Fluctuations of conserved charges in relativistic heavy ion collisions

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- Perspectives

Fluctuations as a probes of the QGP

- Non-standard observables ("the noise is the signal"), but potentially rich of information, e.g. Brownian motion and CMB (tiny fluctuations in the Temperature of the Cosmic Microwave Background).

  In RHIC observed by measuring event-by-event fluctuations, e.g. in the net-electric charge and net-proton number. However N_B is a conserved quantity, the net-proton number is not and fluctuations are affected by final state effects. Fluctuations proposed to probe the freeze-out conditions (T and μ_B).

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- Formally obtained from the cumulant of some statistical distribution and related to generalized susceptibilities χ^n:

$$\chi_B^n(T, \mu_B/T) = \frac{d^n P(T, \mu_B/T)}{d(\mu_B/T)^n} \quad (1)$$

  with: μ_B baryon-chemical potential, χ_B^n n-th susceptibility of net-baryon number and $P = -\Omega/V$ pressure.

Theoretical Models

PNJL (Nambu-Jona Lasinio with Polyakov Loop)

NJL with Polyakov loop is based on the Lagrangian (written for 3-flavor quark field):

$$\mathcal{L} = \bar{q}(iD - \hat{m})q + G\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma^5\lambda^a q)^2] + \\ + K\{\det[\bar{q}(1+\gamma^5)q] + \det[\bar{q}(1-\gamma^5)q]\} - U[\Phi[A], \bar{\Phi}[A]; T) \quad (2)$$

The covariant derivative is defined as $D^\mu = \partial^\mu - iA^\mu$, with $A^\mu = \delta^\mu_0 A^0$ (Polyakov Gauge); in Euclidean notation $A^0 = -iA^4$. The strong coupling constant G_s is absorbed in the definition of $A^\mu(x) = G_s \mathcal{A}^\mu_a(x)\frac{\lambda^a}{2}$, where $\mathcal{A}^\mu_a(x)$ is the $SU_c(3)$ gauge field.
The Polyakov loop field Φ appearing in the potential term U is related to the gauge field through the gauge covariant average of Polyakov line

$$\Phi(\vec{x}) \equiv \langle\langle l(\vec{x})\rangle\rangle = \frac{1}{N_c}\mathrm{Tr}_c\langle\langle L(\vec{x})\rangle\rangle \quad (3)$$

PNJL (cnt)

The potential for the Polyakov field:

$$\frac{\mathcal{U}(\Phi,\bar{\Phi};T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln[1-6\bar{\Phi}\Phi+4(\Phi^3+\Phi^3)-3(\bar{\Phi}\Phi)^2] \quad (5)$$

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \qquad b(T) = b_3(T_0/T)^3 \quad (6)$$

used to reproduce lattice data and QCD thermodynamics.
In pure gauge fixed to $T_0 = 270$ MeV; with other arguments (e.g. comparison with other models) $T_0 = 182$ MeV.
Treat the model in Mean Field approximation, producing self energies from the 4-fermion interaction term ($\Sigma^{(4)}$) and from the 6-fermion interaction ($\Sigma^{(6)}$):

$$\Sigma^{(4)} = 4iGN_c \mathrm{Tr}S^j, \quad (7)$$

$$\Sigma^{(6)} = K(2N_c^2+3N_c+1)(\mathrm{Tr}S^j)(\mathrm{Tr}S^k) \qquad i \neq j \neq k \quad (8)$$

Quark gap equations read

$$M_i = m_i - 4G\phi_i - 2K\phi_j\phi_k \qquad i \neq j \neq k \quad (9)$$

Fluctuations in PNJL

Gran Canonical Potential density ($\omega = \Omega/V$) in Mean Field approximation

$$\omega(\Phi, \bar{\Phi}, \phi_i; T; \mu_i) = \mathcal{U}(\Phi, \bar{\Phi}; T) + 2G \sum_{i=u,d,s} \phi_i^2 + 4K\phi_u\phi_d\phi_s$$
$$- 2N_c \sum_{i=u,d,s} \int_\Lambda \frac{d^3p}{(2\pi)^3} E_i - -2T \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} [z_\Phi^+(E_i - \mu_i) + z_\Phi^-(E_i + \mu_i)] \tag{10}$$

Φ, $\bar{\Phi}$ Polyakov Fields,
ϕ_u, ϕ_d and ϕ_s chiral condensates of up,down and strange quarks
μ_u, μ_d and μ_s are the quark chemical potentials

$$\mu_i \equiv B_i\mu_B + Q_i\mu_Q + S_i\mu_S \tag{11}$$

In the above $E_i = \sqrt{p^2 + M_i^2}$ is quasi particle energy for quark i
z_Φ^+ and z_Φ^- the partition function densities ($\beta = 1/T$)

$$\begin{aligned} z_\Phi^+(E_i - \mu_i) \equiv & \mathrm{Tr}_c \ln[1 + L^\dagger e^{-\beta(E_i-\mu)}] \\ = & \ln[1 + 3\bar{\Phi} e^{-\beta(E_i-\mu_i)} + 3\Phi e^{-2\beta(E_i-\mu_i)} + e^{-3\beta(E_i-\mu_i)}] \end{aligned} \tag{12}$$

$$\begin{aligned} z_\Phi^-(E_i + \mu_i) \equiv & \mathrm{Tr}_c \ln[1 + L^\dagger e^{-\beta(E_i+\mu)}] \\ = & \ln[1 + 3\Phi e^{-\beta(E_i+\mu_i)} + 3\bar{\Phi} e^{-2\beta(E_i+\mu_i)} + e^{-3\beta(E_i+\mu_i)}] \end{aligned} \tag{13}$$

