CHARACTERIZATION OF WAVEBREAKING TIME AND DISSIPATION OF WEAKLY NONLINEAR WAKEFIELDS DUE TO ION MOTION

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*THE REAL SCIENCE

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ESSENCE IN BRIEF

We suggest a novel method of characterizing the wave lifetime in numerical simulations quantitatively and study how the lifetime scales with the ion mass. It scales as the cubic root of the ion mass to charge ratio.

The novel method is based on the usage of energy flux in co-moving window. This flux $\Psi(t)$ can be defined under the condition of the quasistatic approximation: the drive beam evolves slowly, and the wakefield is almost stationary in the co-moving frame (x, y, ξ). The flux is the difference of the linear energy density multiplied by c and the usual energy flux in the laboratory frame. Its electromagnetic part is

$$\Psi_{em} = \int dS \left(\frac{c}{8\pi} \left(E^2 + B^2 \right) - \frac{c}{4\pi} \left[\vec{E} \times \vec{B} \right]_z \right)$$

where the integration is carried out over the transverse cross-section of the wake. The part associated with the kinetic energy arises from summing the contributions of individual particles that cross this section per unit time Δt

$$\Psi_p = \frac{1}{\Delta t} \sum_j \left(\gamma_j - 1 \right) m_j c^2$$

where m_j and γ_j are the mass and relativistic factor of particles. Alternatively, we can consider plasma species as fluids and calculate the fluid contribution to the kinetic energy

$$\Psi_f = \int dS \sum_{s=i,e} \left(n_s m_s c^2 (\gamma_s - 1)(c - v_{sz}) \right)$$

where n_s, m_s, $\vec{v}_s$ and γ_s are the number density, particle mass, average velocity, and relativistic factor of a species. The total flux

$$\Psi(\xi) = \Psi_{em} + \Psi_p$$

is constant in the absence of energy sources and sinks. The linear energy density

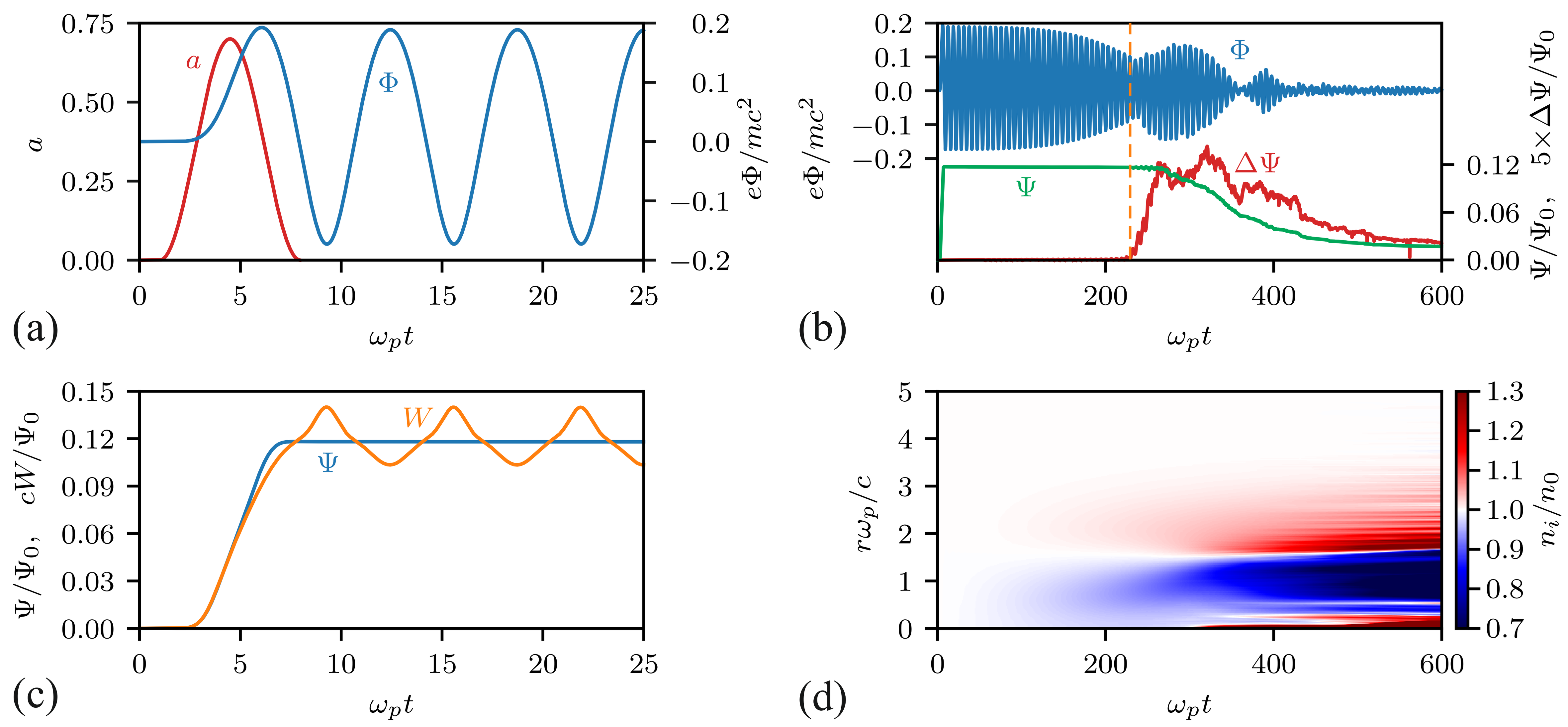
$$W = \int \frac{E^2 + B^2}{8\pi} dS + \frac{1}{\Delta z} \sum_j \left(\gamma_j - 1 \right) m_j c^2$$

