The Type III See-Saw:

the model and updated constraints

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$\Rightarrow$ we need more particles

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Dirac mass: $$-\overline{\nu_L} m_D \nu_R + \text{h.c.}$$

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See-saw mechanism: 1 flavour

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$$\mathcal{L}_M = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{h.c.}$$

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which ones?

$\Rightarrow$ we look at effective operators

dimension 5: Weinberg operator

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$$\mathcal{L}_W = \frac{1}{2} c^{d=5}_{\alpha\beta} \left(\overline{\ell^c_{L\alpha}} \widetilde{\phi}^* \right) \left(\widetilde{\phi}^\dagger \ell_{L\beta} \right) + \text{h.c.}$$

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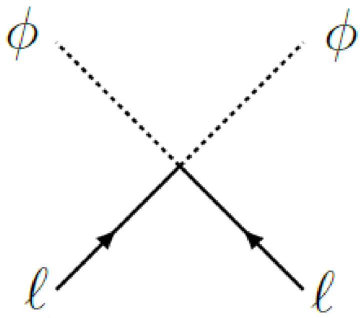

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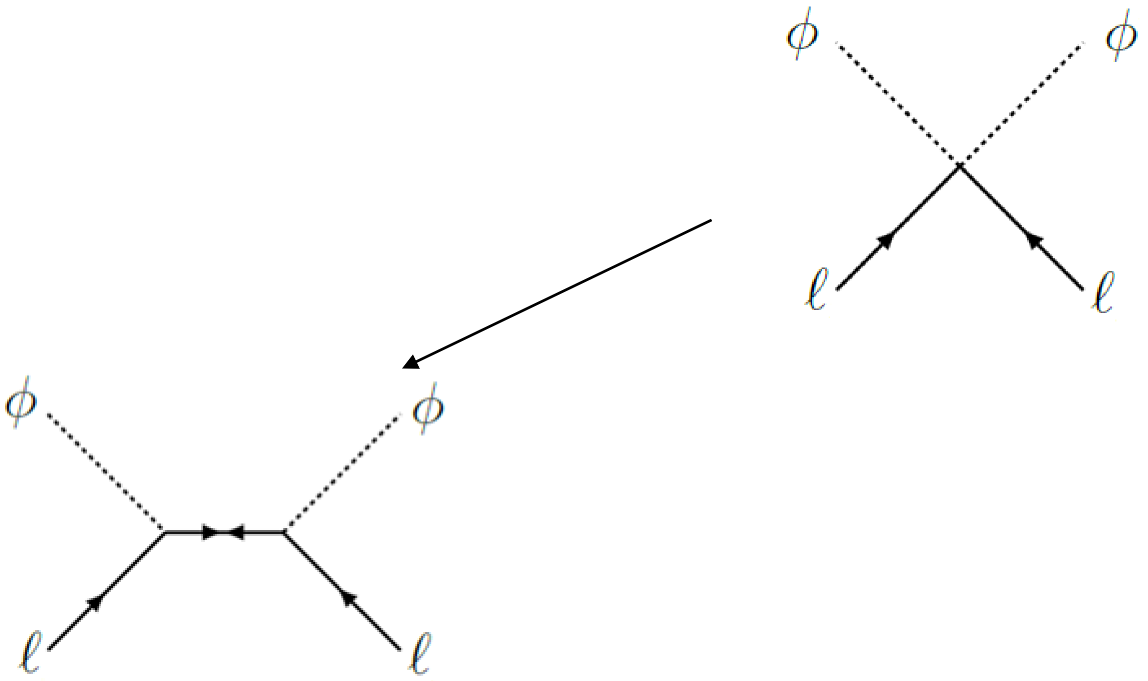

