Non-geometric backgrounds in string theory

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LMU Munich

Fundamental interactions, geometry and topology

Napoli — 25.10.2018

outline

1. introduction
2. non-geometry
3. developments
4. conclusions

outline

String theory is a theory of quantum gravity including gauge interactions.

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- Desirable for the description of our universe.

- Framework to test expectations on a quantum-gravity theory.
- Allows to explore the interplay between gravity and QFT.
- Applications to particle phenomenology and cosmology.

- Contains profound mathematical results.

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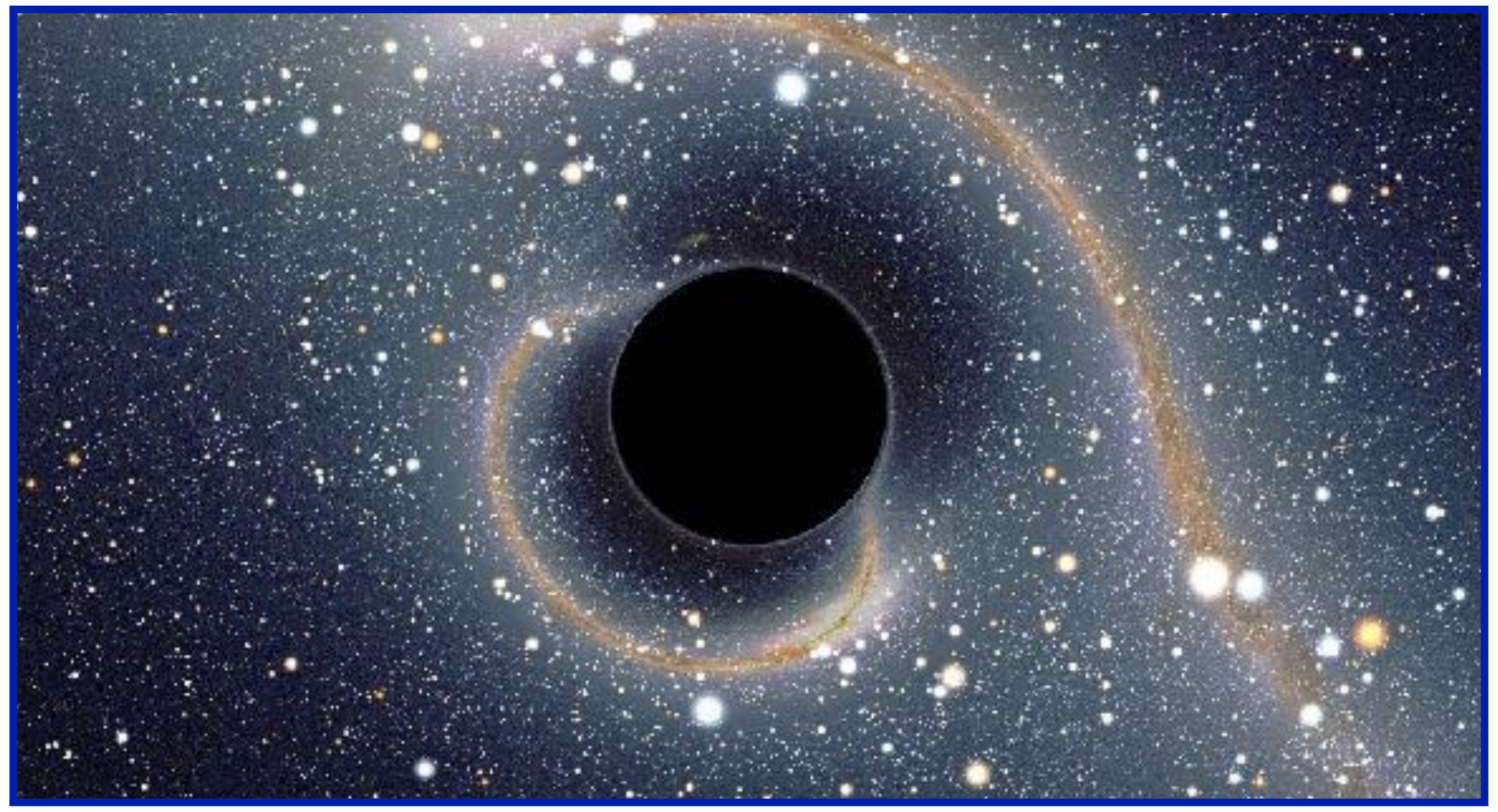

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$$\Lambda = +5.6 \times 10^{-122} t_{\mathrm{P}}^{-2}$$

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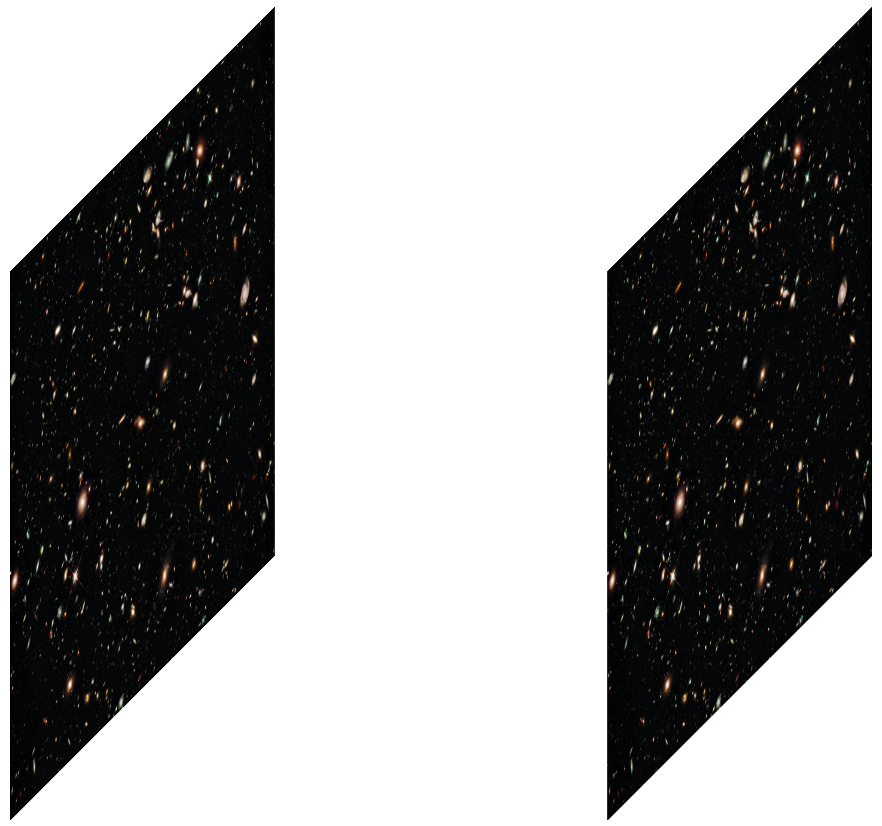
NASA 2009

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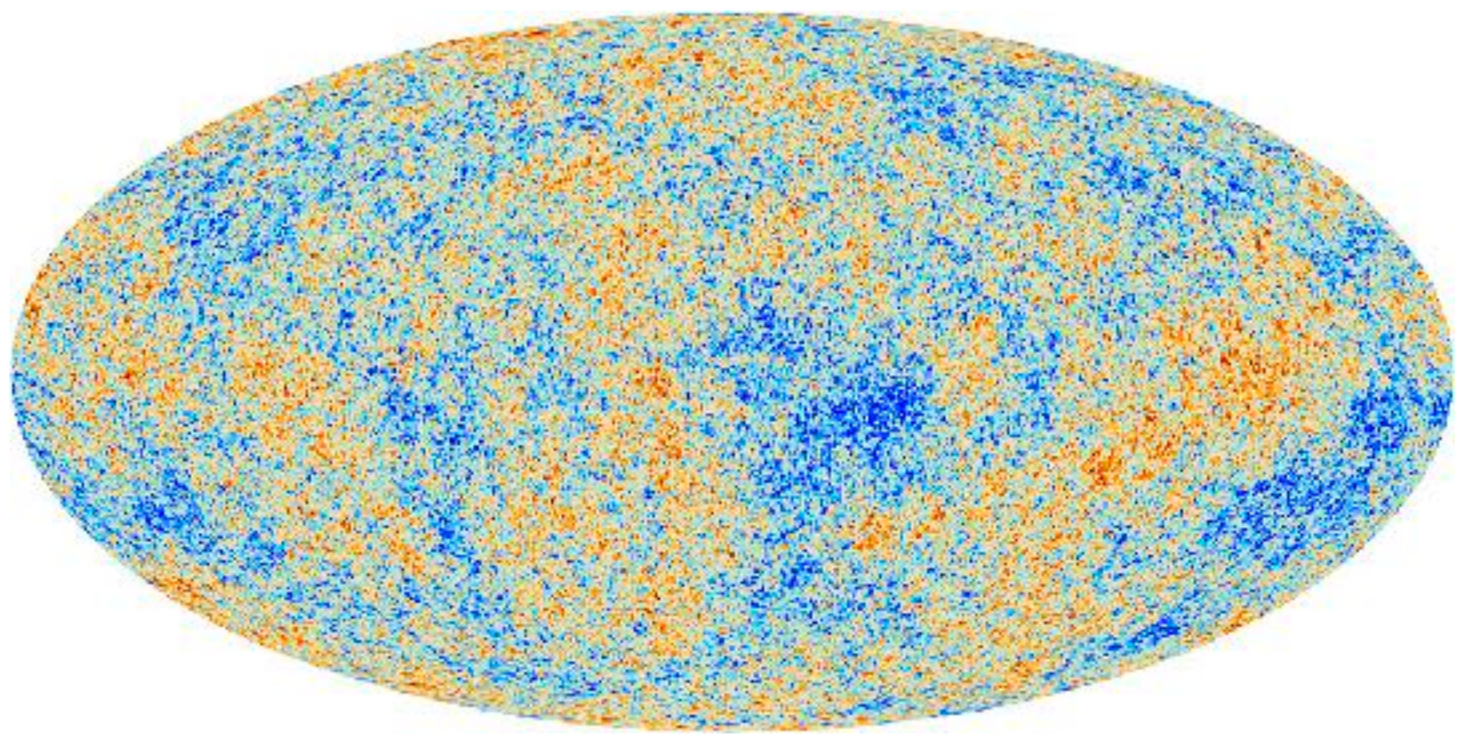