is not constant because plasma wave causes longitudinal energy flows both into and out of the considered layer. As the wave breaks fluid and particle contribution to the energy flux start to differ because of multiple flows

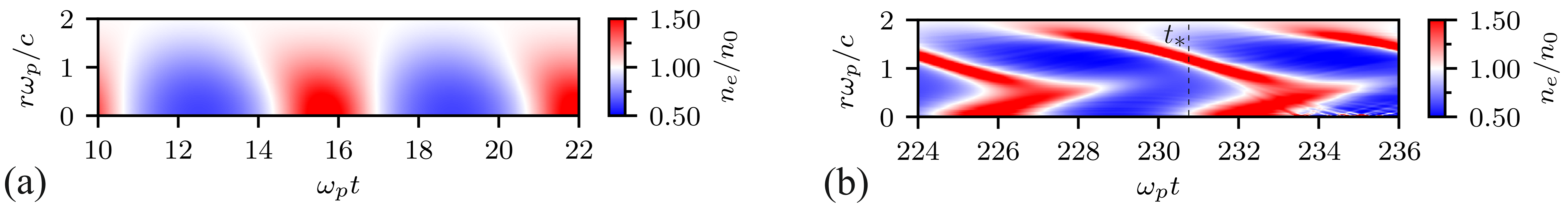
$$\Delta\Psi = \Psi_p - \Psi_f \neq 0.$$

We define the wave lifetime as the moment at which $\Delta\Psi$ exceeds 1% of the maximum value of $\Psi(t)$. This method is more practical than detection of particle trajectories' intersection, as it may be caused by initial plasma temperature of numerical plasma heating.

Parameters of simulation: helium plasma with $n_e = 2.5 \times 10^{18}$ cm^{-3}, laser strength parameter $a_0 = eE_0/(m_e \omega_0 c) = 0.7$, waist size of the laser pulse $\sigma = 2.27\, c/\omega_p$, pulse duration $\tau = 3.5\, \omega_p^{-1}$, laser frequency is $\omega_0 = 25.44\, \omega_p$, the simulation window is $10\, c/\omega_p$ wide, plasma dynamics is simulated up to $1000\, \omega_p^{-1}$, cell size is $0.01\, c/\omega_p$, time step is $0.01\, \omega_p^{-1}$, 10 particles per cell.

