

# Heavy quark dynamics in deconfined matter

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# Outline

① Heavy-quark (HQ) observables: why?

② POWLANG setup:

- ▶  $Q\bar{Q}$  production in  $pp$ ,  $pA$  and  $AA$
- ▶ Heavy-quark propagation in a medium
- ▶ HQ hadronization in the medium
- ▶ Results and comparison to experimental data

③ Discussion and future improvements

Published papers: A.Beraudo et al., Nucl. Phys. A 831 (2009) 59

W.Alberico et al., Eur. Phys. J. C71 (2011) 1666

W.Alberico et al., Eur. Phys. J. C73 (2013) 2481

A.Beraudo et al., Eur. Phys. J. C75 (2015) 121

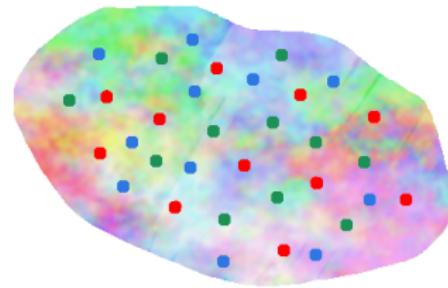
A.Beraudo et al., JHEP 1603 (2016) 123

A.Beraudo et al., JHEP 1802 (2018) 043

The aim of high energy Nucleus-Nucleus Collisions is to create and study the Quark-Gluon Plasma (QGP)

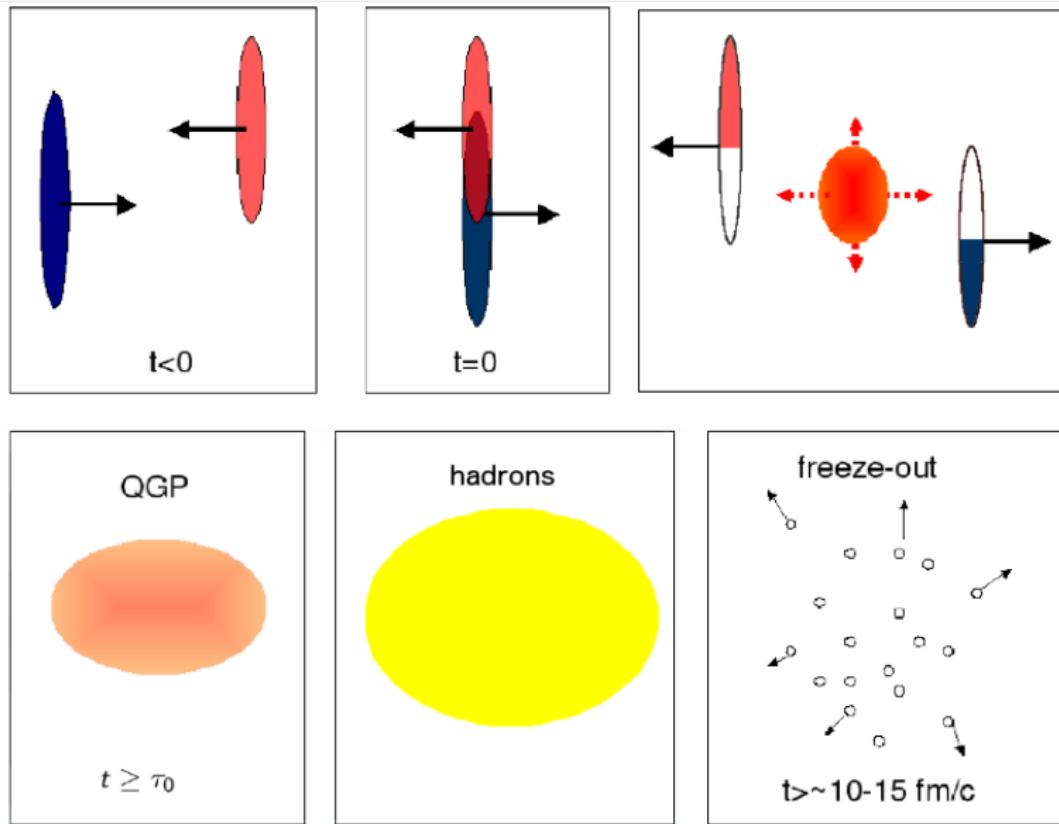


Hadron Gas



QGP

# High-energy Nucleus-Nucleus collision

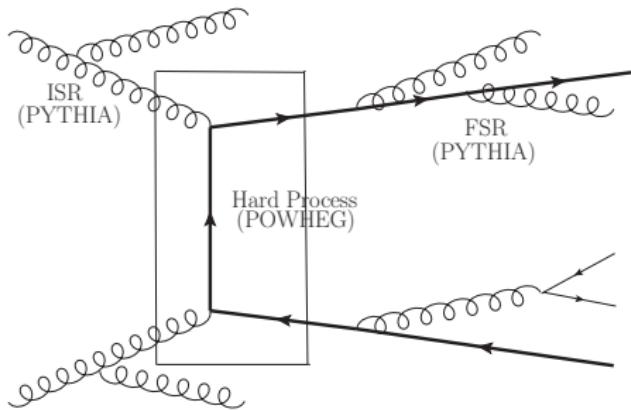


# Why Heavy-quark observables ?

- c- and b-quarks are heavy: their mass  $M$  is larger than  $\Lambda_{QCD}$ , therefore their initial production can be described by pQCD (cross-section and  $p_T$  spectra)
- $M \gg T$ : their thermal production is negligible.
- $M \gg gT$ , with  $gT$  being the typical momentum exchange in the collisions with the plasma particles: many soft scatterings necessary to change significantly the momentum/trajectory of the quark.  
*[caveat: for realistic temperature  $g \sim 2$ , so that one can wonder whether a charm is really "heavy", at least in the initial stage of the evolution]*

**HQ are produced in the very early stage of the collision and they witness the entire space-time evolution of the system: they are good probes to extract information about the bulk medium.**

## HQ production in pp collisions

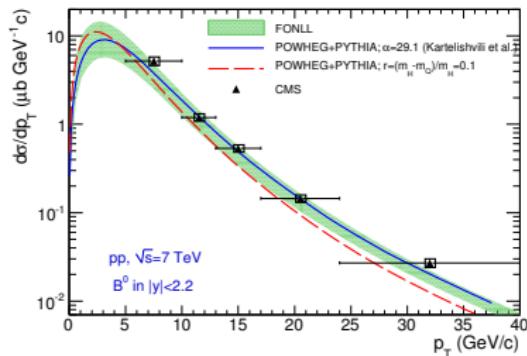
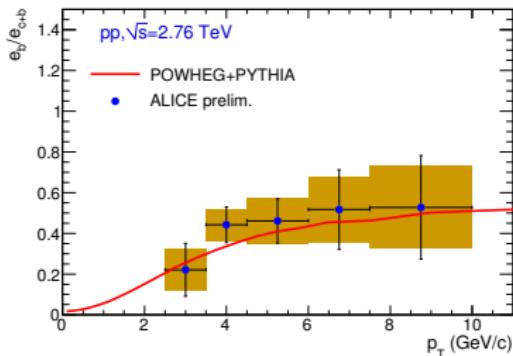
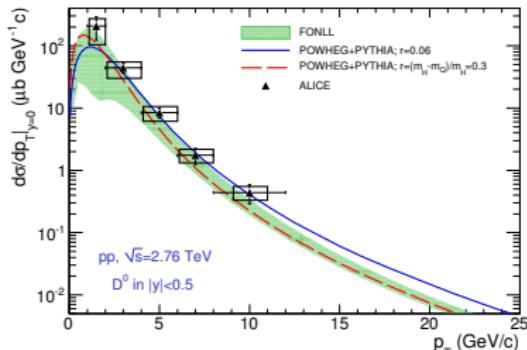
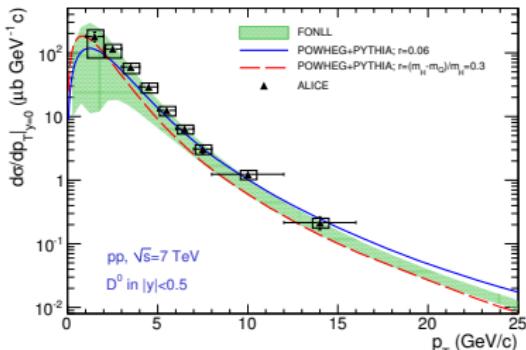


