Single-cycle THz signal accompanying laser wake in photo-ionized plasmas and plasma channels

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Can a Langmuir wave emit or absorb radiation?

In the uniform plasma – NO. Violation of quasi-neutrality there does not generate a rotational current

Electromagnetic radiation in the medium is driven by the *rotational* current:

$$\boxed{\left(\nabla^2 - c^{-2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = -\frac{4\pi}{c}\nabla\times\mathbf{j}}$$

Violation of quasi-neutrality generates potential electric field:

$$n_e - n_{e0} = (4\pi e)^{-1}\nabla\cdot\mathbf{E}_{\text{pot}} = (4\pi e)^{-1}\nabla^2\varphi$$

From a divergence of the Ampère's Law,

$$\frac{\partial\nabla\cdot\mathbf{E}}{\partial t} = \nabla\cdot(c\nabla\times\mathbf{B} - 4\pi\mathbf{j})$$

this potential field sets up a purely potential current, equal to the displacement current:

$$\mathbf{j}_{\text{pot}} = -(4\pi)^{-1}\,\partial\mathbf{E}_{\text{pot}}/\partial t = -(4\pi)^{-1}\nabla(\partial\varphi/\partial t) \quad\Rightarrow\quad \nabla\times\mathbf{j}_{\text{pot}} \equiv 0$$

$$\mathbf{v}_{\text{pot}} = -(4\pi e n_{e0})^{-1}\,\partial\mathbf{E}_{\text{pot}}/\partial t = -(4\pi e n_{e0})^{-1}\nabla(\partial\varphi/\partial t) \quad\Rightarrow\quad \nabla\times\mathbf{v}_{\text{pot}} \equiv 0$$

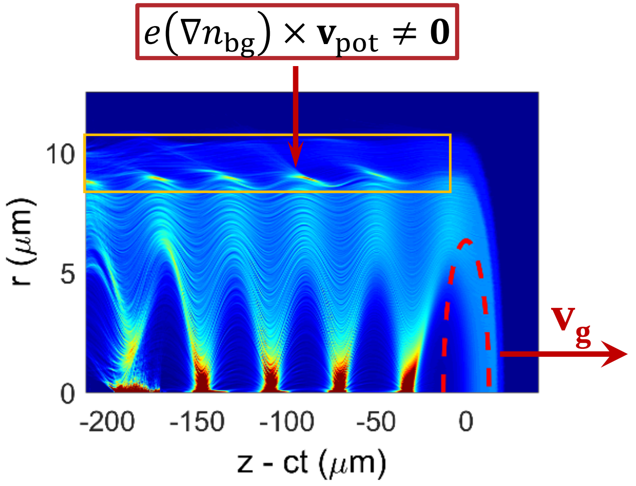
No contribution to the radiation source

In the non-uniform plasma – YES! For instance, $\mathbf{v}_{\text{pot}}$ may contribute to the rotational current:

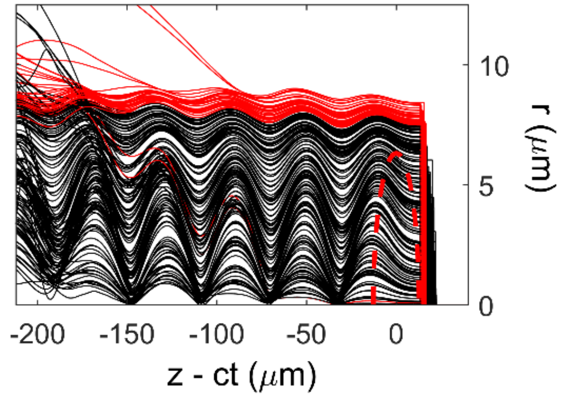
$$\boxed{e\nabla\big(n_{\text{bg}}(\mathbf{r})\mathbf{v}_{\text{pot}}\big) = e\big(\nabla n_{\text{bg}}\big)\times\mathbf{v}_{\text{pot}} \neq \mathbf{0}}$$

Converting optical pulse radiation into low-frequency radiation modes in radially non-uniform plasmas

$n_{\mathrm{bg}} = n_{\mathrm{bg}}(r_\perp),\ \nabla n_{\mathrm{bg}} = \mathbf{e}_r \mathrm{d}n_{\mathrm{bg}}/\mathrm{d}r_\perp$



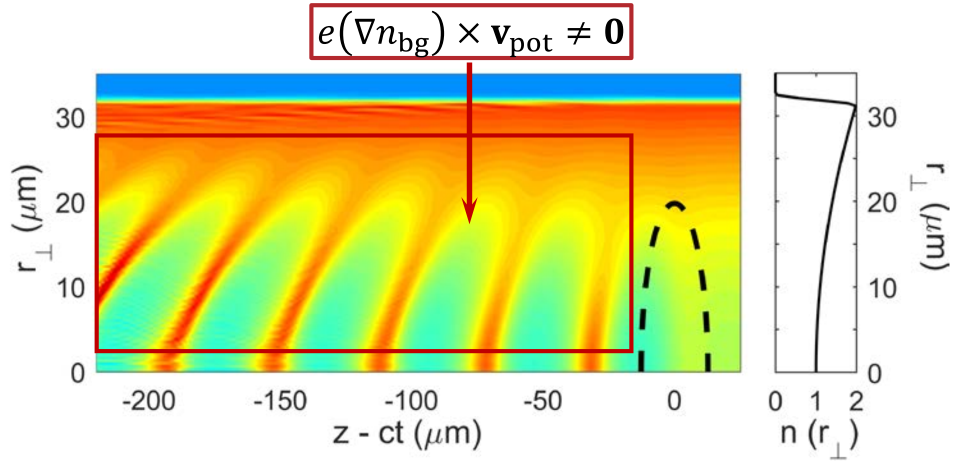
Electron trajectories making up the rotational current



1. *Plasma column created by optical field ionization*

At the column boundary, longitudinal oscillations in electron velocity couple to the sharp gradient in density.