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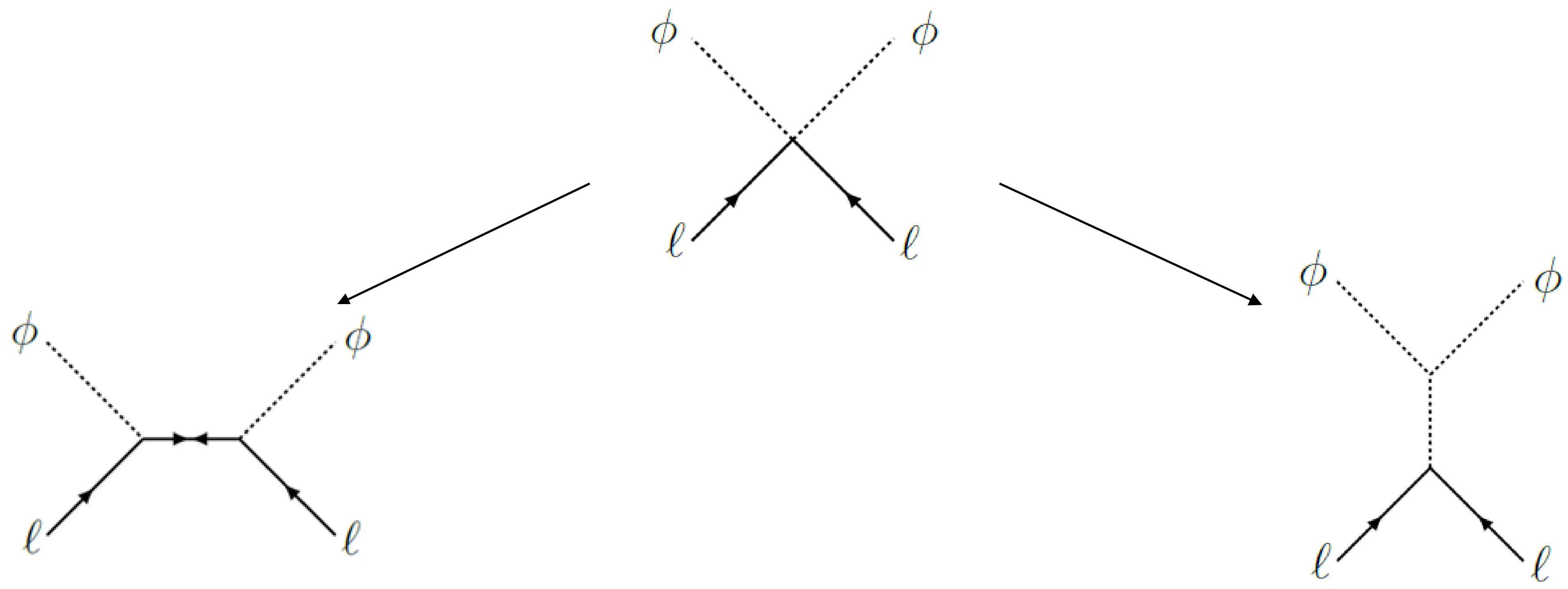

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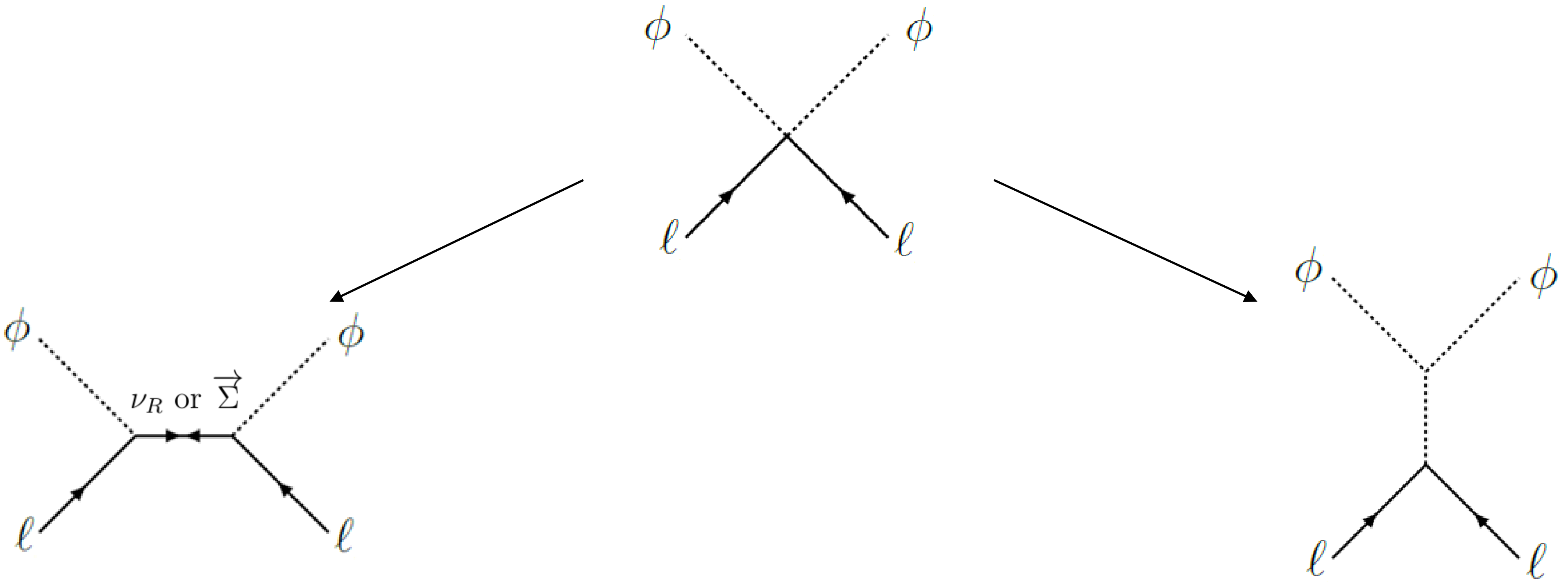