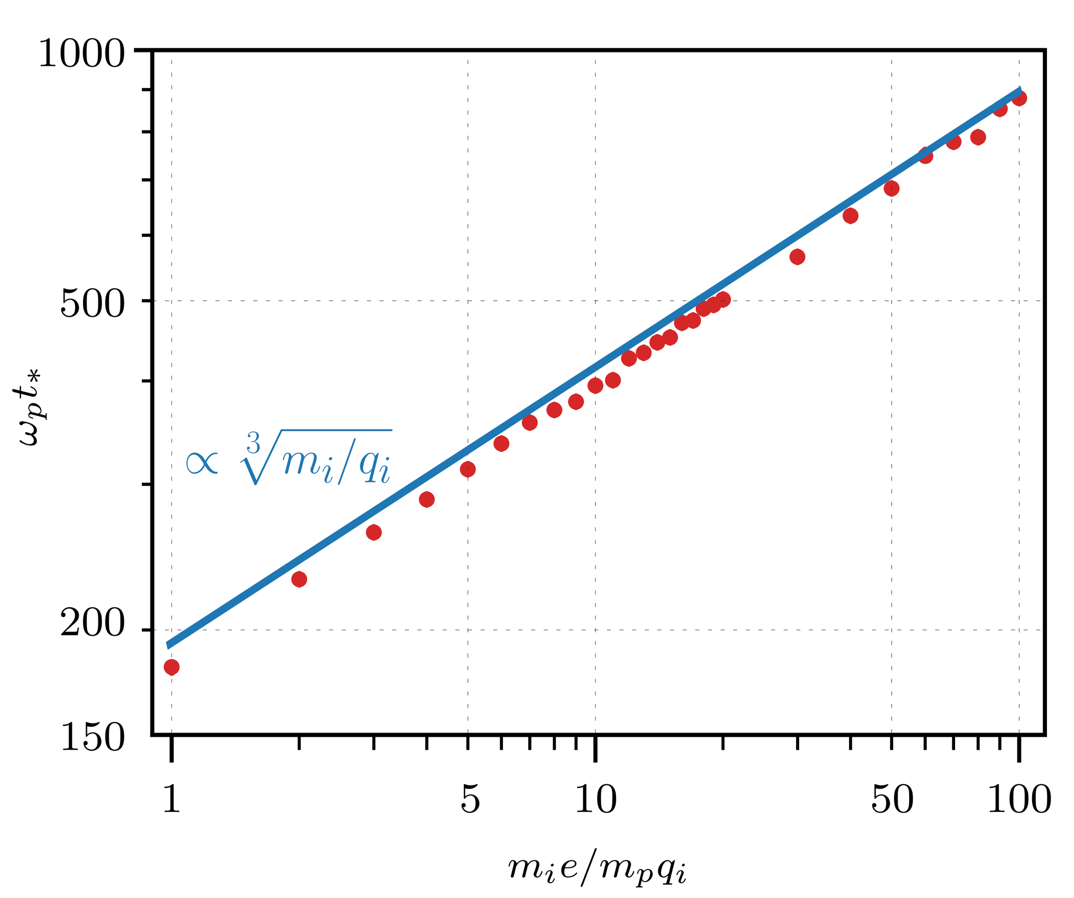


(a) The wakefield potential $\Phi(t)$ (blue) and the laser strength parameter $a(t)$ (red) during the first few oscillation periods.
(b) The wakefield potential $\Phi(t)$ (blue), the energy flux $\Psi(t)$ (green), and the difference $\Delta\Psi(t)$ enlarged 5 times (red) at a long time interval. The vertical dashed line indicates the onset of wave breaking. The energy flux unit $\Psi_0 = m_e^2 c^5/(4\pi e^2)$. (c) The energy flux $\Psi(t)$ (blue) and the linear density $W(t)$ (orange) during the first few oscillation periods. (d) The radial distribution of the ion density versus time.



Radial distribution of the electron density n_e after the driver passage (a) and near the wavebreaking moment $t*$ (b). The perturbation of the ion density results in different electron oscillation frequencies at different radial positions and, therefore, distortion of phase fronts. At the moment $t*$ of wavebreaking, the relative phase shift between the areas of the lowest ion density (at some $r \lesssim c/\omega_p$) and unperturbed density (at large r) approaches 2π.

HOW DOES THE LIFETIME OF THE PLASMA WAVE DEPEND ON THE ION MASS



The wavebreaking time $t*$ versus ion mass-to-charge ratio m_i/q_i (in units of proton mass-to-charge ratio m_p/e) obtained from theory (solid line) and simulations (points).

