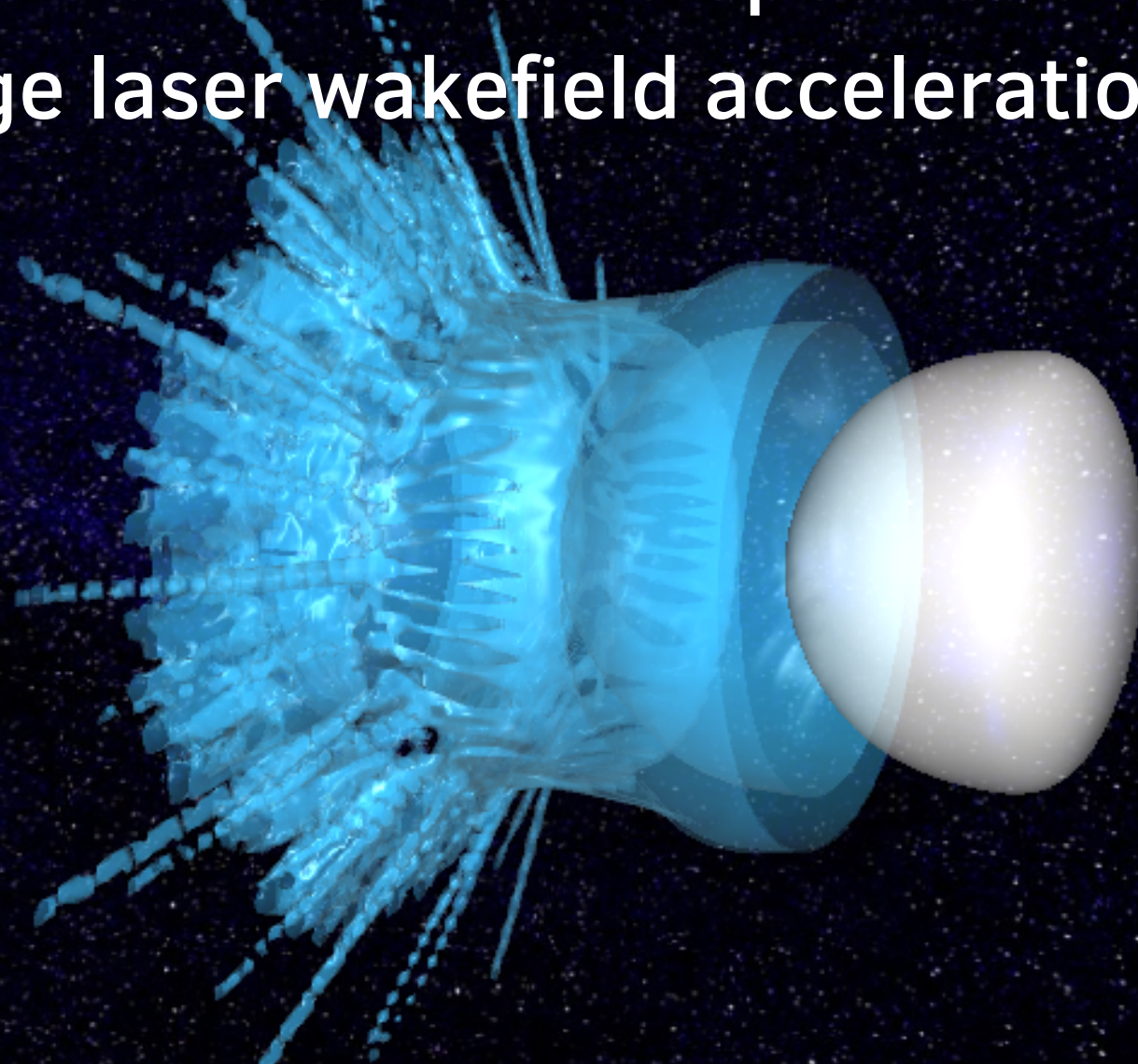


# Efficient 3D envelope modeling for two-stage laser wakefield acceleration experiments



Francesco Massimo



# Outline

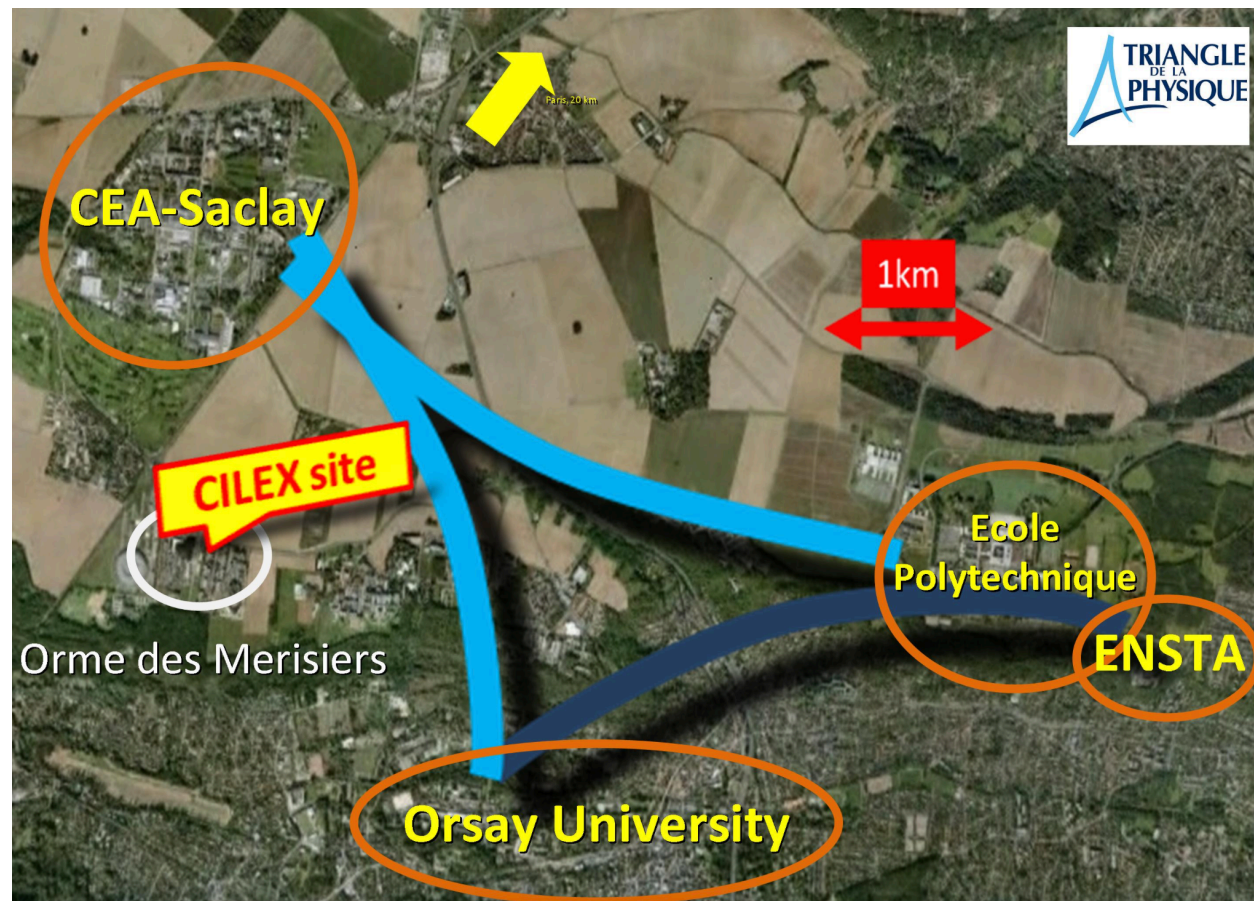
- Context
- Modeling Laser Wakefield Acceleration with a laser envelope
- First stage LWFA simulations
- Second stage LWFA simulations
- Conclusions

# Outline

- **Context**
- Modeling Laser Wakefield Acceleration with a laser envelope
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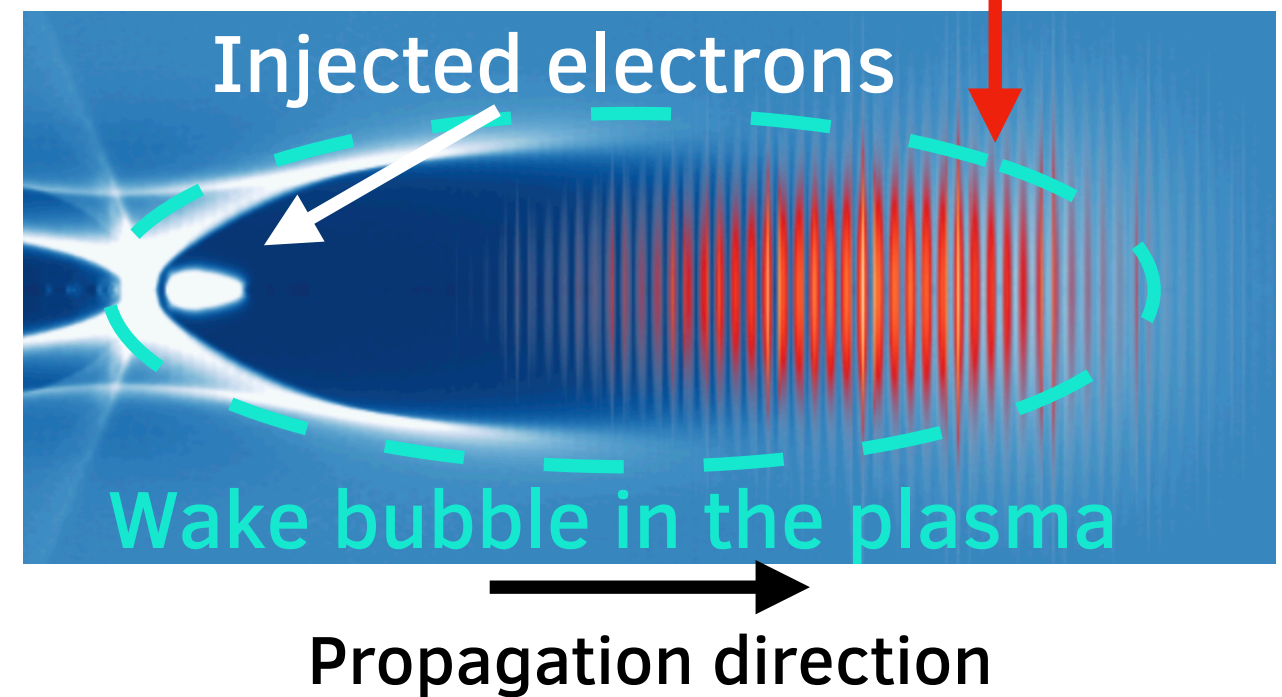


# Centre Interdisciplinaire de la Lumière Extrême (CILEX)



Laser Wakefield  
Acceleration (LWFA)

