Modeling of capillary discharge plasmas for wakefield acceleration and beam transport

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Outline

- Motivation
- Modeling discharge capillary plasmas with FLASH
  - *Solvers, EOS, and boundary conditions*
  - *Discharge current and transport*
- Benchmarks for 2D capillary simulations
  - *Three-phase capillary dynamics*
  - *Comparison of radial density and temperature profiles*
- Additional considerations
  - *Laser heater for sub-channel formation*
  - *Nonlinearities in plasma current response*
- A FLASH interface for capillary modeling

Future compact accelerators rely on plasma stages

- Wakefield acceleration challenges
  - *Plasma stages must reach higher energy (~10 GeV per stage)*
  - *Reproducible and robust at high repetition rate*
  - *Longer targets, lower densities, and better driver coupling are required*

- Beam transport challenges
  - *Collider quality beams must transport cleanly between stages*
  - *Stronger fields, shorter elements, more flexibility are required*

- Capillary discharge plasmas promise solutions to both problems
  - *Low density, parabolic channels with small matched spot sizes for LWFA*
  - *High peak currents for orders-of-magnitude increase in focusing fields*
  - *Flexible ionization profiles for novel plasma columns*
  - *Existing tools are insufficient to model dynamics with require fidelity*

Radial density channels enable laser guiding

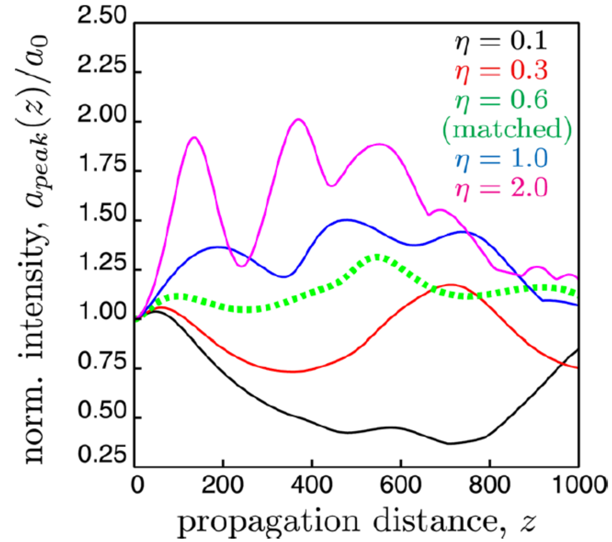
- Maintain laser intensity along channel
  - *Gaussian laser matches to a parabolic channel*

$$n_p(r) = n_0 + \Delta n \frac{r^2}{W_0^2}$$

  - *Mismatch leads to intensity oscillations*

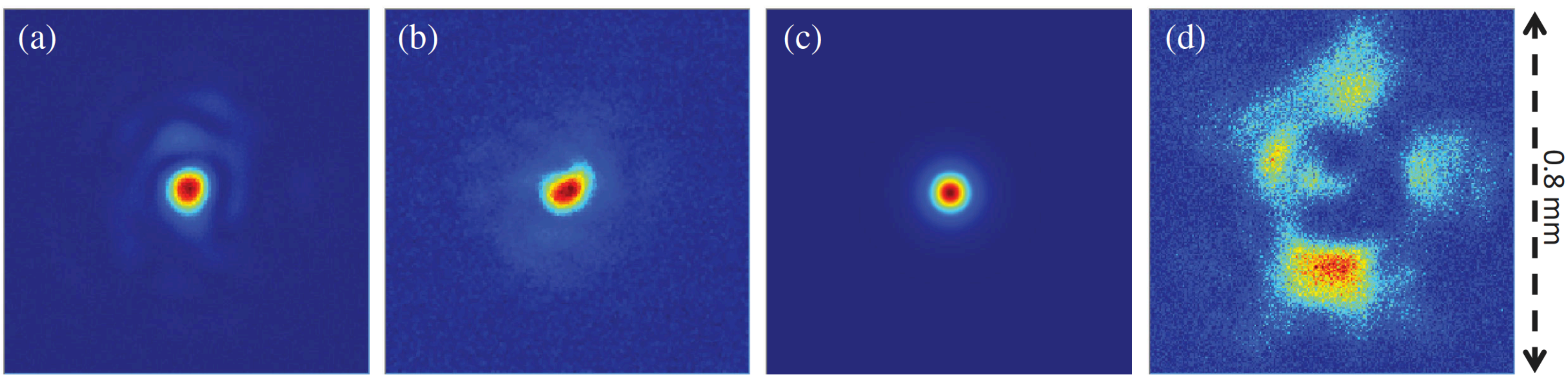
$$\lambda_{\rm osc} = \pi z_{RM} = \pi^2 r_0^2 / \lambda_0$$

- Capillaries constrain profile and density



Phys. Rev. Lett. **122**, 084801 (2019)

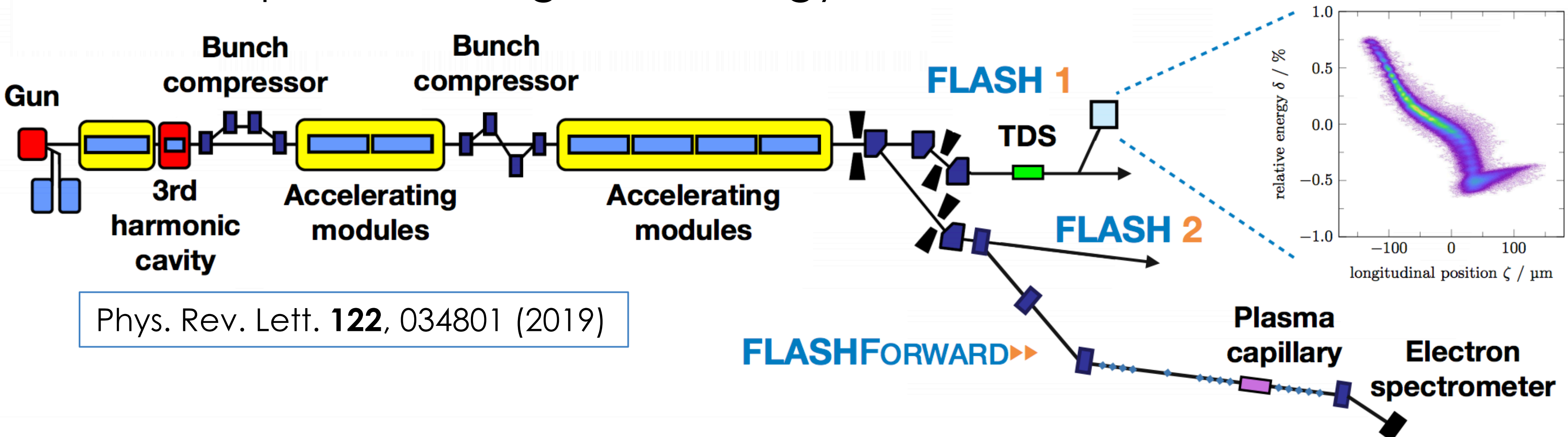
Phys. of Plasmas **19**, 053101 (2012)



Vacuum focus z=0 cm	Capillary exit z=20 cm (experiment)	Capillary exit z=20 cm (simulation)	Vacuum z=5.4 cm

