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Nucleon Electromagnetic Form Factors on GPDs

Presented by

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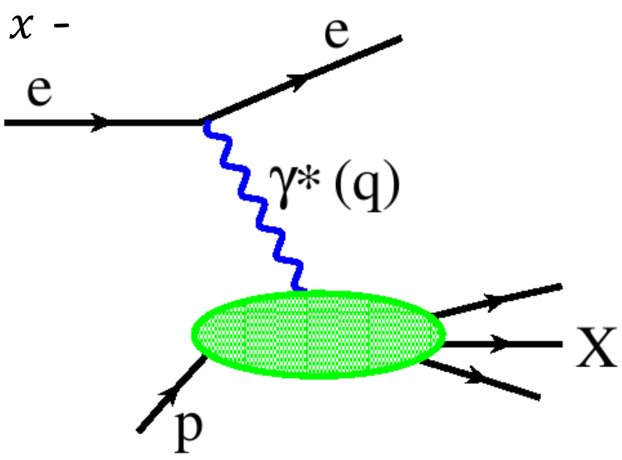
- ✓ Most of what we know on the structure of the nucleon has come from the scattering of high energy leptons. This is due to their structureless nature, their well-known and quantified electromagnetic interaction with matter.

1. Inclusive scattering
   Only the scattered lepton is detected and the target nucleon is unobserved.
2. Exclusive scattering
   The final hadronic state of the reaction is fully determined.

Deep-Inelastic scattering

In DIS, $f_1(x)$ is related to the unpolarized x -momentum distributions of the partons $q(x)$.



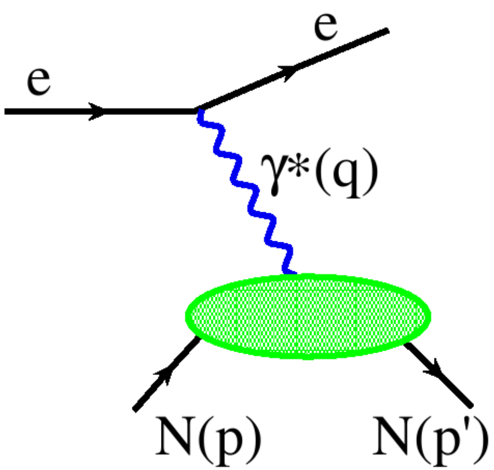
$$f_1(x) = \frac{1}{2}\sum_q e_q^2 q(x)$$

The non-perturbative parton distribution functions (PDFs) is defined via matrix elements of parton operators between nucleon states with equal momenta:

$$q(x) = \frac{p^+}{4\pi}\int dy^- e^{ixp^+y^-} \langle p|\bar{\psi}_q(0)\gamma^+\psi_q(y)|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$

Elastic scattering

The FFs are related to the following QCD matrix element in space-time coordinates



$$\langle p' \,|\bar{\psi}_q(0)\gamma^+\psi_q(0)|p\rangle$$
$$= F_1^q(t)\bar{N}(p')\gamma^+N(p) + F_2^q(t)\bar{N}(p')i\sigma^{+\upsilon}\frac{\Delta_\upsilon}{2m_N}N(p),$$

$F_1^q(t)$ → Dirac form factor

$F_2^q(t)$ → Pauli form factor

Sachs form factors

$$G_E(t) = F_1(t) + \frac{-t}{4m^2{}_N}F_2(t)$$

$$G_M(t) = F_1(t) + F_2(t)$$

Generalized Parton Distributions

Like usual PDFs, GPDs are non-perturbative functions defined via the matrix elements between nucleon states with different momenta [1-3]:

$$\frac{P^+}{2\pi}\int dy^- e^{ixP^+y^-}\langle p'|\bar{\psi}_q(0)\gamma^+\psi_q(y)|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$