Laser  
Pulse



Irfu

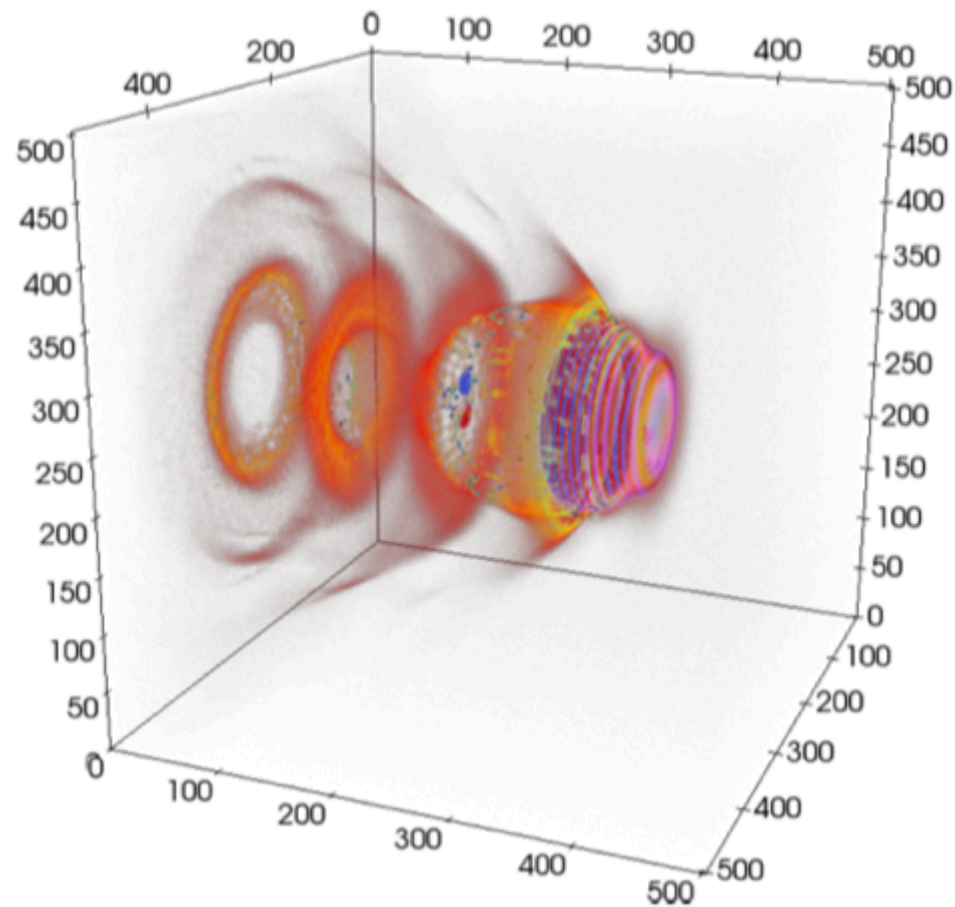


LABORATOIRE  
CHARLES  
FABRY





# PIC simulations need many resources



3D standard LWFA simulations:

1 mm plasma ~ 320 kcpu-hours ~ 10.2 k€  
(36 years on 1 cpu)

→ **Parallelization is mandatory**  
but still 320 kcpu-hours ~ 13 days on 1000 cpus ...

Any trick to speed up the calculation  
is most welcome

Implemented in **Smilei**:

High Performance Computing  
(HPC) techniques

- Parallelization
- Smart Load Balancing
- Vectorisation

Techniques using  
physical approximations

- Azimuthal Fourier decomposition
- Envelope modeling

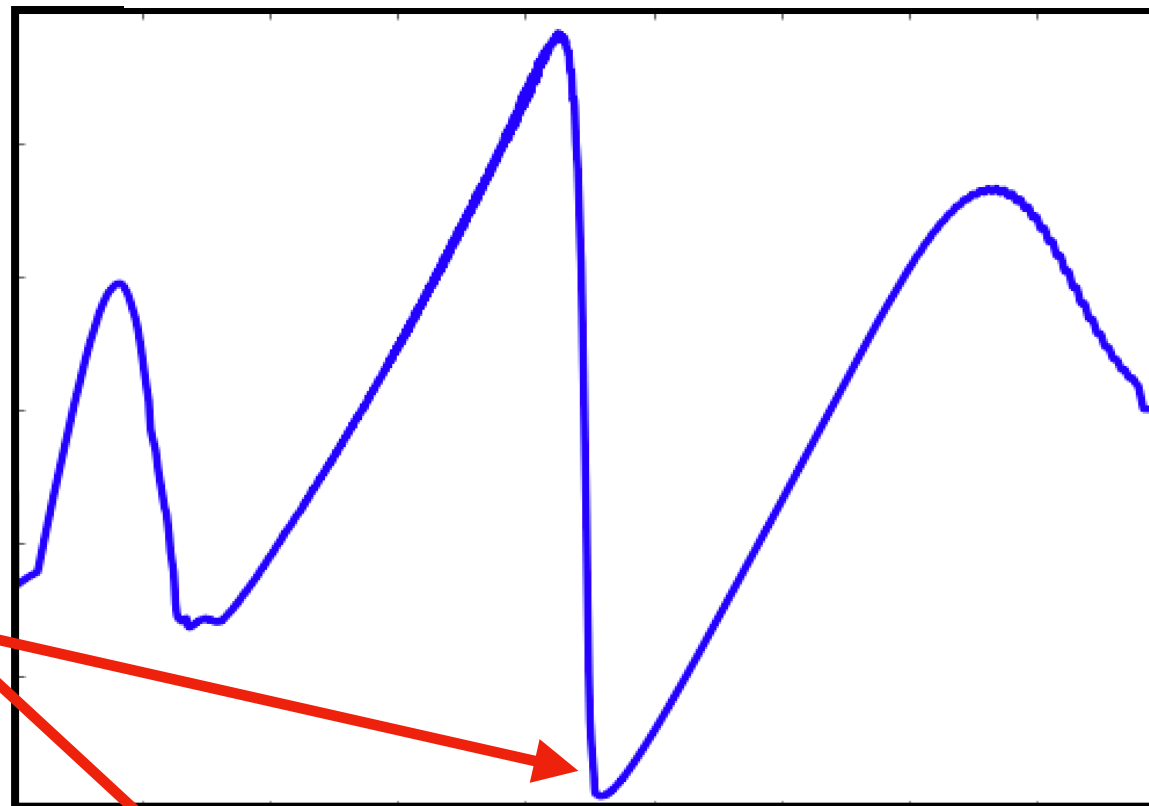
# Outline

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# Physical scales disparity in LWFA

Electrons  
are accelerated  
Here



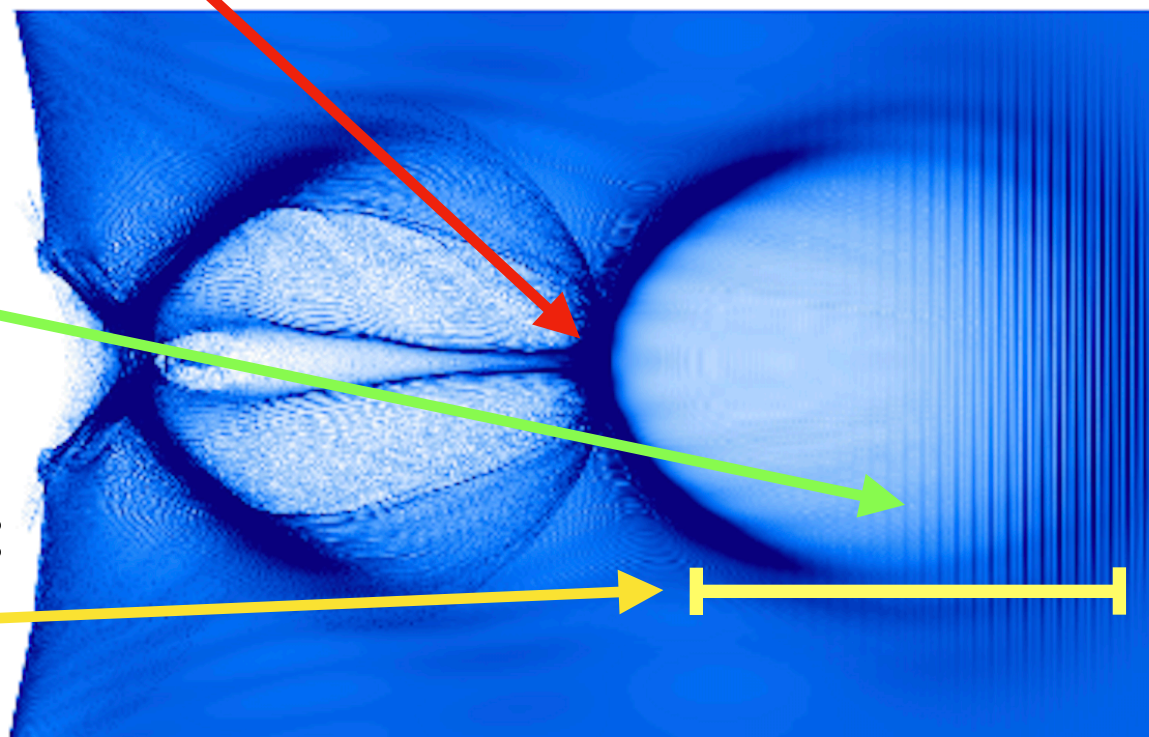
0.04  
Longitudinal  
Electric Field  
(A.U.)  
0.00  
on propagation axis  
-0.04

Laser  
wavelength:

$\sim 1 \mu\text{m}$

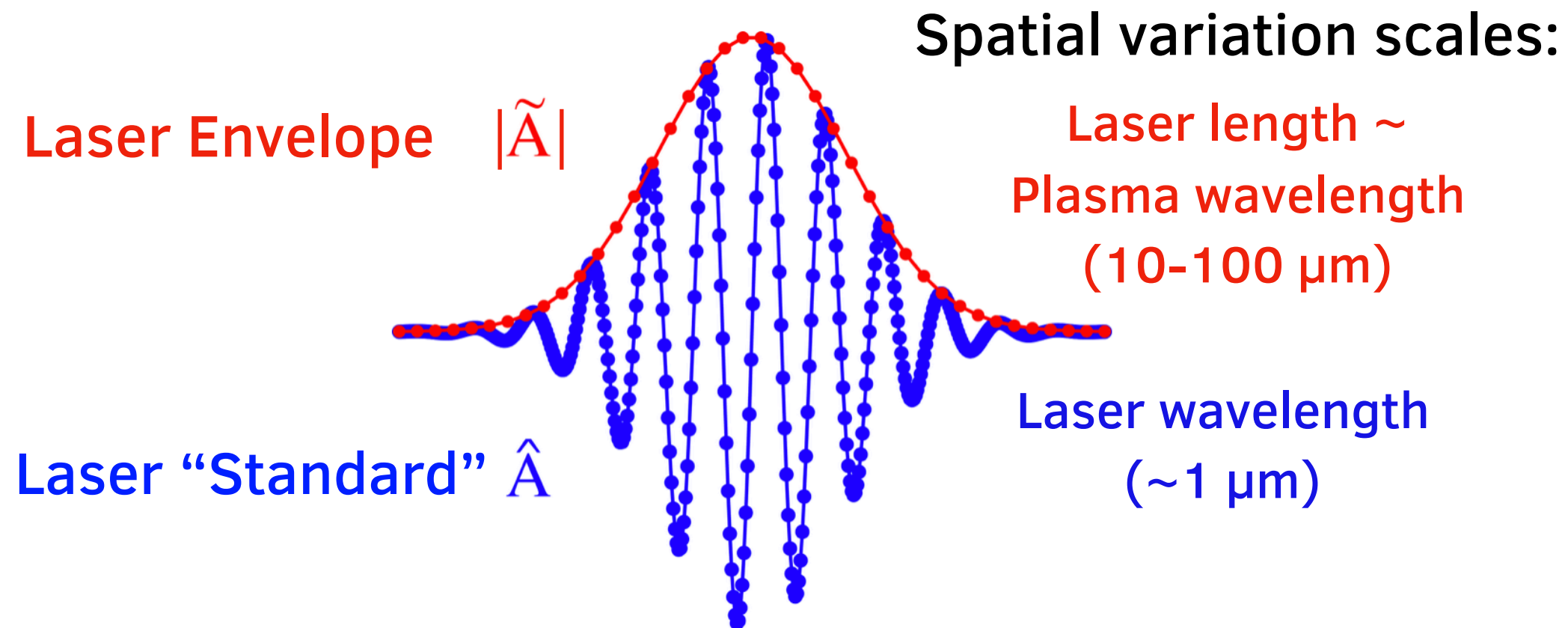
Laser duration ~  
Plasma wavelength:

$\sim 20 \mu\text{m}$



0.003  
0.002  
Electron  
Charge  
Density  
(A.U.)  
0.001  
0.000

# Laser Envelopes need less sampling points



D. Terzani and P. Londrillo,  
Comput. Phys. Comm. (2019)

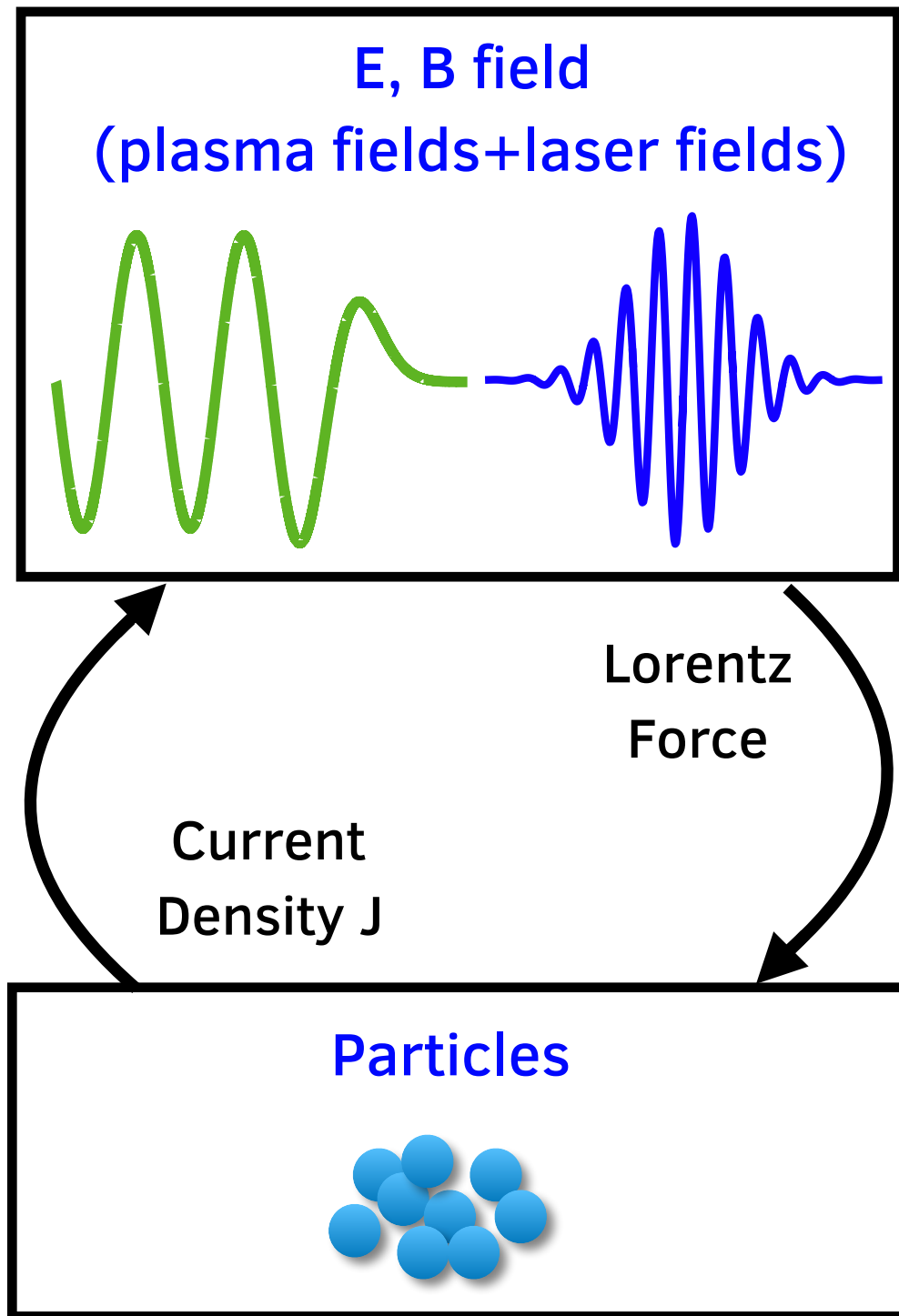
$$\frac{\text{Points sampling Laser "Standard"}}{\text{Points sampling Laser Envelope}} = 10$$

→ Larger  $\Delta x$  and  $\Delta t$  can be used!

