Ion motion and hosing suppression
in plasma-based accelerators

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Overview

- Witness bunch parameters of interest for a plasma-based collider trigger hosing and ion motion

- Mitigation/suppression of hosing by means of change of betatron frequency along the bunch (chirp)

- Bunch-induced ion motion as a way to generate betatron frequency chirp that suppresses hosing
  - analytical expression for wake with ion motion
  - theory describing hosing in presence of ion motion
  - 3D PIC simulations show hosing suppression with ion motion
  - emittance degradation associated with ion motion eliminated by proper bunch shaping

- Summary
Concept for next-generation TeV-class PA-based linear collider: requires bunches with high charge and small emittance

Laser-driven PA-based LC
[all optical, driver=10s J laser, 10 kHz, 50x10GeV PA stages @ $n_0=10^{17}$ cm$^{-3}$]

Leemans, Esarey, Physics Today (2009)
Schroeder et al., PRSTAB (2010)

Beam-driven PA-based LC
[driver=25 GeV e-bunch, 19x25 GeV PA stages @ $n_0=10^{17}$ cm$^{-3}$]

Seryi et al., PAC 2009
Delahaye et al., IPAC 2014

- Compact machine (1 TeV): $\leq$1 Km for PA-based LC VS $\sim$30 Km for RF-based (ILC)
- Requirements on witness bunch:
  $\rightarrow N_b \sim 10^{10}$ part./bunch ($\sim$ nC), $\varepsilon_n < 100$ nm, high wake $\rightarrow$ witness bunch efficiency ($\sim$40%)

Witness bunch parameters of interest for a plasma-based collider induce hosing and ion motion resulting in emittance degradation

**High charge + high-efficiency:**
- large longitudinal wake driven by beam (i.e., high beamloading)
- high beamloading implies strong coupling between beam and wake
- resonance between beam centroid motion and wake centroid → hosing

Whittum et al., PRA (1992)  
Schroeder et al., PRL (1999)  
Huang et al., PRL (2007)  
Lehe et al., PRL (2017)  
Mehrling et al., POP (2018)  
Lebedev et al., PRAB (2017)
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**Low emittance + high charge + high energy:**
→ small matched beam size and high beam density
→ large beam space charge fields → **background ion motion**

\[ \sigma_x^2 = \frac{\epsilon_n}{\gamma k_p} = \sqrt{\frac{2}{\gamma}} \frac{\epsilon_n}{k_p} \]

Condition for ion motion:
\[ \Gamma = \frac{Z_i}{M_i} \frac{n_{b,0}}{n_0} (k_p L_b)^2 \sim 1 \]

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Rosenzweig et al., PRL (2005)
An et al., PRL (2017)
Benedetti et al., PRAB (2017)
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**Transverse wakefields in the blowout regime**
- \( \frac{W_r}{E_0} = \frac{k_p r}{2} \)

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- Small matched beam size and high beam density
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**Condition for ion motion:**
\[
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Chirp of betatron frequency suppresses hosing

- Head-to-tail variation in focusing force affects hosing

\[
\frac{\partial^2 x_c(\xi, z)}{\partial z^2} + k_p^2(\xi) x_c(\xi, z) = k_p^2 \int_0^{\xi} \frac{n_b(\xi')}{n_p} x_c(\xi', z) \sin[\kappa_p(\xi' - \xi)] \kappa_p d\xi'.
\]

- Positive chirp, \(\partial_\xi k_p > 0\)

- Negative chirp, \(\partial_\xi k_p < 0\)

Lehe et al., PRL (2017)
Chirp of betatron frequency suppresses hosing

- Head-to-tail variation in focusing force affects hosing

\[ \partial^2 \chi_c(\xi, z) + k_B^2(\xi) x_c(\xi, z) = k_B^2 \int_0^\xi \frac{n_b(\xi')}{n_p} x_c(\xi', z) \sin[k_p(\xi' - \xi)] \kappa_p d\xi'. \]

Complete solution
Early time solution (exponential)
Asymptotic solution (saturation/damping)

Lehe et al., PRL (2017)

- PA stage operating in the quasilinear regime
  → head-to-tail variation in (beam-loaded) focusing force provides linear (or mostly linear) betatron chirp, largely suppresses hosing
Chirp of betatron frequency suppresses hosing

- Head-to-tail variation in focusing force affects hosing

- PA stage operating in the quasilinear regime
  → head-to-tail variation in (beam-loaded) focusing force provides linear (or mostly linear) betatron chirp, largely suppresses hosing

