Numerical calculation of the Smith-Purcell radiation from dielectric laser acceleration (DLA) structures

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Why study the Smith-Purcell effect when working on DLA?

- Dielectric Laser Acceleration is the inverse S-P effect
- Due to the reciprocity theorem of electromagnetism, a good understanding of the S-P effect brings better understanding of how light accelerates electrons
- Electron beams from DLA may serve as natural sources of S-P radiation

→ Joel England’s talk

Aim of the present work

- Understand how to calculate the intensity of the S-P radiation using a frequency-domain solver (eg. Comsol)
- Verify the results against previously published data
- In perspective, calculate S-P radiation from DLA-compatible structures
• Electron beams from DLA may serve as natural radiation

→ Joel England’s talk

\[ \int \frac{dE}{dz} \]

Aim of the present work

• Understand how to calculate the intensity of the radiation using a frequency-domain solver (eg. Comsol)
If an electron passes close to the surface of a metal diffraction grating, the periodic motion of the charge induced in the grating causes electromagnetic radiation.

\[ \lambda = \frac{a}{m} \left( \frac{1}{\beta} - \cos \theta \right) \]

Decomposition of the electromagnetic field of a travelling charge into evanescent waves – work of Toraldo di Francia (1960) [2]

Di Francia treats the Smith-Purcell and Cherenkov radiation within the same formalism: the field of a travelling charge is expanded into a superposition of evanescent waves which are reflected and refracted at the boundary.

On the Theory of some Čerenkovian Effects (*)

G. Toraldo di Francia

Istituto di Fisica della Radiazione - Università di Firenze

(received il 21 Dicembre 1959)

Summary. - The field generated by a charged particle in uniform straight motion is expanded into a set of evanescent waves. The expansion is valid in any half-space with no points in common with the path of the particle. The evanescent waves may impinge on the surface of an optical diffraction grating and be diffracted. Some of the diffracted waves turn out to be ordinary plane waves, which carry energy away from the grating. It is possible in this way to explain the Smith and Purcell effect and to derive some quantitative conclusions.
Calculation of the S–P radiation from a metallic grating \( (\sigma = 0) \) – the work of van den Berg (1973) [3]

- Method valid for arbitrary metal surface profile;
- Concrete numbers for sinusoidal surface profile – possible benchmark;
- Travelling line charge (sheet current pulse) can be treated within a 2D model.

Smith–Purcell radiation from a line charge moving parallel to a reflection grating

P. M. van den Berg

Department of Electrical Engineering, Division of Electromagnetic Research, Delft University of Technology, Delft, The Netherlands

(Received 16 November 1972)

A rigorous solution is obtained for the problem of radiation from an electric line charge that moves, at a constant speed, parallel to an electrically perfectly conducting grating. The relevant vectorial electromagnetic problem is reduced to a two-dimensional scalar one. With the aid of a Green’s-function formulation of the problem, an integral equation of the second kind for the surface current density on a single period of the grating surface is derived. This integral equation is solved numerically by a method of moments. Some numerical results pertaining to the radiation from a moving line charge above a sinusoidal grating are presented.

Another paper by van den Berg [4] similarly solves the problem for a travelling point charge.
S-P radiation from dielectric gratings in Yang et al. (2018) [5]

This is another possible benchmark. They use both 3D models with a point charge and 2D models with a line charge.