Planck 2013

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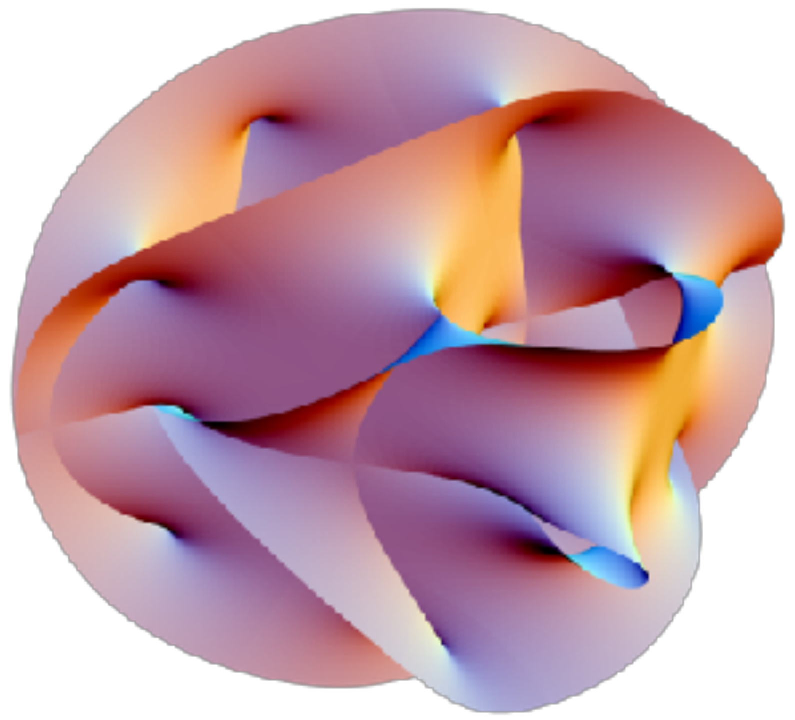

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$$
\begin{aligned}
1 &= r_1 \\
196884 &= r_1 + r_2 \\
21493760 &= r_1 + r_2 + r_3 \\
864299970 &= 2r_1 + 2r_2 + r_3 + r_4 \\
20245856256 &= 3r_1 + 3r_2 + r_3 + 2r_4 + r_5 \\
&\dots
\end{aligned}
$$

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Escher
Wikimedia Commons

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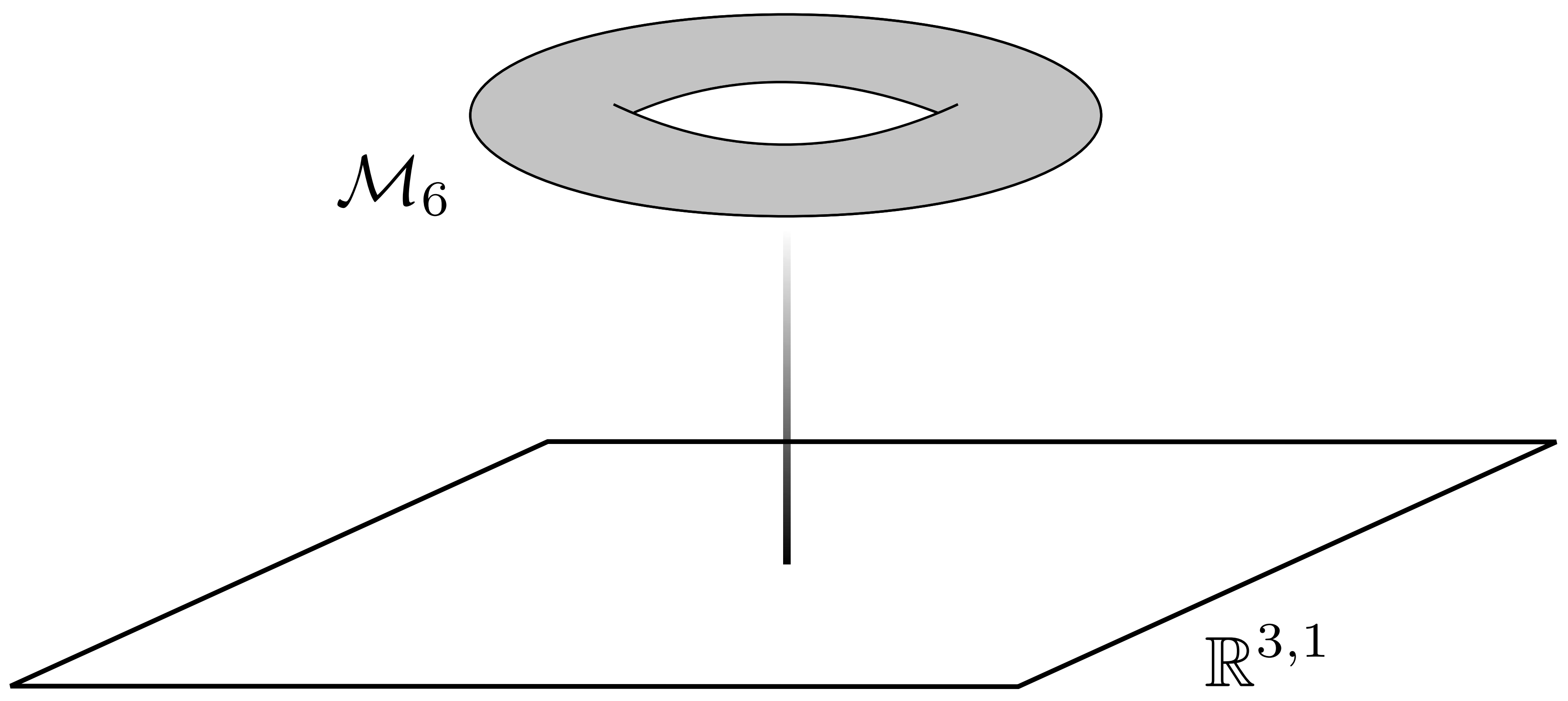
String theory unifies gravity and gauge interactions ⟶ employ it for the description of our universe.

String theory is consistent only in ten dimensions — need to compactify from 10D to 4D ::

- The compact space $\mathcal{M}_6$ strongly affects the 4D theory, so understanding such spaces is important.
- There is a large number of choices for the compact space (string-theory landscape).

→ When are two compact spaces considered to be "*different*" …?



Duality :: two different theories are dual to each other, if they describe the same physics.

Example :: electrodynamics in four dimensions (without sources) ::

- action functional $$\mathcal{S} = -\frac{1}{2}\int F \wedge \star F \,,$$
- Bianchi identity $$dF = 0 \,,$$
- equation of motion $$d \star F = 0 \,,$$
- duality transformation $$F \to \star F \,.$$

String theory has a rich structure of dualities ::

S-duality

U-duality

T-duality

AdS/CFT correspondence

mirror symmetry

heterotic/type IIA duality

open-/closed-string duality

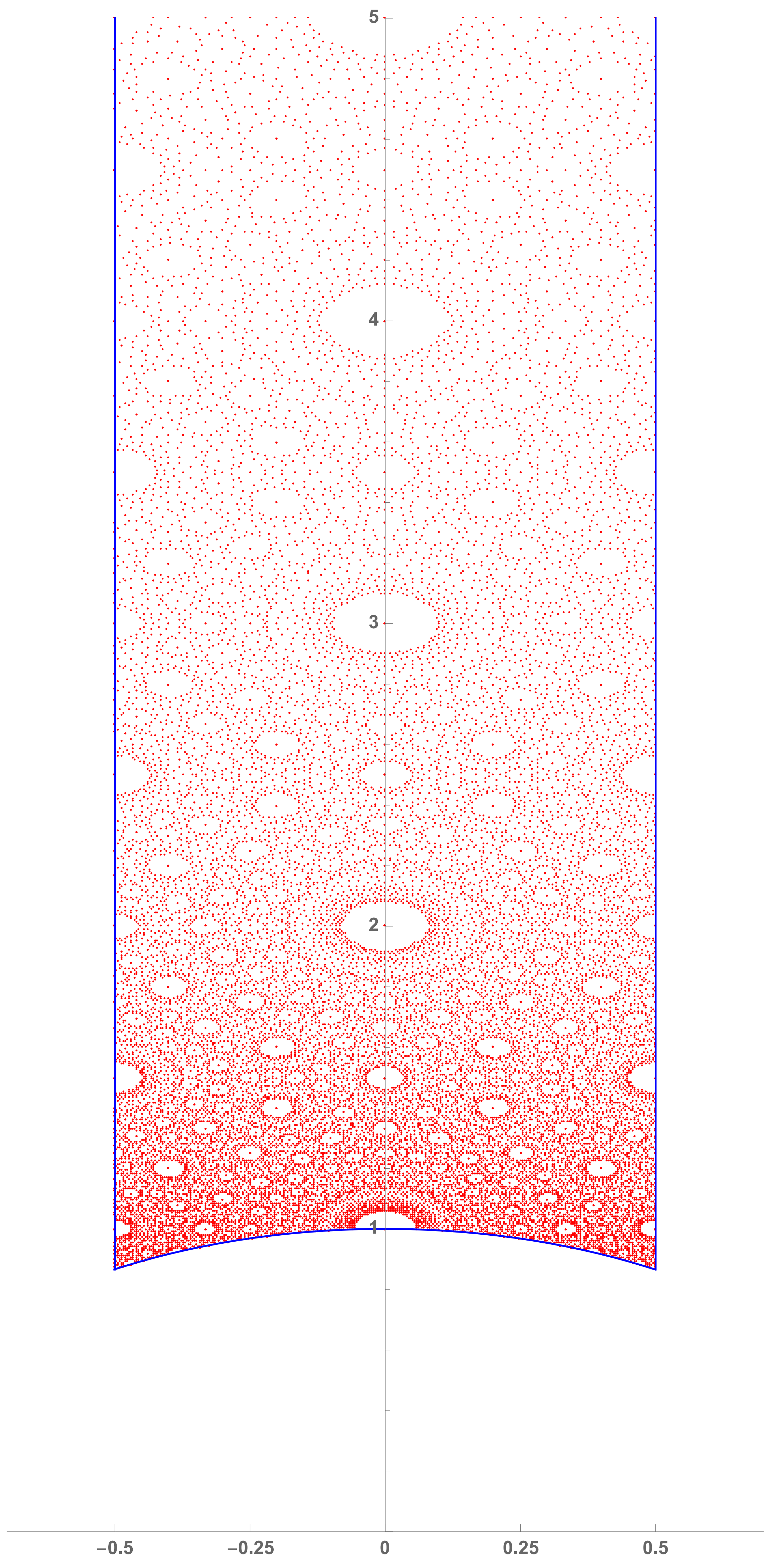
heterotic/M-theory duality

type IIA/M-theory duality

When are two compact spaces considered to be "*different*" …?

↓

… when they are **not** related via a duality transformations.



