

Modeling of capillary discharge plasmas for wakefield acceleration and beam transport

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Outline

- Motivation
- Modeling discharge capillary plasmas with FLASH
 - *Solvers, EOS, and boundary conditions*
 - *Discharge current and transport*
- Benchmarks for 2D capillary simulations
 - *Three-phase capillary dynamics*
 - *Comparison of radial density and temperature profiles*
- Additional considerations
 - *Laser heater for sub-channel formation*
 - *Nonlinearities in plasma current response*
- A FLASH interface for capillary modeling

Future compact accelerators rely on plasma stages

- Wakefield acceleration challenges
 - Plasma stages must reach higher energy (~10 GeV per stage)
 - Reproducible and robust at high repetition rate
 - Longer targets, lower densities, and better driver coupling are required
- Beam transport challenges
 - Collider quality beams must transport cleanly between stages
 - Stronger fields, shorter elements, more flexibility are required
- Capillary discharge plasmas promise solutions to both problems
 - Low density, parabolic channels with small matched spot sizes for LWFA
 - High peak currents for orders-of-magnitude increase in focusing fields
 - Flexible ionization profiles for novel plasma columns
 - Existing tools are insufficient to model dynamics with require fidelity

Radial density channels enable laser guiding

- Maintain laser intensity along channel
 - *Gaussian laser matches to a parabolic channel*

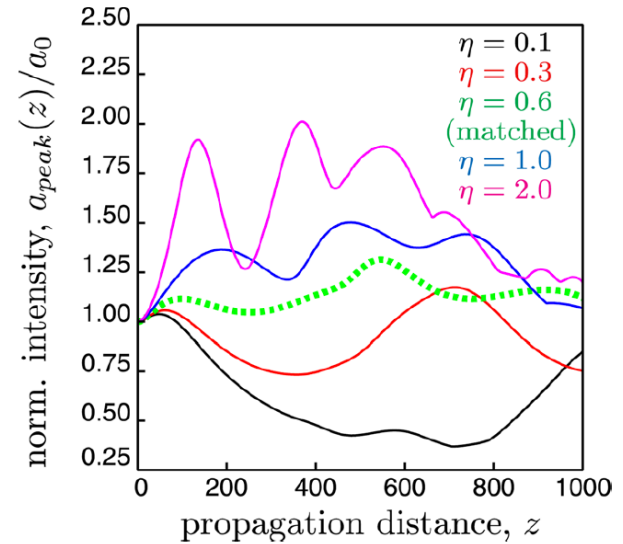
$$n_p(r) = n_0 + \Delta n \frac{r^2}{W_0^2}$$

- *Mismatch leads to intensity oscillations*

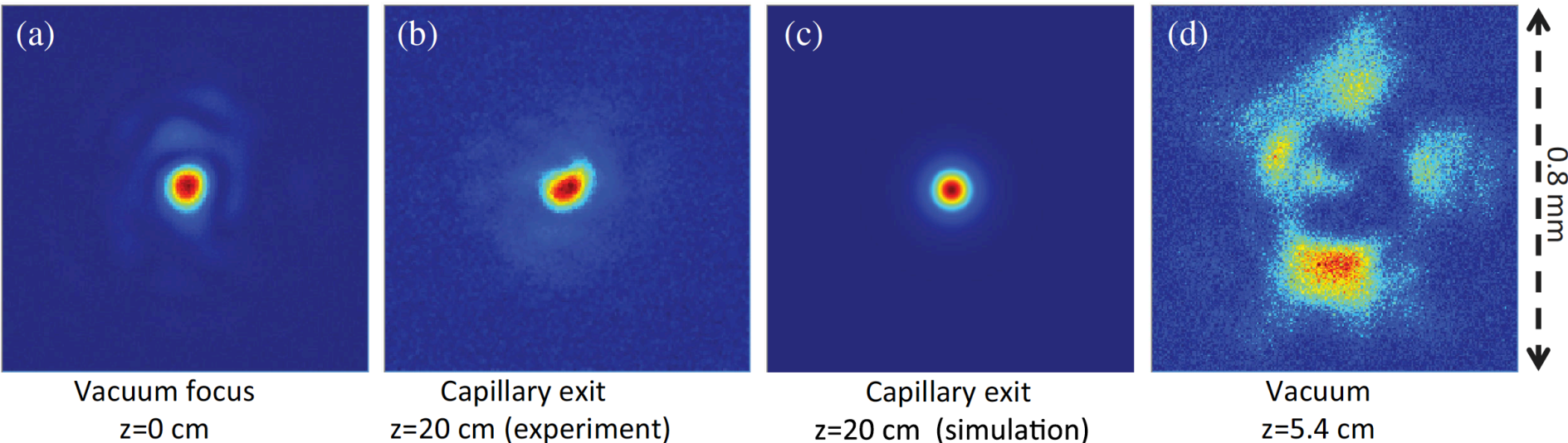
$$\lambda_{\text{osc}} = \pi z_{RM} = \pi^2 r_0^2 / \lambda_0$$

- Capillaries constrain profile and density

Phys. Rev. Lett. **122**, 084801 (2019)

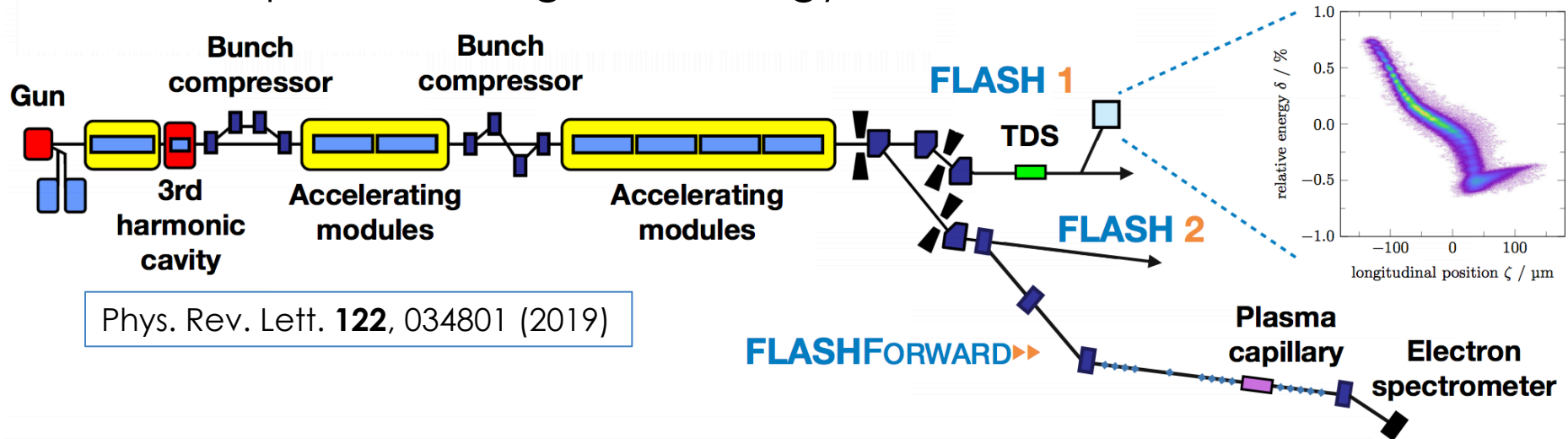


Phys. of Plasmas **19**, 053101 (2012)

