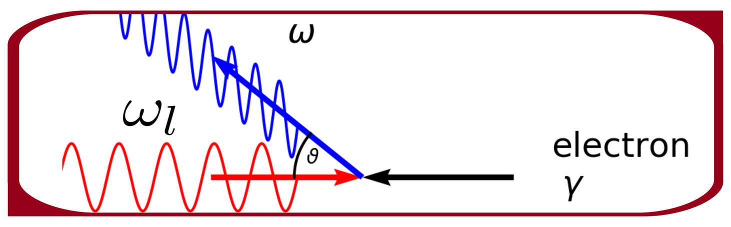
Matching laser frequency to electron energy for a Thomson source

Marcel Ruijter[1,2,3]
Vittoria Pettrilo[2,3]
Matt Zepf[4]

1) Università La Sapienza Roma,
2) Università Degli Studi di Milano,
3) INFN Milano
4) Helmholtz Institute Jena

Introduction to Thomson sources

A Thomson source is based upon the scattering of photons on electrons. In the event of a collision a low energy photon is converted into a high energetic one due to the transfer of *energy* and *momentum*.

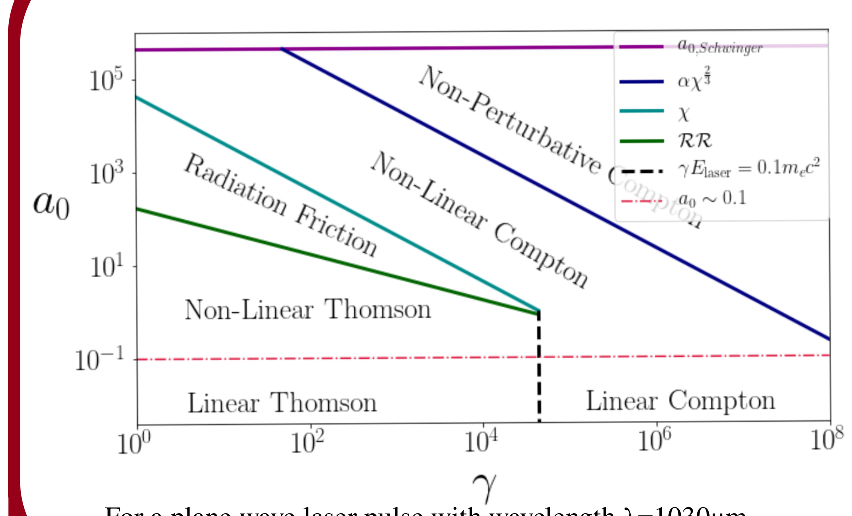


$$\frac{\omega}{c} = \omega_l \frac{\gamma^2(1+\beta)^2}{1+(a)^2}$$

A Thomson source is the *low energy regime* of inverse Compton scattering, where the whole process can be described using *classical electrodynamics* [2].

Up to the Non-linear Thomson (NLT) regime the equation of motion for the electron is described by the *Lorentz force*. Then by using *Liénard–Wiechert potentials* the emitted radiation is calculated.

The different energy regimes for the scattering event [2-7]



For a plane wave laser pulse with wavelength λ=1030μm , with a top hat temporal profile of length L_{pulse}= 160μm

$$a_0 = \frac{eA_0}{mc^2}$$ Laser strength parameter

The double differential for calculating the spectrum [1]

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\omega\int_{-\infty}^{\infty} dt\ \hat{n}\times\hat{n}\times\vec{\beta}\exp\left[i\,\frac{\omega}{c}(ct-\hat{n}\cdot\vec{r})\right]\right|^2$$

Matching of laser frequency to electron energy

The double differential can be solved analytically (for some specific cases) using the *stationairy phase approximation* (SPA). Through the SPA a relation is found for the emitted frequency during the collision. The relation can be generalized for a particle distribution.

$$\frac{\omega}{c} = \frac{\partial\eta}{\partial\zeta}\frac{\gamma^2(1+\beta)^2}{1+(a)^2}$$, where $\frac{\partial\eta}{\partial\zeta}$ is the instantaneous laser frequency.

