Poisson-Lie T-duality in Double Field Theory

Falk Hassler

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based on

1707.08624, 1611.07978

and

181?.????? with Saskia Demulder and Daniel Thompson

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Universidad de Oviedo
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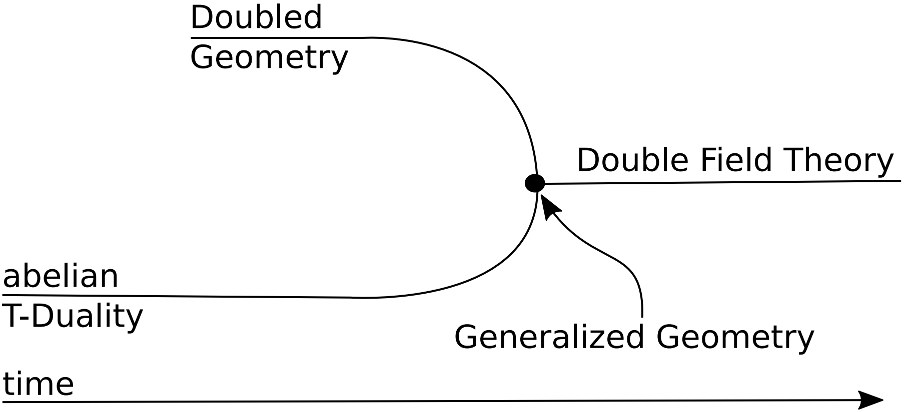
Motivation

Chris Hull
Barton Zwiebach
Olaf Hohm



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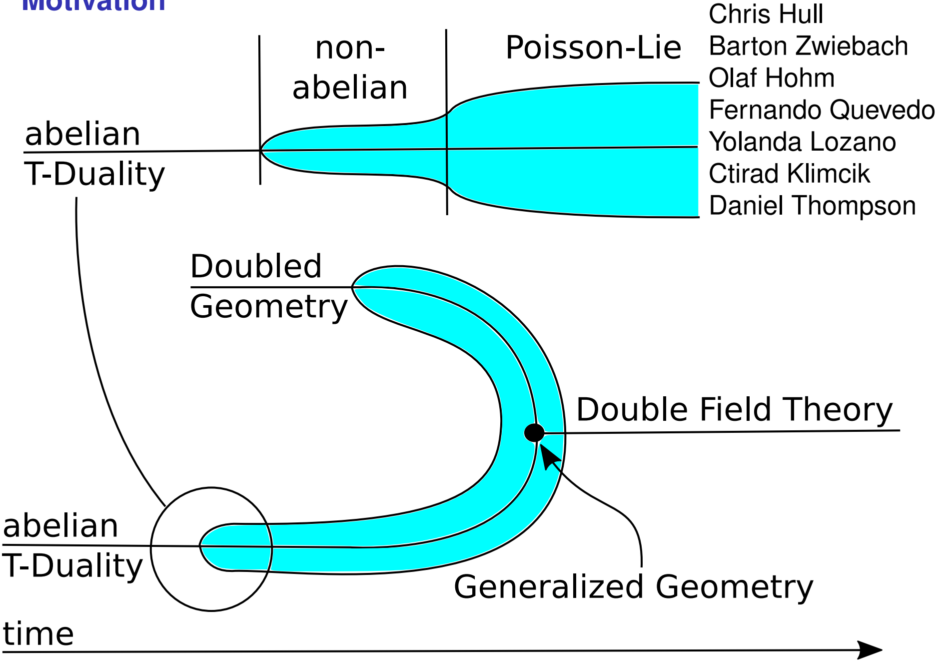
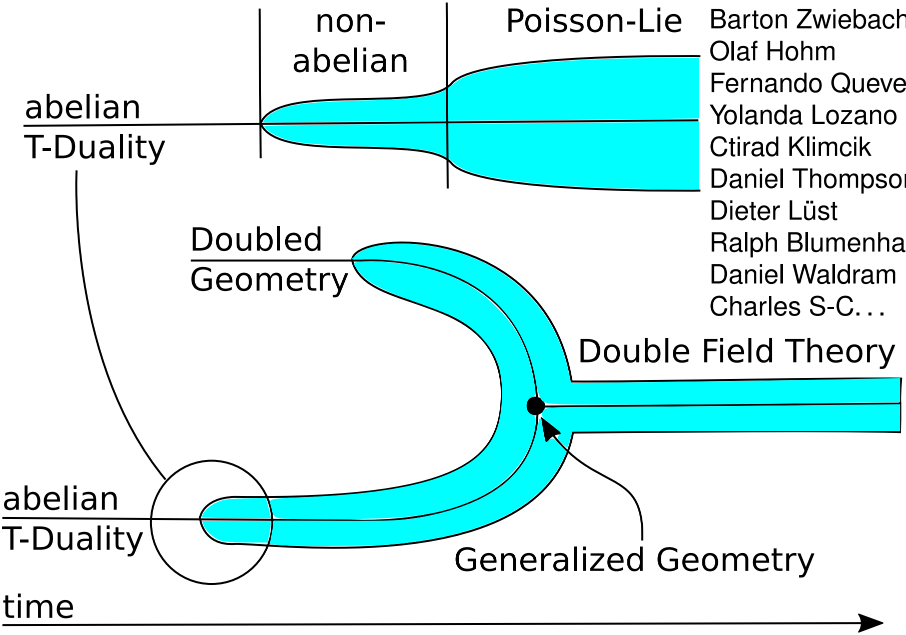

