Multihadron Molecules

M. Bayar

Physics Department
Kocaeli University

Women in Nuclear and Hadron Theoretical Physics: the last frontier-WTPLF 2018
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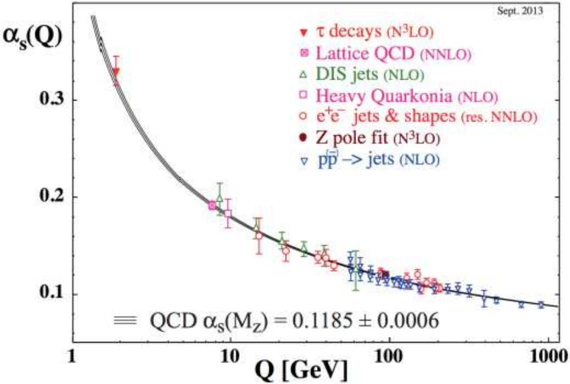
Outline

- Introduction
- Formalism
  The Faddeev equations under the Fixed Center Approximation
- The Light Unflavored and Stange Sectors
- The Charm Sector
- The Beauty Sector

Introduction

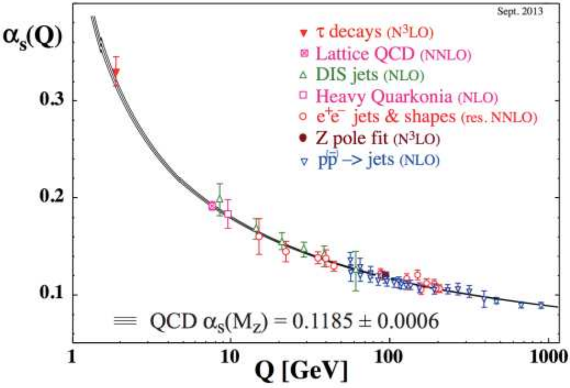
Quantum Cromodynamics (QCD) $\Rightarrow$ the theory behind the strong interactions.



At large distances ($\geq$ 0.1 fm):
- Confinement
- Quarks build coherent bound states - hadrons
- QCD perturbation theory is inapplicable

At small distances:
- Asymptotic freedom
- Quasi-free quark propagation
- QCD perturbation theory

Quantum Cromodynamics (QCD) ⇒ the theory behind the strong interactions.

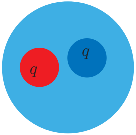


Non-perturbative methods:

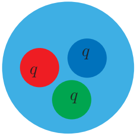
- Phenomenological quark models
- Lattice QCD
- Effective lagrangian methods
- QCD sum rules
- Chiral Perturbation theory
- .....

Hadron Spectroscopy

- Ordinary hadrons

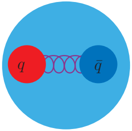


Meson($q\bar{q}$)

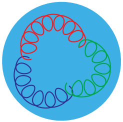


Baryon (qqq)

- Gluonic excitations

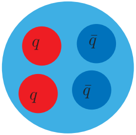


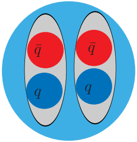
Hybrid meson:
$q\bar{q}$ with gluonic excitations,



Glueball:
only gluons, no valence quarks

- Multiquark states





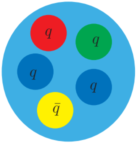
Tetraquark:

Two quarks and two antiquarks,

Hadronic molecule:

composed of two or more color-neutral hadrons

- Pentaquark

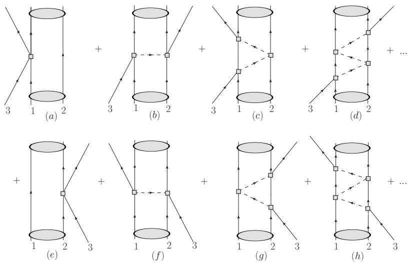


four quarks and one antiquark bound together

Formalism

The Faddeev equations under the Fixed Center Approximation (FCA)

The FCA to the Faddeev equations is an effective tool to deal with multi-hadron interaction



- T_1: all diagrams beginning with interaction in particle 1.
- T_2: all diagrams beginning with interaction in particle 2.

