Ion motion and hosing suppression in plasma-based accelerators

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Overview

- Witness bunch parameters of interest for a plasma-based collider trigger hosing and ion motion

- Mitigation/suppression of hosing by means of change of betatron frequency along the bunch (chirp)

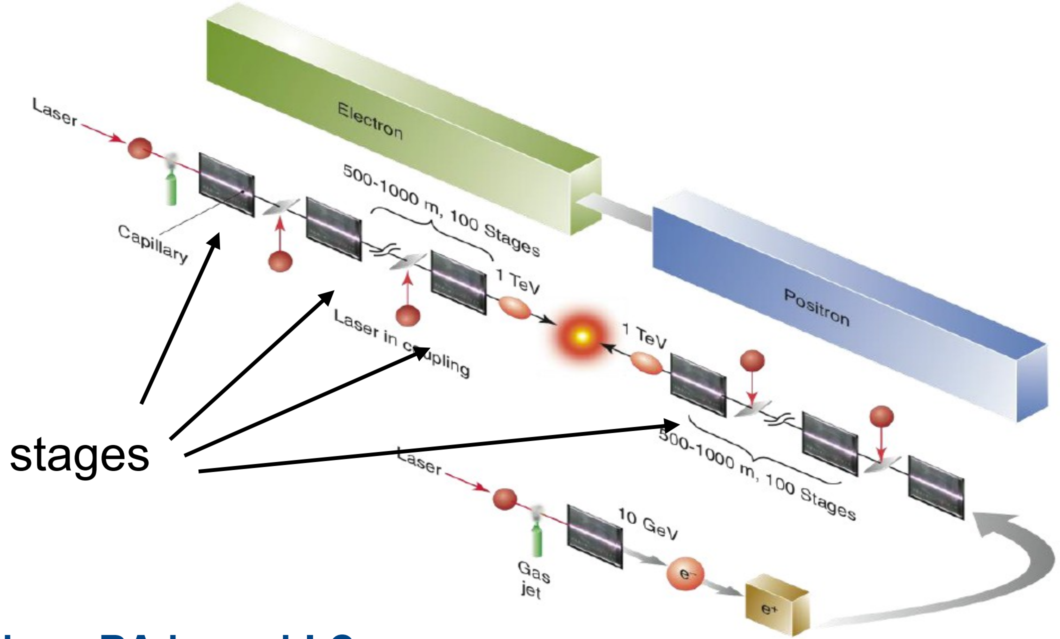
- Bunch-induced ion motion as a way to generate betatron frequency chirp that suppresses hosing
  → analytical expression for wake with ion motion
  → theory describing hosing in presence of ion motion
  → 3D PIC simulations show hosing suppression with ion motion
  → emittance degradation associated with ion motion eliminated by proper bunch shaping

- Summary

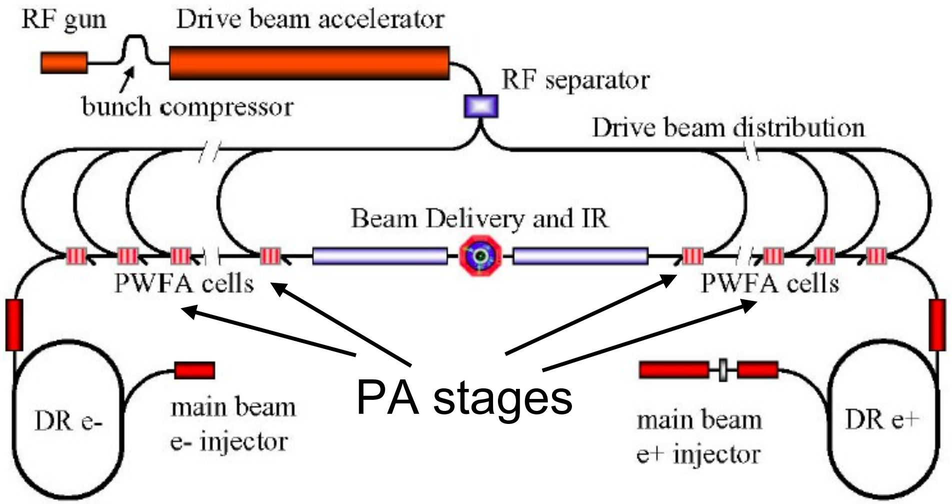
Concept for next-generation TeV-class PA-based linear collider: requires bunches with high charge and small emittance

Laser-driven PA-based LC ————>
[all optical, driver=10s J laser, 10 kHz, 50x10GeV PA stages @ $n_0=10^{17}$ cm^{-3}]

Leemans, Esarey, Physics Today (2009)
Schroeder *et al.*, PRSTAB (2010)



PA stages



<—— Beam-driven PA-based LC
[driver=25 GeV e-bunch, 19x25 GeV PA stages @ $n_0=10^{17}$ cm^{-3}]

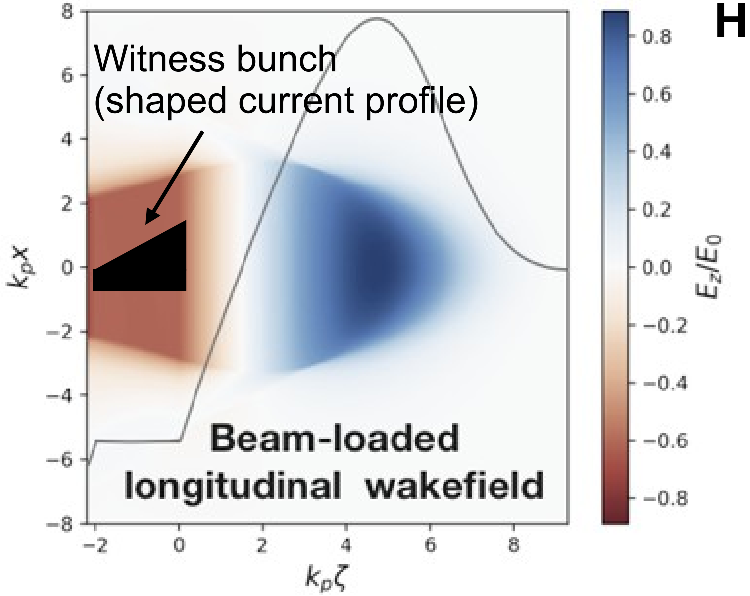
Seryi *et al.*, PAC 2009
Delahaye *et al.*, IPAC 2014

- Compact machine (1 TeV): ≤1 Km for PA-based LC VS ~30 Km for RF-based (ILC)
- Requirements on witness bunch:

→ N_b~ 10^{10} part./bunch (~ nC), ε_n < 100 nm, high wake → witness bunch efficiency (~40%)

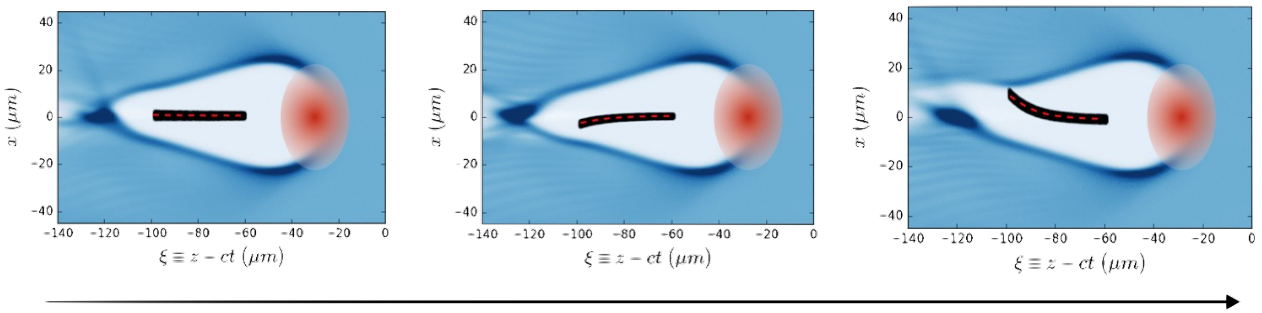
*Ellis and Wilson, Nature (2001); Hinchliffe and Battaglia, Phys. Today (2004)

Witness bunch parameters of interesest for a plasma-based collider induce hosing and ion motion resulting in emittance degradation



High charge + high-efficiency:

→ large longitudinal wake driven by beam (i.e., high beamloading)
→ high beamloading implies strong coupling between beam and wake
→ resonance between beam centroid motion and wake centroid → **hosing**



Whittum et al., PRA (1992)
Schroeder et al., PRL (1999)
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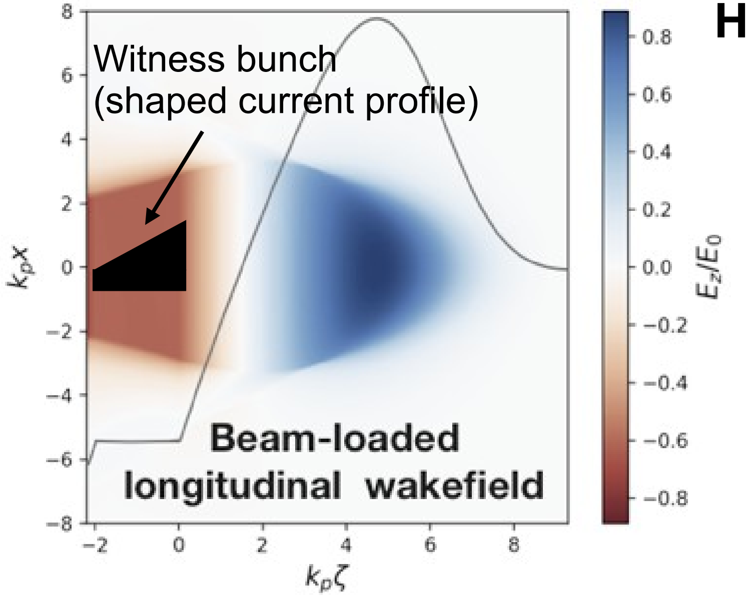

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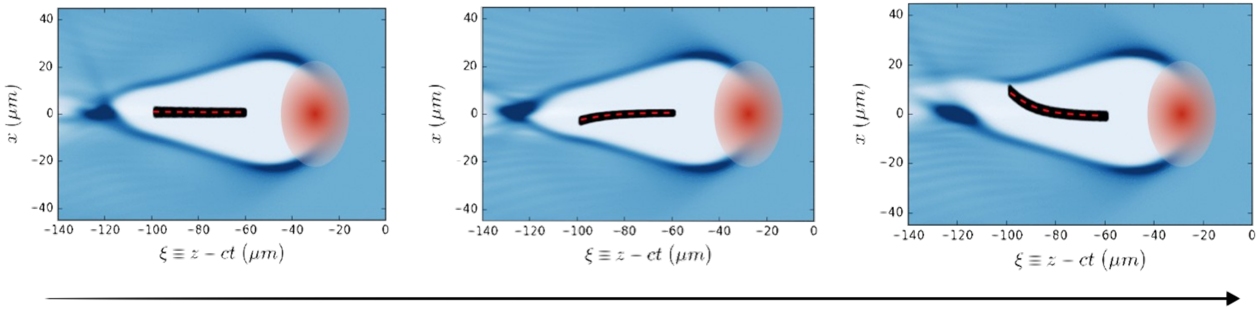