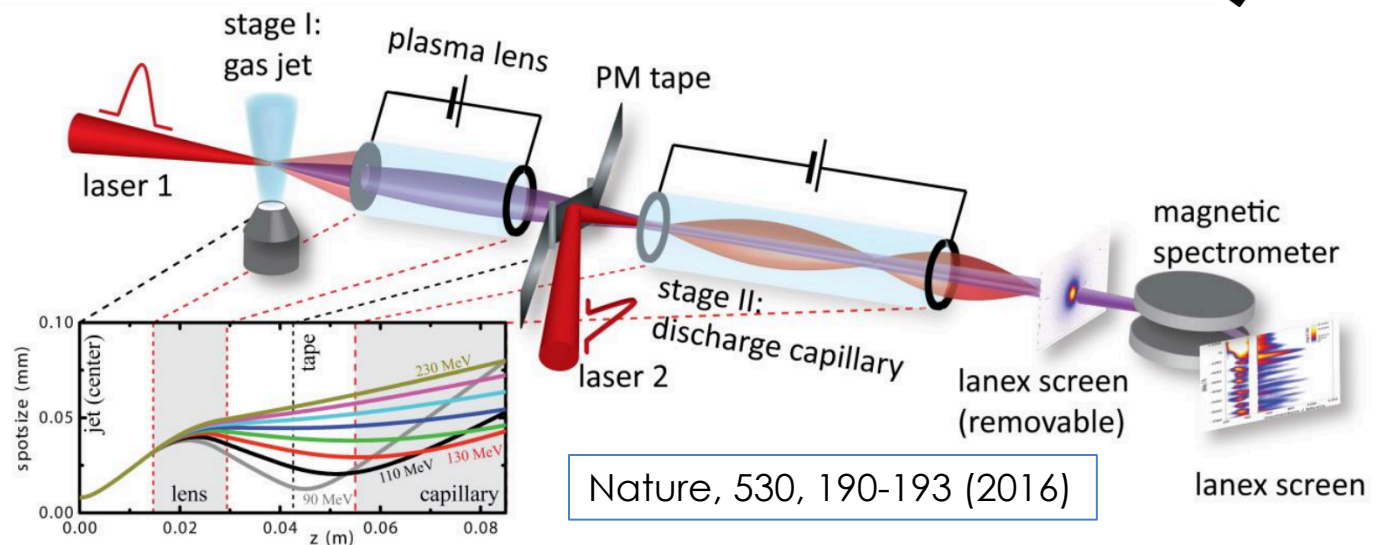


High field strengths enable beam transport

Beam transport, focusing, and energy decorrelation



Staging



Deconstructing a Capillary Discharge Plasma

- Narrow insulating tube with controlled gas flow
 - *Hydrogen, Helium, Argon*
 - *Length from ~1-30 cm*
Radius from ~0.1-1 mm
- Applied voltage to rings to drive discharge
 - *Vary density, voltage to adjust*
- Many computational Complexities
 - *High Aspect Ratio*
 - *High temporal resolution required*
 - Transport timescales (0.01-10 ps) are small compared to discharge (>100 ns)
 - Magnetic field effects require explicit integration
 - *Time Dependent boundaries require special treatment*
 - Discharge representation influences choice of boundary conditions
 - Electrical and Thermal conductivities must change self-consistently

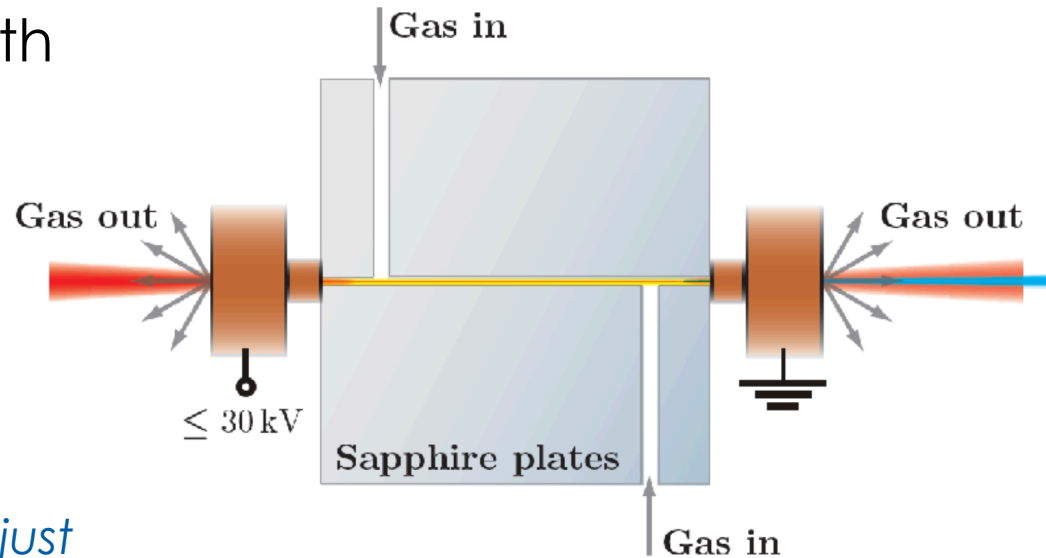
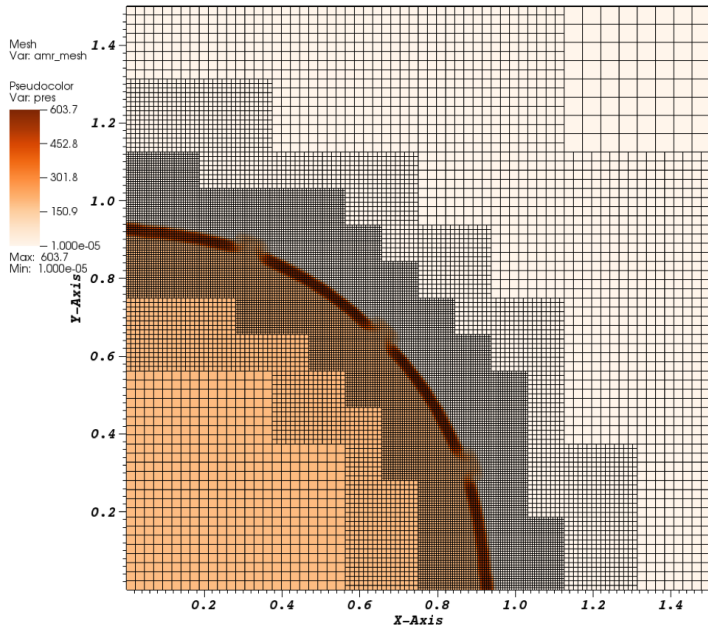


Figure Credit: Jens Osterhoff

Adopting the FLASH code

- Publicly available multi-physics application developed at University of Chicago – Flash Center for Computational Science (1997)
 - *Compressible flow problems from astrophysics to HEDP*
 - *Additional Physics specific to HEDP*
 - *Heat exchange, ionization, conductivity/resistivity/viscosity/opacity, radiative transfer, inverse Bremsstrahlung, gravity, nuclear burning, NEI, and more*
- Modular design written in Fortran
 - *Include only necessary physics within each executable*
 - *Pre-tabulate user configurable input/output parameters*
 - *Separate runtime configuration options*
- External solver libraries
 - *HYPRE provides general, implicit diffusion solves*
 - *PARAMESH and Chombo for AMR*
 - *Gamma law or pre-tabulated equations of state*
- Data Analysis Tools
 - *Parallel I/O support via HDF5 and NetCDF*
 - *VisIt plug-in for HPC visualization pipeline*
 - *Python YT, IDL, OpenPMD ("Rad-Hydro") APIs*