- PA stage operating in the nonlinear/blowout regime
  → head-to-tail variation achieved by energy chirp (similar to BNS damping): \( k_\beta(\xi) = k_p / \sqrt{2\gamma(\xi)} \)

\[ \partial^2_{\xi} x_c(\xi, z) + k_\beta^2(\xi) x_c(\xi, z) = k_\xi \int_{\xi}^{0} \frac{n_b(\xi')}{n_p} x_c(\xi', z) \sin[k_\rho(\xi' - \xi)] k_\rho d\xi'. \]

Lehe et al., PRL (2017)
Modeling tools: INF&RNO and HiPACE

Modeling performed with quasi-static codes
(optimal tool for this problem)

**INF&RNO/QS**

- 2D axisymmetric
- PIC or fluid for plasma
- Quasi-static modality
- Dynamic time step adjustment + subcycling

**HiPACE**

- 3D Cylindrical
- Quasi-static PIC
- Dynamic time step adjustment + subcycling
- High resolution subgrid in witness bunch domain to correctly model ion motion
- Parallelized with MPI

Benedetti et al., AAC2010, AAC2012, ICAP2012, AAC2016, PPCF2017

Mehrling et al., PPCF (2014), AAC2018
An analytical expression for the perturbed wakefield in presence of ion motion has been derived.

- Assuming a bunch density of the form

\[ n_b(\zeta, r) = n_{b,0} g_{\parallel}(\zeta) g_{\perp}(r; \zeta) \]

the perturbed transverse wakefield is

\[ \frac{W_r}{E_0} = \frac{k_p r}{2} - Z_i \frac{m}{M_i} \frac{n_{b,0} k_p^3}{n_0} \int_0^r d\zeta' (\zeta - \zeta') g_{\parallel}(\zeta') \int_0^r g_{\perp}(r'; \zeta') r' dr'. \]

(valid as long as ion velocity remains non-relativistic, \(\Gamma^2 \leq 1\))

- For \(g_{\parallel}(\zeta)=1\) for \(-L_b \leq \zeta \leq 0\) and \(g_{\perp}(r)=\exp(-r^2/2\sigma_x^2)\)

\[ \frac{W_r(\zeta, r)}{E_0} = \frac{k_p r}{2} \left[ 1 + Z_i \frac{m}{M_i} \frac{n_{b,0} (k_p \zeta)^2}{n_0} 2 \frac{1 - \exp(-r^2/2\sigma_x^2)}{r^2/2\sigma_x^2} \right] \]

Parameters:
- \(n_0 = 10^{17}\) cm\(^{-3}\) (Hydrogen)
- \(k_p L_b = 1\) (flat-top)
- \(k_p \sigma_x = 0.015\) (Gaussian)

Transverse lineout of \(W_r\) at \(\zeta = -L_b\)

Benedetti et al., PRAB (2017)
Slice-dependent and nonlinear nature of wake perturbation from ion motion results in chirp of the betatron frequency

- Betatron frequency chirp:
  \[
  \frac{k^2_B(\zeta)}{k^2_B(\zeta = 0)} = 1 + \log(2) \frac{Z_i}{2} \frac{m}{M_i} \frac{n_{b,0}}{n_0} (k_p \zeta)^2
  \]

Parameters:
- \(n_0 = 10^{17}\text{cm}^{-3}\) (Hydrogen)
- \(E = 25\text{ GeV}, k_p L_b = 1.25\) (flat-top)
- \(n_{b,0}/n_0 = 500, k_p \sigma_x = 0.015\) (Gaussian)
For a bunch initially matched in the linear (unperturbed) wakefield ion motion results in bunch emittance growth

Bunch: $E=25$ GeV, $\varepsilon_{n,0} = (\varepsilon_{n,x} \varepsilon_{n,y})^{1/2} = 0.6$ um, $L_b = 20$ um, $N_b = 10^{10}$ ($n_{b,0}/n_0 = 12000 \rightarrow \Gamma = 10$) \text{Delahaye et al., IPAC 2014}

Background: Hydrogen, $n_0 = 10^{17}$ cm$^{-3}$

Projected emittance evolution

*no acceleration

$\sim +20\%$ projected emittance growth!

Slice emittance @ $k_p z = 4000$
For a bunch initially matched in the linear (unperturbed) wakefield ion motion results in bunch emittance growth

Bunch: $E=25$ GeV, $\varepsilon_{n,0} = (\varepsilon_{n,x} \varepsilon_{n,y})^{1/2} = 0.6$ um, $L_b = 20$ um, $N_b = 10^{10} \left( n_{b,0}/n_0 = 12000 \rightarrow \Gamma = 10 \right)$  

Background: Hydrogen, $n_0 = 10^{17}$ cm$^{-3}$

Projected emittance evolution

- *no acceleration
- $\sim +20\%$ projected emittance growth!