$$T_1 = t_1 + t_1 G_0 T_2, \;\; T_2 = t_2 + t_2 G_0 T_1, \;\; T = T_1 + T_2$$

$$T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\,\tilde{t}_1\,\tilde{t}_2\,G_0}{1 - \tilde{t}_1\,\tilde{t}_2\,G_0^2}.$$

The function G_0 :

$$G_0(s) = \int \frac{d^3\vec{q}}{(2\pi)^3} F_R(q) \frac{1}{q^{02} - \vec{q}^{\,2} - m_3^2 + i\,\epsilon}, \qquad q^0(s) = \frac{s + m_3^2 - M_R^2}{2\sqrt{s}}.$$

$F_R(q)$ is the cluster form factor

$$F_R(q) = \frac{1}{\mathcal{N}} \int_{|\vec{p}|<\Lambda', |\vec{p}-\vec{q}|<\Lambda'} d^3\vec{p} \frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})} \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})}$$
$$\frac{1}{2E_1(\vec{p}-\vec{q})} \frac{1}{2E_2(\vec{p}-\vec{q})} \frac{1}{M_R - E_1(\vec{p}-\vec{q}) - E_2(\vec{p}-\vec{q})}, \tag{1}$$

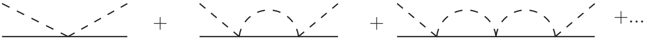
$$\mathcal{N} = \int_{|\vec{p}|<\Lambda'} d^3\vec{p} \left(\frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})} \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \right)^2,$$

(J.Yamagata-Sekihara, J. Nieves, E. Oset Phys. Rev. D 83,014003 (2011))

Two Body Scattering

The Bethe-Salpeter equation in coupled channels

$$t = V + VGt$$



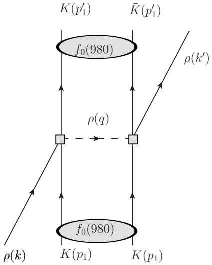
A loop function of pseudoscalar and vector mesons G_l:

$$G_l(\sqrt{s}) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

The Light Unflavered and Stange Sectors

Description of $\rho(1700)$ as a $\rho K\bar{K}$ system

(M. Bayar, W. H Liang, T. Uchino and C. W Xiao, Eur.Phys.J. A50 (2014))



- A cluster of two bound particle ($K\bar{K}$ ($I = 0$), $f_0(980)$)
- $K\bar{K}$+coupled channel $\longrightarrow$ $f_0(980)$ (J. A. Oller, E. Oset, Nuclear Physics A **620**,438 (1997))
  $\Rightarrow$ $\rho - (K\bar{K}) \longrightarrow$ one needs t for $\rho K(\rho\bar{K})$
- $\rho K(\rho\bar{K})$ unitarized scattering amplitude, (L. S. Geng, E. Oset, L. Roca and J. A. Oller, Phys. Rev. D **75**, 014017 (2007), L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005).)

$\rho K(\rho\bar{K})$ unitarized scattering amplitude

- The Bethe-Salpeter equation in coupled channels: $t = [1 + V\hat{G}]^{-1}(-V)\vec{\epsilon}\cdot\vec{\epsilon}\,'$
- The two meson loop function:

  $G_l(\sqrt{s}) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2-M_l^2+i\epsilon}\frac{1}{q^2-m_l^2+i\epsilon}$
- In the dimensional regularization scheme the loop function:

$$
\begin{aligned}
G_l(\sqrt{s}) \quad = \quad & \frac{1}{16\pi^2}\Big\{a(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} \\
& + \frac{q_l}{\sqrt{s}}\Big[\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\
& + \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\
& - \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\
& - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})\Big]\Big\}
\end{aligned}
$$

- q_l determined at the center of mass frame, $q_l = \frac{\sqrt{[s-(M_l-m_l)^2][s-(M_l+m_l)^2)]}}{2\sqrt{s}}$
- $\mu \Rightarrow$ a scale parameter in this scheme, $a(\mu)\Rightarrow$the subtraction constant

For the normalization

The S-matrix for the single scattering

$$S^{(1)} = -i(2\pi)^4\delta^4(k_\rho + k_{f_0} - k_\rho^{'} - k_{f_0}^{'}) \times \frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_\rho}}\frac{1}{\sqrt{2\omega_\rho^{'}}}\frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_K^{'}}}t,$$

