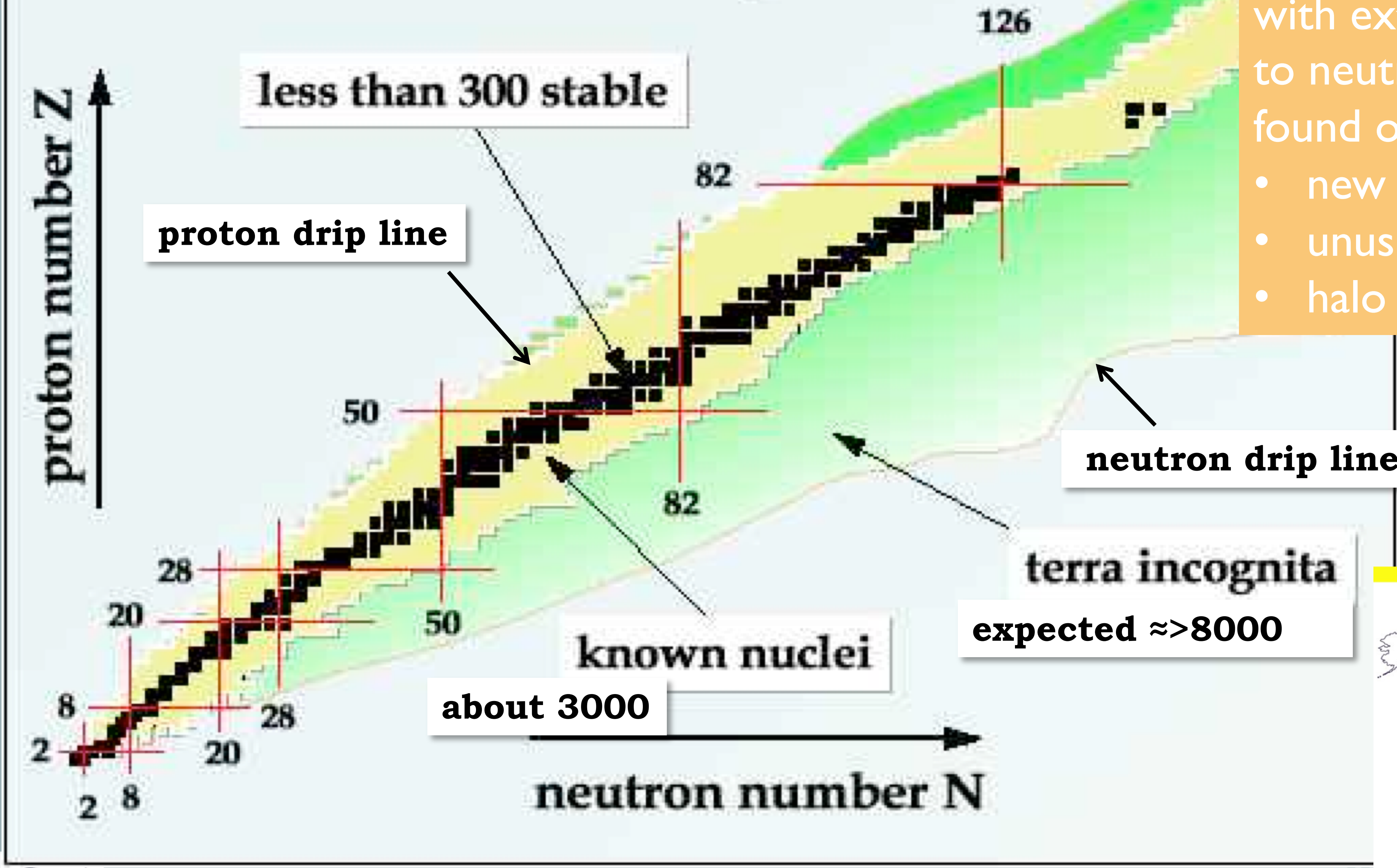
Nuclear structure and reactions

INVESTIGATING THE NUCLEAR MANY BODY PROBLEM



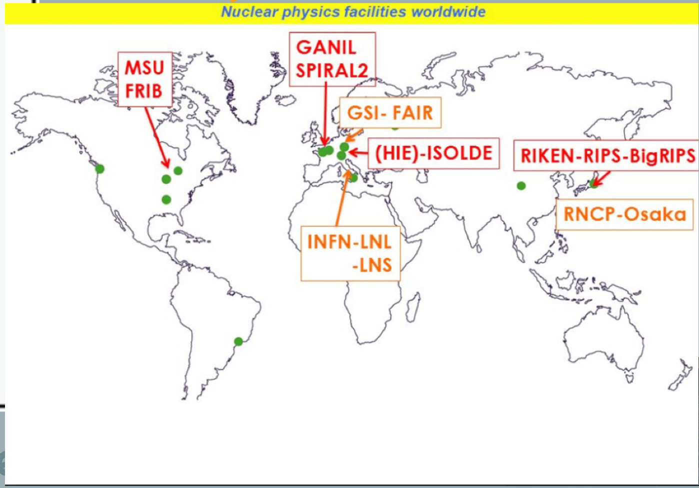
Nuclei are many-body quantal systems (up to ~300 nucleons) showing a large variety of quantum phenomena, as
shell structure (like atoms),
collective behavior (like rigid rotor),
clustering

NUCLEAR LANDSCAPE



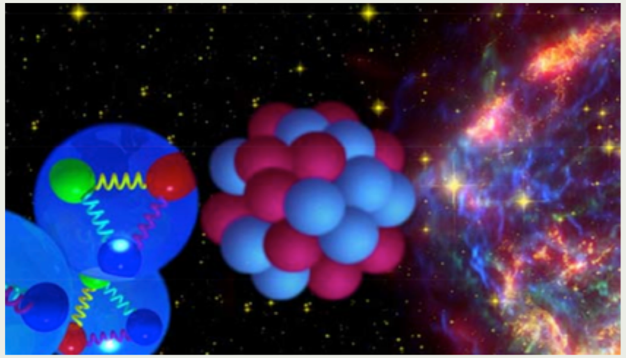
Key new data are obtained for nuclei very far from stability, with extreme ratios of protons to neutrons (that cannot be found on the Earth) →

- new shell configurations
- unusual shapes
- halo nuclei…



The theory of atomic nuclei is intimately tied to other problems beyond nuclear physics

from QCD



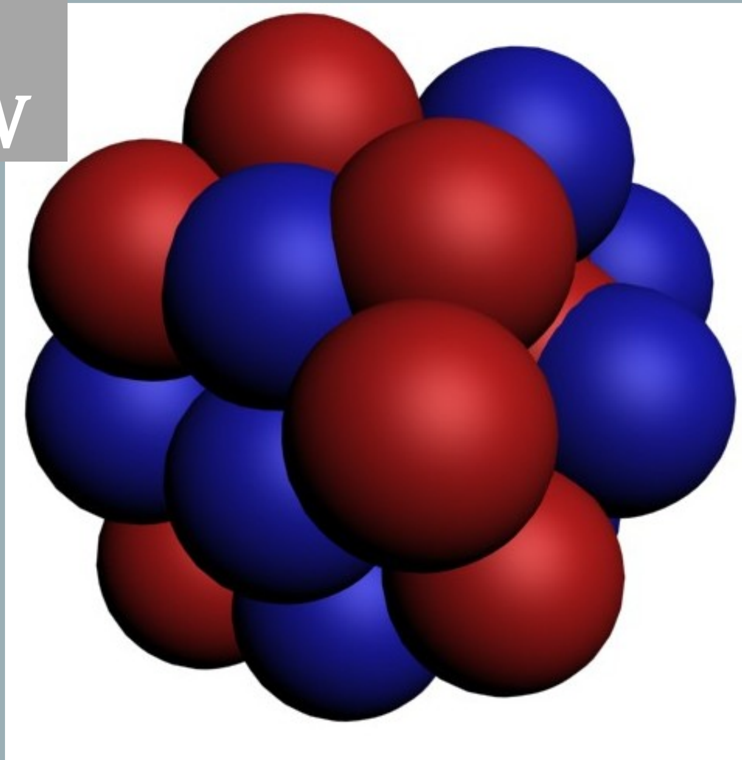
to astrophysics

- A deep understanding of nuclei and nucleonic matter also allows us to use nuclear physics to probe neutrino physics and fundamental symmetries → physics beyond the standard model