Explanation of $m_i^{1/3}$ scaling

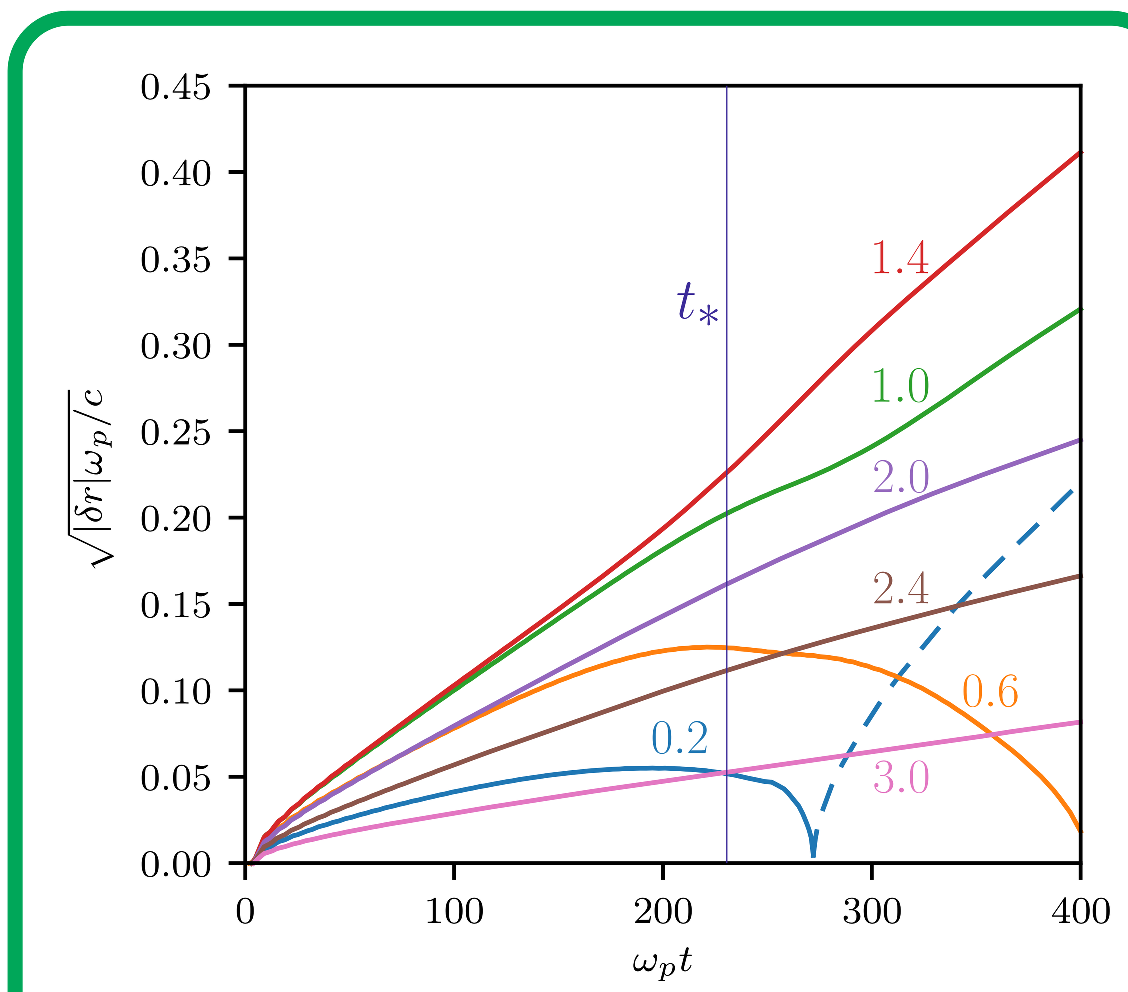
The radial force acting on the ions does not depend on the ion mass. This force of charge separation field is equal to the ponderomotive force exerted on electrons. It linearly accelerates the ions in the positive r direction and changes their positions by $\delta r \propto t^2/m_i$. Despite the fact that the ions experience different acceleration at different radii, the change in the ion density δn_i and the elongation of the wave period $\delta\lambda_p$ have the same scaling $|\delta n_i| \propto \delta\lambda_p \propto t^2/m_i$ as long as the perturbations remain small. The wave breaks when the cumulative phase advance becomes of the order of 2π, or if

$$\int_0^{t*} \delta\lambda_p(t)dt \propto \frac{t^3}{m_i}$$

reaches some threshold value, whence

$$t* \propto m_i^{1/3}$$

After the wave breaks, the field structure changes, and near-axis ions reverse the direction of motion and proceed to form a density increase near the axis.



Time dependencies of ion displacement. Numbers next to the curves correspond to initial radii of ions in units of c/ω_p.

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