Denef, Douglas - 2004

Summary ::

- String theory is a theory of quantum gravity including gauge interactions.
- For describing our universe, need to compactify to four dimensions.
- Dualities are an integral part of string theory.

outline

outline

1. introduction
2. non-geometry
   a) generalities
   b) t-duality
   c) example
   d) summary
3. developments
4. conclusions

The equation of motion for a closed string (in the simplest setting) is the wave equation in two dimensions

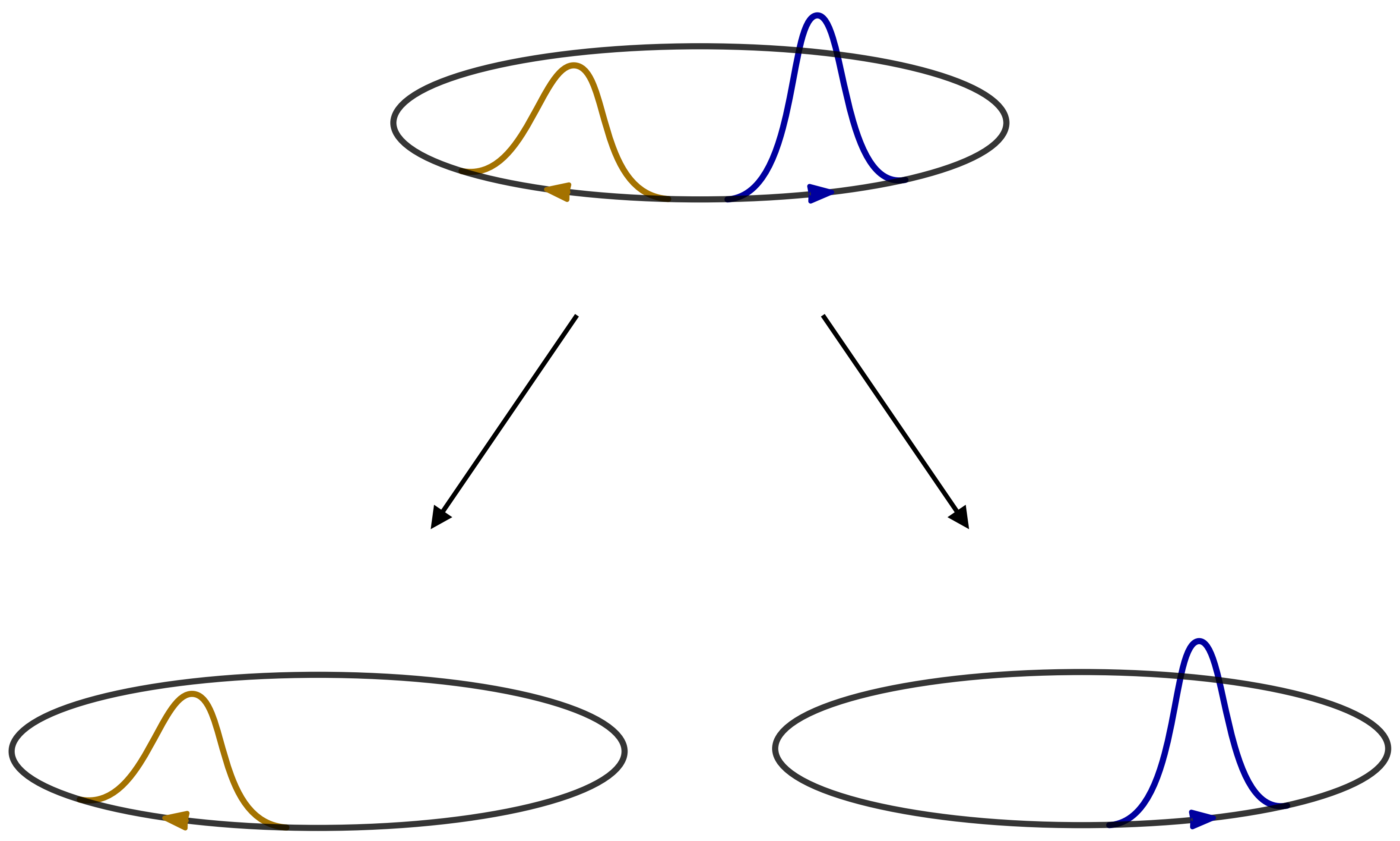
$$0 = \partial_\alpha \partial^\alpha X^\mu(\sigma)\,.$$

- The general solution splits into a left-moving and right-moving part

$$X^\mu(\sigma) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-)\,.$$

- If both parts *see* the same geometry, the space is geometric.

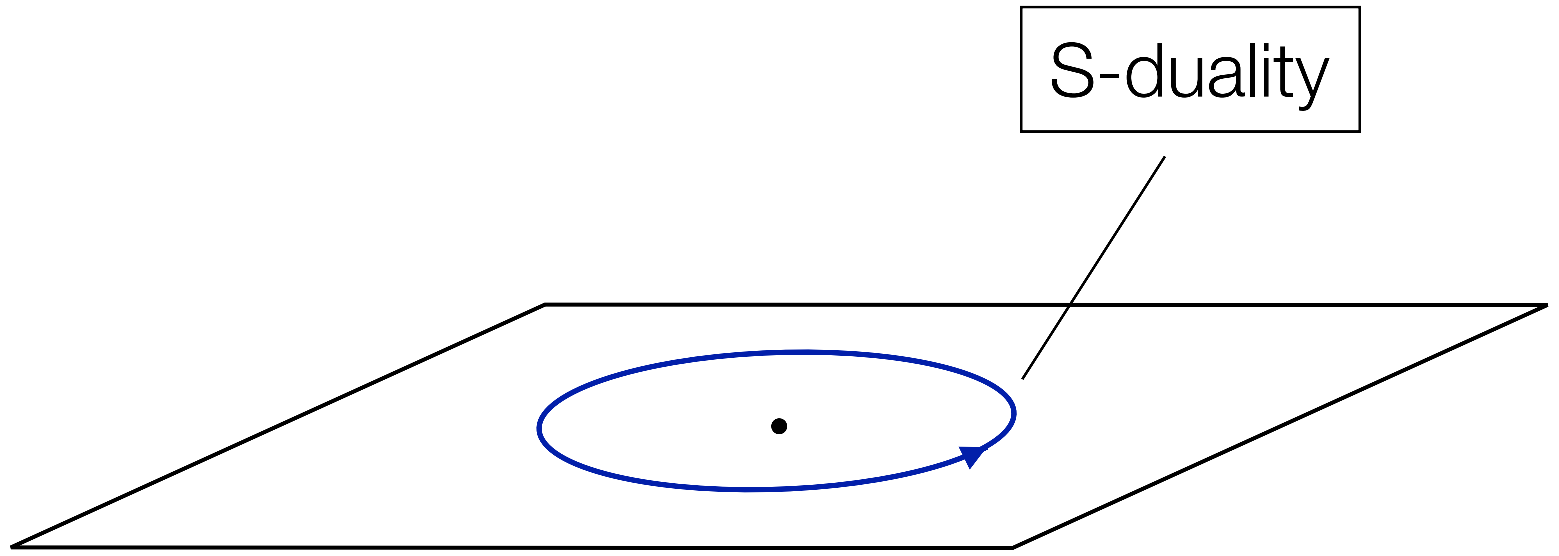
- If the two parts *see* different geometries, the space is non-geometric (but well-defined for a string).



Non-geometric spaces can be constructed using duality transformations ::

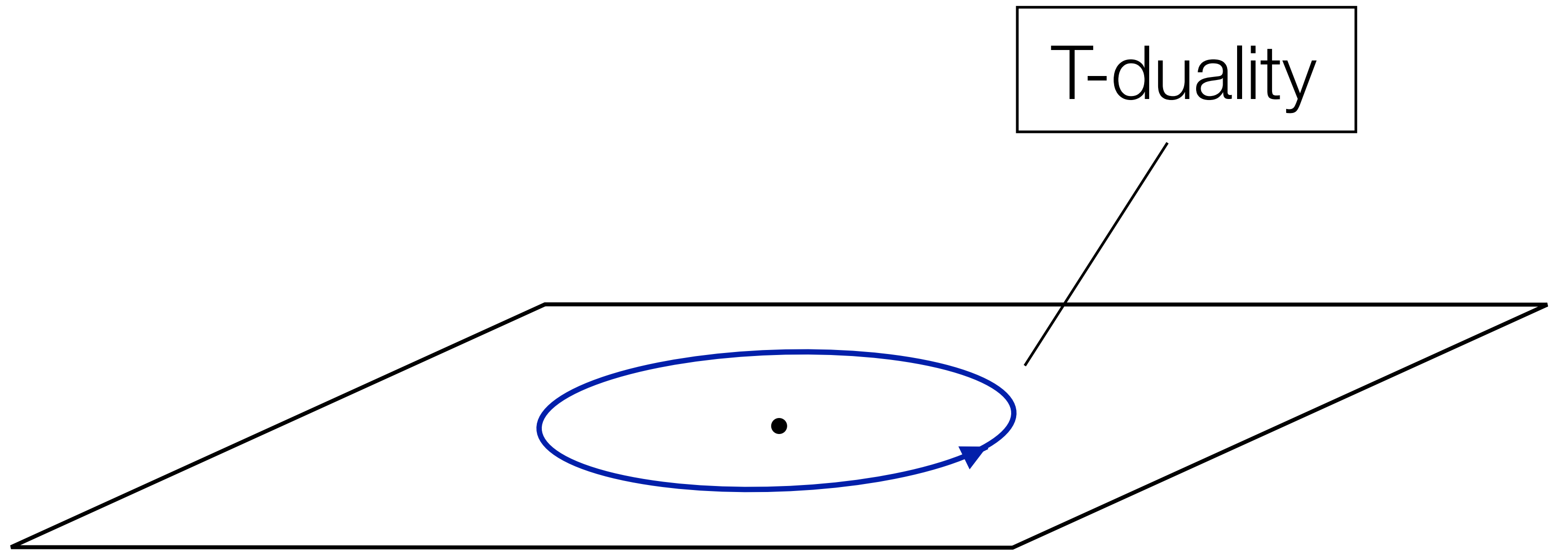
S-duality

- duality transformation $g_s \to 1/g_s$
- monodromy around *(p,q)*-branes contains S-duality



T-duality

- duality transformation $R \to 1/R$
- monodromy around defects may contain T-duality