- Nuclei are the core of matter and their properties govern the evolution of the universe and the abundance of the elements

NUCLEAR MANY-BODY PROBLEM

$^{A}_{Z}X_{N}$



$$H = T + V_{NN} + V_{3N} + \ldots$$

$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

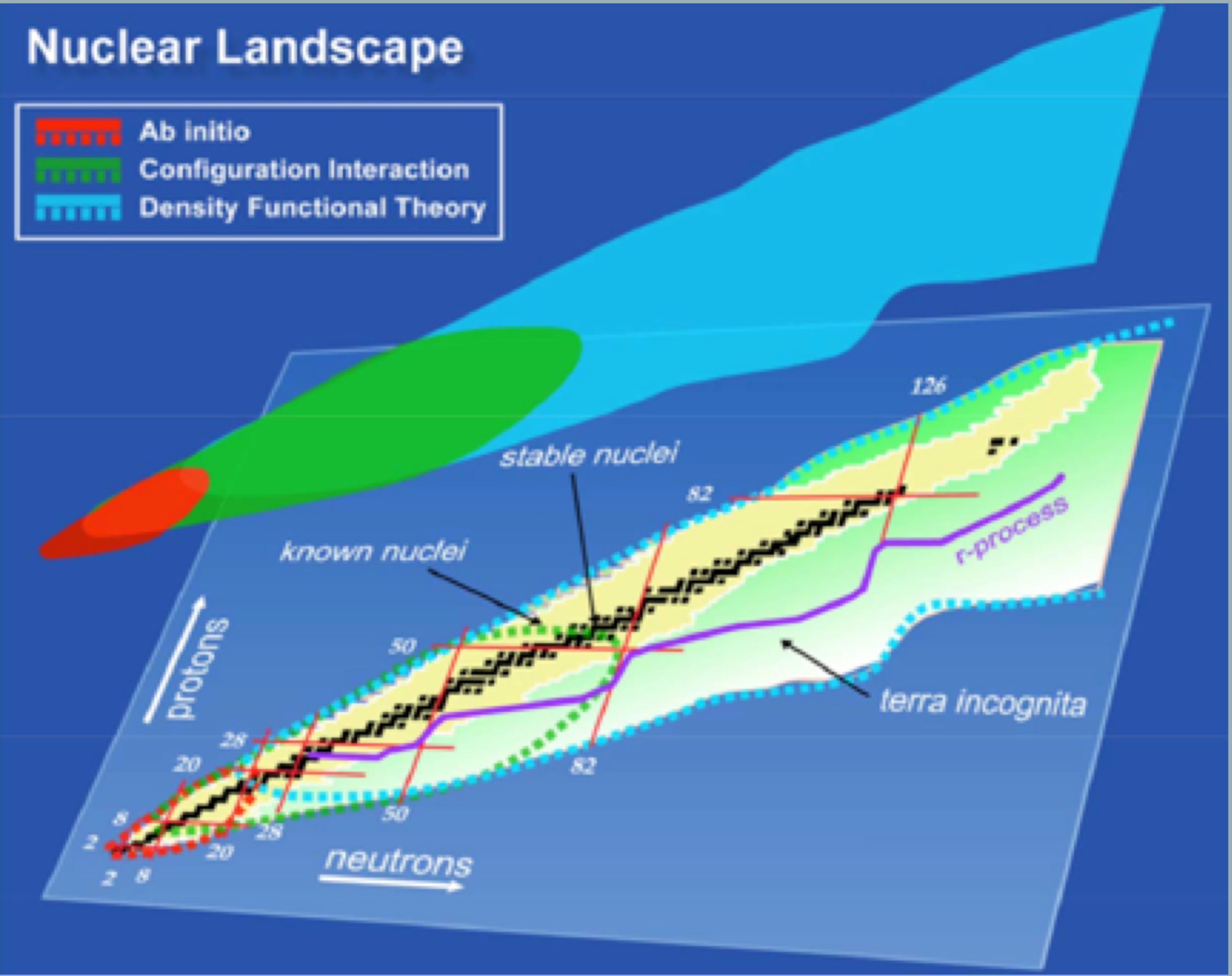
$$\langle\psi_j|O|\psi_i\rangle$$

The "exact" solution of the Schrödinger equation for any value of A is unfeasible

only for light-mass nuclei ($A \leq 12$)

→ NUCLEAR MODELS

NUCLEAR MODELS



- Each model has strengths and limitations
- Accuracy or more microscopic vs larger application domain
- No systematic connections between the models
- Overlapping regions to cross-check

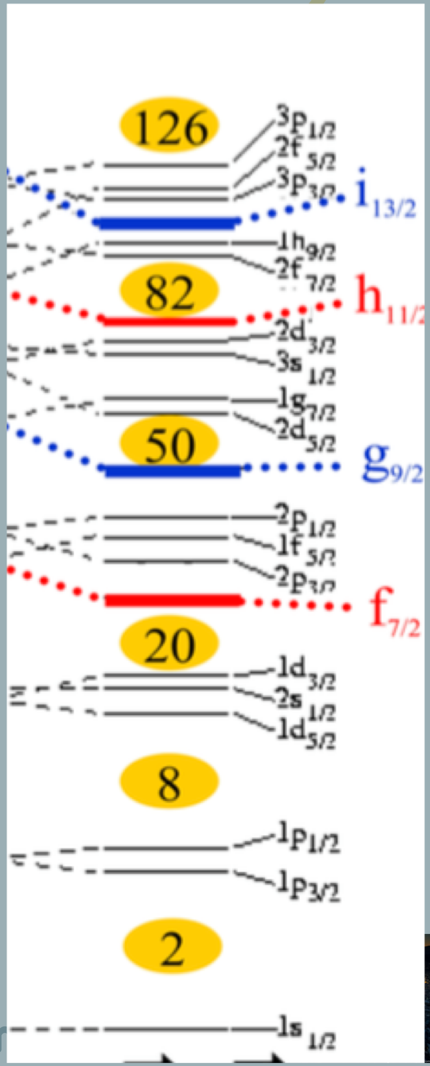
SHELL MODEL (NON INTERACTING)

Basic idea:

Mean-field ansatz, namely each nucleon is assumed to be moving in an external field created by the remaining nucleons

$$U=U(r) + U_{ls}\, \boldsymbol{l} \cdot \boldsymbol{s}$$

magic numbers!