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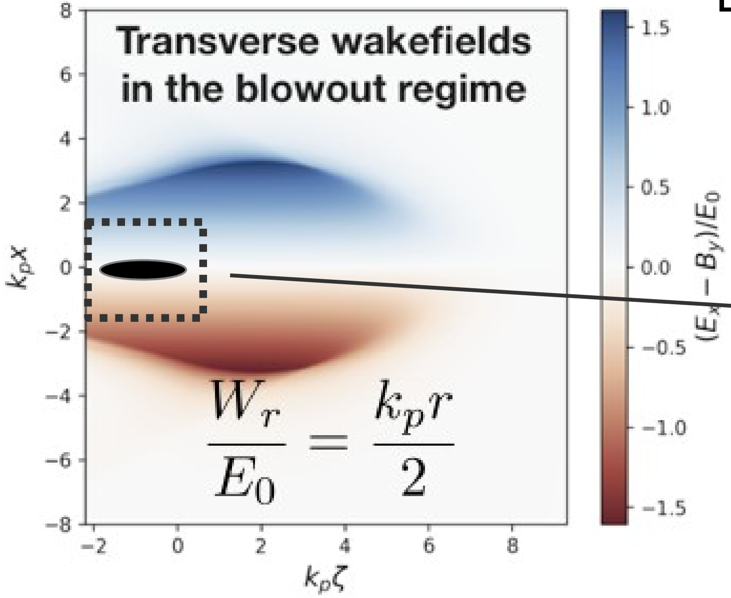
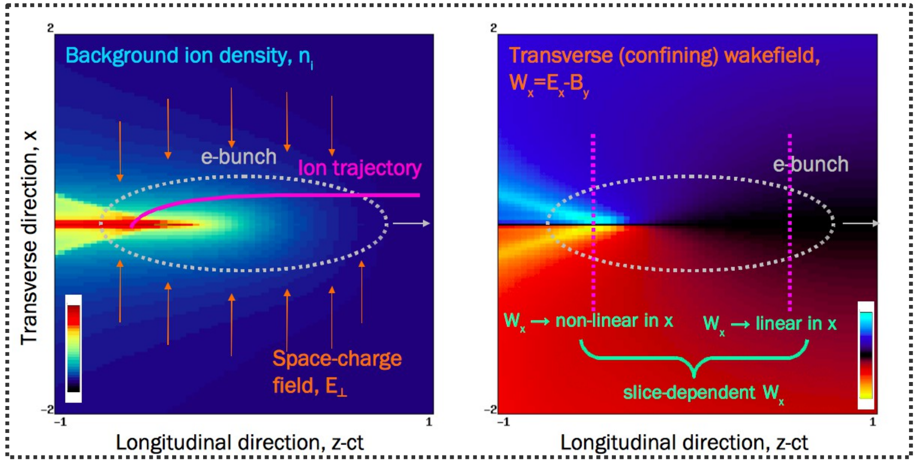


Low emittance + high charge + high energy:

→ small matched beam size and high beam density
→ large beam space charge fields → **background ion motion**

$$\sigma_x^2 = \frac{\epsilon_n}{\gamma k_\beta} = \sqrt{\frac{2}{\gamma}} \frac{\epsilon_n}{k_p}$$



Condition for ion motion:

$$\Gamma = Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} (k_p L_b)^2 \sim 1$$

Rosenzweig et al., PRL (2005)
An et al., PRL (2017)
Benedetti et al., PRAB (2017)

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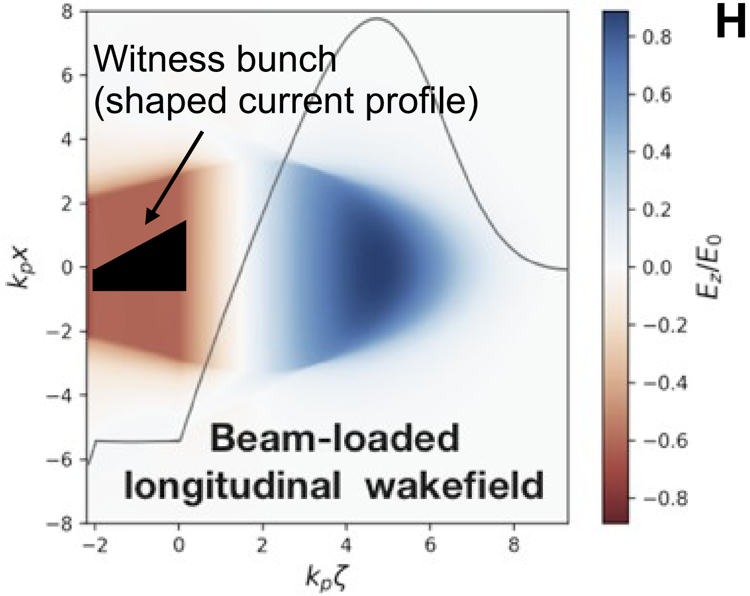

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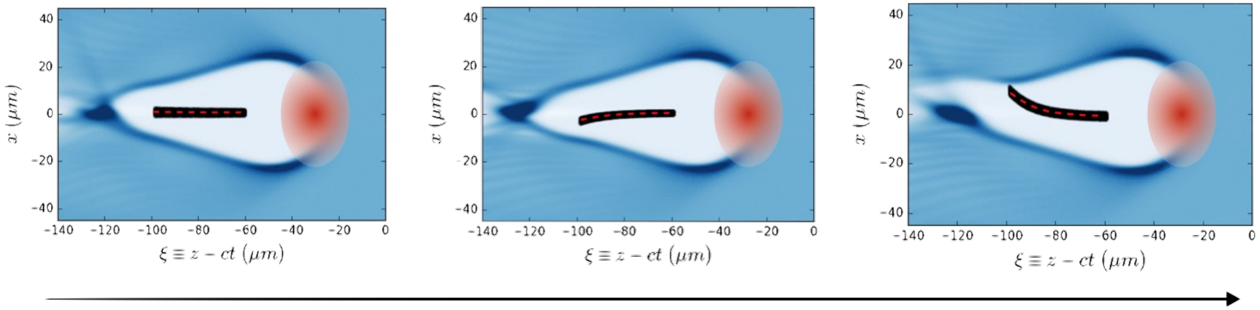

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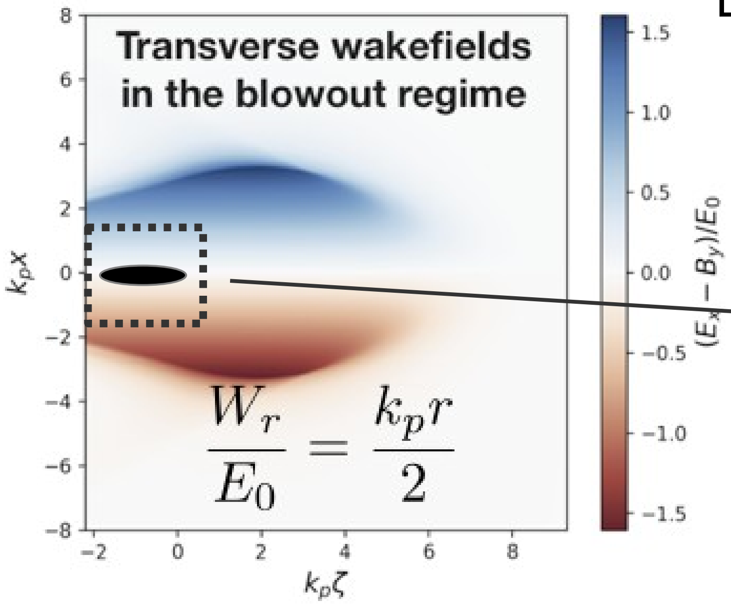

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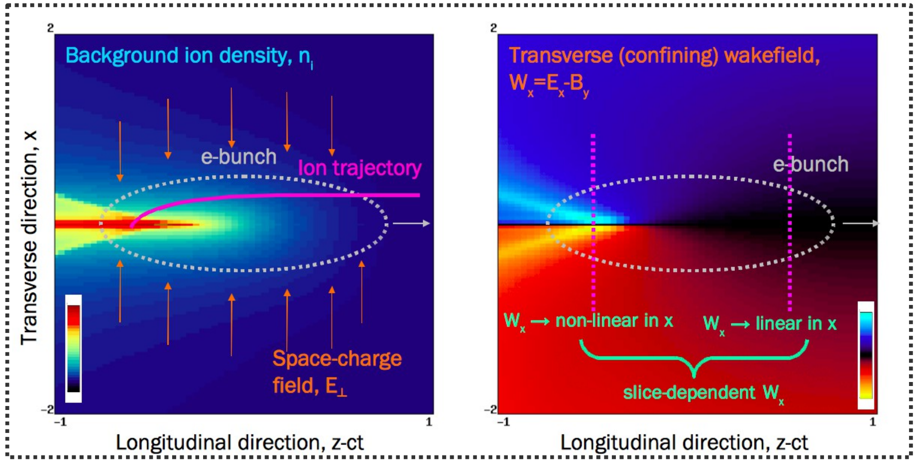

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Emittance growth!