S-matrix for the double scattering,

$$\begin{aligned} S^{(2)} &= -i(2\pi)^4\delta^4(k_\rho + k_{f_0} - k_\rho^{'} - k_{f_0}^{'}) \times \frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_\rho}}\frac{1}{\sqrt{2\omega_\rho^{'}}}\frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_K^{'}}} \\ &\times \frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_K^{'}}}\int\frac{d^3q}{(2\pi)^3}F_{f_0}(q)\frac{1}{q^{02} - \vec{q}^2 - m_\rho^2 + i\epsilon}tt, \end{aligned}$$

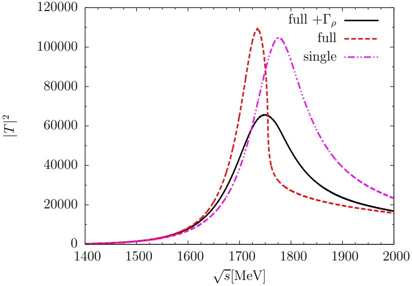
The full three-body scattering is given by

$$S = -i(2\pi)^4\delta^4(k_\rho + k_{f_0} - k_\rho^{'} - k_{f_0}^{'})\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_\rho}}\frac{1}{\sqrt{2\omega_\rho^{'}}}\frac{1}{\sqrt{2\omega_{f_0}}}\frac{1}{\sqrt{2\omega_{f_0}^{'}}}T.$$

Using the low energy reduction, $\sqrt{2\omega} \sim \sqrt{2m} \Rightarrow \tilde{t} = \frac{2m_{f_0}}{2m_K}t$.

- Finally $T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\ \tilde{t}_1\ \tilde{t}_2\ G_0}{1 - \tilde{t}_1\ \tilde{t}_2\ G_0^2}$.

Description of $\rho(1700)$ as a $\rho K\bar{K}$ system

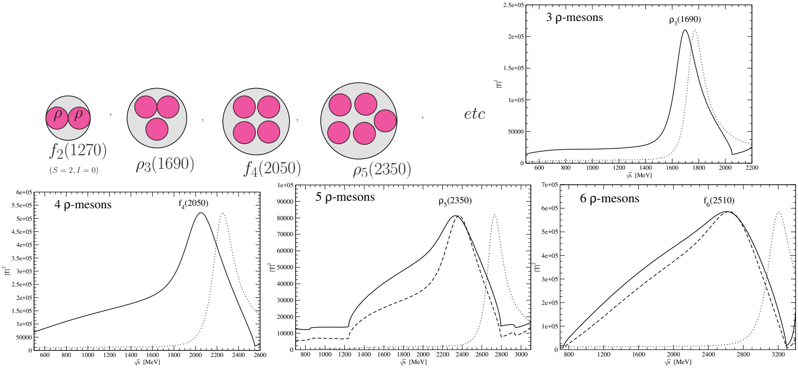


	single	full	full + Γ_ρ	PDG
Mass (MeV)	1777.9	1734.8	1748.0	1720 ± 20
Width (MeV)	144.4	63.7	160.8	250 ± 100

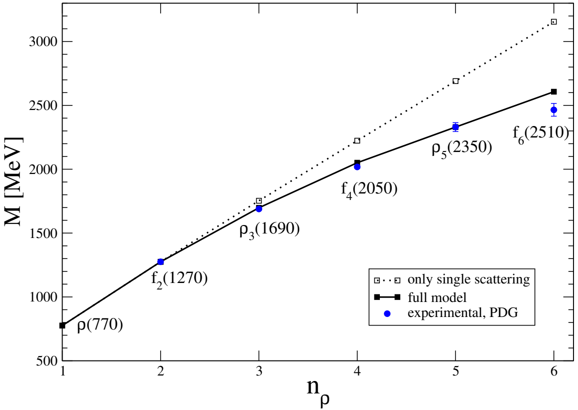
A description of the $f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$ resonances as multi-$\rho(770)$ states (L. Roca and E. Oset PRD 82 054013 (2010).)