non-
abelian
Poisson-Lie
Chris Hull
Barton Zwiebach
Olaf Hohm
Fernando Quevedo
Yolanda Lozano
Ctirad Klimcik
Daniel Thompson
abelian
T-Duality
Doubled
Geometry
Double Field Theory
abelian
T-Duality
Generalized Geometry
time

Motivation



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Ctirad Klimcik
Daniel Thompson
Dieter Lüst
Ralph Blumenhagen
Daniel Waldram
Charles S-C. . .

Outline

1. Motivation

2. Poisson-Lie T-duality

3. Double Field Theory on Drinfeld doubles

4. Application: 1. Dilaton transformation

5. Summary

Drinfeld double [Drinfeld, 1988]

Definition: A **Drinfeld double** is a $2D$-dimensional Lie group $\mathcal{D}$, whose Lie-algebra $\mathfrak{d}$

1. has an ad-invariant bilinear for $\langle\cdot\,,\,\cdot\rangle$ with signature (D, D)
2. admits the decomposition into two maximal isotropic subalgebras $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$

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- $\begin{pmatrix} t^a & t_a \end{pmatrix} = t_A \in \mathfrak{d}\,, \quad t_a \in \mathfrak{g}$ and $t^a \in \tilde{\mathfrak{g}}$
- $\langle t_A, t_B \rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix}$

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- $\langle t_A, t_B\rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix}$
- $[t_A, t_B] = F_{AB}{}^C t_C$ with non-vanishing commutators

$$[t_a, t_b] = f_{ab}{}^c t_c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$
$$[t^a, t^b] = \tilde{f}^{ab}{}_c t^c$$

- ad-invariance of $\langle\cdot\,,\,\cdot\rangle$ implies $F_{ABC} = F_{[ABC]}$

Poisson-Lie T-duality: 1. Definition [Klimcik and Severa, 1995]

- 2D σ-model on target space M with action

  $S(E, M) = \int dzd\bar{z}\, E_{ij}\partial x^i \bar{\partial} x^j$
- $E_{ij} = g_{ij} + B_{ij}$ captures metric and two-from field on M
- inverse of E_{ij} is denoted as E^{ij}