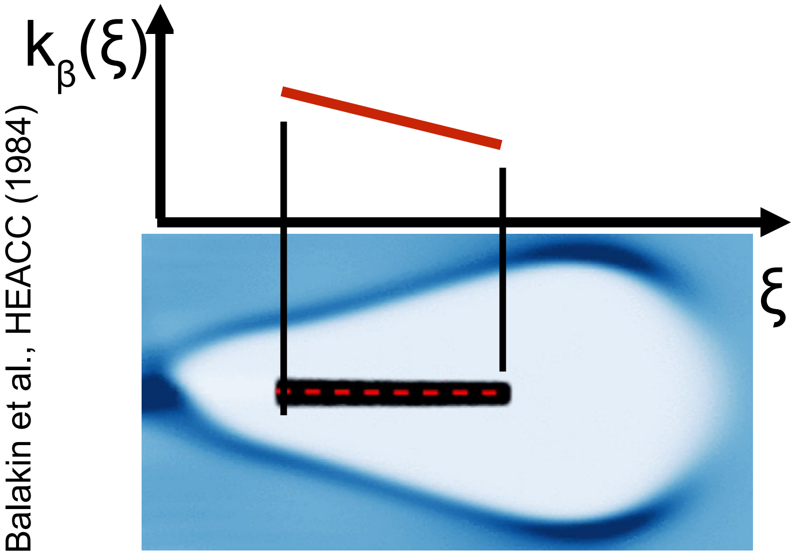
Chirp of betatron frequency suppresses hosing

- Head-to-tail variation in focusing force affects hosing

$$\partial_z^2 x_c(\xi, z) + k_\beta^2(\xi) x_c(\xi, z) = k_c^2 \int_\xi^0 \frac{n_b(\xi')}{n_p} x_c(\xi', z) \sin[k_p(\xi' - \xi)] k_p d\xi',$$

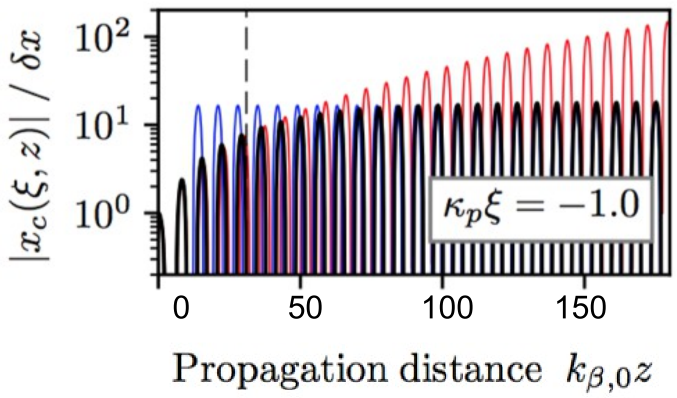
Lehe et al., PRL (2017)

Lau, PRL (1989)
Balakin et al., HEACC (1984)

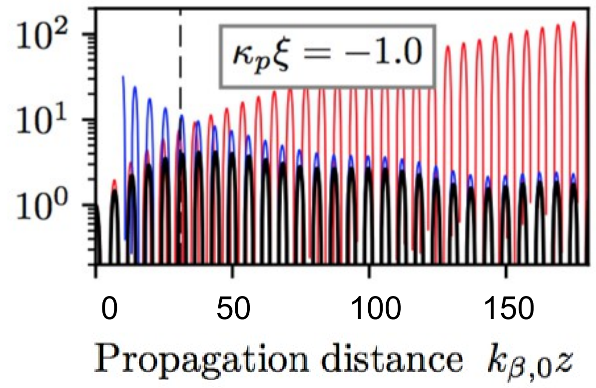


Complete solution
Early time solution (exponential)
Asymptotic solution (saturation/damping)

Positive chirp, $\partial_\xi k_\beta > 0$



Negative chirp, $\partial_\xi k_\beta < 0$



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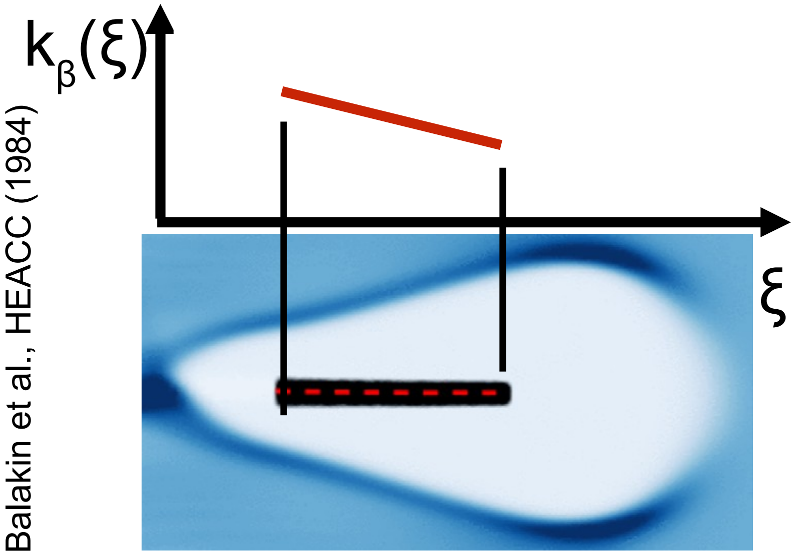

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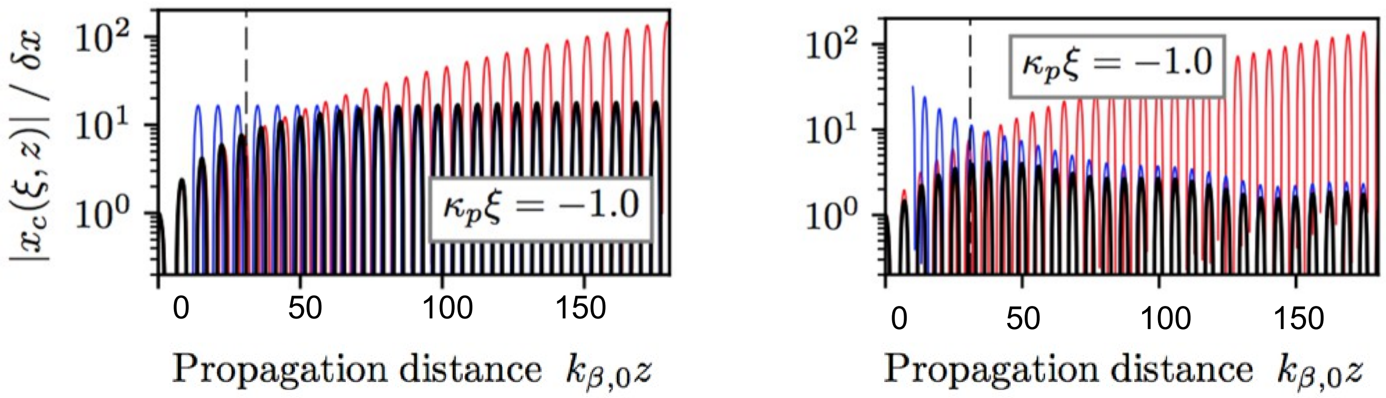
Positive chirp, $\partial_\xi k_\beta > 0$ | Negative chirp, $\partial_\xi k_\beta < 0$



- PA stage operating in the quasilinear regime
  → head-to-tail variation in (beam-loaded) focusing force provides linear (or mostly linear) betatron chirp, largely suppresses hosing

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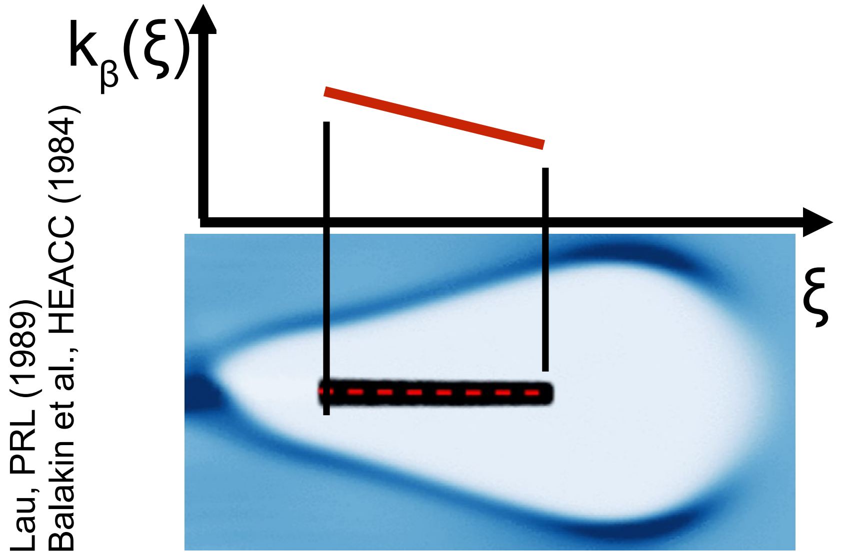
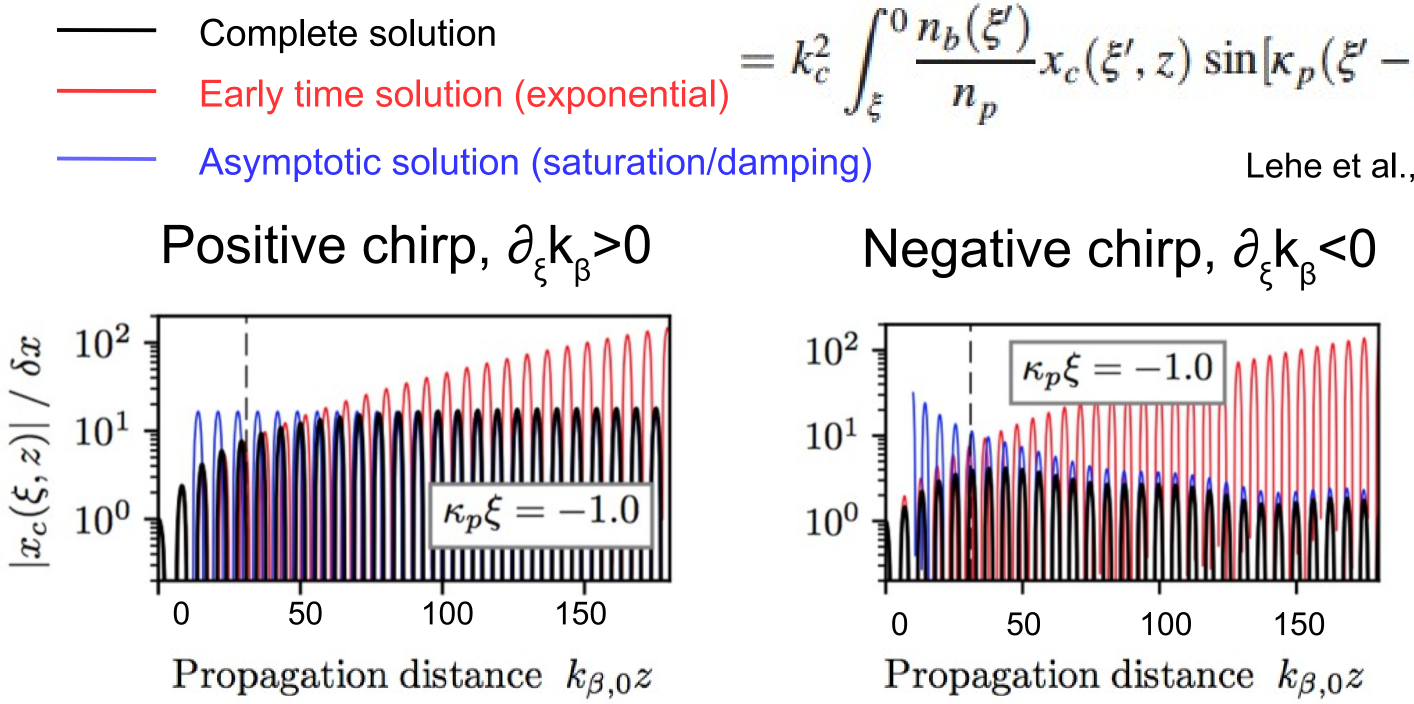