Yang et al. (2018) [5], Fig. 1e. (3D model) → → → →

Analysis of units:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>SI unit</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{d\Gamma}{dx} / \hbar )</td>
<td>[Meter^{-1} * Joule^{-1}]</td>
<td>? (any ideas?)</td>
</tr>
<tr>
<td>( \frac{d\Gamma}{dx} )</td>
<td>[Second * Meter^{-1}]</td>
<td>Spectral probability of photon emission per electron per unit travelled length</td>
</tr>
<tr>
<td>( \hbar \omega \frac{d\Gamma}{dx} )</td>
<td>[Joule * Second * Meter^{-1}]</td>
<td>Energy emitted by the electron per unit travelled length per unit of angular frequency</td>
</tr>
<tr>
<td>( \hbar \omega \frac{d\Gamma}{dx} \Delta \omega )</td>
<td>[Joule * Meter^{-1}]</td>
<td>Energy emitted by the electron per unit travelled length in the frequency range ((\omega, \omega + \Delta \omega))</td>
</tr>
<tr>
<td>( \int \hbar \omega \frac{d\Gamma}{dx} d\omega )</td>
<td>[Joule * Meter^{-1}]</td>
<td>Energy emitted by the electron per unit travelled length</td>
</tr>
<tr>
<td>dx</td>
<td>Emission per electron per unit travelled length</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-----------------------------------------------</td>
<td></td>
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Energy emitted by the electron per unit travelled length per unit of angular frequency

Energy emitted by the electron per unit travelled length in the frequency range $(\omega, \omega + \Delta \omega)$

Energy emitted by the electron per unit travelled length

\[ \Delta \omega \cdot \frac{2 \pi}{\Delta \omega} \cdot \frac{1}{\lambda p} \cdot \frac{1}{\Delta \omega} \] \[ \frac{\omega^2}{m} \cdot \frac{1}{\pi} \cdot \frac{1}{\Delta \omega} \] \[ \frac{\omega^2}{m} \cdot \frac{1}{\pi} \cdot \frac{1}{\Delta \omega} \]

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Frequency domain calculation of S–P radiation

Constructing a 3D Comsol model

A point charge, forming a line current, requires a 3D model

| Au slab inside the unit cell | Line current in frequency domain [A] \( I_0 = \frac{1}{2\pi} e^{\exp(-j\omega z/v)} \Delta \omega \) [ \( \times \exp(j\omega t) \) ] | Radiated energy [W] through a surface \( \rightarrow \) convert to [J/m] |

2D model:
Reading values from Fig 2b. of Yang et al. (2018)
Constructing a 2D Comsol model

Expressions for current density $J_z$:

<table>
<thead>
<tr>
<th></th>
<th>Delta pulse</th>
<th>Gaussian pulse, duration $\sigma_t$</th>
</tr>
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<tbody>
<tr>
<td><strong>Time domain</strong></td>
<td>$J_z(\vec{r},t) = (-e) \delta(x) \delta(y) \delta(t-z/v_0)$ [A/m^2]</td>
<td>$J_z(\vec{r},t) = (-e) \delta(x) \delta(y) \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-(t-z/v_0)^2/2\sigma_t^2\right]$ [A/m^2]</td>
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<tr>
<td><strong>Frequency domain</strong></td>
<td>$J_z(\vec{r},\omega) = \frac{1}{2\pi i}(-e) \delta(x) \delta(y) \exp\left[-j(\omega/v_0)z\right]$ [C/m^2]</td>
<td>$J_z(\vec{r},\omega) = \frac{1}{2\pi}(-e) \delta(x) \delta(y) \exp\left[-j(\omega/v_0)z\right] \exp\left[-\sigma_t^2\omega^2/2\right]$ [C/m^2]</td>
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Note different units of $J_z(\vec{r},t)$ and $J_z(\vec{r},\omega)$.

Decomposition of a travelling charge into harmonic charge oscillations

Problem – a paradox: although an electron (localized charge density) radiates a finite amount of joules per unit travelled length [per unit frequency or total], a single Fourier component of this charge distribution appears to radiate with power which is constant in time (as FD software appears to tells us), meaning an infinite amount of joules per unit cell per unit frequency $\Delta\omega$.)
Resolution of the paradox: harmonics from the frequency range \((\omega, \omega + \Delta \omega)\) add up coherently only over a finite \(z\) range, so the energy in this range is finite → see Calculations

Set the sheet current:

\[
\begin{array}{c|c|c}
\text{Surface current density:} & x & y \\
J_0 & 0 & 0 \\
\end{array}
\]