Powerful pQCD tools<sup>1</sup> are available to simulate the initial QQ production. We rely on a standard pQCD public tool, **POWHEG-BOX**, based on collinear factorization, in which the hard  $Q\bar{Q}$  event (computed at NLO) is interfaced with a parton-shower (PYTHIA), describing Initial and Final State Radiation and non-perturbative effects (intrinsic- $k_T$ , MPI, hadronization). It agrees with FONLL and provides a fully exclusive information on the final state.

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<sup>1</sup>For a systematic comparison (POWHEG vs MC@NLO vs FONLL): M. Cacciari et al., JHEP 1210 (2012) 137

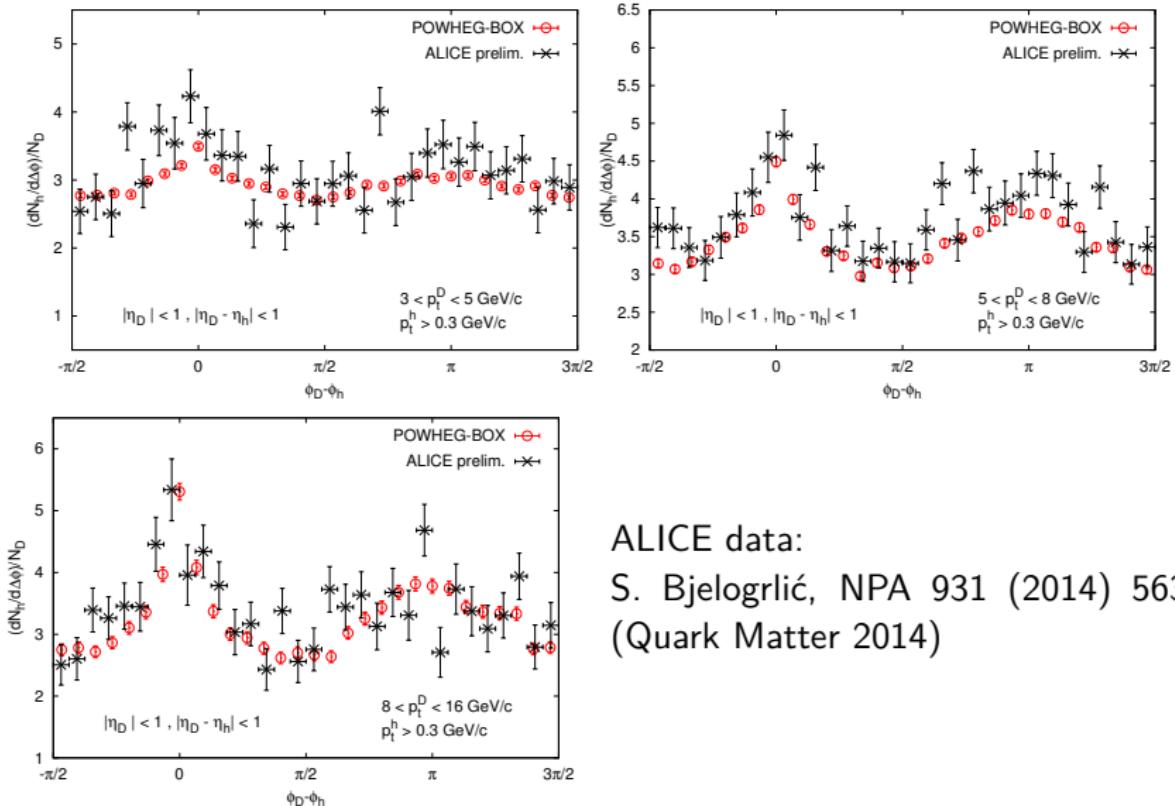
# HQ production in pp: the baseline



ALICE Collab., JHEP 1201, 128 (2012); JHEP 1201, 191 (2012); arXiv:1208.5411.; JHEP 1201, 128 (2012); JHEP 1201, 191 (2012); arXiv:1208.5411.

CMS Collab., Phys. Rev. Lett. 106, 252001 (2011)

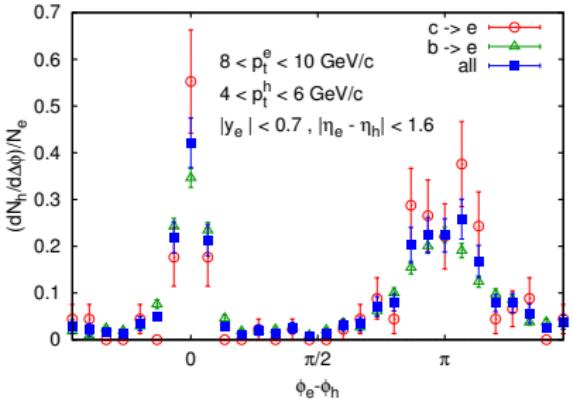
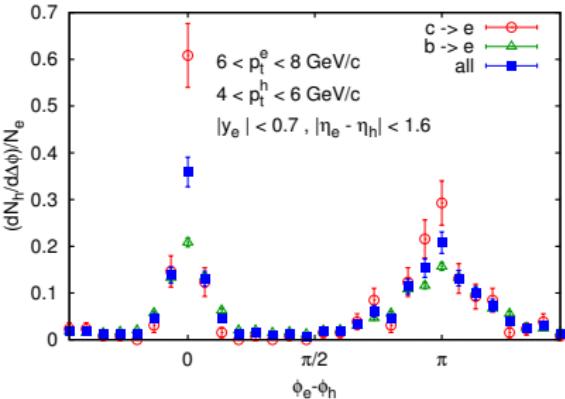
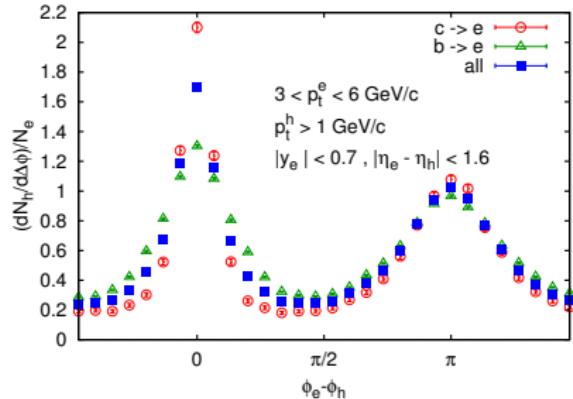
# D-h azimuthal correlations in pp collisions at $\sqrt{s}=7$ TeV



ALICE data:  
S. Bjelogrlić, NPA 931 (2014) 563  
(Quark Matter 2014)

A.Beraudo et al., Eur. Phys. J. C75 (2015) 121

# e-h azimuthal correlations in pp collisions at $\sqrt{s}=7$ TeV



Red points: e's from D;

Green points: e's from B (incl.  $B \rightarrow D \rightarrow e$ );

Blue points: all e's.