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- *left* invariant vector field $v_a{}^i$ on G is the inverse transposed of *right* invariant Maurer-Cartan form $t_a v^a{}_i dx^i = dg\, g^{-1}$
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Definition: $S(E, \mathcal{D}/\tilde{G})$ and $S(\tilde{E}, \mathcal{D}/G)$ are **Poisson-Lie T-dual** if

$$E^{ij} = v_c{}^i M_a{}^c (M^{ae} M^b{}_e + E_0^{ab}) M_b{}^d v_d{}^j$$
$$\tilde{E}^{ij} = \tilde{v}^{ci} \tilde{M}^a{}_c (\tilde{M}_{ae} \tilde{M}_b{}^e + E_{0\,ab}) \tilde{M}^b{}_d \tilde{v}^{dj}$$

holds, where E_0^{ab} is constant and invertible with the inverse $E_{0\,ab}$.

Poisson-Lie T-duality: 2. Properties

- captures $\left\{ \begin{array}{llll} \text{abelian T-d.} & G \text{ abelian} & \text{and} & \tilde{G} \text{ abelian} \\ \text{non-abelian T-d.} & G \text{ non-abelian} & \text{and} & \tilde{G} \text{ abelian} \end{array} \right.$

  [Ossa and Quevedo, 1993;Giveon and Rocek, 1994; Alvarez, Alvarez-Gaume, and Lozano, 1994;. . .]

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- dual σ-models related by canonical transformation

  [Klimcik and Severa, 1995;Klimcik and Severa, 1996;Sfetsos, 1998]

$\rightarrow$ equivalent at the classical level

- preserves conformal invariance at one-loop

  [Alekseev, Klimcik, and Tseytlin, 1996;Sfetsos, 1998;. . . ;Jurco and Vysoky, 2017]

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  [Alekseev, Klimcik, and Tseytlin, 1996;Sfetsos, 1998;...;Jurco and Vysoky, 2017]
- dilaton transformation [Jurco and Vysoky, 2017]

$$\phi = -\frac{1}{2}\log\left|\det\left(1+\tilde{g}_0^{-1}(\tilde{B}_0+\Pi)\right)\right|$$

$$\tilde{\phi} = -\frac{1}{2}\log\left|\det\left(1+g_0^{-1}(B_0+\tilde{\Pi})\right)\right| \qquad \text{details later}$$

2D σ-model perspective

SUGRA perspective

Additional structure on the Drinfeld double

[Blumenhagen, Hassler, and Lüst, 2015, Blumenhagen, Bosque, Hassler, and Lüst, 2015]

- *right* invariant vector $E_A{}^I$ field on $\mathcal{D}$ is the inverse transposed of *left* invariant Maurer-Cartan form $t_A E^A{}_I dX^I = g^{-1} dg$

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- two η-compatible, covariant derivatives[1]
  1. flat derivative

     $$D_A V^B = E_A{}^I \partial_I V^B$$

  2. convenient derivative

     $$\nabla_A V^B = D_A V^B + \frac{1}{3} F_{AC}{}^B V^C - w F_A \qquad F_A = D_A \log \left|\det(E^B{}_I)\right|$$

[1] definitions here just for quantities with flat indices

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- generalized metric $\mathcal{H}_{AB}$ ($w = 0$)

$$\mathcal{H}_{AB} = \mathcal{H}_{(AB)}, \qquad \mathcal{H}_{AC} \eta^{CD} H_{DB} = \eta_{AB}$$

- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$

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- triple $(\mathcal{D}, \mathcal{H}_{AB}, d)$ captures the doubled space of DFT

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Double Field Theory for $(\mathcal{D}, \mathcal{H}_{AB}, d)$ [Blumenhagen, Bosque, Hassler, and Lüst, 2015]

see also [Vaisman, 2012; Hull and Reid-Edwards, 2009;Geissbuhler, Marques, Nunez, and Penas, 2013; Cederwall, 2014; . . .]