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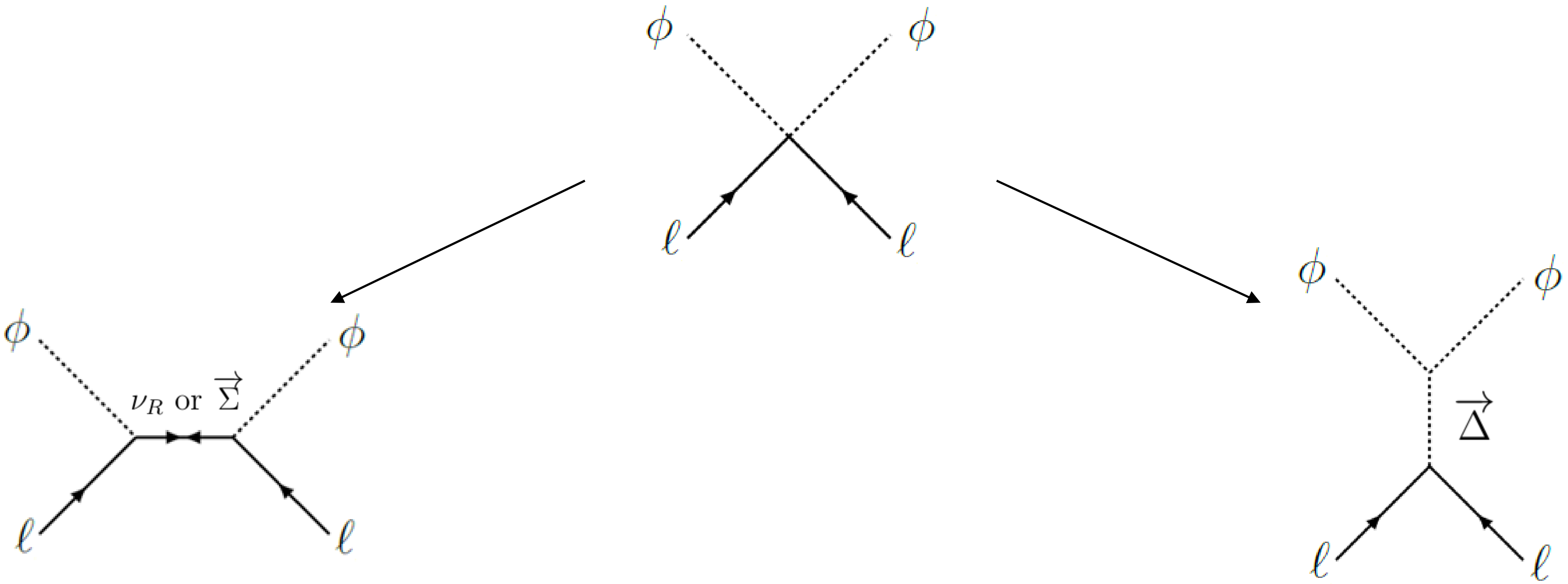

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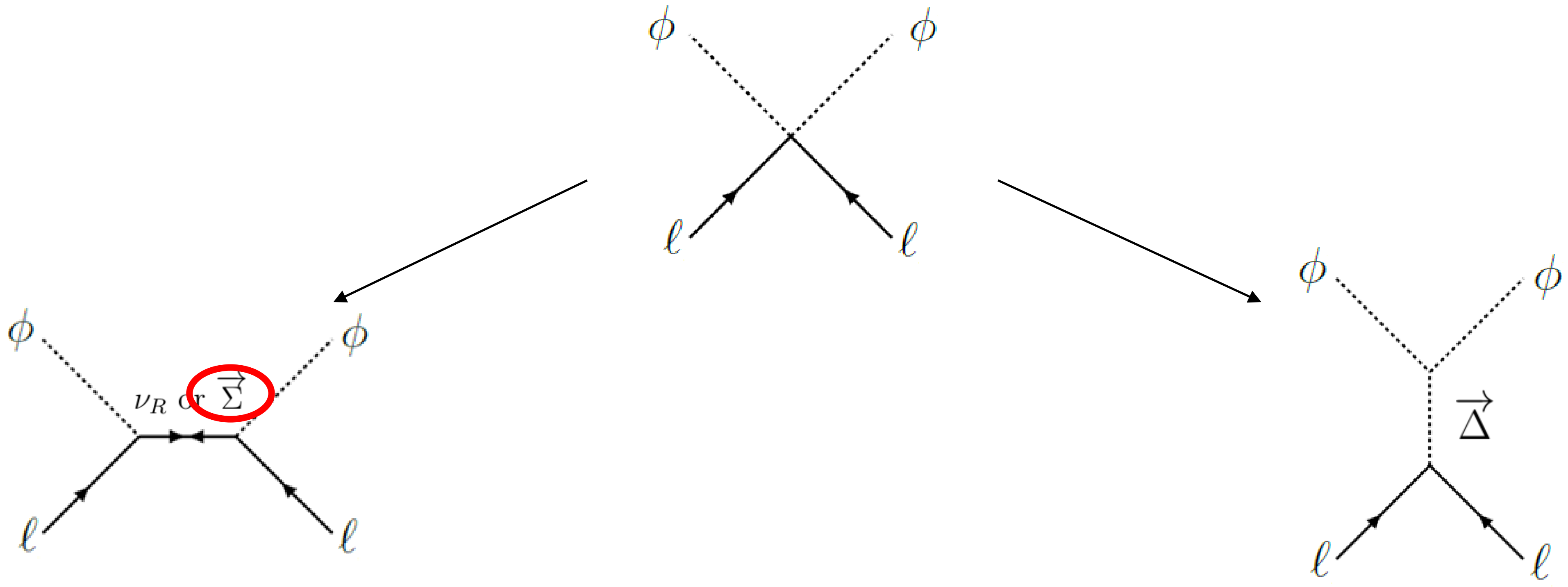