2. *Pre-formed leaky channel – a plasma string with a density depression on axis, and, possibly, with a direct current flowing along it.*



Density profile of a single-mode leaky channel:

$$\frac{n_{\mathrm{bg}}}{n_0} \equiv n(r_\perp) = \begin{cases} 1 + r_\perp^2 / r_{\mathrm{ch}}^2, & r_\perp \le r_{\mathrm{ch}} \\ 0, & r_\perp > r_{\mathrm{ch}} \end{cases} \qquad r_{\mathrm{ch}} = \frac{\omega_{\mathrm{pe0}}}{2c} r_0^2$$

Direct current causes magnetization of plasma channel

External voltage $\mathbf{E}_{\mathrm{dc}}$ applied to the plasma channel causes electron drift along the channel – direct current (DC)

$$\boxed{\mathbf{j}_{\mathrm{dc}} = en_{\mathrm{bg}}(r_\perp)\mathbf{v}_{\mathrm{dc}} = \mathbf{e}_z\sigma_S(r_\perp)E_{\mathrm{dc}} \approx \mathbf{e}_z\sigma_S(0)E_{\mathrm{dc}}}$$

$$v_{\mathrm{dc}} \ll v_{Te} = 2.45\times10^{-3}c\sqrt{T_e[\mathrm{eV}]},\qquad E_{\mathrm{dc}} < E_D = 5.6\times10^{-12}\frac{n_0[\mathrm{cm}^{-3}]}{T_e[\mathrm{eV}]}\frac{\lambda_{ei}Z}{\gamma(Z)}\left(\frac{\mathrm{V}}{\mathrm{m}}\right) \Longrightarrow$$

no runaway electrons

Electrons streaming along the channel generate radially non-uniform, constant azimuthal magnetic field:

$$\boxed{\mathbf{B}_{\mathrm{dc}}(r_\perp \le r_{\mathrm{ch}}) \approx \mathbf{e}_\phi B_{\mathrm{dc}}^{\max}\frac{r_\perp}{r_{\mathrm{ch}}} = \mathbf{e}_\phi \alpha E_{\mathrm{dc}}\frac{r_\perp}{r_{\mathrm{ch}}},}$$

$$\alpha = 2\pi\sigma_S(0)\,r_{\mathrm{ch}}/c$$

$$B_{\mathrm{dc}}^{\max} < \alpha E_D \approx 10^{-11}n_0[\mathrm{cm}^{-3}]T_e^{1/2}[\mathrm{eV}]r_{\mathrm{ch}}[\mu\mathrm{m}](\mathrm{V/m})$$

The limiting value of magnetic field is usually much lower than the cold wave-breaking field of the plasma wave

Both electron drift velocity and static magnetic field generate rotational current,

- $\mathbf{v}_{\mathrm{dc}}$ *through coupling to the density perturbations in the laser wake*
- $\mathbf{B}_{\mathrm{dc}}$ *through changing vorticity*

First-order rotational fields and currents in non-uniform plasma

Electromagnetic radiation (e.g. that of the laser pulse) is driven in the plasma by a *rotational current* :

$$\boxed{\left(\nabla^2 - c^{-2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B}_1 = -\frac{4\pi}{c}\nabla\times\mathbf{j} = -4\pi e\nabla\times\left(n_{\mathrm{bg}}\mathbf{a}\right)}$$

$$\mathbf{a} = \mathbf{v}_{\mathrm{rot}}^{(1)}/c \sim O(\mathbf{E}_1) \sim O(\mathbf{B}_1)$$

Conservation of the 1st – order vorticity expresses rotational velocity through the laser magnetic field:

$$\boxed{\mathbf{W}_1 = \nabla\times\mathbf{a} + \frac{e\mathbf{B}_1}{m_e c^2} \equiv 0} \Rightarrow \boxed{\left(\nabla^2 - c^{-2}\left(\frac{\partial^2}{\partial t^2} + \omega_{\mathrm{pe}}^2(\mathbf{r})\right)\right)\mathbf{B}_1 = -4\pi e\left(\nabla n_{\mathrm{bg}}\right)\times\mathbf{a}}$$

$$\Downarrow \qquad \mathbf{B}_1(t,\mathbf{r}) = \int \mathbf{B}_1(\omega,\mathbf{r})\,\mathrm{e}^{-i\omega t}\mathrm{d}t$$

$$\boxed{\left(\nabla^2 + \frac{\omega^2}{c^2}\varepsilon(\omega,\mathbf{r})\right)\mathbf{B}_1(\omega,\mathbf{r}) + \frac{\nabla\varepsilon}{\varepsilon}\times\nabla\times\mathbf{B}_1(\omega,\mathbf{r}) = 0}$$

$$\varepsilon = 1 - \omega_{\mathrm{pe}}^2(\mathbf{r})/\omega^2 = 1 - n_{\mathrm{bg}}(\mathbf{r})/n_c(\omega)$$

NB: Presence in the plasma of the DC does not alter the 1st – order vorticity as long as the plasma is transparent, $\omega \gg \omega_{\mathrm{pe}}$