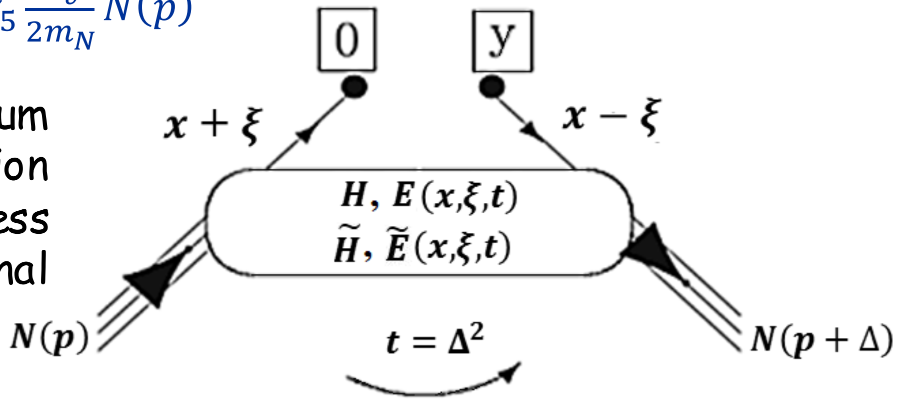
$$= H_q(x,\xi,t)\bar{N}(p')\gamma^+N(p) + E_q(x,\xi,t)\bar{N}(p')i\sigma^{+\upsilon}\frac{\Delta_\upsilon}{2m_N}N(p)$$

[1] D. Muller, et.al Phys. **42**, 101 (1994).
[2] X. D. Ji, Phys. Rev. Lett. **78**, 610 (1997).
[3] A. V. Radyushkin, Phys. Rev. D **56**, 5524 (1997).

$$\frac{P^+}{2\pi}\int dy^- e^{ixP^+y^-}\langle p'|\bar{\psi}_q(0)\gamma^+\gamma_5\psi_q(y)|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$

$$=\tilde{H}_q(x,\xi,t)\bar{N}(p')\gamma^+\gamma_5N(p) + \tilde{E}_q(x,\xi,t)\bar{N}(p')\gamma_5\frac{\Delta_\upsilon}{2m_N}N(p)$$

GPDs depend on the squared 4-momentum transfer t, on the average momentum fraction x of the active quark, and on the skewness parameter ξ, which is the longitudinal momentum transfer.



Relation between GPDs, PDFs, and FFs

- Traditionally, elastic form factors and parton distribution functions (PDFs) were considered totally unrelated;
- Elastic form factors give information on the charge and magnetization distributions in transverse plane.
- PDFs describe the distribution of partons in the longitudinal direction.

Forward limit $(p = \acute{p}, t = 0)$ $\longrightarrow$ $H_q(x,0,0) = \begin{cases} q(x) & x > 0 \\ -\bar{q}(-x) & x < 0 \end{cases}$

✓ At finite momentum transfer, there are model independent sum rules which relate the first moments of the GPDs to the elastic form factors

$$\int_{-1}^{+1} dx\, H^q(x,\xi,t) = F_1^q(t) \qquad \int_{-1}^{+1} dx\, E^q(x,\xi,t) = F_2^q(t)$$

$$F_1^u(0) = 2 \quad F_1^d(0) = 1 \qquad F_2^u(0) = \kappa_u \quad F_2^d(0) = \kappa_d$$

$$F_1(t) = \sum_q e_q F_1^q(t) \qquad F_2(t) = \sum_q e_q F_2^q(t)$$

$$F_1^p(0) = 1 \quad F_1^n(0) = 0 \qquad F_2^p(0) = 1.79 \quad F_2^n(0) = -1.91$$

If the momentum transfer $t = \Delta_\perp^2$ is transverse, then ξ=0. The integration region can be reduced to the interval $0 < x < 1$. So, We can rewrite $F_1(t)$ and $F_2(t)$ as follow:

$$F_1(t) = \sum_q e_q \int_0^1 dx \mathcal{H}^q(x,t) \qquad F_2(t) = \sum_q e_q \int_0^1 dx \mathcal{E}^q(x,t)$$

In some studies [4-6], the dependence on t and x for $\mathcal{H}^q(x,t)$ has been considered separately

$$\mathcal{H}^q(x,t) = q_v(x)\, G(t)$$

[4] M. Guidal *et al.*, Phys. Rev. D **72**, 054013 (2005).
[5] A. V. Radyushkin, Phys. Rev. D 58, 114008 (1998).
[6] O. V. Selyugin, Phys. Rev. D 89, 093007 (2014).