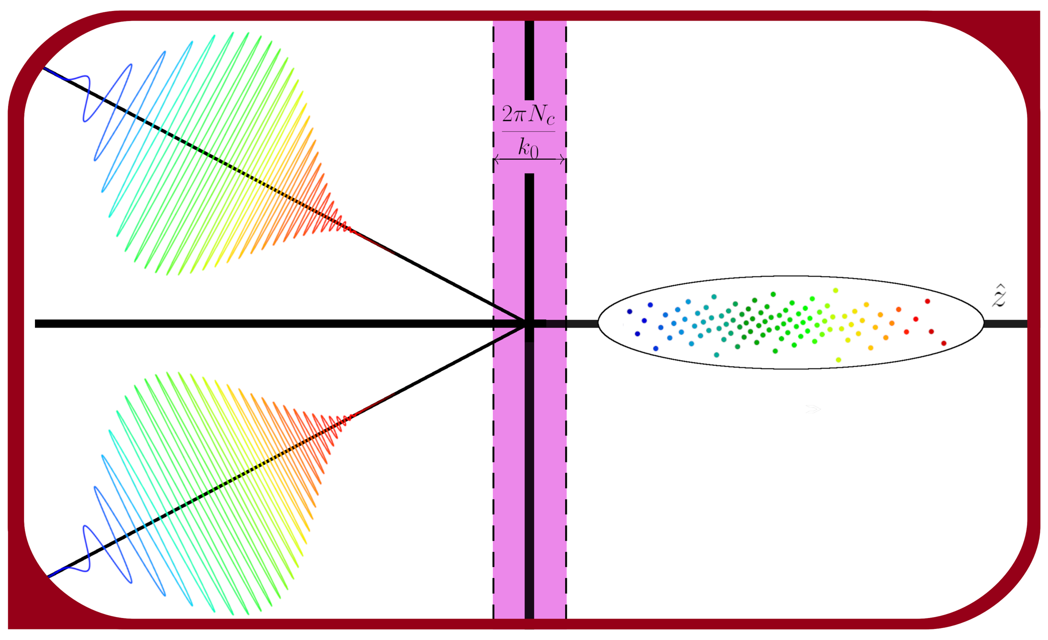
For an electron bunch with a *correlated energy spread* the emitted frequency can be held constant by introducing a *frequency modulation* on the laser pulse.

$$\frac{\partial\eta}{\partial\zeta} = \left(\frac{\langle\gamma\rangle}{\gamma}\right)^2 (1+(a)^2)\frac{\omega_{l,0}}{c}$$

This frequency modulation is than *linearized* to keep a simple solution. The *direction* of the correlation of the electron energy fixes the direction of the frequency modulation. Two geometries have been investigated according to the correlation of the electron energy: 1) along the *longitudinal*- and 2) along the *transverse* direction.

Longitudinal Chirp

The electron bunch needs to have its energy distribution along the axis of propagation. At the *interaction region* an electron experiences a *plane wave* with a frequency matched to its energy. This plane wave is achieved by *combining two chirped laser pulses* coming in under an *angle*.



- This scheme works in the *linear Thomson regime*. The *angle of incidence* is related to the *width* of the laser pulse and the *interaction length* (pink shaded area).

$$\alpha = \sin^{-1}\left(\frac{2\left(\frac{3}{\sqrt{2}}W_0\right)}{L_I}\right)$$

