Quark (ud, s) mass dependence of the rho meson revisited

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WTPLF 2018

Introduction

- ⋆ The rho meson is one of the most extensively studied hadrons
- ⋆ Its properties fit very well into a Breit-Wigner parameterization and its nature is believed to be a resonance made of light u and d quarks
  Example of non Breit-Wigner amplitude: Scalar σ meson, D. Guo, A. Alexandru, R. Molina, M. Mai and M. Doring, *Extraction of isoscalar $\pi\pi$ phase shifts from latticeQCD*, PRD98 (2018)
- ⋆ Its properties, as decay width or coupling to two-pions, are tight to f_π. Example: Hidden gauge lagrangian,

$$\mathcal{L}=\mathcal{L}^{(2)}+\mathcal{L}_{III},\quad \begin{cases} \mathcal{L}^{(2)}=\frac{1}{4}f^2\langle D_\mu U D^\mu U^\dagger+\chi U^\dagger+\chi^\dagger U\rangle \\ \mathcal{L}_{III}=-\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle+\frac{1}{2}M_V^2\langle[V_\mu-\frac{i}{g}\Gamma_\mu]^2\rangle \end{cases}$$

$$U=e^{i\sqrt{2}P/f}\qquad V_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu-ig[V_\mu,V_\nu]$$

$$u^2=U\qquad \Gamma_\mu=\frac{1}{2}[u^\dagger(\partial_\mu-ieQA_\mu)u+u(\partial_\mu-ieQA_\mu)u^\dagger]$$

$$\Longrightarrow \mathcal{L}_{VPP}=-ig\langle V^\mu[P,\partial_\mu P]\rangle;\ \mathcal{L}_{V\gamma}=-M_V^2\frac{e}{g}A_\mu\langle V^\mu Q\rangle;\ g=\frac{m_\rho}{2f_\pi}$$

Introduction

- ⋆ Dispersion-relation treatment of f_π, transitions from off-shell pion to a complete set of intermediate states dominated by $N\bar{N}$,

  Goldberger-Treiman relation: $\boxed{f_\pi = M_N\, g_A/g_{\pi NN}}$

  g_A: axial vector coupling in the nucleon β decay,

  $g_{\pi NN}$: pion-nucleon coupling.

- ⋆ f_π is also conected to the sponeaous chiral symmetry breaking. Chiral condensate (chiral limit $m_{u,d,s} = 0$):

$$\Longrightarrow \boxed{\Sigma_0 = B_0 f_0^2 = -\langle 0|\bar{q}q|0\rangle_0}$$

  $\langle \bar{q}q\rangle \neq 0$, and $f_0 \neq 0$,

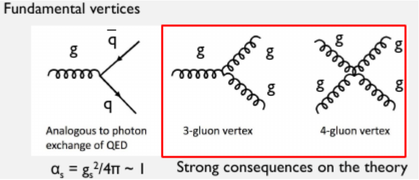
  $\langle 0|A_a^\mu(0)|\phi_b(p)\rangle = ip^\mu f_0 \delta_{ab}$.

Introduction

* ⋆ Chiral Perturbation Theory studies the interactions between pseudoscalar mesons and the quark mass dependence of the pseudoscalar decay constants (Gasser and Leutwyler)
  or Partially Quenched ChPT (finite volume effects),
  Unitary Chiral Perturbation Theory (extension to higher energies) (Pelaez, Oset, Oller),
  Inverse Amplitude Method...(Truon, Dobado, Gomez-Nicola, Pelaez, Oller)
* ⋆ The predictions can be tested by latticeQCD simulations...

LatticeQCD: Non-perturbative approach to solve QCD

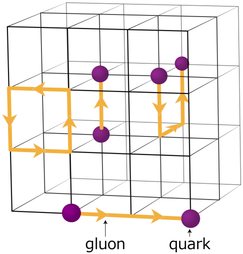
$$\bar{\psi}_i(i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij})\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu_a} \qquad (1)$$



Discretization of space and time,

$$\langle \mathcal{O}(A_\mu, q, \bar{q}) \rangle = \frac{1}{Z}\int \mathcal{D}A_\mu \mathcal{D}q\mathcal{D}\bar{q}\mathcal{O}(A_\mu, q, \bar{q})\exp\left(-\int d^4x\mathcal{L}\right)$$

Feynmann path integral formulation



Introduction

However,

- Lattice computations are computationally expensive.
- Results depend on lattice artefacts as volume size or/and lattice spacing.
- Becomes especially complicated when dealing with excited states partly because the number of operators needed in the basis increases. Also, threshold effects, multi-channel treatment are tough.
- Even for one of the "simplest" hadrons, the ρ meson, few points over a chiral trajectory (one or two masses).
- Till now, there was only data on chiral trajectories $m_s = m_{s,\mathrm{phys}}$ for f_π and phase-shifts in the isovector channel.