- PA stage operating in the quasilinear regime
  → head-to-tail variation in (beam-loaded) focusing force provides linear (or mostly linear) betatron chirp, largely suppresses hosing

- PA stage operating in the nonlinear/blowout regime
  → head-to-tail variation achieved by energy chirp (similar to BNS damping): $k_\beta(\xi) = k_p/\sqrt{2\gamma(\xi)}$
  [not desirable for collider applications]

Modeling tools: INF&RNO and HiPACE

Modeling performed with quasi-static codes
(optimal tool for this problem)

INF&RNO/QS

- 2D axisymmetric
- PIC or fluid for plasma
- Quasi-static modality
- Dynamic time step adjustment + subcycling

Benedetti at al., AAC2010, AAC2012, ICAP2012, AAC2016, PPCF2017

HiPACE

- 3D Cylindrical
- Quasi-static PIC
- Dynamic time step adjustment + subcycling
- High resolution subgrid in witness bunch domain to correctly model ion motion
- Parallelized with MPI

Mehrling et al., PPCF (2014), AAC2018

An analytical expression for the perturbed wakefield in presence of ion motion has been derived

- Assuming a bunch density of the form

Longitudinal profile

$$n_b(\zeta, \mathbf{r}) = n_{b,0}\, g_{\parallel}(\zeta) g_{\perp}(\mathbf{r}; \zeta)$$

Transverse profile

the perturbed transverse wakefield is

$$\frac{W_r}{E_0} = \frac{k_p r}{2} - Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} \frac{k_p^3}{r} \int_{\zeta}^{0} d\zeta' (\zeta - \zeta') g_{\parallel}(\zeta') \int_0^r g_{\perp}(r'; \zeta') r' dr'.$$

(valid as long as ion velocity remains non-relativistic, $\Gamma^2 \leq 1$)

- For $g_{\parallel}(\zeta)=1$ for $-L_b \leq \zeta \leq 0$ and $g_{\perp}(r)=\exp(-r^2/2\sigma_x^2)$

$$\frac{W_r(\zeta, r)}{E_0} = \frac{k_p r}{2} \left[1 + Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} \frac{(k_p \zeta)^2}{2} \frac{1 - \exp(-r^2/2\sigma_x^2)}{r^2/2\sigma_x^2} \right]$$

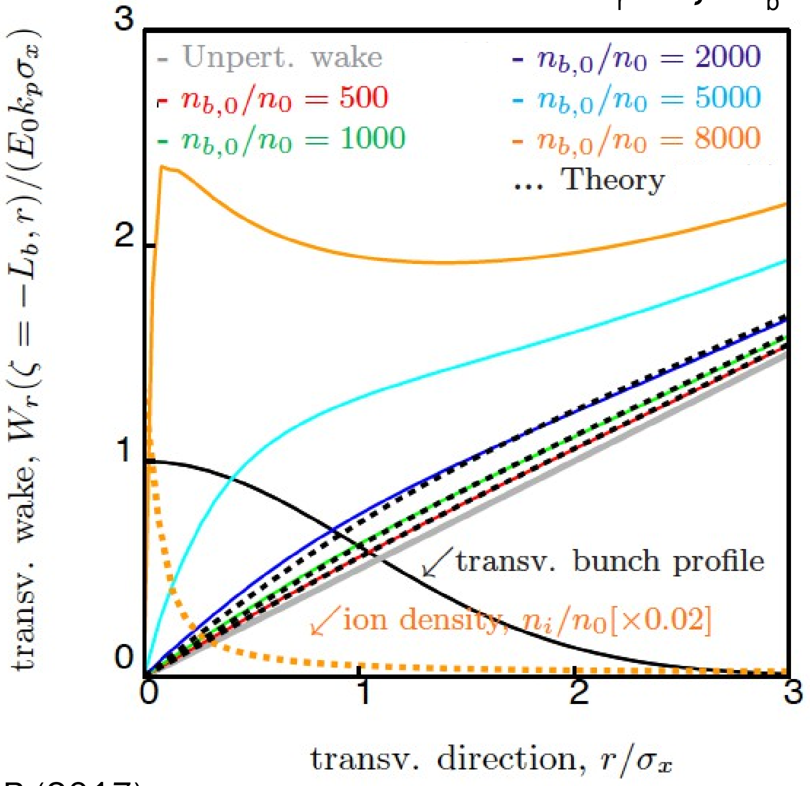
Unperturbed wakefield

Slice-dependent confining force

Wakefield acquires nonlinear dependence on transverse coordinate *r*

Parameters:
$n_0=10^{17}$cm^{-3} (Hydrogen)
$k_p L_b=1$ (flat-top)
$k_p \sigma_x=0.015$ (Gaussian)

Transverse lineout of W_r at $\zeta=-L_b$

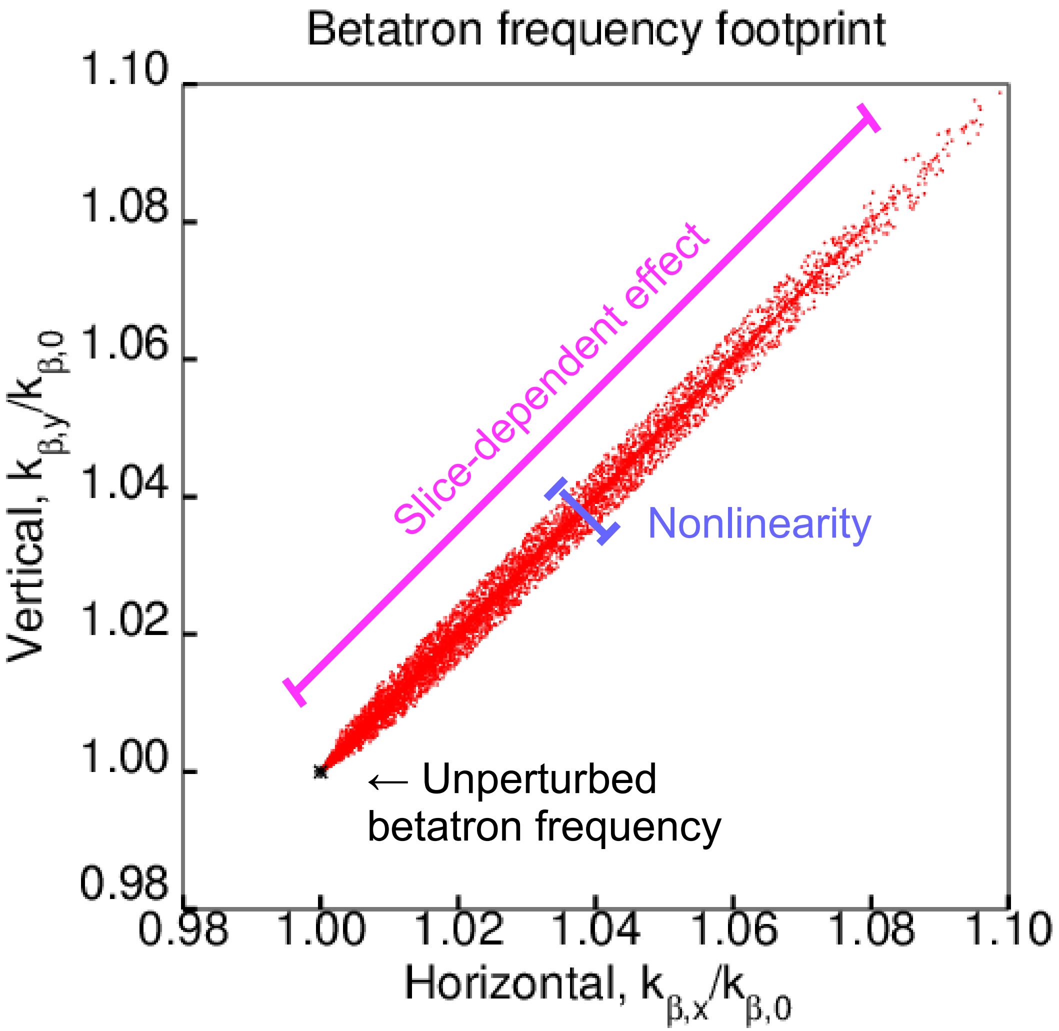


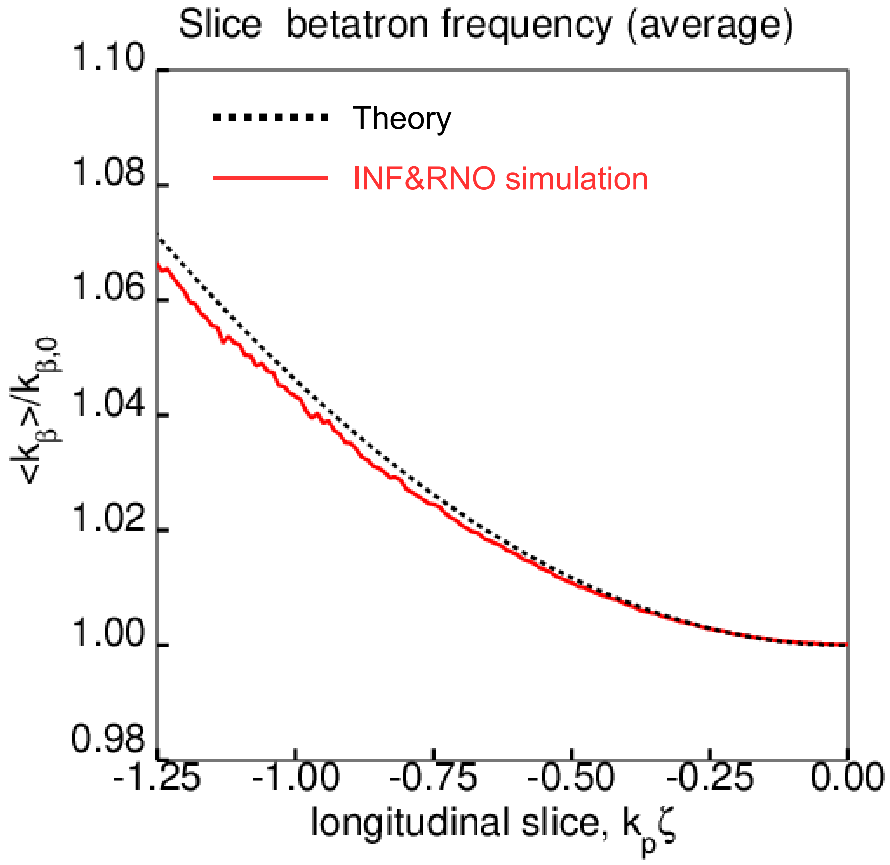
Benedetti et al., PRAB (2017)