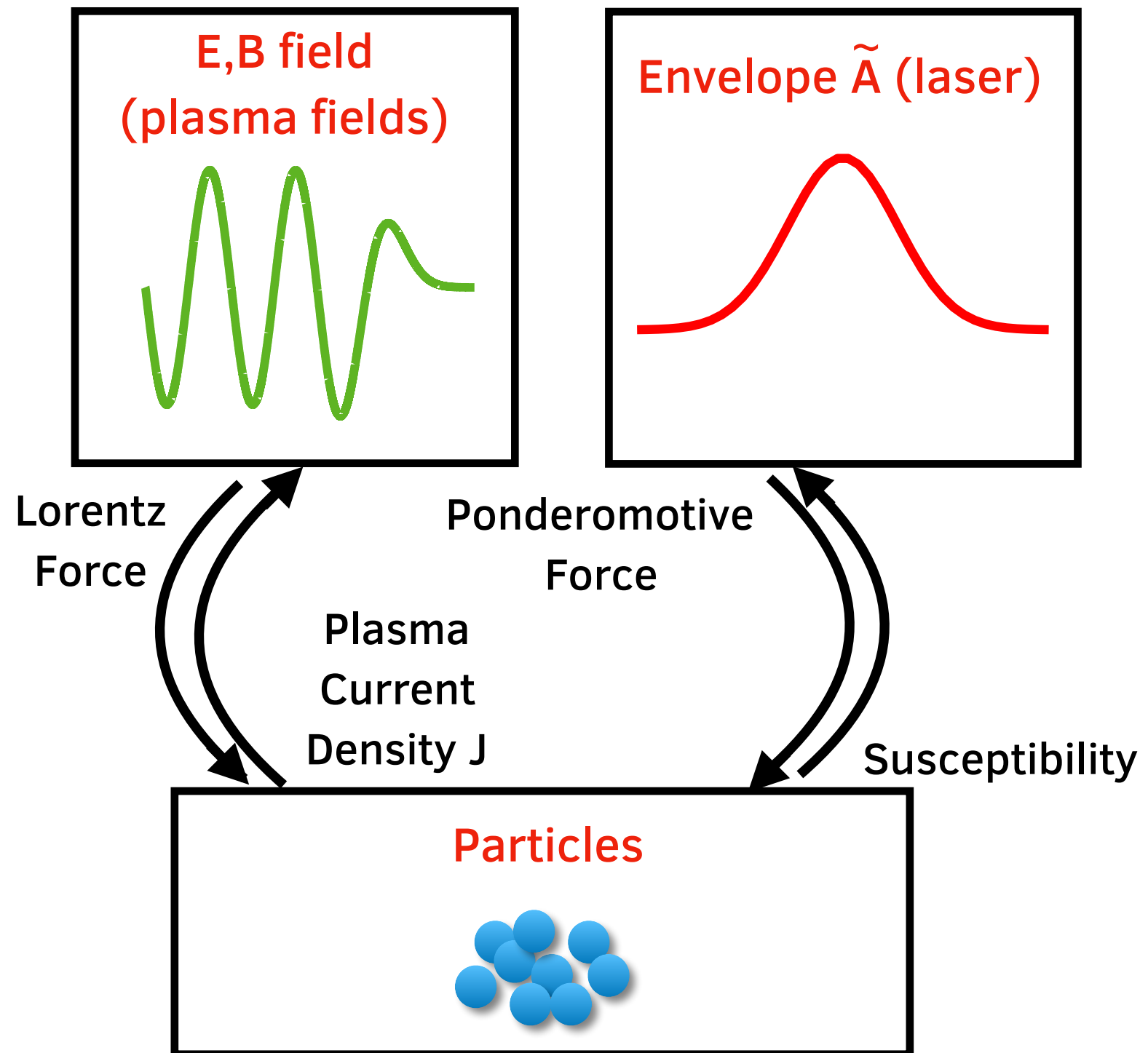


# Envelope model: separate the laser field

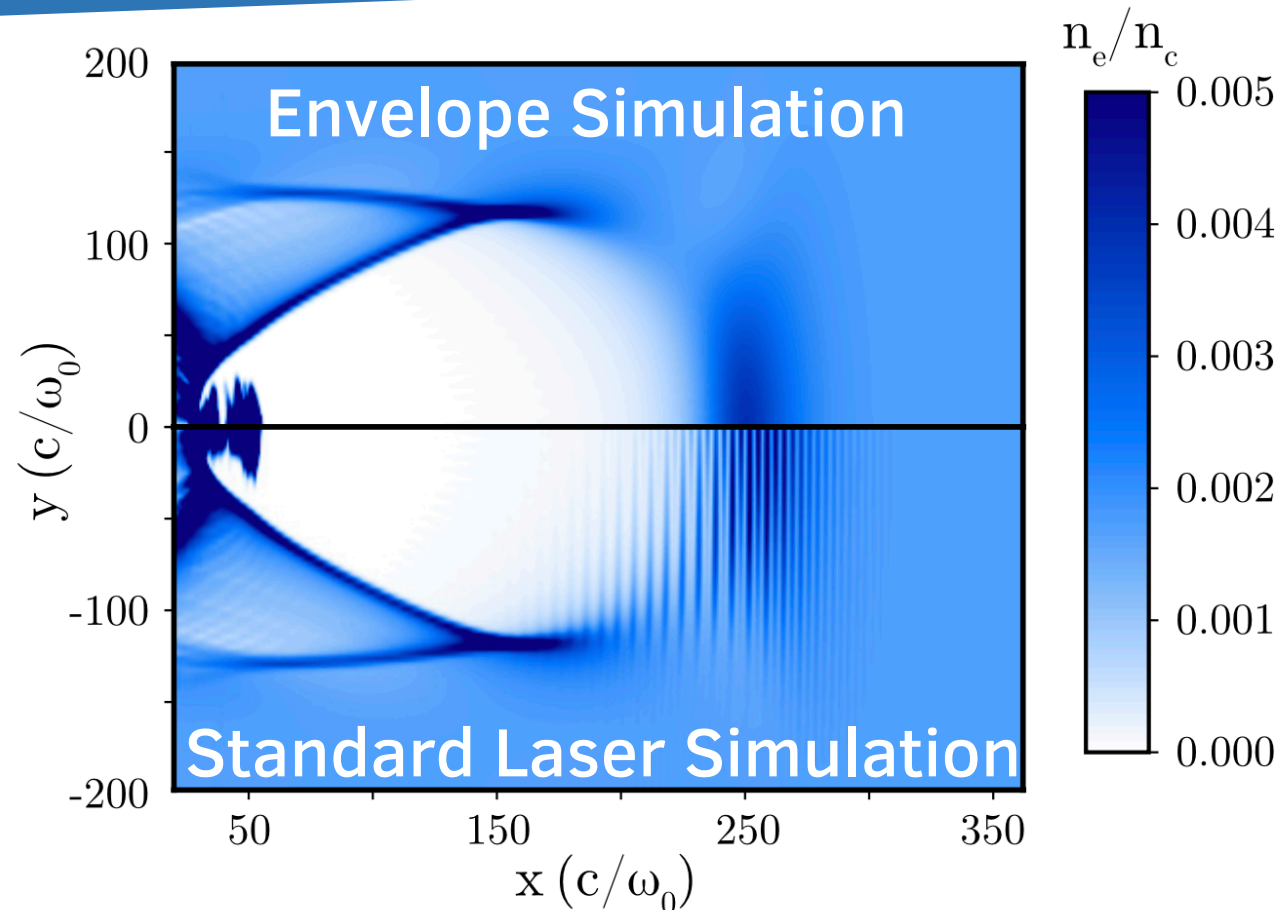
## Standard PIC



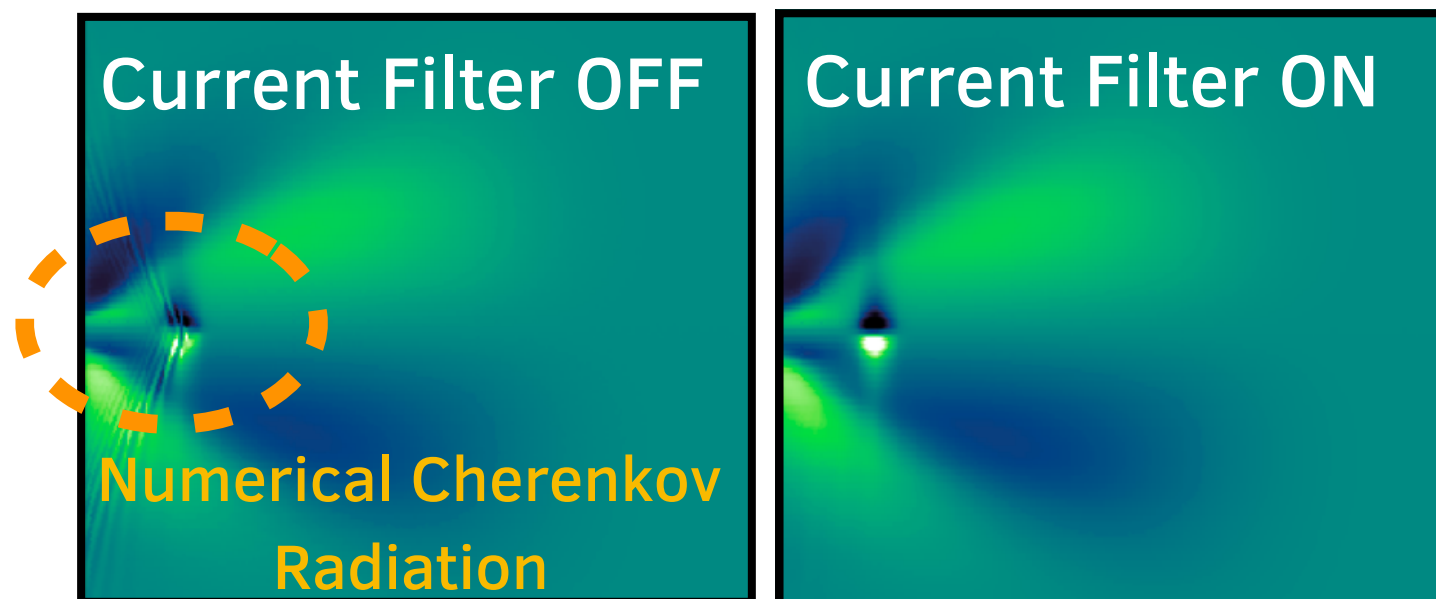
## Envelope (/Ponderomotive) PIC



# Envelope modeling has multiple advantages



Transverse E field, Apollon Simulations



J.-L.Vay, JCP 230 (2011)

**T<sub>Standard PIC</sub>**  
(1 mm of propagation):

~ 320 kcpu-hours

~ 10.2 k€

$$\frac{T_{\text{Standard PIC}}}{T_{\text{Envelope PIC}}} > 20$$

Advantages of the envelope  
over standard Yee solver:

- Quicker (speedup > 20)
- Safe Filtering
- More accurate laser speed



# Outline

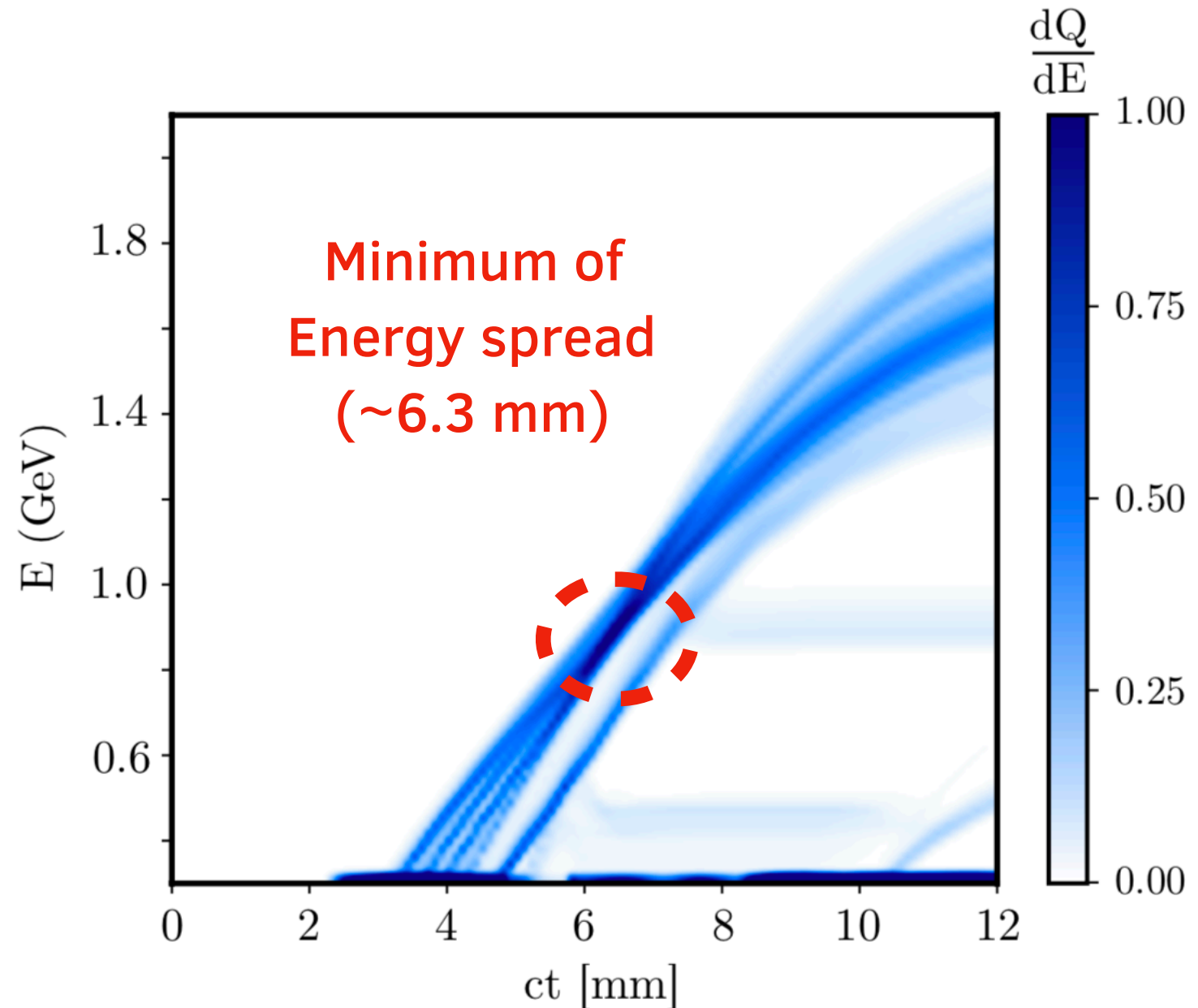
- Context
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# CILEX 1st stage: quick envelope simulations

## Apollon 1<sup>st</sup> Stage

Example of simulation with the envelope model

Simulation	1
$E_{\text{Laser}}$ [J]	10
$P/P_{\text{cr}}$	18.3
Laser $w_0$ [ $\mu\text{m}$ ]	40
$n_{\text{plasma}}$ [ $10^{18}\text{cm}^{-3}$ ]	1.1
$L_{\text{FWHM, Laser}}$ [fs]	20
$Q_{\text{beam}}$ [pC]	263
$E_{\text{peak}}$ [GeV]	870
$\Delta E/E$ (%)	8.3
Distance [mm]	6.3





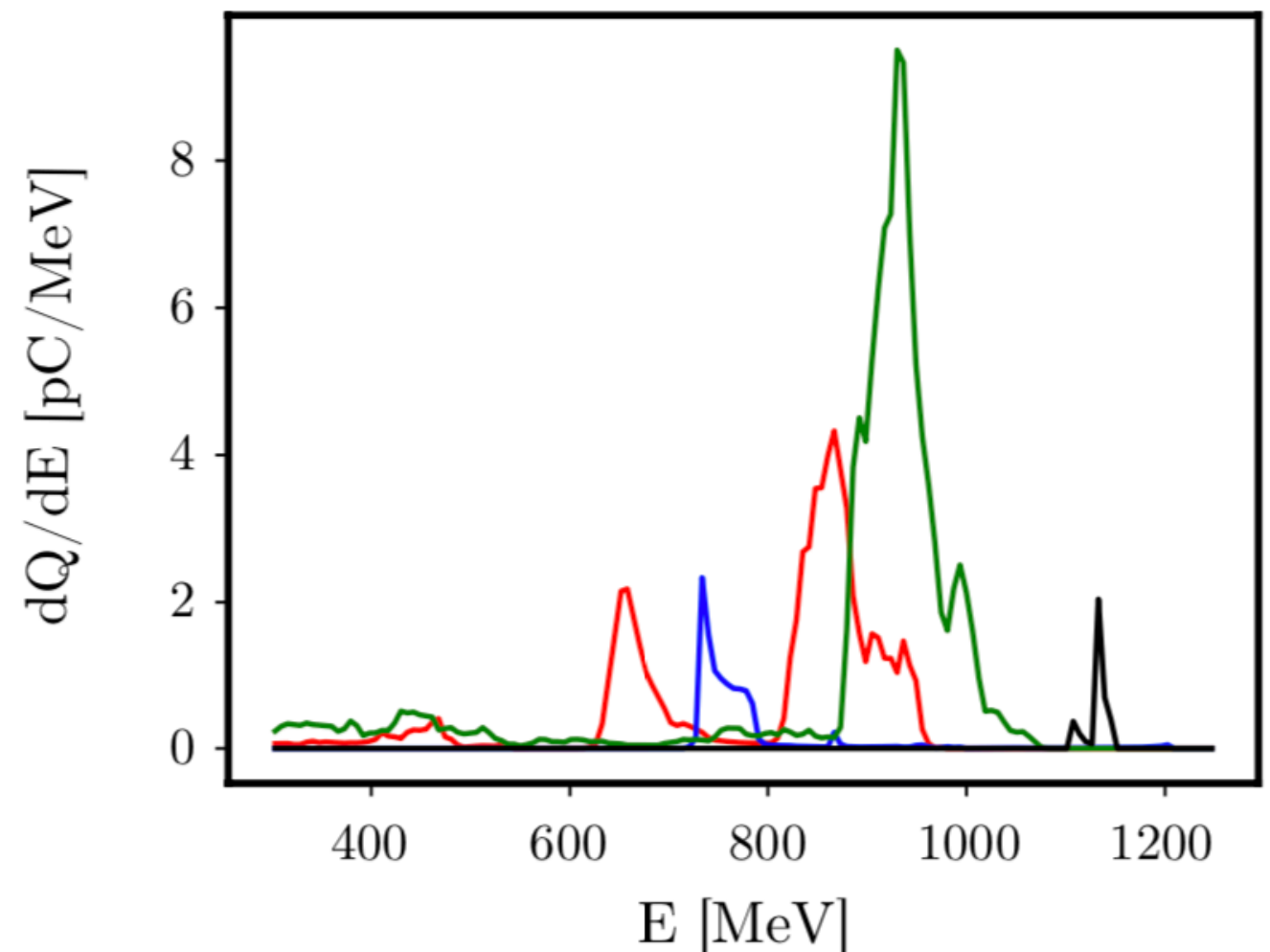
# CILEX 1st stage: quick envelope simulations

## Apollon 1<sup>st</sup> Stage

Possible working points studied with the envelope model

Simulation	1	2	3	4
$E_{\text{Laser}}$ [J]	10	10	15	15
$P/P_{\text{cr}}$	18.3	12.0	22.0	13.3
Laser $w_0$ [ $\mu\text{m}$ ]	40			
$n_{\text{plasma}}$ [ $10^{18}\text{cm}^{-3}$ ]	1.1	0.9	1.1	0.8
$L_{\text{FWHM, Laser}}$ [fs]	20	25	25	30
$Q_{\text{beam}}$ [pC]	263	48	543	24
$E_{\text{peak}}$ [GeV]	870	740	930	1130
$\Delta E/E$ (%)	8.3	3.2	6.4	2.0
Distance [mm]	6.3	7.2	6.5	7.6