- action ($\nabla_A d = -\frac{1}{2} e^{2d} \nabla_A e^{-2d}$)

$$S_{\mathrm{NS}} = \int_{\mathcal{D}} d^{2D} X e^{-2d} \Big(\frac{1}{8} \mathcal{H}^{CD} \nabla_C \mathcal{H}_{AB} \nabla_D \mathcal{H}^{AB} - \frac{1}{2} \mathcal{H}^{AB} \nabla_B \mathcal{H}^{CD} \nabla_D \mathcal{H}_{AC}$$
$$- 2 \nabla_A d \nabla_B \mathcal{H}^{AB} + 4 \mathcal{H}^{AB} \nabla_A d \nabla_B d + \frac{1}{6} F_{ACD} F_B{}^{CD} \mathcal{H}^{AB} \Big)$$

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- $2D$-diffeomorphisms

$$L_\xi V^A = \xi^B D_B V^A + w D_B \xi^B V^A$$

- global O(D,D) transformations

$$V^A \to T^A{}_B V^B \quad \text{with} \quad T^A{}_C T^B{}_D \eta^{CD} = \eta^{AB}$$

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- generalized diffeomorphisms

$$\mathcal{L}_\xi V^A = \xi^B \nabla_B V^A + \left(\nabla^A \xi_B - \nabla_B \xi^A \right) V^B + w \nabla_B \xi^B V^A$$

- section condition (SC)

$$\eta^{AB} D_A \cdot D_B \cdot = 0$$

Symmetries of the action

- S_{NS} invariant for $X^I \rightarrow X^I + \xi^A E_A{}^I$ and
  1. $\mathcal{H}^{AB} \rightarrow \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$ and $e^{-2d} \rightarrow e^{-2d} + \mathcal{L}_\xi e^{-2d}$
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object	gen.-diffeomorphisms	$2D$-diffeomorphisms	global O(D,D)
$\mathcal{H}_{AB}$	tensor	scalar	tensor
$\nabla_A d$	not covariant	scalar	1-form
e^{-2d}	scalar density (w=1)	scalar density (w=1)	invariant
η_{AB}	invariant	invariant	invariant
$F_{AB}{}^C$	invariant	invariant	tensor
$E_A{}^I$	invariant	vector	1-form
S_{NS}	invariant	invariant	invariant
SC	invariant	invariant	invariant
D_A	not covariant	covariant	covariant
∇_A	not covariant	covariant	covariant

manifest

Poisson-Lie T-duality: 1. Solve SC [Hassler, 2016]

- fix D physical coordinates x^i from $X^I = \begin{pmatrix} x^i & x^{\tilde{i}} \end{pmatrix}$ on $\mathcal{D}$

  such that $\eta^{IJ} = E_A{}^I \eta^{AB} E_B{}^J = \begin{pmatrix} 0 & \cdots \\ \cdots & \cdots \end{pmatrix} \to$ SC is solved
- fields and gauge parameter depend just on x^i

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- only *two* SC solutions, relate them by symmetries of DFT

$$d(X^I) = g(x^i)\tilde{g}(x^{\tilde{i}}) \qquad\qquad t_A = \begin{pmatrix} t^a & t_a \end{pmatrix}$$

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- duality target spaces arise as different solutions of the SC

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$$\underbrace{\text{• 2D-diffeomorphisms } X^I \to X'^I(X^1, \dots X^{2D}) \text{ with } d(X^I) = d(X'^I) \quad \text{• global O}(D,D) \text{ transformation } t_A \to \eta^{AB} t_B}_{\text{manifest symmtries of DFT}}$$

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$\rightarrow$ no 2D-diffeomorphisms needed, only global O(D,D) transformation

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- for abelian T-duality $X^I \to X'^I = X^I$

$\to$ no 2D-diffeomorphisms needed, only global O(D,D) transformation

Poisson-Lie T-duality is a manifest symmetry of DFT

Equivalence to supergravity: 1. Generalized parallelizable spaces

[Lee, Strickland-Constable, and Waldram, 2014]