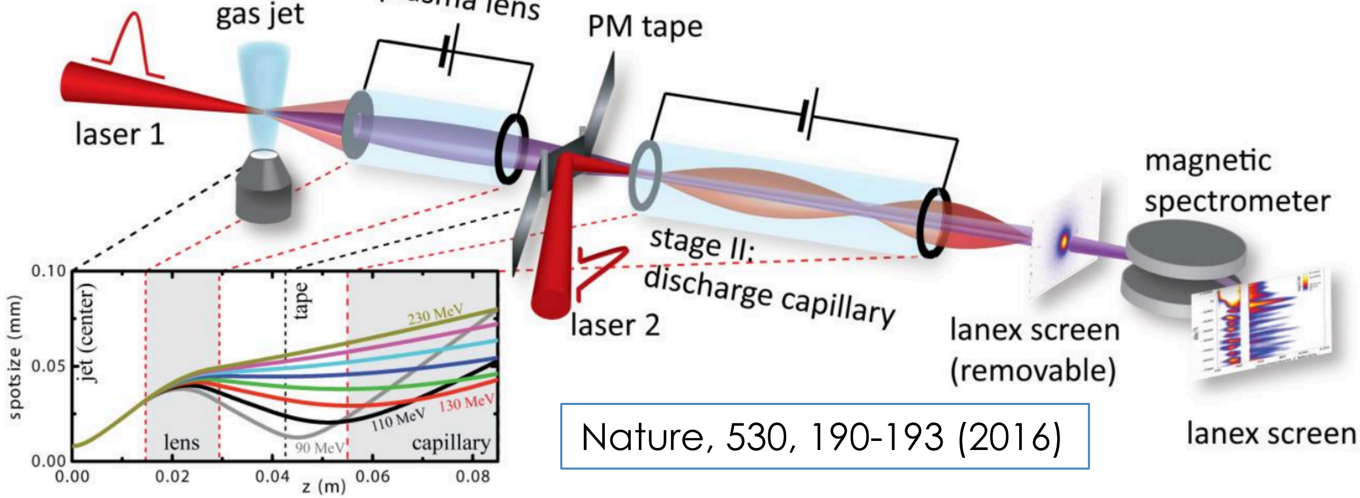
High field strengths enable beam transport

Beam transport, focusing, and energy decorrelation



Phys. Rev. Lett. **122**, 034801 (2019)

Staging



Nature, 530, 190-193 (2016)

Deconstructing a Capillary Discharge Plasma

- Narrow insulating tube with controlled gas flow
  - *Hydrogen, Helium, Argon*
  - *Length from ~1-30 cm*
    Radius from ~0.1-1 mm
- Applied voltage to rings to drive discharge
  - *Vary density, voltage to adjust*

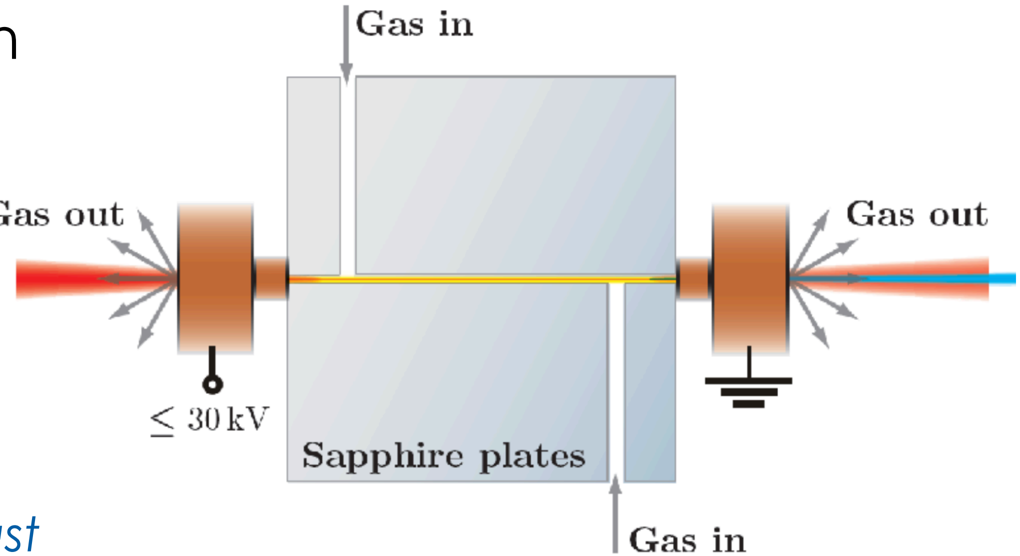


Figure Credit: Jens Osterhoff

- Many computational Complexities
  - *High Aspect Ratio*
  - *High temporal resolution required*
    - Transport timescales (0.01-10 ps) are small compared to discharge (>100 ns)
    - Magnetic field effects require explicit integration
  - *Time Dependent boundaries require special treatment*
    - Discharge representation influences choice of boundary conditions
    - Electrical and Thermal conductivities must change self-consistently

Adopting the FLASH code

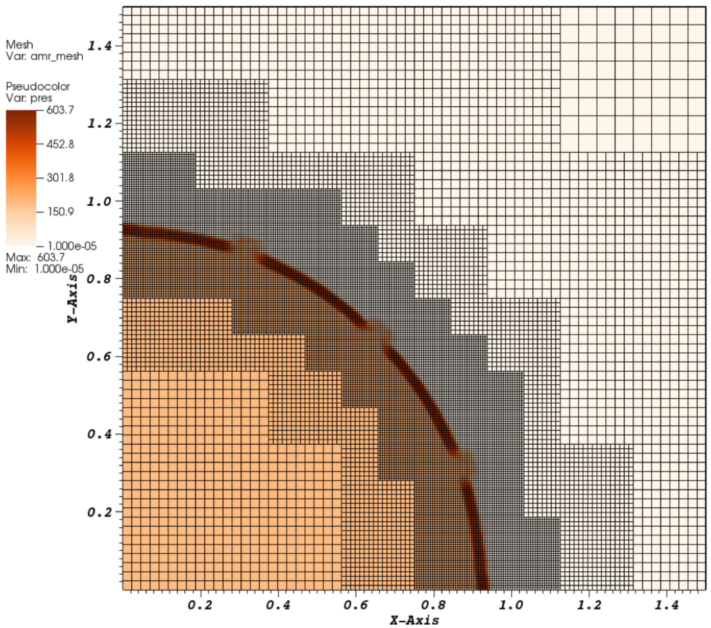
- Publicly available multi-physics application developed at University of Chicago – Flash Center for Computational Science (1997)
  - *Compressible flow problems from astrophysics to HEDP*
  - *Additional Physics specific to HEDP*
    - Heat exchange, ionization, conductivity/resistivity/viscosity/opacity, radiative transfer, inverse Bremmsstrahlung, gravity, nuclear burning, NEI, and more

- Modular design written in Fortran
  - *Include only necessary physics within each executable*
  - *Pre-tabulate user configurable input/output parameters*
  - *Separate runtime configuration options*

- External solver libraries
  - *HYPRE provides general, implicit diffusion solves*
  - *PARAMESH and Chombo for AMR*
  - *Gamma law or pre-tabulated equations of state*

- Data Analysis Tools
  - *Parallel I/O support via HDF5 and NetCDF*
  - *VisIt plug-in for HPC visualization pipeline*
  - *Python YT, IDL, OpenPMD ("Rad-Hydro") APIs*

FLASH Fundamentals



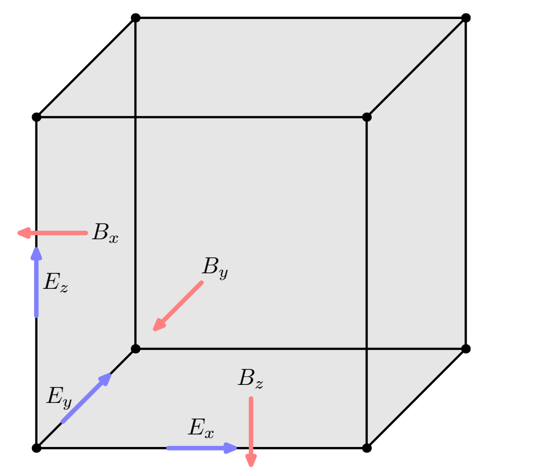
- Uniform, block-structured grid
  - *AMR with user-specified refinement*
- 1/2/3D geometries
  - *Cartesian, Spherical, Cylindrical, Polar*
- Solves fluid evolution with convection

$$\left\{\begin{array}{c}\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v})=0\\ \frac{\partial}{\partial t}(\rho\boldsymbol{v})+\nabla\cdot(\rho\boldsymbol{v}\boldsymbol{v})+\nabla P_{\text{tot}}=0\\ \frac{\partial}{\partial t}(\rho E_{\text{tot}})+\nabla\cdot[(\rho E_{\text{tot}}+P_{\text{tot}})\boldsymbol{v}]=Q_{\text{las}}-\nabla\cdot\boldsymbol{q}\end{array}\right\}$$