J. Hans D. Jensen
(1907 – 1973)
***PR* 75, 1766 (1949)**

Maria Goeppert Mayer
1906 - 1972
***PR* 75, 1969 (1949)**

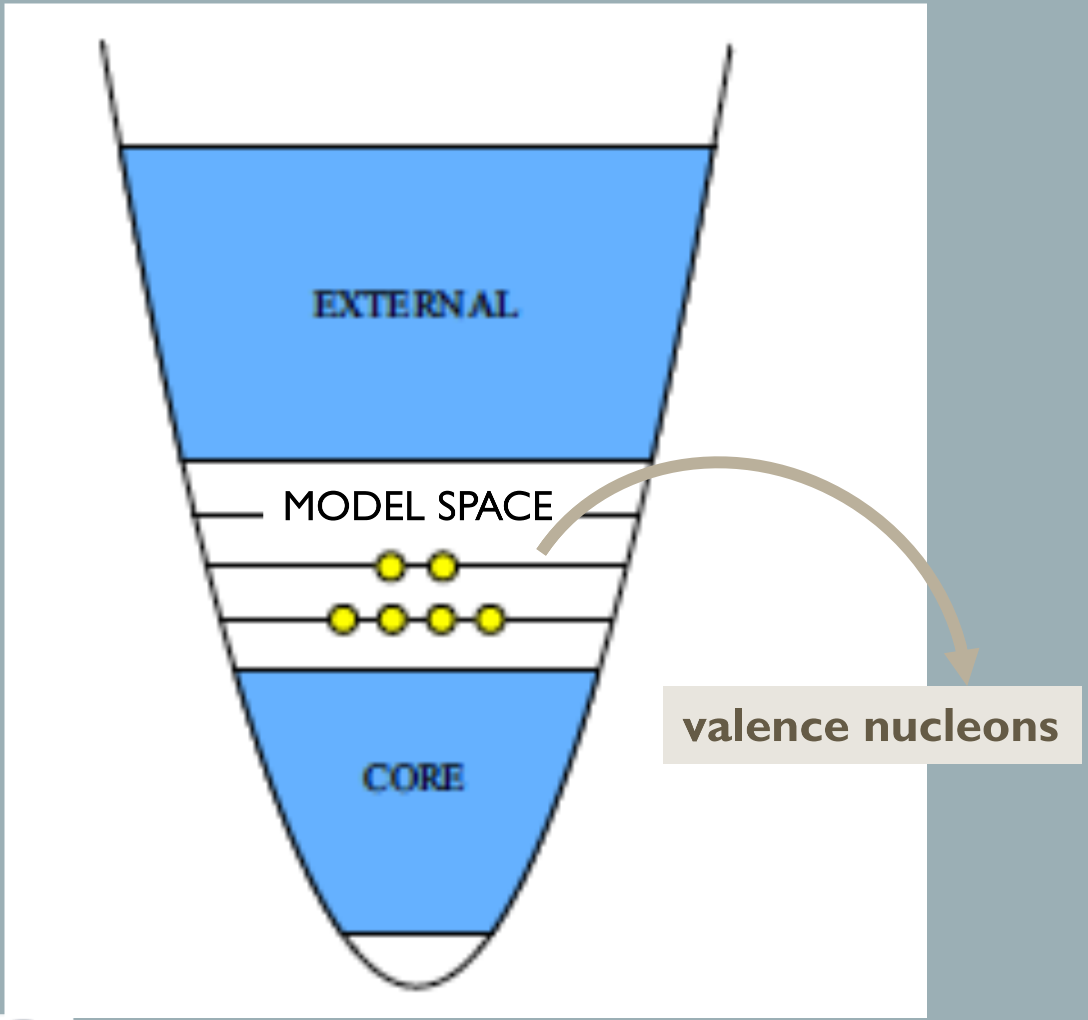
The shell model has initiated a large field of research. It has served as the starting point for more refined calculations. There are enough nuclei to investigate so that the shell modellists will not soon be unemployed.

from the Nobel Lecture of MGM, December 12,1963

Noble Prize in 1963
for their descoveries concerning nuclear shell structure

SHELL MODEL (INTERACTING)

Starting from the mean-field ansatz, we obtain a set of single-particle states, where all nucleons of the nucleus are distributed starting from the lowest energy level



SP states are assumed to be separated into 3 spaces, well separated in energy:

– inert core
– model space
– external space

the valence nucleons are the only active/interacting degrees of freedom

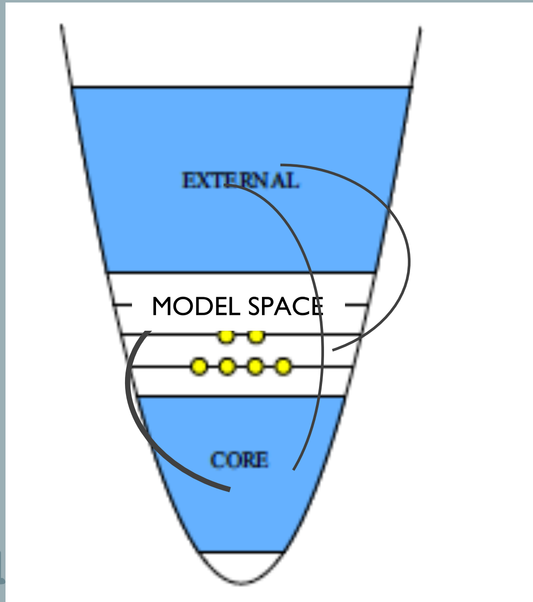
core nucleons and the excitations of valence nucleons above the model space are "*frozen*"

SHELL MODEL HAMILTONIAN

$$H_{\rm eff} = \sum_i U_i + \sum_{i<j} V_{ij}$$

defined in the model space for only valence nucleons

$H_{\rm eff}$
should take into account
in an effective way
all the degrees of freedom not considered explicitly
namely excitations of the core nucleons into the model and external spaces and of the valence nucleons into the external space



SHELL-MODEL HAMILTONIAN

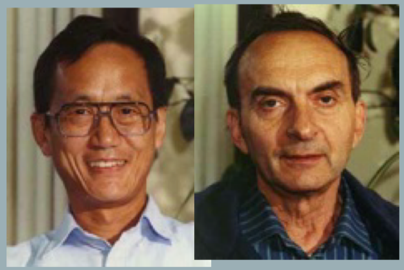
Two alternative approaches

- ***empirical*** - MEs of interaction are considered as parameters or contain adjustable parameters fitted on the experimental data

- ***microscopic***

realistic shell-model Hamiltonian

$$V_{NN}\ (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\rm eff}$$



T. T. S. Kuo & G. E. Brown
Nucl. Phys. 85 (1966) 40

- **No adjustable parameters → increase of predictive power**
- **"Bridge" between effective shell-model interactions and underlying nuclear forces**

FLOW CHART OF A REALISTIC SHELL MODEL CALCULATION

- Choose a free *NN (NNN)* potential

- Determine the number of valence nucleons and the model space better tailored to study the system under investigation

- Derive the effective Hamiltonian making use of many-body theory

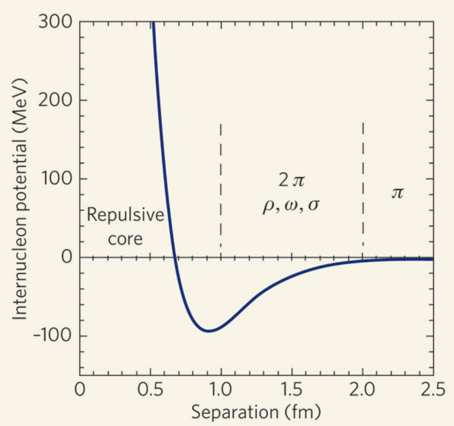
- Diagonalize the Hamiltonian matrix & calculate physical observables as energies, electromagnetic transition probabilities, …

REALISTIC NN POTENTIALS

- CD-Bonn, Argonne V_{18}, Nijmegen,…

Modern Potentials reproduce the two-body data with

$\chi^2/N_{data} \sim 1$



Renormalization of the short-range repulsion through V_{low-k} approach: low-momentum potentials confined with a momentun space defined by the cutoff Λ [S. Bogner et al. ,Phys. Rev. C 65 , 051301(R) (2002)]

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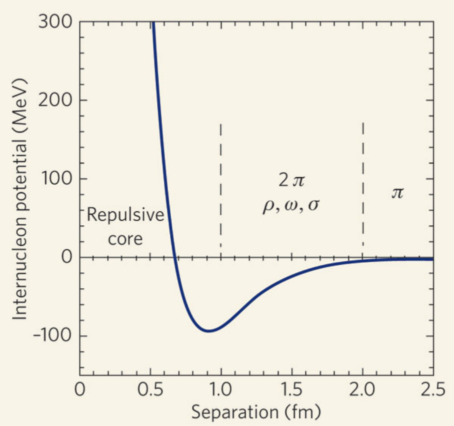

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- Chiral potentials rooted in χEFT

✓ pions are added to the nucleons as fundamental degrees of freedom

✓ long-range forces are ruled by the symmetries of low-energy QCD, while short-range dynamics is absorbed in LEC (fitted on data of the few-body systems)

✓ two- and many-body forces are generated on the same footing

[see for instance R. Machleidt, D. R Entem, Phys. Rep. 503, 1 (2011)]

SHELL-MODEL EFFECTIVE HAMILTONIAN

$$H = \mathrm{T} + V_{NN} = (T + U) + (V_{NN} - U) = H_0 + H_1$$

where the auxiliary one-body potential U is introduced to break up the Hamiltonian into a sum of a one body term H_0 and a residual interaction H_1

$$\Rightarrow \qquad H_{\mathrm{eff}} = H_0 + V_{\mathrm{eff}}$$

with the requirement that the eigenvalues of H_{eff} should belong to the set of eigenvalues of the full nuclear Hamiltonian