- ✓ Our goal is to calculate electromagnetic form factors in the region of high momentum transfers.
- ✓ As yet, the most common ansatzes used in literature are Gaussian and Regge ansatzes for calculating the electromagnetic form factors.

$$\mathcal{H}^q(x,t) = q_v(x) e^{\frac{(1-x)t}{4x\lambda^2}} \longrightarrow \textbf{Gaussian ansatz}$$

$$\mathcal{H}^q(x,t) = q_v(x) x^{-\alpha' t} \longrightarrow \textbf{Regge ansatz}$$

- ✓ These ansatzes can reproduce experimental electromagnetic form factors in the region of low and medium momentum transfers.

✓ However, by considering Modified Gaussian and Extended Regge ansatzes we can reproduce experimental form factors also in the region of high momentum transfers.

$$\mathcal{H}(x,t) = q(x)\, exp\left(\alpha t \frac{(1-x)^2}{x^m}\right) \longrightarrow$$ **Modified Gaussian ansatz**

$$\mathcal{H}(x,t) = q(x) x^{-\alpha' t(1-x)} \longrightarrow$$ **Extended Regge ansatz**

❑ In the case of the Pauli form factor, $F_2\,(t)$, we take the same parameterization for $\mathcal{E}^q(x,t)$, which is given by:

$$\mathcal{E}^q(x,t) = \mathcal{E}^q(x) x^{-\alpha'(1-x)t}$$ and

$$\mathcal{E}^q(x,t) = \mathcal{E}^q(x) \exp[\alpha \frac{(1-x)^2}{x^m} t]$$

for extended Regge and modified Gaussian ansatzes, respectively.

The experimental proton Pauli form factor at large t exhibits a faster reduction than Dirac form factor. In order to yield a faster reduction with t, one has to multiply an additional power of $(1-x)$ to $\mathcal{E}(x)$. Thus, we have

$$\mathcal{E}^u(x) = \frac{\kappa_u}{N_u}(1-x)^{\eta_u} u_v(x) \qquad \text{and}$$

$$\mathcal{E}^d(x) = \frac{\kappa_d}{N_d}(1-x)^{\eta_d} d_v(x)$$

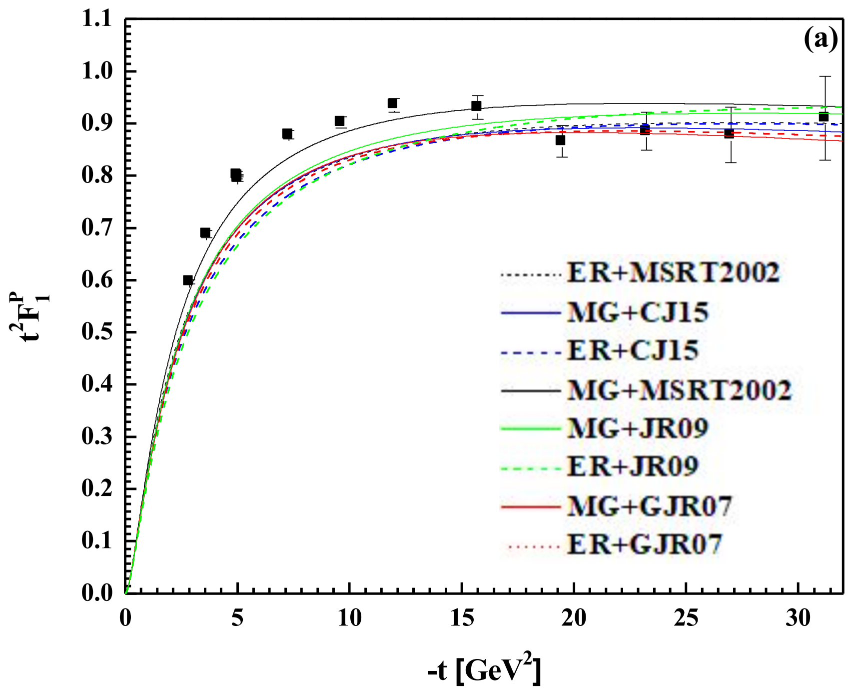
- ✓ In some previous studies [4, 7], Dirac and Pauli form factors were extracted by choosing **MSRT 2002** PDFs.
- ✓ In Ref. [8], we used **CJ15** , **JR09** , and **GJR07** PDFs to extract the Dirac and Pauli form factors.