- 3T capabilities (per species)

$$\left\{\begin{array}{c}\frac{\partial}{\partial t}(\rho e_{\text{i}})+\nabla\cdot(\rho e_{\text{i}}\boldsymbol{v})+P_{\text{i}}\nabla\cdot\boldsymbol{v}=\rho\frac{c_{v,\text{e}}}{\tau_{ei}}(T_{\text{e}}-T_{\text{i}})\\ \frac{\partial}{\partial t}(\rho e_{\text{e}})+\nabla\cdot(\rho e_{\text{e}}\boldsymbol{v})+P_{\text{e}}\nabla\cdot\boldsymbol{v}=\rho\frac{c_{v,\text{e}}}{\tau_{ei}}(T_{\text{i}}-T_{\text{e}})-\nabla\cdot\boldsymbol{q}_{\text{e}}+Q_{\text{abs}}-Q_{\text{emis}}+Q_{\text{las}}\\ \frac{\partial}{\partial t}(\rho e_{\text{r}})+\nabla\cdot(\rho e_{\text{r}}\boldsymbol{v})+P_{\text{r}}\nabla\cdot\boldsymbol{v}=\nabla\cdot\boldsymbol{q}_{\text{r}}-Q_{\text{abs}}+Q_{\text{emis}}\end{array}\right\}$$

- Electromagnetic fields defined on a Yee mesh
  - *Secondary, uniform mesh overlapping fluid domain*
- Second-order accurate predictor-corrector updates both fluid and field quantities
  - *Divergence-free condition enforced*
  - *Explicit integration scheme*



FLASH – Transport and Boundaries

- Dissipation described by conduction and heat exchange

$$\frac{\partial e_\mathrm{e}}{\partial t} = \nabla \cdot K_\mathrm{e} \nabla T_\mathrm{e}$$

$$\frac{\partial e_\mathrm{i}}{\partial t} = \frac{c_{v,\mathrm{e}}}{\tau_{ei}}(T_\mathrm{e} - T_\mathrm{i})$$
$$\frac{\partial e_\mathrm{e}}{\partial t} = \frac{c_{v,\mathrm{e}}}{\tau_{ei}}(T_\mathrm{i} - T_\mathrm{e})$$

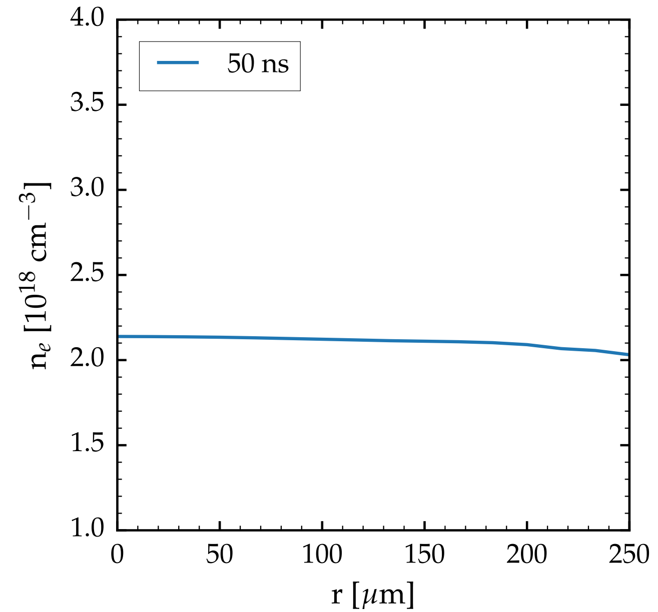
- Spitzer model describes plasma resistivity (and thermal conduction)

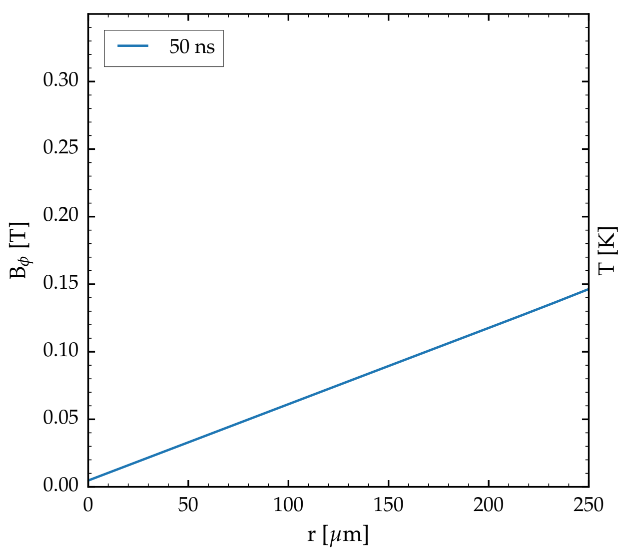
$$\eta_\perp = \frac{4\sqrt{2\pi}}{3} \frac{Z e^2 m_e^{1/2} \ln \Lambda}{(4\pi\epsilon_0)^2 (k_B T_e)^{3/2}} F(Z)$$

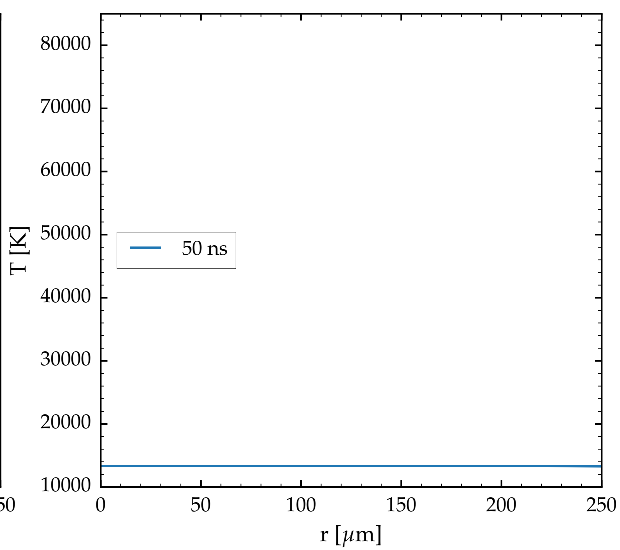
- Magnetic field specified along boundary via Ampere's Law
  - *Discharge is modelled via changing field at the boundary*
  - *Insufficient to describe breakdown, but acceptable for slowly varying current*
- Notable Boundary Conditions
  - ***At r=0:*** *Axisymmetric (reflecting) conditions for transverse fluid flow*
  - ***At r=R:*** *Dirichlet conditions for transverse fluid flow* $v_\perp = 0$
    - Need to zero fluxes at outer radius to prevent outflow
  - ***At r=R:*** *Dirichlet condition for thermal conductivity* ($K_e(r=R)$)
    - Differs from analytical solutions specifying temperature at boundary

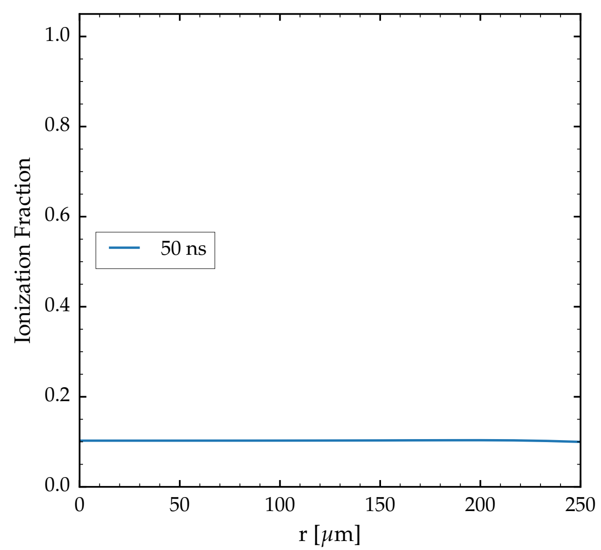
Capillary Benchmark Simulations in R-Z

- Phase I: Uniform Ohmic heating
  - *Rising, linear magnetic field profile*
    - Some initial ionization used to bootstrap field penetration into the plasma
  - *Uniform Heating*
  - *Increasing ionization*
  - *No distinctive plasma motion*
- Period of~100 ns assuming comparable current rise time