Strange quark mass dependence of the pseudoscalar decay constants and $\pi\pi$ phase shifts is still unknown

Lattice data analyzed

Andersen, Christian and Bulava, John and Hörz, Ben and Morningstar, Colin, arXiv: 1808.05007 (2018) [CLS]

Bruno, Mattia and Korzec, Tomasz and Schaefer, Stefan, Phys. Rev. D95, 074504 (2017) [CLS]

Wilson, David J. and Briceno, Raul A. and Dudek, Jozef J. and Edwards, Robert G. and Thomas, Christopher E., Phys. Rev. D92, 094502 (2015) [HS]

Dudek, Jozef J. and Edwards, Robert G. and Thomas, Christopher E., Phys. Rev. D87, 034505 (2013) [HS]

Bulava, John and Fahy, Brendan and Hörz, Ben and Juge, Keisuke J. and Morningstar, Colin and Wong, Chik Him, Nucl. Phys. B910, 842-867 (2016) [JB]

Feng, Xu and Aoki, Sinya and Hashimoto, Shoji and Kaneko, Takashi, Phys. Rev. D91, 054504 (2015) [PACS-CS]

Blum, T. and others, Phys. Rev. D93, 074505 (2016) [UKQCD]

Bazavov, A. and others, PoS LATTICE2010, 074 (2010), Rev. Mod. Phys. 82, 1349-1417 (2010) [MILC]

Aubin, Christopher and Laiho, Jack and Van de Water, Ruth S., PoS LATTICE2008, 105 (2008) [Laiho]

Noaki, J. and others, PoS LATTICE2009, 096 [JL/TWQCD]

Aoki, S. and others, Phys. Rev. D79,034503 (2009) [PACS-CS]

Chiral Perturbation Theory

⋆ ChPT expansion of the amplitude for meson-meson scattering

$$t(s) = t_2(s) + t_4(s) + ...t_{2k} = \mathcal{O}(p^{2k}) \qquad (2)$$

⋆ Lowest-order Chiral Lagrangian

$$\mathcal{L}_2 = \frac{f^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U + M(U + U^\dagger)\rangle \qquad (3)$$

$$\begin{aligned}\mathcal{L}_4 = {} & L_1\langle \partial_\mu U^\dagger \partial^\mu U\rangle^2 + L_2\langle \partial_\mu U^\dagger \partial_\nu U\rangle\langle \partial^\mu U^\dagger \partial^\nu U\rangle \\ & + L_3\langle \partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\rangle + L_4\langle \partial U^\dagger \partial^\mu U\rangle\langle U^\dagger M + M^\dagger U\rangle \\ & + L_5\langle \partial_\mu U^\dagger \partial^\mu U(U^\dagger M + M^\dagger U)\rangle + L_6\langle U^\dagger M + M^\dagger U\rangle^2 \\ & + L_7\langle U^\dagger M - M^\dagger U\rangle^2 + L_8\langle M^\dagger U M^\dagger U + U^\dagger M U^\dagger M\rangle \qquad (4)\end{aligned}$$

Chiral Perturbation Theory

where $U(\phi) = \exp(i\sqrt{2}\Phi/f)$, and

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}_\mu \quad (5)$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix} \quad (6)$$

- J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984)
- J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985)
- J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999)