[7] O. V. Selyugin and O. V. Teryaev, Phys. Rev. D **79**, 033003 (2009).
[8] Negin Sattary Nikkhoo and M. R. Shojaei, Phys. Rev. C 97, 055211 (2018).

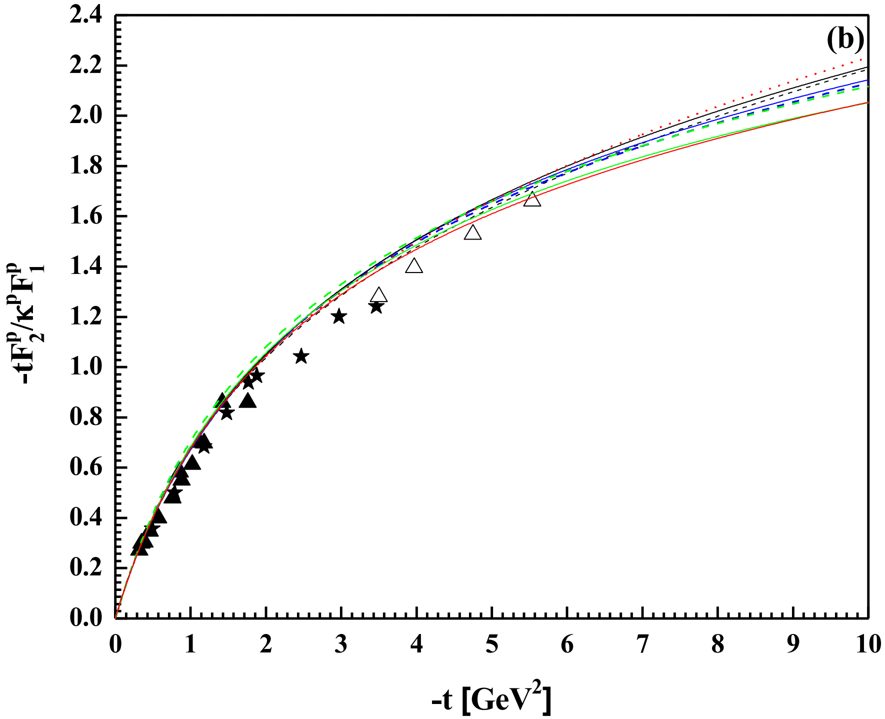
Modified Gaussian (MG)	α	η_u	η_d	**m**
MSRT2002	$1.1\ GeV^{-2}$	1.71	0.56	0.4
CJ15	$1.09 GeV^{-2}$	1.41	0.61	0.39
JR09	$1.25 GeV^{-2}$	1.89	-0.11	0.42
GRJ07	$1.345\ GeV^{-2}$	1.7	0.35	0.36

Extended Regge (ER)	α'	η_u	η_d
MSRT2002	$1.105\ GeV^{-2}$	1.713	0.556
CJ15	$1.05\ GeV^{-2}$	1.56	0.19
JR09	$1.22\ GeV^{-2}$	1.51	0.31
GRJ07	$1.29\ GeV^{-2}$	1.84	-0.05

