



Phenomenology of an excited pseudoscalar glueball and charmed mesons in a chiral symmetric model

Walaa I. Eshraim

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Introduction

Quantum Chromodynamics (QCD)
Symmetries of the QCD Lagrangian. if all quark massless then we have chiral symmetry

 $U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$

- Spontaneous breaking of chiral symmetry by quark condensates.
- Explicit breaking of global chiral symmetry by quark masses and chiral anomaly.
- Effective chiral models of (QCD).

Decay modes of the excited pseudoscalar glueball

Interaction Lagrangain for the excited pseudoscalar glueball

with a pseudoscalar glueball and the ordinary scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}\tilde{G}'}^{int} = c_{\tilde{G}\tilde{G}'}\tilde{G}\tilde{G}'\,Tr\left(\Phi^{\dagger}\Phi\right)$$

with a scalar glueball and the pseudo(scalar) mesons

$$\mathcal{L}_{\tilde{G}G}^{int} = ic_{\tilde{G}G\Phi}\tilde{G}G\left(\det\Phi - \det\Phi^{\dagger}\right)$$

with scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}\Phi}^{int} = ic_{\tilde{G}\Phi}\tilde{G}\left(\det\Phi - \det\Phi^{\dagger}\right)$$

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

where $C_{\tilde{G}\Phi}$ is a dimensionless coupling constant and Φ reads for three flavours, $N_f = 3$:

$$\Phi = (S^{a} + iP^{a}) t^{a} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_{N} + a_{0}^{0}) + i(\eta_{N} + \pi^{0})}{\sqrt{2}} & a_{0}^{+} + i\pi^{+} & K_{S}^{+} + iK^{+} \\ a_{0}^{-} + i\pi^{-} & \frac{(\sigma_{N} - a_{0}^{0}) + i(\eta_{N} - \pi^{0})}{\sqrt{2}} & K_{S}^{0} + iK^{0} \\ K_{S}^{-} + iK^{-} & \bar{K}_{S}^{0} + i\bar{K}^{0} & \sigma_{S} + i\eta_{S} \end{pmatrix}$$

Glueballs

Lattice QCD calculation



[C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688, 220 (2004)

[arXiv:nucl-th/0309068]];

The pseudoscalar glueball $\breve{G} \equiv |gg\rangle$ at the border within light and heavy

$$M_{\tilde{G}} = 2.6 \text{ GeV}, J^{PC} = 0^{-+}, I = 0.$$

The first excited pseudoscalar Glueball

$$M_{\tilde{G}} = 3.7 \text{ GeV}, J^{PC} = 0^{*-+}.$$

Branching ratios for the decay of the excited pseudoscalar glueball into the pseudoscalar glueball

Quantity	The theoretical result
$\Gamma_{\tilde{G} \to \tilde{G}' K K} / \Gamma_{\tilde{G}}^{tot}$	0.0277
$\Gamma_{\tilde{G} \to \tilde{G}' \pi \pi} / \Gamma_{\tilde{G}}^{tot}$	0.9697
$\Gamma_{\tilde{G} \to \tilde{G}' \eta \eta'} / \Gamma_{\tilde{G}}^{tot}$	0.0026
$\Gamma_{\tilde{G} \to \tilde{G}' \eta \eta} / \Gamma_{\tilde{G}}^{tot}$	0.000012

The branching ratio for the decay of the excited pseudoscalar glueball into charmonium state $\Gamma_{\tilde{C}} = 0.001$

$$G \gamma \eta e \pi \pi \gamma G_3$$

Could be measured by **BESIII** and **PANDA**!

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball into *PS* and scalar-isoscalar states as well as η and η'