FLASH Fundamentals



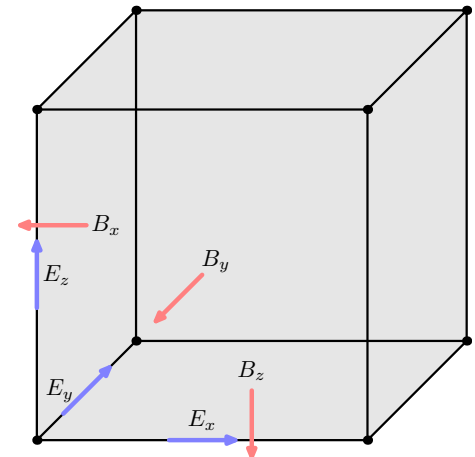
- Uniform, block-structured grid
 - *AMR with user-specified refinement*
- 1/2/3D geometries
 - *Cartesian, Spherical, Cylindrical, Polar*
- Solves fluid evolution with convection

$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \\ \frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla P_{\text{tot}} = 0 \\ \frac{\partial}{\partial t} (\rho E_{\text{tot}}) + \nabla \cdot [(\rho E_{\text{tot}} + P_{\text{tot}}) \mathbf{v}] = Q_{\text{las}} - \nabla \cdot \mathbf{q} \end{array} \right\}$$

- 3T capabilities (per species)

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} (\rho e_i) + \nabla \cdot (\rho e_i \mathbf{v}) + P_i \nabla \cdot \mathbf{v} = \rho \frac{c_{v,e}}{\tau_{ei}} (T_e - T_i) \\ \frac{\partial}{\partial t} (\rho e_e) + \nabla \cdot (\rho e_e \mathbf{v}) + P_e \nabla \cdot \mathbf{v} = \rho \frac{c_{v,e}}{\tau_{ei}} (T_i - T_e) - \nabla \cdot \mathbf{q}_e + Q_{\text{abs}} - Q_{\text{emis}} + Q_{\text{las}} \\ \frac{\partial}{\partial t} (\rho e_r) + \nabla \cdot (\rho e_r \mathbf{v}) + P_r \nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{q}_r - Q_{\text{abs}} + Q_{\text{emis}} \end{array} \right\}$$

- Electromagnetic fields defined on a Yee mesh
 - *Secondary, uniform mesh overlapping fluid domain*
- Second-order accurate predictor-corrector updates both fluid and field quantities
 - *Divergence-free condition enforced*
 - *Explicit integration scheme*



FLASH – Transport and Boundaries

- Dissipation described by conduction and heat exchange

$$\frac{\partial e_e}{\partial t} = \nabla \cdot K_e \nabla T_e$$

$$\begin{aligned}\frac{\partial e_i}{\partial t} &= \frac{c_{v,e}}{\tau_{ei}} (T_e - T_i) \\ \frac{\partial e_e}{\partial t} &= \frac{c_{v,e}}{\tau_{ei}} (T_i - T_e)\end{aligned}$$

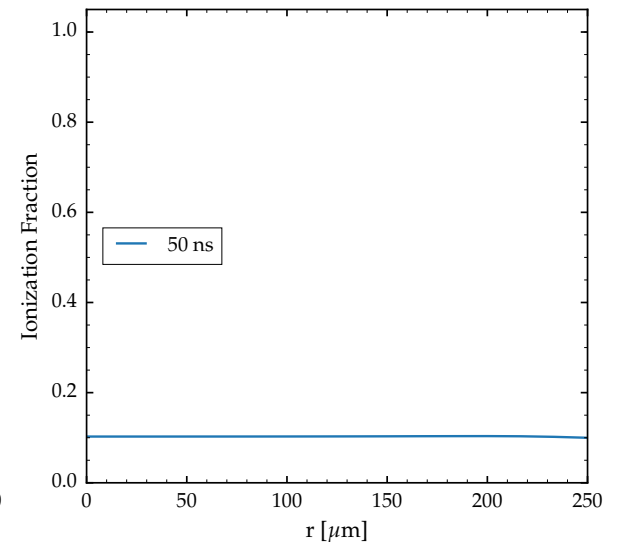
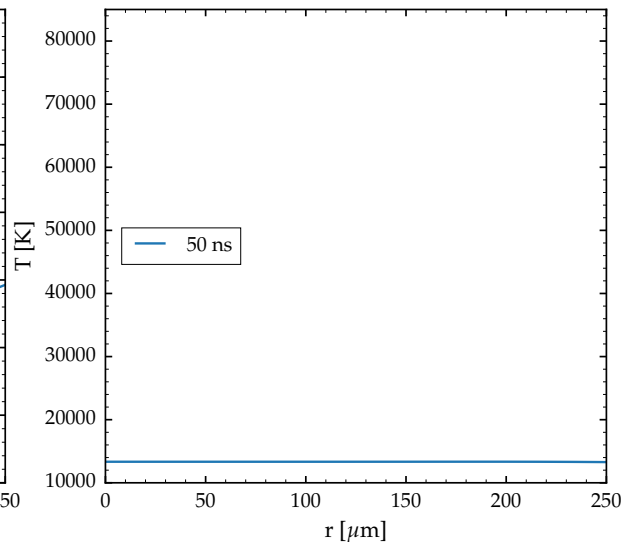
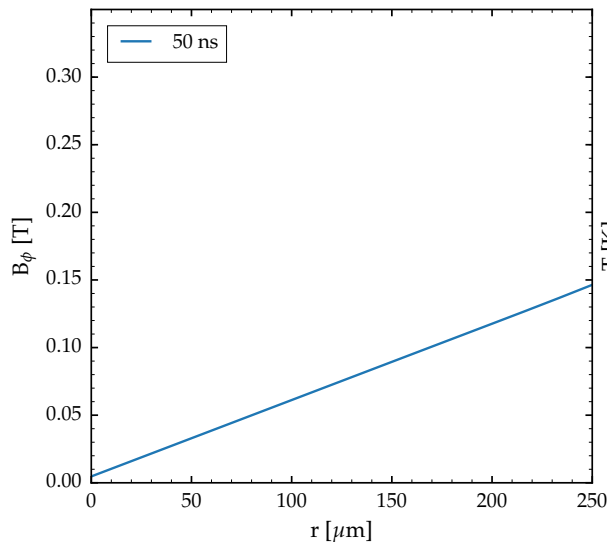
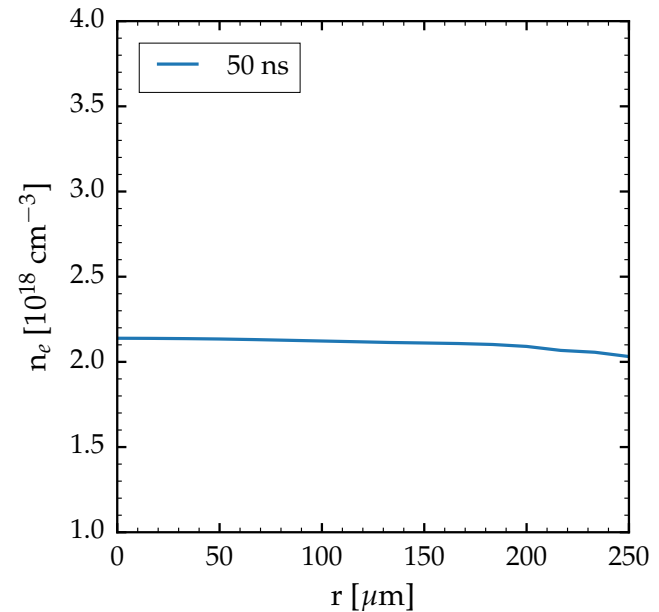
- Spitzer model describes plasma resistivity (and thermal conduction)

$$\eta_{\perp} = \frac{4\sqrt{2\pi}}{3} \frac{Ze^2 m_e^{1/2} \ln \Lambda}{(4\pi\epsilon_0)^2 (k_B T_e)^{3/2}} F(Z)$$