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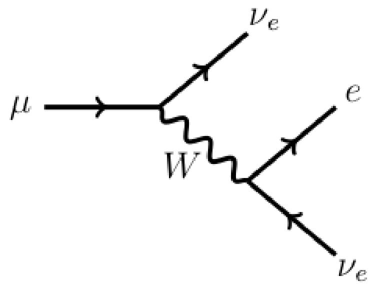

$$\Longrightarrow G_F^{\mu} = G_F^{SM} \sqrt{(NN^\dagger)_{ee}\,(NN^\dagger)_{\mu\mu}}$$

measured through

$\mu^- \to \nu_\mu e^- \overline{\nu_e}$

$$G_F^{SM} = \frac{\sqrt{2}g^2}{8M_W^2} = \frac{1}{\sqrt{2}v^2}$$

Phenomenology:

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<u>LFC:</u>

- $\mu \to e\nu\overline{\nu}$
- $l_\alpha \to l_\beta\nu\overline{\nu}$
- $\pi^+ \to \bar{l}\nu$
- $W \to l\overline{\nu}$
- $Z \to l_\alpha\overline{l_\alpha}$
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$$\Gamma \sim \Gamma_{SM}\left(1 + \eta_{\alpha\alpha}\right)$$

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<u>LFV:</u>

- $l_\alpha \to l_\beta l_\beta \overline{l_\beta}$
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- $Z \to l_\alpha \overline{l_\beta}$
- $H \to l_\alpha \overline{l_\beta}$

$$\Gamma \sim f\left(SM\right)\left|\eta_{\alpha\beta}\right|^2$$

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can one have

$$c^{d=5} \sim m_\nu = small$$

but also

$$c^{d=6} \sim \eta = big?$$

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$m_D, \ M$ are matrices

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$\Rightarrow$ cancellations in $c^{d=5} = Y_\Sigma^T \frac{1}{M} Y_\Sigma$

but NOT in $c^{d=6} = Y_\Sigma^\dagger \frac{1}{M^\dagger} \frac{1}{M} Y_\Sigma$

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but why?

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Weinberg operator violates L $\Longrightarrow c^{d=5} \sim m_\nu = small$

d=6 operator preserves L $\Longrightarrow c^{d=6} \sim \eta = whatever$

3 triplets:

L exact: $L_e = L_\mu = L_\tau = L_{\Sigma_1} = -L_{\Sigma_2} = 1, \quad L_{\Sigma_3} = 0$

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$$
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& + Y_\mu \left(m_{e\mu} m_{e\tau} - m_{ee} m_{\mu\tau} \right) - \sqrt{Y_e^2 m_{\mu\mu} - 2 Y_e Y_\mu m_{e\mu} + Y_\mu^2 m_{ee}} \times \\
& \left. \times \sqrt{m_{e\tau}^2 m_{\mu\mu} - 2 m_{e\mu} m_{e\tau} m_{\mu\tau} + m_{ee} m_{\mu\tau}^2 + m_{e\mu}^2 m_{\tau\tau} - m_{ee} m_{\mu\mu} m_{\tau\tau}} \right)
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constrains Majorana phase φ :