Hellerman, McGreevy, Williams - 2003

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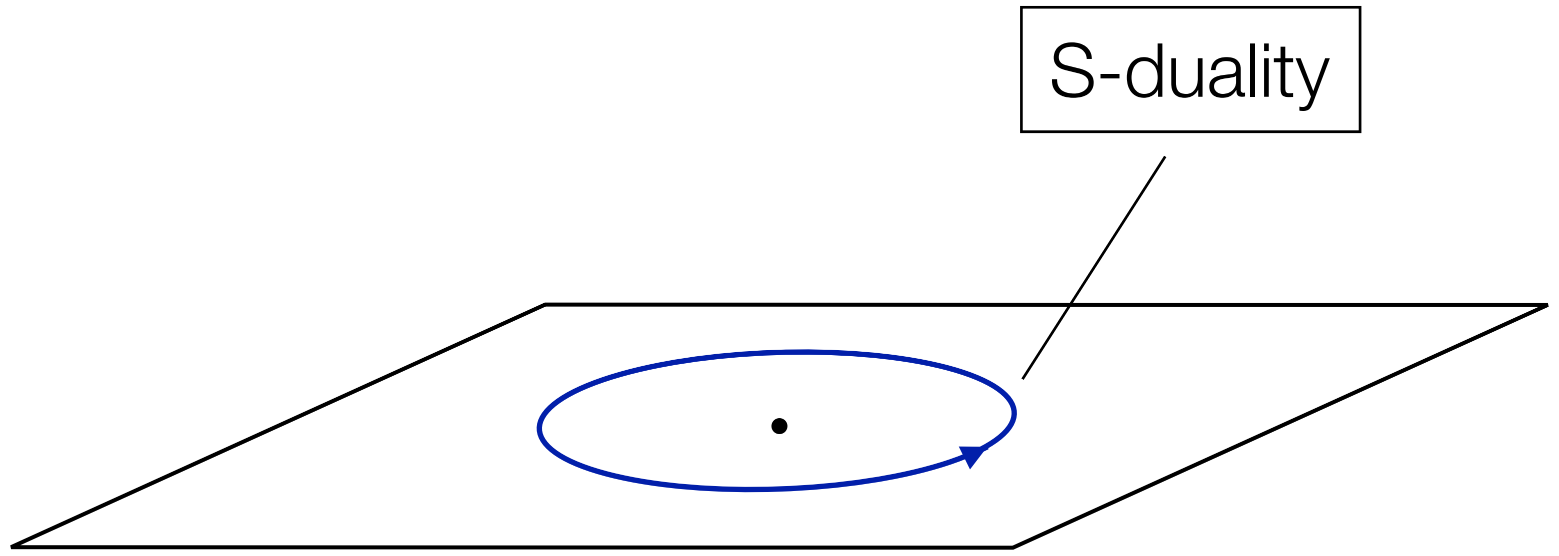

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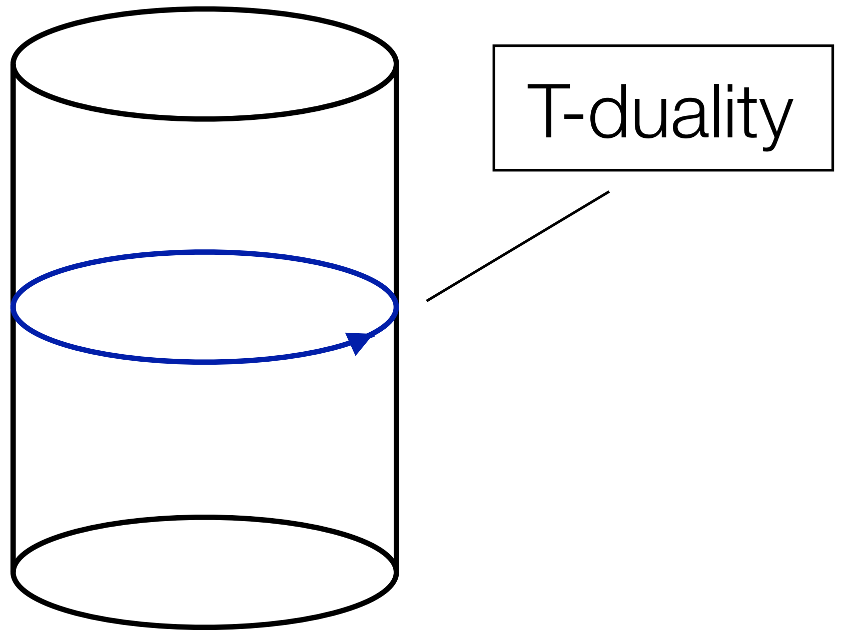


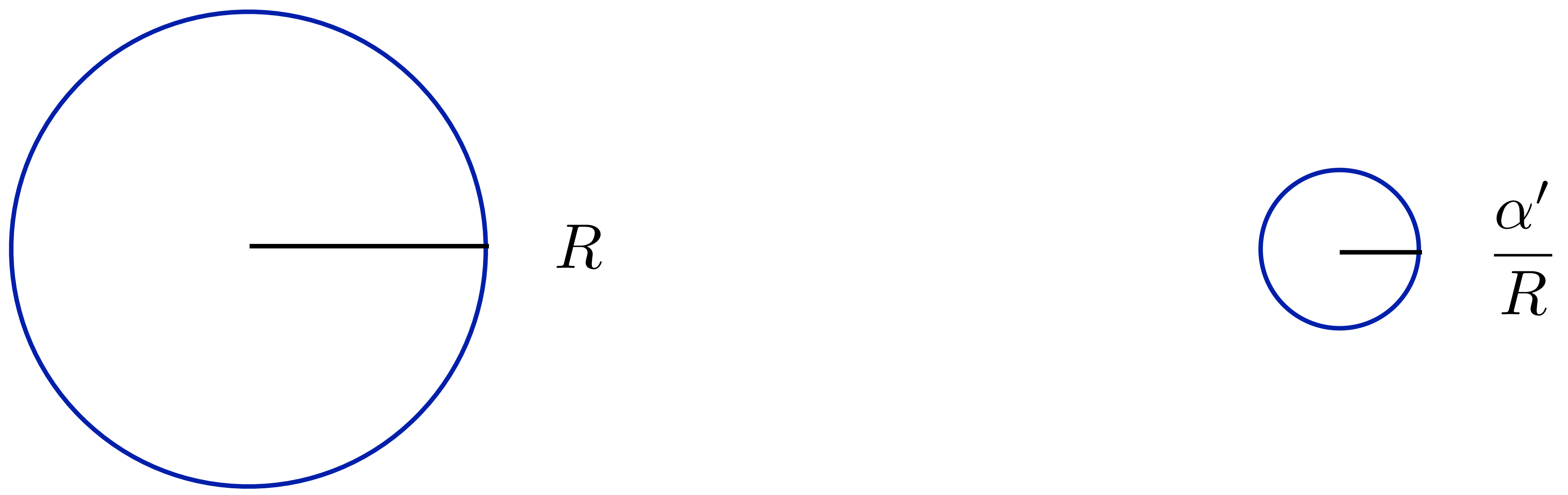
→ non-geometric background

Hellerman, McGreevy, Williams - 2003
Hull - 2004

outline

T-duality ::

- Two string-theory compactifications on dual circles cannot be distinguished.



- The duality group for the circle is $\mathbb{Z}_2$.
- T-duality is a string-theory duality — not existing for point particles.

A string-theory background (in the NS-NS sector) is characterized by a choice of

- metric $G_{\mu\nu}$,
- anti-symmetric two-form $B_{\mu\nu}$,
- dilaton Φ .

T-duality transformations act on (G, B, Φ) in a non-trivial way.

For D-dimensional toroidal compactifications the duality group is $O(D, D; \mathbb{Z})$,

- which for $\mathcal{O} \in O(D, D; \mathbb{Z})$ is specified by $\mathcal{O}^T \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \mathcal{O} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$.

The duality group $O(D,D;\mathbb{Z})$ contains the elements ::

- A-transformations ($\mathsf{A} \in GL(D,\mathbb{Z})$)

$$\mathcal{O}_{\mathsf{A}} = \begin{pmatrix} \mathsf{A}^{-1} & 0 \\ 0 & \mathsf{A}^{T} \end{pmatrix} \longrightarrow \text{diffeomorphisms}$$

- B-transformations (B_{ij} an anti-symmetric matrix)

$$\mathcal{O}_{\mathsf{B}} = \begin{pmatrix} \mathbb{1} & 0 \\ \mathsf{B} & \mathbb{1} \end{pmatrix} \longrightarrow \text{gauge transformations } B \to B + \alpha'\mathsf{B}$$

- β-transformations (β^{ij} an anti-symmetric matrix)

$$\mathcal{O}_{\beta} = \begin{pmatrix} \mathbb{1} & \beta \\ 0 & \mathbb{1} \end{pmatrix}$$

- factorized duality (E_{i} with only non-zero $E_{\mathsf{ii}} = 1$)