- $Q = 1 - P$

SHELL-MODEL EFFECTIVE HAMILTONIAN: $\hat{Q}$ + folded diagram expansion

L. Coraggio et al, Part. Nucl. Phys. 62 , 135 (2009)
L. Coraggio et al, Ann. Phys. 327, 2125 (2012)

$$H_{\text{eff}} = \hat{Q} + \sum_{i=1}^{\infty} F_i$$

$$F_1 = \hat{Q}_1 \hat{Q}$$

$$F_2 = \hat{Q}_2 \hat{Q} + \hat{Q}_1 \hat{Q}_1 \hat{Q}$$

$$\vdots$$

$$\hat{Q} = PH_1P + PH_1Q \frac{1}{\varepsilon_0 - QHQ} QH_1P$$

$$\hat{Q}_n = \frac{1}{n!} \left. \frac{d^n \hat{Q}(\varepsilon)}{d\varepsilon^n} \right|_{\varepsilon=\varepsilon_0}$$

- ε_0 is the unperturbed energy of two nucleons in the model space
- P projection operator onto the model space
- $Q = 1 - P$

SHELL-MODEL EFFECTIVE HAMILTONIAN: $\hat{Q}$ + folded diagram expansion

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The series for H_{eff} is summed up by using Lee-Suzuki iteration method

Prog. Theor. Phys. 64, 2091 (1980)

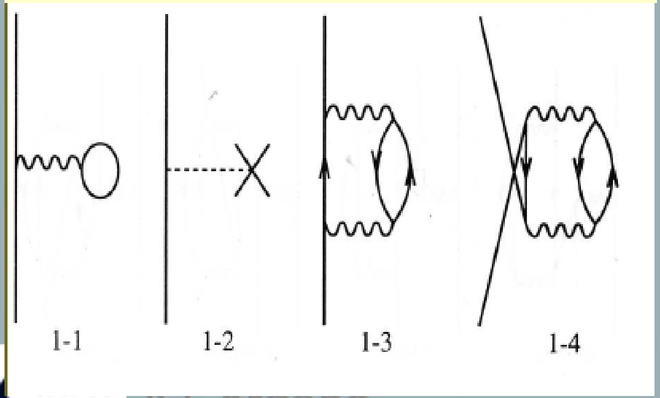
$\hat{Q}$-BOX CALCULATION

The $\hat{Q}$-box is calculated perturbatively by using a diagrammatic expansion

$$\hat{Q} = PH_1P + PH_1Q \frac{1}{E_0 - QHQ} QH_1P$$

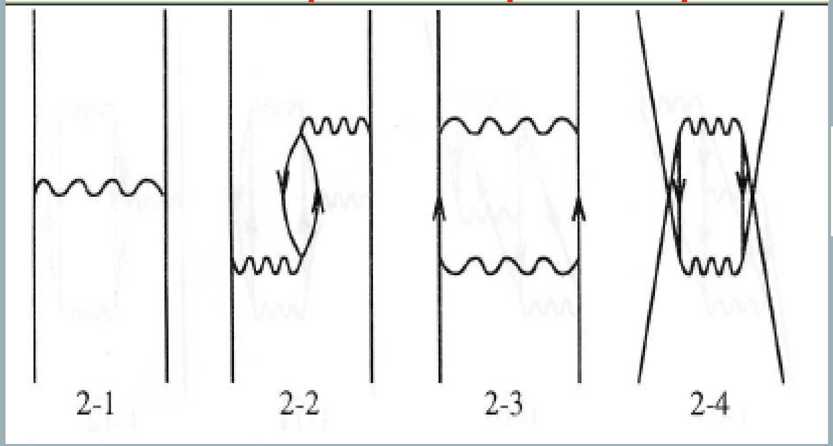
diagrammatic expansion

1-body diagrams up to 2nd order S-box



2-body diagrams up to 2nd order:

V V_{1p1h} V_{2p} V_{2p2h}



+ …

Modern calculations do not go beyond third order (~200 diagrams)

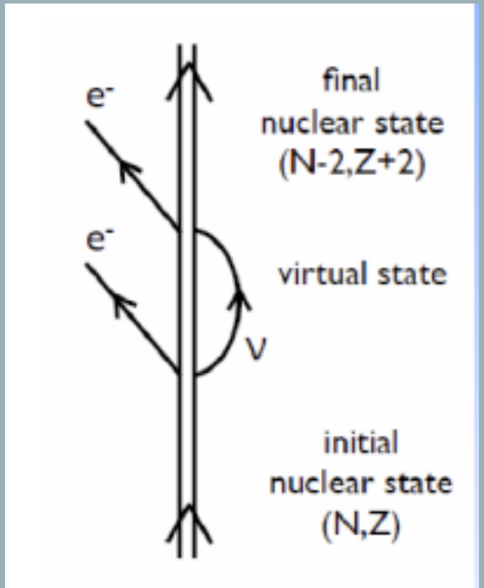
EFFECTIVE OPERATORS

Calculations of observables (electromagnetic transitions, β decay,…) require effective operators that in realistic shell model should be derived within an approach consistent with the derivation of the Hamiltonian

K. Suzuki and R. Okamoto, Prog. Theor. Phys. 93 , 905 (1995)

0νββ DECAY

The 0νββ decay is a hypothetical very rare transitions in which two neutrons undergo β decay simultaneously without the emission of neutrinos.



$$(A, Z) \rightarrow (A, , Z + 2) + e^{-} + e^{-}$$

Its detection is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.
Detection of this decay would

- correspond to a violation of the conservation of the leptonic number
- imply that neutrino is its own antiparticle, namely is a Majorana particle
- provide information on the neutrino masses

0νββ DECAY & NME

The information to be extracted from the 0νββ experiments is subject to the NME of the 0νββ transition operator between the ground states of the decaying nucleus (parent) and its decay product (grand-daughter).

The rate of the 0νββ decay is proportional to the squared NME

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2$$

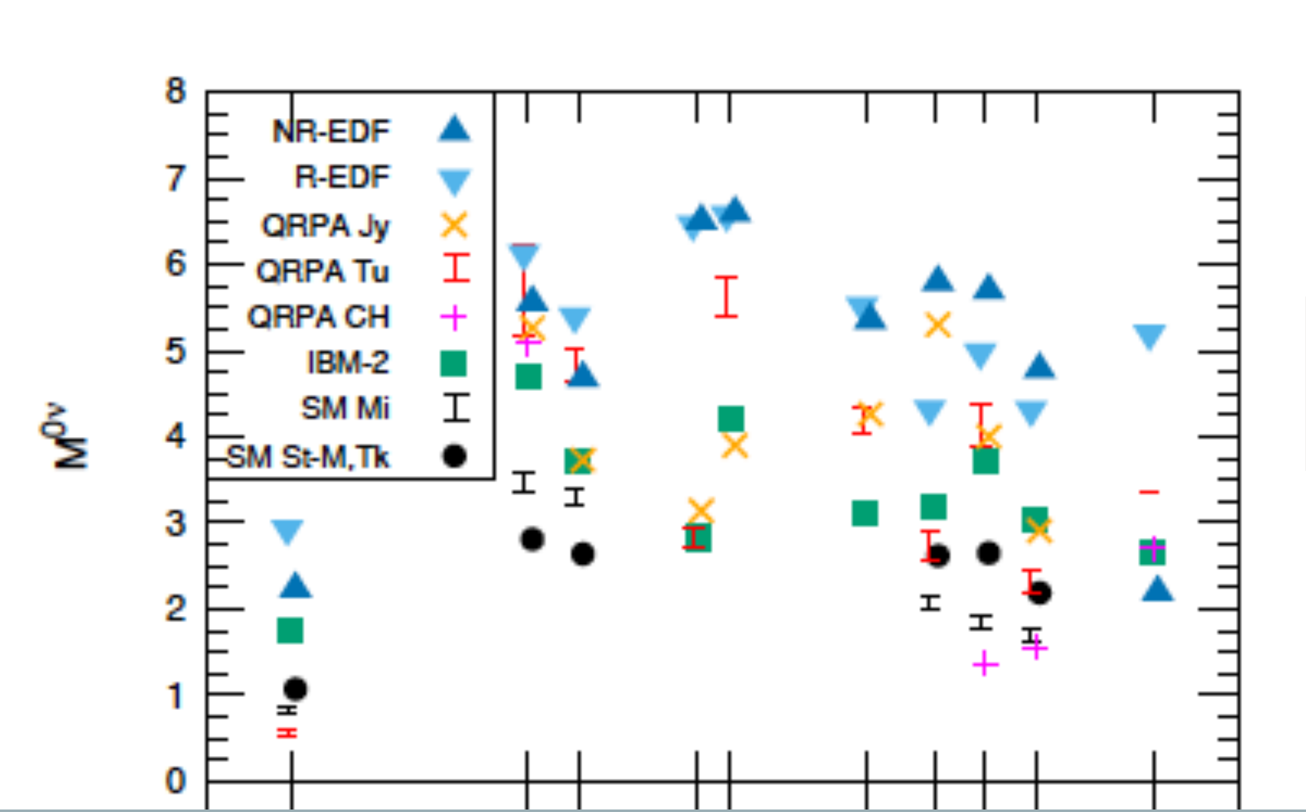
- $G^{0\nu}$ ≡ phase-space factor
- $\langle m_\nu \rangle = \sum_k m_k U_{ek}^2$ ⇒ effective neutrino mass
- $M^{0\nu}$ ≡ NME