- generalized tangent space element $V^{\widehat{I}} = \begin{pmatrix} V^i & V_i \end{pmatrix}$
- generalized Lie derivative

$$\widehat{\mathcal{L}}_\xi V^{\widehat{I}} = \xi^{\widehat{J}} \partial_{\widehat{J}} V^{\widehat{I}} + \left(\partial^{\widehat{I}} \xi_{\widehat{J}} - \partial_{\widehat{J}} \xi^{\widehat{I}}\right) V^{\widehat{J}} \qquad \text{with} \qquad \partial_{\widehat{I}} = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$$

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Definition: A manifold M which admits a globally defined generalized frame field $\widehat{E}_A{}^{\hat{I}}(x^i)$ satisfying

1. $\widehat{\mathcal{L}}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} = F_{AB}{}^C \widehat{E}_C{}^{\hat{I}}$

   where $F_{AB}{}^C$ are the structure constants of a Lie algebra $\mathfrak{h}$
2. $\widehat{E}_A{}^{\hat{I}} \eta^{AB} \widehat{E}_B{}^{\hat{J}} = \eta^{\hat{I}\hat{J}} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^i_j & 0 \end{pmatrix}$

is a **generalized parallelizable space** $(M, \mathfrak{h}, \widehat{E}_A{}^{\hat{I}})$.

Equivalence to supergravity: 2. Generalized metric and dilaton

[Klimcik and Severa, 1995; Hull and Reid-Edwards, 2009; du Bosque, Hassler, Lüst, 2017]

- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\widehat{I}})$$

$$\widehat{E}_A{}^{\widehat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\widehat{I}}$$

and

$$(D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\widehat{I}})$$

$$\widetilde{\widehat{E}}_A{}^{\widehat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\widehat{I}}$$

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$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\hat{I}}) \quad \text{and} \quad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\hat{I}})$$

$$\widehat{E}_A{}^{\hat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\hat{I}} \qquad \widetilde{\widehat{E}}_A{}^{\hat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\hat{I}}$$

- express $\mathcal{H}^{AB}$ in terms of the generalized $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}$ on $TD/\tilde{G} \oplus T^*D/\tilde{G}$

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- express d in terms of the standard generalized dilaton $\widehat{d}$

$$d = \widehat{d} - \frac{1}{2} \log |\det \tilde{v}_{ai}|$$

$$\widehat{d} = \phi - 1/4 \log \left|\det g_{ij}\right|$$

Equivalence to supergravity: 2. Generalized metric and dilaton

[Klimcik and Severa, 1995; Hull and Reid-Edwards, 2009; du Bosque, Hassler, Lüst, 2017]

- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\hat{I}}) \qquad \text{and} \qquad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\hat{I}})$$

$$\widehat{E}_A{}^{\hat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\hat{I}} \qquad \widetilde{\widehat{E}}_A{}^{\hat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\hat{I}}$$

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- plug into the DFT action S_{NS}

Equivalence to supergravity: 3. IIA/B bosonic sector action

- if G and $\tilde{G}$ are unimodular

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int d^D x \, e^{-2\widehat{d}} \Big(\frac{1}{8} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{K}} \widehat{\mathcal{H}}_{\hat{I}\hat{J}} \partial_{\hat{L}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - 2 \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - \frac{1}{2} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{L}} \widehat{\mathcal{H}}_{\hat{I}\hat{K}} + 4 \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{d} \Big)$$

- $V_{\tilde{G}} = \int_{\tilde{G}} d\tilde{x}^D \, \det \tilde{v}_{ai}$ volume of group $\tilde{G}$

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[Hohm, Hull, and Zwiebach, 2010; Hohm, Hull, and Zwiebach, 2010]

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int \mathrm{d}^D x \sqrt{\det(g_{ij})} e^{-2\phi} \big(\mathcal{R} + 4 \partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \big)$$

- holds for all $\mathcal{H}_{AB}(x^i)$ / $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}(x^i)$
- only D-diffeomorphisms and B-field gauge trans. as symmetries

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- similar story for R/R sector