Meson decay constants from SU(3) Ch PT

One loop SU(3) UChPT,

$$f_\pi = f_0\left[1 - 2\mu_\pi - \mu_K + \frac{4M_{0\,\pi}^2}{f_0^2}(L_4 + L_5) + \frac{8M_{0\,K}^2}{f_0^2}L_4\right],$$

$$f_K = f_0\left[1 - \frac{3\mu_\pi}{4} - \frac{3\mu_K}{2} - \frac{3\mu_\eta}{4} + \frac{4M_{0\,\pi}^2}{f_0^2}L_4 + \frac{4M_{0\,K}^2}{f_0^2}(2L_4 + L_5)\right],$$

$$f_\eta = f_0\left[1 - 3\mu_K + \frac{4L_4}{f_0^2}\left(M_{0\,\pi}^2 + 2M_{0\,K}^2\right) + \frac{4M_{0\,\eta}^2}{f_0^2}L_5\right] .$$

J. Nebreda and J. R. Pelaez., Phys. Rev. D **81**, 054035 (2010)

Chiral trajectories

★ $m_s = k$:

$$M_{0K}^2 = +\frac{1}{2} M_{0\pi}^2 + k\, B_0 \ ,$$

where $k = m_{s,\mathrm{phys}}$ or $0.6\, m_{s,\mathrm{phys}}$.

★ $\mathrm{Tr\ M} = c$:

$$M_{0K}^2 = -\frac{1}{2} M_{0\pi}^2 + c\, B_0 \ ,$$

We also show predictions for the trajectories,

★ $m_s = m_{ud}$,

$$M_{0k}^2 = M_{0\pi}^2 \ ,$$

★ $m_{ud} = m_{ud\,\mathrm{phys}}$,

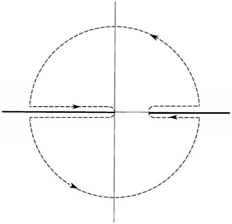
$$M_{0\pi}^2 = M_{0\pi,\mathrm{phys}}^2 ;\ M_{0K}^2 = M_{0K,\mathrm{phys}}^2 + (m_s - m_{s,\mathrm{phys}})\, B_0$$

Free parameters: $L_{12} = 2\, L_1 - L_2$ and L_i, $i = 3, 8$, and $c\, B_0$, $k\, B_0$, adjusted to the the chiral trajectories. $\mu = 770$ MeV, $f_0 = 80$ MeV.

Inverse Amplitude Method

Dispersion relations

$$t(s) = \frac{1}{2\pi i} \int_C \frac{t(s')}{s' - s} ds'$$



If we assume that $t \to 0$ as $|s| \to \infty$, then

$$\boxed{t(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{Im}\, t(s')}{s' - s} ds' + \frac{1}{\pi} \int_{-\infty}^{0} \frac{\mathrm{Im}\, t(s')}{s' - s} ds'} \qquad (7)$$

with $t(s) \equiv \mathrm{Im}\, t(s + i\epsilon) = \frac{1}{2i}\left[t(s + i\epsilon) - t(s - i\epsilon)\right]$.

Inverse Amplitude Method

If t does not goes to zero when $|s| \to \infty$ sufficiently fast to apply the Cauchy Theorem,

$$G(s) = \frac{t(s)}{(s-s_1)(s-s_2)....(s-s_N)}$$

For some N we can apply a dispersion relation for G.
Subtraction points: $s_1, s_2, ...s_N$
Unitarity condition:
(physical cut)$\mathrm{Im}\, G = t_2^2 \mathrm{Im} \frac{1}{t} = -t_2^2 \sigma$
Left cut (perturbative expansion)
$\mathrm{Im}\, G = t_2^2\, \mathrm{Im} \frac{1}{t} \simeq t_2^2\, \mathrm{Im} \frac{1}{t_2+t_4} \simeq -\mathrm{Im}\, t_4.$

$$\boxed{t_{\mathrm{IAM}}(s) = \frac{t_2(s)^2}{t_2(s) - t_4(s)}} \tag{8}$$

Inverse Amplitude Method

S-matrix

$$S_{ij} = \delta_{ij} + 2\,i\sqrt{\sigma_i \sigma_j}\,t_{ij} \tag{9}$$

with

$$\sigma_i = \begin{cases} \sqrt{1 - 4\,m_i^2/s} & \sqrt{s} > 2\,m_i \\ 0 & \text{else} \end{cases} \tag{10}$$

and the S-matrix is parametrized as

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} . \tag{11}$$

Fitting procedure

⋆ In general, energy measurements in the lattice are correlated:

$$\boxed{\chi^2 = (\vec{W}_1 - \vec{W}_0)^T C^{-1} (\vec{W}_1 - \vec{W}_0)}\,, \qquad (12)$$

$\vec{W}_0$: eigenenergies measured on the lattice,
C: covariance matrix,
$\vec{W}_1$:energies of the fit function.