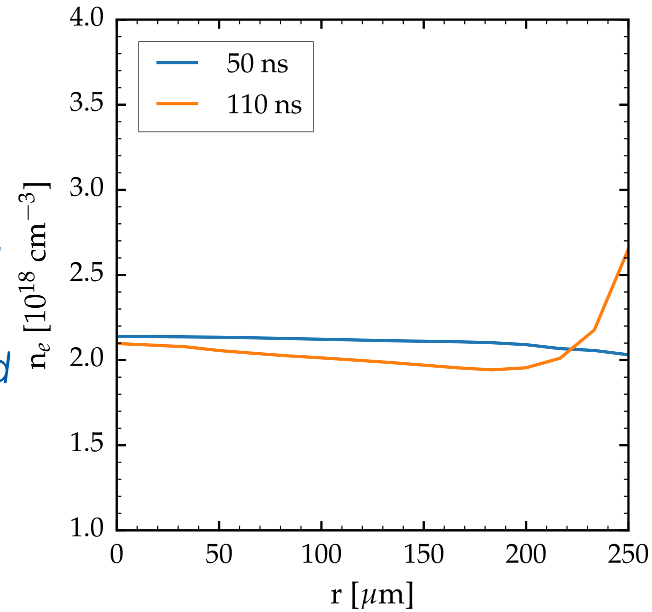


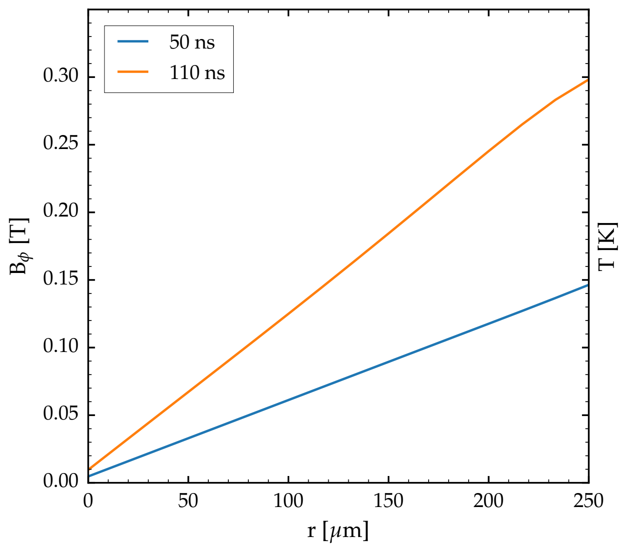


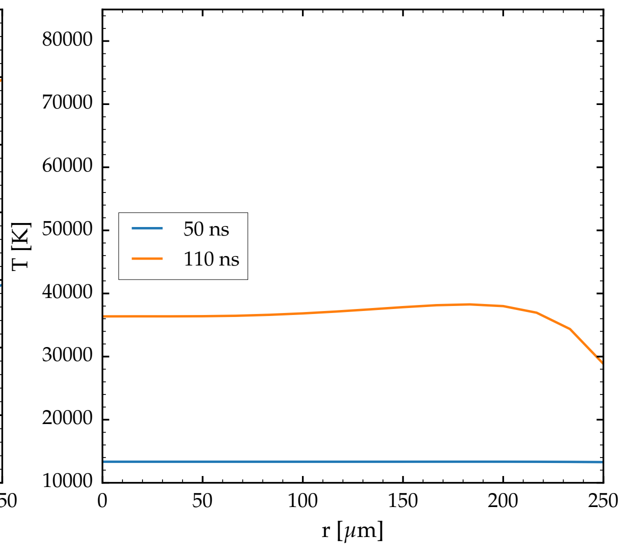


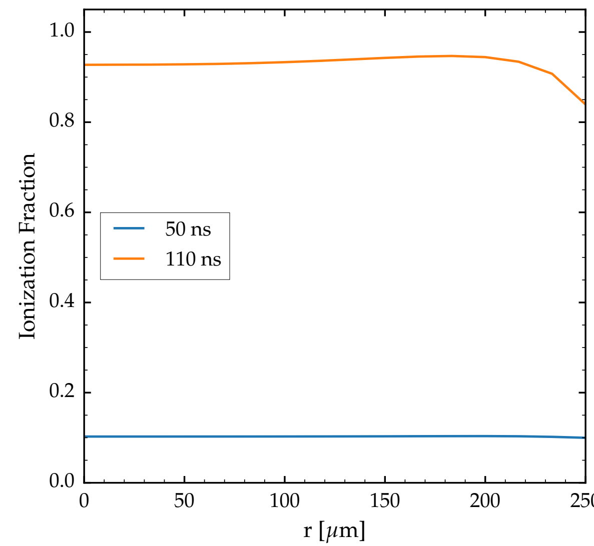
Capillary Benchmark Simulations in R-Z

- Phase II: Conductive cooling at wall
  - *Almost total ionization*
  - *Strong cooling at channel wall*
  - *Significant transverse velocity drives plasma redistribution*
  - *Nearly constant (rising linear) magnetic field profile*
- Roughly 20 ns for channel formation