## Heavy flavours in $AA$ and $pA$ collisions

# Initial $Q\bar{Q}$ production in p-A and A-A collisions

The initial hard  $Q\bar{Q}$  production in AA collisions was simulated through the POWHEG+PYTHIA setup described previously for  $pp$ , with some differences:

- We include the EPS09 nuclear corrections to the PDFs.
- The position of each  $Q\bar{Q}$  pair is distributed in the transverse plane according to the local density of binary collisions, taken from an optical Glauber calculation<sup>2</sup>, in the case of A-A collisions. In p-A simulations a Glauber-MC is more suitable to handle event-by-event fluctuations.
- The colliding partons acquire, on the average, a larger transverse momentum, proportional to the size of the traversed medium. To get a realistic estimate for  $\langle k_T^2 \rangle_{AA}$  we have adopted the same Glauber approach.

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<sup>2</sup>W.M. Alberico, A. Beraudo, A. De Pace, A. Molinari, M. Monteno, M.N. and F. Prino, Eur.Phys.J. C71 (2011) 1666

## Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution  $f_Q(t, \mathbf{x}, \mathbf{p})$ :

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}}$$

Neglecting  $\mathbf{x}$ -dependence and mean fields:  $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$ .

Collision integral:

$$C[f_Q] = \int d\mathbf{k} \left[ \underbrace{w(\mathbf{p} + \mathbf{k}, \mathbf{k}) f_Q(\mathbf{p} + \mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}} \right]$$

where  $w(p, k)$  is the transition rate for a HQ changing its momentum from  $\mathbf{p}$  to  $\mathbf{p} - \mathbf{k}$ .

# From Boltzmann to Fokker-Planck

Expanding the collision integral for small momentum exchange<sup>3</sup> (Landau approx.)

$$C[f_Q] \simeq \int d\mathbf{k} \left[ k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})].$$

The Boltzmann equation reduces to the Fokker-Planck equation

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k} k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(p)}_{friction} = \mathbf{A}(p) p^i$$

$$B^{ij}(p) = \frac{1}{2} \int d\mathbf{k} k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p})}_{momentum\ broadening} = \hat{p}^i \hat{p}^j \mathbf{B}_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) \mathbf{B}_1(p)$$

The problem is reduced to the evaluation of three transport coefficients:  $A(p)$ ,  $B_0(p)$  and  $B_1(p)$ .

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<sup>3</sup>B. Svetitsky, PRD 37, 2484 (1988)

# The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the [Langevin equation](#)

$$\frac{\Delta p^i}{\Delta t} = - \underbrace{\eta_D(p)p^i}_{determ.} + \underbrace{\xi^i(t)}_{stochastic},$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t) \xi^j(\mathbf{p}_{t'}) \rangle = b^{ij}(\mathbf{p}_t) \frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) = \kappa_{\parallel}(p) \hat{p}^i \hat{p}^j + \kappa_{\perp}(p) (\delta^{ij} - \hat{p}^i \hat{p}^j)$$

Transport coefficients (to be derived from theory):

- Momentum diffusion  $\kappa_{\perp} = \frac{1}{2} \frac{\langle \Delta p_{\perp}^2 \rangle}{\Delta t}$  and  $\kappa_{\parallel} = \frac{\langle \Delta p_{\parallel}^2 \rangle}{\Delta t}$ ;
- Friction term

$$\eta_D^{Ito} = \frac{\kappa_{\parallel}(p)}{2TE_p} + \dots \text{ (dicr. scheme)}$$

fixed in order to assure approach to equilibrium (Einstein relation)<sup>4</sup>

<sup>4</sup>A.Beraudo et al., NPA 831, 59 (2009)

The time step in the rest frame, which enters in updating the quark position and also the quark momentum through the Langevin equation, in our calculations has the value  $\Delta\bar{t} = 0.02$  fm.

The heavy-flavour transport coefficients  $\kappa_{L/T}(p)$  are, in principle, obtained from first-principle calculations.

We have tested two different approaches:

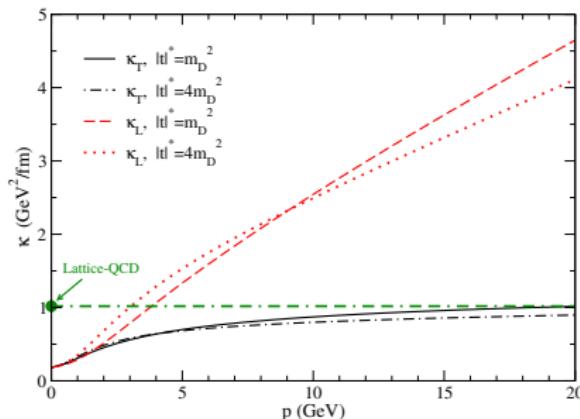
- ① within a weakly-coupled scenario (pQCD + HTL)<sup>5</sup>;
- ② with non-perturbative lattice-QCD simulations<sup>6</sup>.

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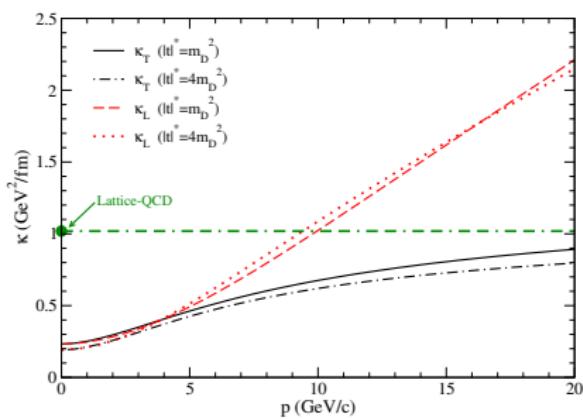
<sup>5</sup>A. Beraudo et al., Nucl.Phys. A **831** 59 (2009)  
W.M. Alberico et al., Eur. Phys. J. C **71** 1666 (2011)

<sup>6</sup>A. Francis et al., PoS LATTICE2011 (2011) 202  
D. Banerjee et al., Phys.Rev. D85 (2012) 014510

## $\kappa_{L/T}(p)$ : comparisons



$c$  quarks



$b$  quarks

Transport coefficients for heavy quarks in the QGP. Weak coupling (HTL+pQCD) results for  $\kappa_{L/T}(p)$  are compared to the data provided by the lattice-QCD calculations at  $p = 0$  (and arbitrarily extrapolated at finite  $p$ ).

The curves refer to the temperature  $T = 400$  MeV

# Heavy flavor in p-A and A-A collisions: POWLANG setup

In order to investigate medium effects on heavy-quark production it is necessary

- to have a realistic description of the **background medium** and of its evolution (hydrodynamics validated with experimental soft-physics data)<sup>7,8</sup>
- to simulate the **heavy quark transport** in a hot (and small) medium (relativistic Langevin equation)
- to account for possible **medium modifications** of heavy quark hadronization

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<sup>7</sup>P. Romatschke, U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007)

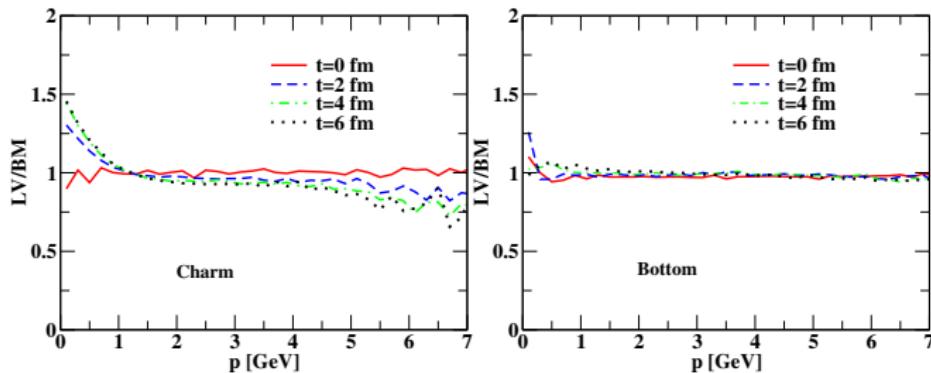
<sup>8</sup>ECHO-QGP: <http://theory.fi.infn.it/echoqgp/>