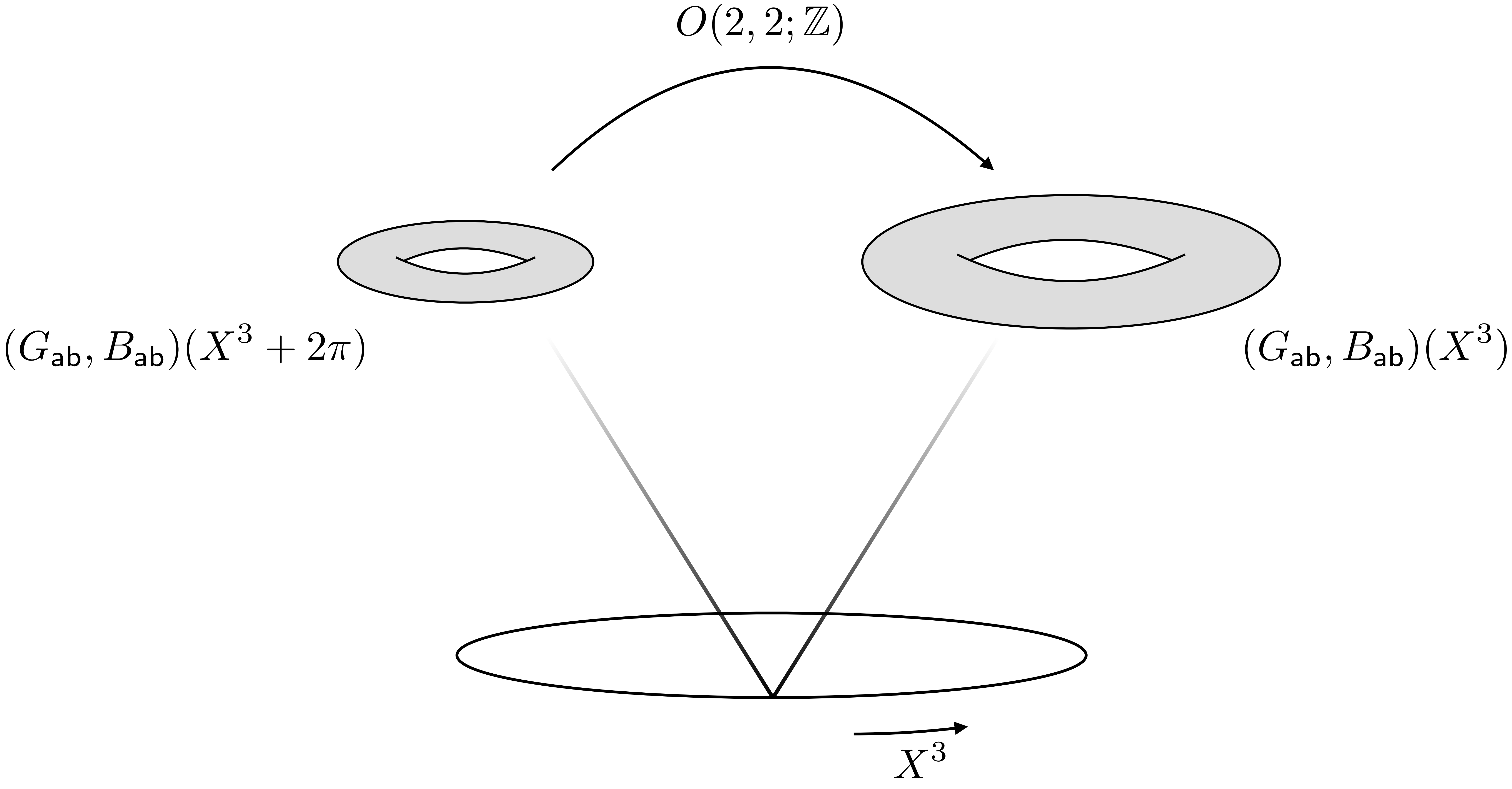
$$\mathcal{O}_{\pm\mathsf{i}} = \begin{pmatrix} \mathbb{1} - E_{\mathsf{i}} & \pm E_{\mathsf{i}} \\ \pm E_{\mathsf{i}} & \mathbb{1} - E_{\mathsf{i}} \end{pmatrix} \longrightarrow \text{T-duality transformations } G_{\mathsf{ii}} \to \frac{\alpha'^2}{G_{\mathsf{ii}}}$$

outline

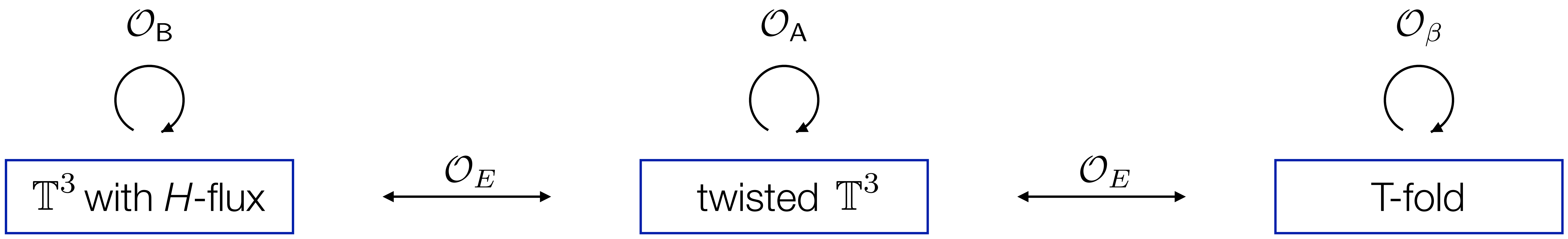
The standard example for a non-geometric background is a $\mathbb{T}^2$-fibration over a circle.

$$G_{ij} = \begin{pmatrix} G_{\mathsf{ab}}(X^3) & 0 \\ 0 & R_3^2 \end{pmatrix}$$

$$B_{ij} = \begin{pmatrix} B_{\mathsf{ab}}(X^3) & 0 \\ 0 & 0 \end{pmatrix}$$



The non-geometric background is part of a **family** of $\mathbb{T}^2$-fibrations ::



A three-torus with *H*-flux is characterized as follows ::

1. Metric and *B*-field

$$G_{ij} = \begin{pmatrix} R_1^2 & 0 & 0 \\ 0 & R_2^2 & 0 \\ 0 & 0 & R_3^2 \end{pmatrix}, \qquad B_{ij} = \begin{pmatrix} 0 & +\frac{\alpha'}{2\pi} h X^3 & 0 \\ -\frac{\alpha'}{2\pi} h X^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad h \in \mathbb{Z}\,.$$

2. The background is well-defined under $X^3 \to X^3 + 2\pi$ using a gauge transformation.

3. The *H*-flux $H = dB$ can be expressed in a vielbein basis as

$$H = \frac{\alpha'}{2\pi} \frac{h}{R_1 R_2 R_3}\, e^1 \wedge e^2 \wedge e^3\,, \qquad H_{123} = \frac{\alpha'}{2\pi} \frac{h}{R_1 R_2 R_3}\,.$$

After a T-duality along X^1 one obtains a twisted three-torus ::

1. Metric and *B*-field

$$G_{ij} = \begin{pmatrix} \frac{\alpha'^2}{R_1^2} & -\frac{\alpha'^2}{R_1^2}\frac{h}{2\pi}X^3 & 0 \\ -\frac{\alpha'^2}{R_1^2}\frac{h}{2\pi}X^3 & R_2^2 + \frac{\alpha'^2}{R_1^2}\left[\frac{h}{2\pi}X^3\right]^2 & 0 \\ 0 & 0 & R_3^2 \end{pmatrix}, \qquad B_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad h \in \mathbb{Z}\,.$$

2. The background is well-defined under $X^3 \to X^3 + 2\pi$ using a diffeomorphism.

3. A geometric *f*-flux is defined via a vielbein basis as

$$de^a = \frac{1}{2} f_{bc}{}^a\, e^b \wedge e^c\,, \qquad f_{23}{}^1 = \frac{\alpha'}{2\pi}\,\frac{h}{R_1\,R_2\,R_3}\,.$$

Kachru, Schulz, Tripathy, Trivedi - 2002

A second T-duality along X^2 gives the T-fold background ::

1. Metric and *B*-field

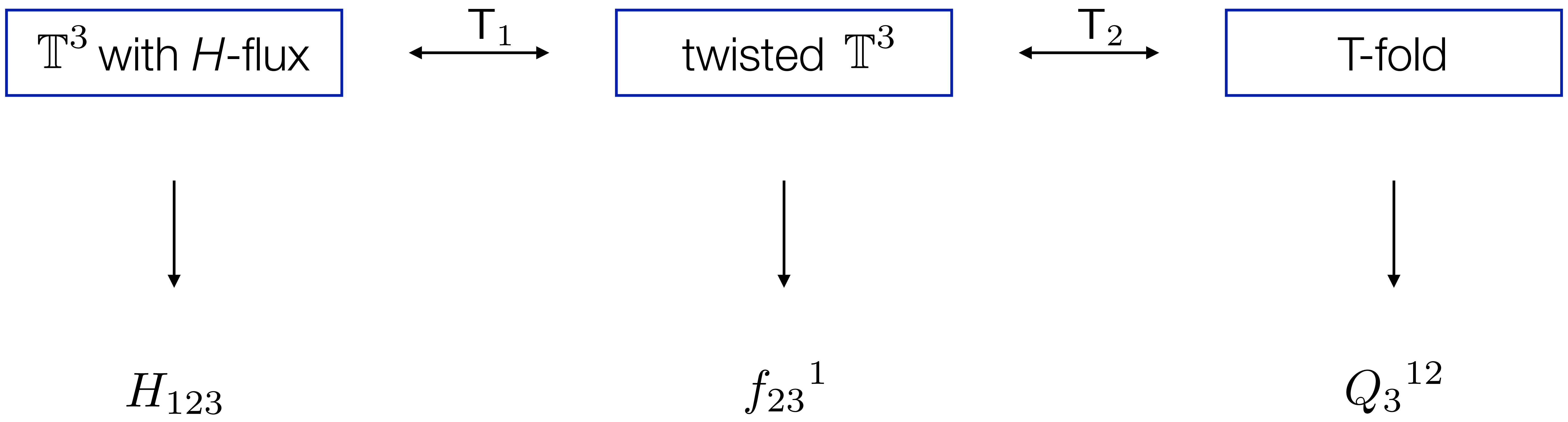
$$G_{ij} = \begin{pmatrix} \frac{R_2^2}{\rho} & 0 & 0 \\ 0 & \frac{R_1^2}{\rho} & 0 \\ 0 & 0 & R_3^2 \end{pmatrix}, \qquad B_{ij} = \frac{1}{\rho}\begin{pmatrix} 0 & -\frac{\alpha'}{2\pi} h\, X^3 & 0 \\ +\frac{\alpha'}{2\pi} h\, X^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \begin{aligned} &\rho = \tfrac{R_1^2 R_2^2}{\alpha'^2} + \left[\tfrac{h}{2\pi}\, X^3\right]^2 , \\ &h \in \mathbb{Z}\,. \end{aligned}$$

2. The background is well-defined under $X^3 \to X^3 + 2\pi$ using a β-transformation.

3. A non-geometric Q-flux is defined via a vielbein basis and $(G-B)^{-1} = g - \beta$ as

$$Q_i{}^{jk} = \partial_i \beta^{jk}\,, \qquad Q_3{}^{12} = \frac{\alpha'}{2\pi}\,\frac{h}{R_1\, R_2\, R_3}\,.$$

The above family of backgrounds is characterized by (non-)geometric **fluxes** ::



non-geometry :: fluxes

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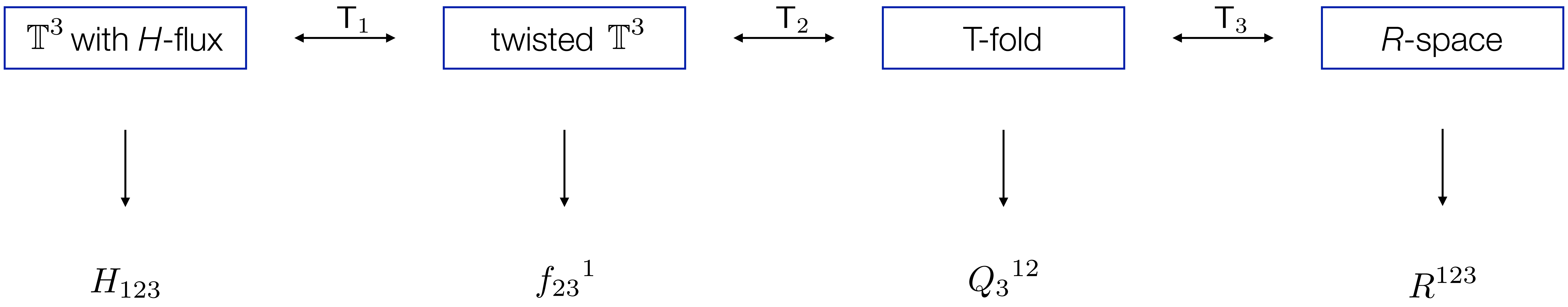

Shelton, Taylor, Wecht - 2005

outline

Summary ::

- T-duality is a string-theory duality — not present for point-particle theories.
- Non-geometric backgrounds are well-defined using T-duality.
- Non-triviality of the background is encoded in (non-)geometric fluxes.