Integrate the component of the Poynting vector \(S_y\) (emw.Poavy)
Calculate \(S_y \cdot \frac{2\pi}{\Delta \omega} / a / \Delta \omega\) → see Calculations

Expression:
\[
(((\text{emw.Poavy*1[\text{nm}]*(2*pi)/(2*pi/1[s])})/\text{gm_a})/(2*pi/1[s])/\text{photon_energy/\text{hbar_const})*(1[\text{nm}]^1[\text{eV}])}
\]

Moderately successful benchmarking ;-) 

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<th>Yang et al. (2018), Fig 2b</th>
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<td>(~ 1 \cdot 10^{-6} / \text{nm/eV})</td>
<td>(~ 20 \cdot 10^{-6} / \text{nm/eV})</td>
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Close in orders of magnitude, but 20-fold discrepancy. Any ideas for improvement?

Resonant enhancement of S-P radiation (see [5])

At certain \(\omega\), the field from the source current matches the eigenmode
Resolution of the paradox: harmonics from the frequency range \((\omega, \omega + \Delta\omega)\) add up coherently only over a finite \(z\) range, so the energy in this range is finite \(\rightarrow\) see Calculations.

Set the sheet current:

\[
J_{s,0} = \text{const}/2\pi i^{\omega} \exp(i2\pi \beta \omega / \hbar m_{\beta,\omega} c) x / [1 \text{nm}] (2\pi i / [1 \text{m}])
\]

Integrate the component of the Poynting vector \(S_y\) (emw.Poavy)

Calculate \(S_y \cdot \Delta \omega / c \Delta \omega \rightarrow\) see Calculations

Expression:

\[
(\text{emw.Poavy}^{1\text{nm}}) (2\pi i / [2\pi i / [1 \text{m}]]) / \text{photocurrent} / \hbar \text{bar const}^{1\text{nm}} [1 \text{eV}]
\]

Moderately successful benchmarking ;-) (Yang et al. 2018)

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Resonant enhancement of S-P radiation (see [5])

At certain \(\omega\), the field from the source current matches the eigenmode.
Calculations

Superposition of pure harmonic oscillations in time

\[ \text{current pulse } = j_2 = e^{j \omega (\frac{z}{v_0} - t)} = \int_{-\infty}^{\infty} \Delta \omega \cdot e^{-j \frac{\Delta \omega}{2} \omega \cdot \Delta \omega} \cdot e^{j \omega t} \]

divide the frequency range into \( \Delta \omega \) intervals

\[ j_2 = \sum \int_{\omega_i + \Delta \omega}^{\omega_i} \omega e^{j \omega (t - \frac{z}{v_0})} \cdot e^{j \omega t} \cdot \text{sinc} \left[ \frac{\Delta \omega}{2} (t - \frac{z}{v_0}) \right] \]

\[ = \sum e^{j (\omega_i + \frac{\Delta \omega}{2})} (t - \frac{z}{v_0}) \cdot \text{sinc} \left[ \frac{\Delta \omega}{2} (t - \frac{z}{v_0}) \right] \]

\[ \text{discrete superposition of harmonic oscillations in time} \]

freq. domain code will consider one such harmonic

\[ E \sim \int (j_2 \cdot \Delta \omega) \, dt \]

is arbitrarily large, depending on time of integration

energy from a (consol) f.d. calculation

necessary correction factor for the time-varying amplitude

\[ \int_{-\infty}^{\infty} \sin^2 \left[ \frac{\Delta \omega}{2} (t - \frac{z}{v_0}) \right] \, dt = \frac{2 \pi}{\Delta \omega} \]

now energy emitted per unit frequency is finite even for time integration over infinite time \( \int_{-\infty}^{\infty} \)
Acknowledgements

I am grateful to Joel England, Levi Schächter and Avraham Gover for helpful comments and inspiring discussions.

Literature