$$M_\nu^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} - M_T^{0\nu}$$

0νββ transition operator

$$\mathcal{O}_{GT} = \tau_{1-}\tau_{2-}\ (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\ H_{GT}(r, E_\kappa),$$
$$\mathcal{O}_F = \tau_{1-}\tau_{2-}\ H_F(r, E_\kappa),$$
$$\mathcal{O}_T = \tau_{1-}\tau_{2-}\ S_{12}\ H_T(r, E_\kappa),$$

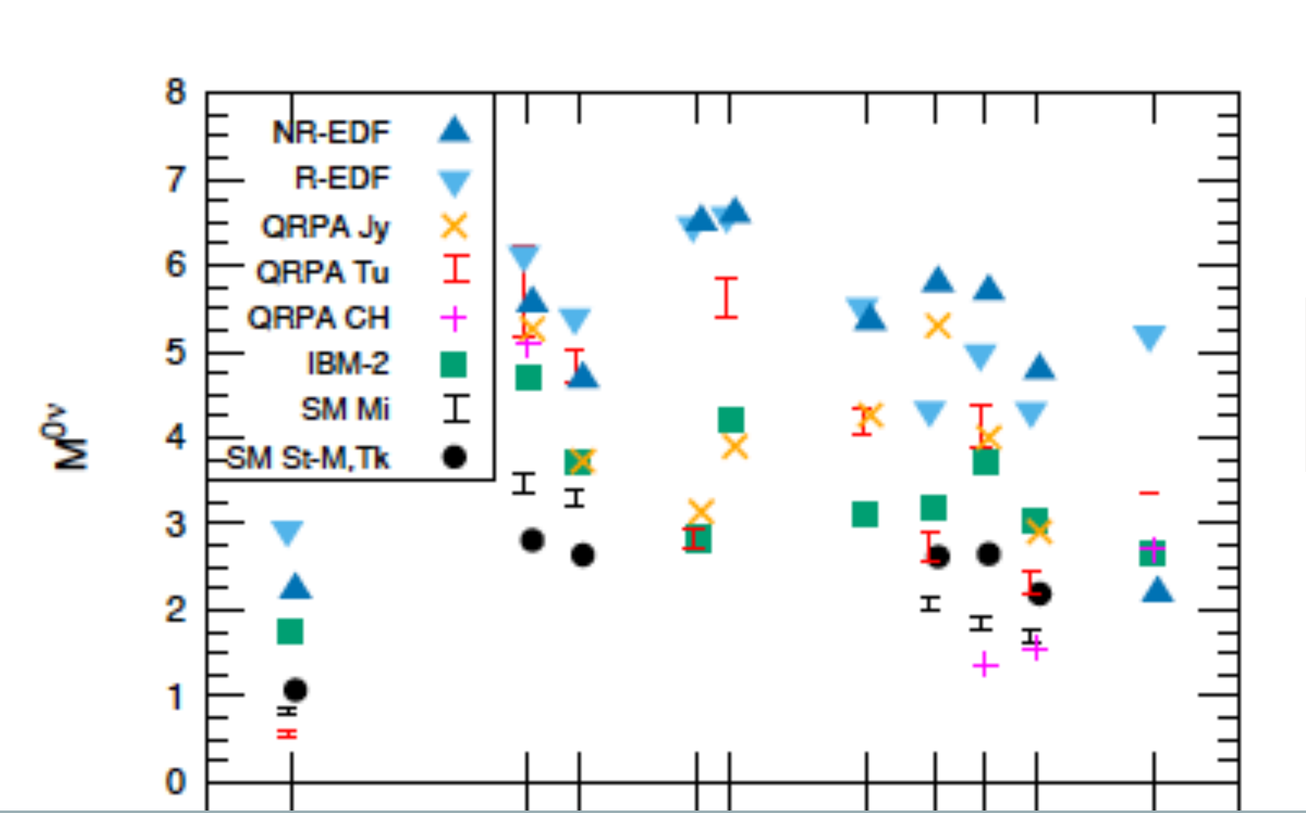
$0\nu\beta\beta$ NME PREDICTIONS



From J. Engel & J. Menéndez Rep. Prog. Phys. **80**, 046301 (2017)

Results produced by different models show a large spread

$0\nu\beta\beta$ NME PREDICTIONS



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Results produced by different model produce a large spread

Matrix elements obtained with different nuclear-structure approaches differ by factors from 2 to 3 corresponding to nearly an order of magnitude in the rate of $0\nu\beta\beta$ decay →

- no insight about of the sensitivity of the experimental devices needed to detect the $0\nu\beta\beta$ (which and how much material should be used)
- no useful information on the effective neutrino mass can be extracted

g_A QUENCING

In almost all calculations the strength of the free axial coupling g_A^{free} = 1.27 is reduced by introducing a quenching factor $q = \frac{g_A^{\text{eff}}}{g_A^{\text{free}}}$, that is fixed from the observed GT and 2νββ decays

In fact the predicted single β and 2νββ decay lifetimes are almost always shorter than measured lifetimes, i.e. corresponding matrix elements are too large

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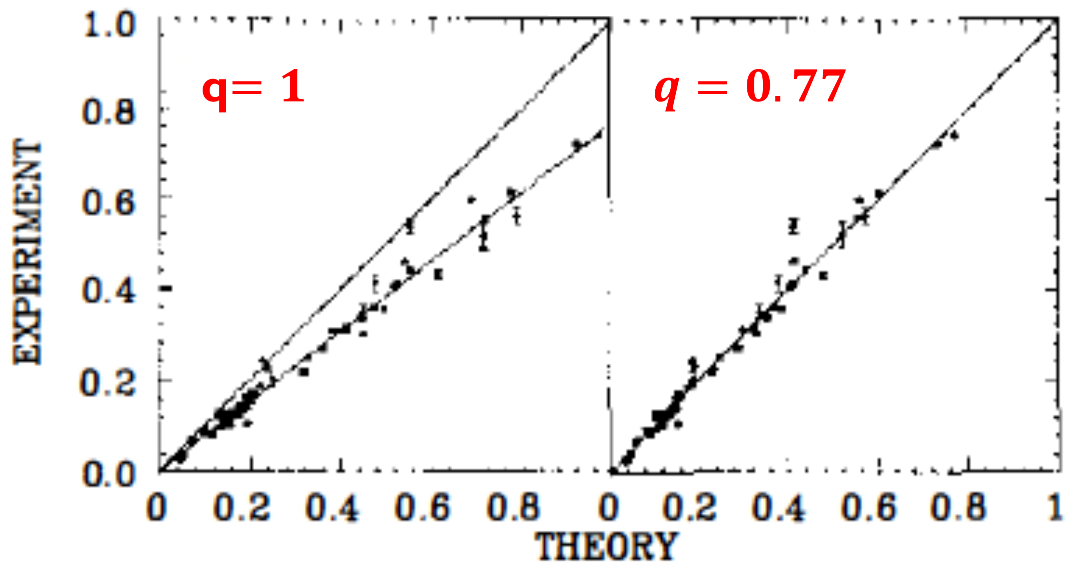
β^- $\quad {}^AZ \rightarrow {}^A(Z+1) + e^- + \bar{\nu}_e$