- Magnetic field specified along boundary via Ampere's Law
 - Discharge is modelled via changing field at the boundary
 - Insufficient to describe breakdown, but acceptable for slowly varying current
- Notable Boundary Conditions
 - **At $r=0$:** Axisymmetric (reflecting) conditions for transverse fluid flow
 - **At $r=R$:** Dirichlet conditions for transverse fluid flow $v_{\perp} = 0$
 - Need to zero fluxes at outer radius to prevent outflow
 - **At $r=R$:** Dirichlet condition for thermal conductivity ($K_e(r=R)$)
 - Differs from analytical solutions specifying temperature at boundary

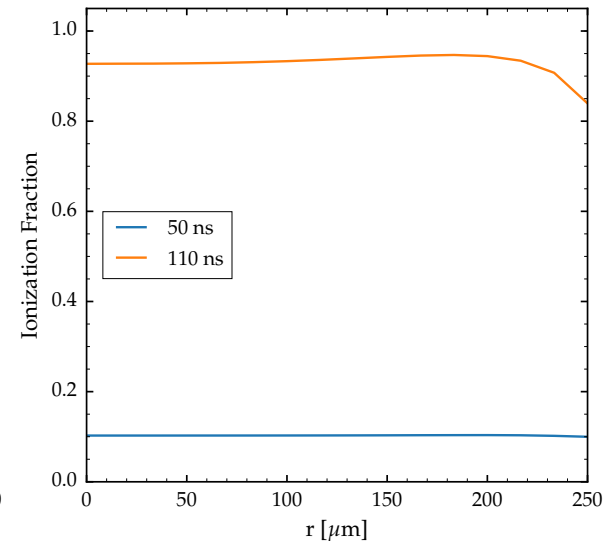
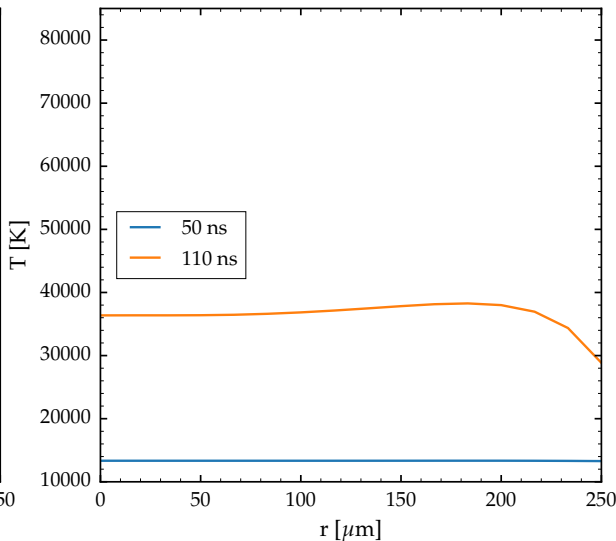
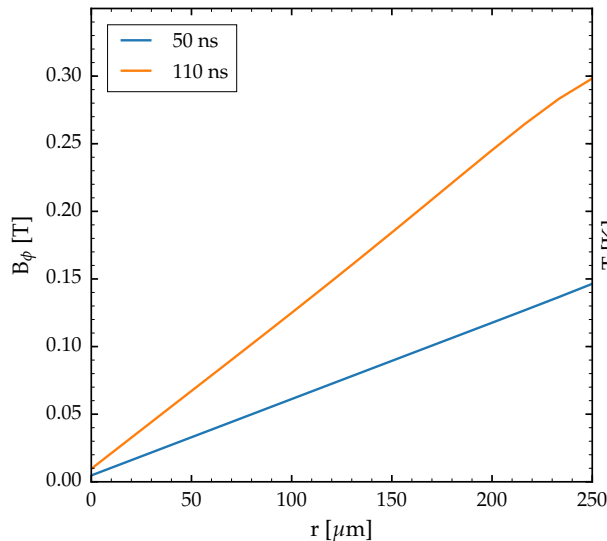
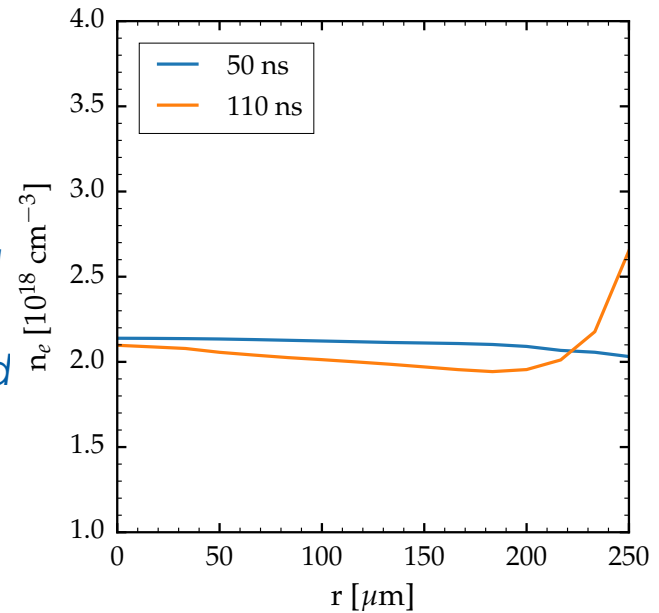
Capillary Benchmark Simulations in R-Z

- Phase I: Uniform Ohmic heating
 - Rising, linear magnetic field profile
 - Some initial ionization used to bootstrap field penetration into the plasma
 - Uniform Heating
 - Increasing ionization
 - No distinctive plasma motion
- Period of ~100 ns assuming comparable current rise time