NB in PNJL the Fermi-Dirac distribution function is modified to

$$f_\Phi^+(E_i - \mu_i) = \frac{\bar{\Phi} e^{-\beta(E_i-\mu_i)} + 2\Phi e^{-2\beta(E_i-\mu_i)} + e^{-3\beta(E_i-\mu_i)}}{1 + 3\bar{\Phi} e^{-\beta(E_i-\mu_i)} + 3\Phi e^{-2\beta(E_i-\mu_i)} + e^{-3\beta(E_i-\mu_i)}} \tag{14}$$

and analogous for $f_\Phi^-(E_i + \mu_i)$.

Variance, skewness, kurtosis

From susceptibilities (again)

$$\chi_B^n(T, \mu_B/T) = \frac{d^n P(T, \mu_B/T)}{d(\mu_B/T)^n} \tag{15}$$

one can define variance

$$\sigma^2 = \chi^2$$

skewness

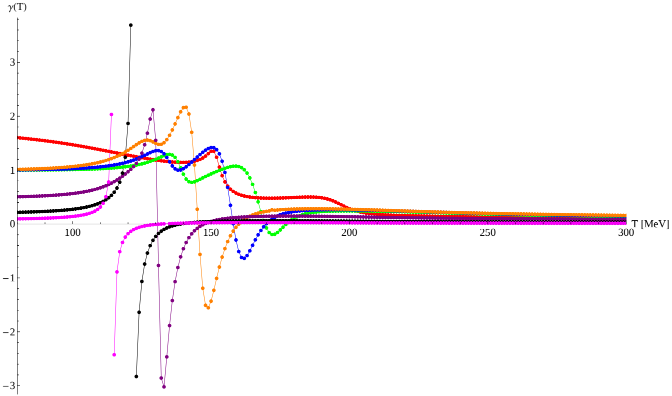
$$\gamma = \frac{\chi^3}{\chi^2}$$

kurtosis

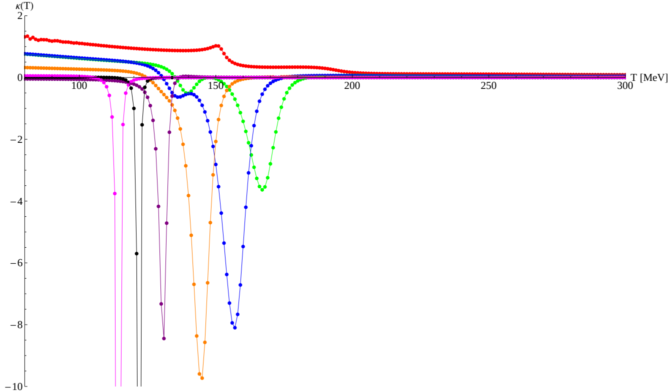
$$\kappa = \frac{\chi^4}{\chi^2}$$

NB discontinuities/divergencies of these quantities should signal Critical End Point temperature.

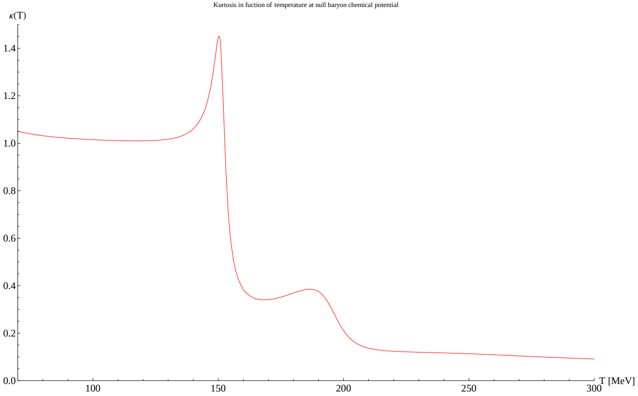
Skewness (results at finite μ_B)



Kurtosis (results at finite μ_B)



Kurtosis (result at $\mu_B = 0$)



Hadron Resonance Gas Model

Partition function of the HRG model can be split into mesonic and baryonic contributions,

$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^{M}_{M_i}(T,V,\mu_Q,\mu_S) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^{B}_{M_i}(T,V,\mu_B,\mu_Q,\mu_S)\ ,$$

where (particle species i with mass M_i)

$$\ln \mathcal{Z}^{M/B}_{M_i} = \mp\frac{Vd_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1\mp z_i e^{-\varepsilon_i/T}) = \frac{VT^3}{2\pi^2} d_i \left(\frac{M_i}{T}\right)^2 \sum_{k=1}^{\infty}(\pm 1)^{k+1}\frac{z_i^k}{k^2}K_2(kM_i/T) \quad .$$

Upper signs correspond to mesons and lower signs to baryons; $\varepsilon_i = \sqrt{k^2+M_i^2}$ is the energy of particle i, d_i its degeneracy factor and its fugacity is given by $z_i = \exp\left((B_i\mu_B + Q_i\mu_Q + S_i\mu_S)/T\right)$.