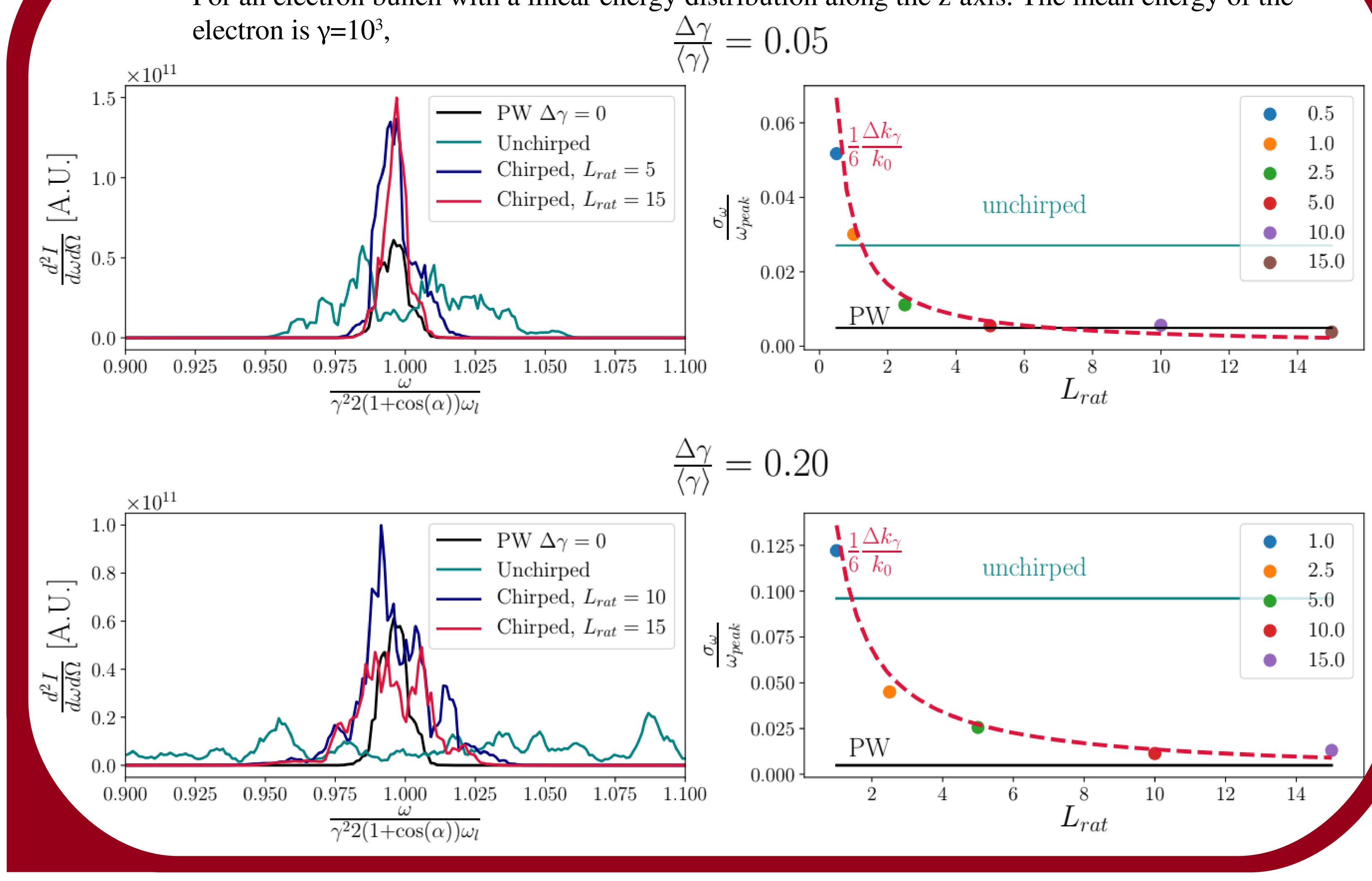
The bandwidth of the emitted radiation decreases as the electron bunch is stretched more because of the amount of frequencies a single electron experiences.

$$\frac{\Delta k_\gamma}{k_0} = \left(\left(\frac{\langle\gamma\rangle}{\gamma_{front}}\right)^2 - \left(\frac{\langle\gamma\rangle}{\gamma_{rear}}\right)^2\right)\frac{1+\beta}{\beta}\frac{1}{L_{rat}}$$

$$\frac{\sigma_{\omega,LC}}{\omega} = \sqrt{\left(\Theta + \frac{\sigma_\epsilon}{\sigma_{W_{bunch}}}\right)^2 + \left(\frac{\Delta k_{\gamma,\epsilon}}{6k_{laser}}\right)^2_{LC} + \left(\frac{\sigma_{\omega_{l,0}}}{\omega_{l,0}}\right)^2 + \left(\frac{a_0^2/3}{1+a_0^2}\right)^2}$$ Bandwidth for the longitudinal chirp scheme

On axis emitted radiation for longitudinal chirp

For an electron bunch with a linear energy distribution along the z-axis. The mean energy of the electron is γ=10³,