⋆ Inclined error bars in the $(W, \delta(W))$ plane, i. e., correlations from the Lüscher formula, $\delta_L = g(W)$, are considered reconstructing the energies, W_{1i}, from the fit function, $\delta_{\rm fit} = f(W_{1i})$, by means of a Taylor expansion near the measured energies W_{0i} (linear order),

$$W_{1i} = \frac{g(W_{0i}) - f(W_{0i})}{f'(W_{0i}) - g'(W_{0i})} + W_{0i}\,. \qquad (13)$$

Fitting procedure

⋆ The set of LECs is chosen according to S-matrix unitarity:

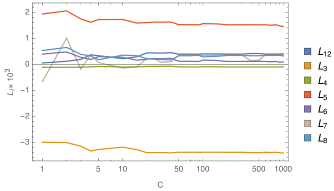
$$\hat{\chi}^2 = \chi_e^2 + \lambda \sum_{ij} \int |(S\,S^\dagger)_{ij} - \delta_{ij}|^2\, dE + \chi_f^2 \qquad (14)$$

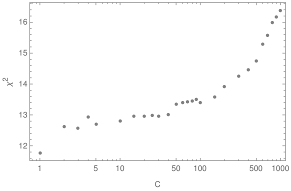
$$\chi_f^2 = \sum_{ij} (h_{ij} - h_{ij}^l)^2 / e_{ij}^{l\,2}$$

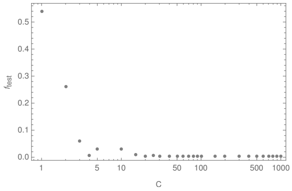
$i = 1, 3,\ j = 1, n.$

$h_1 = m_\pi / f_\pi,\ h_2 = m_K / f_K$

and $h_3 = m_K / f_\pi$.







Global fit to decay constants/phase shifts

LEC$\times 10^3$	Fit I	Fit III
L_{12}	-	$0.35(1)$
L_3	-	$-3.4(1)$
L_4	$-0.06(1)$	$-0.09(2)$
L_5	$0.91(2)$	$0.95(1)$
L_6	$0.15(2)$	$0.19(1)$
L_7	-	$0.007(2)$
L_8	$0.03(3)$	$0.11(2)$

Tabela: Values of LECs in Fits I and III.

Global fit to decay constants/phase shifts

$c(k)\, B_0 \times 10^{-3}(\mathrm{MeV}^2)$	Fit I	Fit III
$[c\, B_0]_{\beta=3.4}$	316(6)	275(4)
$[c\, B_0]_{\beta=3.55}$	295(6)	260(4)
$[c\, B_0]_{\beta=3.7}$	298(6)	265(4)
$[k\, B_0]_{m_{s,\mathrm{phys}}}$	257(6)	222(4)

Tabela: Values of parameters in Fits I and III.

$\Sigma_0 = B_0 f_0^2 = -\langle 0|\bar{q}q|0\rangle_0 \Longrightarrow \boxed{\Sigma_0^{1/3} = 245 - 280 MeV}$

Flag review: 214 – 290 MeV.

MILC: $\Sigma_0^{1/3} = 245(5)(4)(4)$ MeV. We obtain 258 and 245 MeV for Fits I and III in the $m_s = m_{s,\mathrm{phys}}$ trajectory, respectively, which are compatible.

Chiral trajectories: m_K^2 vs. m_π^2

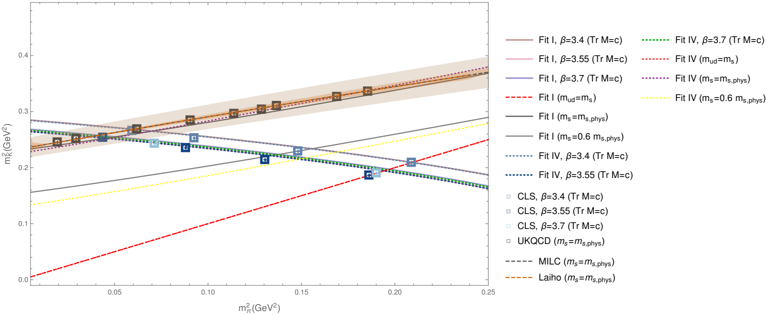


Figura: Chiral trajectories Fits I (decay constant data) and III (decay constant plus rho phase shift data) over both kind of chiral trajectories, $m_s = k$, and $\mathrm{Tr\,M} = \mathrm{c}$.