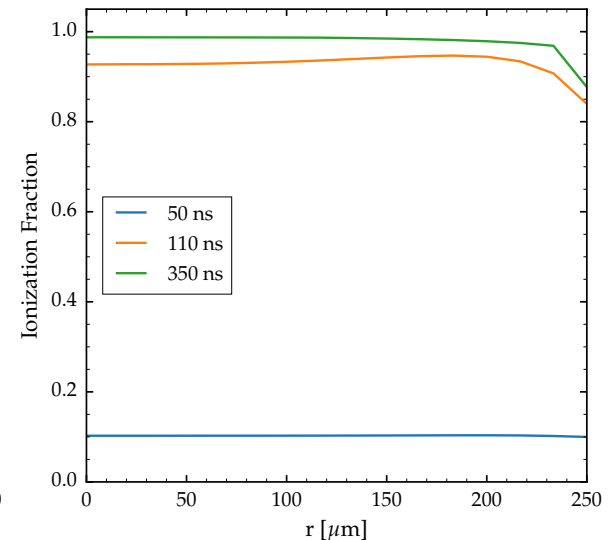
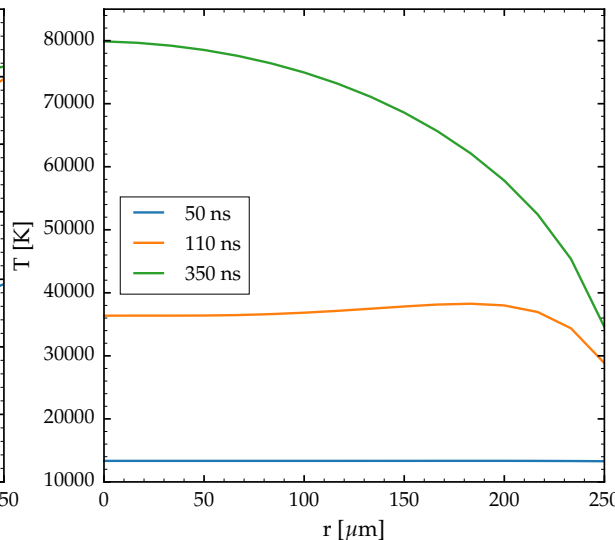
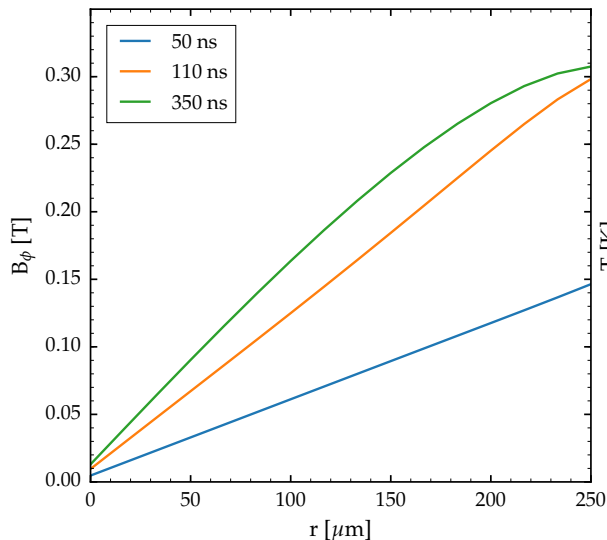
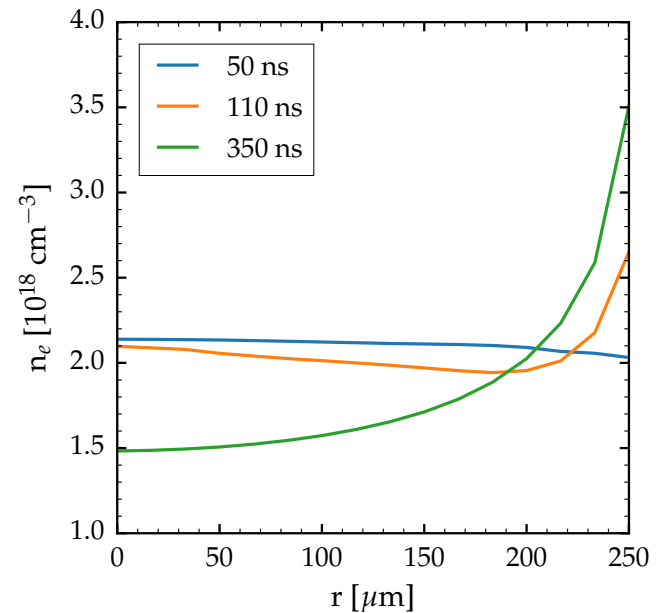
Capillary Benchmark Simulations in R-Z

- Phase II: Conductive cooling at wall
 - Almost total ionization
 - Strong cooling at channel wall
 - Significant transverse velocity drives plasma redistribution
 - Nearly constant (rising linear) magnetic field profile
- Roughly 20 ns for channel formation



Capillary Benchmark Simulations in R-Z

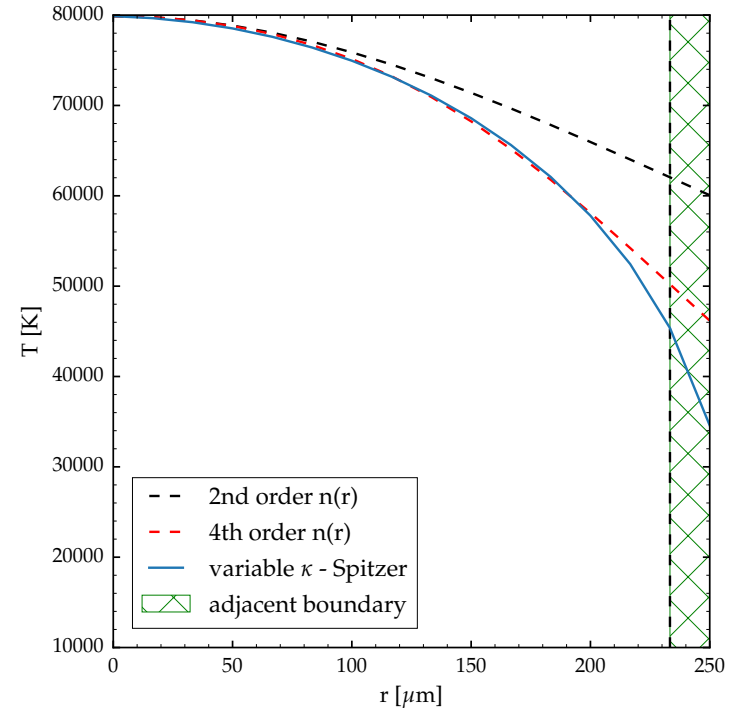
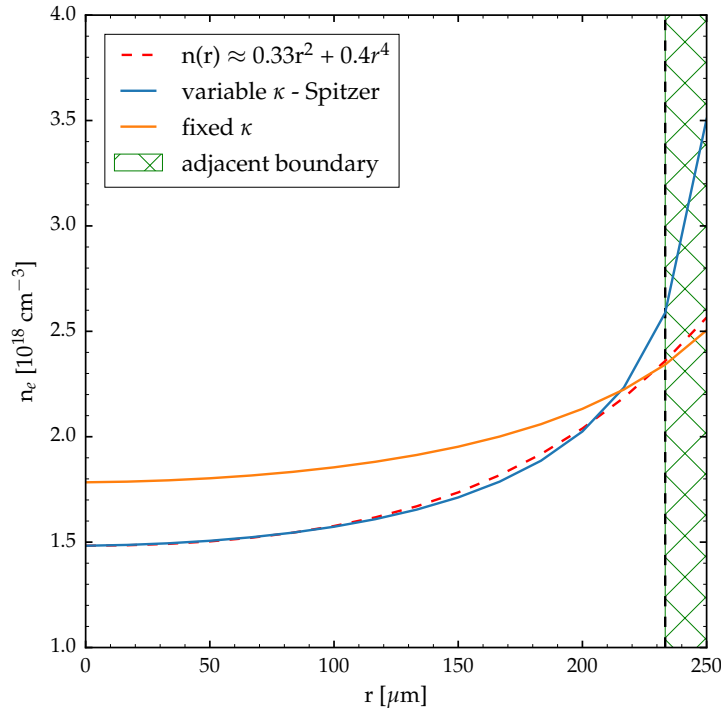
- Phase III: Steady state channeling
 - Cooling at channel wall balances Ohmic heating along central axis
 - Quadratic reduction in temperature yields parabolic radial density profile
 - Complete ionization of plasma
 - Drift away from uniform current distribution
- Sustained through discharge duration



Comparison to analytical model

- A 1D model* predicts an approximate series solution for the density and temperature

*Phys. Rev. E, **65**, 016407 (2001)