ρ-ρ interaction in the hidden gauge approach (R.Molina, D. Nicmorus, E. Oset PRD78 (2008))

- scattering of f_2 with f_2 $\Rightarrow$ the f_4 • ρ interaction with f_4 $\Rightarrow \rho_5$ • f_2 with f_4 $\Rightarrow$ f_6



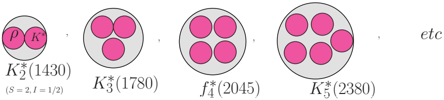
Modulus squared of the unitarized multi-ρ amplitudes. Solid line: full model $\Lambda'|_{f_4} = 1500$ MeV; dashed line: full model $\Lambda'|_{f_4} = 875$ MeV; dotted line: only single-scattering contribution.

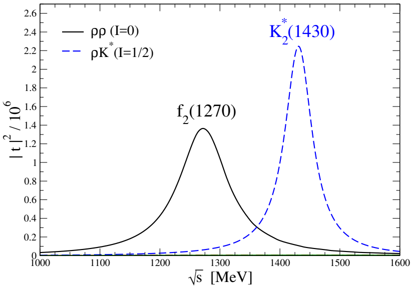


Masses of the dynamically generated states as a function of the number of constituent $\rho(770)$ mesons, n_ρ.

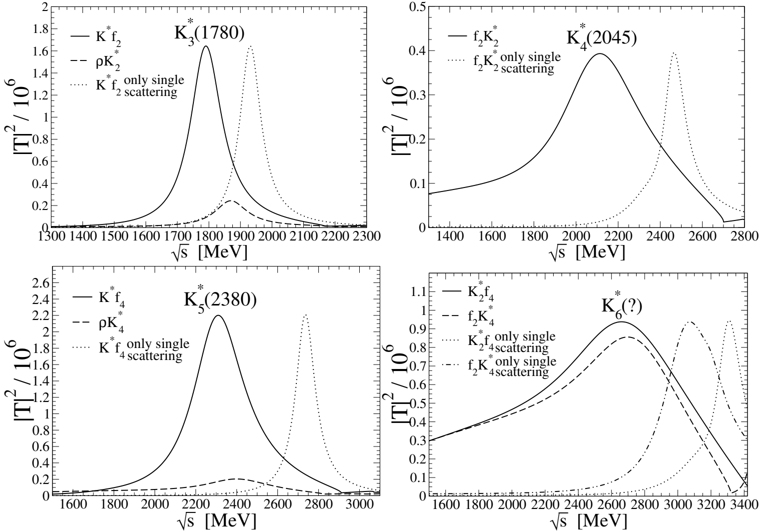
On the nature of the $K_2^*(1430)$, $K_3^*(1780)$, $K_4^*(2045)$, $K_5^*(2380)$ and K_6^* as K^*–multi-ρ states, (J. Yamagata, L.Roca and E. Oset PRD 82 094017 (2010).)

ρK^* interaction $\Rightarrow$ $K_2^*(1430)$ (L. S. Geng and E. Oset, Phys. Rev. D 79 (2009))





	A	B $(b_1 b_2)$
two-body	ρ	K^*
three-body	K^* ρ	f_2 $(\rho\rho)$ K_2^* (ρK^*)
four-body	f_2	K_2^* (ρK^*)
five-body	K^* ρ	f_4 $(f_2 f_2)$ K_4^* $(f_2 K_2^*)$
six-body	K_2^* f_2	f_4 $(f_2 f_2)$ K_4^* $(f_2 K_2^*)$



K*3(1780)
K*f2
ρK*2
K*f2 only single scattering
|T|^2 / 10^6
√s [MeV]
K*4(2045)
f2K*2
f2K*2 only single scattering
|T|^2 / 10^6
√s [MeV]
K*5(2380)
K*f4
ρK*4
K*f4 only single scattering
|T|^2 / 10^6
√s [MeV]
K*6(?)
K*2f4
f2K*4
K*2f4 only single scattering
f2K*4 only single scattering
|T|^2 / 10^6
√s [MeV]

Results for the masses of the dynamically generated states:

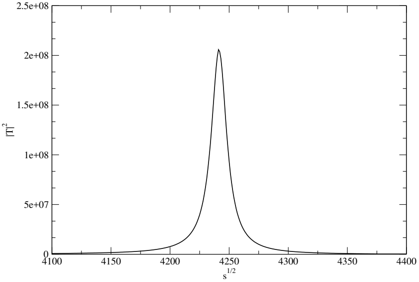
generated resonance	amplitude	mass (MeV), PDG	mass (MeV) only single scatt.	mass (MeV) full model
$K_2^*(1430)$	ρK^*	1429 ± 1.4	–	1430
$K_3^*(1780)$	$K^* f_2$	1776 ± 7	1930	1790
$K_4^*(2045)$	$f_2 K_2^*$	2045 ± 9	2466	2114
$K_5^*(2380)$	$K^* f_4$	$2382 \pm 14 \pm 19$	2736	2310
K_6^*	$K_2^* f_4$-$f_2 K_4^*$	–	3073-3310	2661-2698

The Charm Sector

The $\rho D\bar{D}$ system

M. Bayar, B. Durkaya Phys.Rev. D92 2015

- Clusters : $\rho D \Rightarrow D_1(2420)(I=1/2)$ $\bar{D}(\rho D)_{D_1(2420)}$
- $\rho D(D_1(2420)) \Rightarrow \pi D^*$, $D\rho$, KD_s^*, $D_s K^*$,ηD^*, $D\omega$, $\eta_c D^*$, DJ/ψ, $(I=1/2)$ and πD^*, $D\rho$, $(I=3/2)$ (D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, PRD76(2007), D. Gamermann and E. Oset,EPJA33(2007))
- $D\bar{D} \Rightarrow$ $D\bar{D}$, $K\bar{K}$, $\pi\bar{\pi}$, $\eta\eta$, $\eta_c\eta$, $D_s\bar{D}_s$, $(I=0)$, $D\bar{D}$, $K\bar{K}$, $\pi\bar{\pi}$, $\pi\eta$, $\eta_c\pi$, $(I=1)$ (J. A. Oller and E. Oset,Nucl. Phys. A620, 438 (1997);A652, 407 (1999))



$\bar{D}(\rho D)_{D_1(2420)}$, $I=0$, $m \sim 4241$ MeV, $\Gamma \sim 25$ MeV

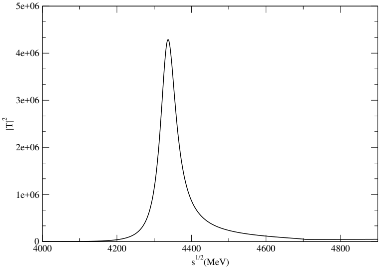
PDG: the $X(4260)$, $I^G(J^{PC}) = 0^-(1^{--})$ $m \simeq 4230$ MeV, $\Gamma \sim 55 \pm 19$ MeV

The $\rho D^* \bar{D}^*$ System with $J = 3$

M. Bayar, X. L. Ren, E. O, Eur. Phys. J. A 2015

$D^* \bar{D}^*$ cluster with $J^P = 2^+ \Rightarrow$ peak around 3920 MeV $\Rightarrow$ the $X(3915)$ or the $Z(3940)$ (with $I = 0$)

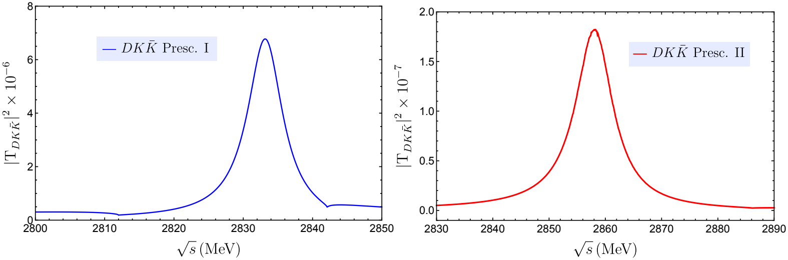
the ρ interact with $D^* \bar{D}^*$



A state with the $I = 1$, $J^P = 3^-$ and hidden charm is predicted around 4330 MeV.