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie T-duality

- in general $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality } (2D\text{-diff.})} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
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A doubled space $(\mathcal{D}, \mathcal{H}_{AB}, d)$ admits Poisson-Lie T-dual (is PL symmetric) supergravity descriptions iff

1. $L_\xi \mathcal{H}_{AB} = 0 \quad \forall \xi \quad \to \quad D_A \mathcal{H}_{AB} = 0$
2. $L_\xi d = 0 \quad \forall \xi \quad \to \quad (D_A - F_A) e^{-2d} = 0$

Application: Dilaton transformation

- $(D_A - F_A)e^{-2d} = 0 \quad \rightarrow \quad \partial_I(\underbrace{2d + \log|\det v| + \log|\det \tilde{v}|}_{= 2\phi_0 = \text{const.}}) = 0$

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- $g = v^T e^T e v \quad$ with $\begin{cases} (\tilde{B}_0 + \tilde{g}_0)^{ab} = E^{0\,ab} \\ \Pi^{ab} = M^{ac} M^b{}_c \\ e^{-1}e^{-T} = \tilde{g}_0 - (\tilde{B}_0 + \Pi)\tilde{g}_0^{-1}(\tilde{B}_0 + \Pi) \\ \tilde{e}_0^T \tilde{e}_0 = \tilde{g}_0 \\ e^{-T} = \tilde{e}_0 + \tilde{e}_0^{-T}(\tilde{B}_0 + \Pi) \end{cases}$

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- $\phi = \phi_0 + \frac{1}{2}\log|\det e| = \phi_0 - \frac{1}{2}\log|\det \tilde{e}_0| - \frac{1}{2}\log\left|\det\left(1 + \tilde{g}_0^{-1}(\tilde{B}_0 + \Pi)\right)\right|$
- reproduces [Jurco and Vysoky, 2017]

2. R/R sector transformation: O(D,D) Majorana-Weyl spinors on $\mathcal{D}$

[Hohm, Kwak, and Zwiebach, 2011,Hassler, 2016]

- Γ-matrices: $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$
- chirality Γ_{2D+1} with $\{\Gamma_{2D+1}, \Gamma_A\} = 0$
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- spinor can be expressed as $\chi = \sum_{p=0}^{D} \frac{1}{p!2^{p/2}} C^{(p)}_{a_1\dots a_p} \Gamma^{a_1\dots a_p}|0\rangle$
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- O(D,D) transformation in spinor representation

  $$\mathcal{S}_\mathcal{O}\Gamma_A\mathcal{S}_\mathcal{O}^{-1} = \Gamma_B\mathcal{O}^B{}_A \qquad \mathcal{O}^T\eta\mathcal{O} = \eta$$

R/R sector of DFT on $\mathcal{D}$ [Hassler, 2017]

- action $S_{\mathrm{RR}} = \frac{1}{4} \int d^{2d} X \, (\nabla\!\!\!/ \chi)^\dagger \, S_{\mathcal{H}} \, \nabla\!\!\!/ \chi$
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- transport χ to the generalized tangent space:

  $$\widehat{\chi} = |\det \tilde{e}_{ai}|^{-1/2} S_{\widehat{E}} \chi \qquad (\, t^a \tilde{e}_{ai} = \tilde{g}^{-1} d\tilde{g} \,)$$

- same for covariant derivative

  $$|\det \tilde{e}_{ai}|^{-1/2} S_{\widehat{E}} \nabla\!\!\!/ \chi = \partial\!\!\!/ \widehat{\Gamma}^{\hat{I}} \widehat{\chi} \qquad S_{\widehat{E}} \Gamma^A S_{\widehat{E}}^{-1} \widehat{E}_A{}^{\hat{I}} = \widehat{\Gamma}^{\hat{I}} \quad \text{and} \quad \partial\!\!\!/ = \widehat{\Gamma}^i \partial_i$$

Equivalence to SUGRA [Hassler, 2017]