β^+ $\quad {}^AZ \rightarrow {}^A(Z-1) + e^+ + \nu_e$

GT strength

$$B(GT^{\pm}) = q^2 \frac{\left|\langle \Phi_f || \sum_j \vec{\sigma}_j \tau_j^{\pm} | \Phi_i \rangle\right|^2}{2J_i + 1}$$



Experimental versus theoretical strengths for GT transitions in nuclei with A=16-40

B.A, Brown, B. H.Wildenthal, Ann. Rev. Nucl. Part. Sci. 38, 29 (1988)

A.

g_A QUENCING

Possible sources of the quencing:

- Corrections due to the truncation of the Hilbert space
- Corrections due to the subnucleonic structure of nucleons

What's their role in determining the quencing is still unknown → it's difficult to establish what are the consequences on the 0νββ decay and if 0νββ NMEs need a similar quenching

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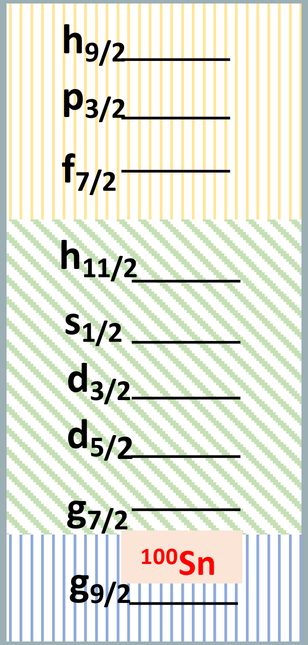
<u>To investigate the renormalization of GT operator:</u>
we consistently derive the effective SM Hamiltonian and the effective decay operators by many-body perturbation theory, starting from the free nuclear potential → we take into account the reduced SM model space without resorting to an empirical quenching.
We do not consider corrections arising from the subnucleonic structure of nucleons

RESULTS

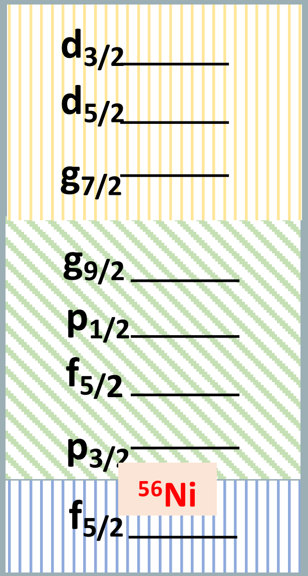
1 ^{130}Te & ^{136}Xe

^{130}Te CUORE – Italy
^{136}Xe EXO – USA
KamLAND – Japan



2 ^{76}Ge & ^{82}Se

^{76}Ge GERDA/Genius – Italy
IGEX – Spain; Russia
Majorana – USA
^{82}Se NEMO – France



CD-Bonn

↓

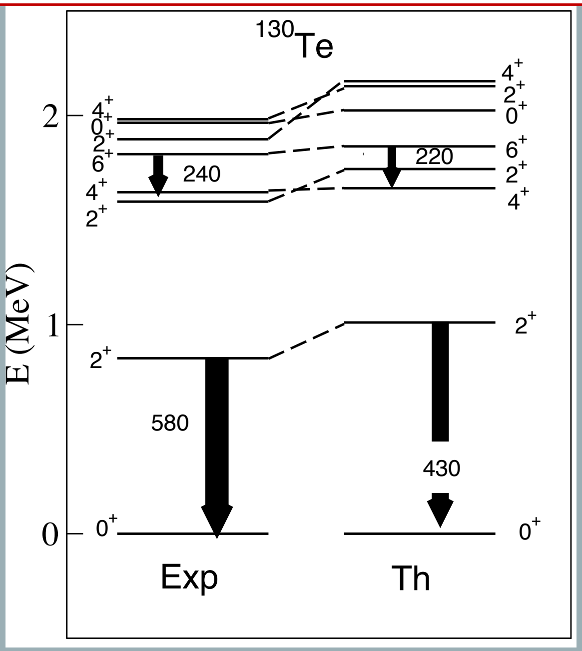
$V_{low\text{-}k}$ with Λ=2.6 fm^{-1} + Coulomb force for protons

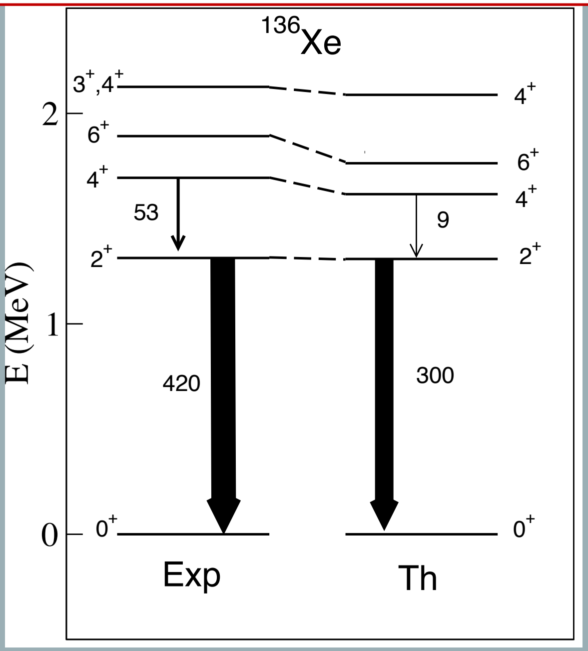
↓

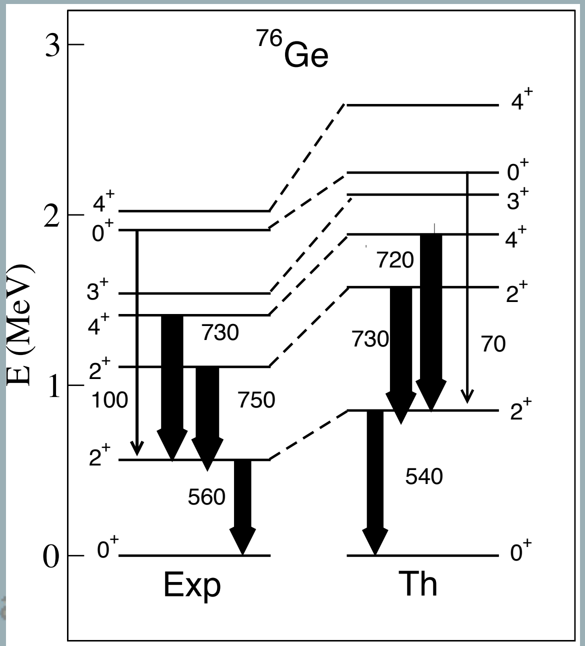
V_{eff} & Θ_{eff} @ third order

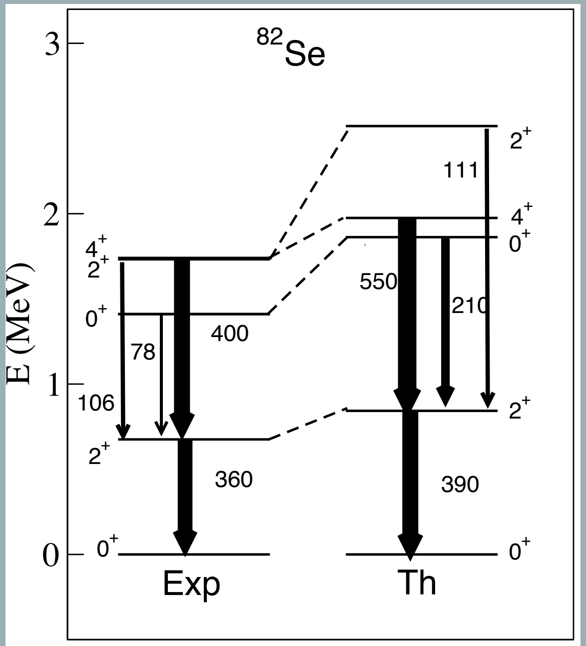
L.Coraggio et al, Phys. Rev. C 95, 064324 (2017)