## To study the propagation of a heavy quark through the QGP:

- ① We determine the initial four-momentum  $p^\mu$  and the initial space-time position  $x^\mu$  of the heavy quark (in the laboratory system) (POWHEG-BOX)
- ② Given the position  $x^\mu$ , we use the information from the hydrodynamic simulation to obtain the fluid local temperature  $T(x)$ , velocity  $u^\mu(x)$  and energy density  $\varepsilon(x)$ .
- ③ We check whether the conditions for hadronization apply: in this case the procedure is ended; otherwise
- ④ we make a Lorentz transformation ( $p^\mu \rightarrow \bar{p}^\mu$ ) to the fluid rest frame, employ the Langevin Eq. to update the quark momentum ( $\bar{p}^\mu \rightarrow \bar{p}'^\mu$ ) and boost it back to the laboratory ( $\bar{p}'^\mu \rightarrow p'^\mu$ ).
- ⑤ We update the space-time step made by the quark in the fluid rest frame ( $\Delta \bar{x}^\mu = (\bar{p}^\mu / E_{\bar{p}}) \Delta \bar{t}$ ), boost it to the laboratory ( $\Delta \bar{x}^\mu \rightarrow \Delta x^\mu$ ) and use it to update the quark position ( $x^\mu \rightarrow x'^\mu$ ).
- ⑥ Given the new momentum  $p'^\mu$  and the new position  $x'^\mu$  the procedure is started again until the conditions for hadronization are met.

# The Langevin/FP approach: a critical perspective

Although the Langevin approach is a very convenient numerical tool and allows one to establish a link between observables and transport coefficients derived from QCD... it was nevertheless derived starting from a soft-scattering expansion of the collision integral  $C[f]$  truncated at second order (friction and diffusion terms), which may be not always justified, in particular for charm.



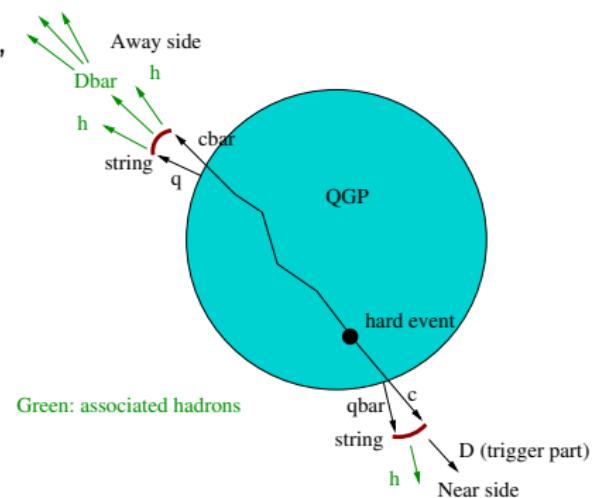
Ratios between the Langevin (LV) and Boltzmann (BM) spectra for charm and bottom quark as a function of momentum for at different time  
(V. Greco et al., Phys.Rev. C90 (2014) 4, 044901)

# HQ in-medium hadronization

In-medium hadronization may affect the momentum distribution of D mesons due to the collective flow of light quarks. We estimate the effect through this model interfaced to our POWLANG transport code:

- At  $T_{dec}$  a  $c(\bar{c})$  is coupled to a light  $\bar{q}(q)$  from a local thermal distribution, boosted to the lab frame ( $u_{fluid}^\mu \neq 0$ );
- A string is formed and given to PYTHIA 6.4 to simulate their fragmentation and produce the final hadrons ( $D + \pi + \dots$ )

This model can explain some features of the  $D$  mesons spectra at low and moderate  $p_T$ ; it allows also to calculate particle correlations.

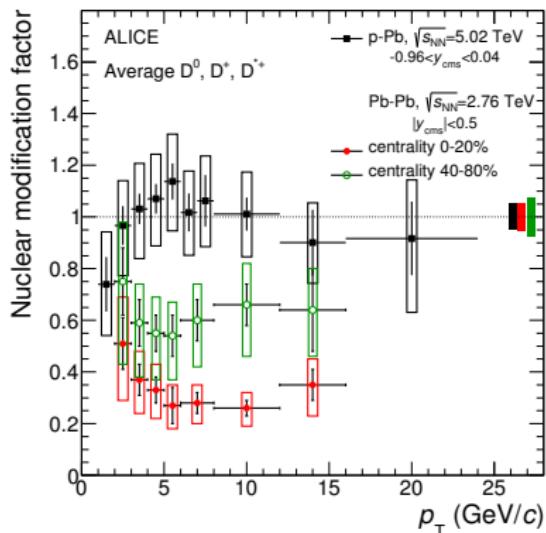
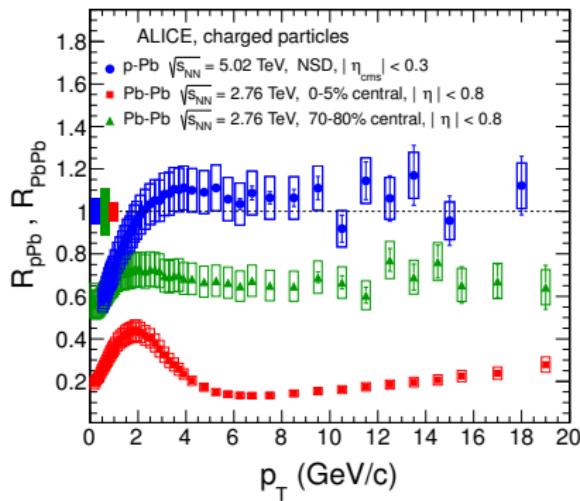


# p-A collisions

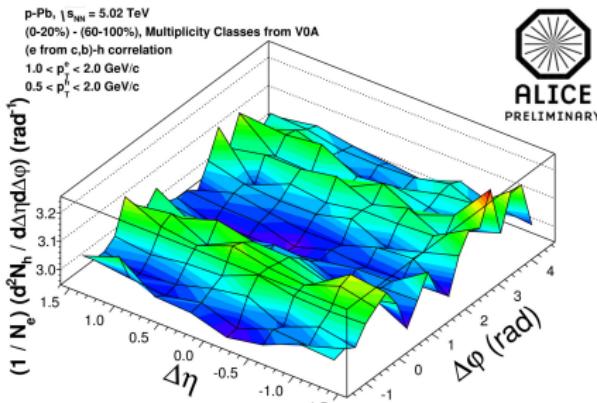
# p-A collisions: medium effects ?

**There is no evidence of hot medium effects in the nuclear modification factor...**

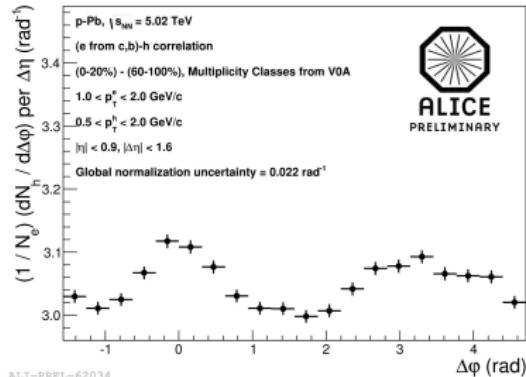
$$R_{AA} = \frac{dN_{AA}/dp_T}{N_{coll} dN_{pp}/dp_T}$$



... but a double-ridge structure appears in (heavy-flavor)e-h correlations!