- Non-geometric backgrounds are examples for compactification spaces.

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Non-geometric fluxes can be described using the framework of generalised geometry ::

- The generalised tangent-bundle E over a manifold M is locally $E = TM \oplus T^*M$.
- Sections of E are given by $X = x + \xi$ with $x \in \Gamma(TM)$ and $\xi \in \Gamma(T^*M)$.
- A bi-linear pairing on E (invariant under $O(D,D)$ transformations) is

$$\big\langle X, Y \big\rangle = \big\langle x + \xi, y + \chi \big\rangle = \frac{1}{2}\big(\iota_x\chi + \iota_y\xi\big) .$$

- A natural bracket on E is the Courant bracket

$$\big[X, Y\big]_{\mathrm{C}} = [x, y]_{\mathrm{L}} + \mathcal{L}_x\chi - \mathcal{L}_y\xi - \frac{1}{2}d\big(\iota_x\chi - \iota_y\xi\big) .$$

- The Courant bracket is in general not a Lie bracket, since the Jacobiator satisfies

$$\mathrm{Jac}\big(X, Y, Z\big)_{\mathrm{C}} = d\,\mathrm{Nij}\big(X, Y, Z\big)_{\mathrm{C}} .$$

Hitchin - 2002
Gualtieri - 2004

On the generalised tangent-bundle, a generalised metric can be introduced ::

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & +BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}.$$

A corresponding generalised vielbein

$$\mathcal{E} = \{\mathcal{E}^A{}_I\} = \begin{pmatrix} \mathcal{E}^a{}_i & \mathcal{E}^{a\,i} \\ \mathcal{E}_{a\,i} & \mathcal{E}_a{}^i \end{pmatrix}$$

(leaving invariant the pairing) is defined via the relations

$$\eta = \mathcal{E}^T \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \mathcal{E}\,, \qquad \mathcal{H} = \mathcal{E}^T \begin{pmatrix} \delta & 0 \\ 0 & \delta^{-1} \end{pmatrix} \mathcal{E}\,.$$

The dual vielbein $\overline{\mathcal{E}}_A = \{\overline{\mathcal{E}}_A{}^I\}$ satisfies ab algebra $[\overline{\mathcal{E}}_A, \overline{\mathcal{E}}_B]_{\mathrm{C}} = F_{AB}{}^C\, \overline{\mathcal{E}}_C$.

- The structure constant are identified with the fluxes

$$F_{abc} = H_{abc}\,, \qquad F_{ab}{}^c = f_{ab}{}^c\,, \qquad F_a{}^{bc} = Q_a{}^{bc}\,, \qquad F^{abc} = R^{abc}\,.$$

- Bianchi identities (for constant fluxes) are determined from the Jacobiator

$$\begin{aligned}
0 &= -\,3\, f_{[\underline{ab}}{}^m\, H_{m|\underline{cd}]}\,,\\
0 &= -\,3\, f_{[\underline{ab}}{}^m\, f_{m|\underline{c}]}{}^d + 3\, H_{[\underline{ab}|m}\, Q_{\underline{c}]}{}^{md}\,,\\
0 &= -\, f_{ab}{}^m\, Q_m{}^{cd} - 4\, f_{m[\underline{a}}{}^{[\underline{c}}\, Q_{\underline{b}]}{}^{m|\underline{d}]} - H_{abm}\, R^{mcd}\,,\\
0 &= +\,3\, Q_a{}^{m[\underline{b}}\, Q_m{}^{\underline{cd}]} + 3\, f_{am}{}^{[\underline{b}}\, R^{m|\underline{cd}]}\,,\\
0 &= +\,3\, Q_m{}^{[\underline{ab}}\, R^{m|\underline{cd}]}\,.
\end{aligned}$$

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2. non-geometry
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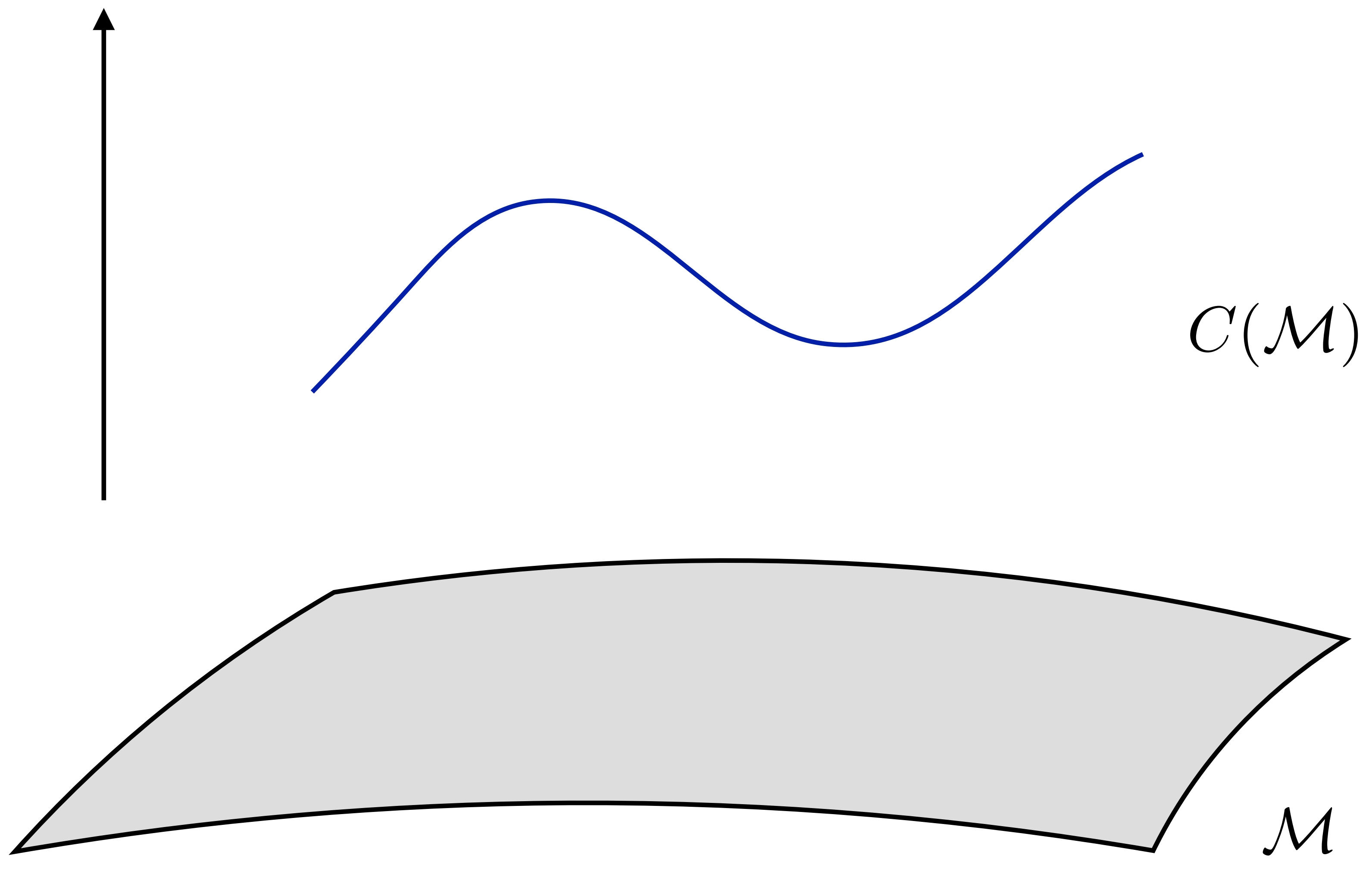
   a) non-geometric fluxes
   b) non-commuting structures
   c) compactifications
   d) outlook
4. conclusions

A geometric space $\mathcal{M}$ can be described in terms of the algebra of functions $C(\mathcal{M})$ on $\mathcal{M}$.

Gelfand, Naimark - 1943



A non-commutative space is characterized through a non-commutative algebra of functions (e.g. Moyal-Weyl $\star$-product).

Connes - 1986

A non-associative space is described by a non-associative algebra of functions.

Result :: A non-geometric *R*-flux gives rise to a non-associative structure.

Consider a three-sphere with *H*-flux ($SU(2)$ WZW model) ::

- Determine the following equal-time Jacobiator

$$\big[X^\mu, X^\nu, X^\rho\big] := \lim_{\sigma_i \to \sigma} \big[[X^\mu(\tau,\sigma_1), X^\nu(\tau,\sigma_2)], X^\rho(\tau,\sigma_3)\big] + \text{cyclic}\,.$$