Slice emittance @ $k_p z = 4000$

Saturated emittance VS bunch density

Final (saturated) emittance:

$$\frac{\varepsilon_{n,x}^*}{\varepsilon_{n,0}} = \frac{\langle x^2 \rangle}{\langle u_x^2 \rangle} \approx 1 + 0.0015 \Gamma + 0.001 \Gamma^2$$

Delahaye et al., IPAC 2014
A class of initial beam distributions with constant slice-by-slice emittance enabling ion motion without emittance growth has been derived.

Equilibrium beam distribution: $f_{0,\perp}(r, u; \zeta) \propto F[H_{\perp}(r, u; \zeta)/H_0(\zeta)]$

→ arbitrary longitudinal current profile possible

(phase space distribution is, slice-by-slice, a stationary solution of Vlasov equation, $\partial f/\partial z = 0$, including ion motion)
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\[ \rightarrow \text{arbitrary longitudinal current profile possible} \]

Bunch: \( E=25 \text{ GeV}, \epsilon_{n,0}=(\epsilon_{n,x} \epsilon_{n,y})^{1/2}=0.6 \text{ um}, L_b=20 \text{ um}, N_b=10^{10} (n_{b,0}/n_0=12000 \rightarrow \Gamma=10) \)

Background: Hydrogen, \( n_0=10^{17} \text{ cm}^{-3} \)

Bunch density (equilibrium solution)

\[ n_{b,0}/n_0 = 12000 \]

→ Tapered bunch profile
→ Constant slice emittance along the beam
→ Transverse distribution is, in general, non-Gaussian

Benedetti et al., PRAB (2017)
A class of initial beam distributions with constant slice-by-slice emittance enabling ion motion without emittance growth has been derived.

Equilibrium beam distribution: \( f_{0,\perp}(r, u; \zeta) \propto F[H_{\perp}(r, u; \zeta)/H_0(\zeta)] \)

\( \rightarrow \) arbitrary longitudinal current profile possible

Bunch: \( E=25 \) GeV, \( \varepsilon_{n,0}=(\varepsilon_{n,x} \varepsilon_{n,y})^{1/2}=0.6 \) um, \( L_b=20 \) um, \( N_b=10^{10} \) (\( n_{b,0}/n_0=12000 \rightarrow \Gamma=10 \))

Background: Hydrogen, \( n_0=10^{17} \) cm\(^{-3}\)

\( \rightarrow \) Tapered bunch profile
\( \rightarrow \) Constant slice emittance along the beam
\( \rightarrow \) Transverse distribution is, in general, non-Gaussian

Projected emittance evolution

\( \Delta \varepsilon_n(\zeta, k_p) = \frac{\Delta \varepsilon_n(\zeta, k_p)}{\varepsilon_{n,0}} \)

\( \rightarrow \) No emittance growth! [for matched solution slice emittance \( \neq \) projected emittance]
A model describing evolution of beam envelope and centroid in a nonlinear wake in presence of ion motion has been derived (no energy gain)

- **Equation for beam moments:**
  \[
  \frac{d^2 X_b}{dz^2} = -\frac{k_p}{\gamma} \frac{\langle W_x \rangle}{E_0},
  \]
  \[
  \frac{d^2 \sigma_x}{dz^2} = \frac{\sigma_x^2}{\gamma^2 \sigma_x^3} - \frac{k_p}{E_0 \gamma \sigma_x} \langle (x - X_b)W_x \rangle,
  \]

- **Equation for wakefield with ion motion:**
  \[
  \frac{W_x}{E_0} = \frac{k_p}{2} \frac{(x - X_p)}{E_0} - Z_i M_i k_p^2 \int_{\zeta}^{\zeta'} d\zeta' (\zeta - \zeta') \frac{E_{b,x}(\zeta')}{E_0},
  \]
  \[
  \Rightarrow \frac{\langle W_x \rangle}{E_0} \approx \frac{k_p}{2} \frac{[X_b(\zeta) - X_p(\zeta)]}{E_0}
  \]
  \[
  + Z_i M_i \frac{\hat{I}_b}{I_A} k_p \int_{\zeta}^{\zeta'} d\zeta' (\zeta - \zeta') g_{\parallel} (\zeta') \frac{X_b(\zeta) - X_b(\zeta')}{\sigma_x^2(\zeta) + \sigma_x^2(\zeta')}
  \]

- **Equation for wake centroid, \(X_p(\zeta)\), from Mehrling et al., POP (2018)**
Model predicts suppression of hosing due to ion motion in agreement with 3D PIC simulations with HiPACE (non relativistic ion motion)

Bunch: $E_b=1$ GeV, $\varepsilon_{n,0}=1.1$ um, $\sigma_b=0.79$ um, $L_b=20$ um, $N_b=0.71 \cdot 10^{10} (n_{b,0}/n_0=800 \rightarrow \Gamma=0.7)$, $X_{b,0} = 0.1\sigma_b$ (displ.)