Case (i): $\mathcal{L}_{\tilde{G}G}^{int}$	The theoretical result	Case (ii): $\mathcal{L}_{\tilde{G}\Phi}^{int}$	The theoretical result
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma_{\tilde{G}_2}^{tot}$	0.0325	$\Gamma_{\tilde{G} \to a_0 \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.0313
$\Gamma_{\tilde{G}\to KK_S}/\Gamma_{\tilde{G}_2}^{tot}$	0.032	$\Gamma_{\tilde{G}\to KK_S}/\Gamma_{\tilde{G}_3}^{tot}$	0.001
$\Gamma_{\tilde{G}\to\eta f_0(1370)}/\Gamma_{\tilde{G}_2}^{tot}$	0.00004	$\left \Gamma_{\tilde{G} \to \eta f_0(1370)}/\Gamma_{\tilde{G}_3}^{tot}\right $	0.0014
$\left[\Gamma_{\tilde{G}\to\eta'f_0(1370)}/\Gamma_{\tilde{G}_2}^{tot}\right]$	0.048	$\left \Gamma_{\tilde{G}\to\eta'f_0(1370)}/\Gamma_{\tilde{G}_3}^{tot}\right $	0.031
$\Gamma_{\tilde{G} \to \eta f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0068	$ \Gamma_{\tilde{G} \to \eta f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot} $	0.0067
$\left[\Gamma_{\tilde{G}\to\eta'f_0(1500)}/\Gamma_{\tilde{G}_2}^{tot}\right]$	0.0219	$\left \Gamma_{\tilde{G}\to\eta'f_0(1500)}/\Gamma_{\tilde{G}_3}^{tot}\right $	0.0214
$\Gamma_{\tilde{G} \to \eta f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0008	$\Gamma_{\tilde{G} \to \eta f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0007
$\Gamma_{\tilde{G} \to \eta' f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	0.001	$\Gamma_{\tilde{G}\to\eta'f_0(1710)}/\Gamma_{\tilde{G}_3}^{tot}$	0.001

Could be measured by BESIII and PANDA!

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball

into scalar-isoscalar states and (pseudo)scalar mesons

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Could be measured by BESIII and PANDA!

	Case (i): $\mathcal{L}_{\tilde{G}G}^{int}$	The theoretical result	Case (ii): $\mathcal{L}_{\tilde{G}\Phi}^{int}$	The theoretical result
	$\Gamma_{ ilde{G} o \eta \pi \pi} / \Gamma_{ ilde{G}_2}^{tot}$	0.095	$\Gamma_{\tilde{G} o \eta \pi \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.1376
5	$\Gamma_{\tilde{G} o \eta' \pi \pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.111	$\Gamma_{\tilde{G} o \eta' \pi \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.1069
	$\Gamma_{\tilde{G} \to a_0 K K_S} / \Gamma_{\tilde{G}_2}^{tot}$	0.0026	$\Gamma_{\tilde{G} \to a_0 K K_S} / \Gamma_{\tilde{G}_3}^{tot}$	0.0025
	$\Gamma_{\tilde{G} o \eta a_0 a_0} / \Gamma_{\tilde{G}_2}^{tot}$	0.0001	$\Gamma_{\tilde{G} o \eta a_0 a_0} / \Gamma_{\tilde{G}_3}^{tot}$	0.0001
	$\Gamma_{\tilde{G} \to a_0 \pi f_0(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0003	$\Gamma_{\tilde{G} \to a_0 \pi f_0(1370)} / \tilde{\Gamma}_{\tilde{G}_3}^{tot}$	0.0003
	$\Gamma_{\tilde{G} \to a_0 \pi f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0034	$\Gamma_{\tilde{G} \to a_0 \pi f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0032
	$\Gamma_{\tilde{G} \to a_0 \pi f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0001	$\Gamma_{\tilde{G} \to a_0 \pi f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0001
	$\Gamma_{\tilde{G} o \eta f_0^2(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0003	$\Gamma_{\tilde{G} o \eta f_0^2(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.001
	$\Gamma_{\tilde{G} \to \eta' f_0^2(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.03×10^{-6}	$\Gamma_{\tilde{G} \to \eta' f_0^2(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.006×10^{-6}
	$\Gamma_{\tilde{G} o \eta f_0^2(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00004	$\Gamma_{\tilde{G} o \eta f_0^2(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.00001
	$\Gamma_{\tilde{G} \to \eta f_0(1370) f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00003	$\Gamma_{\tilde{G} \to \eta f_0(1370) f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0001
	$\Gamma_{\tilde{G} \to \eta f_0(1370) f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	3.798×10^{-6}	$\Gamma_{\tilde{G} \to \eta f_0(1370) f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	7.25×10^{-6}
	$\Gamma_{\tilde{G} \to KK_S f_0(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0025	$\Gamma_{\tilde{G} \to KK_S f_0(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0025
	$\Gamma_{\tilde{G} \to KK_S f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00013	$\Gamma_{\tilde{G} \to KK_S f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.00013
	$\Gamma_{\tilde{G} \to KK_S f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	6.2×10^{-6}	$\Gamma_{\tilde{G} \to KK_S f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	4.75×10^{-6}
	$\Gamma_{\tilde{G}\to KK\eta}/\Gamma_{\tilde{G}_2}^{tot}$	0.0668	$\Gamma_{\tilde{G}\to KK\eta}/\Gamma_{\tilde{G}_3}^{tot}$	0.0643
	$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma_{\tilde{G}_2}^{tot}$	0.045	$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma_{\tilde{G}_3}^{tot}$	0.044
	$\Gamma_{\tilde{G} \to K_S K_S \eta} / \Gamma_{\tilde{G}_2}^{tot}$	0.0002	$\Gamma_{\tilde{G} \to K_S K_S \eta} / \Gamma_{\tilde{G}_3}^{tot}$	0.0002
	$\Gamma_{\tilde{G} \to \eta^3} / \Gamma_{\tilde{G}_2}^{tot}$	0.024	$\Gamma_{\tilde{G} o \eta^3} / \Gamma_{\tilde{G}_3}^{tot}$	0.0233
	$\Gamma_{\tilde{G} \to \eta'^3} / \Gamma_{\tilde{G}_2}^{tot}$	0.0048	$\Gamma_{\tilde{G} \to \eta'^3} / \Gamma_{\tilde{G}_3}^{tot}$	0.0046
	$\frac{\Gamma_{\tilde{G} \to \eta' \eta^2} / \Gamma_{\tilde{G}_2}^{tot}}{\Gamma_{\tilde{G}_2}^{tot}}$	0.005	$\frac{\Gamma_{\tilde{G} \to \eta' \eta^2} / \Gamma_{\tilde{G}_3}^{tot}}{\Gamma_{\tilde{G}_3}^{tot}}$	0.0048
	$\Gamma_{\tilde{G} \to \eta'^2 \eta} / \Gamma_{\tilde{G}_2}^{tot}$	0.0035	$\Gamma_{\tilde{G} \to \eta'^2 \eta} / \Gamma_{\tilde{G}_3}^{tot}$	0.0034
	$\Gamma_{\tilde{G}\to KK\pi}/\Gamma_{\tilde{G}_2}^{tot}$	0.489	$\Gamma_{\tilde{G}\to KK\pi}/\Gamma_{\tilde{G}_3}^{tot}$	0.471
	$\Gamma_{\tilde{G} \to K_S K_S \pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.002	$\Gamma_{\tilde{G} \to K_S K_S \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.0057