- introduce field strength $\widehat{F} = e^{\phi} S_B \not{\partial} \widehat{\chi}$ and $\mathbf{d} = e^{\phi} S_B \not{\partial} S_B^{-1} e^{-\phi}$
- DFT R/R field equations: $\not{\nabla}(\mathcal{K}\not{\nabla})\chi = 0$
- rewrite them as:

  $\mathbf{d}(\star \mathbf{d}\widehat{F}) = 0 \qquad \star = C^{-1} S_g^{-1}$
- plus Bianchi identity (BI)

  $\mathbf{d}\widehat{F} = 0$
- action on polyforms

  $$\begin{array}{ccc} \mathbf{d} & \leftrightarrow & d + H \wedge - d\phi \\ \star & \leftrightarrow & \star \end{array}$$
- matches the R/R sector of SUGRA

Transformation rules

- again require PL symmetric for G: $(D_A - \frac{1}{2}F_A)G = 0$
- on the gen. tangent space $\widehat{G} = |\det v|^{1/2} S_{\widehat{E}} G_0$, G_0 = constant
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R/R fields transform under Poisson-Lie T-duality as

$$\widetilde{\widehat{G}} = |\det v^{-1} \tilde{v}|^{1/2} S_{\widetilde{\widehat{E}} \widehat{E}^{-1}} \widehat{G}$$

Remarks:

- generalized frame fields $\widehat{E}$ and $\widetilde{\widehat{E}}$ are know explicitly
- transform to differential forms with the O(D,D) spinor map
- first derivation of the R/R rules for PL TD

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- various new directions for research in DFT
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  - Drinfeld doubles $\to$ quantum groups $\to$ rich mathematical structure
  - new way to organized α' corrections?
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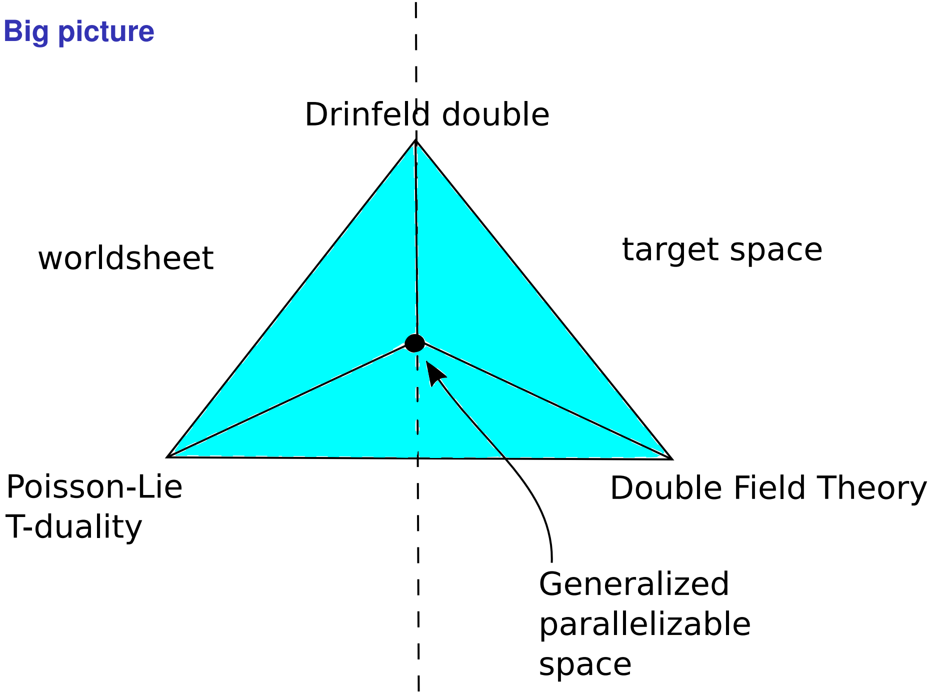
- facilitates new applications
  - integrable deformations of 2D σ-models
  - solution generating technique
  - explore underlying structure of AdS/CFT

Big picture


Drinfeld double
worldsheet
target space
Poisson-Lie
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Double Field Theory
Generalized
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