The $DK\bar{K}$ system

V. R. Debastiani, J. M. Dias, E. Oset, Phys.Rev. D96 (2017)



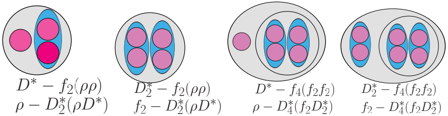
Narrow bound state around $Df_0(980)$ threshold (2855 MeV) in both prescriptions.
$M_{DK\bar{K}} = 2833 - 2858$ MeV,
QCD Sum Rules: $M_{Df_0} = (2926 \pm 237)$ MeV , Full Faddeev equations: $M_{Df_0} = 2890$ MeV , (A. Martinez Torres, K. P. Khemchandani, M. Nielsen and F. S. Navarra, Phys. Rev. D 87, no. 3, 034025 (2013))

A prediction of D^*-multi-ρ Molecular States

C. W. Xiao, M. Bayar, E. Oset, Phys.Rev. D86 (2012) 094019

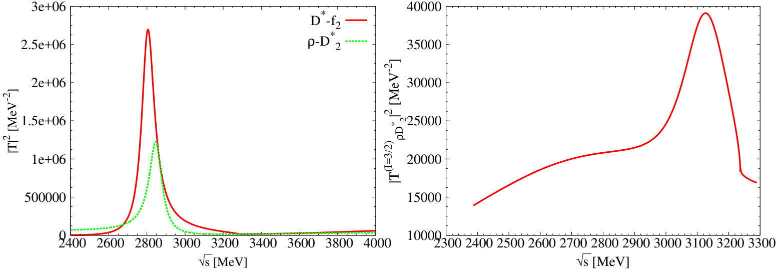
- $\rho\rho$ interaction in $I = 0$ and $S = 2$ is very strong
  $\Rightarrow f_2(1270)$ is a molecule of two $\rho(770)$
- ρD^* interaction in $I = 1/2$ and $S = 2$ is also very strong
  $\Rightarrow D_2^*(2460)$ is a molecule of $\rho(770)$ and D^*

  $\Rightarrow$ Hence, $\rho\rho$ and ρD^* are the clusters



particles:	3	R (1,2)	amplitudes
Two-body	ρ	D^*	$t_{\rho D^*}$
	ρ	ρ	$t_{\rho\rho}$
Three-body	D^*	f_2 $(\rho\rho)$	$T_{D^*-f_2}$
	ρ	D_2^* (ρD^*)	$T_{\rho-D_2^*}$
Four-body	D_2^*	f_2 $(\rho\rho)$	$T_{D_2^*-f_2}$
	f_2	D_2^* (ρD^*)	$T_{f_2-D_2^*}$
Five-body	D^*	f_4 $(f_2 f_2)$	$T_{D^*-f_4}$
	ρ	D_4^* $(f_2 D_2^*)$	$T_{\rho-D_4^*}$
Six-body	D_2^*	f_4 $(f_2 f_2)$	$T_{D_2^*-f_4}$
	[illegible]	[illegible]	[illegible]

Three-body interaction:

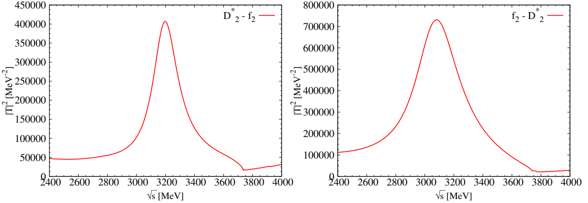


2800-2850 MeV ($I_{total} = \frac{1}{2}$) —— 3120 MeV ($I_{total} = \frac{3}{2}$)

$\sim$ 400 MeV below $D^* - f_2$ thr. —— $|T^{I=3/2}_{\rho-D_2^*}|^2$ «(30 times)$|T^{I=1/2}_{\rho-D_2^*}|^2$

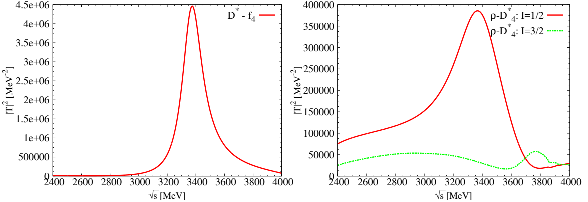
New D_3^* state ; mixture of $D^* - f_2$ and $\rho - D_2^*$ m $\sim$ 2800-2850 MeV $\Gamma \sim$ 60-100 MeV