ALICE-PREL-62026



ALICE-PREL-62034

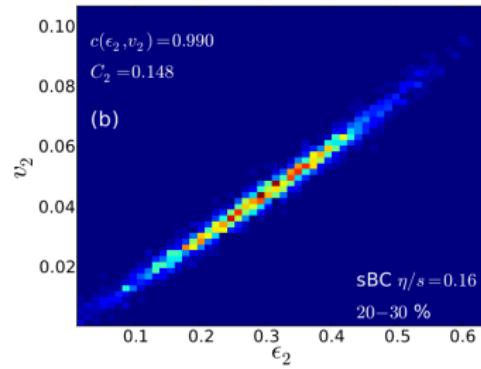
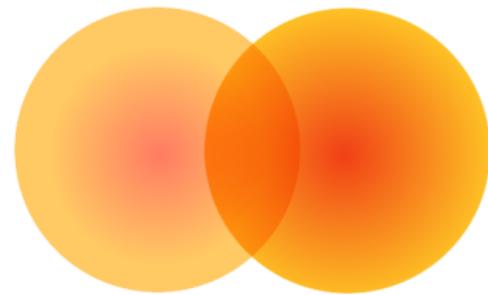
So far, experimental data don't allow one to draw firm conclusions

# Event-by-event fluctuations

Besides the finite impact parameter  $b$ , event-by-event fluctuations (e.g. in the nucleon positions) leads to an initial **eccentricity**

$$s(\vec{x}) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{coll}} \exp \left[ -\frac{(\vec{x} - \vec{x}_i)^2}{2\sigma^2} \right] \implies \epsilon_2 = \frac{\sqrt{\{y^2 - x^2\}^2 + 4\{xy\}^2}}{\{y^2 + x^2\}}$$

which translates into a non-vanishing elliptic flow



H. Niemi et al.,  
arXiv:1505.02677

$$\frac{dN}{p_t dp_t dp_z d\phi} = \frac{dN}{p_t dp_t dp_z} (1 + 2 v_2 \cos 2(\phi - \psi_2))$$

$v_2$  grows linearly with the initial eccentricity.

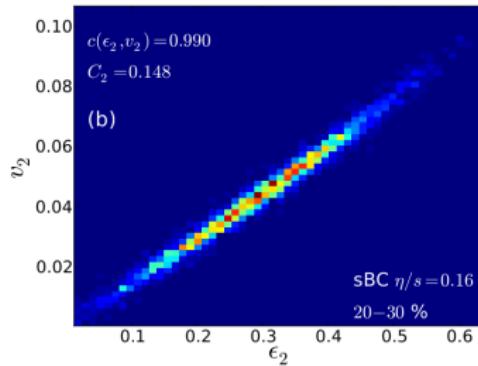
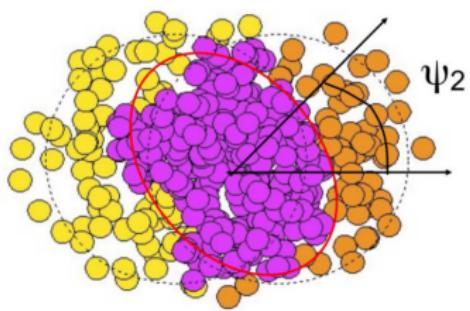
**It is crucial in p-Pb collisions to take into account these fluctuations!**

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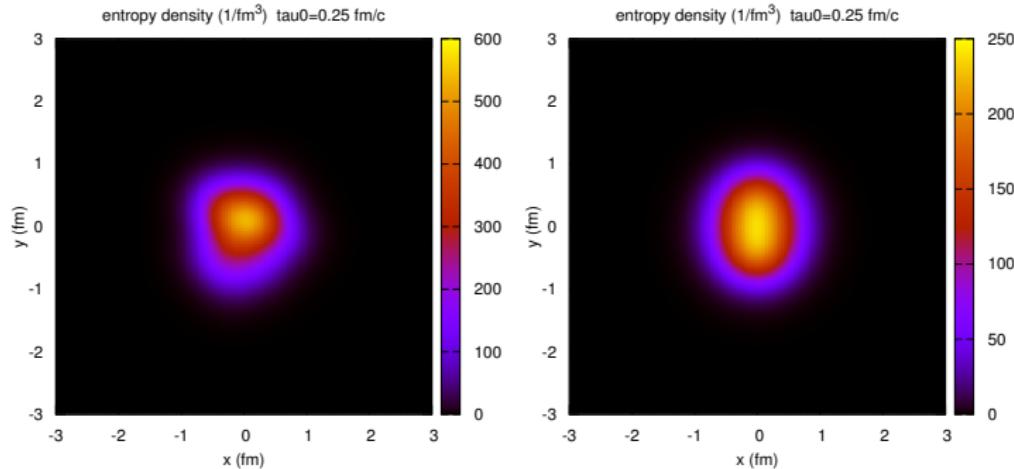
$v_2$  grows linearly with the initial eccentricity.

**It is crucial in p-Pb collisions to take into account these fluctuations!**

# Medium evolution in p-A: Relativistic Hydrodynamics

A full event-by event hydro+transport study requires huge computing resources (time and storage).

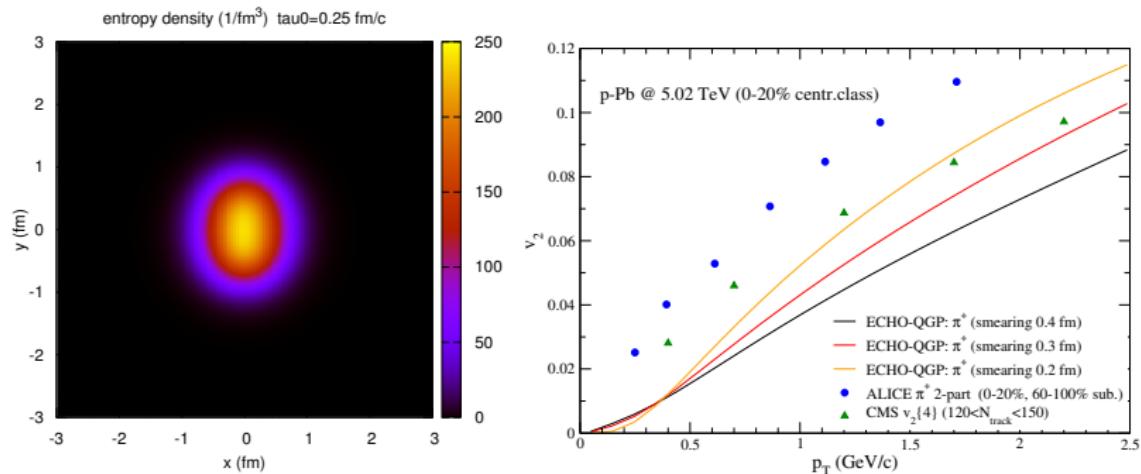
To compute  $v_2$  one can exploit the strong correlation  $v_2 \sim \epsilon_2$  considering an **average background** obtained summing all the events of a given centrality class rotated of the event-plane angle  $\psi_2$



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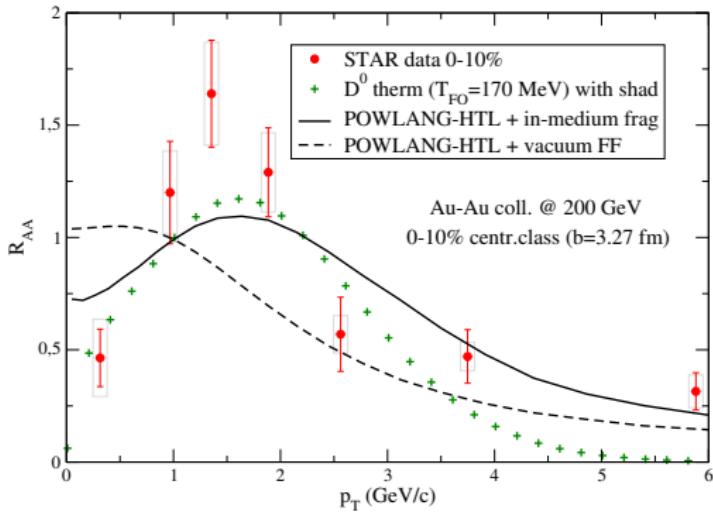


The hydrodynamical evolution in p-A is simulated with ECHO-QGP

# Results / 1

## Pb-Pb collisions at RHIC and LHC

A.Beraudo et al., Eur. Phys. J. C75 (2015) 121



The  $R_{AA}$  of  $D^0$  mesons in central Au-Au collisions at  $\sqrt{s_{NN}} = 200$  GeV.