Bandwidth Dependency

Laser
- Natural bandwidth laser
- Intensity of laser pulse

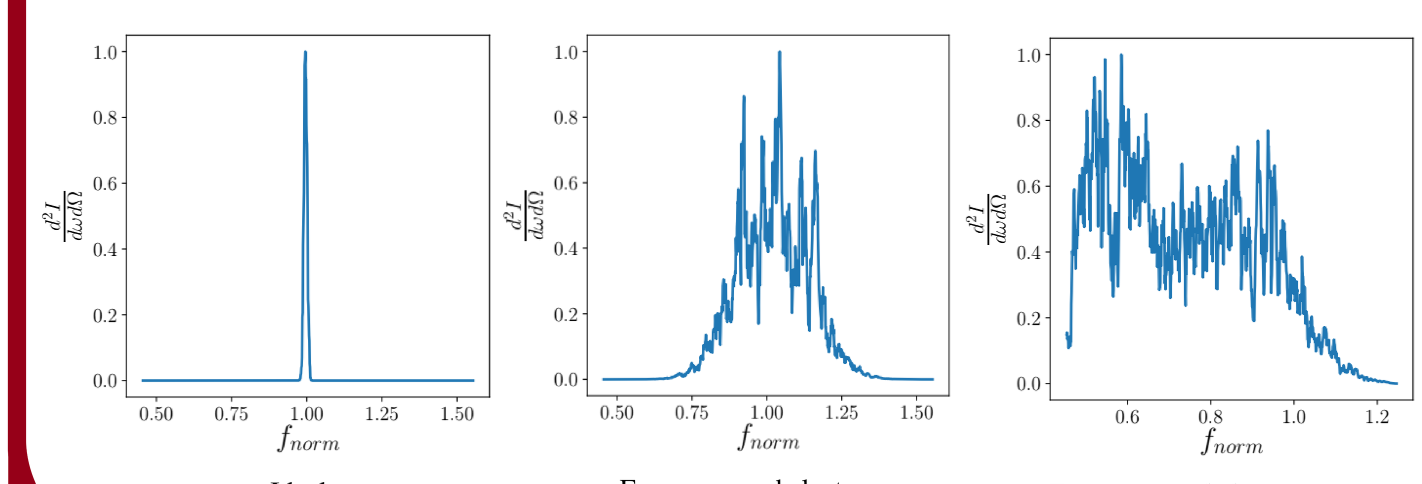
Electron beam
- Energy spread electrons
- Transverse momentum electrons

Other
- Acceptance Angle for emitted radiation

General bandwidth dependency [8]

$$\frac{\sigma_\omega}{\omega} = \sqrt{\left(\Theta + \frac{\sigma_\epsilon}{\sigma_{W_{bunch}}}\right)^2 + \left(2\frac{\sigma_\gamma}{\gamma}\right)^2 + \left(\frac{\sigma_{\omega_l}}{\omega_l}\right)^2 + \left(\frac{a_0^2/3}{1+a_0^2}\right)^2}$$

Examples of bandwidth dependency for on axis radiation

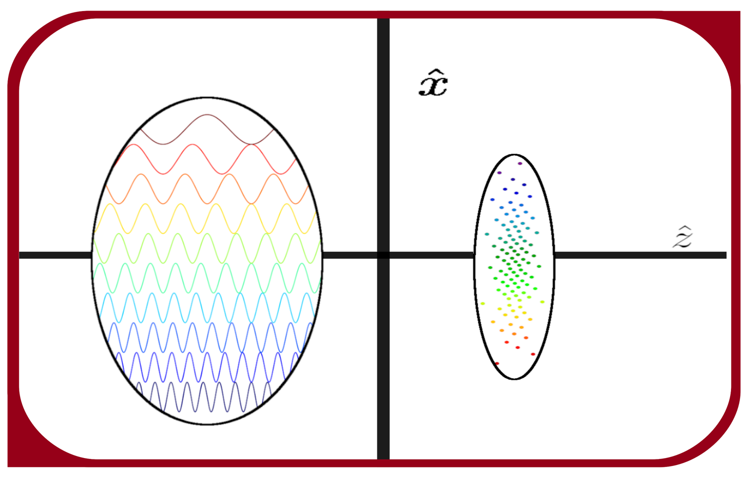


Transverse Chirp

This scheme requires an electron bunch with its energy distribution in a transverse direction (here along the x-axis).

This scheme can operate in both the *linear*- and *non-linear Thomson regime*. The latter is achieved by imposing a non-linear chirp as given in [9, 10] along the z-axis.

The frequencies experienced by a single electron in the linear regime depends amongst others on its transverse momentum.