Charmed mesons in the extended Linear Sigma Model

Charmed mesons in the model

The number of fields in the model

 $4N_f^2 + 2$ fields

• For $N_f = 4$ there are 66 fields: 64 quark-antiquark fields + one pseudoscalar glueball \tilde{G} +one scalar glueball G



W. I. Eshraim, F. Giacosa, and D. H. Rischke, Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Including charm degree of freedom



3) Vector fields:

4) Axial vector fields:

$$V^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} (\omega_{N} + \rho^{0}) & \rho^{+} & K^{*} (892)^{+} & D^{*0} \\ \rho^{-} & \frac{1}{\sqrt{2}} (\omega_{N} - \rho^{0}) & K^{*} (892)^{0} & D^{*-} \\ K^{*} (892)^{-} & \bar{K}^{*} (892)^{0} & \omega_{S} & D^{*-}_{S} \\ \bar{D}^{*0} & D^{*+} & D^{*+}_{S} & J/\psi \end{pmatrix}^{\mu} A^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} (f_{1,N} + a_{1}^{0}) & a_{1}^{+} & K_{1}^{+} & D_{1}^{0} \\ a_{1}^{-} & \frac{1}{\sqrt{2}} (f_{1,N} - a_{1}^{0}) & K_{1}^{0} & D_{1}^{-} \\ K_{1}^{-} & \bar{K}_{1}^{0} & f_{1,S} & D_{S1}^{-} \\ \bar{D}_{1}^{0} & D_{1}^{+} & D_{S1}^{+} & \chi_{c,1} \end{pmatrix}^{\mu} A^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} (f_{1,N} + a_{1}^{0}) & a_{1}^{+} & K_{1}^{+} & D_{1}^{0} \\ a_{1}^{-} & \frac{1}{\sqrt{2}} (f_{1,N} - a_{1}^{0}) & K_{1}^{0} & D_{1}^{-} \\ K_{1}^{-} & \bar{K}_{1}^{0} & f_{1,S} & D_{S1}^{-} \\ \bar{D}_{1}^{0} & D_{1}^{+} & D_{S1}^{+} & \chi_{c,1} \end{pmatrix}^{\mu} A^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} (f_{1,N} + a_{1}^{0}) & a_{1}^{+} & K_{1}^{+} & D_{1}^{0} \\ a_{1}^{-} & \frac{1}{\sqrt{2}} (f_{1,N} - a_{1}^{0}) & K_{1}^{0} & D_{1}^{-} \\ K_{1}^{-} & \bar{K}_{1}^{0} & f_{1,S} & D_{S1}^{-} \\ \bar{D}_{1}^{0} & D_{1}^{+} & D_{S1}^{+} & \chi_{c,1} \end{pmatrix}^{\mu} A^{\mu} A^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} (f_{1,N} + a_{1}^{0}) & a_{1}^{+} & K_{1}^{+} & D_{1}^{0} \\ K_{1}^{-} & \bar{K}_{1}^{0} & D_{1}^{-} \\ K_{1}^{-} & \bar{K}_{1}^{0} & f_{1,S} & D_{S1}^{-} \\ \bar{K}_{1}^{0} & D_{1}^{+} & D_{S1}^{+} & \chi_{c,1} \end{pmatrix}^{\mu} A^{\mu} A^{\mu}$$