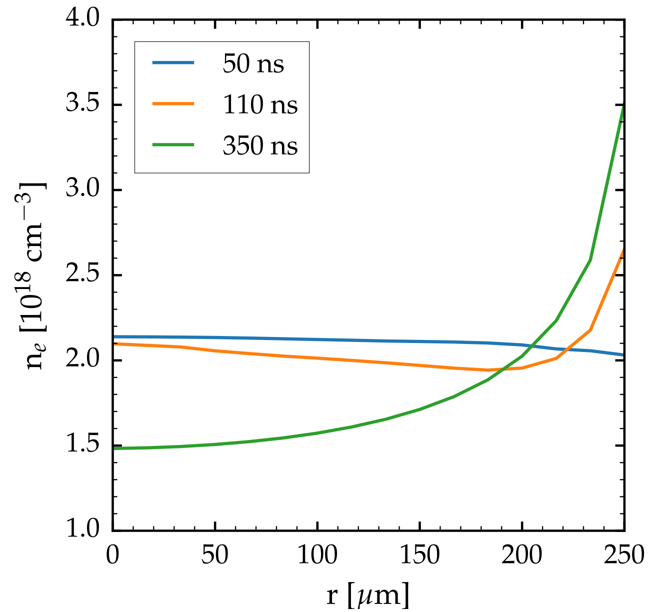


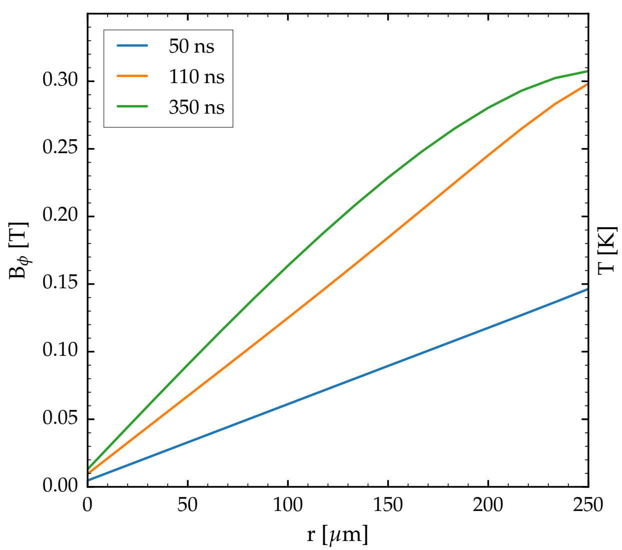


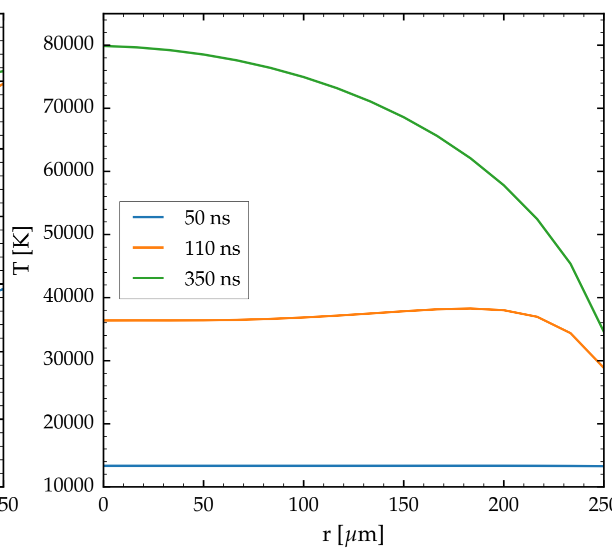


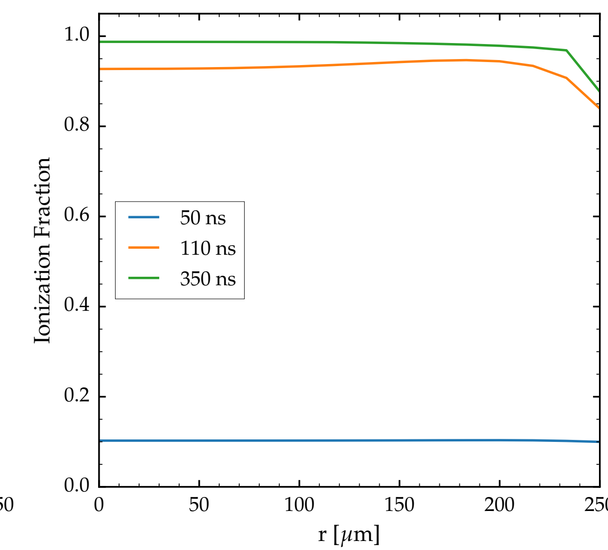
Capillary Benchmark Simulations in R-Z

- Phase III: Steady state channeling
  - *Cooling at channel wall balances Ohmic heating along central axis*
  - *Quadratic reduction in temperature yields parabolic radial density profile*
  - *Complete ionization of plasma*
  - *Drift away from uniform current distribution*
- Sustained through discharge duration





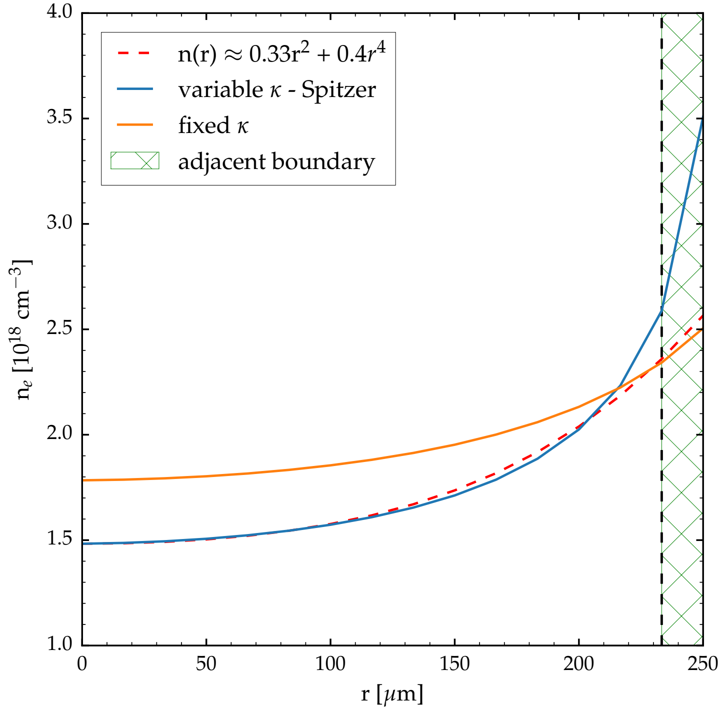


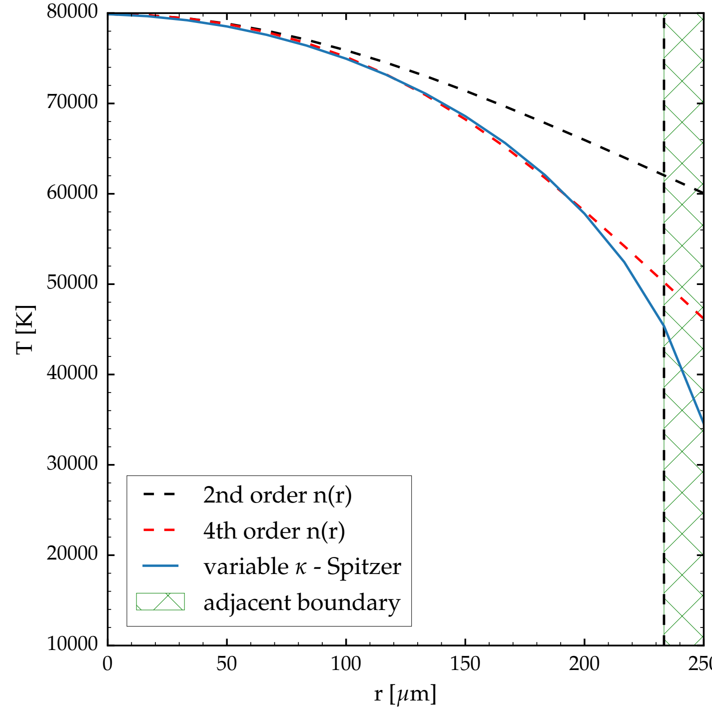


Comparison to analytical model

- A 1D model* predicts an approximate series solution for the density and temperature

*Phys. Rev. E, **65**, 016407 (2001)





- *Constant conductivity approximation is insufficient to develop channel depth*

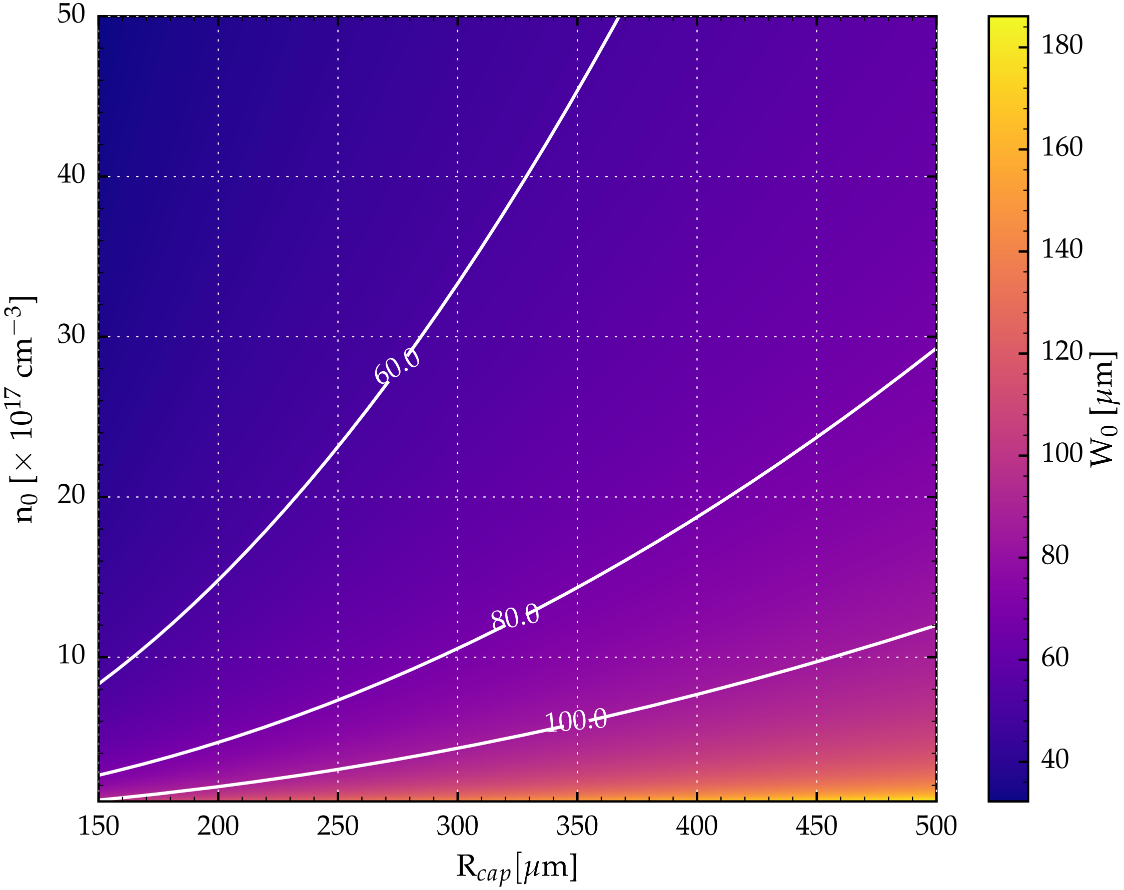
$$K_{\mathrm{ele}} = \left(\frac{8}{\pi}\right)^{3/2} \frac{k_B^{7/2}}{e^4\sqrt{m_e}} \left(\frac{1}{1+3.3/\mathbf{z}}\right) \frac{T_e^{5/2}}{\mathbf{z}\ln\Lambda_{ei}}$$