Driving second-order perturbations of velocity

First-order electric field and rotational velocity in the under-dense plasma: Gaussian optical pulse

$$\begin{aligned}\mathbf{E}_1(t,r_\perp,z) &= \mathbf{e}_x E_0(r_0/\tilde{r}_0)\mathrm{e}^{-(r_\perp/\tilde{r}_0)^2}\mathrm{e}^{-\xi^2/(2L)^2}\cos[k_0 z-\omega_0 t+\Psi(r_\perp,z)]\\ \mathbf{v}_1(t,r_\perp,z) &= -\mathbf{e}_x c a\sin[k_0 z-\omega_0 t+\Psi(r_\perp,z)]\\ a(t,r_\perp,z) &= a_0(r_0/\tilde{r}_0)\mathrm{e}^{-(r_\perp/\tilde{r}_0)^2}\mathrm{e}^{-\xi^2/(2L)^2}\end{aligned}$$

$$\xi = z - v_{g0}t = z-\sqrt{\varepsilon_L}ct$$

$$\varepsilon_L = 1-(\omega_{pe0}/\omega_0)^2 \lesssim 1$$

$$\nabla = (\partial/\partial\xi\,,\partial/\partial r_\perp)$$

$z=0$: $$\boxed{a(t,r_\perp) = a_0\mathrm{e}^{-(r_\perp/r_0)^2}\mathrm{e}^{-\xi^2/(2L)^2}}$$

The pulse ponderomotive force ($\sim O(a^2)$) generates a 2^{nd} *– order current*, $\mathbf{j}_2 = en_{bg}\mathbf{v}_2 + en_{bg}(\mathbf{v}_{dc}\delta n) \sim O(a^2)$

The velocity splits into two parts, $\mathbf{v}_2 = \mathbf{v}_S + \mathbf{v}_B$, the first driving the magnetic field, the second being driven by it:

$$\boxed{\hat{\mathcal{L}}\mathbf{v}_S = (\sqrt{\varepsilon_L}\,c/4)\nabla(\partial a^2/\partial\xi)}\qquad \hat{\mathcal{L}}\mathbf{v}_B = [e/(m_e c)]\nabla\times\mathbf{B}_2 \qquad \hat{\mathcal{L}} = \varepsilon_L\big(\partial/\partial\xi-\mu(r_\perp)\big)^2 + k_p^2(r_\perp)$$

$\mathbf{v}_S$ is not potential at all: $$\varepsilon_L\partial^2/\partial\xi^2\,\nabla\times\mathbf{v}_S = -[e/(m_e c)]\mathbf{S}_1, \qquad \boxed{\mathbf{S}_1 = (4\pi e/c)\big(\nabla n_{bg}\big)\times\mathbf{v}_S}$$

In fact, $\mathbf{S}_1(\xi, r_\perp)$ is the main contributor to the source of magnetic field

Forced wave equation for the wake magnetic field

Magnetization of the background plasma by DC alters 2nd – order vorticity, adding component $\mathbf{S}_2$ *to the source*

$$\mathbf{W}_2 = c^{-1}\nabla\times\mathbf{B}_2 + \frac{e}{m_e c^2}(\mathbf{B}_2 - \delta n\mathbf{B}_{\mathrm{dc}}) \equiv \mathbf{0}$$

All source terms have azimuthal polarization $\Rightarrow$ $\mathbf{B}_2 = \mathbf{e}_\phi B_{2\phi}$

$$\boxed{(\nabla_\phi^2 - \hat{\mathcal{L}})B_{2\phi} - \frac{4\pi e}{c}\left(\frac{\mathrm{d}n_{\mathrm{bg}}}{\mathrm{d}r_\perp}\mathrm{v}_{Bz}\right) = \overbrace{\frac{4\pi e}{c}\left(\frac{\mathrm{d}n_{\mathrm{bg}}}{\mathrm{d}r_\perp}\mathrm{v}_{Sz}\right)}^{\mathbf{S}_1} - \mathrm{S}_2 - \mathrm{S}_3}$$

The form of sources S_2 and S_3 holds in a wide channel, $(k_{\mathrm{p0}}r_{\mathrm{ch}})^{1/2} \sim k_{\mathrm{p0}}r_0 \gg 1$

in the absence of runaway electrons,

$$B_{\mathrm{dc}} \ll \alpha E_D \ll E_{\mathrm{br}}$$

$$S_2 = k_{\mathrm{p}}^2(r_\perp)B_{\mathrm{dc}}\delta n$$

→ Modification of 2nd – order vorticity

$$S_3 = -\frac{4\pi}{c}j_{\mathrm{dc}}\frac{\partial\delta n}{\partial r_\perp}$$

→ Coupling DC to the radial gradient of density perturbation in the wake

Equation for electron density perturbation in the non-uniform plasma, cf. N. E. Andreev *et al*., Phys. Plasmas **4**, 1145-53 (1997)

In a wide channel, $\mathbf{S}_1$ is almost always dominant

$\mathbf{S}_2$ becomes comparable with $\mathbf{S}_1$ if a channel is sufficiently wide,

$$k_{p0}r_{\mathrm{ch}} > (v_{\mathrm{dc}}/c)^{-1/2} > 20T_e^{-1/4}[\mathrm{eV}]$$

and $\mathbf{S}_3$ is always a small correction

Electron oscillations at the border of plasma column produced by photo-ionization: Cylindrical PIC simulations

Parameters of the plasma and Gaussian drive pulse:

$\lambda_0 = 0.8\ \mu\mathrm{m}$ (carrier frequency $\omega_0 = 2\pi c/\lambda_0 = 2.356\times10^{15}\ \mathrm{s}^{-1}$)

$a_0 = 0.5$ (peak intensity $5.4\times10^{17}\ \mathrm{W/cm}^2$)

$n_{e0} = 7\times10^{17}\ \mathrm{cm}^{-3}$ [gas : heluim (fully stripped), argon (8 shells stripped)]