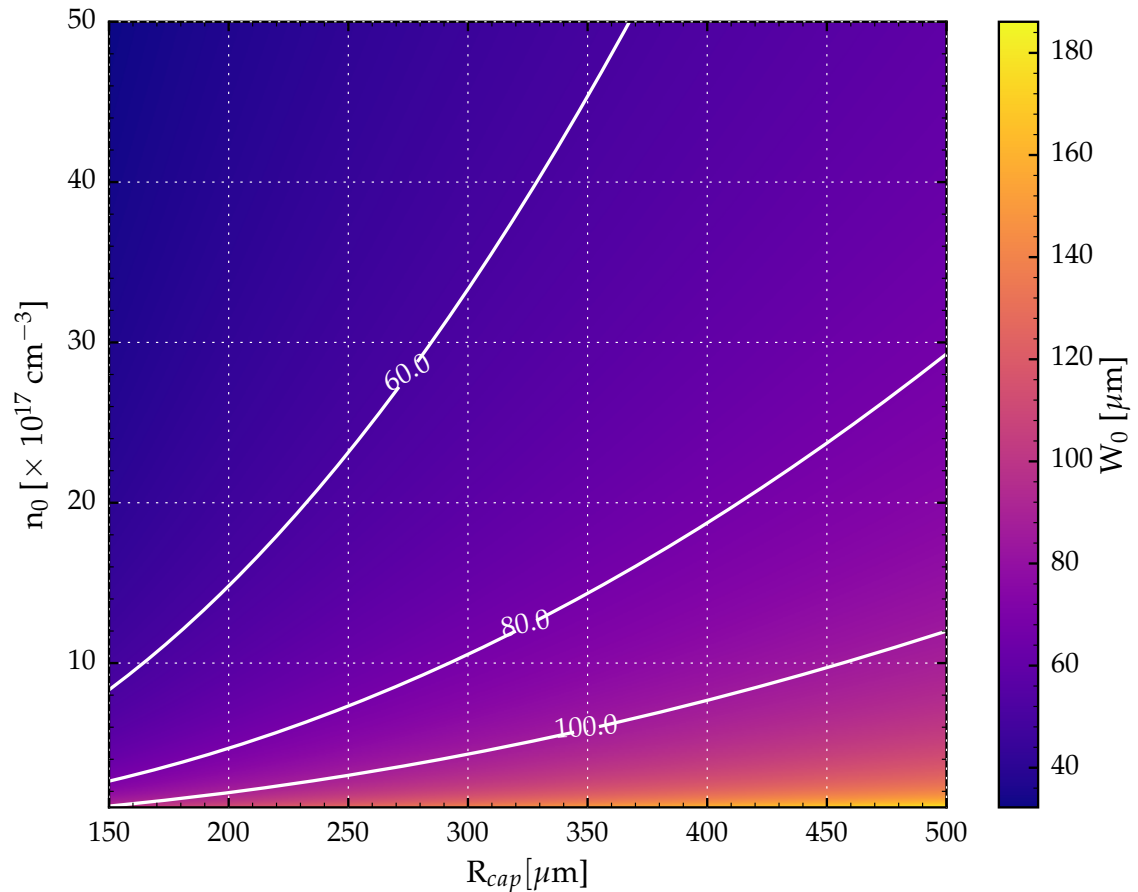


- Constant conductivity approximation is insufficient to develop channel depth

$$K_{\text{ele}} = \left(\frac{8}{\pi} \right)^{3/2} \frac{k_B^{7/2}}{e^4 \sqrt{m_e}} \left(\frac{1}{1 + 3.3/z} \right) \frac{T_e^{5/2}}{z \ln \Lambda_{ei}}$$

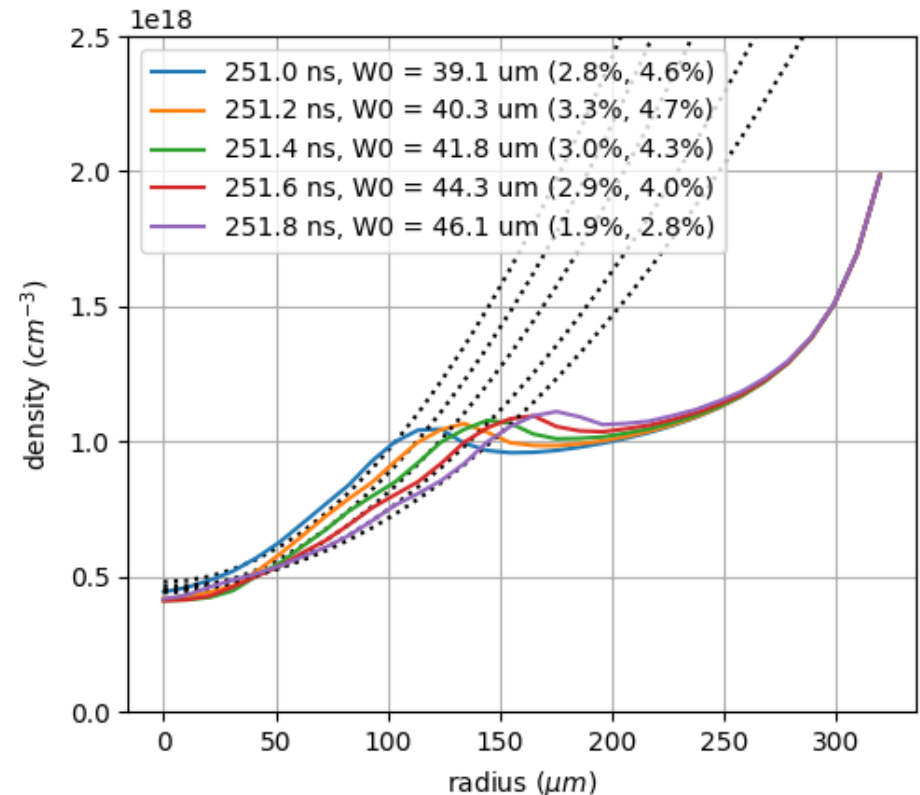
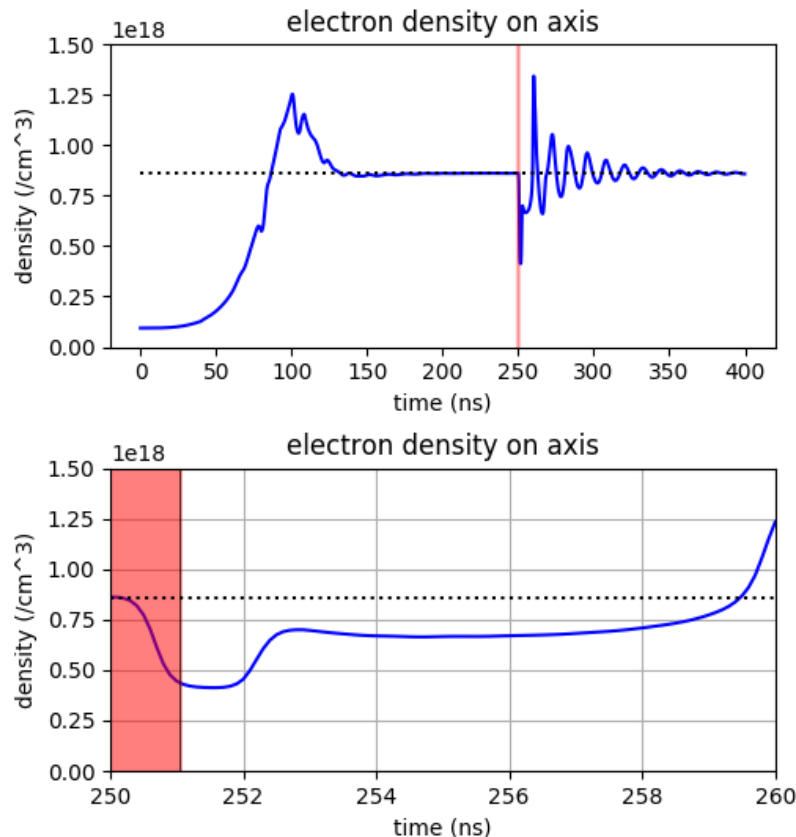
Laser Deposition