- After T-duality, one obtains

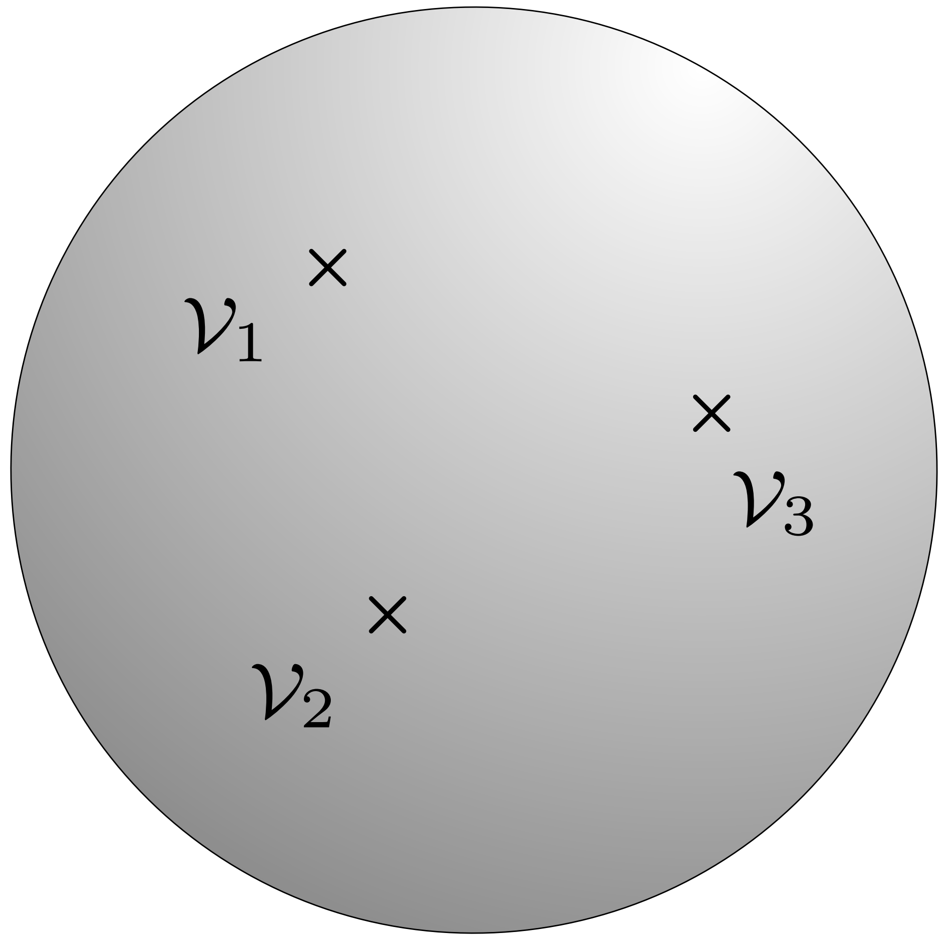
$$\big[X^\mu, X^\nu, X^\rho\big] = 3\pi^2 R^{\mu\nu\rho}\,.$$

- This behaviour is described by a non-associative tri-product of functions

$$\big(f_1 \vartriangle f_2 \vartriangle f_3\big)(x) := \exp\Big(\tfrac{\pi^2}{2}\, R^{\mu\nu\rho}\, \partial^{x_1}_\mu\, \partial^{x_2}_\nu\, \partial^{x_3}_\rho\Big)\, f_1(x_1)\, f_2(x_2)\, f_3(x_3)\Big|_{x_1=x_2=x_3=x}\,.$$

Blumenhagen, Plauschinn - 2010

The non-associative property can also been seen from the behaviour of vertex operators $\mathcal{V}_i$ under permutations.



World-sheet sphere diagram with three vertex-operator insertions.

In an *R*-flux background one finds for $\sigma \in S_3$

$$\left\langle \mathcal{V}_{\sigma(1)}\mathcal{V}_{\sigma(2)}\mathcal{V}_{\sigma(3)}\right\rangle = \exp\left[i\pi^2 R^{\mu\nu\rho}(p_1)_\mu(p_2)_\nu(p_3)_\rho\,\eta_\sigma\right]\left\langle \mathcal{V}_1\,\mathcal{V}_2\,\mathcal{V}_3\right\rangle .$$

This is an off-shell result — on-shell (that means $p_1 + p_2 + p_3 = 0$) the phase factor is trivial.

Blumenhagen, Deser, Lüst, Plauschinn, Rennecke - 2011

Application ::

- The tri-product can be used for a non-associative differential geometry,
- and the construction of a non-associative gravity theory.

→ Space-time is non-associative at small distances.

Blumenhagen, Plauschinn - 2010
Blumenhagen, Fuchs - 2016
Aschieri, Ciric, Szabo - 2017

Result :: A non-geometric Q-flux gives rise to a non-commutative structure.

Consider non-trivial $\mathbb{T}^2$-fibrations over the circle, parametrized by Θ^{ab} ::

- The fiber coordinates can be written as $X^{\mathsf{a}} = X_L^{\mathsf{a}} + X_R^{\mathsf{a}}$, with relations

$$\left[X_L^{\mathsf{a}}, X_L^{\mathsf{b}}\right] = \tfrac{i}{2}\,\Theta_1^{\mathsf{ab}}\,, \qquad \left[X_L^{\mathsf{a}}, X_R^{\mathsf{b}}\right] = 0\,, \qquad \left[X_R^{\mathsf{a}}, X_R^{\mathsf{b}}\right] = \tfrac{i}{2}\,\Theta_2^{\mathsf{ab}}\,.$$

- For a geometric background one finds $\Theta_1^{\mathsf{ab}} = -\Theta_2^{\mathsf{ab}} = \Theta^{\mathsf{ab}}$ and $\left[X^{\mathsf{a}}, X^{\mathsf{b}}\right] = 0\,.$

- For a non-geometric background one obtains $\Theta_1^{\mathsf{ab}} = +\Theta_2^{\mathsf{ab}} = \Theta^{\mathsf{ab}}$ and hence

$$\left[X^{\mathsf{a}}, X^{\mathsf{b}}\right] = i\,\Theta^{\mathsf{ab}}\,.$$

Define an equal-time commutator between two closed-string coordinates in the following way

$$\left[X^1, X^2\right] = \lim_{\sigma_i \to \sigma} \left[X^1(\tau, \sigma_1), X^2(\tau, \sigma_2)\right] .$$

- For the T-fold background (parabolic monodromy) one finds a non-commutative behaviour controlled by the Q-flux

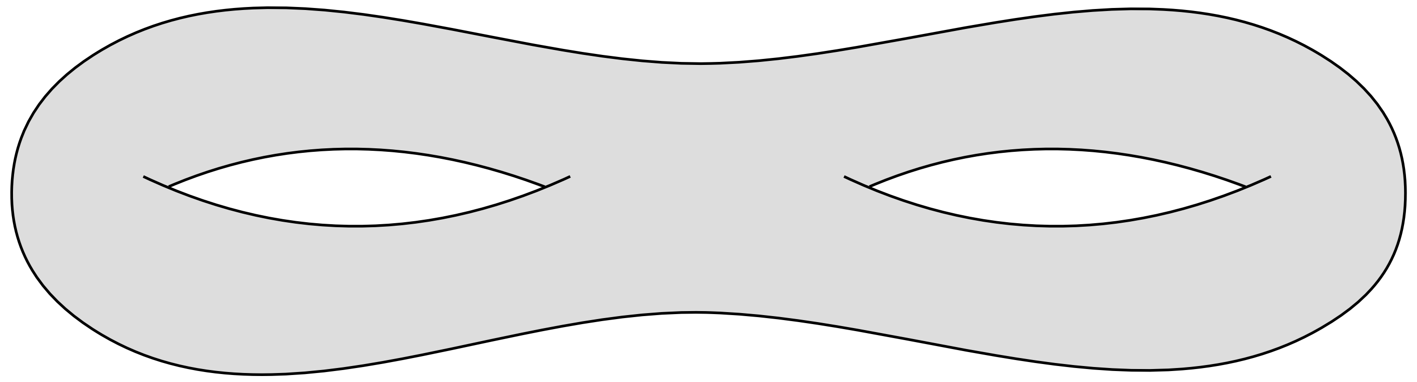
$$\left[X^1, X^2\right] = -\frac{i\pi^2}{6} \oint Q_3{}^{12} dX^3 .$$

- Other examples (elliptic monodromies) are provided by asymmetric orbifolds.

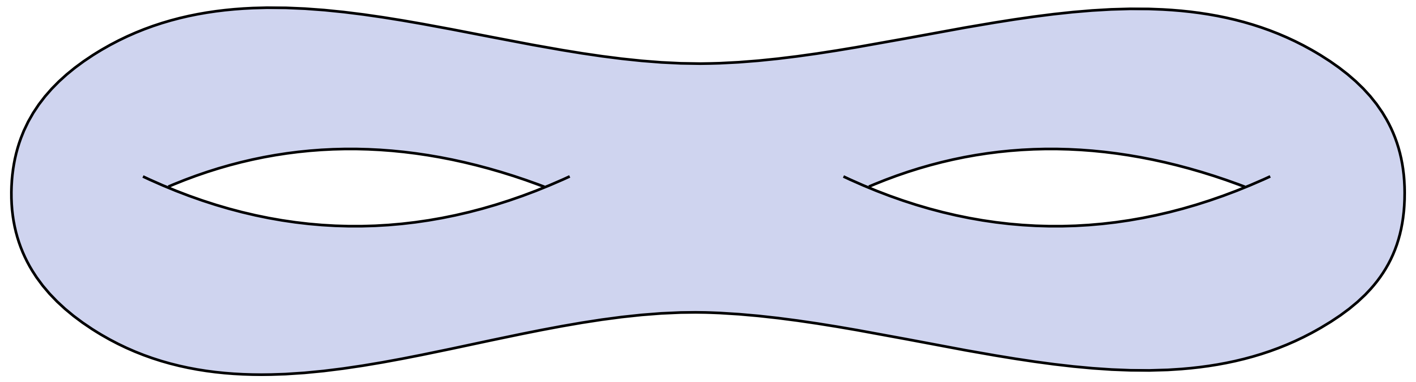
Andriot, Larfors, Lüst, Patalong - 2012

String-theory compactifications on Calabi-Yau three-folds ::

compact space



CY3



CY3 + fluxes H, f, Q, R

↓ ↓

4D theory

- no potential generated
→ massless scalar fields T^a, U^A, S

- potential generated by fluxes
→ massive scalar fields T^a, U^A, S

The scalar potential in 4D (for a $N=1$ supergravity theory) is in general given by

$$V = V_F + V_D$$

$$V_F = e^{\mathcal{K}} \left(\mathcal{G}^{\alpha\overline{\beta}} D_\alpha W \, D_{\overline{\beta}} \overline{W} - 3 \big| W \big|^2 \right)$$

$$\mathcal{G}_{\alpha\overline{\beta}} = \partial_\alpha \partial_{\overline{\beta}} \mathcal{K}$$

$$D_\alpha W = \partial_\alpha W + \big(\partial_\alpha \mathcal{K}\big) W$$

Kähler potential $\mathcal{K}$

superpotential W

$$V_D = \frac{1}{2} \left[(\mathrm{Re}\, f)^{-1} \right]^{ij} \mathcal{P}_i \mathcal{P}_j$$

gauge-kinetic function f_{ij}

moment maps $\mathcal{P}_i$

For type IIB compactifications on Calabi-Yau orientifolds $\mathcal{M}$, fluxes contribute to the superpotential ::

$$W = \int_{\mathcal{M}} \Phi^- \wedge \left(F_3 + \mathcal{D}\Phi^+ \right)$$

- Φ^- depends on U^A,
- Φ^+ depends on T^a, S,
- F_3 is a R-R flux,
- $\mathcal{D}$ is a twisted differential.