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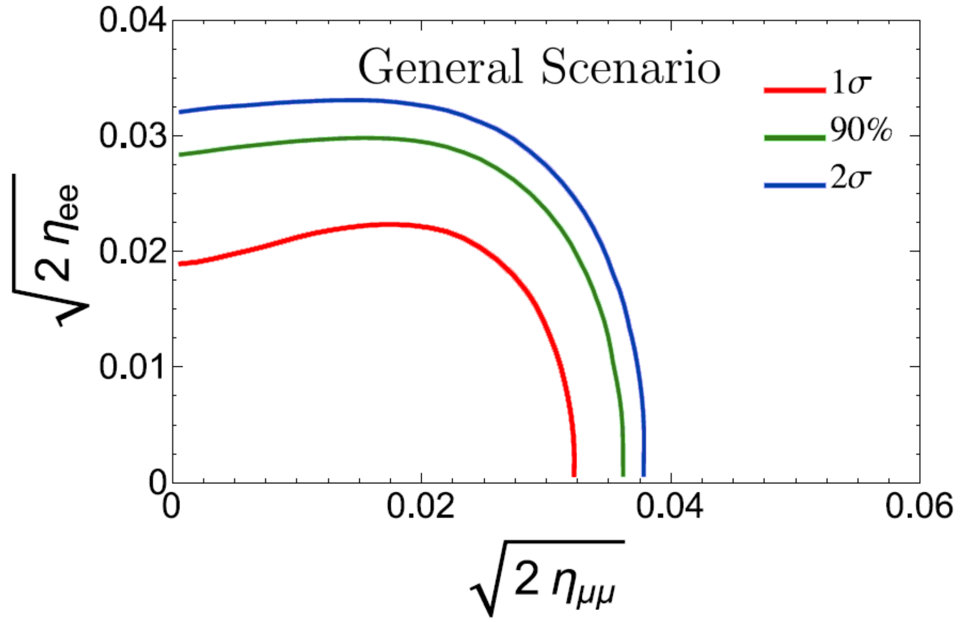