Ratio m_π/f_π vs. m_π

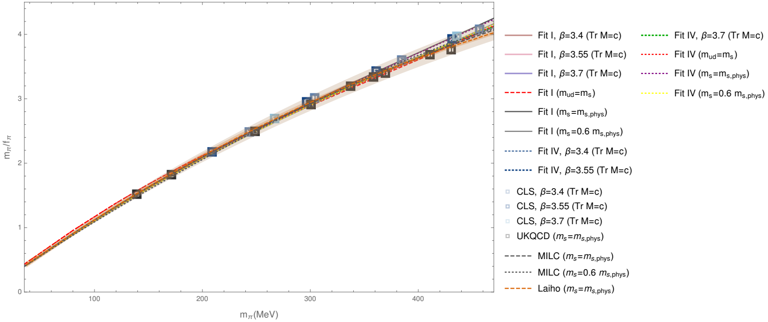
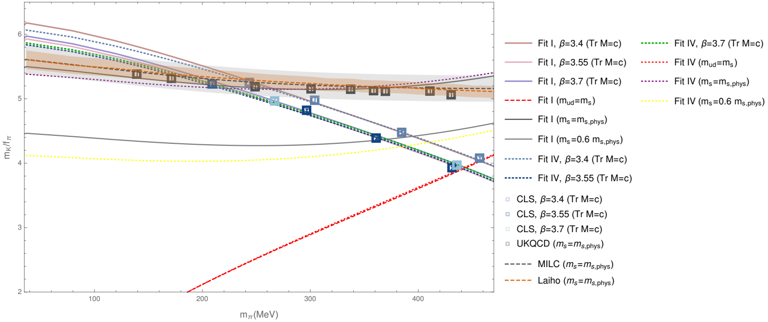


Figura: Ratio m_π/f_π obtained in Fits I (decay constant data) and III (decay constant plus rho phase shift data) over both kind of chiral trajectories, $m_s = k$, and $\mathrm{Tr\,M} = \mathrm{c}$.

Ratio m_K/f_π vs. m_π



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Ratio m_K/f_K vs. m_π

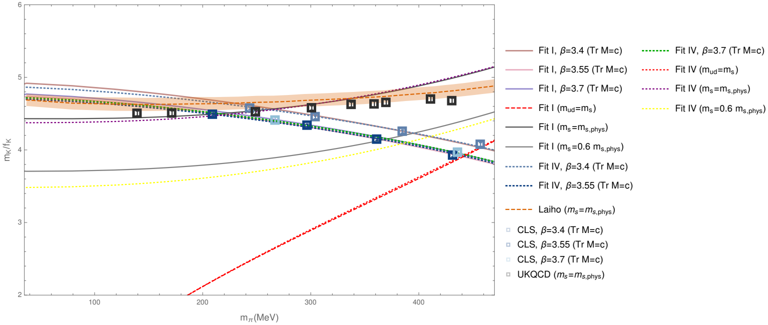


Figura: Ratio m_K/f_K obtained in Fit I (decay constant data) and III (decay constant plus rho phase shift data) over both kind of chiral trajectories, $m_s = k$, and $\mathrm{Tr\,M} = \mathrm{c}$.

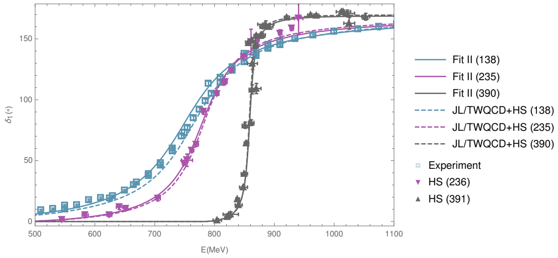
Chiral trajectories $m_s = k$

HS: rho phase shift data of HadSpec(236,391), JB: Bulava(235)
MILC,Laiho,UKQCD: decay constant data on $m_s = k$,
$k = 0.6, 1\, m_{s,\mathrm{phys}}$. We find these data compatible (excludes JL/TWQCD, PACS-CS).