The left-handed matrix: $L^{\mu} = V^{\mu} + A^{\mu}$ and the right-handed matrix: $R^{\mu} = V^{\mu} - A^{\mu}$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]];
W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Linear Sigma Model Lagrangian with (axial-)vector mesons

$$\begin{split} \mathcal{L} &= \mathcal{L}_{dil} + \text{Tr}[(D^{\mu}\Phi)^{\dagger}(D^{\mu}\Phi)] - m_{0}^{2} \left(\frac{G}{G_{0}}\right)^{2} \text{Tr}(\Phi^{\dagger}\Phi) - \lambda_{1}[\text{Tr}(\Phi^{\dagger}\Phi)]^{2} - \lambda_{2} \text{Tr}(\Phi^{\dagger}\Phi)^{2} \\ &+ \text{Tr} \left\{ \left[\left(\frac{G}{G_{0}}\right)^{2} \frac{m_{1}^{2}}{2} + \Delta \right] \left[(L^{\mu})^{2} + (R^{\mu})^{2} \right] \right\} + \text{Tr}[H(\Phi + \Phi^{\dagger})] - 2 \text{Tr}[\varepsilon \Phi^{\dagger}\Phi] \\ &- \frac{1}{4} \text{Tr}[(L^{\mu\nu})^{2} + (R^{\mu\nu})^{2}] + c(\det \Phi - \det \Phi^{\dagger})^{2} + i\tilde{c}\tilde{G}\left(\det \Phi - \det \Phi^{\dagger}\right) + i\frac{g_{2}}{2} \{\text{Tr}(L_{\mu\nu}[L^{\mu}, L^{\nu}]) \\ &+ \frac{h_{1}}{2} \text{Tr}(\Phi^{\dagger}\Phi) \text{Tr}[(L^{\mu})^{2} + (R^{\mu})^{2}] + h_{2} \text{Tr}[(\Phi R^{\mu})^{2} + (L^{\mu}\Phi)^{2}] + 2h_{3} \text{Tr}(\Phi R_{\mu}\Phi^{\dagger}L^{\mu}) + \text{Tr}(R_{\mu\nu}[R^{\mu}, R^{\nu}])\} + \dots , \end{split}$$
where \mathcal{L}_{dil} is the dilaton Lagrangian,

$$\mathcal{L}_{dil} = \frac{1}{2} (\partial_{\mu} G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \frac{G^2}{\Lambda^2} - \frac{G^4}{4} \right)$$

D.Parganlija, P.Kovacs, G.Wolf, F.Giacosa and D.H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]]; W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]].

Spontaneous Symmetry Breaking (SSB)

Shifting the fields

$$G \to G + G_0, \quad \sigma_N \to \sigma_N + \phi_N, \quad \sigma_S \to \sigma_S + \phi_S$$

where,
$$\phi_N = Z_\pi f_\pi, \qquad \phi_S = \frac{2Z_k f_k - \phi_N}{\sqrt{2}}$$

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph].