Slice-dependent and nonlinear nature of wake perturbation from ion motion results in chirp of the betatron frequency

- Betratron frequency chirp: $$\frac{k_\beta^2(\zeta)}{k_\beta^2(\zeta=0)} = 1 + \log(2)\frac{Z_i}{2}\frac{m}{M_i}\frac{n_{b,0}}{n_0}(k_p\zeta)^2$$

Parameters:
$n_0=10^{17}\,\mathrm{cm}^{-3}$ (Hydrogen)
E=25 GeV, $k_pL_b=1.25$ (flat-top)
$n_{b,0}/n_0=500$, $k_p\sigma_x=0.015$ (Gaussian)



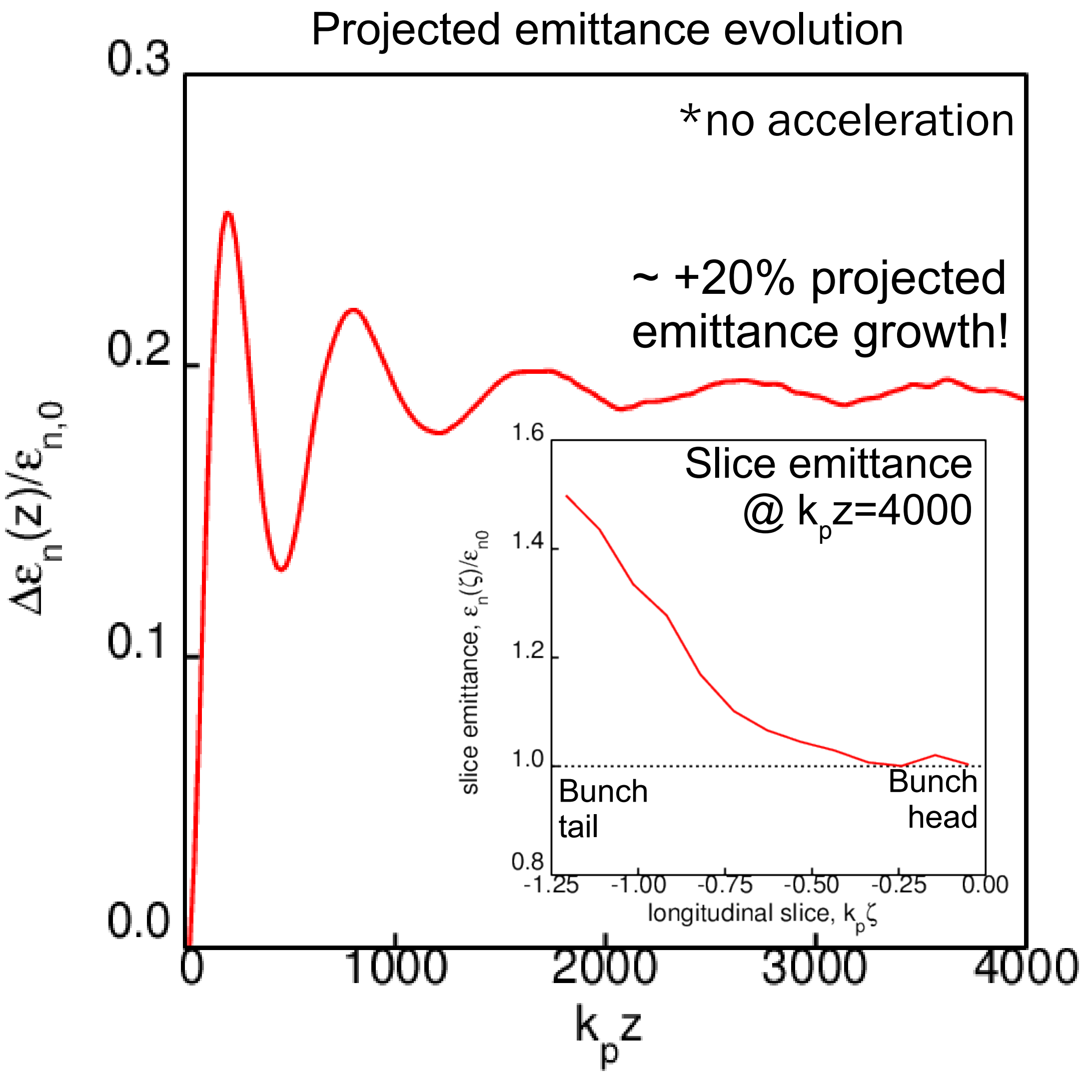


Burov et al., arXiv:1808.03860 (2018)
Pukhov and Farmer, PRL (2018)

For a bunch initially matched in the linear (unperturbed) wakefield ion motion results in bunch emittance growth

Bunch: E=25 GeV, $\varepsilon_{n,0}=(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2}$=0.6 um, L_b=20 um, $N_b=10^{10}$ ($n_{b,0}/n_0$=12000 → Γ=10) Delahaye *et al.*, IPAC 2014

Background: Hydrogen, $n_0=10^{17}$ cm^{-3}



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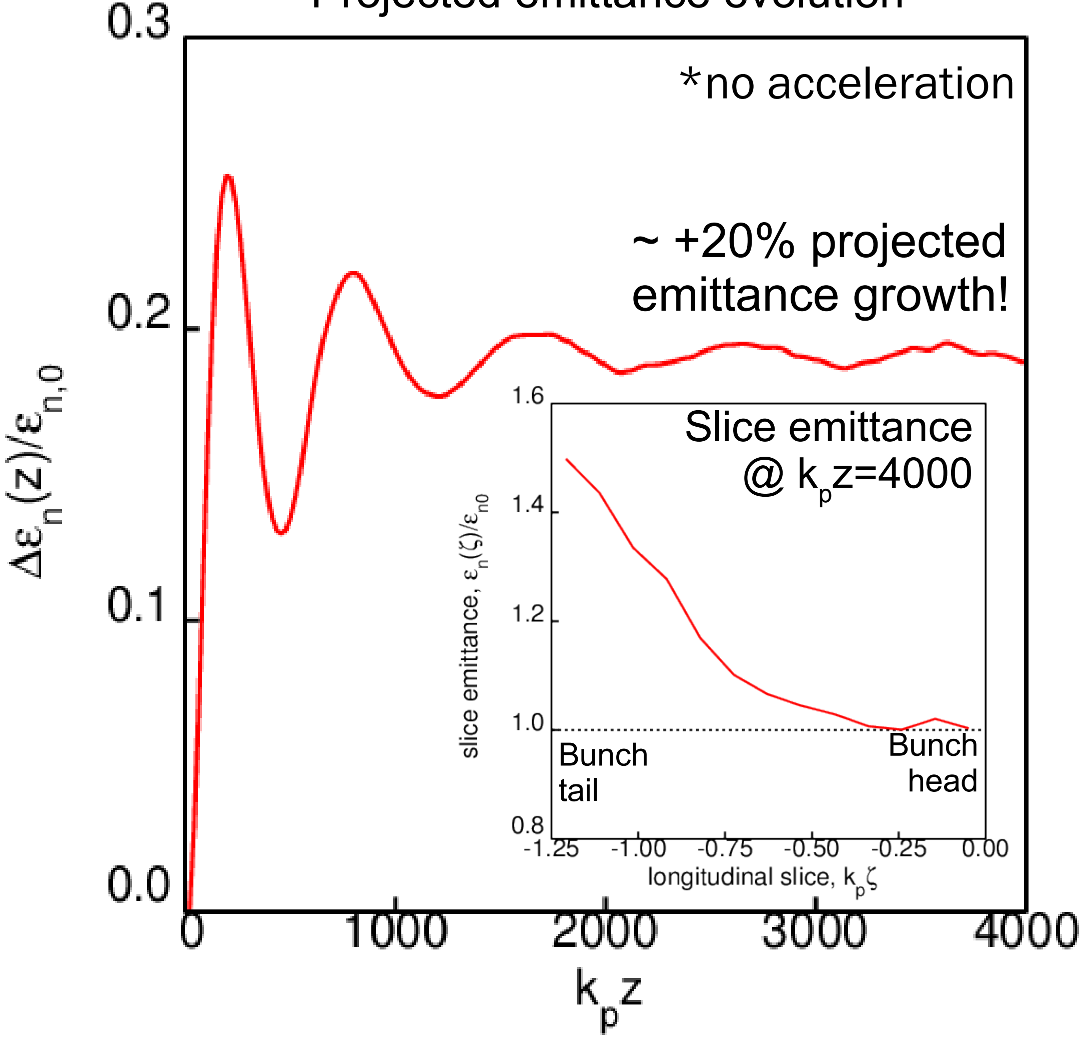
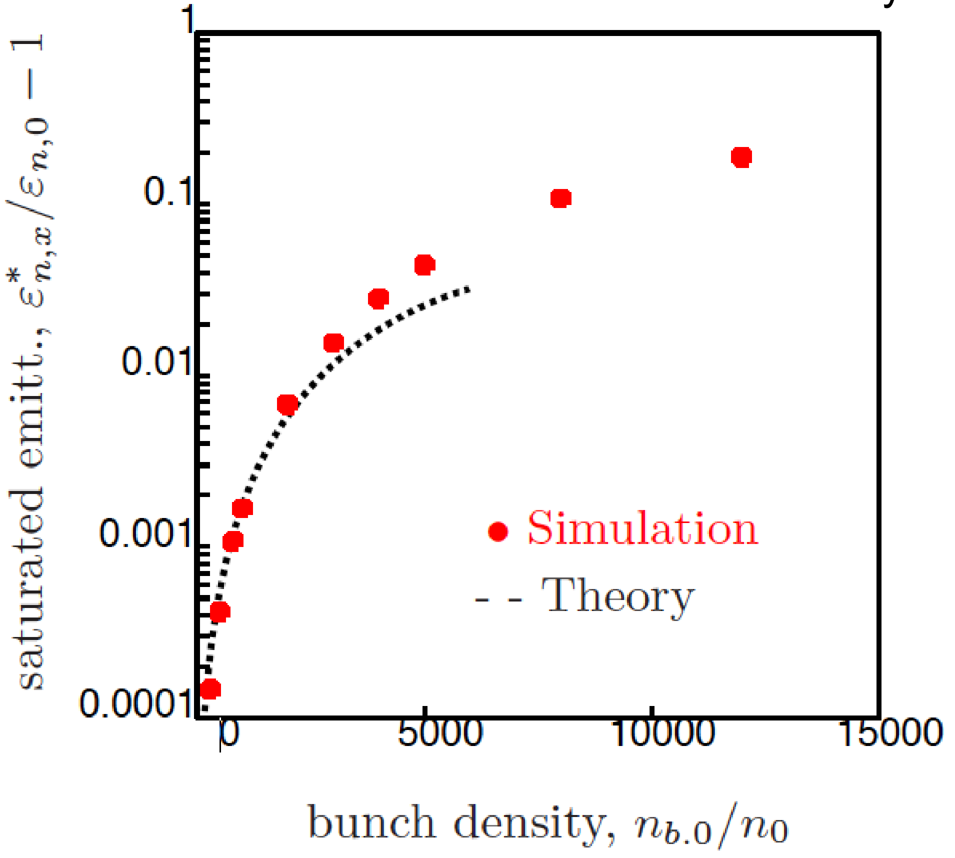
Projected emittance evolution