The twisted differential $\mathcal{D}$ contains the NS-NS fluxes (derived via T-duality) ::

$$\mathcal{D} = d \; + H$$

Shelton, Taylor, Wecht - 2005

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$$\mathcal{D} = D + H - F + Q - R$$

$$\begin{aligned} H &= \tfrac{1}{6}\, H_{ijk}\, dx^i \wedge dx^j \wedge dx^k \,, \\ F &= \tfrac{1}{2}\, F_{ij}{}^k\, dx^i \wedge dx^j \wedge \iota_k \,, \\ Q &= \tfrac{1}{2}\, Q_i{}^{jk}\, dx^i \wedge \iota_j \wedge \iota_k \,, \\ R &= \tfrac{1}{6}\, R^{ijk}\, \iota_i \wedge \iota_j \wedge \iota_k \,. \end{aligned}$$

Shelton, Taylor, Wecht - 2005

The fluxes in the twisted differential can be interpreted as operators

$$\mathcal{D} = d + H - F + Q - R\,,$$

$$\begin{array}{lcccl} H & : & p\text{-form} & \rightarrow & (p+3)\text{-form}\,, \\ F & : & p\text{-form} & \rightarrow & (p+1)\text{-form}\,, \\ Q & : & p\text{-form} & \rightarrow & (p-1)\text{-form}\,, \\ R & : & p\text{-form} & \rightarrow & (p-3)\text{-form}\,. \end{array}$$

Aldazabal, Camara, Font, Ibanez - 2006
Villadoro, Zwirner - 2006
Shelton, Taylor, Wecht - 2006

When requiring nil-potency $\mathcal{D}^2 = 0$, Bianchi identities for the fluxes can be derived.

Shelton, Taylor, Wecht - 2006
Robins, Wrase - 2007

Summary ::

- Non-geometric fluxes are a natural part of string-theory compactifications.
- T-duality (mirror symmetry) requires geometric and non-geometric fluxes.
- Fluxes give masses to scalar fields.

outline

Outlook ::

- Non-commutative & non-associative structures.

  Blumenhagen, Plauschinn - 2010
  Lüst - 2010
  Mylonas, Schupp, Szabo - 2012

- Moduli stabilization and inflation.

  Shelton, Taylor, Wecht - 2006
  e.g. Blumenhagen, Herschmann, Plauschinn - 2014

- Origin for gauged supergravity theories.

  Grana, Louis, Waldram - 2005
  Cassani - 2008

- Description via generalized & doubled geometry.

  Hull - 2004
  Grana, Minasian, Petrini, Waldram - 2008

- More general non-geometric torus-fibrations.

  Hellermann, McGreevy, Williams - 2002
  de Boer, Shigemori - 2012
  Lüst, Massai, Vall-Camell - 2015

- …

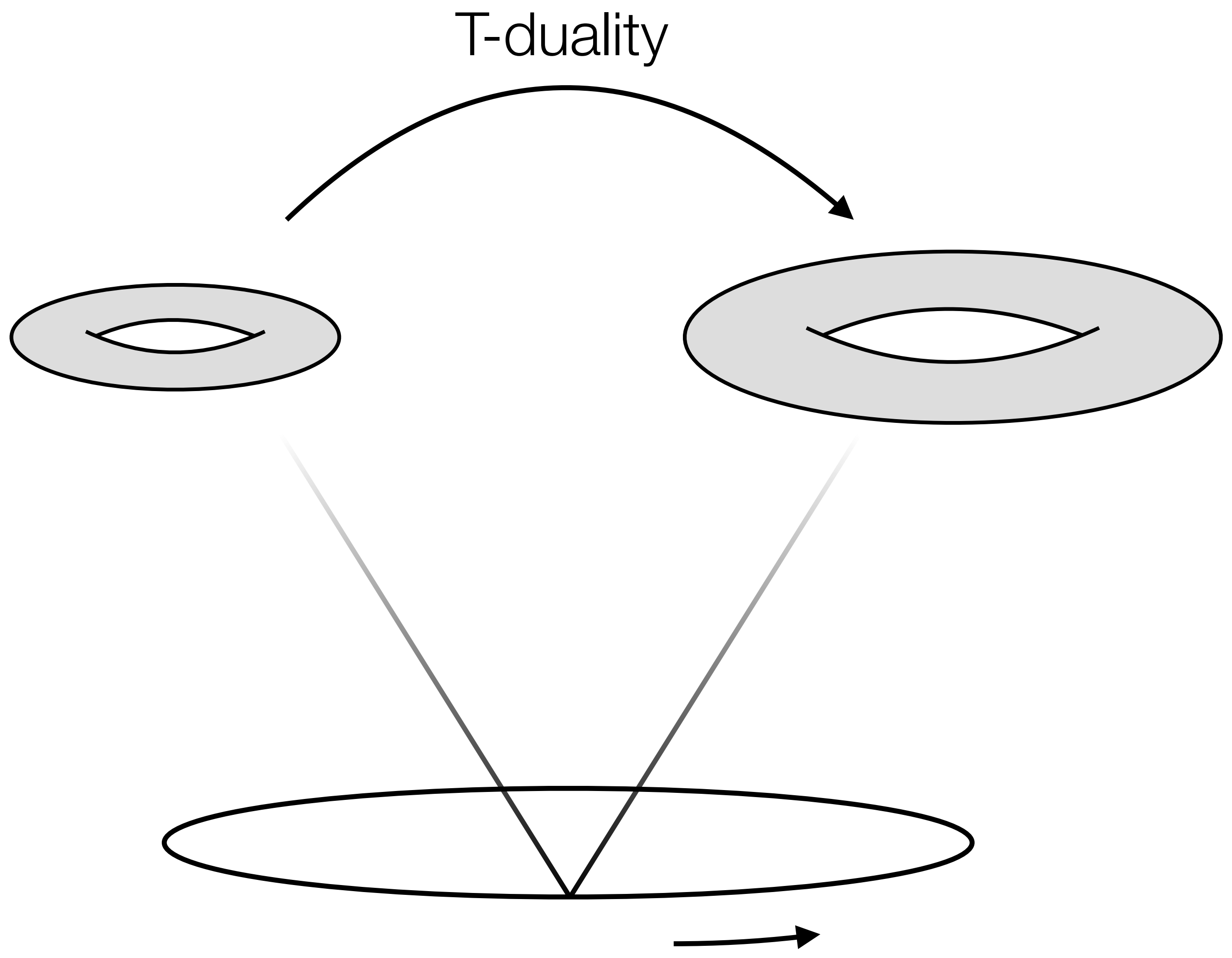
outline

String theory ::

- String theory is a theory of quantum gravity including gauge interactions.
- The theory features a large number of dualities — including T-duality.
- For a description of our universe, need to compactify.

Non-geometry ::

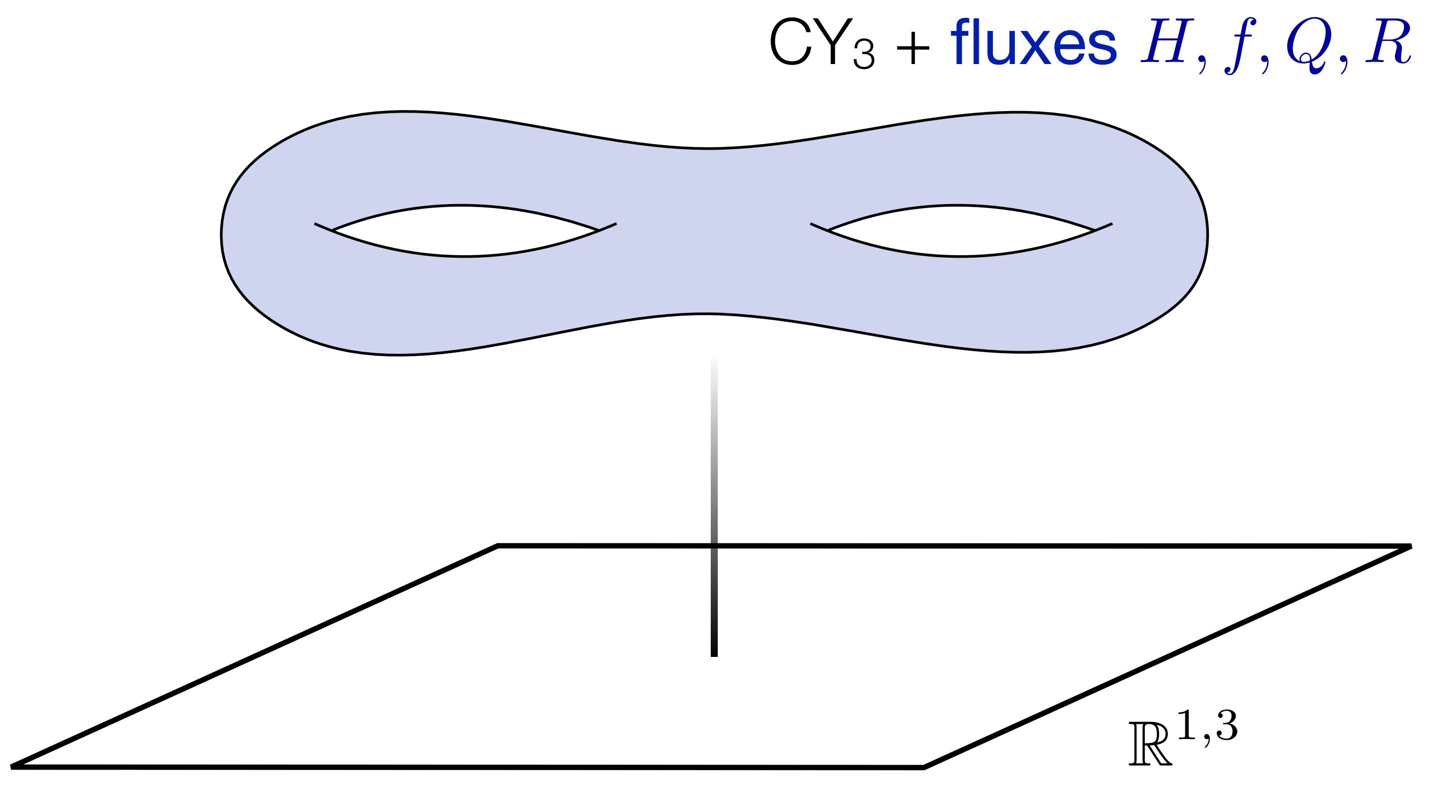
- Non-geometric backgrounds are well-defined using T-duality transformations.
- Such spaces are natural in string theory, but inconsistent for point particles.
- The non-triviality of such backgrounds is encoded in non-geometric fluxes.



- Non-geometric fluxes give rise non-commutative & non-associative structures.

Compactification ::

- Non-geometric backgrounds are examples for compactification spaces.
- Fluxes generate masses for scalar fields to match with experimental constraints.



- T-duality (mirror symmetry) relates different compactifications to each other.