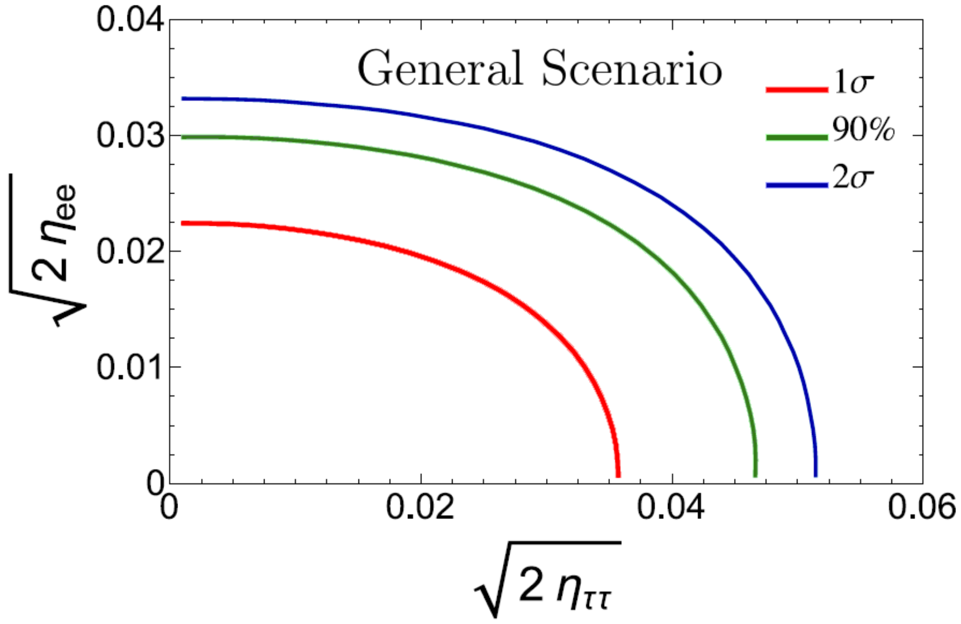
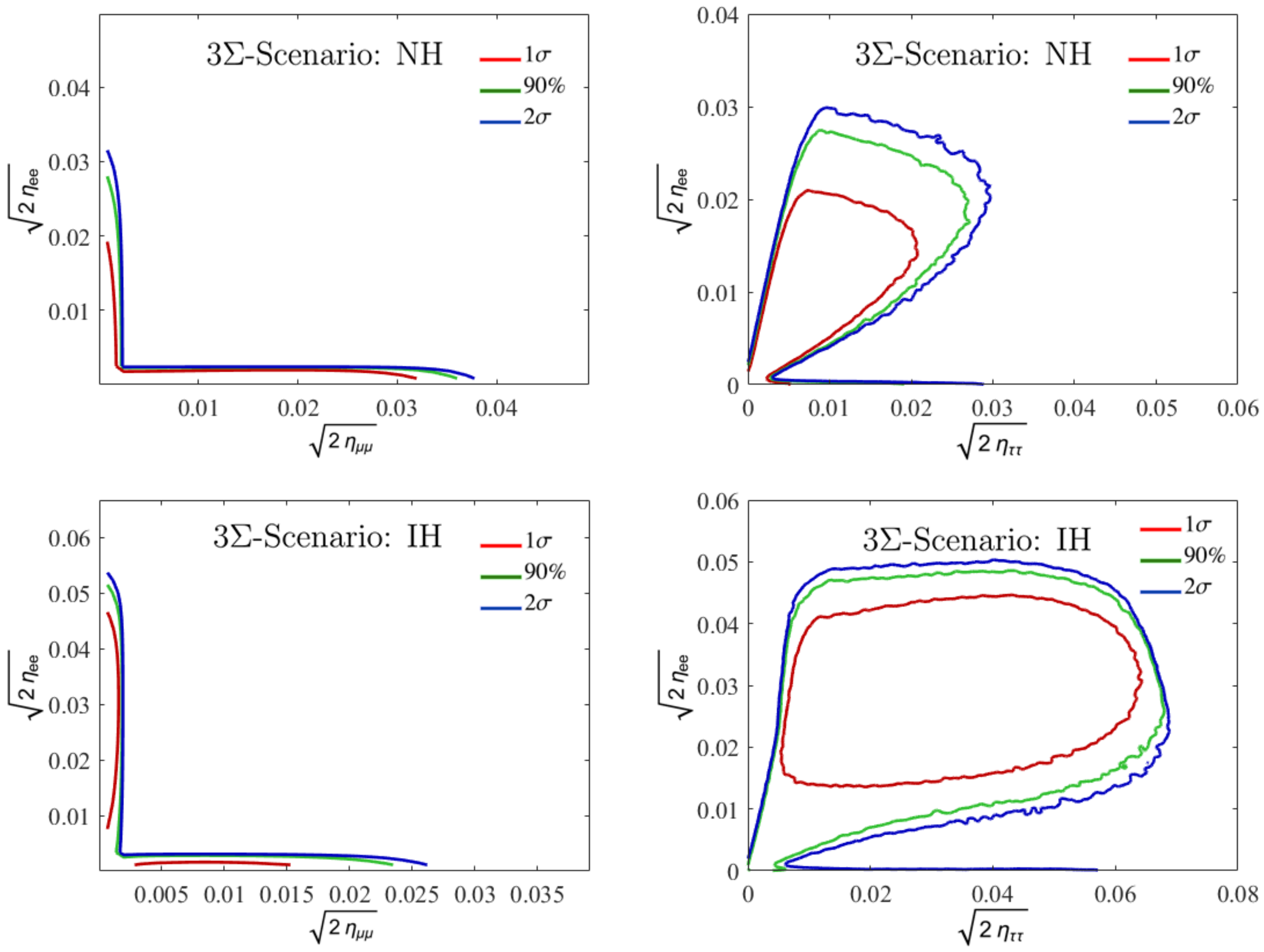
Preliminary results: General scenario