Four and Five-body interactions:



3200 MeV, $\Gamma \sim$ 200 MeV —— 3075 MeV, $\Gamma \sim$ 400 MeV

New D_4^* resonance ; m $\sim$ 3075-3200 MeV, $\Gamma \sim$ 200-400 MeV



3375 MeV, $\Gamma \sim$ 200 MeV —— 3360 MeV, $\Gamma \sim$ 400 MeV

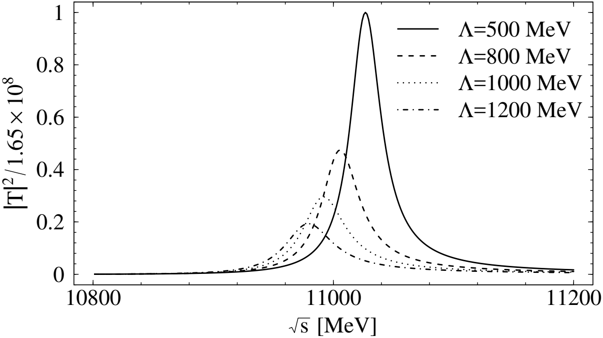
New D_5^* resonance ; m $\sim$ 3360-3375 MeV, $\Gamma \sim$ 200-400 MeV

The Beauty Sector

States of $\rho B^* \bar{B}^*$ with $J = 3$

M. Bayar, P. Fernandez-Soler, Zhi-Feng Sun, E. Oset, Eur.Phys.J. A52 (2016) no.4, 106

- A ρ meson and a $B^* \bar{B}^*$ cluster
- The $B^* \bar{B}^*$ cluster $\Rightarrow$ the $J = 2$, $I = 0$ (A. Ozpineci, C.W. Xiao, E. Oset, Phys.Rev. D88 (2013) 034018)

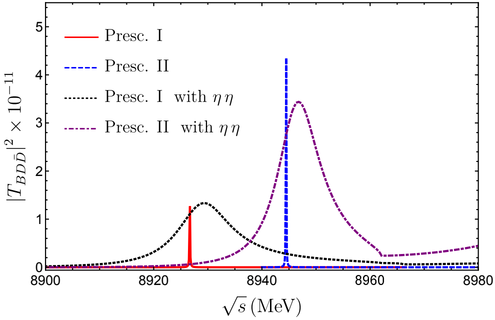


- Taking the ρB^* interaction in $J = 2$ (P. F. Soler, Z. F. Sun, J. Nieves and E. Oset, Eur.Phys.J. C76 (2016))
- We find a $I(J^{PC}) = 1(3^{--})$ state of mass 10987 ± 40 MeV and width 40 ± 15 MeV

On the binding of the $BD\bar{D}$ system

J. M. Dias, V. R. Debastiani, L. Roca, S. Sakai, E. Oset, Phys.Rev. D96 (2017) no.9, 094007

- BD cluster and $\bar{D}$ scattering from that cluster
- $I(J^P) = 1/2(0^-)$ bound state for the $BD\bar{D}$ system at an energy around $8925 - 8985$ MeV



$|T_{BD\bar{D}}|^2$ with prescriptions I, II for $\sqrt{s_{\bar{D}B}}$, $\sqrt{s_{\bar{D}D}}$ and $q_{\max} = 600$ MeV with and without considering width (from $\eta\eta$ channel) for the $X(3700)$ through the $D\bar{D}$ interaction. The two curves with $\eta\eta$ channel were multiplied by a factor 10^4 for comparison.

Summary

- The FCA to the Faddeev equations is an effective tool to deal with multi-hadron interaction
- $\rho(1700)$ appears as resonance of $\rho K\bar{K}$
- Multirho states could be identified with meson states of increasing spin
- K^* -multirho states can also be identified with K^* states of increasing spin
- In the charm sector the method is repeated and new charmed resonances, D_3^*, D_4^*, D_5^* and D_6^* are predicted
- The method is expanded to the beauty sector ....

Thank you for your attention!