Efficient 3D envelope modeling for two-stage laser wakefield acceleration experiments

Francesco Massimo
Outline

- Context

- Modeling Laser Wakefield Acceleration with a laser envelope

- First stage LWFA simulations

- Second stage LWFA simulations

- Conclusions
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Centre Interdisciplinaire de la Lumière Extrême (CILEX)

Laser Wakefield Acceleration (LWFA)

Injected electrons

Wake bubble in the plasma

Propagation direction
PIC simulations need many resources

3D standard LWFA simulations:
1 mm plasma ~ 320 kcpu-hours ~ 10.2 k€
(36 years on 1 cpu)

Parallelization is mandatory
but still 320 kcpu-hours ~ 13 days on 1000 cpus ...

Any trick to speed up the calculation is most welcome

Implemented in **Smilei**

- High Performance Computing (HPC) techniques
  - Parallelization
  - Smart Load Balancing
  - Vectorisation

- Techniques using physical approximations
  - Azimuthal Fourier decomposition
  - Envelope modeling
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Electrons are accelerated here.

Laser wavelength: \(~1 \mu m\)

Laser duration \(~\) 

Plasma wavelength: \(~20 \mu m\)

Physical scales disparity in LWFA
Laser Envelopes need less sampling points

Laser Envelope \(|\tilde{A}|\)

Laser “Standard” \(\hat{A}\)

Spatial variation scales:

Laser length \(\sim\)
Plasma wavelength
\((10-100 \, \mu\text{m})\)

Laser wavelength
\((\sim 1 \, \mu\text{m})\)

Points sampling Laser “Standard”

Points sampling Laser Envelope = 10

\(\rightarrow\) Larger \(\Delta x\) and \(\Delta t\) can be used!

D. Terzani and P. Londrillo,
Envelop model: separate the laser field

**Standard PIC**
- E, B field (plasma fields + laser fields)
- Lorentz Force
- Current Density $J$
- Particles

**Envelope (Ponderomotive) PIC**
- E, B field (plasma fields)
- Lorentz Force
- Plasma Current Density $J$
- Ponderomotive Force
- Susceptibility
- Envelope $\tilde{\alpha}$ (laser)
- Particles
Envelope modeling has multiple advantages

Transverse E field, Apollon Simulations

Advantages of the envelope over standard Yee solver:
- Quicker (speedup $>20$)
- Safe Filtering
- More accurate laser speed

Standard PIC (1 mm of propagation):
- $\sim 320$ kcpu-hours
- $\sim 10.2$ k€

Envelope PIC $> 20$

J.-L. Vay, JCP 230 (2011)
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CILEX 1st stage: quick envelope simulations

Apollon 1st Stage
Example of simulation with the envelope model

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{Laser}}$ [J]</td>
<td>10</td>
</tr>
<tr>
<td>$P/P_{cr}$</td>
<td>18.3</td>
</tr>
<tr>
<td>Laser $w_0$ [μm]</td>
<td>40</td>
</tr>
<tr>
<td>$n_{\text{plasma}}$ [10^{18} \text{cm}^{-3}]</td>
<td>1.1</td>
</tr>
<tr>
<td>$L_{\text{FWHM, Laser}}$ [fs]</td>
<td>20</td>
</tr>
<tr>
<td>$Q_{\text{beam}}$ [pC]</td>
<td>263</td>
</tr>
<tr>
<td>$E_{\text{peak}}$ [GeV]</td>
<td>870</td>
</tr>
<tr>
<td>$\Delta E/E$ (%)</td>
<td>8.3</td>
</tr>
<tr>
<td>Distance [mm]</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Minimum of Energy spread (~6.3 mm)
CILEX 1st stage: quick envelope simulations

Apollon 1st Stage
Possible working points studied with the envelope model

<table>
<thead>
<tr>
<th>Simulation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{Laser}}$ [J]</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$P/P_{\text{cr}}$</td>
<td>18.3</td>
<td>12.0</td>
<td>22.0</td>
<td>13.3</td>
</tr>
<tr>
<td>Laser $w_0$ [μm]</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_{\text{plasma}}$ [$10^{18}$cm$^{-3}$]</td>
<td>1.1</td>
<td>0.9</td>
<td>1.1</td>
<td>0.8</td>
</tr>
<tr>
<td>$L_{\text{FWHM, Laser}}$ [fs]</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>$Q_{\text{beam}}$ [pC]</td>
<td>263</td>
<td>48</td>
<td>543</td>
<td>24</td>
</tr>
<tr>
<td>$E_{\text{peak}}$ [GeV]</td>
<td>870</td>
<td>740</td>
<td>930</td>
<td>1130</td>
</tr>
<tr>
<td>$\Delta E/E$ (%)</td>
<td>8.3</td>
<td>3.2</td>
<td>6.4</td>
<td>2.0</td>
</tr>
<tr>
<td>Distance [mm]</td>
<td>6.3</td>
<td>7.2</td>
<td>6.5</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Simulation 1, Propagation distance = 6.3 mm
Simulation 2, Propagation distance = 7.2 mm
Simulation 3, Propagation distance = 6.5 mm
Simulation 4, Propagation distance = 7.6 mm
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Case Study: Multistage LWFA experiments