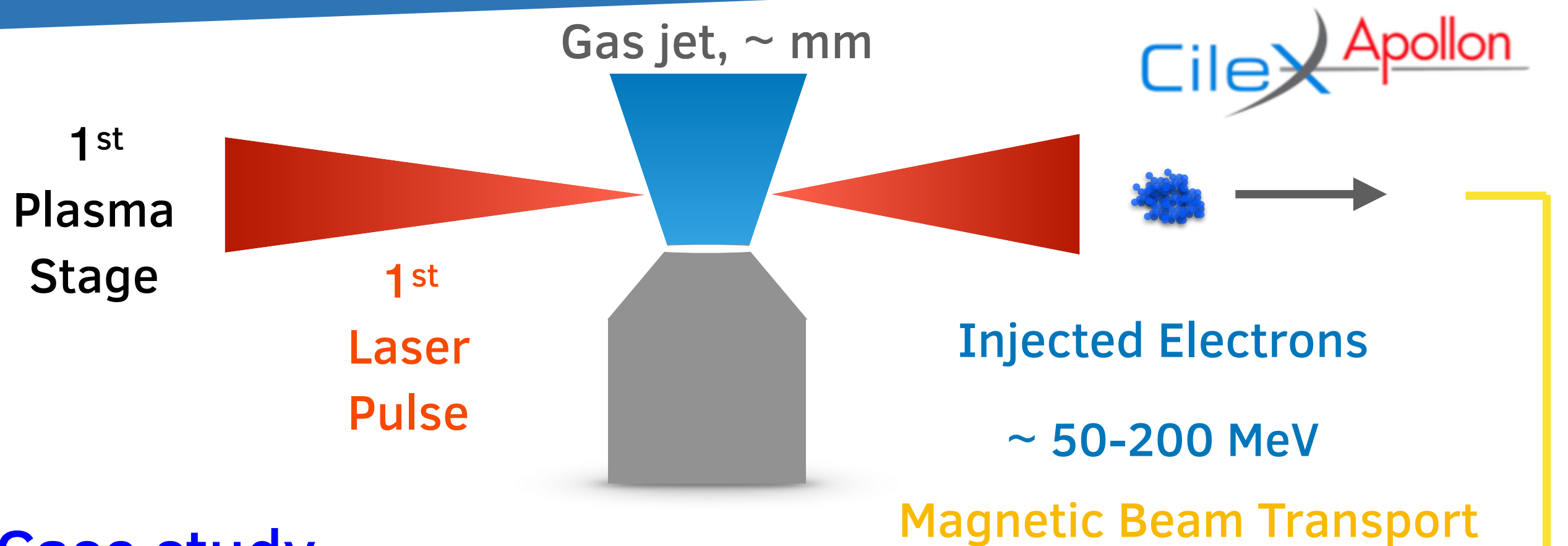
- Simulation 1, Propagation distance = 6.3 mm
- Simulation 2, Propagation distance = 7.2 mm
- Simulation 3, Propagation distance = 6.5 mm
- Simulation 4, Propagation distance = 7.6 mm



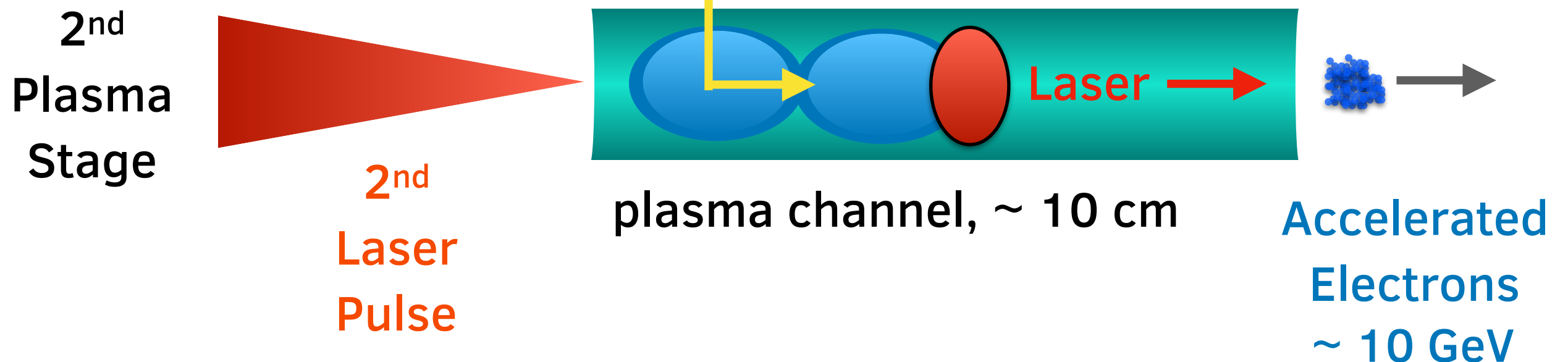
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# Case Study: Multistage LWFA experiments



## Case study

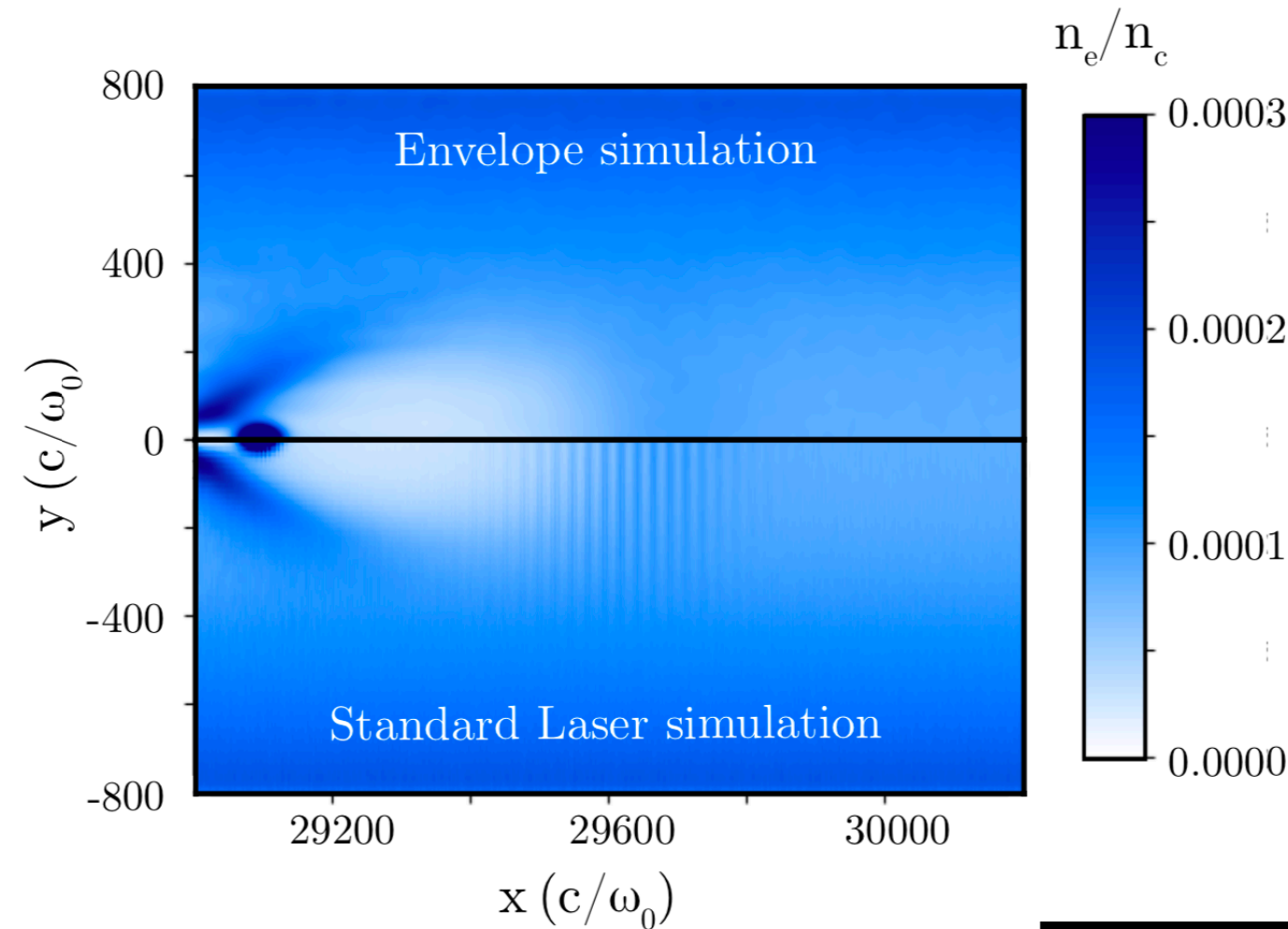




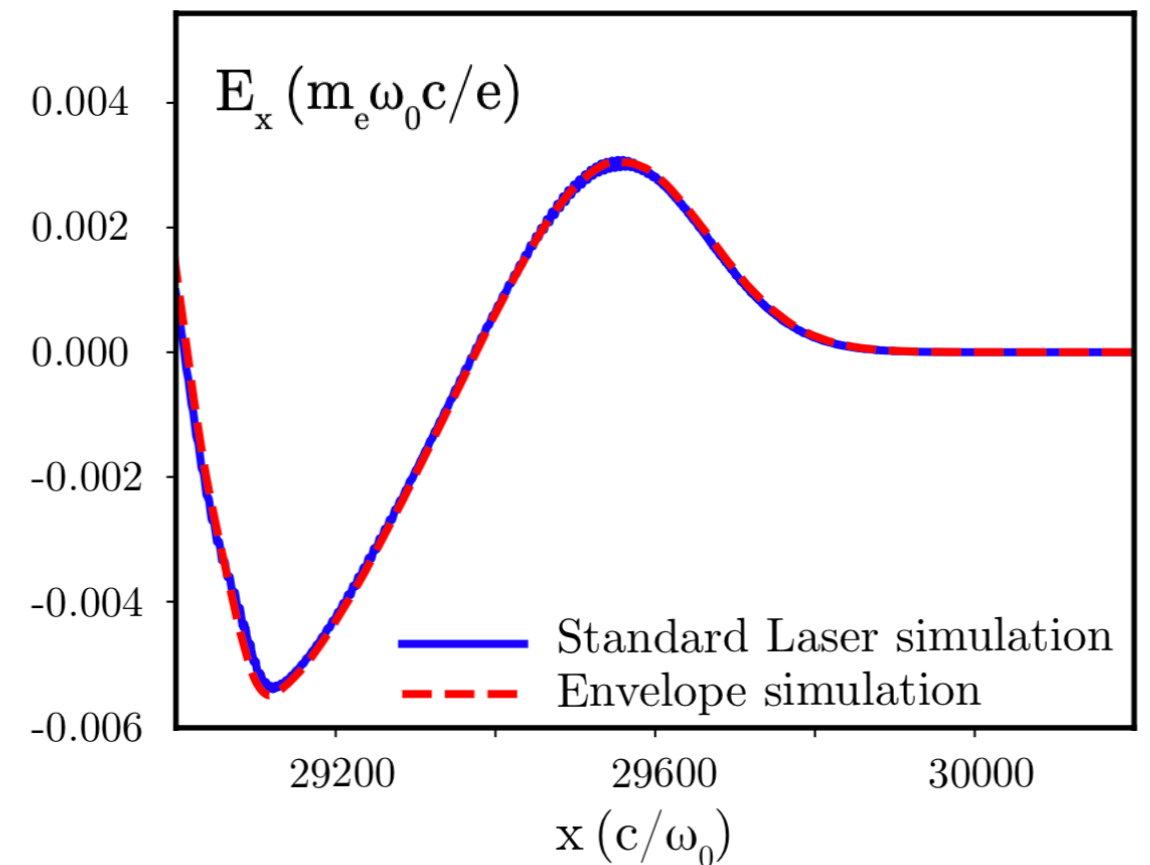
# Envelope Benchmark: External injection in LWFA