- Lower densities diminish dephasing and depletion
 - *At these densities, matched spot size may be too large to reach necessary intensity*
 - *e.g. At 10^{17} cm^{-3} , matched spot w_0 twice BELLA laser spot size*
- Achieving 60-70 m guided spots requires either small capillary or high density.
- Introduction of “heater laser” further reduces axial density and increases matched spot size



Modeling sub-channel formation via laser “heater”

- Proof of principle studies of laser deposition in pre-formed channel
 - *Gaussian laser, $\lambda_R \gg L_z$ produces collisional heating*
- For large pulse energies (~ 1 J), significant channeling observed, even at large background densities
 - $\rho = 8 \times 10^{17} \text{ cm}^{-3} \rightarrow \rho = 4 \times 10^{17} \text{ cm}^{-3}$, reduction of spot size from 75 to < 50 micron
 - *Density reduction scales laser intensity*



Capturing Nonlinear Current Distributions

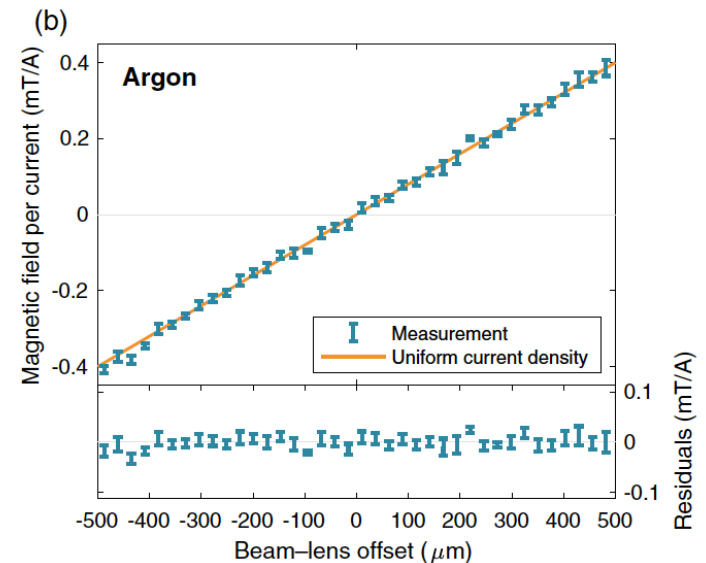
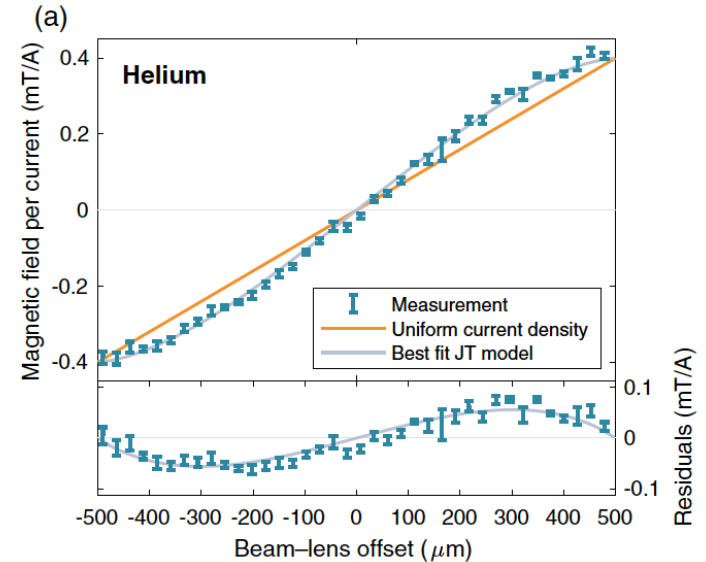
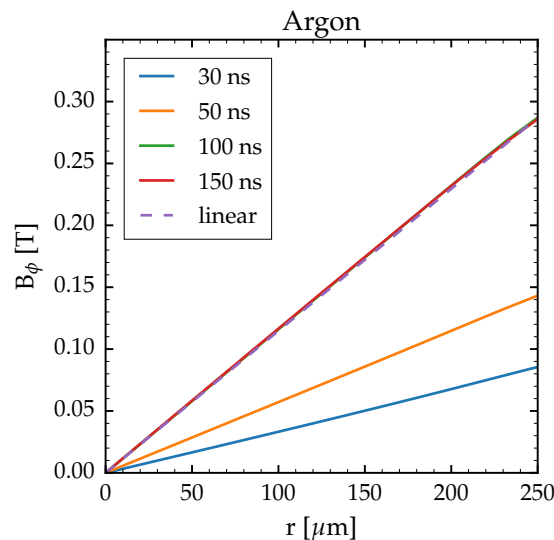
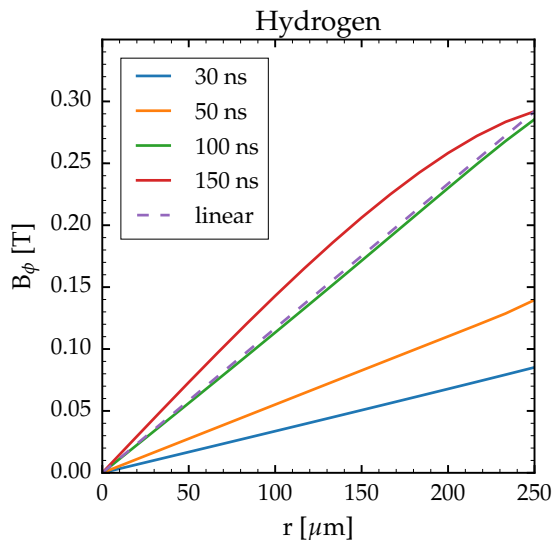
- Experimentally observed nonlinearities in field reproduced by simulations

- Temperature deviation drives current deviation*

$$J(r) \propto T_e(r)^{3/2}$$

*PRAB 20, 032803 (2017)

- Timescale of deviation from linearity is a function of mass
 - ex. Argon maintains linear profile after 150 ns, whereas Hydrogen evolves



Phys. Rev. Lett. 121, 194801 (2018)

A prototype, modular FLASH interface (1)

- Tailored interfaces for different classes of problems
 - Existing templates include pre-configured setup routines
 - Capillary discharge and Rayleigh-Taylor flame examples
 - Configure simulation geometry, species, and grid parameters

The screenshot displays the FLASHCap web interface. At the top, a navigation bar includes the FLASHCap logo, 'Simulations', and a 'test' button. On the right, there are tabs for 'Source', 'Physics', and 'Visualization', along with settings and help icons.

The main content area is divided into two panels:

Simulation Settings

This panel has two tabs: 'FLASHCap' and 'Multispecies'. The 'FLASHCap' tab is active, showing a 'Target Geometry' section with a dropdown menu set to 'slab' (with 'sphere' as an alternative). Below this are input fields for 'Target Height' (0.025), 'Target Radius' (0.005), 'Target Minimum Zbar' (0), and 'Vacuum Height' (0.02). The 'EOS Type' section has two columns: 'Chamber' and 'Target'. The 'Chamber' column has a dropdown set to 'eos_gam' and input fields for 'Initial Density' (2.655e-7), 'Electron Temp.' (290.11375), 'Ion Temp.' (290.11375), and 'Radiation Temp.' (290.11375). The 'Target' column has a dropdown set to 'eos_tab' and corresponding input fields for 'Initial Density' (2.7), 'Electron Temp.' (290.11375), 'Ion Temp.' (290.11375), and 'Radiation Temp.' (290.11375).