1st Plasma Stage

1st Laser Pulse

Injected Electrons

~ 50-200 MeV

Magnetic Beam Transport

2nd Plasma Stage

2nd Laser Pulse

plasma channel, ~ 10 cm

Accelerated Electrons

~ 10 GeV
Envelope Benchmark: External injection in LWFA

Comparison @15 mm of propagation

Electron density

Longitudinal electric field

Propagation Direction

\[ \frac{T_{\text{Standard Laser}}}{T_{\text{Envelope}}} = 20 \]
Envelope Benchmark: External injection in LWFA

Further improvement of numerical dispersion:
Spectral solvers
(I. Zemzemi’s thesis)

Envelope PIC less dispersive than standard PIC
Simulation of External injection LWFA

Comparison @15 mm of propagation, Preliminary Results

Electron density

Envelope simulation

Standard Laser simulation

Longitudinal electric field

Envelope simulation

Standard Laser simulation

Propagation Direction

Envelope PIC less dispersive than standard PIC

\[
\frac{T_{\text{Standard Laser}}}{T_{\text{Envelope}}} = 20
\]
Comparison between Standard PIC and Envelope PIC

**Electron Beam Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values @Injection</th>
<th>Standard Simulation @15mm</th>
<th>Envelope Simulation @15mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q [pC]</td>
<td>30</td>
<td>29.98</td>
<td>29.94</td>
</tr>
<tr>
<td>E [MeV]</td>
<td>150</td>
<td>427</td>
<td>438</td>
</tr>
<tr>
<td>ΔE/E [%]</td>
<td>0.5</td>
<td>4.7</td>
<td>6.4</td>
</tr>
<tr>
<td>σ_x, σ_y, σ_z [μm]</td>
<td>2.0, 1.3, 1.3</td>
<td>2.0,1.5,1.4</td>
<td>2.0,1.0, 1.0</td>
</tr>
<tr>
<td>ε_n,y, ε_n,z [mm-mrad]</td>
<td>1.0, 1.0</td>
<td>2.0, 2.1</td>
<td>1.0, 1.0</td>
</tr>
</tbody>
</table>

**Numerical Cherenkov**
- Emittance conserved
- Beam stays focused

**Champ E longitudinal**

- More accurate laser speed:
- More accurate phase and Longitudinal phase space evolution
Conclusions and perspectives

- Time explicit (non quasi-static) 3D envelope model for the laser now available in Smilei.

- Benchmarked on long second stage simulation

- Used to study possible working points for Apollon LWFA experiments

Future developments:

- Envelope model + cylindrical geometry
- Envelope model + ionization
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The Laser Envelope evolution: wave equation

D. Terzani and P. Londrillo,

Hypothesis:
\[ \hat{A}(x, t) = \text{Re} \left[ \hat{\tilde{A}}(x, t) e^{i(x-ct)} \right] \]

Laser Complex Envelope

D’Alembert Equation:
\[ \nabla^2 \hat{A} - \partial_t^2 \hat{A} = -\hat{J} \]

Envelope Equation:
\[ \nabla^2 \hat{A} + 2i \left( \partial_x \hat{A} + \partial_t \hat{A} \right) - \partial_t^2 \hat{A} = \chi \hat{\tilde{A}} \]

Plasma Susceptibility
Ponderomotive Equations of motion

Ponderomotive force acts as a radiation pressure on charged particles: it expels the electrons from high-intensity zones

\[ \mathbf{F}_{\text{ponderomotive}} \]

Motion Equations for the macroparticles (here electrons):

\[
\frac{d\mathbf{x}_p}{dt} = \frac{\mathbf{u}_p}{\gamma_p} \\
\frac{d\mathbf{u}_p}{dt} = \left( \mathbf{E}_p + \frac{\mathbf{u}_p}{\gamma_p} \times \mathbf{B}_p \right) - \frac{1}{4\gamma_p} \nabla \left( |\mathbf{A}_p|^2 \right)
\]


Lorentz Force (plasma fields) + Ponderomotive Force (laser envelope)
Electromagnetic field initialization: Relativistic electron

\( \gamma_0 = 1 \)  
(immobile electron)

\( \gamma_0 = 200 \)  
(\( \sim 100 \) MeV)

\( E \)  
\( B = 0 \)  
\( v = 0 \)

\( E \)  
\( B \neq 0 \)  
\( v \sim c \)
Initialisation of Electromagnetic Fields

Immobiles Species: Poisson’s Equation
\[ \nabla^2 \Phi = -\rho \]

Relativistic Species: “Relativistic” Poisson’s Equation
\[ \left( \frac{1}{\gamma_0^2} \partial_x^2 + \nabla_\perp^2 \right) \Phi = -\rho \]
\[ \mathbf{E} = \left( -\frac{1}{\gamma_0^2} \partial_x \Phi, -\partial_y \Phi, -\partial_z \Phi \right) \]
\[ \mathbf{B} = \frac{\beta_0}{c} \hat{x} \times \mathbf{E} \]

Hypothesis:
Negligible energy spread

If non-negligible energy spread:
Repeat for each energy “slice”