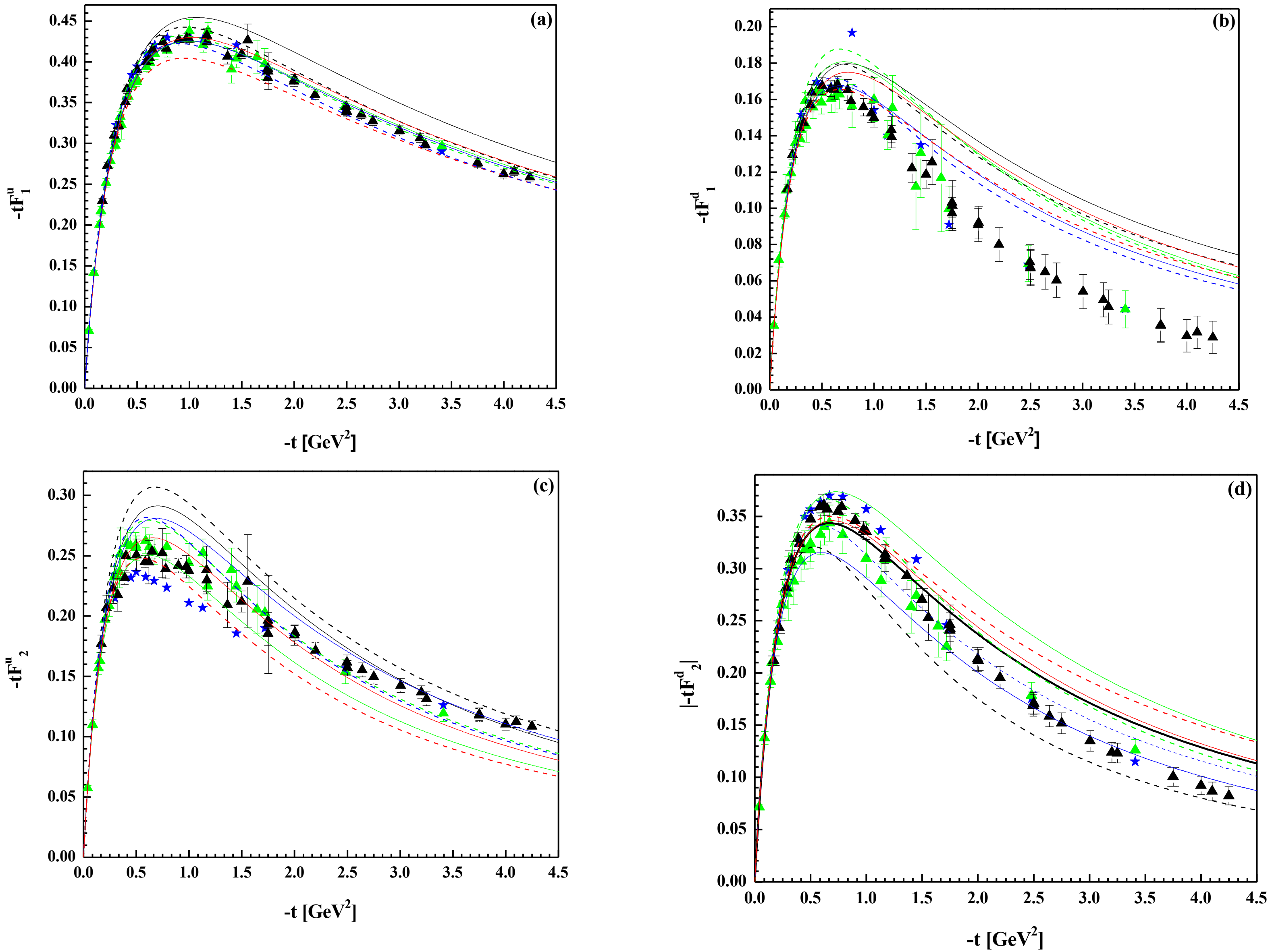
The proton Dirac form factor F_1 multiplied by t^2