Laser Deposition

- Lower densities diminish dephasing and depletion
  - *At these densities, matched spot size may be too large to reach necessary intensity*
  - *e.g. At 10^{17} cm^{-3}, matched spot w_0 twice BELLA laser spot size*

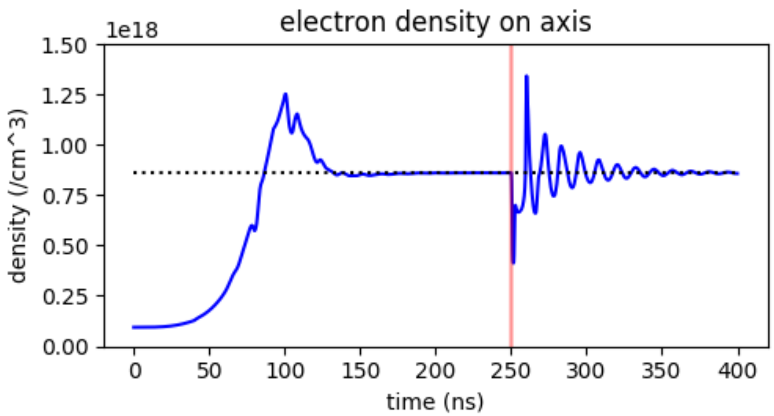
- Achieving 60-70 m guided spots requires either small capillary or high density.

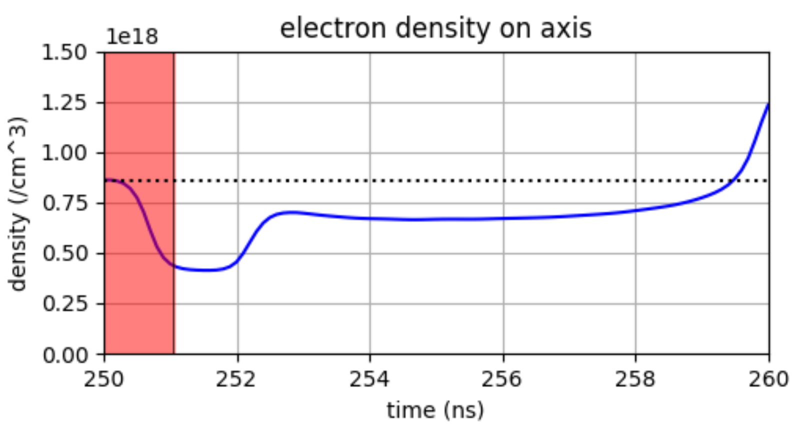
- Introduction of "heater laser" further reduces axial density and increases matched spot size

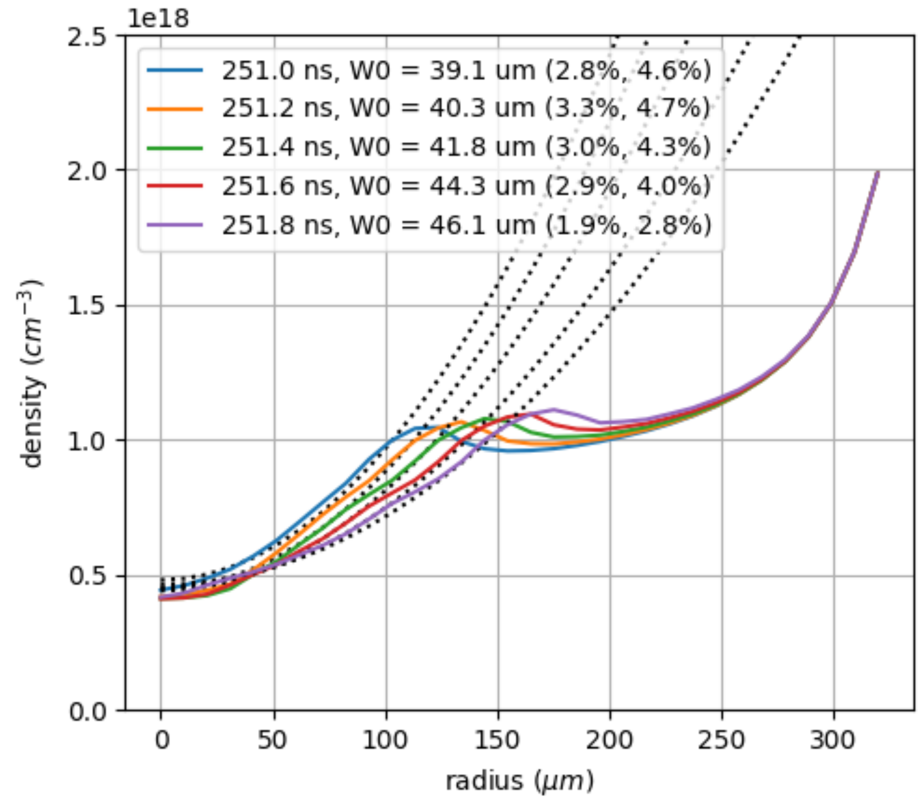


Modeling sub-channel formation via laser "heater"

- Proof of principle studies of laser deposition in pre-formed channel
  - *Gaussian laser, $\lambda_R >> L_Z$ produces collisional heating*
- For large pulse energies (~1 J), significant channeling observed, even at large background densities
  - *$\rho = 8 \times 10^{17}\ cm^{-3} \rightarrow \rho = 4 \times 10^{17}\ cm^{-3}$, reduction of spot size from 75 to <50 micron*
  - *Density reduction scales laser intensity*







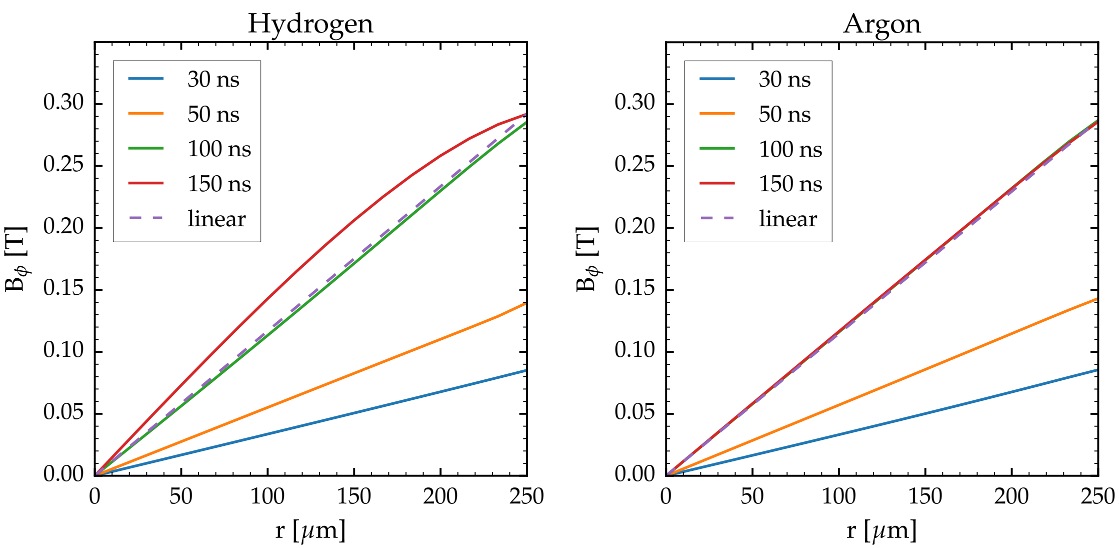
Capturing Nonlinear Current Distributions