SPECTROSCOPIC PROPERTIES: ^{130}Te, ^{136}Xe, ^{76}Ge, ^{82}Se



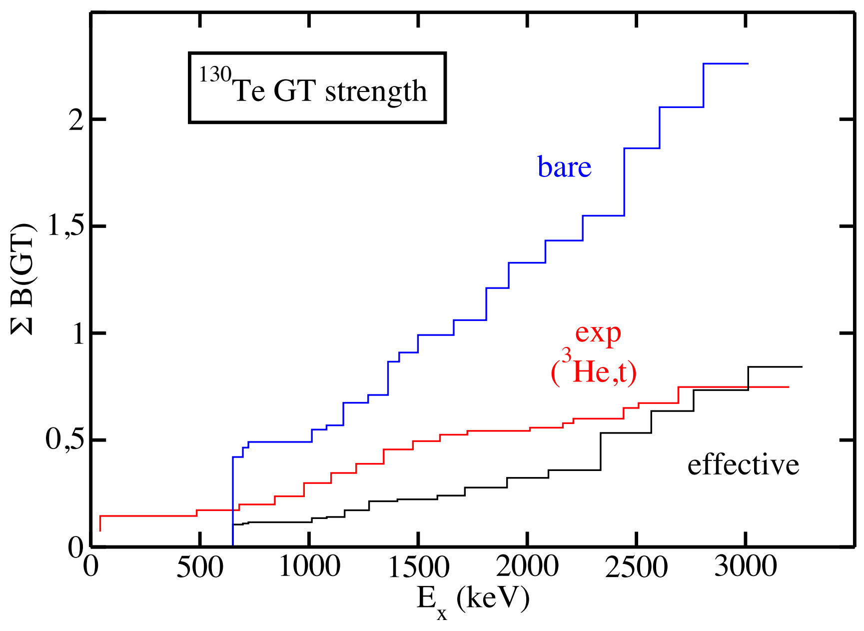


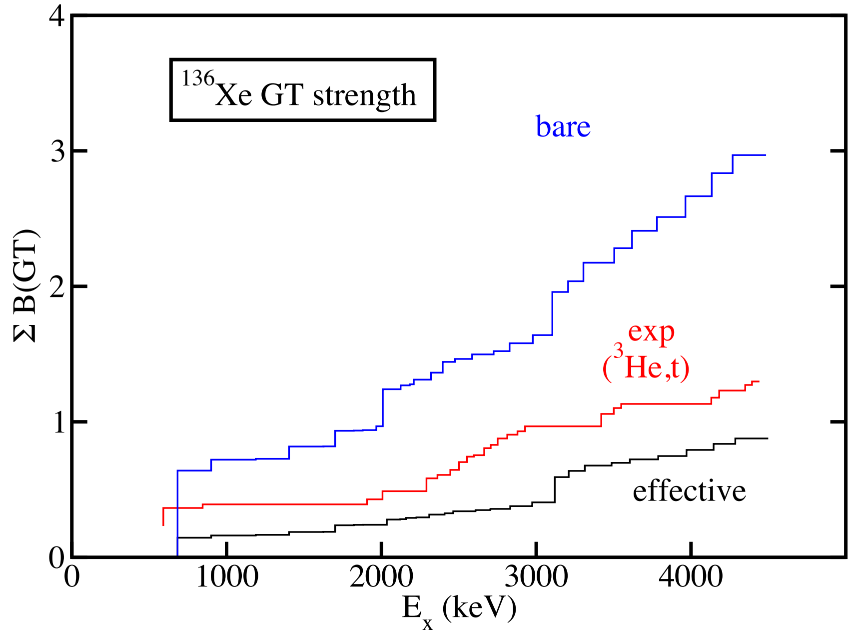


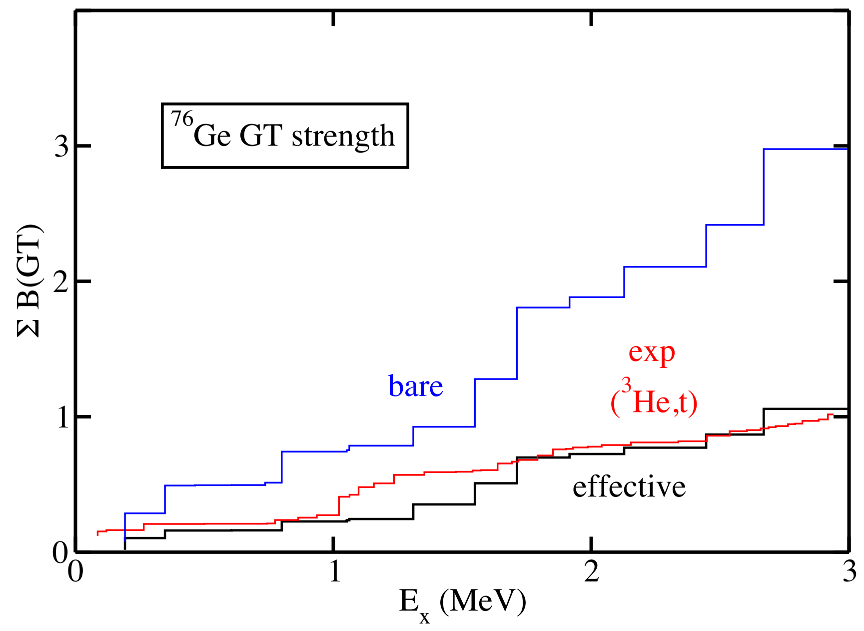


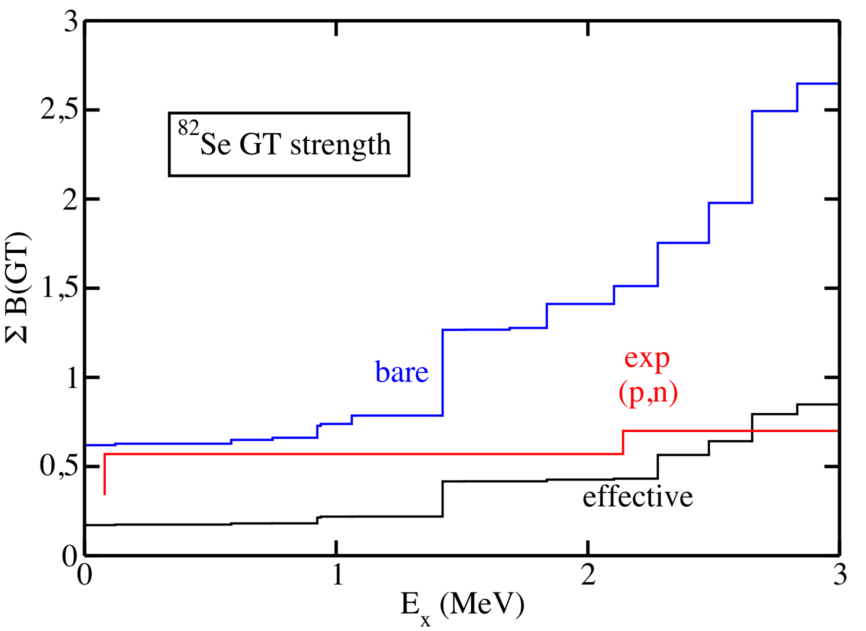
B(E2)s in e^2fm^4

GT^- RUNNING SUM: ^{130}Te & ^{136}Xe - ^{76}Ge & ^{82}Se









$$\textbf{Running Sum} = \sum_f \frac{\left|\langle \Phi_f || \sum_j \vec{\sigma}_j \tau_j^- \,|\Phi_i\rangle\right|^2}{2J_i + 1}$$

Expt from the cross section of one-charge exchange reaction

EFFECTIVE VERSUS BARE OPERATOR

^{100}Sn core

Comparison of the neutron-proton matrix elements of the effective and bare GT^{-} operators