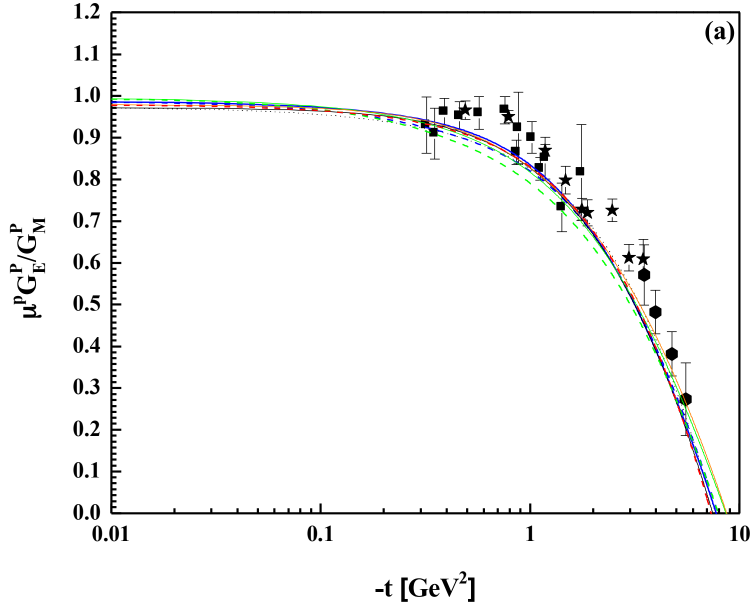
The ratio of Pauli to Dirac form factor



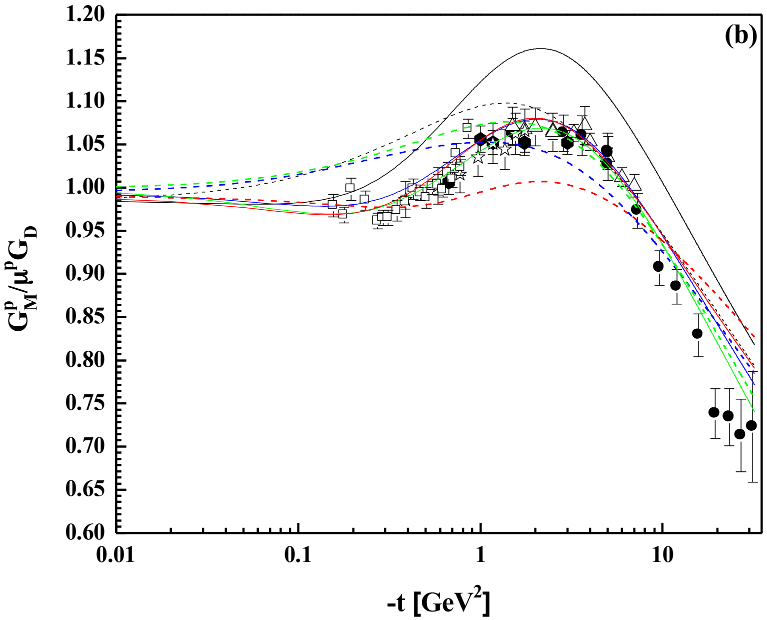
F_1 and F_2 form factors of u and d quarks



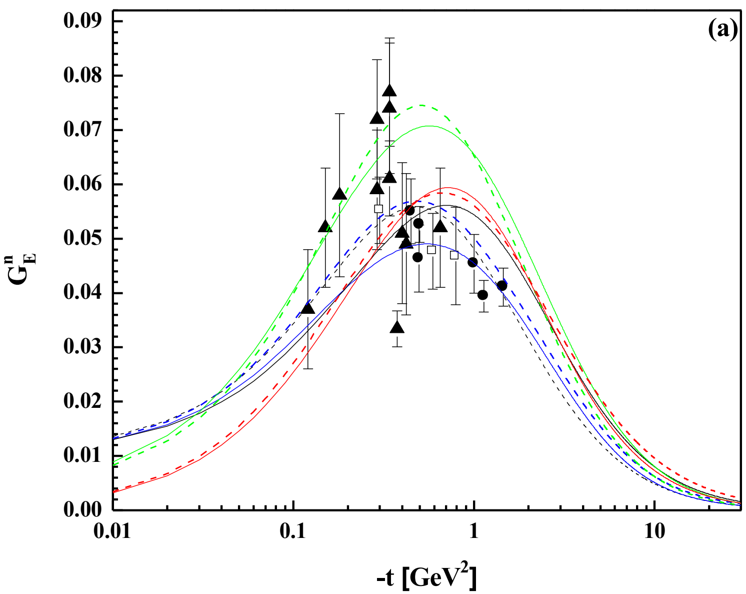
The ratio of the proton electric and magnetic form factors