Saturated emittance VS bunch density



Final (saturated) emittance:

$$\frac{\varepsilon^*_{n,x}}{\varepsilon_{n,0}} = \frac{[\langle x^2\rangle\langle u_x^2\rangle]^{1/2}}{\sigma_x\sigma_{u_x}} \simeq 1 + 0.0015\,\Gamma + 0.001\,\Gamma^2$$

Benedetti et al., PRAB (2017)

A class of initial beam distributions with constant slice-by-slice emittance enabling ion motion without emittance growth has been derived

Equilibrium beam distribution: $f_{0,\perp}(\mathbf{r},\mathbf{u};\zeta) \propto F[H_\perp(\mathbf{r},\mathbf{u};\zeta)/H_0(\zeta)]$

Arbitrary function

Single particle Hamiltonian for transverse motion

Slice-dependent scale parameter

(phase space distribution is, slice-by-slice, a stationary solution of Vlasov equation, *∂f/∂z=0,* including ion motion)

→ arbitrary longitudinal current profile possible

Benedetti et al., PRAB (2017)

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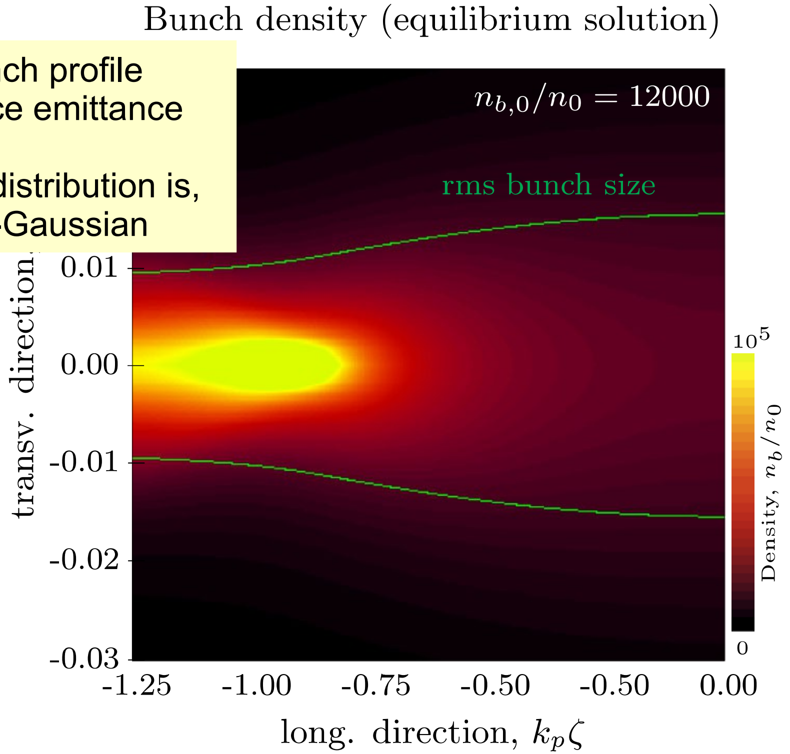
Background: Hydrogen, $n_0=10^{17}$ cm^{-3}

→ Tapered bunch profile
→ Constant slice emittance along the beam
→ Transverse distribution is, in general, non-Gaussian



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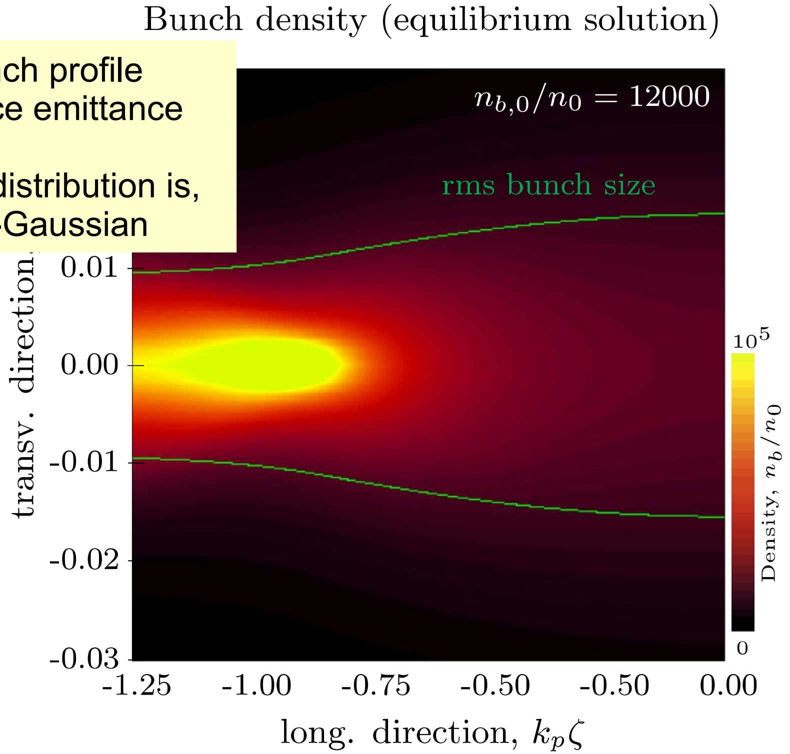

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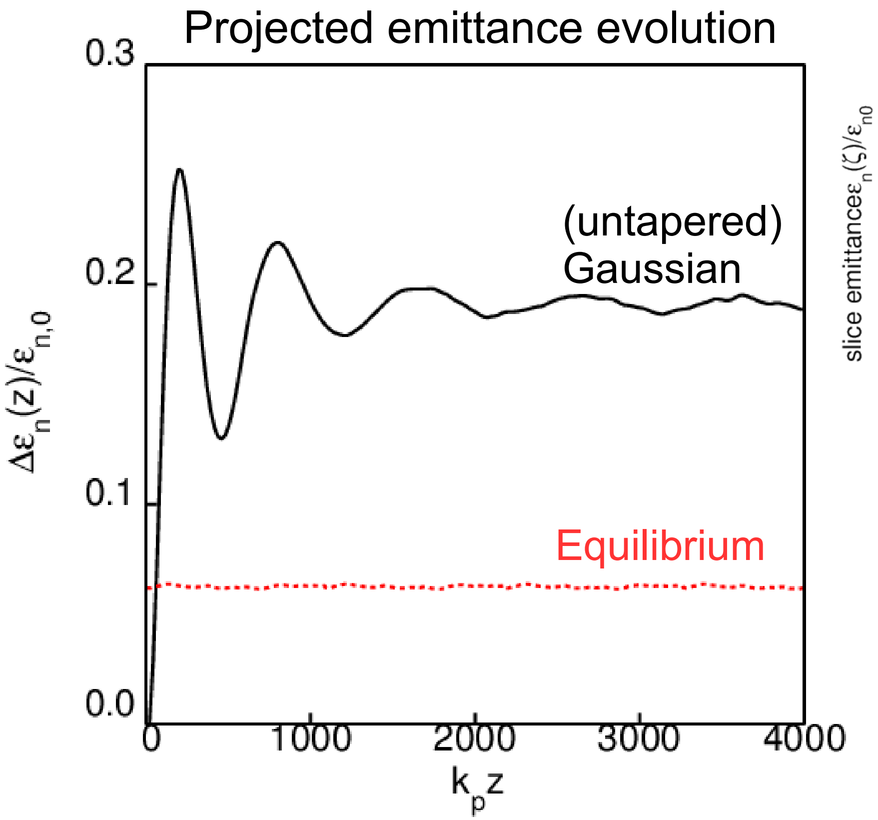
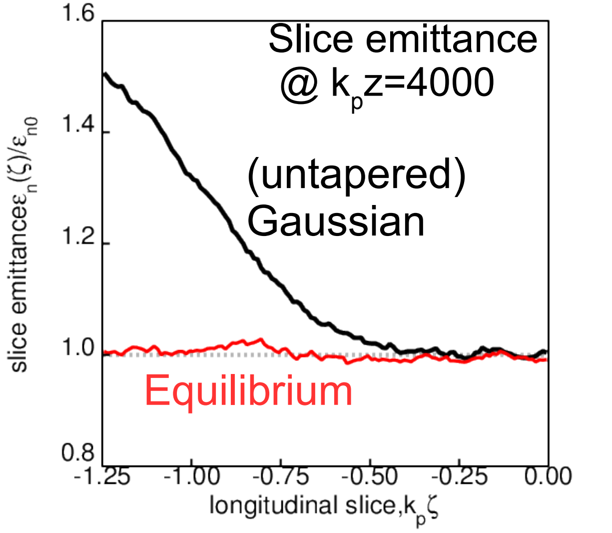
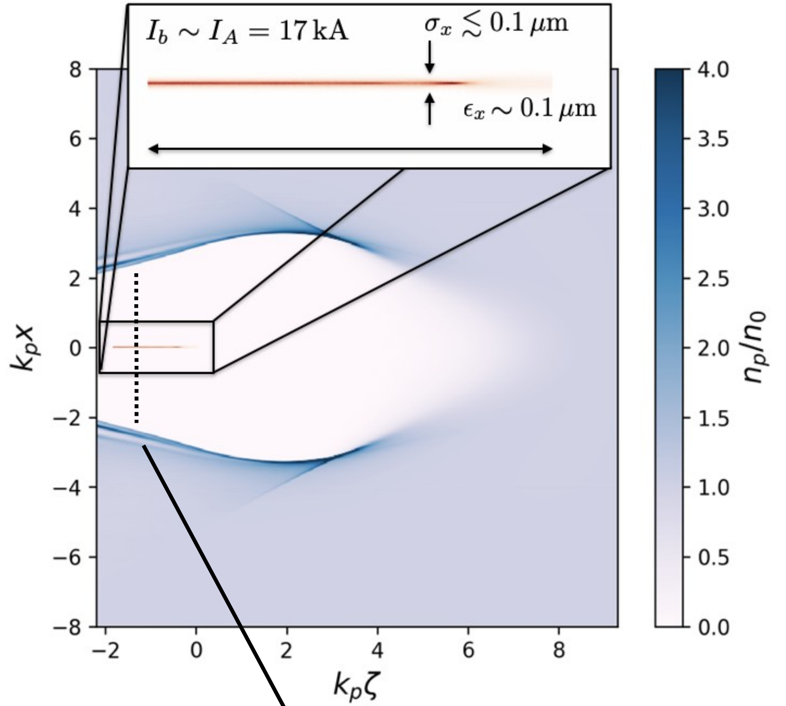
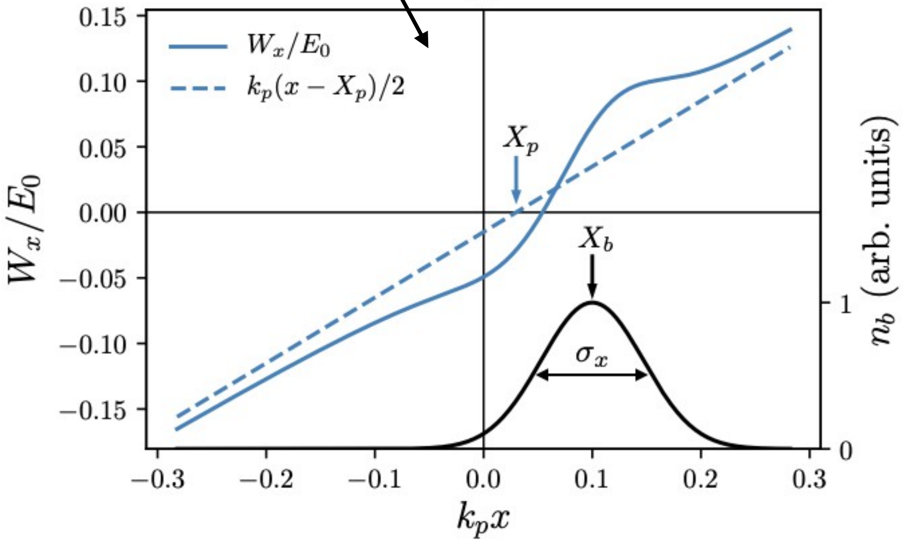