Comparison @15 mm of propagation

Electron density



Longitudinal electric field

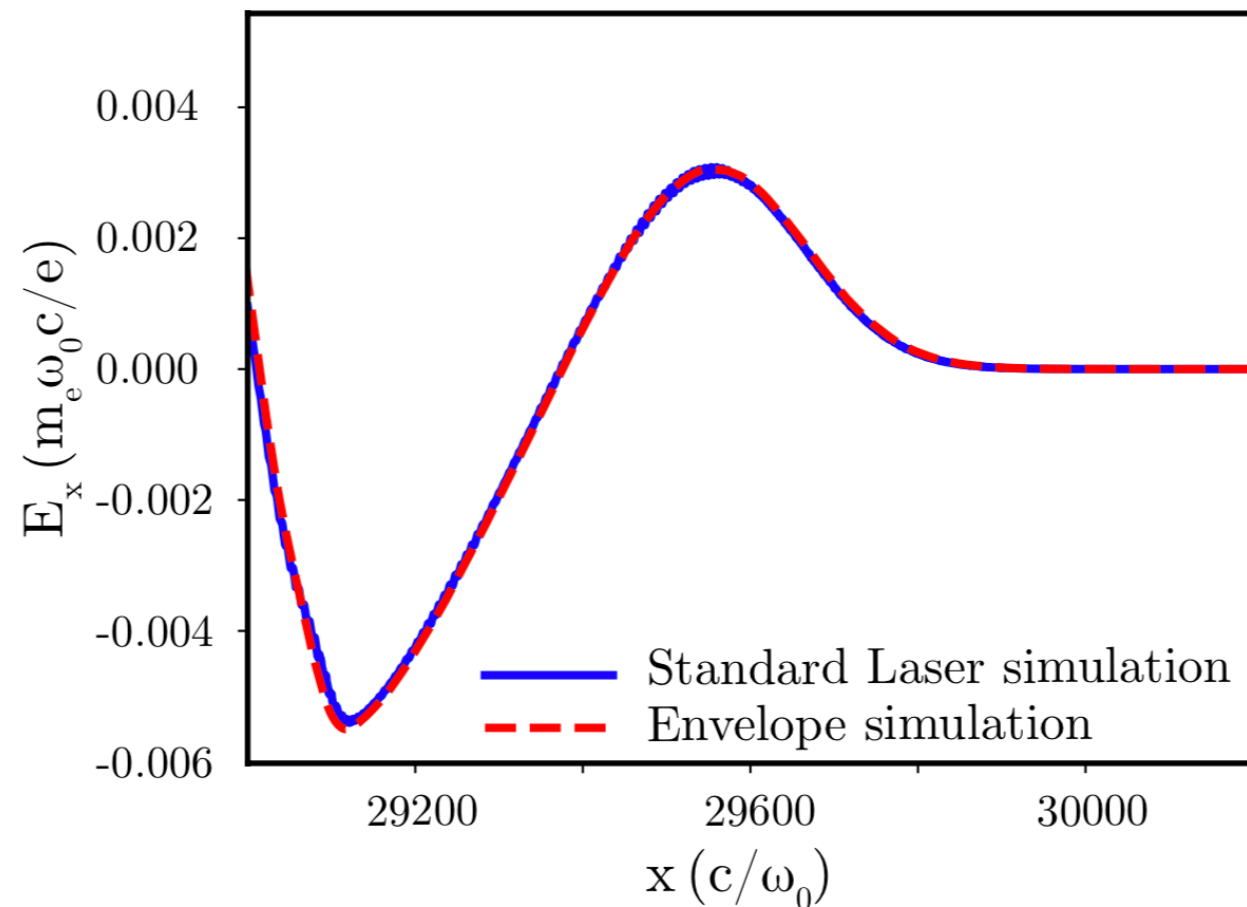


Propagation Direction

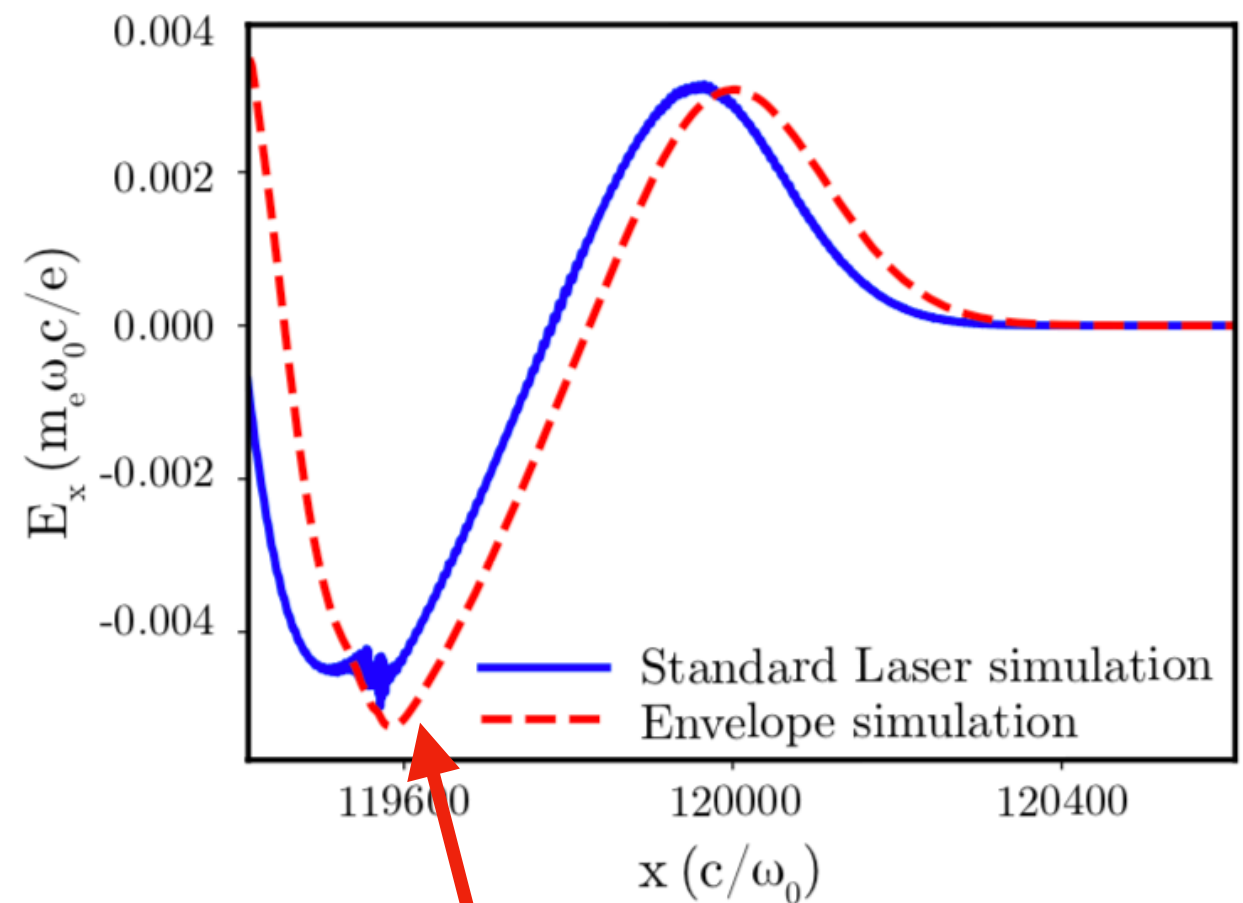
$$\frac{T_{\text{Standard Laser}}}{T_{\text{Envelope}}} = 20$$

# Envelope Benchmark: External injection in LWFA

@3.7 mm



@15 mm



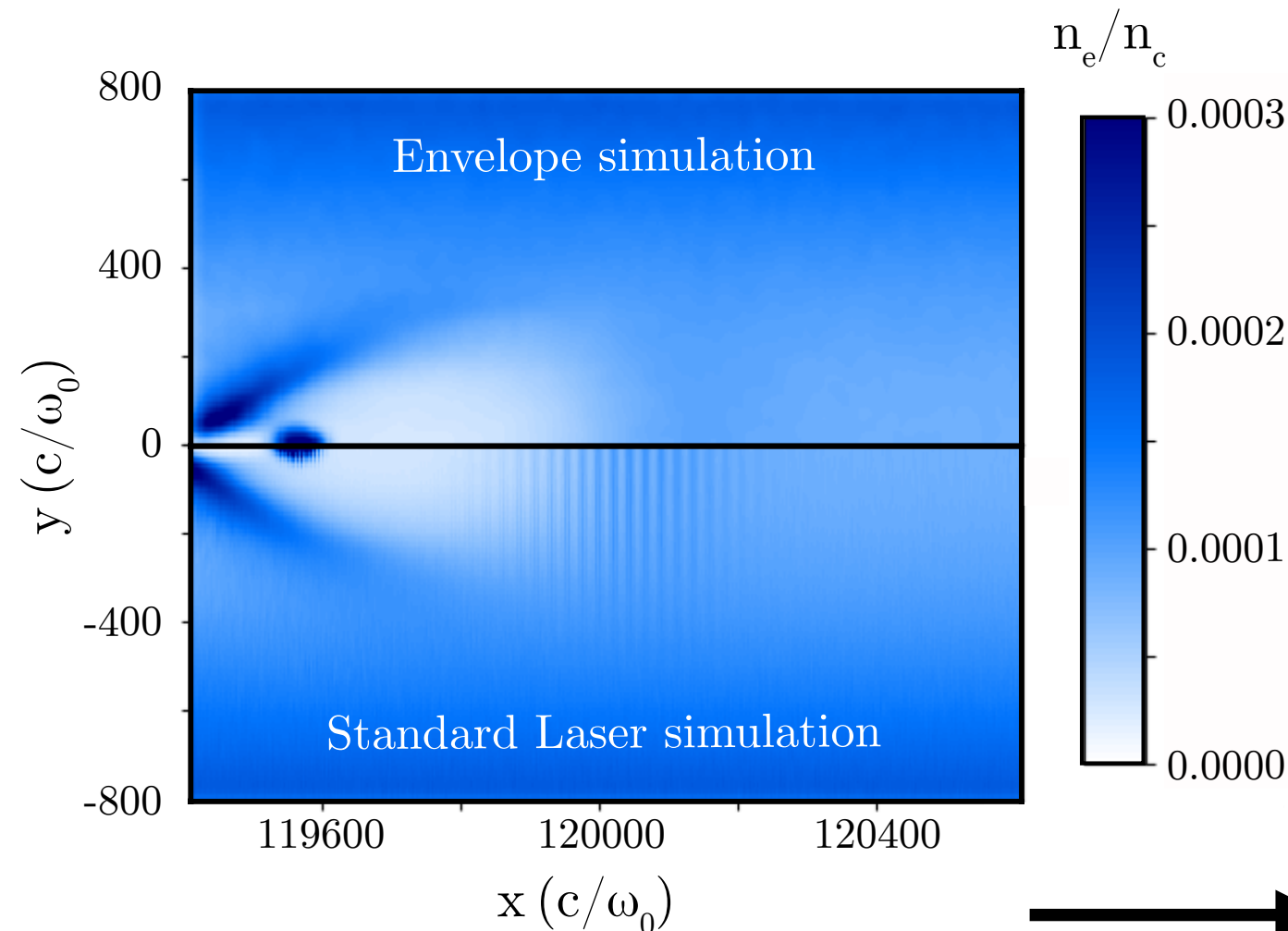
Further improvement  
of numerical dispersion:  
Spectral solvers  
(I. Zemzemi's thesis)