For $N_f = 4$ new shift $\chi_{C0} \rightarrow \chi_{C0} + \phi_C$ where

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]]; W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

$$\phi_C = \frac{2Z_D f_D - \phi_N}{\sqrt{2}} = \sqrt{2}Z_{D_s} f_{D_s} - \phi_S = \frac{Z_{\eta_C} f_{\eta_C}}{\sqrt{2}}$$

There are 29 eqs. of square masses of mesons with 15 unknown parameters.

Parameters

The values of the $N_f = 3$ parameters :

Parameter	Value	Parameter	Value	
m_1^2	$0.413 \times 10^6 \text{ MeV}^2$	m_0^2	$\left -0.918 \times 10^6 \text{ MeV}^2\right $	[D.
$\phi_C^2 c/2$	$450 \cdot 10^{-6} \text{ MeV}^{-2}$	δ_S	$0.151 \times 10^6 \mathrm{MeV^2}$	Wo Ris
g_1	5.84	h_1	0	(20 [ar]
h_2	9.88	h_3	3.87	
ϕ_N	$164.6 { m ~MeV}$	ϕ_S	$126.2 { m ~MeV}$	
λ_1	0	λ_2	68.3	

[D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]].

$$\lambda \chi^2 / d.o.f = 1.23$$

The new three parameters for $N_f = 4$ are $\phi_C, \delta_C, \varepsilon_C$. By fit with $\chi^2 / d.o.f = 1$:

$$\phi_C = (176 \pm 28) \text{ MeV}, \ \delta_C = (3.91 \pm 0.36) \times 10^6 \text{ MeV}^2, \ \varepsilon_C = (2.23 \pm 0.71) \times 10^6 \text{ MeV}^2$$

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Results

Masses of (open and hidden) charmed mesons:

Resonance	Quark content	J^P	Our Value [MeV]	Experimental Value [MeV]
D^0	$uar{c},ar{u}c$	0^{-}	1981 ± 73	1864.86 ± 0.13
D_S^{\pm}	$sar{c},ar{s}c$	0^{-}	2004 ± 74	1968.50 ± 0.32
$\eta_c(1S)$	$c\bar{c}$	0-	2673 ± 118	2983.7 ± 0.7
$D_0^*(2400)^0$	$uar{c},ar{u}c$	0^{+}	2414 ± 77	2318 ± 29
$D_{S0}^{*}(2317)^{\pm}$	$sar{c},ar{s}c$	0^{+}	2467 ± 76	2317.8 ± 0.6
$\chi_{c0}(1P)$	$c\bar{c}$	0^{+}	3144 ± 128	3414.75 ± 0.31
$D^*(2007)^0$	$uar{c},ar{u}c$	1^{-}	2168 ± 70	2006.99 ± 0.15
D_s^*	$sar{c},ar{s}c$	1-	2203 ± 69	2112.3 ± 0.5
$J/\psi(1S)$	$c\bar{c}$	1-	2947 ± 109	3096.916 ± 0.011
$D_1(2420)^0$	$uar{c},ar{u}c$	1^{+}	2429 ± 63	2421.4 ± 0.6
$D_{S1}(2536)^{\pm}$	$sar{c},ar{s}c$	1^{+}	2480 ± 63	2535.12 ± 0.13
$\chi_{c1}(1P)$	$c\bar{c}$	1^{+}	3239 ± 101	3510.66 ± 0.07

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]]. W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph].

Mass difference and decay constants

The mass difference of the squared charmed (axial-)vector mesons:

mass difference	theoretical value MeV^2	experimental value MeV^2
$m_{D_1}^2 - m_{D^*}^2$	$(1.2 \pm 0.6) \times 10^{6}$	$1.82 imes 10^6$
$m^2_{\chi_{C1}}-m^2_{J/\psi}$	$(1.8 \pm 1.3) \times 10^{6}$	$2.73 imes 10^6$
$m_{D_{S1}}^2 - m_{D_S^*}^2$	$(1.2 \pm 0.6) \times 10^6$	$1.97 imes 10^6$

Weak decay constant of D, D_S , and f_{η_C} .

$$f_D = (254 \pm 17) \text{ MeV}$$
, $f_{D_S} = (261 \pm 17) \text{ MeV}$, $f_{\eta_C} = (314 \pm 39) \text{ MeV}$.
[Exp. value = 206.7 ± 8.9] MeV , [Exp. value = 260.5 ± 5.4] MeV , [Exp. value = 335 ± 75] MeV