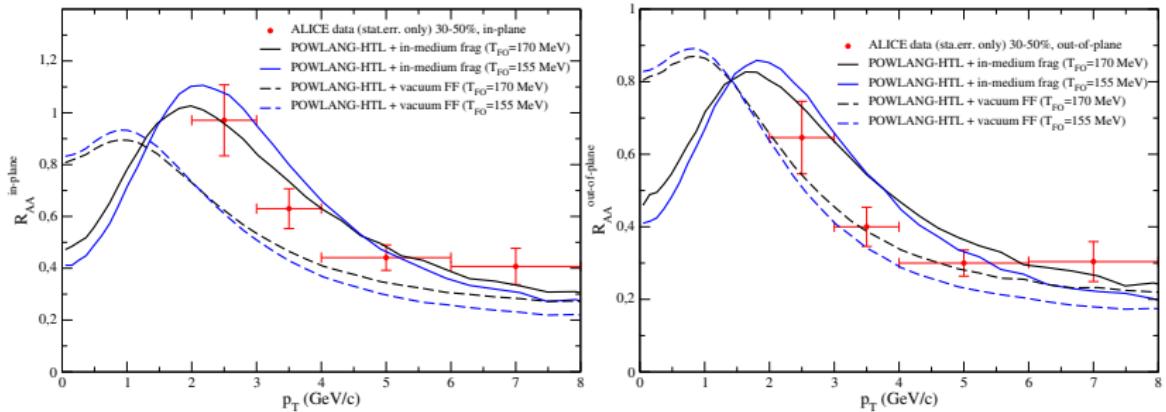
POWLNG results obtained with HTL transport coefficients and a decoupling temperature  $T_{dec} = 170$  MeV.

Also shown, for comparison, the limiting case of full kinetic thermalization of  $D$  mesons.

Theory curves are compared to STAR data <sup>9</sup>.

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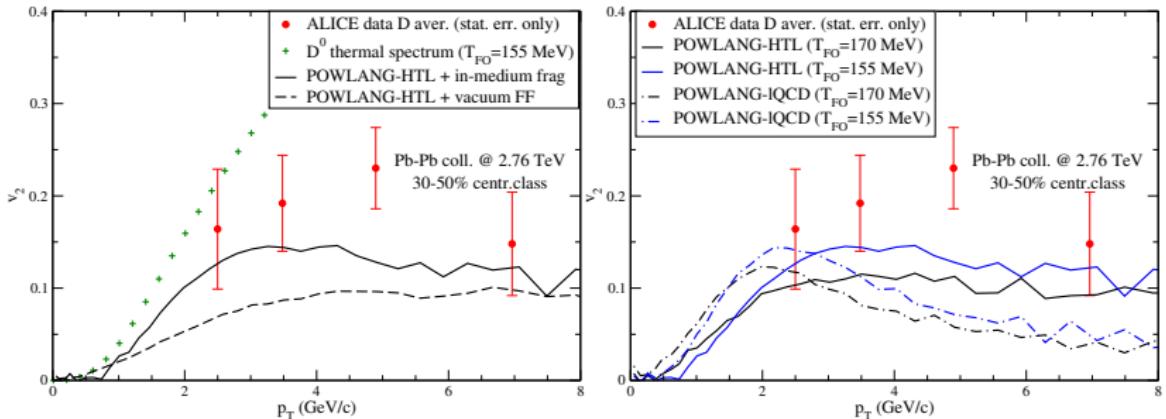
<sup>9</sup>STAR Collaboration (L. Adamczyk *et al.*), arXiv:1404.6185 [nucl-ex]



The  $R_{AA}$  in-plane (left) and out-of-plane (right) of  $D$  mesons.

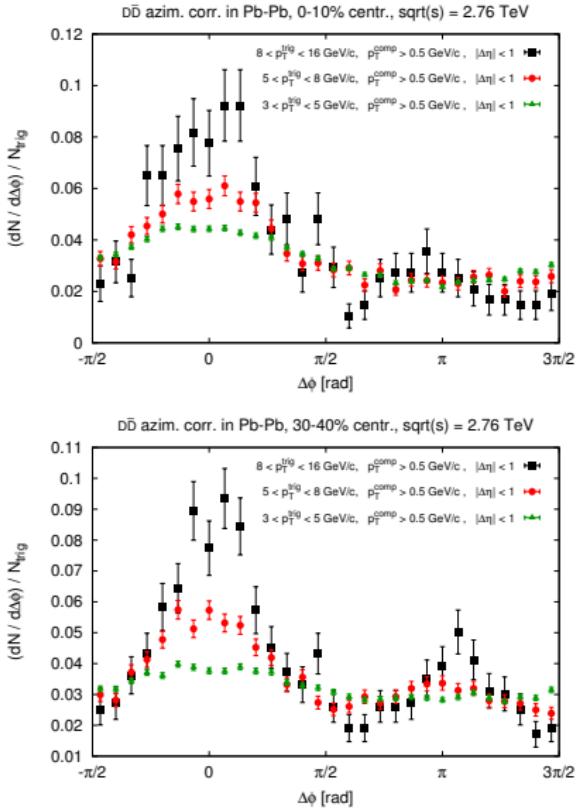
ALICE data in the 30–50% centrality class<sup>10</sup> are compared to POWLANG results obtained with different hadronization mechanisms (in-medium, solid curves, and vacuum fragmentation, dashed curves) and decoupling temperatures ( $T_{dec} = 170$ , black curves, and 155 MeV, blue curves).

<sup>10</sup>ALICE Collaboration (B. Abelev *et al.*), arXiv:1405.2001 [nucl-ex]

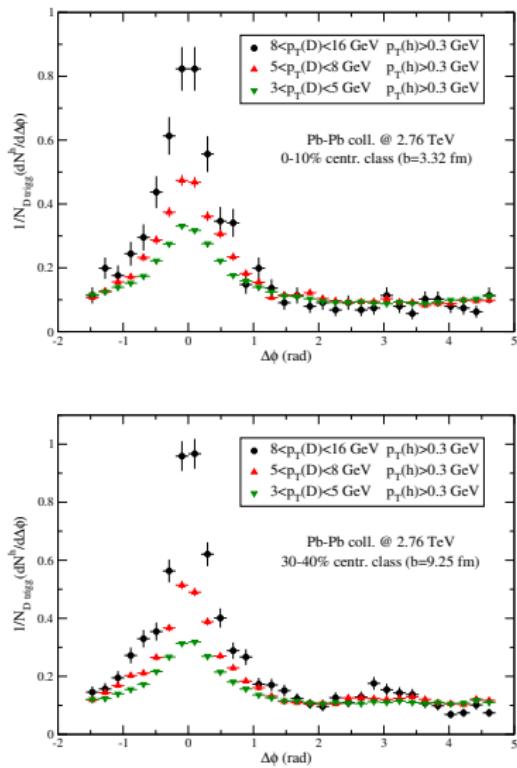


The  $v_2$  of  $D$  mesons in Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV.  
POWLANG results (with HTL transport coefficients) with in-vacuum and  
in-medium HQ fragmentation at the decoupling temperature  $T_{dec} = 155$  MeV  
compared to ALICE data in the 30-50% centrality class and to the limit of kinetic  
thermalization.