Envelope PIC less dispersive  
than standard PIC

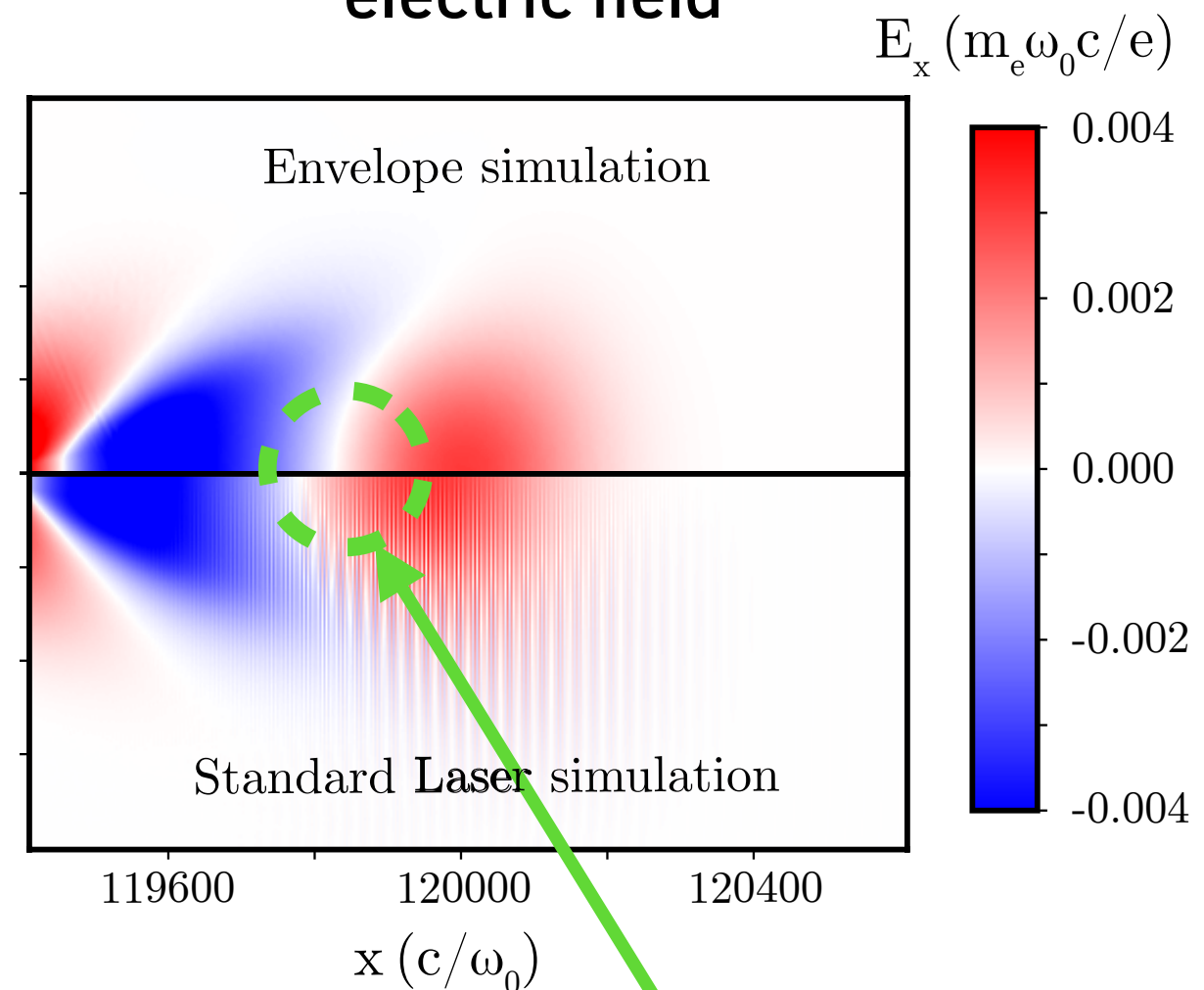
# Simulation of External injection LWFA

Comparison @15 mm of propagation, Preliminary Results

Electron density



Longitudinal electric field



Propagation Direction

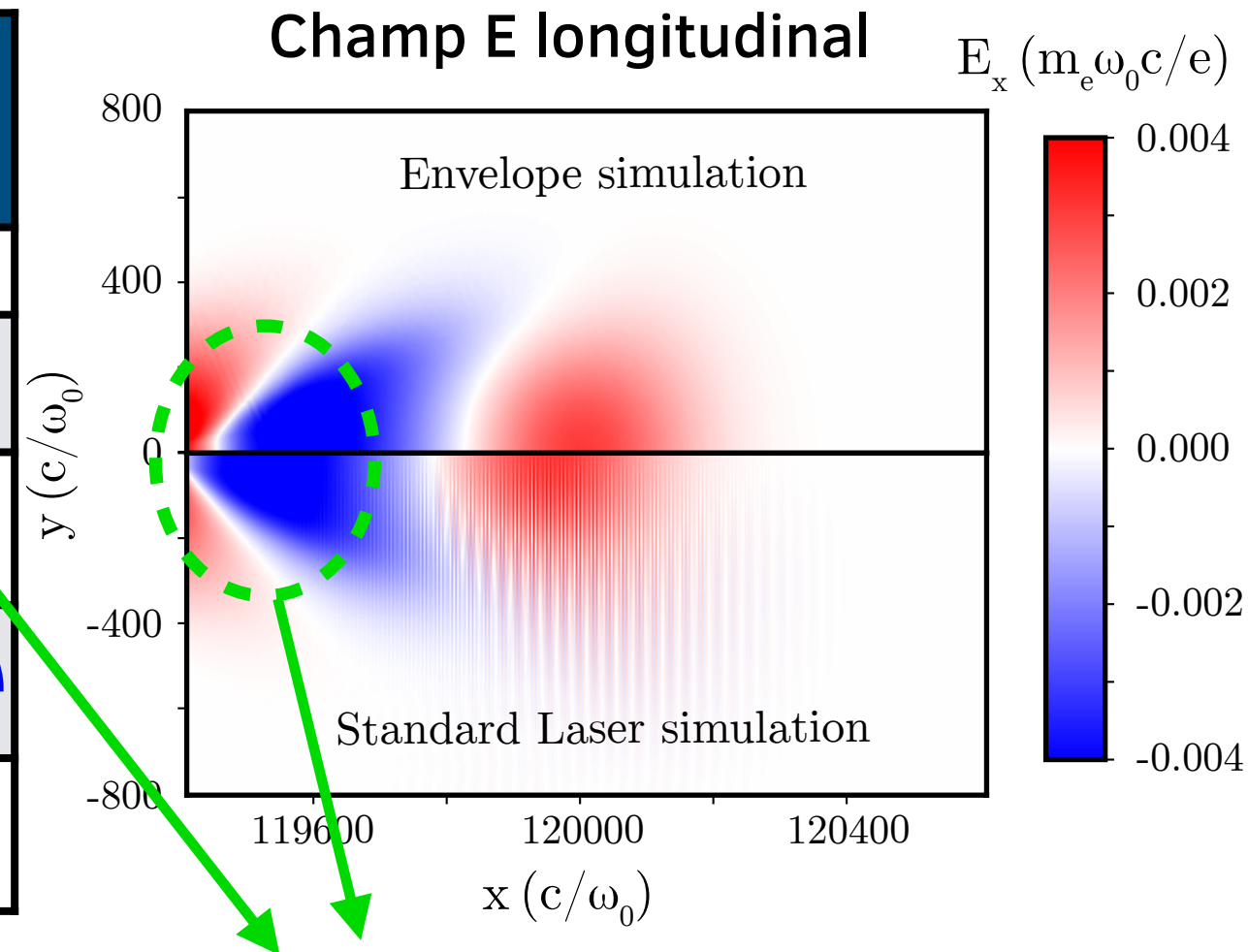
$$\frac{T_{\text{Standard Laser}}}{T_{\text{Envelope}}} = 20$$

Envelope PIC less dispersive than standard PIC

# Simulation of External injection LWFA: envelope advantages

## 2<sup>ème</sup> Stage of Apollon, Comparison between Standard PIC and Envelope PIC

Electron Beam Parameters	Values @Injection	Standard Simulation @15mm	Envelope Simulation @15 mm
Q [pC]	30	29.98	29.94
E [MeV]	150	427	438
$\Delta E/E$ [%]	0.5	4.7	6.4
$\sigma_x, \sigma_y, \sigma_z$ [ $\mu\text{m}$ ]	2.0, 1.3, 1.3	2.0, 1.5, 1.4	2.0, 1.0, 1.0
$\epsilon_{n,y}, \epsilon_{n,z}$ [mm-mrad]	1.0, 1.0	2.0, 2.1	1.0, 1.0



Numerical Cherenkov  
reduced by filtering:  
- Emittance conserved  
- Beam stays focused

More accurate laser speed:  
More accurate phase and  
Longitudinal phase space evolution



# Conclusions and perspectives

- Time explicit (non quasi-static) 3D envelope model for the laser now available in **Smilei**)
- Benchmarked on long second stage simulation
- Used to study possible working points for Apollon LWFA experiments

## Future developments:

- Envelope model + cylindrical geometry
- Envelope model + ionization

# Acknowledgements

## Group GALOP



- Arnaud Beck, Imen Zemzemi, A. Specka

## Developers of Smilei)

- Arnaud Beck, Imen Zemzemi
- Frédéric Pérez, Mickael Grech
- Julien Derouillat, Mathieu Lobet



## Developers of ALaDyn

- Alberto Marocchino
- Stefano Sinigardi,
- Davide Terzani



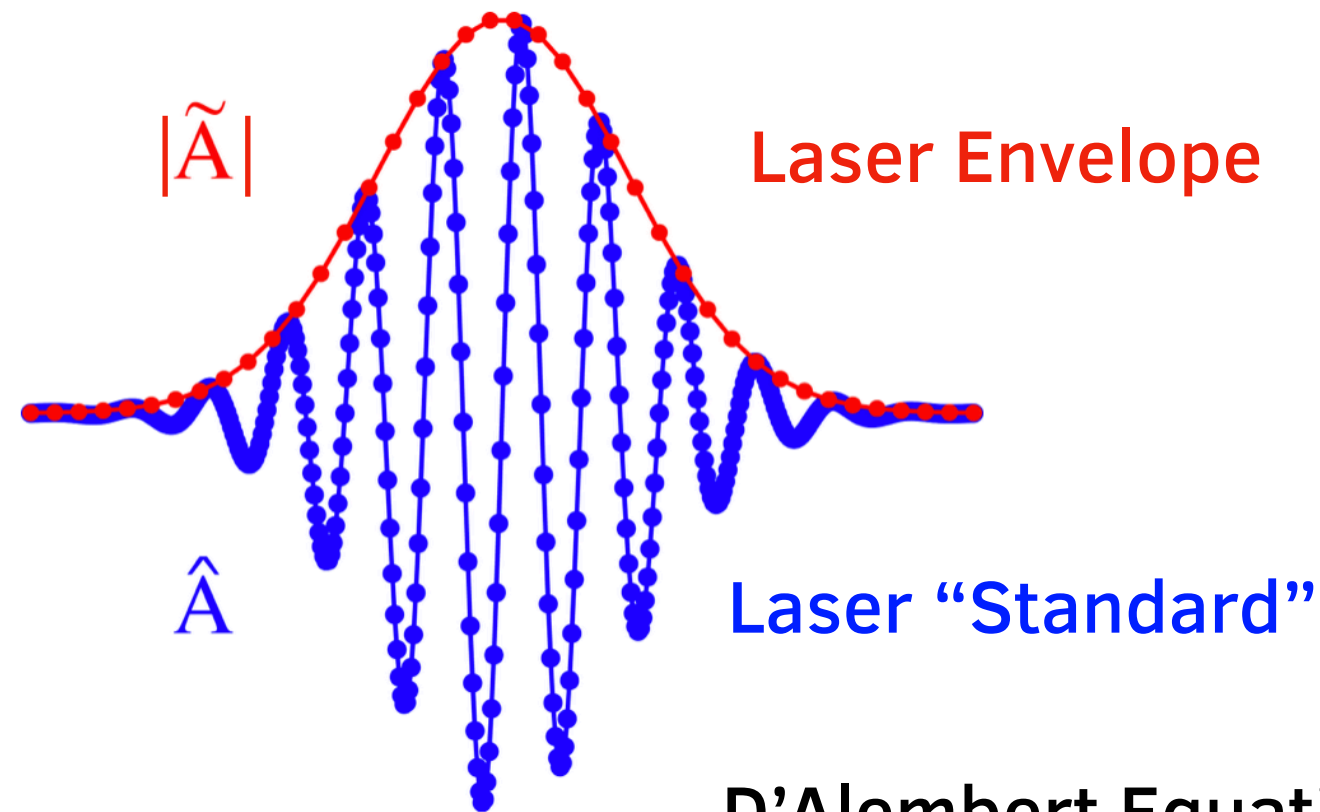
This work used computational resources of TGCC, CINES, through the allocation of resources 2018-A0010510062 granted by GENCI (Grand Equipement National de Calcul Intensif) and Grand Challenge "Irene" 2018 project gch0313 made by GENCI.