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Decay widths of open charmed mesons:

Decay Channel	Theoretical result [MeV]	Experimental result [MeV]
$D_0^*(2400)^0 \rightarrow D\pi = D^+\pi^- + D^0\pi^0$	139^{+243}_{-114}	$D^+\pi^-$ seen; full width $\Gamma = 267 \pm 40$
$D_0^*(2400)^+ \to D\pi = D^+\pi^0 + D^0\pi^+$	51^{+182}_{-51}	$D^+\pi^0$ seen; full width: $\Gamma = 283 \pm 24 \pm 34$
$D^*(2007)^0 \to D^0 \pi^0$	0.025 ± 0.003	seen; < 1.3
$D^*(2007)^0 \to D^+\pi^-$	0	not seen
$D^*(2010)^+ \to D^+\pi^0$	$0.018^{+0.002}_{-0.003}$	0.029 ± 0.008
$D^*(2010)^+ \to D^0 \pi^+$	$0.038^{+0.005}_{-0.004}$	0.065 ± 0.017
$D_1(2420)^0 \to D^*\pi = D^{*+}\pi^- + D^{*0}\pi^0$	65^{+51}_{-37}	$D^{*+}\pi^{-}$ seen; full width: $\Gamma = 27.4 \pm 2.5$
$D_1(2420)^0 \to D^0 \pi \pi = D^0 \pi^+ \pi^- + D^0 \pi^0 \pi^0$	0.59 ± 0.02	seen
$D_1(2420)^0 \to D^+\pi^-\pi^0$	$0.21^{+0.01}_{-0.015}$	seen
$D_1(2420)^0 \to D^+\pi^-$	0	not seen; $\Gamma(D^+\pi^-)/\Gamma(D^{*+}\pi^-) < 0.24$
$D_1(2420)^+ \to D^*\pi = D^{*+}\pi^0 + D^{*0}\pi^+$	65^{+51}_{-36}	$D^{*0}\pi^+$ seen; full width: $\Gamma = 25 \pm 6$
$D_1(2420)^+ \to D^+\pi\pi = D^+\pi^+\pi^- + D^+\pi^0\pi^0$	0.56 ± 0.02	seen
$D_1(2420)^+ \to D^0 \pi^0 \pi^+$	0.22 ± 0.01	seen
$D_1(2420)^+ \to D^0 \pi^+$	0	not seen; $\Gamma(D^0\pi^+)/\Gamma(D^{*0}\pi^+) < 0.18$
$D_{S1}(2536)^+ \to D^*K = D^{*0}K^+ + D^{*+}K^0$	25^{+22}_{-15}	seen; full width $\Gamma = 0.92 \pm 0.03 \pm 0.04$
$D_{S1}(2536)^+ \to D^+ K^0$	Ũ	not seen
$D_{S1}(2536)^+ \to +D^0K^+$	0	not seen

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Decay widths of hidden charmed mesons:

1) Decay widths of (axial-)vector charmonium states: $\Gamma_{J/\psi} = 0$ and $\Gamma_{\chi_{c1}} = 0$

2) Decay widths of a pseudoscalar charmonium state (η_c):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\eta_c \to \overline{K}_0^* K}$	0.01	-
$\Gamma_{\eta_c \to a_0 \pi}$	0.01	-
$\Gamma_{\eta_c \to f_0(1370)\eta}$	0.00018	-
$\Gamma_{\eta_c \to f_0(1500)\eta}$	0.006	-
$\Gamma_{\eta_c \to f_0(1710)\eta}$	0.000032	-
$\Gamma_{\eta_c \to f_0(1370)\eta'}$	0.027	-
$\Gamma_{\eta_c \to f_0(1500)\eta'}$	0.024	-
$\Gamma_{\eta_c \to f_0(1710)\eta'}$	0.0006	-
$\Gamma_{\eta_c \to \eta \eta \eta}$	0.052	-
$\Gamma_{\eta_c \to \eta' \eta' \eta'}$	0.0023	-
$\Gamma_{\eta_c \to \eta' \eta \eta}$	0.44	-
$\Gamma_{\eta_c \to \eta' \eta' \eta}$	0.0034	
$\Gamma_{\eta_c \to \eta K \overline{K}}$	0.15	0.32 ± 0.17
$\Gamma_{\eta_c \to \eta' KK}$	0.41	
$\Gamma_{\eta_c \to \eta \pi \pi}$	0.12	$0.54{\pm}0.18$
$\Gamma_{\eta_c \to \eta' \pi \pi}$	0.08	$1.3 \pm 0.0.6$
$\Gamma_{\eta_c \to KK\pi}$	0.095	_