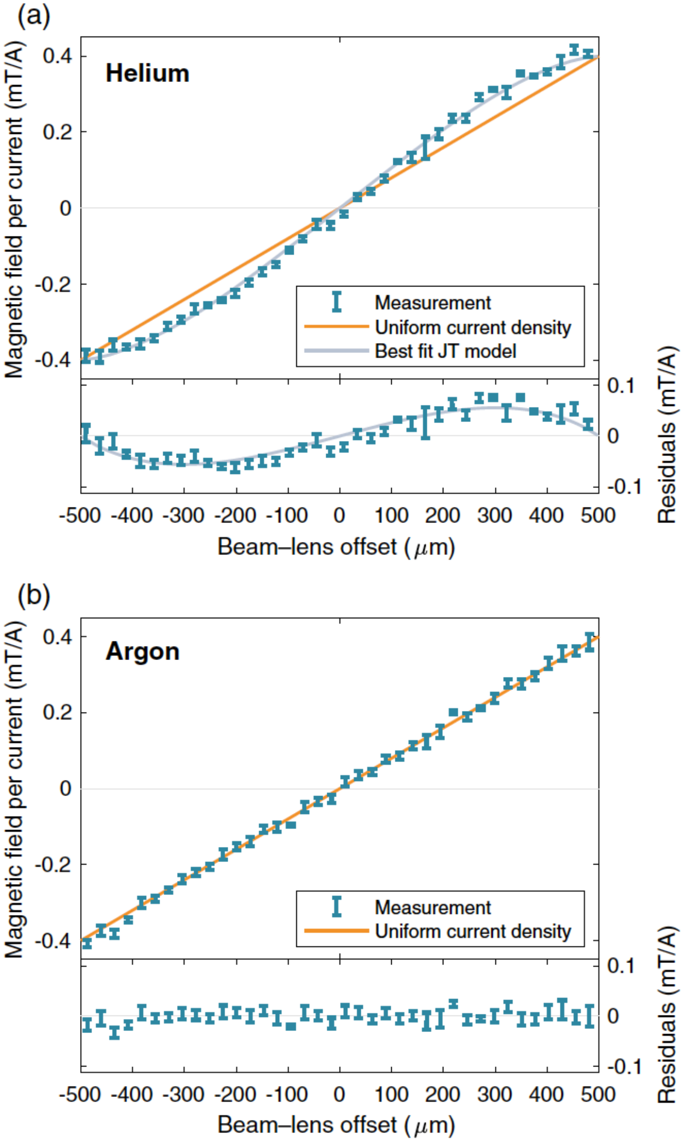
- Experimentally observed nonlinearities in field reproduced by simulations
  - *Temperature deviation drives current deviation**

$$J(r) \propto T_e(r)^{3/2}$$

*PRAB 20, 032803 (2017)

  - *Timescale of deviation from linearity is a function of mass*
    - ex. Argon maintains linear profile after 150 ns, whereas Hydrogen evolves

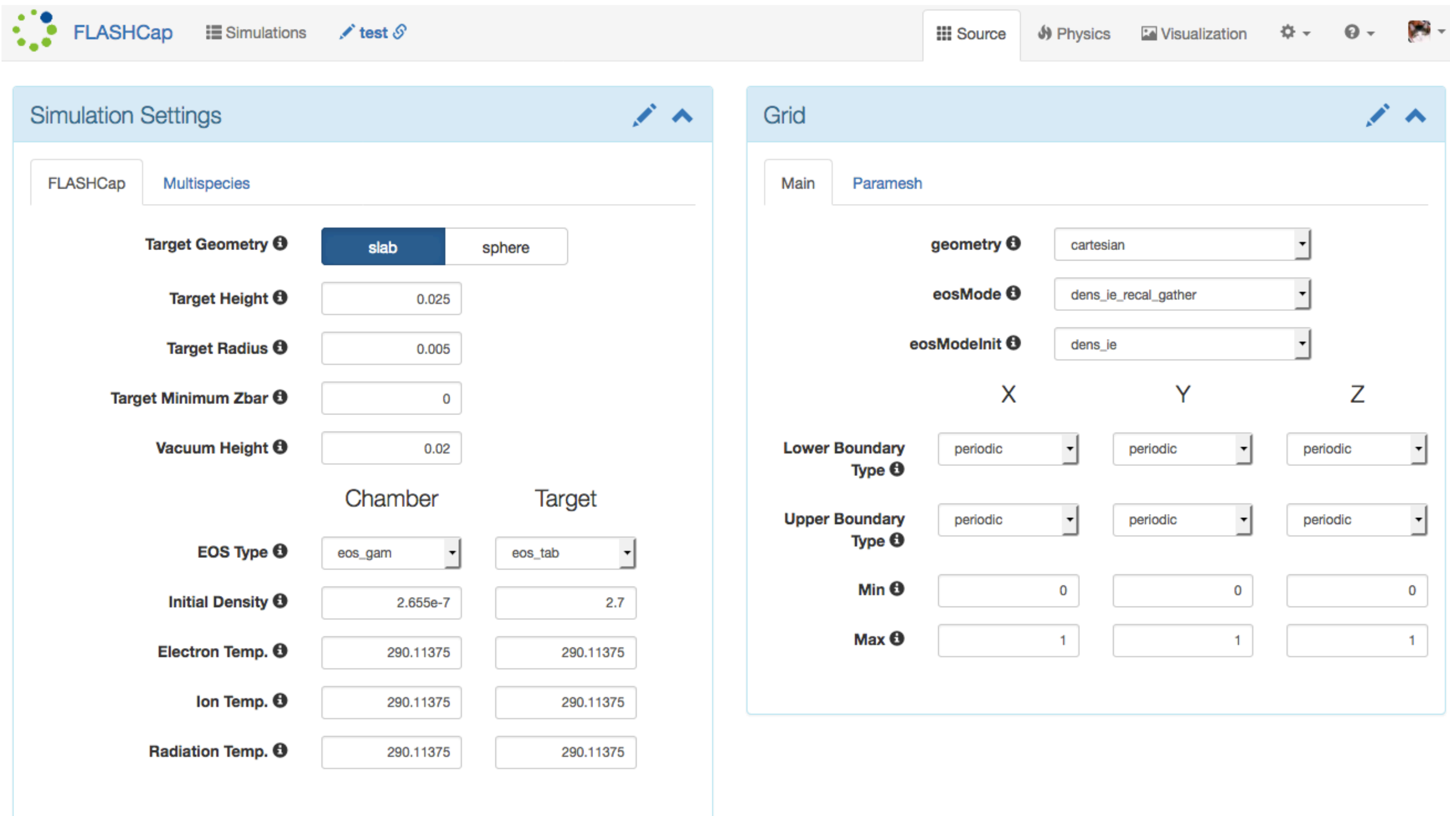




Phys. Rev. Lett. 121, 194801 (2018)