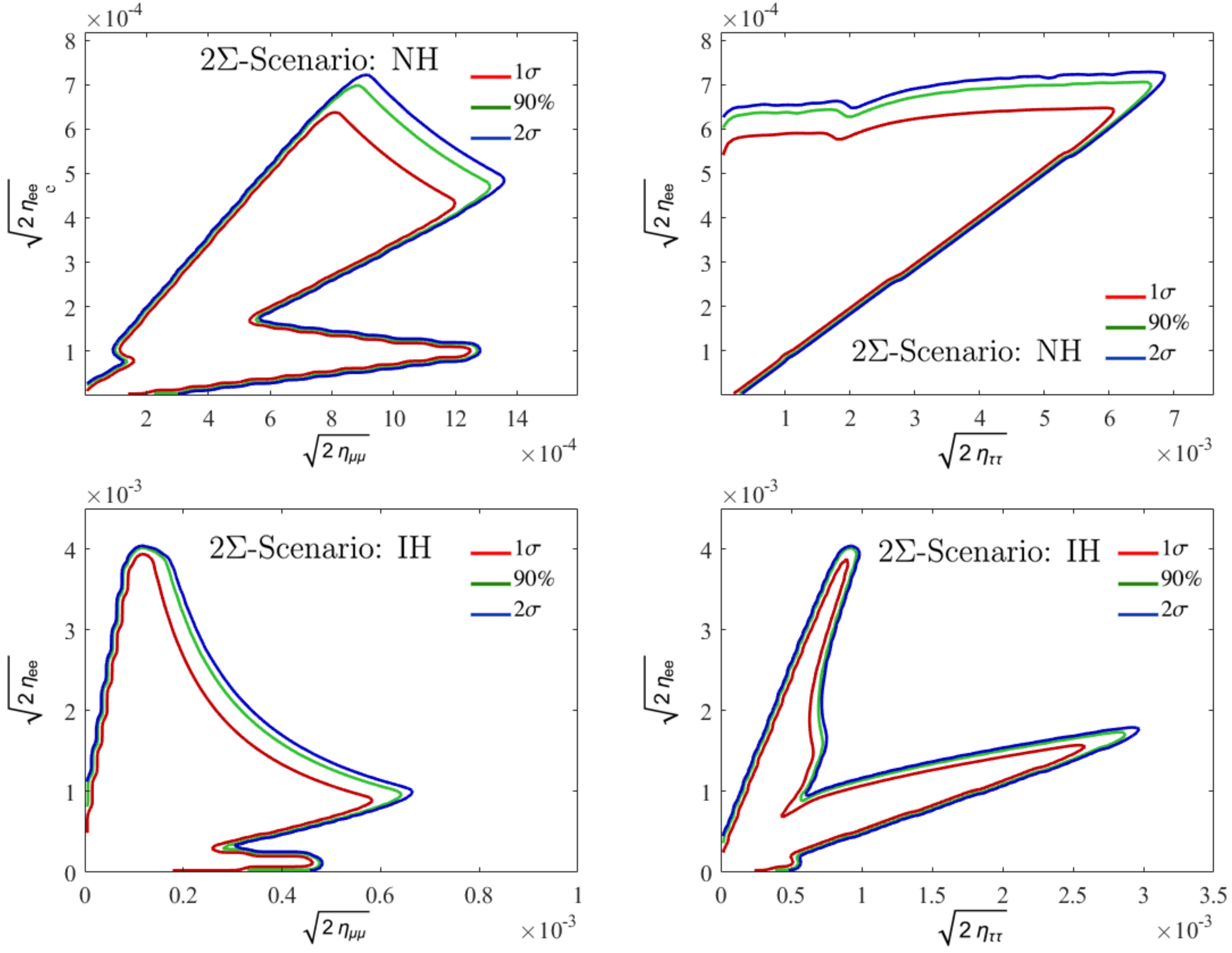


$$\sqrt{2\eta_{e\mu}} < 7.7 \cdot 10^{-4} \qquad \sqrt{2\eta_{e\tau}} < 0.024 \qquad \sqrt{2\eta_{\mu\tau}} < 0.033 \qquad (2\sigma)$$

Preliminary results: 3 triplets scenario



Preliminary results: 2 triplets scenario



Preliminary results:

		General	3 triplets		2 triplets	
			NH	IH	NH	IH
$\sqrt{2\eta_{ee}}$	1σ	< 0.015	< 0.0037	$0.032^{+0.010}_{-0.012}$	$< 6.9 \cdot 10^{-4}$	< 0.0026
	2σ	$< \mathbf{0.027}$	$< \mathbf{0.025}$	$< \mathbf{0.025}$	$< \mathbf{0.084}$	$< \mathbf{0.0032}$
$\sqrt{2\eta_{\mu\mu}}$	1σ	< 0.028	< 0.027	$< 3.7 \cdot 10^{-4}$	< 0.0010	$< 5.5 \cdot 10^{-4}$
	2σ	$< \mathbf{0.034}$	$< \mathbf{0.034}$	$< \mathbf{0.020}$	$< \mathbf{0.0012}$	$< \mathbf{6.6 \cdot 10^{-4}}$
$\sqrt{2\eta_{\tau\tau}}$	1σ	< 0.024	< 0.024	$0.040^{+0.018}_{-0.036}$	< 0.0077	< 0.0020
	2σ	$< \mathbf{0.043}$	$< \mathbf{0.042}$	$< \mathbf{0.066}$	$< \mathbf{0.0093}$	$< \mathbf{0.0023}$
$\sqrt{2\eta_{e\mu}}$	1σ	$< 6.5 \cdot 10^{-4}$	$< 6.4 \cdot 10^{-4}$	$< 6.4 \cdot 10^{-4}$	$< 6.5 \cdot 10^{-4}$	$< 6.5 \cdot 10^{-4}$
	2σ	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.6 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$
$\sqrt{2\eta_{e\tau}}$	1σ	< 0.012	< 0.0019	$0.036^{+0.010}_{-0.023}$	< 0.023	< 0.0016
	2σ	$< \mathbf{0.024}$	$< \mathbf{0.023}$	$< \mathbf{0.052}$	$< \mathbf{0.0028}$	$< \mathbf{0.0019}$
$\sqrt{2\eta_{\mu\tau}}$	1σ	< 0.022	< 0.023	$< 9.3 \cdot 10^{-7}$	< 0.021	< 0.0082
	2σ	$< \mathbf{0.033}$	$< \mathbf{0.032}$	$< \mathbf{0.032}$	$< \mathbf{0.025}$	$< \mathbf{0.0097}$

Preliminary results:

		General	3 triplets		2 triplets	
			NH	IH	NH	IH
$\sqrt{2\eta_{ee}}$	1σ	< 0.015	< 0.0037	$0.032^{+0.010}_{-0.012}$	$< 6.9 \cdot 10^{-4}$	< 0.0026
	2σ	$< \mathbf{0.027}$	$< \mathbf{0.025}$	$< \mathbf{0.025}$	$< \mathbf{0.084}$	$< \mathbf{0.0032}$
$\sqrt{2\eta_{\mu\mu}}$	1σ	< 0.028	< 0.027	$< 3.7 \cdot 10^{-4}$	< 0.0010	$< 5.5 \cdot 10^{-4}$
	2σ	$< \mathbf{0.034}$	$< \mathbf{0.034}$	$< \mathbf{0.020}$	$< \mathbf{0.0012}$	$< \mathbf{6.6 \cdot 10^{-4}}$
$\sqrt{2\eta_{\tau\tau}}$	1σ	< 0.024	< 0.024	$0.040^{+0.018}_{-0.036}$	< 0.0077	< 0.0020
	2σ	$< \mathbf{0.043}$	$< \mathbf{0.042}$	$< \mathbf{0.066}$	$< \mathbf{0.0093}$	$< \mathbf{0.0023}$
$\sqrt{2\eta_{e\mu}}$	1σ	$< 6.5 \cdot 10^{-4}$	$< 6.4 \cdot 10^{-4}$	$< 6.4 \cdot 10^{-4}$	$< 6.5 \cdot 10^{-4}$	$< 6.5 \cdot 10^{-4}$
	2σ	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.6 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$
$\sqrt{2\eta_{e\tau}}$	1σ	< 0.012	< 0.0019	$0.036^{+0.010}_{-0.023}$	< 0.023	< 0.0016
	2σ	$< \mathbf{0.024}$	$< \mathbf{0.023}$	$< \mathbf{0.052}$	$< \mathbf{0.0028}$	$< \mathbf{0.0019}$
$\sqrt{2\eta_{\mu\tau}}$	1σ	< 0.022	< 0.023	$< 9.3 \cdot 10^{-7}$	< 0.021	< 0.0082
	2σ	$< \mathbf{0.033}$	$< \mathbf{0.032}$	$< \mathbf{0.032}$	$< \mathbf{0.025}$	$< \mathbf{0.0097}$

Preliminary results:

		General	3 triplets		2 triplets	
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$\sqrt{2\eta_{ee}}$	1σ	< 0.015	< 0.0037	$0.032^{+0.010}_{-0.012}$	$< 6.9 \cdot 10^{-4}$	< 0.0026
	2σ	$< \mathbf{0.027}$	$< \mathbf{0.025}$	$< \mathbf{0.025}$	$< \mathbf{8.4 \cdot 10^{-4}}$	$< \mathbf{0.0032}$
$\sqrt{2\eta_{\mu\mu}}$	1σ	< 0.028	< 0.027	$< 3.7 \cdot 10^{-4}$	< 0.0010	$< 5.5 \cdot 10^{-4}$
	2σ	$< \mathbf{0.034}$	$< \mathbf{0.034}$	$< \mathbf{0.020}$	$< \mathbf{0.0012}$	$< \mathbf{6.6 \cdot 10^{-4}}$
$\sqrt{2\eta_{\tau\tau}}$	1σ	< 0.024	< 0.024	$0.040^{+0.018}_{-0.036}$	< 0.0077	< 0.0020
	2σ	$< \mathbf{0.043}$	$< \mathbf{0.042}$	$< \mathbf{0.066}$	$< \mathbf{0.0093}$	$< \mathbf{0.0023}$
$\sqrt{2\eta_{e\mu}}$	1σ	$< 6.5 \cdot 10^{-4}$	$< 6.4 \cdot 10^{-4}$	$< 6.4 \cdot 10^{-4}$	$< 6.5 \cdot 10^{-4}$	$< 6.5 \cdot 10^{-4}$
	2σ	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.6 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$	$< \mathbf{7.7 \cdot 10^{-4}}$
$\sqrt{2\eta_{e\tau}}$	1σ	< 0.012	< 0.0019	$0.036^{+0.010}_{-0.023}$	< 0.0023	< 0.0016
	2σ	$< \mathbf{0.024}$	$< \mathbf{0.023}$	$< \mathbf{0.052}$	$< \mathbf{0.0028}$	$< \mathbf{0.0019}$
$\sqrt{2\eta_{\mu\tau}}$	1σ	< 0.022	< 0.023	$< 9.3 \cdot 10^{-7}$	< 0.021	< 0.0082
	2σ	$< \mathbf{0.033}$	$< \mathbf{0.032}$	$< \mathbf{0.032}$	$< \mathbf{0.025}$	$< \mathbf{0.0097}$

Conclusions

- Neutrinos are massive, but we don't know why
- The see-saw mechanism can explain small neutrino masses
- We studied the type III see-saw
- If L is an approximate symmetry:
  - Interesting correlations in mass matrix
  - Potentially large (detectable!) effects
- We are placing new bounds on the parameters of the models with 3 or 2 triplets (for both Normal and Inverted Hierarchy)