$r_0 = 9.5\ \mu\mathrm{m}$ ($k_{p0}r_0 = 1.5$)

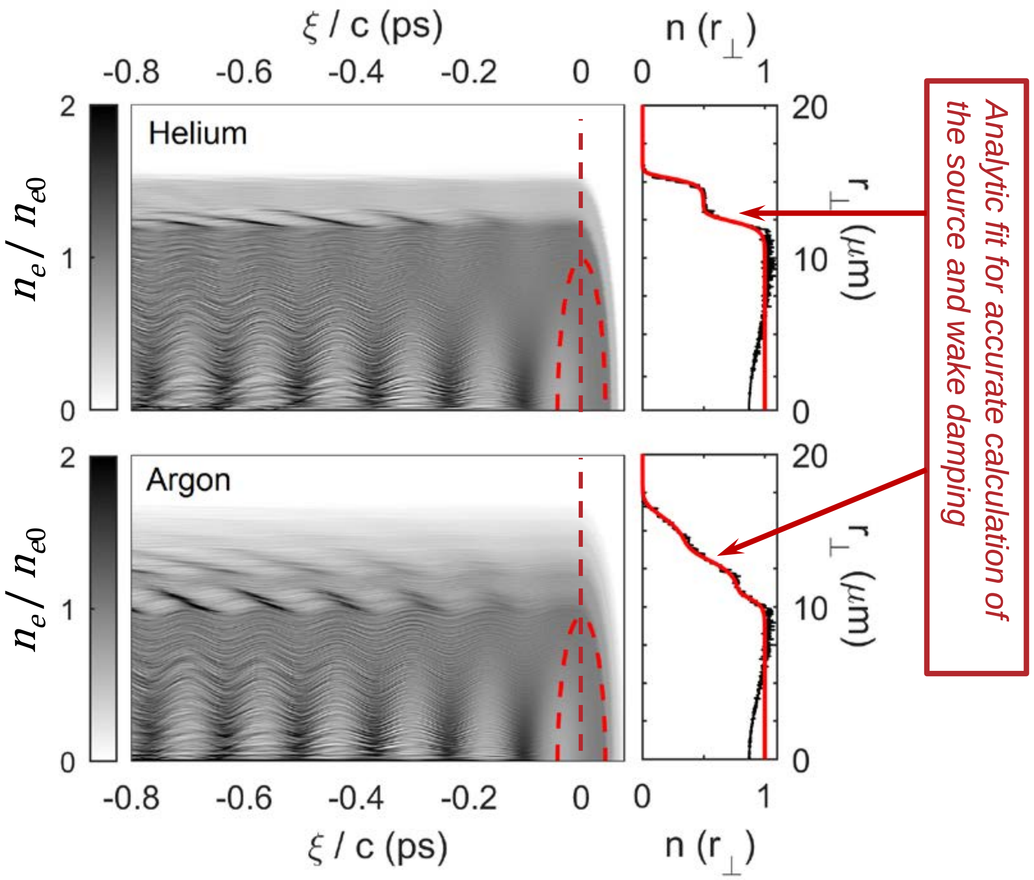
$L = 6.36\ \mu\mathrm{m}$ (FWHM in intensity $\tau_L = \sqrt{8\ln 2}\, L/c = 50$ fs)

Simulation code:

WAKE – P. Mora & T. M. Antonsen Jr., Phys. Plasmas **4**, 217 (1997)

(quasi-static PIC, ponderomotive particle push, cylindrical symmetry, extended paraxial solver for the pulse, OFI – Keldysh model)

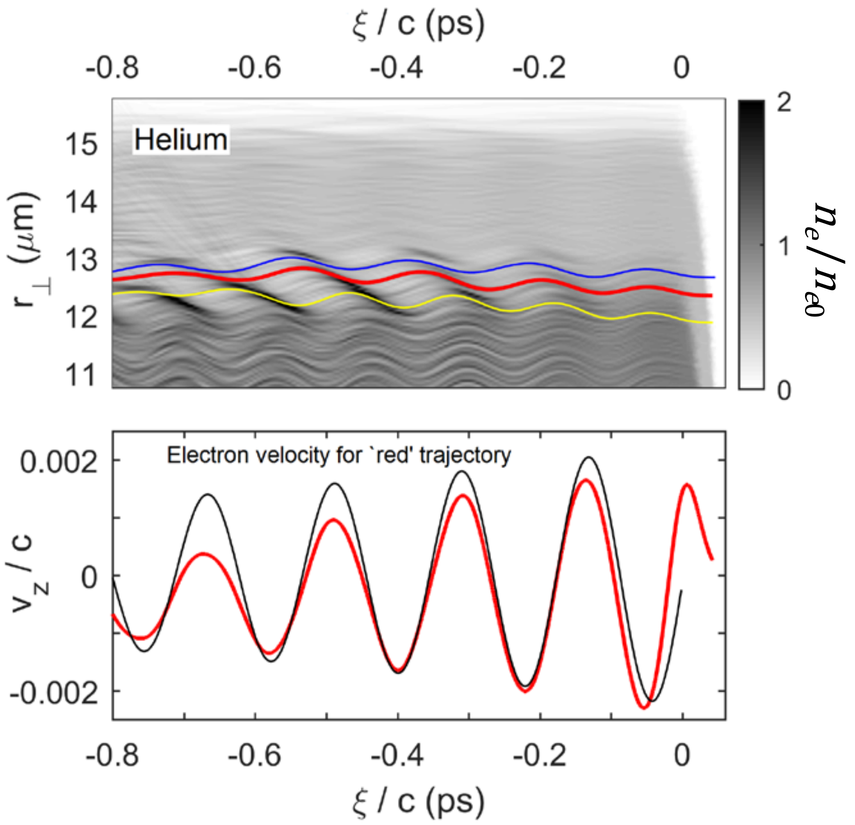
Laser wake in the vicinity of the focal plane