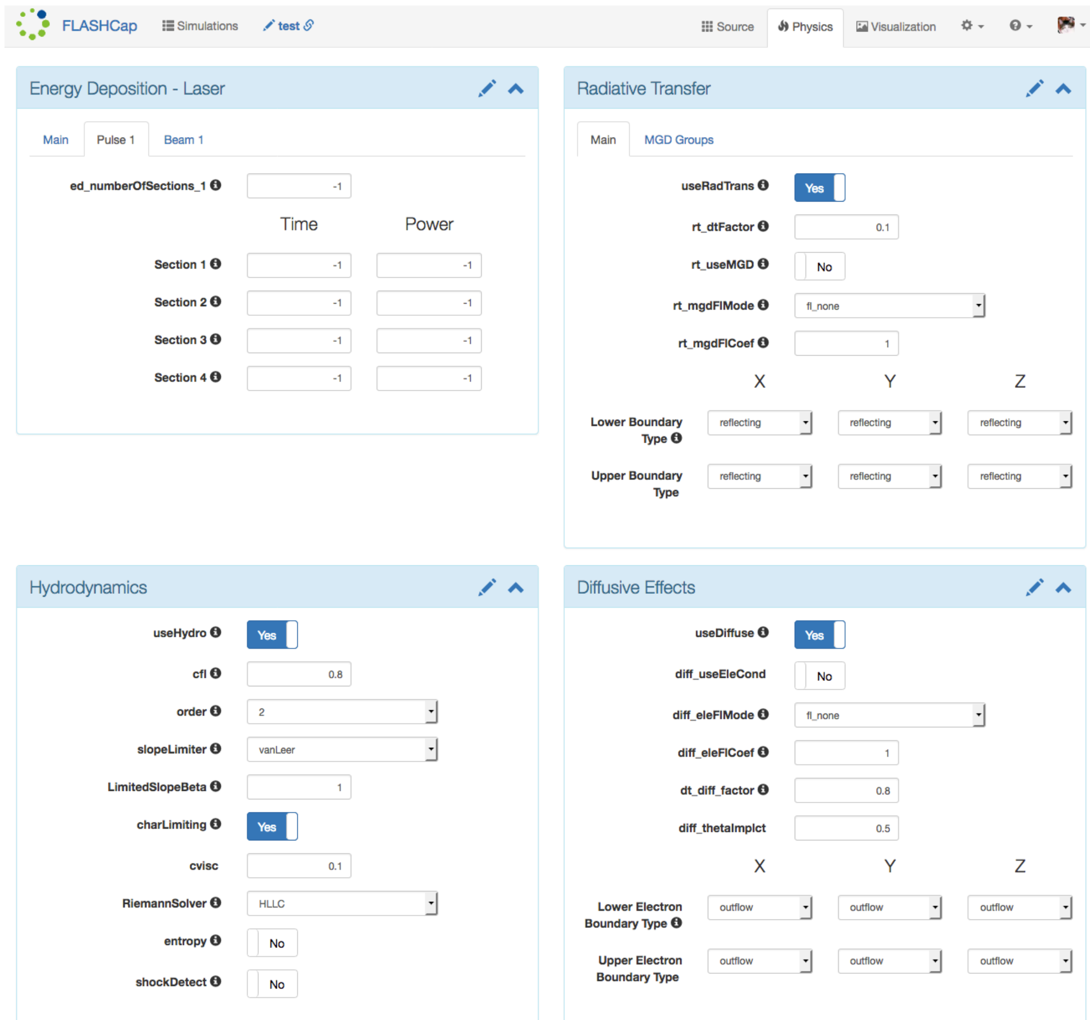
A prototype, modular FLASH interface (1)

- Tailored interfaces for different classes of problems
  - *Existing templates include pre-configured setup routines*
    - Capillary discharge and Rayleigh-Taylor flame examples
  - *Configure simulation geometry, species, and grid parameters*



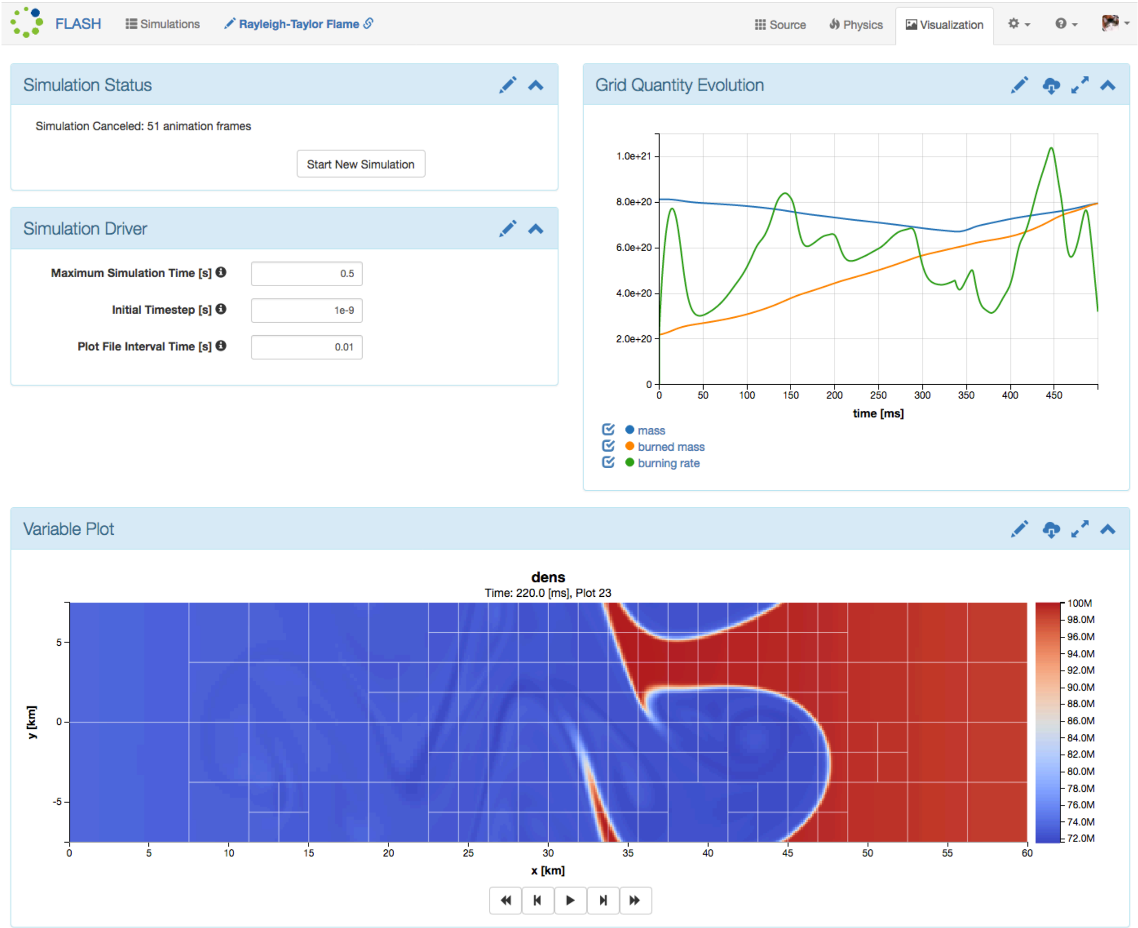
A prototype, modular FLASH interface (2)

- Tailored interfaces for different classes of problems
  - *Specify physics modules and associated transport coefficients*



A prototype, modular FLASH interface (3)

- Tailored interfaces for different classes of problems
  - *Visualize specified plot variables and integrated grid quantities*
  - *Dynamic visualization updates at runtime*



Making FLASH accessible through Sirepo

- Sirepo is a cloud-based platform for scientific computing

- Supported Codes include:
  - *Particle Tracking: elegant, Synergia, Zgoubi,*
  - *Radiation: SRW (Synchrotron Radiation Workshop)*
  - *Particle-in-cell: Warp (Vacuum Nanoelectronic Devices + Plasma-Based Accelerators)*
  - *Electron Cooling: JSPEC*

- Advantages:
  - *No installation required*
  - *Share you work with a simple link*
  - *Archive and save simulations online*
  - *Export files for command-line execution*

- Try it out at https://sirepo.com/

Conclusions

- Capillary discharge plasmas promise a breadth of applications for advanced accelerators
  - *LWFA waveguide, active plasma lens, plasma de-chirper*
- FLASH code benchmarked for simple capillary dynamics
  - *Three phase evolution produces channel in agreement with analytic prediction*
- Additional characteristics explored
  - *Laser heater for sub-channel formation*
  - *Current/Magnetic Field aberrations observed with different species*
- Development of user-friendly interface to FLASH
  - *Plans to be made available online in the next year*
- Future Work will consider more exotic geometries and schemes
  - *3D geometries, density downramps, Bessel laser deposition under consideration*