P2IO LabEx (ANR-10-LABX-0038) in the Framework "Investissements d'Avenir" (ANR-11-IDEX-0003-01) managed by Agence Nationale de la Recherche (ANR, France) provided financial support for F. Massimo

# Additional slides

# The Laser Envelope evolution: wave equation

D. Terzani and P. Londrillo,  
Comput. Phys. Comm. (2019)



Hypothesis:

$$\hat{A}(\mathbf{x}, t) = \text{Re} \left[ \tilde{A}(\mathbf{x}, t) e^{i(x-ct)} \right]$$

Laser Complex Envelope

+

D'Alembert Equation:

$$\nabla^2 \hat{A} - \partial_t^2 \hat{A} = -\hat{J} \quad =$$

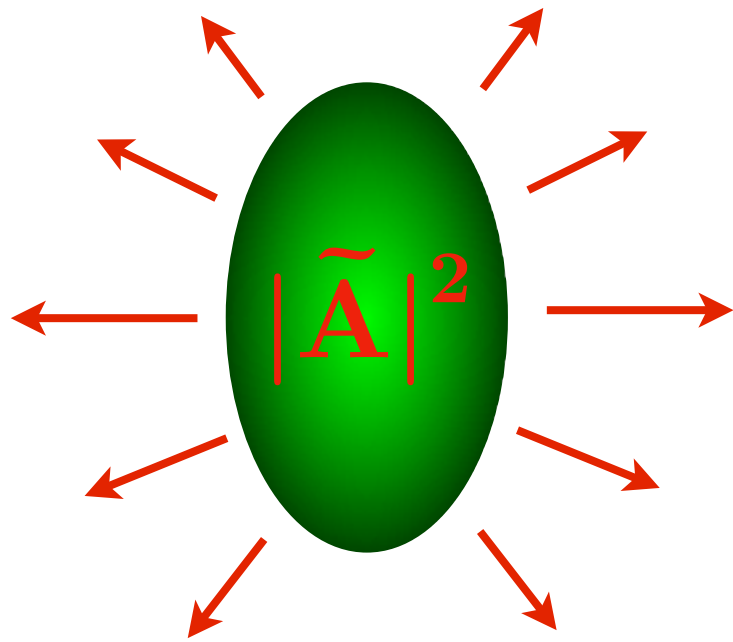
Envelope Equation:

$$\nabla^2 \tilde{A} + 2i \left( \partial_x \tilde{A} + \partial_t \tilde{A} \right) - \partial_t^2 \tilde{A} = \chi \tilde{A}$$

Plasma  
Susceptibility



# Ponderomotive Equations of motion



Ponderomotive force acts as a radiation pressure on charged particles : it expels the electrons from high-intensity zones

$F_{\text{ponderomotive}}$

Motion Equations for the macroparticles (here electrons):

$$\frac{d\bar{\mathbf{x}}_p}{dt} = \frac{\bar{\mathbf{u}}_p}{\bar{\gamma}_p}$$

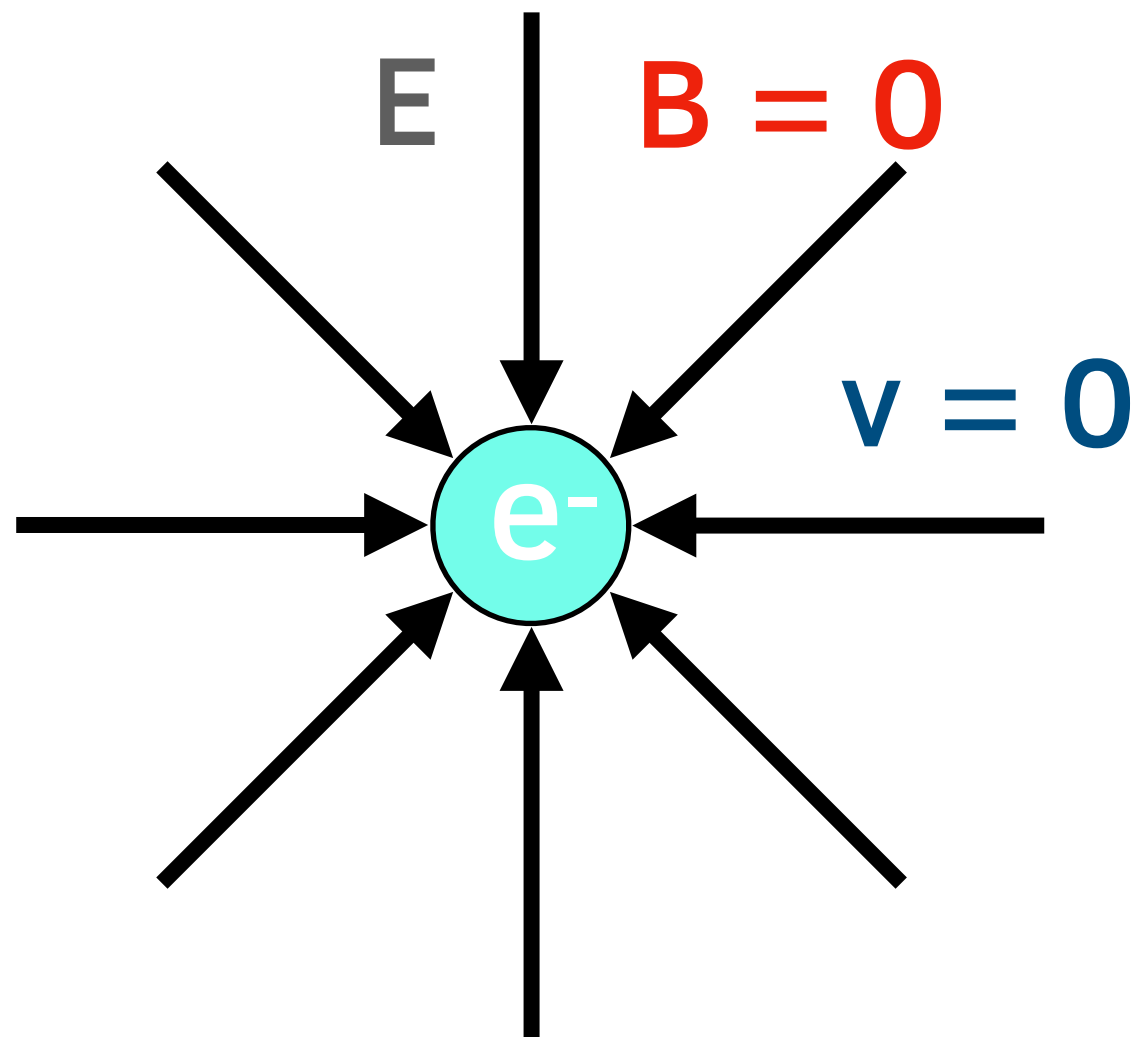
$$\frac{d\bar{\mathbf{u}}_p}{dt} = \left( \bar{\mathbf{E}}_p + \frac{\bar{\mathbf{u}}_p}{\bar{\gamma}_p} \times \bar{\mathbf{B}}_p \right) - \frac{1}{4\bar{\gamma}_p} \nabla \left( |\tilde{A}_p|^2 \right)$$

Lorentz Force  
(plasma fields)
+ Ponderomotive Force  
(laser envelope)

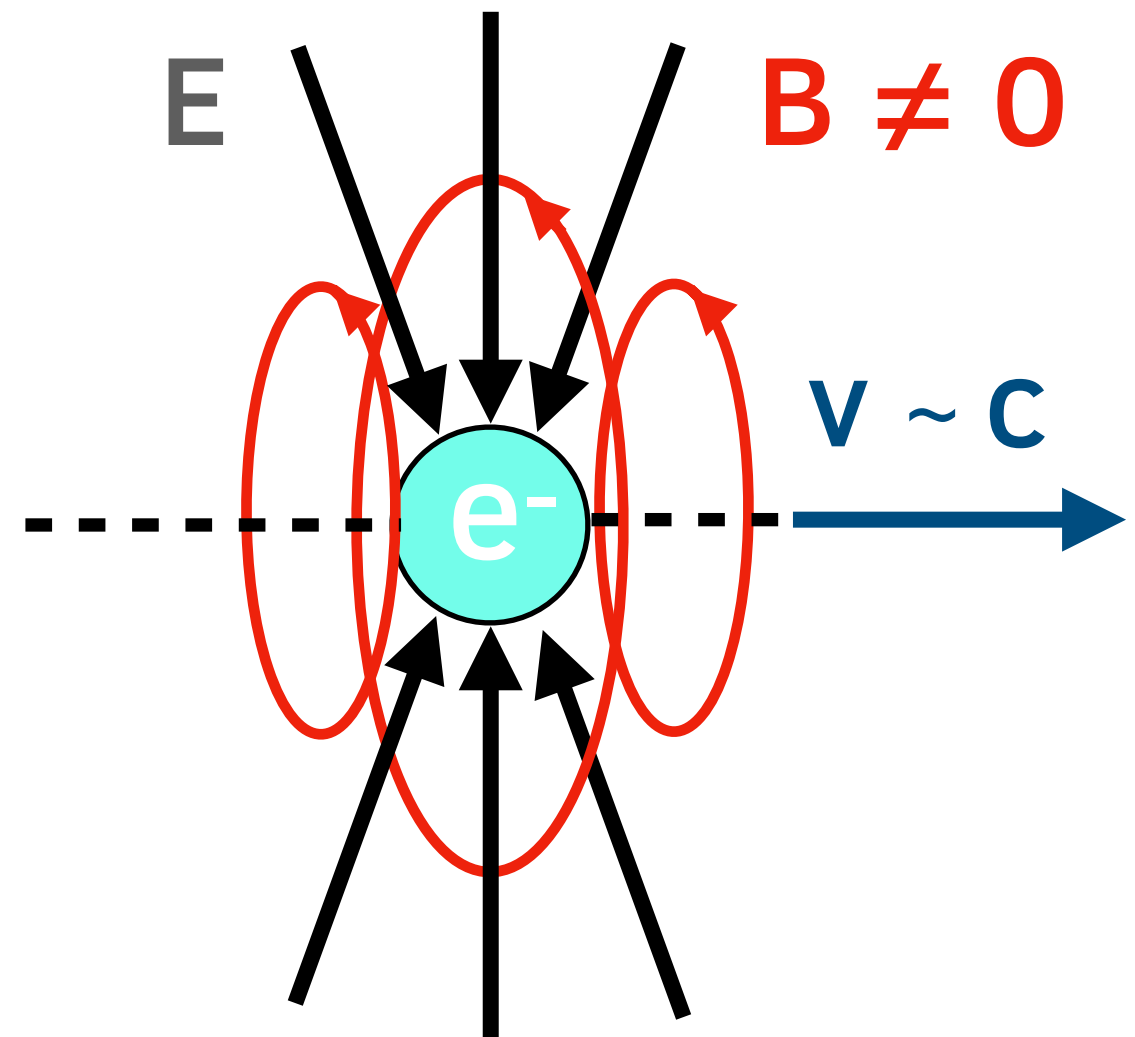
B. Quesnel and P. Mora,  
Physics Review E 58,  
3719 (1998)

# Electromagnetic field initialization: Relativistic electron

$\gamma_0 = 1$   
(immobile electron)



$\gamma_0 = 200$   
( $\sim 100$  MeV)



# Initialisation of Electromagnetic Fields

## Immobiles Species: Poisson's Equation

$$\nabla^2 \Phi = -\rho$$

## Relativistic Species: “Relativistic” Poisson's Equation

$$\left( \frac{1}{\gamma_0^2} \partial_x^2 + \nabla_{\perp}^2 \right) \Phi = -\rho$$

$$\mathbf{E} = \left( -\frac{1}{\gamma_0^2} \partial_x \Phi, -\partial_y \Phi, -\partial_z \Phi \right)$$

$$\mathbf{B} = \frac{\beta_0}{c} \hat{\mathbf{x}} \times \mathbf{E}$$

Hypothesis:  
Negligible energy spread

If non-negligible energy spread:  
Repeat for each energy “slice”

J.-L. Vay, Physics of Plasmas 15, 056701 (2008)

P. Londrillo, C. Gatti and M. Ferrario, Nucl. Instr. and Meth. A 740, 236-241 (2014)

F. Massimo, A. Marocchino and A. R. Rossi, Nucl. Instr. and Meth. A 829, 378-382 (2016)