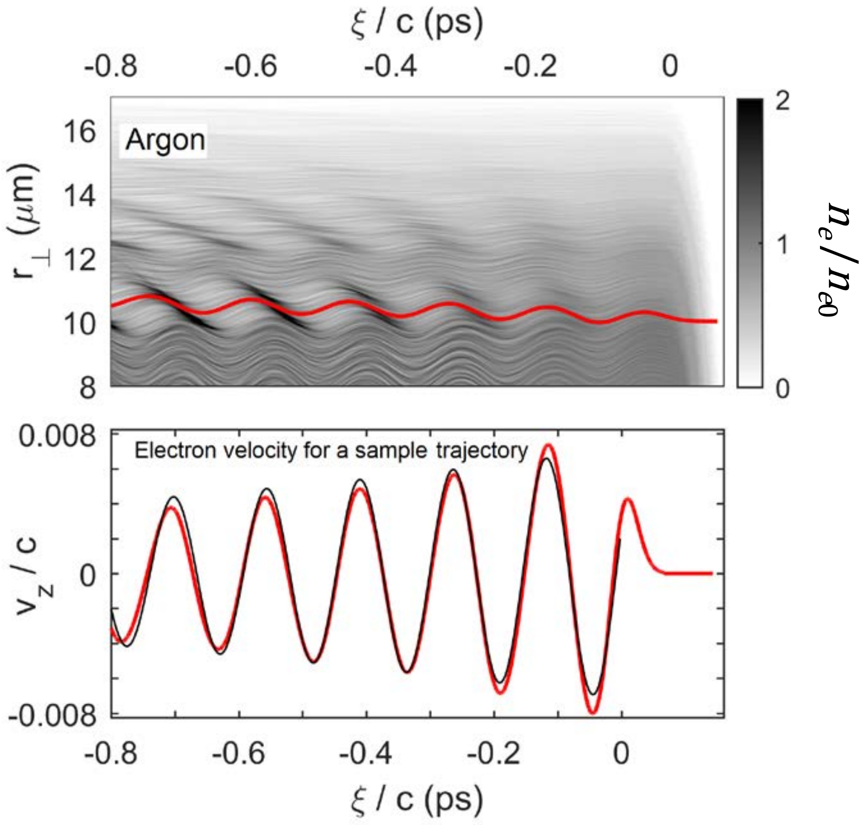


Phase mixing and damping of electron plasma wave in the column boundary layer

Tracking WAKE macro-particles yields the decrement of electron velocity oscillations as a function of radius.

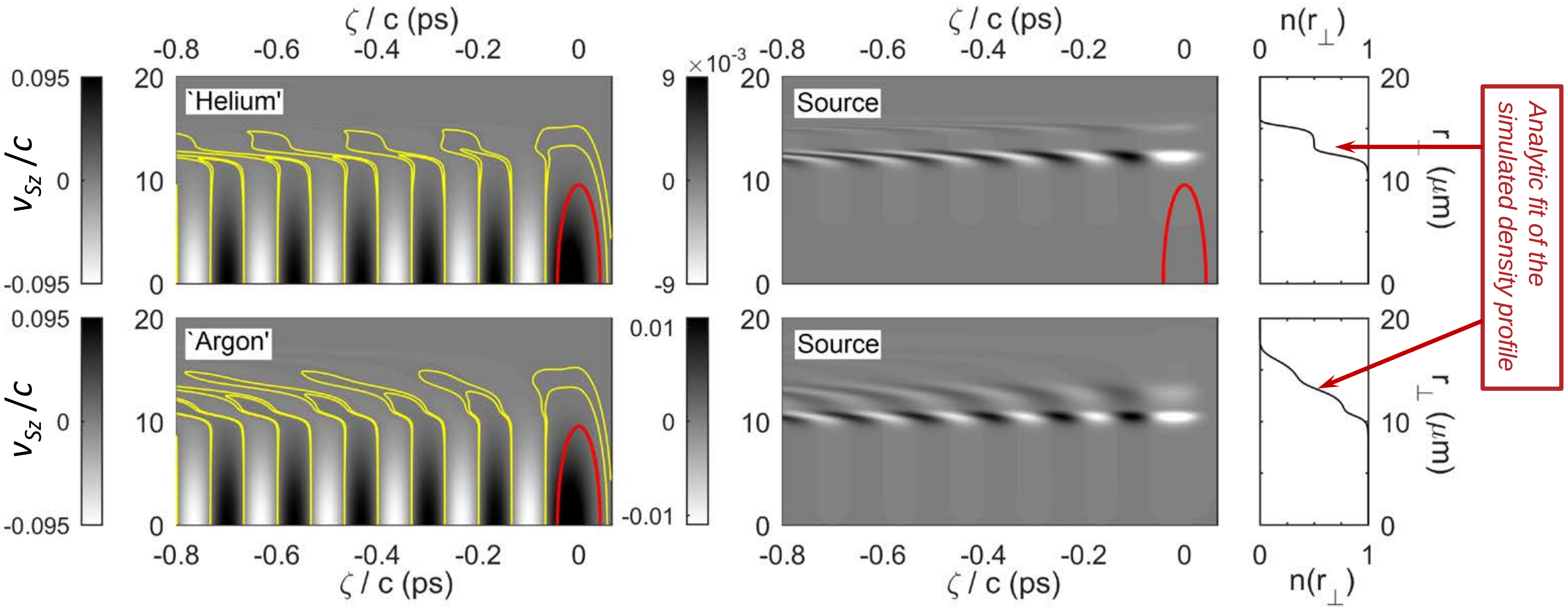
Linear analytic solution with this phenomenological decrement (black) agrees with the PIC simulation fairly well.