## $D-\bar{D}$ azim. correlations in Pb-Pb

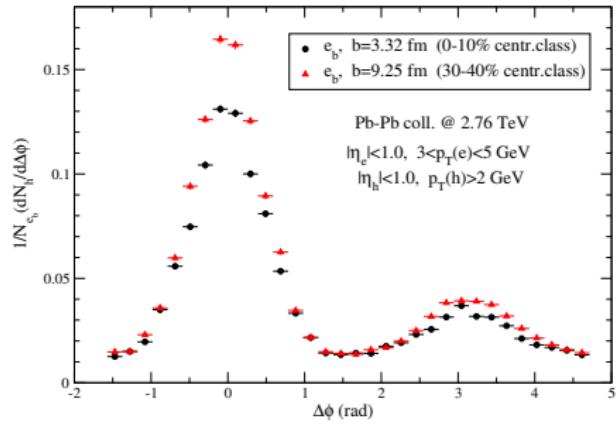
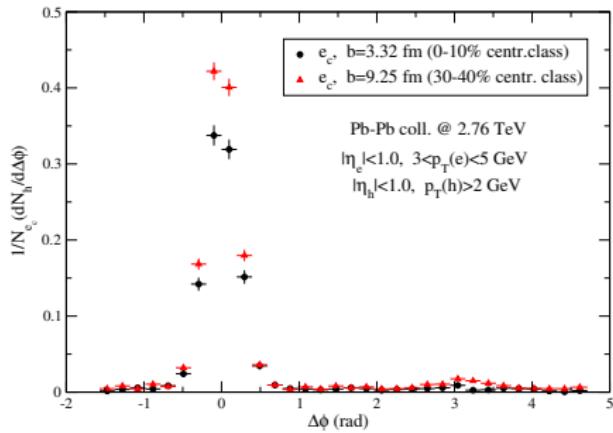


## $D-h$ azim. correlations in Pb-Pb



e's from D

e's from B  
(incl.  $B \rightarrow D \rightarrow e$ )

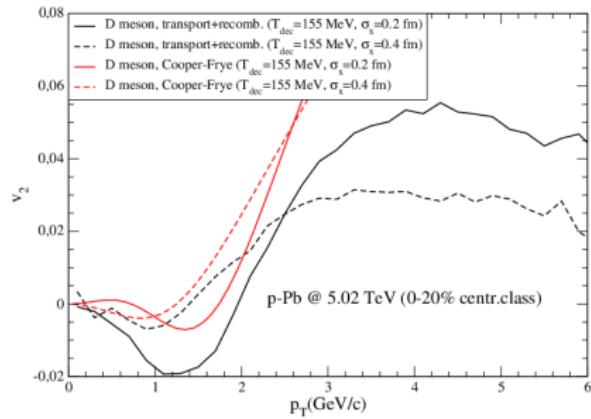
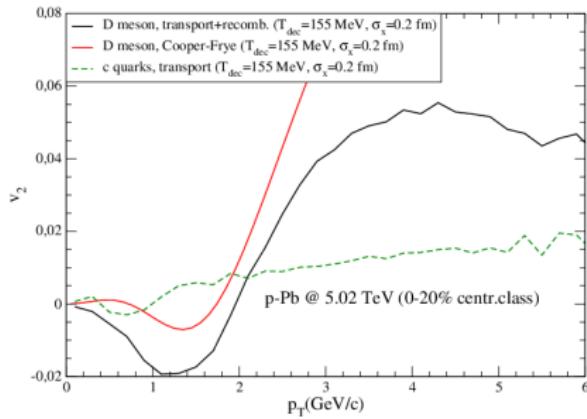


Azimuthal  $e-h$  correlations in  $Pb - Pb$  collisions at  $\sqrt{s}=2.76 \text{ TeV}$  for different centrality classes.

# Results / 2

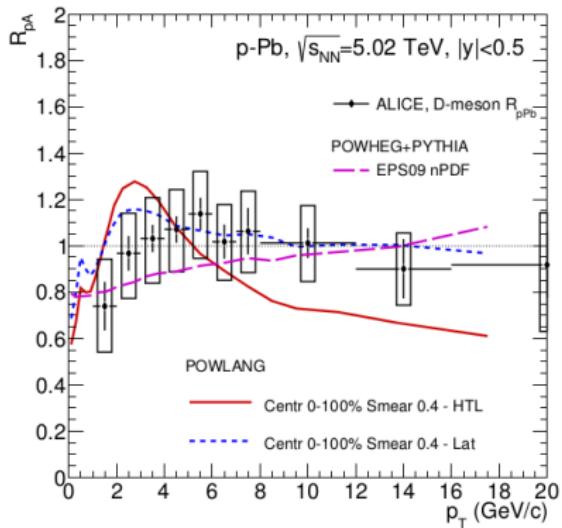
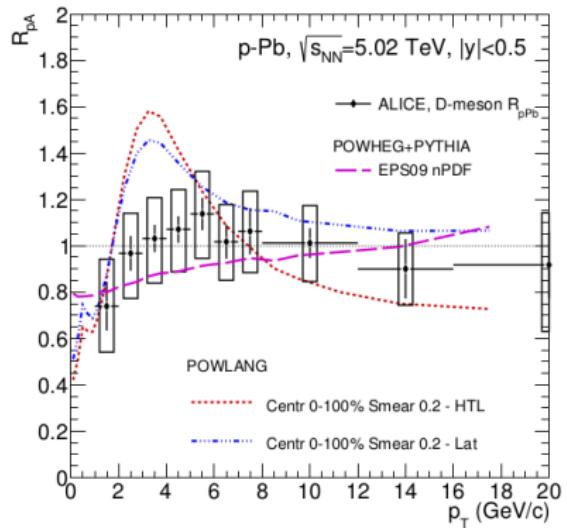
p-Pb collisions at LHC energy

# Elliptic flow coefficient in p-Pb



Preliminary results obtained with ECHO-QGP + Langevin + in-medium hadronization are shown. All the flow of D-mesons comes from the one of light partons.

# Nuclear modification factor in p-Pb



Nuclear modification factor for D mesons, in p-Pb at LHC.

# Conclusions

The implementation of a new, in-medium hadronization routine has improved the agreement between POWLANG results and the experimental data, both at RHIC and LHC energies:  $R_{AA}$  peak at small  $p_T$ , larger  $v_2$ , better in-plane out-of-plane description.

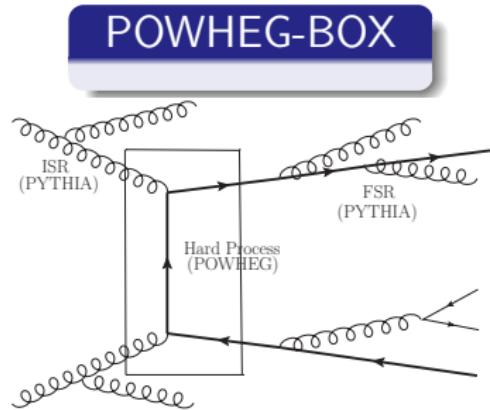
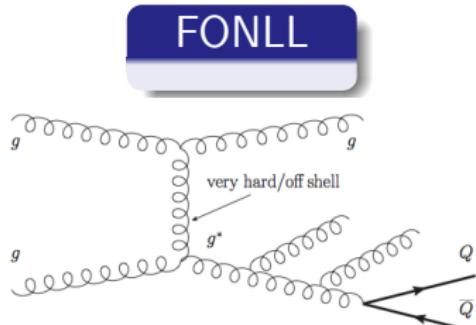
Preliminary results for p-Pb are available.