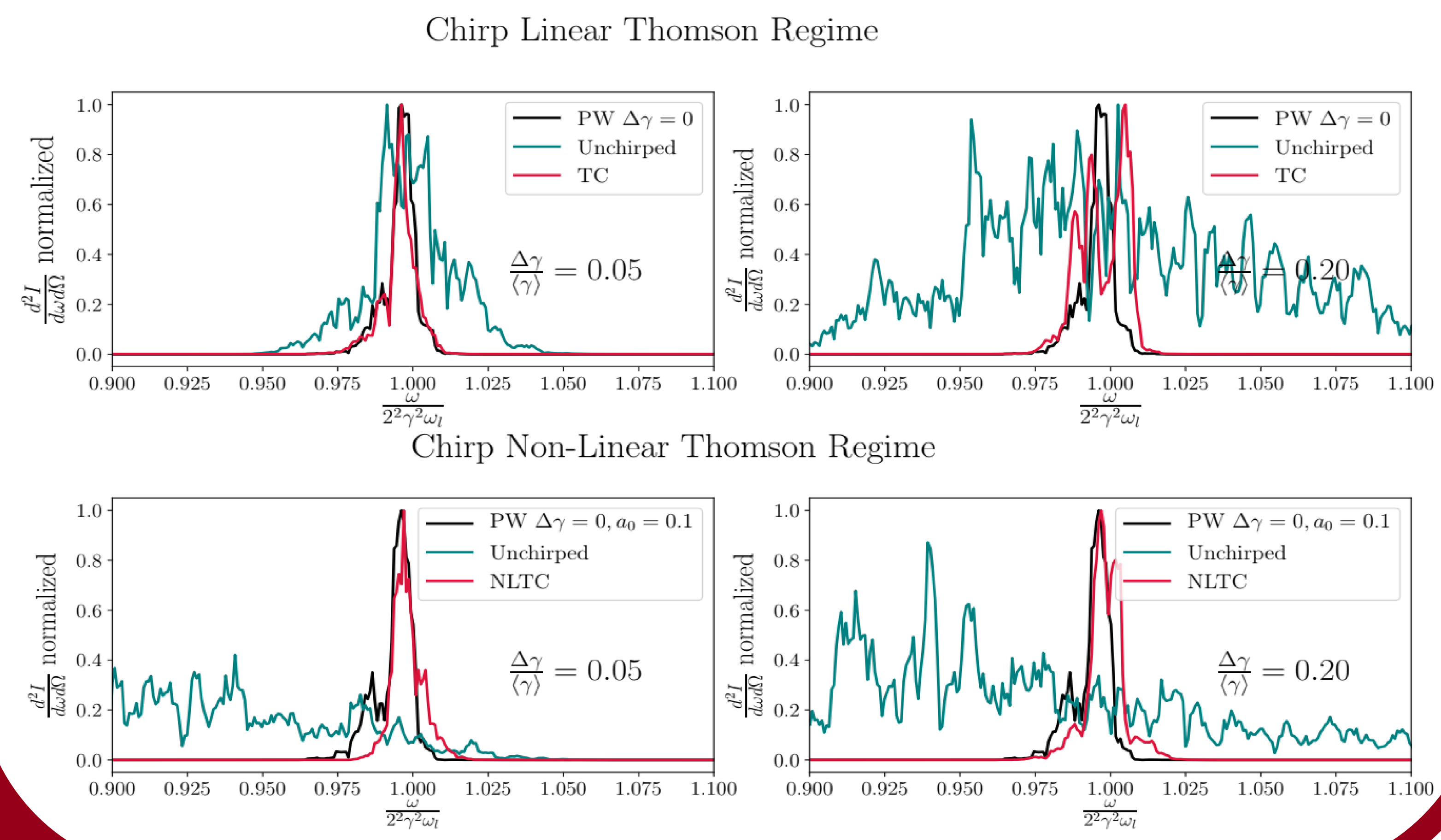


$$\Delta k_\epsilon = \left(\left(\frac{\langle\gamma\rangle}{\gamma_{top}}\right)^2 - \left(\frac{\langle\gamma\rangle}{\gamma_{bottom}}\right)^2\right)\frac{L_I}{W_{bunch}}\frac{\beta_x}{\beta_z}$$

Bandwidth for the transverse chirp scheme $$\frac{\sigma_{\omega,LC}}{\omega} = \sqrt{\left(\Theta + \frac{\sigma_\epsilon}{\sigma_{W_{bunch}}}\right)^2 + \left(\frac{\Delta k_{\gamma,\epsilon}}{6k_{laser}}\right)^2_{TC} + \left(\frac{\sigma_{\omega_{l,0}}}{\omega_{l,0}}\right)^2}$$

On axis emitted radiation for transverse chirp

For an electron bunch with a linear energy distribution along the x-axis. The mean energy of the electron is γ=10³,



Conclusion

A general relation to match the laser frequency to the electron energy is found by taking the SPA of the classical formula for the energy radiated per unit solid angle per unit frequency. By correlating the energy spread of the electron bunch along an axis, a geometry for the scattering can be devised.

The two schemes investigated are the longitudinal and transverse chirp. In both schemes the bandwidth of the emitted radiation for an electron bunch with an energy spread can be reduced down to the monochromatic case.

Matching the laser frequency to the electon energy is especially interesting for electron bunches with large energy spread, like the ones created with laser wake field acceleration.

References

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Marcel Ruijter
Marcel.Ruijter@mi.infn.it