HRG (cnt.)

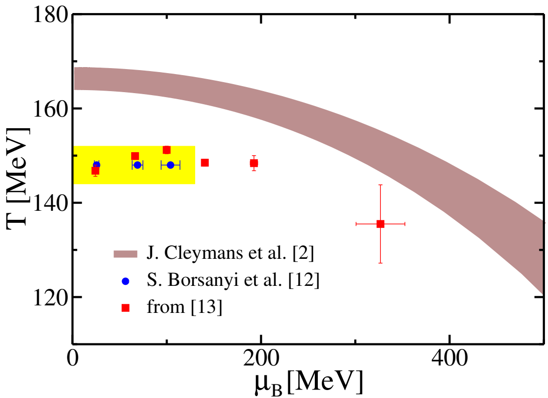
Fluctuations of conserved charges and their correlations in a thermalized medium are given by derivatives evaluated at $\vec{\mu} = (\mu_B, \mu_Q, \mu_S) = 0$,

$$\hat{\chi}_2^X \equiv \frac{\chi_2^X}{T^2} = \left.\frac{\partial^2 p/T^4}{\partial \hat{\mu}_X^2}\right|_{\vec{\mu}=0}, \tag{16}$$

$$\hat{\chi}_{11}^{XY} \equiv \frac{\chi_{11}^{XY}}{T^2} = \left.\frac{\partial^2 p/T^4}{\partial \hat{\mu}_X \partial \hat{\mu}_Y}\right|_{\vec{\mu}=0}, \tag{17}$$

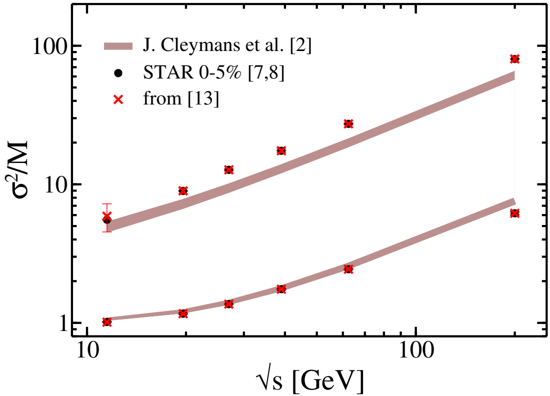
with $\hat{\mu}_X \equiv \mu_X/T$ and $X,\ Y = B,\ Q,\ S$.

Freeze-out conditions from HRG (I)



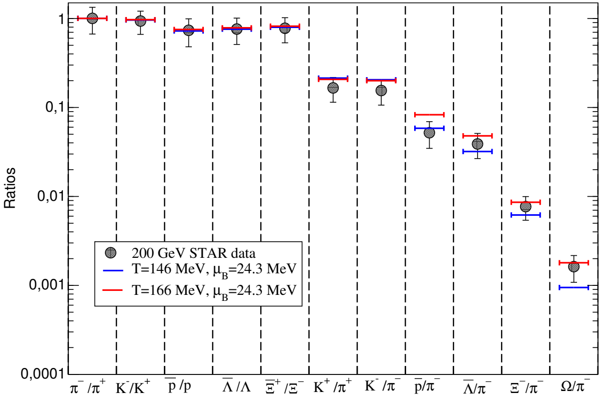
Comparison of freeze-out parameters ($\mu_{B,ch}$, T_{ch}). The curved band is a summary of freeze-out conditions obtained from various SHM fits. The horizontal band and the circles are the lattice QCD results (from Borsanyi et al.2014) The squares show freeze-out conditions found by Alba et al. (2014)

Freeze-out conditions from HRG (II)



Comparison of the experimental data with model results for the cumulant ratio σ^2/M of net-electric charge (top) and net protons (bottom) as a function of $\sqrt{s}$: the circles show the efficiency corrected STAR data for most central collisions and the crosses the results of Alba et al. (2014).

Freeze-out conditions from HRG (III)



Comparison between STAR particle ratio data for central events at $\sqrt{s} = 200$ GeV and HRG model results for the specified chemical freeze-out parameters. frem Alba et al. PLB738 (2014).

Conclusions and perspectives

- Fluctuations of conserved charges contain important informations on properties of the QGP and on the yet unknown phase transition toward hadronization
- Quite encoraging results from several well-known effective models (HRG, PNJL, PQM)
- Work in progress to compare PNJL and PQM (Polyakov Quark Meson model), and their outcomes for fluctuations.
- Recently brought to attention the relevance of isoentropic lines.