Analytical map of longitudinal electron velocity and localization of the source of rotational fields

- The source is concentrated in the column boundary layer (the shell is ~ 2 μm-thick for He, ~ 8 μm-thick for Ar)
- Velocities contributing to the source are lower than $2\times10^{-3}c$



This source can produce a detectable EM signal

Calculating EM signal supported by the plasma wake

Recalling the model equations:

$$(\nabla_\phi^2 - \hat{\mathcal{L}})B_{2\phi} - \frac{4\pi e}{c}\left(\frac{\mathrm{d}n_{\mathrm{bg}}}{\mathrm{d}r_\perp}\mathrm{v}_{Bz}\right) = \frac{4\pi e}{c}\left(\frac{\mathrm{d}n_{\mathrm{bg}}}{\mathrm{d}r_\perp}\mathrm{v}_{Sz}\right)$$

$$\hat{\mathcal{L}}v_{Bz} = [e/(m_e c)]\, r_\perp^{-1}\,\partial(r_\perp B_{2\phi})/\partial r_\perp$$

$$\hat{\mathcal{L}}v_{Sz} = (\sqrt{\varepsilon_L}\, c/4)\,\partial^2 a^2/\partial\xi^2$$

$$\hat{\mathcal{L}} = \varepsilon_L\big(\partial/\partial\xi - \mu(r_\perp)\big)^2 + k_\mathrm{p}^2(r_\perp)$$

Normalizing all lengths to $k_{\mathrm{p0}}^{-1} = c/\omega_{\mathrm{pe0}}$, wave numbers to k_{p0}, and passing into Fourier domain,

$$B_{2\phi}^k(r_\perp) = \frac{e}{m_e c\,\omega_{\mathrm{pe0}}}\int B_{2\phi}(\xi, r_\perp)\,\mathrm{e}^{\mathrm{i}k\xi}\,\mathrm{d}\xi$$

yields equations for Fourier images of the azimuthal magnetic and longitudinal & radial electric fields

Local decrement empirically derived from WAKE simulations:

$$\mu(r_\perp) = \mu_0\big(\mathrm{d}n_{\mathrm{bg}}/\mathrm{d}r_\perp\big)\big(\max(\mathrm{d}n_{\mathrm{bg}}/\mathrm{d}r_\perp)\big)^{-1}$$

$$\mu_0 = k_{\mathrm{p0}}/(10\pi) \;-\; \text{helium}$$
$$\mu_0 = k_{\mathrm{p0}}/(20\pi) \;-\; \text{argon}$$

Bi-Gaussian pulse in time and Fourier domain:

$$\begin{aligned} a^2(\xi, r_\perp) &= a_0^2\mathrm{e}^{-2(r_\perp/r_0)^2}\mathrm{e}^{-\xi^2/2L^2} \\ a_k^2(r_\perp) &= \sqrt{2\pi}a_0^2 L\mathrm{e}^{-(kL)^2/2}\mathrm{e}^{-2(r_\perp/r_0)^2} \end{aligned}$$

Fourier components of the EM signal inside the plasma

$$\boxed{\left(\frac{\mathrm{d}}{\mathrm{d}r_\perp}+\frac{\mathrm{d}n/\mathrm{d}r_\perp}{k^2\varepsilon_k\varepsilon_L}\right)\frac{1}{r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp B_{2\phi}^k\right)-k^2(1-\varepsilon_k\varepsilon_L)B_{2\phi}^k=\frac{\mathrm{d}n/\mathrm{d}r_\perp}{4\varepsilon_k\sqrt{\varepsilon_L}}a_k^2}$$

$$E_{2r}^k=\frac{1}{k^2\varepsilon_k\varepsilon_L}\left[\sqrt{\varepsilon_L}(1+\mathrm{i}\,\mu/k)^2B_{2\phi}^k+(r_\perp/r_0^2)n(r_\perp)a_k^2\right]$$

$$E_{2z}^k=\frac{\mathrm{i}}{k\varepsilon_k\varepsilon_L}\left[\sqrt{\varepsilon_L}(1+\mathrm{i}\,\mu/k)^2\frac{1}{r_\perp}\frac{\mathrm{d}}{\mathrm{d}r_\perp}\left(r_\perp B_{2\phi}^k\right)-\frac{1}{4}n(r_\perp)a_k^2\right]$$

These are the images of ponderomotively driven currents

They vanish in the plasma-free area

Electric field in the plasma-free area is purely rotational

Dielectric function with local attenuation:

$$\boxed{\varepsilon_k=[1+\mathrm{i}\,\mu(r_\perp)/k]^2-n(r_\perp)/(\varepsilon_L k^2)} \qquad n(r_\perp)=n_{\mathrm{bg}}(r_\perp)/n_{e0}$$

For any k from the band $0<k<\varepsilon_L^{-1/2}$ there exists a critical surface $r_\perp=R_{\mathrm{cr}}$ where $\varepsilon_k(0,R_{\mathrm{cr}})=0$

Thus the task at hand is transformation of the potential field of a Langmuir oscillation into an electromagnetic vacuum eigenmode

Fourier components of the EM signal in plasma-free area

Inside the plasma, the solution must be proportional to the source. Magnetic field and its derivative vanish on axis:

$$B_{2\phi}^{k}(r_{\perp}=0)=\frac{\mathrm{d}B_{2\varphi}^{k}}{\mathrm{d}r_{\perp}}(r_{\perp}=0)=0$$