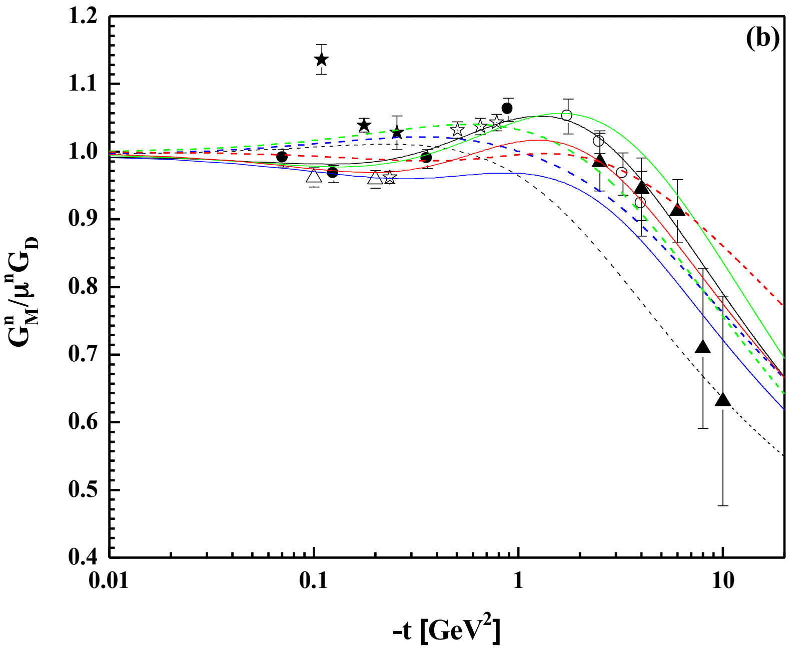
The ratio of the proton magnetic and dipole form factors



The neutron electric form factor



The ratio of the neutron magnetic and dipole form factors



Nucleon electric mean squared radius

The electric mean squared radius of proton and neutron are determined as

$$r_{E,N}^2 = r_{1,N}^2 + \frac{3}{2}\frac{\kappa_N}{M_N^2}$$

$6\left.\frac{dG_E^N}{dt}\right|_{t=0}$ $\quad$ $6\left.\frac{dF_1^N}{dt}\right|_{t=0}$ $\quad$ Foldy term

The Dirac mean radii squared is calculated both with the Extended Regge ansatz:

$$r_{1,p}^2 = -6\alpha' \int_0^1 dx[e_u u_v(x) + e_d d_v(x)](1-x)\ln(x)\,,$$

$$r_{1,n}^2 = -6\alpha' \int_0^1 dx[e_u d_v(x) + e_d u_v(x)](1-x)\ln(x)\,,$$

and with the Modified Gaussian ansatz:

$$r_{1,p}^2 = -6\alpha \int_0^1 dx[e_u u_v(x) + e_d d_v(x)]\frac{(1-x)^2}{x^m}\,,$$

$$r_{1,n}^2 = -6\alpha \int_0^1 dx[e_u d_v(x) + e_d u_v(x)]\frac{(1-x)^2}{x^m}\,.$$

The proton and neutron electric radius

	$r_{E,p}$	$r^2_{E,n}$
Experimental data	0.877 fm	$-0.1161\ fm^2$
ER+MSRT2002	0.818 fm	$-0.101\ fm^2$
ER+CJ15	0.839 fm	$-0.1055\ fm^2$
ER+JR09	0.857 fm	$-0.1401\ fm^2$
ER+GRJ07	0.871 fm	$-0.1118\ fm^2$
MG+MSRT2002	0.866 fm	$-0.0896\ fm^2$
MG+CJ15	0.899 fm	$-0.1003\ fm^2$
MG+JR09	0.943 fm	$-0.1559\ fm^2$
MG+GRJ07	0.897 fm	$-0.1070\ fm^2$

Thanks for your attention