Background: Hydrogen, $n_0=10^{17}$ cm$^{-3}$

Two-particles model:

$\rightarrow$ decoherence length

$k_{\beta 0} L_d = \frac{8 \pi}{\Gamma} \simeq 36$

($\sim 6$ betatron periods)
3D PIC simulations show suppression of hosing in presence of ion motion. Emittance is preserved if tapered beams are used.

Bunch: $E_b = 25$ GeV, $\varepsilon_{n_0} = 0.26$ um, $\sigma_b = 0.17$ um, $L_b = 33$ um, $N_b = 1.5 \cdot 10^{10}$ [trapezoidal] $(n_{b,0}/n_0 = 31000 \rightarrow \Gamma = 68)$, initial displacement: $X_{b,0} = \sigma_b$

Background: Hydrogen, $n_0 = 10^{17}$ cm$^{-3}$

Centroid evolution

Emittance evolution

- NO ion motion $\rightarrow$ hosing, large emittance growth (beam breakup)
- WITH ion motion + LIN. matched $\rightarrow$ hosing suppressed, but emittance growth (+60%) from ion motion
- WITH ion motion + NONLIN. matched (equilibrium bunch) $\rightarrow$ hosing suppressed and no emittance growth
Tapered beams which are equilibrium solution in presence of ion motion can be obtained by means of an adiabatic matching procedure

Adiabatic beam matching during plasma acceleration of initially low-energy beams:

- At low energy ion motion is negligible ($\Gamma \sim \gamma^{1/2}$)
- Energy gain adiabatically compresses beam
- Gradual compression of beam triggers ion motion
- Wakefield perturbation is gradually enhanced
- Beam distribution adiabatically adjusts (with emittance preserved at % level) to ion-motion-perturbed wakefields

Bunch: $\varepsilon_{n,0} = 1.2 \text{ um, } L_b = 33 \text{ um, } N_b = 1.5 \cdot 10^{10}$ [trapezoidal]
Background: Hydrogen, $n_0 = 10^{17} \text{ cm}^{-3}$

E=50 MeV

initial (linearly-matched) beam distribution,

E=15 GeV

final (non-lin. matched) beam distribution

initial (unperturbed) transverse wakefield

final (ion perturbed) transverse wakefield
• Witness bunch parameters of interest for future plasma-based colliders trigger ion motion and hosing that lead, potentially, to severe emittance degradation and beam breakup;

• Analytic model describing hosing in presence of ion motion has been derived:
  → model predicts suppression of hosing due to detuning associated with ion motion;

• 3D PIC simulations confirm suppression of hosing in presence of ion motion for collider relevant witness beam parameters;

• Beam quality preservation possible with slice-by-slice matching of witness bunch:
  → stable, high-efficiency acceleration possible for HEP applications;
  → strategy to produce tapered beams based on adiabatic matching presented.
Expressions for the perturbed wakefield and emittance growth at saturation have been derived in the case of flat beams (proposed to reduce beamstrahlung)

- Perturbed wakefield for a flat beam (i.e., $\sigma_x >> \sigma_y$, $\epsilon_x >> \epsilon_y$)

\[
\left\{ \begin{array}{l}
\frac{W_x}{E_0} \sim \frac{k_p x}{2} \\
\frac{W_y}{E_0} \sim \frac{k_p y}{2} \left[ 1 + Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_{0}} (k_p \zeta)^2 \exp\left(-\frac{x^2}{2\sigma_x^2}\right) K\left(y/\sqrt{2}\sigma_y\right) \right], \quad K(q) = (\sqrt{\pi}/2)\text{erf}(q)/q
\end{array} \right.
\]

- Projected emittance growth at saturation (final)

\[
\left\{ \begin{array}{l}
\frac{\epsilon^*_{n,x}}{\epsilon_{n,x}} \sim 1 \\
\frac{\epsilon^*_{n,y}}{\epsilon_{n,y}} \sim 1 + 0.0027 \Gamma + 0.0053 \Gamma^2
\end{array} \right.
\]

(unif. longitudinal profile assumed)

Horizontal wake essentially unperturbed

Wake perturbation twice as large compared to the round beam case

Horizontal emittance preserved

Vertical emittance growth twice as large compared to the round beam case