Decay width of η_c into a pseudoscalar glueball



Decay widths of a pseudoscalar charmonium state (χ_{c0}):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]			
$\Gamma_{\chi_{c0} \to a_0 a_0}$	0.004	-	Decay Channel	theoretical result [MoV]	Experimental result MeV
$\Gamma_{\chi_{c0} \to k_1 \overline{K}_1}$	0.005	-	Decay Channel		Experimental result [wev]
$\Gamma_{\chi_{c0} \to \eta\eta}$	0.022	$0.031 {\pm} 0.0039$	$\Gamma_{\chi_{c0}\to\overline{K}_0^{*0}K_0^{*0}}$	0.01	0.01 ± 0.0047
$\Gamma_{\chi_{c0} \to \eta' \eta'}$	0.02	0.02 ± 0.0035	$\Gamma_{\gamma \rightarrow K^-K^+}$	0.059	0.061 ± 0.007
$\Gamma_{\chi_{c0} \to \eta \eta'}$	0.004	< 0.0024	Γ	0.089	0.088 ± 0.0002
$\Gamma_{\chi_{c0}\to K^*K_0^*}$	0.00007	-	$\chi_{c0} \rightarrow \pi\pi$	0.005	0.000±0.0032
$\Gamma_{\chi_{c0}\to\rho\rho}$	0.01	-	$\Gamma_{\chi_{c0}\to\overline{K}^{*0}K^{*0}}$	0.0175	0.017 ± 0.0072
$\Gamma_{\chi_{c0} \to f_0(1370) f_0(1370)}$	0.005	< 0.003	$\Gamma_{\chi_{c0} \to ww}$	0.01	0.0099 ± 0.0017
$\Gamma_{\chi_{c0} \to f_0(1500) f_0(1500)}$	0.004	< 0.0005	En 144	0.004	0.0081 ± 0.0013
$\Gamma_{\chi_{c0} \to f_0(1370) f_0(1500)}$	0.000004	< 0.001	Γ	0.005	0.062 ± 0.0222
$\Gamma_{\chi_{c0} \to f_0(1370) f_0(1710)}$	0.0003	0.0069 ± 0.004	$\chi_{c0} \rightarrow k_1^+ K^-$	0.005	0.003±0.0233
$\Gamma_{\chi_{c0} \to f_0(1500) f_0(1710)}$	0.00004	< 0.0007			
$\Gamma_{\chi_{c0}\to K_0^*K\eta}$	0.008	-			
$\Gamma_{\chi_{c0}\to K_0^*K\eta'}$	0.004	-			
$\Gamma_{\chi_{c0} \to f_0(1370)\eta\eta}$	0.0004	_			
$\Gamma_{\chi_{c0} \to f_0(1500)\eta\eta}$	0.003	-			
$\Gamma_{\chi_{c0}\to f_0(1370)\eta'\eta'}$	0.0027	-			
$\Gamma_{\chi_{c0} \to f_0(1370)\eta\eta'}$	0.000089	-	W. I. Eshraim	and C. S. Fischer, Eur.Pl	nys.J. A54 (2018) no.8, 139
$\Gamma_{\chi_{c0} \to f_0(1500)\eta\eta'}$	0.011	-			
$\Gamma_{\chi_{c0} \to f_0(1710)\eta\eta}$	0.00008	-			
$\Gamma_{\chi_{c0} \to f_0(1710)\eta\eta'}$	0.00003	-			

Conclusions

- 2. Decay widths of the first excited pseudoscalar glueball in cases of $N_f = 3$ and $N_f = 4$.
- 3. Linear sigma model with $N_f = 4$ and vector and axial-vector mesons.
- 3. Masses of (open and hidden) charmed mesons.
- 4. Decay widths of (open and hidden) charmed mesons.

Outlook

- 1. Study of the chirally symmetric model with vector and axial-vector mesons in the case of isospin breaking for $N_f = 3$ at nonzero temperature (ongoing).
- 2. Hybrid phenomenology in a chiral symmetric model (ongoing).
- 3. Study of the light tetraquark nonet and its extension to $N_f = 4$ (ongoing).
- 4. Study the scattering of glueballs.