The bounded at infinity asymptotic is evanescent (not surprisingly, as the source is subluminal):

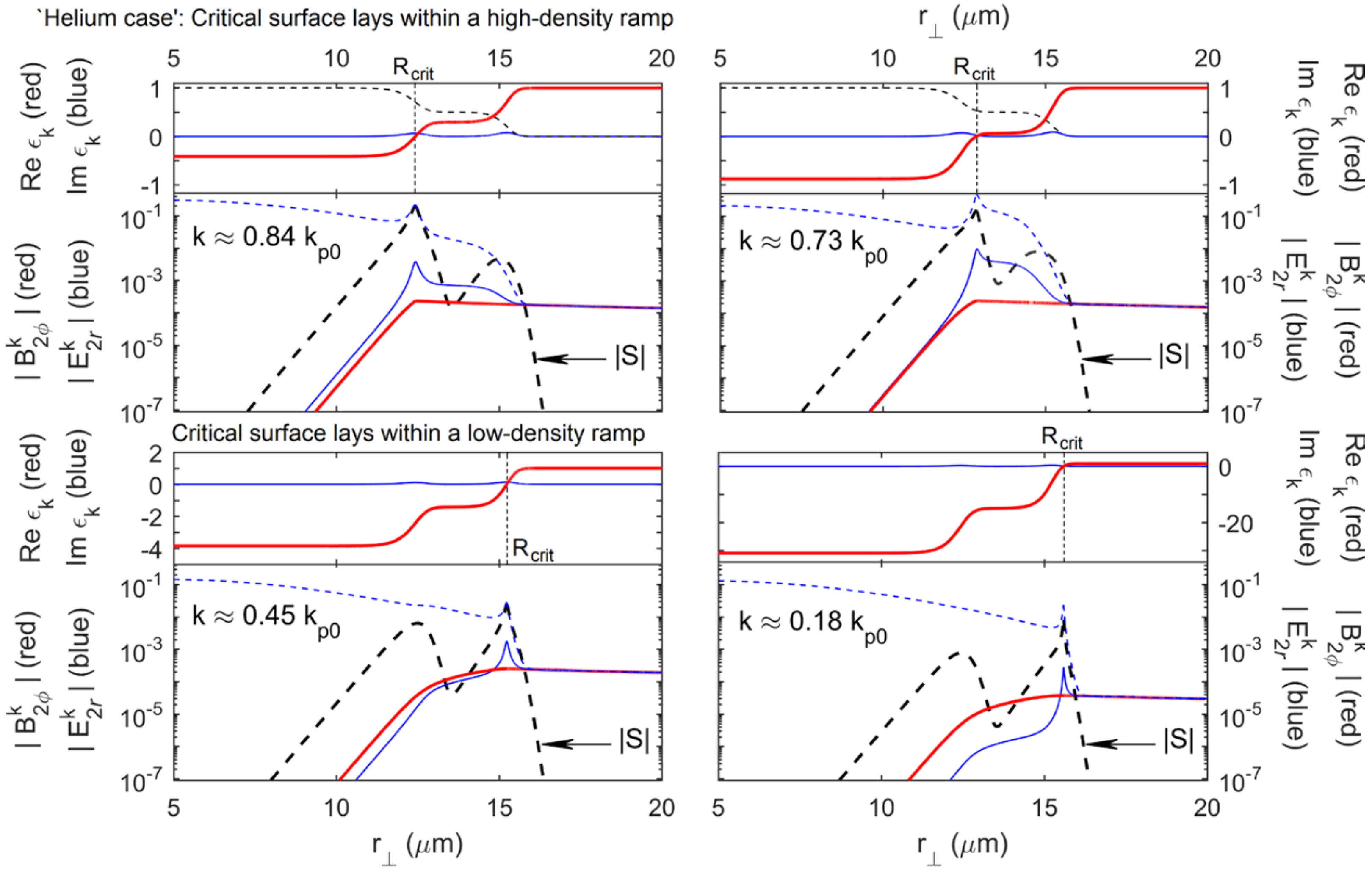
$$B_{2\phi}^{k}(r_{\perp}\to\infty)=CK_{1}\left(kr_{\perp}\sqrt{1-\varepsilon_{L}}\right)\underset{r_{\perp}>k^{-1}(1-\varepsilon_{L})^{-1/2}}{\xrightarrow{\text{'far field zone'}}}\sqrt{\frac{\pi}{2kr_{\perp}\sqrt{1-\varepsilon_{L}}}}\,\mathrm{e}^{-kr_{\perp}\sqrt{1-\varepsilon_{L}}}$$

$$\frac{\mathrm{d}B_{2\phi}^{k}}{\mathrm{d}r_{\perp}}(r_{\perp}\to\infty)=-\frac{C}{2}k_{\perp}\sqrt{1-\varepsilon_{L}}\left[K_{0}\left(kr_{\perp}\sqrt{1-\varepsilon_{L}}\right)+K_{2}\left(kr_{\perp}\sqrt{1-\varepsilon_{L}}\right)\right]$$

The `in' and `out' solutions are matched (eliminating the complex constant C) at the critical surface

Fourier images of the fields near the column surface

- Fourier images of radial electric field and the source spike at the critical surface
- The lack of attenuation would turn these peaks into singularities
- Magnetic field has no anomaly at the critical surface
- Fields slowly decay towards plasma-free space
- Magnetic field and rotational part of electric field sharply fall off towards axis