Grid

This panel has two tabs: 'Main' and 'Paramesh'. The 'Main' tab is active, showing a 'geometry' dropdown set to 'cartesian'. Below this are 'eosMode' (dens_ie_recal_gather) and 'eosModelInit' (dens_ie) dropdowns. The 'Lower Boundary Type' and 'Upper Boundary Type' sections each have three dropdowns for X, Y, and Z dimensions, all set to 'periodic'. The 'Min' and 'Max' sections each have three input fields for X, Y, and Z dimensions, with 'Min' values set to 0 and 'Max' values set to 1.

A prototype, modular FLASH interface (2)

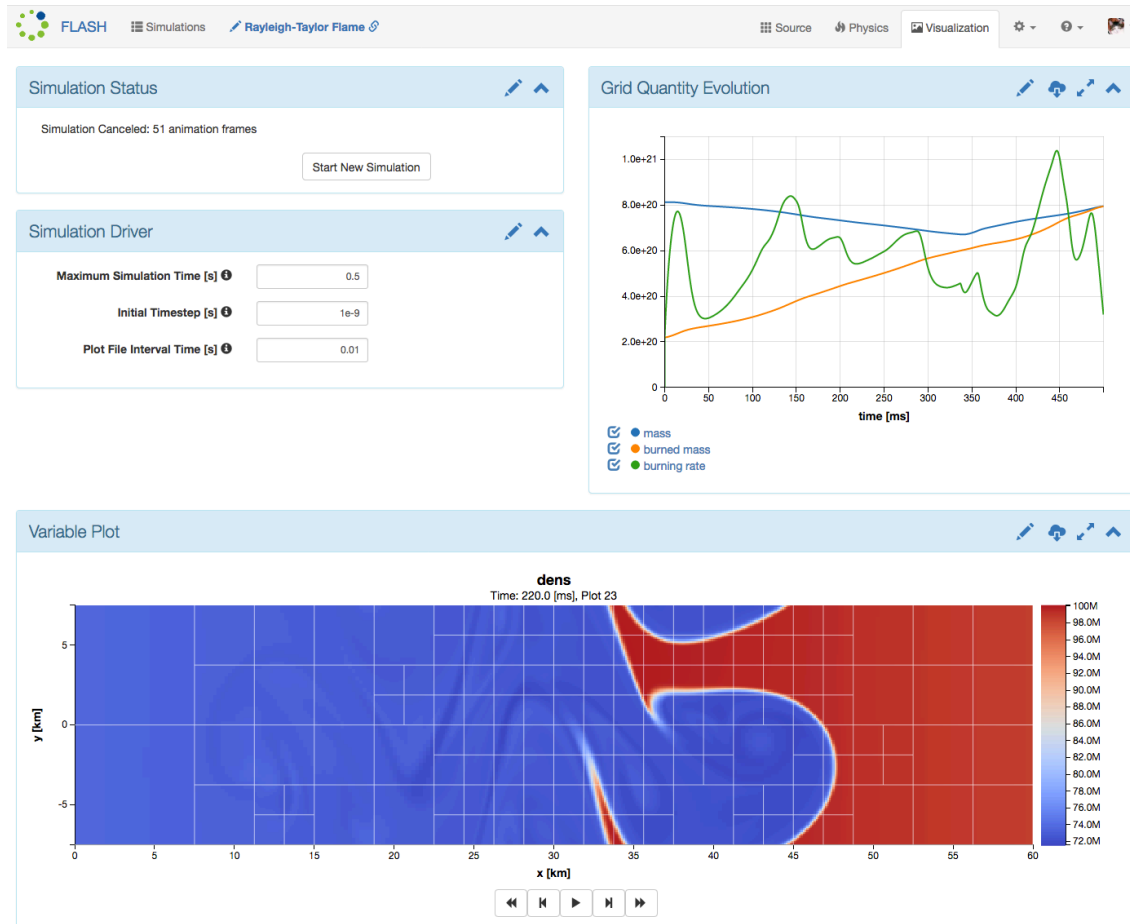
- Tailored interfaces for different classes of problems
 - Specify physics modules and associated transport coefficients*

The image displays the FLASHCap interface, a modular tool for configuring FLASH simulations. It features four main panels, each with a 'Main' tab and a 'test' button. The panels are:

- Energy Deposition - Laser:** Contains a 'Main' tab and a 'Beam 1' sub-tab. It includes a table for 'ed_numberOfSections_1' with columns for 'Time' and 'Power'. The table has four rows, each labeled 'Section 1' through 'Section 4', with values of '-1' in both columns.
- Radiative Transfer:** Contains a 'Main' tab and an 'MGD Groups' sub-tab. It includes a 'useRadTrans' checkbox (checked), a 'rt_dtFactor' input (0.1), a 'rt_useMGD' dropdown (No), a 'rt_mgdFIMode' dropdown (fl_none), and a 'rt_mgdFICoef' input (1). It also has three columns for 'X', 'Y', and 'Z' boundary types, each with a 'Lower Boundary Type' and 'Upper Boundary Type' dropdown (all set to 'reflecting').
- Hydrodynamics:** Contains a 'Main' tab. It includes a 'useHydro' checkbox (checked), a 'cfl' input (0.8), an 'order' dropdown (2), a 'slopeLimiter' dropdown (vanLeer), a 'LimitedSlopeBeta' input (1), a 'charLimiting' checkbox (checked), a 'cvisc' input (0.1), a 'RiemannSolver' dropdown (HLLC), an 'entropy' dropdown (No), and a 'shockDetect' dropdown (No).
- Diffusive Effects:** Contains a 'Main' tab. It includes a 'useDiffuse' checkbox (checked), a 'diff_useEleCond' dropdown (No), a 'diff_eleFIMode' dropdown (fl_none), a 'diff_eleFICoef' input (1), a 'dt_diff_factor' input (0.8), and a 'diff_thetaImpct' input (0.5). It also has three columns for 'X', 'Y', and 'Z' boundary types, each with a 'Lower Electron Boundary Type' and 'Upper Electron Boundary Type' dropdown (all set to 'outflow').

A prototype, modular FLASH interface (3)

- Tailored interfaces for different classes of problems
 - Visualize specified plot variables and integrated grid quantities
 - Dynamic visualization updates at runtime



Making FLASH accessible through Sirepo

- Sirepo is a cloud-based platform for scientific computing
- Supported Codes include:
 - Particle Tracking: elegant, Synergia, Zgoubi,
 - Radiation: SRW (Synchrotron Radiation Workshop)
 - Particle-in-cell: Warp (Vacuum Nanoelectronic Devices + Plasma-Based Accelerators)
 - Electron Cooling: JSPEC
- Advantages:
 - No installation required
 - Share you work with a simple link
 - Archive and save simulations online
 - Export files for command-line execution
- Try it out at <https://sirepo.com/>



Conclusions

- Capillary discharge plasmas promise a breadth of applications for advanced accelerators
 - *LWFA waveguide, active plasma lens, plasma de-chirper*
- FLASH code benchmarked for simple capillary dynamics
 - *Three phase evolution produces channel in agreement with analytic prediction*
- Additional characteristics explored
 - *Laser heater for sub-channel formation*
 - *Current/Magnetic Field aberrations observed with different species*
- Development of user-friendly interface to FLASH
 - *Plans to be made available online in the next year*
- Future Work will consider more exotic geometries and schemes
 - *3D geometries, density downramps, Bessel laser deposition under consideration*