← No emittance growth! [for matched solution slice emittance ≠ projected emittance]

A model describing evolution of beam envelope and centroid in a nonlinear wake in presence of ion motion has been derived (no energy gain)





Mehrling et al., PRL (2018)

- Equation for beam moments:

$$\frac{d^2 X_b}{dz^2} = -\frac{k_p}{\gamma}\frac{\langle W_x \rangle}{E_0},$$
$$\frac{d^2 \sigma_x}{dz^2} = \frac{\epsilon_x^2}{\gamma^2 \sigma_x^3} - \frac{k_p \langle (x - X_b) W_x \rangle}{E_0 \gamma \sigma_x},$$

- Equation for wakefield with ion motion:

Depends on X_b and σ_x

$$\frac{W_x}{E_0} = \frac{k_p(x - X_p)}{2} - Z_i \frac{m}{M_i} k_p^2 \int_\infty^\zeta d\zeta' (\zeta - \zeta') \frac{E_{b,x}(\zeta')}{E_0},$$

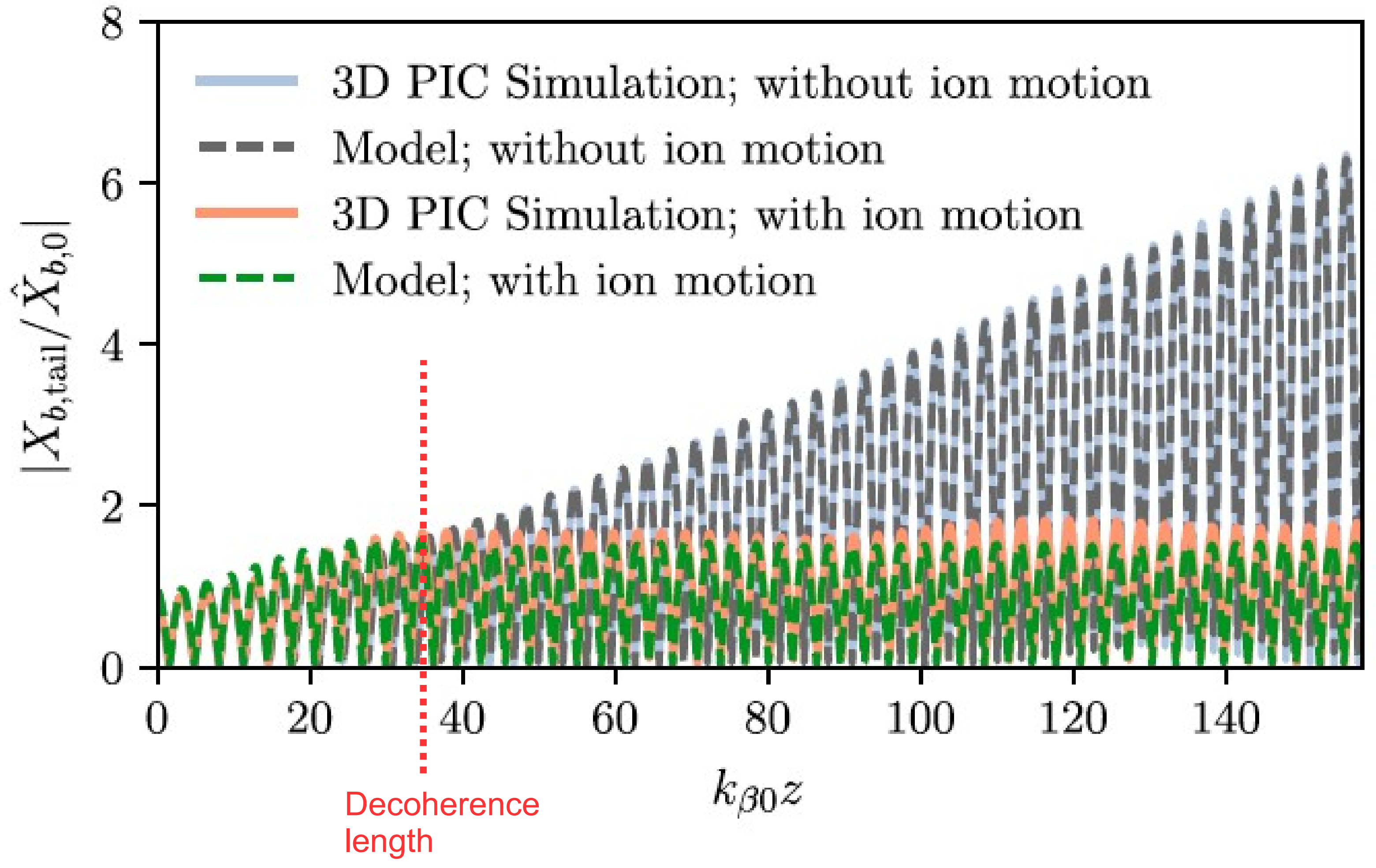
$$\rightarrow \frac{\langle W_x \rangle}{E_0} \simeq \frac{k_p[X_b(\zeta) - X_p(\zeta)]}{2} + Z_i \frac{m}{M_i}\frac{\hat{I}_b}{I_A} k_p \int_\infty^\zeta d\zeta' (\zeta - \zeta') g_\parallel(\zeta') \frac{X_b(\zeta) - X_b(\zeta')}{\sigma_x^2(\zeta) + \sigma_x^2(\zeta')}$$

- Equation for wake centroid, $X_p(\zeta)$, from Mehrling et al., POP (2018)

Model predicts suppression of hosing due to ion motion in agreement with 3D PIC simulations with HiPACE (non relativistic ion motion)

Bunch: E_b=1 GeV, $\varepsilon_{n,0}$=1.1 um, σ_b=0.79 um, L_b=20 um, N_b=0.71 · 10^{10} ($n_{b,0}/n_0$=800 → Γ=0.7), $X_{b,0}$ = 0.1σ_b (displ.)

Background: Hydrogen, n_0=10^{17} cm^{-3}



Two-particles model:

→ decoherence length

$$k_{\beta 0} L_d = \frac{8\pi}{\Gamma} \simeq 36$$

(~6 betatron periods)

Mehrling et al., PRL (2018)

3D PIC simulations show suppression of hosing in presence of ion motion. Emittance is preserved if tapered beams are used.

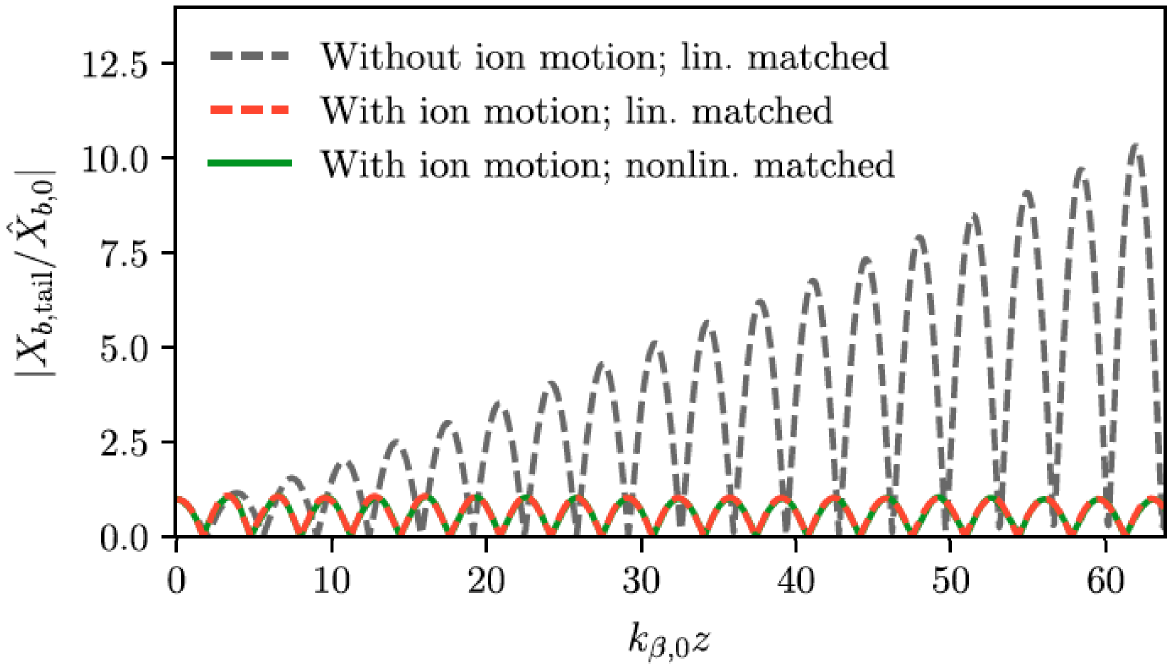
Bunch: E_b=25 GeV, $\varepsilon_{n,0}$=0.26 um, σ_b=0.17 um, L_b=33 um, N_b=$1.5\cdot10^{10}$ [trapezoidal] ($n_{b,0}/n_0$=31000 → Γ=68),
initial displacement: $X_{b,0} = \sigma_b$