LEC$\times 10^3$	MILC,HS	UKQCD,HS	Laiho,HS	MUL,HS	Fit II
L_{12}	0.3(4)	−0.1(4)	0.1(2)	0.3(1)	0.2(1)
L_3	−3.4(4)	−3.1(4)	−3.4(2)	−3.4(1)	−3.4(1)
L_4	0.03(2)	0.04(1)	0.03(2)	0.05(2)	0.04(1)
L_5	1.2(2)	0.90(3)	0.90(5)	0.93(3)	0.94(2)
L_6	0.5(1)	0.24(3)	0.2(1)	0.25(4)	0.24(2)
L_7	0.6(3)	0.5(2)	0.5(1)	0.40(6)	0.44(6)
L_8	−0.5(1)	−0.3(1)	−0.2(1)	−0.3(1)	−0.27(4)

Tabela: Values of the LECs obtained from fits $m_s = k$.

Rho phase shift data on $m_s = k$



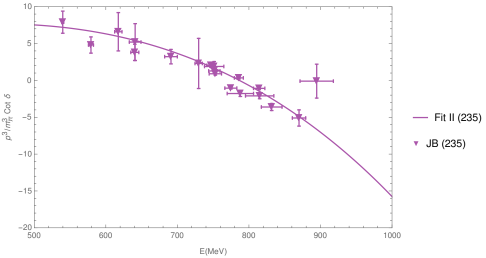
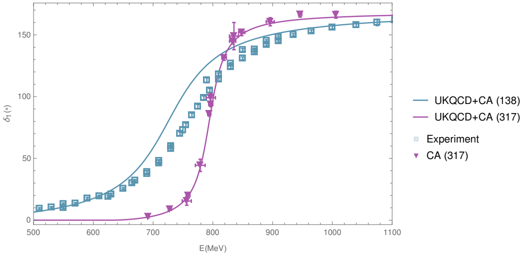
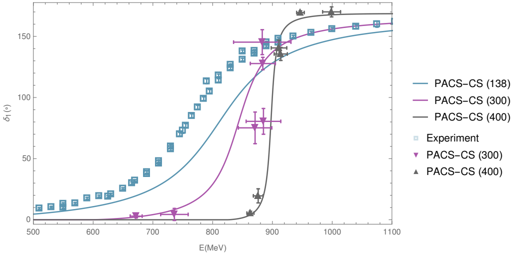


Figura: Phase shifts and extrapolations to the physical point from the fits in comparison with lattice and experimental data.

Rho phase shift data on $m_s = k$





Similar findings than in PRD96 (2017) B. Hu, R. Molina and M. Döring et al.

Rho phase shift data on a global fit ($m_s = k$, TrM$= c$)

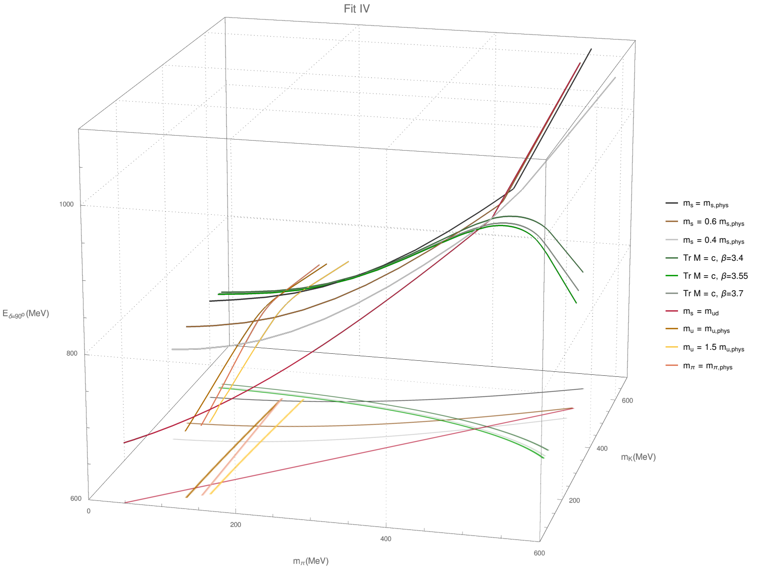


Figura: m_ρ as a function of m_π, m_K.