We plan to make further improvements, in particular

- we will extend our calculations at non-zero rapidity by interfacing our transport setup with the output of a viscous 3+1 hydrodynamical code (ECHO-QGP);
- we will include the study of the transport of  $D$  mesons in the hadronic phase (so far neglected).

more slides...

# FONLL vs POWHEG-BOX



- It is a calculation
- It provides NLL accuracy, resumming large  $\ln(p_T/M)$
- It includes processes missed by POWHEG (hard events with light partons)

- It is an event generator
- Results compatible with FONLL
- It is a more flexible tool, allowing to address more differential observables (e.g.  $QQ$  correlations)

## Transition rate

$$w(p, k) = \int \frac{d^3 q}{(2\pi)^3} f_g(x, p) v_{rel} \frac{d\sigma_{g+Q \rightarrow g+Q}}{d\Omega}$$

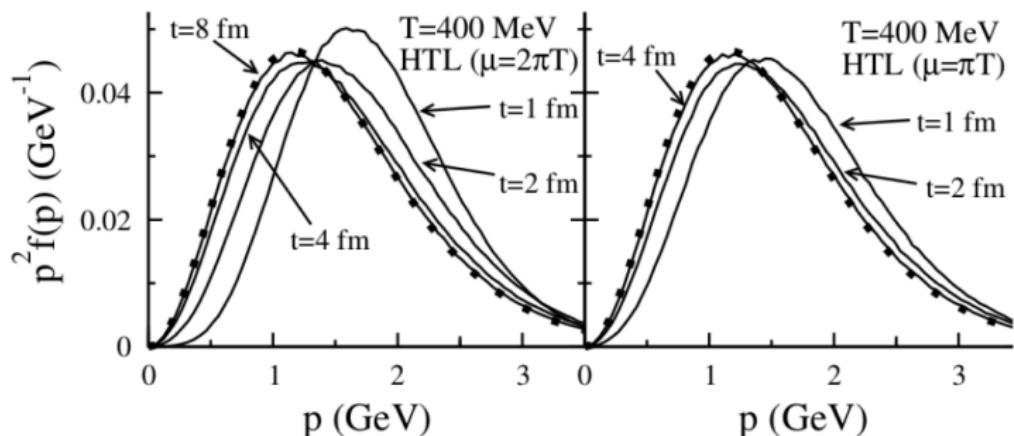
$$v_{rel} \frac{d\sigma_{g+Q \rightarrow g+Q}}{d\Omega} = \frac{1}{d_c} \frac{1}{4E_p E_q} \frac{|\mathcal{M}_{gQ}|^2}{16\pi^2 E_{p-k} E_{q+k}} \delta(E_p + E_q - E_{p-k} - E_{q+k})$$

## A check: thermalization in a static medium

For  $t \gg 1/\eta_D$  one approaches a relativistic Maxwell-Jüttner distribution<sup>11</sup>

$$f_{MJ}(p) \equiv \frac{e^{\hat{L} \tilde{S} E_p / T}}{4\pi M^2 T K_2(M/T)}, \quad \text{with} \quad \int d^3 p f_{MJ}(p) = 1$$

(Test with a sample of c quarks with  $p_0 = 2$  GeV/c and weak-coupling HTL transport coefficients)

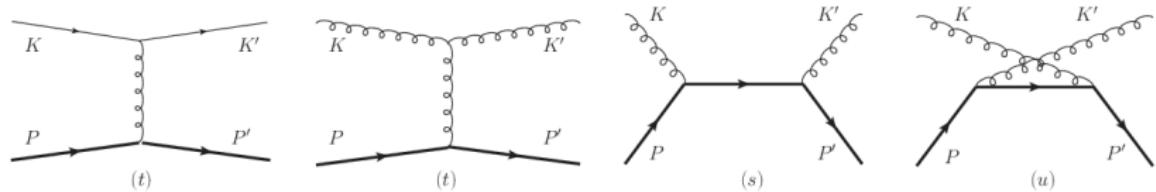


<sup>11</sup>A.Beraudo et al., NPA 831, 59 (2009)

# Perturbative $\kappa_{L/T}(p)$

The momentum broadening (and degradation) of heavy quarks in the medium must arise from their interaction with the other constituents of the plasma: light quarks and gluons.

Within a perturbative setup the lowest order diagrams to consider for the hard scattering of a heavy quark off a light (anti-)quark and a gluon are



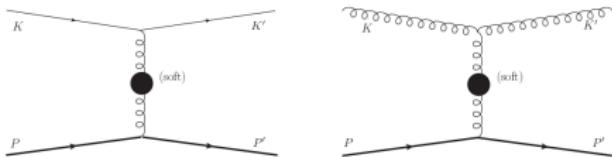
If the four-momentum exchange is sufficiently hard ( $|t| > |t|^*$ , where  $t \equiv \omega^2 - \mathbf{q}^2$ ) one is dealing with a short-distance process and the result is given by a kinetic pQCD calculation:

$$\kappa_{L,\text{hard}}^{g/q} = \frac{1}{2E} \int_k \frac{n_{B/F}(k)}{2k} \int_{k'} \frac{1 \pm n_{B/F}(k')}{2k'} \int_{p'} \frac{\theta(|t| - |t|^*)}{2E'} \\ \times (2\pi)^4 \delta^{(4)}(P + K - P' - K') |\overline{\mathcal{M}}_{g/q}(s, t)|^2 \mathbf{q}_L^2 \quad (1)$$

and

$$\kappa_{T,\text{hard}}^{g/q} = \frac{1}{2E} \int_k \frac{n_{B/F}(k)}{2k} \int_{k'} \frac{1 \pm n_{B/F}(k')}{2k'} \int_{p'} \frac{\theta(|t| - |t|^*)}{2E'} \\ \times (2\pi)^4 \delta^{(4)}(P + K - P' - K') |\overline{\mathcal{M}}_{g/q}(s, t)|^2 \frac{\mathbf{q}_T^2}{2}. \quad (2)$$

If the momentum transfer is soft ( $|t| < |t|^*$ ), the scattering involves the exchange of a long wavelength gluon, which requires the resummation of medium effects, as in



This can be done in hot-QCD within the [Hard Thermal Loop approximation](#).

Eventually, one has to sum-up the soft and hard contributions to the transport coefficients

$$\kappa_{L/T} = \kappa_{L/T}^{\text{soft}} + \kappa_{L/T}^{\text{hard}},$$

checking that the final result is not too sensitive to the artificial intermediate cutoff  $|t|^*$ .

The strong coupling  $g$  (for soft collisions) was evaluated at the scale  $\mu = 1.5\pi T$ , representing the central value of the systematic band explored in our study.

## Non-perturbative $\kappa_{L/T}(p)$

An independent way to extract the transport coefficients from the underlying microscopic theory comes from lattice QCD simulations. The results we will employ in the calculations were obtained in the static ( $m_Q \rightarrow \infty$ ,  $p \rightarrow 0$ ) limit (no information on the momentum-dependence is then available) and refer to the momentum diffusion coefficient  $\kappa$ <sup>12</sup>

We arbitrarily assume a constant value as a function of  $p_T$ .

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<sup>12</sup>A. Francis et al., PoS LATTICE2011 (2011) 202  
D. Banerjee et al., Phys.Rev. D85 (2012) 014510