Background: Hydrogen, $n_0=10^{17}$ cm^{-3}

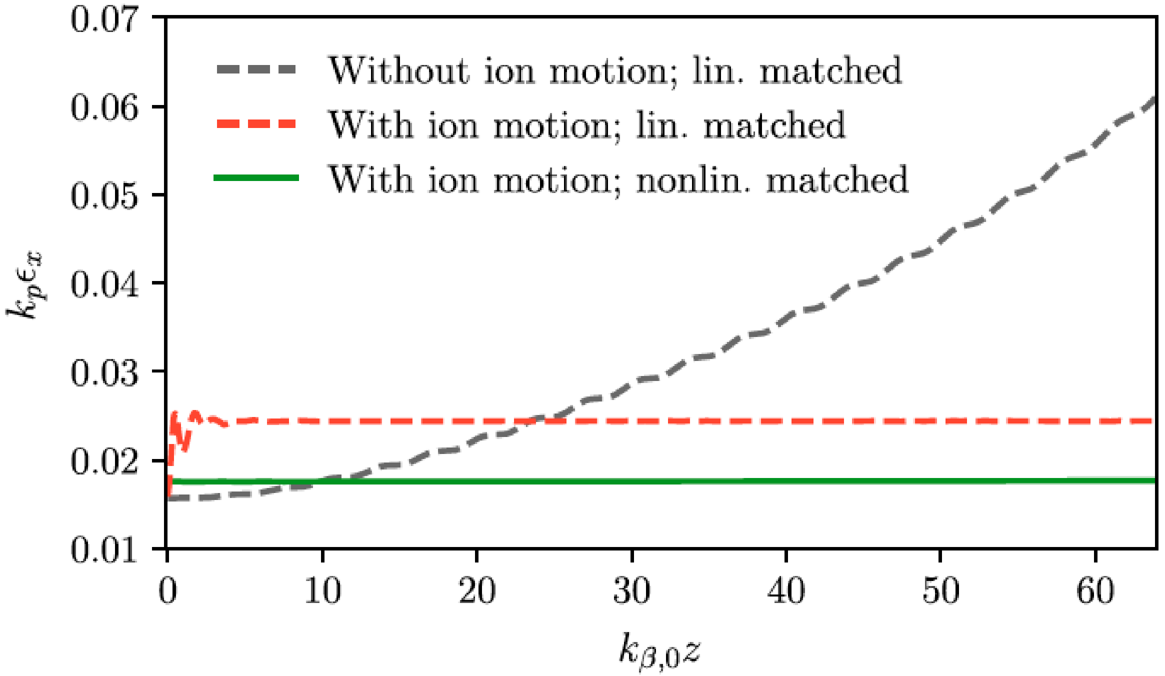
Very rapid (<1 betatron period) decoherence expected from two-particles model

Mehring et al., PRL (2018)

Centroid evolution



Emittance evolution



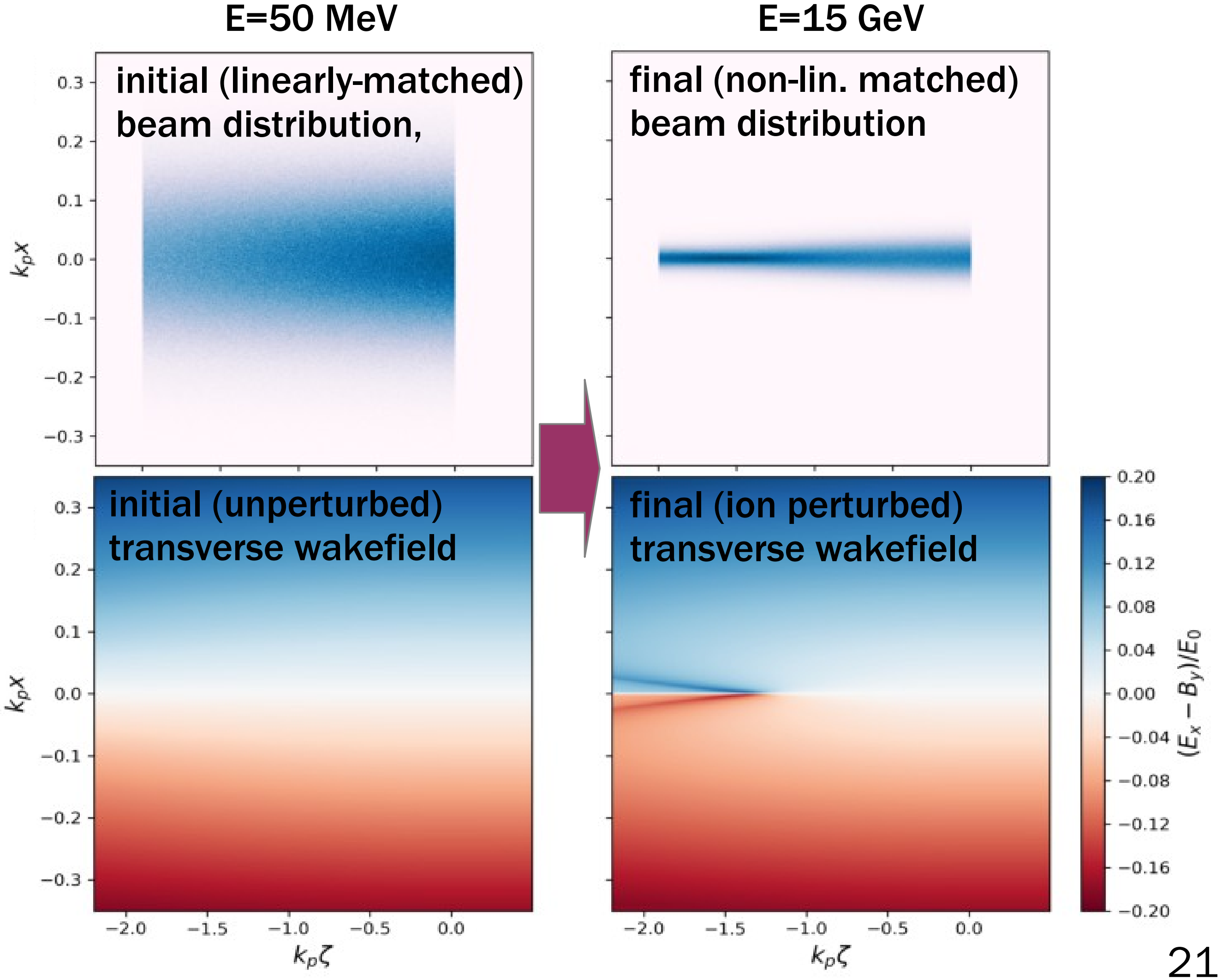
- NO ion motion → hosing, large emittance growth (beam breakup)
- WITH ion motion +LIN. matched → hosing suppressed, but emittance growth (+60%) from ion motion
- WITH ion motion + NONLIN. matched (equilibrium bunch) → hosing suppressed and no emittance growth

Tapered beams which are equilibrium solution in presence of ion motion can be obtained by means of an adiabatic matching procedure

Adiabatic beam matching during plasma acceleration of initially low-energy beams:

- At low energy ion motion is negligible ($\Gamma \sim \gamma^{1/2}$)
- Energy gain adiabatically compresses beam
- Gradual compression of beam triggers ion motion
- Wakefield perturbation is gradually enhanced
- Beam distribution adiabatically adjusts (with emittance preserved at % level) to ion-motion-perturbed wakefields

Bunch: $\varepsilon_{n,0}$=1.2 um, L_b=33 um, N_b=1.5·10^{10} [trapezoidal]
Background: Hydrogen, n_0=10^{17} cm^{-3}



Benedetti et al., in preparation

Summary

- Witness bunch parameters of interest for future plasma-based colliders trigger ion motion and hosing that lead, potentially, to severe emittance degradation and beam breakup;
- Analytic model describing hosing in presence of ion motion has been derived:
  → model predicts suppression of hosing due to detuning associated with ion motion;
- 3D PIC simulations confirm suppression of hosing in presence of ion motion for collider relevant witness beam parameters;
- Beam quality preservation possible with slice-by-slice matching of witness bunch:
  → stable, high-efficiency acceleration possible for HEP applications;
  → strategy to produce tapered beams based on adiabatic matching presented.

Expressions for the perturbed wakefield and emittance growth at saturation have been derived in the case of flat beams (proposed to reduce beamstrahlung)

- Perturbed wakefield for a flat beam (i.e., $\sigma_x >> \sigma_y$, $\varepsilon_x >> \varepsilon_y$)

(unif. longitudinal profile assumed)

$$\begin{cases} \dfrac{W_x}{E_0} \simeq \dfrac{k_p x}{2} \\ \dfrac{W_y}{E_0} \simeq \dfrac{k_p y}{2}\left[1 + \underbrace{Z_i \dfrac{m}{M_i}\dfrac{n_{b,0}}{n_0}(k_p\zeta)^2 \exp(-x^2/2\sigma_x^2) K(y/\sqrt{2}\sigma_y)}_{\text{Wake perturbation twice as large compared to the round beam case}}\right], \quad K(q) = (\sqrt{\pi}/2)\mathrm{erf}(q)/q \end{cases}$$

Horizontal wake essentially unperturbed

- Projected emittance growth at saturation (final)

$$\begin{cases} \dfrac{\epsilon^*_{n,x}}{\epsilon_{n,x}} \simeq 1 \\ \dfrac{\epsilon^*_{n,y}}{\epsilon_{n,y}} \simeq 1 + 0.0027\,\Gamma + 0.0053\,\Gamma^2 \end{cases}$$

Horizontal emittance preserved

Vertical emittance growth twice as large compared to the round beam case