$n_a l_a j_a$	$n_b l_b j_b$	Effective	Bare	quencing
$0g_{7/2}$	$0g_{7/2}$	-1.24	-2.48	0.50
$0g_{7/2}$	$1d_{5/2}$	-0.14	0	
$1d_{5/2}$	$0g_{7/2}$	0.02	0	
$1d_{5/2}$	$1d_{5/2}$	1.86	2.91	0.64
$1d_{5/2}$	$1d_{3/2}$	-1.75	-3.10	0.56
$1d_{3/2}$	$1d_{5/2}$	1.94	3.10	0.63
$1d_{3/2}$	$1d_{3/2}$	-1.02	-1.55	0.66
$1d_{3/2}$	$2s_{1/2}$	-0.12	0	
$2s_{1/2}$	$1d_{3/2}$	0.09	0	
$2s_{1/2}$	$2s_{1/2}$	1.60	2.46	0.65
$0h_{11/2}$	$0h_{11/2}$	2.60	3.76	0.69

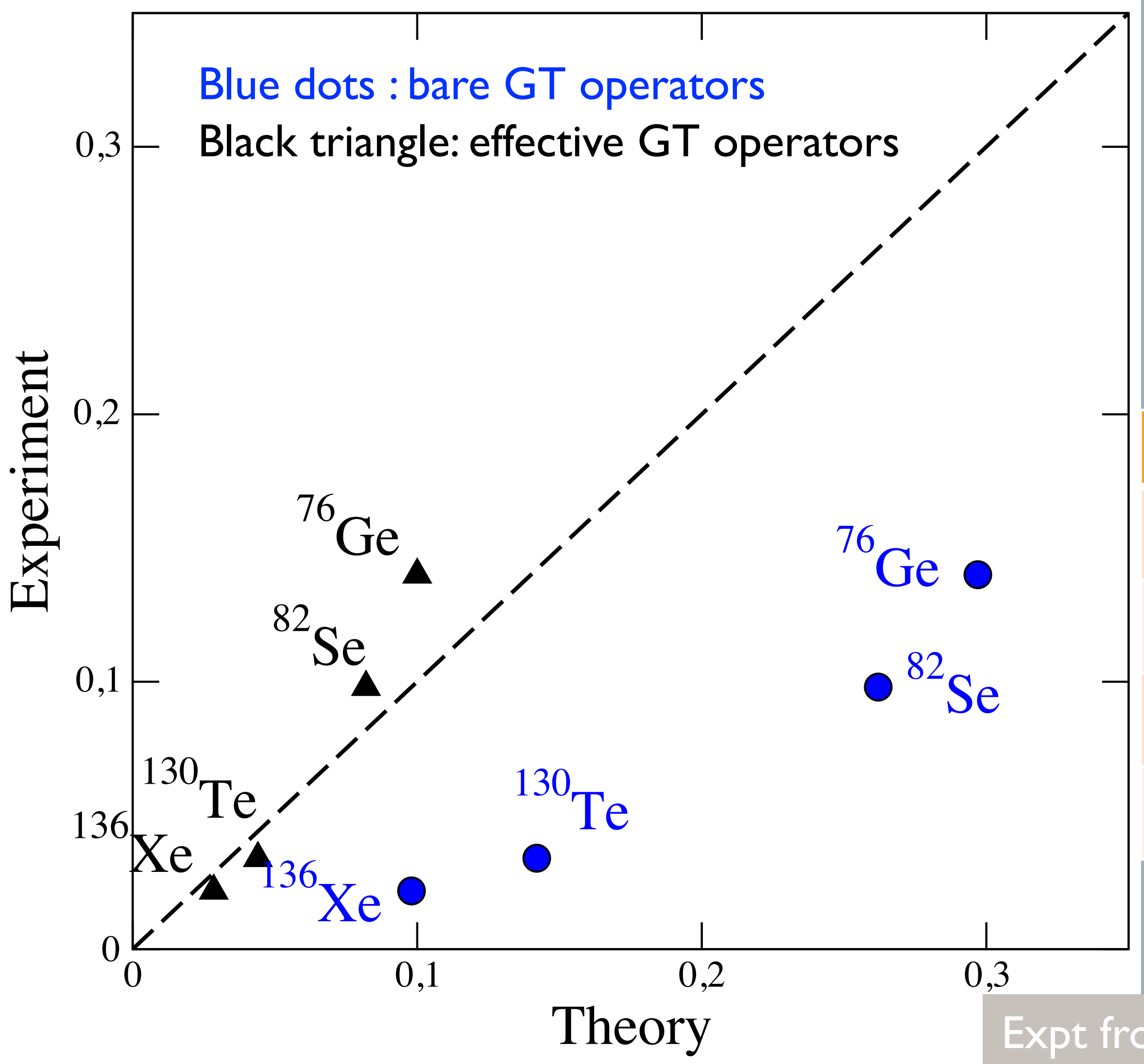
^{56}Ni core

Comparison of the neutron-proton matrix elements of the effective and bare $\mathbf{GT}^{-}$ operators

$n_a l_a j_a$	$n_b l_b j_b$	Effective	Bare	quencing
$0f_{5/2}$	$0f_{5/2}$	-0.69	-1.86	0.37
$0f_{5/2}$	$1p_{3/2}$	-0.10	0	
$1p_{3/2}$	$0f_{5/2}$	0.03	0	
$1p_{3/2}$	$1p_{3/2}$	1.44	2.32	0.62
$1p_{3/2}$	$1p_{1/2}$	-1.15	-2.09	0.55
$1p_{1/2}$	$1p_{3/2}$	1.21	2.09	0.58
$1p_{1/2}$	$1p_{1/2}$	-0.49	-0.73	0.67
$0g_{9/2}$	$0g_{9/2}$	2.21	3.16	0.70

2νββ MATRIX ELEMENTS

2νββ $\quad {}^{A}Z \rightarrow {}^{A}(Z+2) + 2e^{-} + 2\bar{\nu}_e,$



$$M_{2\nu}^{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma}\tau^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma}\tau^- || 0_i^+ \rangle}{E_n + E_0}$$

E_0 depends on the Q value and the mass difference between the parent and daughter nuclei
E_n excitation energy of the daughter nucleus

Decay	Expt	Bare	Effective
$^{76}Ge \rightarrow {}^{76}Se$	0.113±0.006	0.304	0.106
$^{82}Se \rightarrow {}^{82}Kr$	0.083±0.004	0.347	0.114
$^{130}Te \rightarrow {}^{130}Xe$	0.031±0.004	0.131	0.044
$^{136}Xe \rightarrow {}^{136}Ba$	0.0181±0.0007	0.091	0.028

Expt from A. S. Barabash, Nucl. Phys.A 935, 52 (2015)

0νββ MATRIX ELEMENTS

VERY PRELIMINARY

$$M_{\nu}^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu}$$

without tensorial component

Decay	$M_{GT}^{0\nu}$		$M_F^{0\nu}$		$M^{0\nu}$	
	Bare	Eff	Bare	Eff	Bare	Eff
$^{76}Ge \rightarrow {}^{76}Se$	2.76	1.76	-0.52	-0.45	3.09	2.01
$^{82}Se \rightarrow {}^{82}Kr$	2.72	1.70	-0.51	-0.43	3.04	1.97
$^{130}Te \rightarrow {}^{130}Xe$	2.51	1.00	-0.57	-0.34	2.87	1.22
$^{136}Xe \rightarrow {}^{136}Ba$	1.81	0.65	-0.41	-0.26	2.08	0.82

- Renormalization effects affect mainly $M_{GT}^{0\nu}$
- The suppression of $M^{0\nu}$ increases with A from 0.65 to 0.42

SUMMARY AND PERSPECTIVES

- Realistic shell-model calculations allow a fully microscopic description of nuclear structure
  - ✓ no parameters modified ad hoc
  - ✓ enhanced predictive power
- Effective shell-model Hamiltonians derived from realistic NN potentials are reliable and may be employed successfully to describe the properties of nuclei:
- satisfactory description of the experimental data, including level spectra, electric quadrupole transitions, GT strengths and 2νββ ME without quenching the g_A for β decay properties

→ good prospects to the parameter-free calculation of 0νββ NME

- Assess the calculation of the 0νββ NME
- Use H_{eff} derived from chiral two- and three-body potentials

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Thanks for your attention!

→ **good pros ... ree calculation of $0\nu\beta\beta$ NME**

- Assess the calculation of the $0\nu\beta\beta$ NME
- Use H_{eff} derived from chiral two- and three-body potentials

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