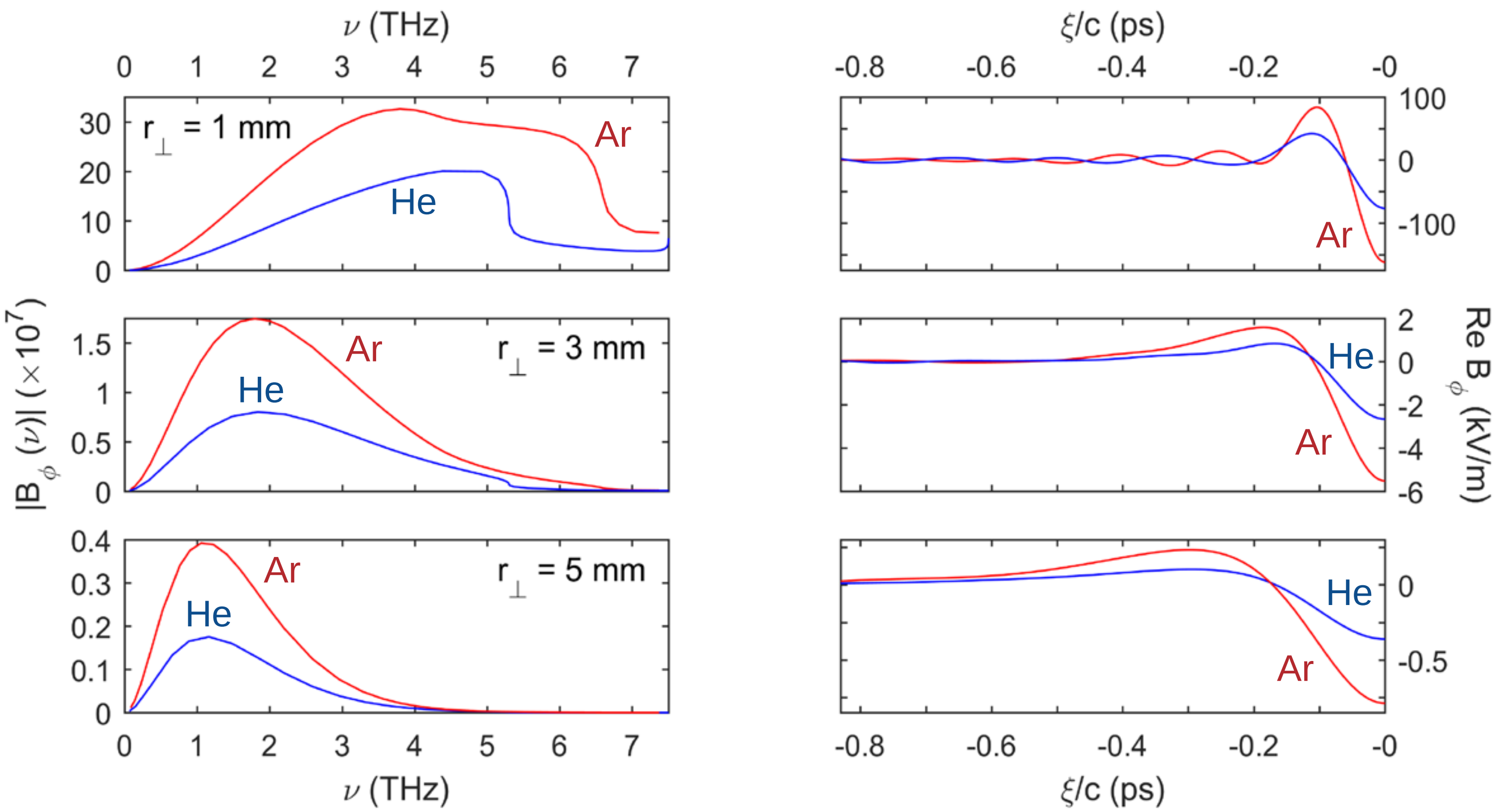
Field in the far-field zone: Formation of single-cycle THz pulse

Far-field zone – fields are exponentially evanescent:

$$B_{2\phi}^{k}\Big(\underbrace{r_\perp > k^{-1}(1-\varepsilon_L)^{-1/2}}_{\text{"Far-field" zone}}\Big) \sim \sqrt{\frac{\pi}{2kr_\perp\sqrt{1-\varepsilon_L}}}\, e^{-kr_\perp\sqrt{1-\varepsilon_L}}$$

$$\sqrt{1-\varepsilon_L} = 0.02$$

- High-frequency components decay rapidly
- The spectrum becomes narrower, it's peak shifting towards 1 THz
- The signal length reduces to a single cycle
- Signal becomes radially polarized
- The peak-to-peak signal voltage exceeds a few 100's V/m



Important observations and conclusions

- Surface of the plasma column, created by OFI of the neutral gas, supports an azimuthally polarized rotational current – a source of high-amplitude EM THz pulse following in the wake of the optical drive pulse
- The rotational current, localized in a μm-thin cylindrical shell, is produced by coupling the longitudinal electron velocity in the plasma wake to the radial gradient of electron density in the ionization front
- The THz signal is evanescent in the radial direction, as dictated by the sub-luminal wake phase velocity
- Faster decay of the higher-frequency components shifts the signal spectrum to 1 THz as the observer moves further away from the column
- At a mm-scale distance from the column surface (100's of the column radii) the THz signal turns into a single-cycle burst, with a peak-to-peak signal voltage on a kV/m level.
- This THz pulse may be captured by electro-optical methods, already used in diagnostics of femtosecond electron beams [A. Curcio, F. Bisesto *et al*., Phys. Rev. Appl. **9**, 024004 (2018)]

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Questions?