Rho phase shift data on a global fit ($m_s = k$, TrM$= c$)

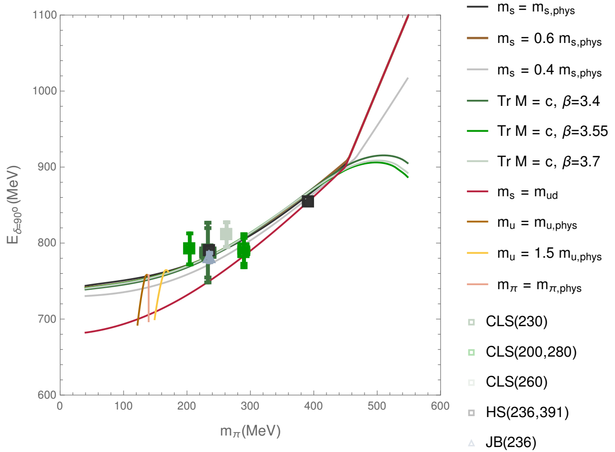


Figura: m_ρ as a function of m_π.

Rho phase shift data on a global fit ($m_s = k$, TrM$= c$)

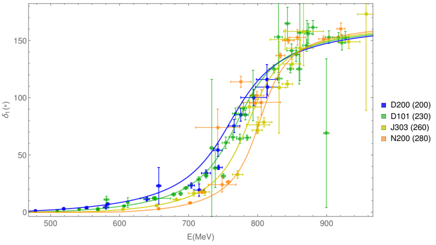


Figura: Phase shifts with TrM$= c$.

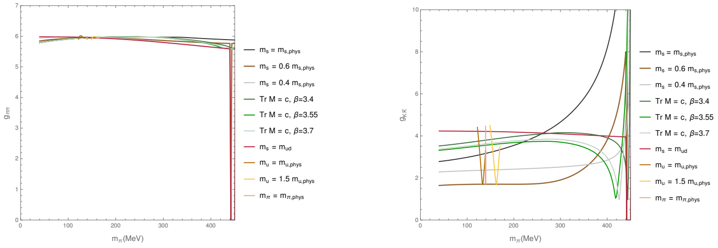


Figura: Couplings vs. m_π, $g_{\pi\pi}$ and $g_{K\bar{K}}$.

Rho phase shift data on a global fit ($m_s = k$, TrM$= c$)

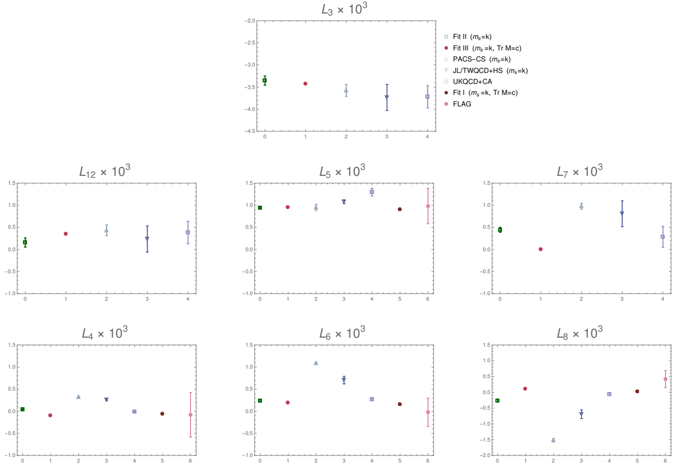


Figura: Values of the LECs obtained in the several fits in comparison with the FLAG average.

Conclusions

- ⋆ The new trajectories TrM$=c$ give insight into the dependence with the strange quark.
- ⋆ Around the physical point the dependence of the rho mass with the strange quark mass is small.
- ⋆ The strange quark starts to play role when the thresholds $\pi\pi$ and $K\bar{K}$ get closer, or when the strange quark mass gets lighter ($m_u = m_{u,\mathrm{phys}}$ trajectories).
- ⋆ We find a transition into a bound state in $m_s = k$ trajectories and in the symmetric around $m_\pi = 450$ MeV. While in the trajectories TrM$=c$, the channels invert and the state starts to decay into $K\bar{K}$.
- ⋆ While $g_{\pi\pi}$ is quite constant before the transition, the coupling to $K\bar{K}$, $g_{K\bar{K}}$, is not small and grows fast when the thresholds get closer.