Numerical calculation of the Smith-Purcell radiation from dielectric laser acceleration (DLA) structures

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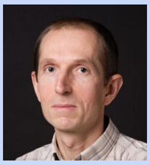

If during the poster session I am not here,
I may be looking at other DLA posters :-)

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Why study the Smith-Purcell effect when working on DLA?

- Dielectric Laser Acceleration is the *inverse* S-P effect
- Due to the reciprocity theorem of electromagnetism, a good understanding of the S-P effect brings better understanding of how light accelerates electrons
- Electron beams from DLA may serve as natural sources of S-P radiation

→ Joel England's talk

139. Compact Radiation Sources Using Dielectric Laser Accelerators
Joel England (SLAC)
17/09/2019, 18:40
WG3-WG4 Joint Session | talk | WG3-WG4 Joint Session

Aim of the present work

- Understand how to calculate the intensity of the S-P radiation using a frequency-domain solver (eg. Comsol)
- Verify the results against previously published data
- In perspective, calculate S-P radiation from DLA-compatible structures

- Electron beams from DLA may serve as natur
radiation

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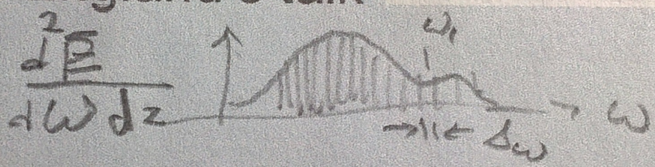
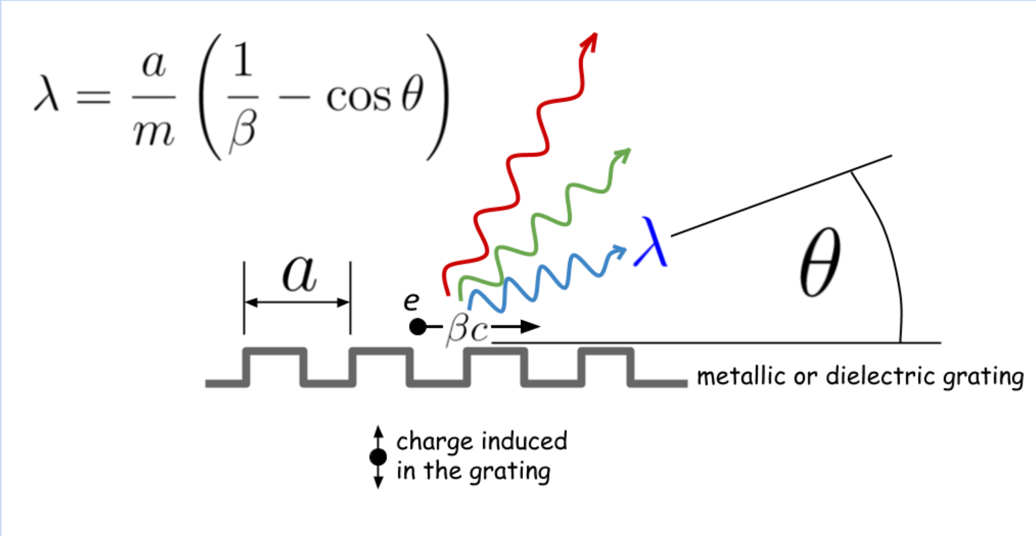


Aim of the present work

- Understand how to calculate the intensity of the
using a frequency-domain solver (eg. Comsol)

Original paper of S.J. Smith and E.P. Purcell (1953) [1]

If an electron passes close to the surface of a metal diffraction grating, the periodic motion of the charge induced in the grating causes electromagnetic radiation.



Decomposition of the electromagnetic field of a travelling charge into evanescent waves – work of Toraldo di Francia (1960) [2]

Di Francia treats the Smith-Purcell and Cherenkov radiation within the same formalism: the field of a travelling charge is expanded into a superposition of evanescent waves which are reflected and refracted at the boundary.

On the Theory of some Čerenkovian Effects (*).

G. TORALDO DI FRANCIA

Istituto di Fisica della Radiazione - Università di Firenze

(ricevuto il 21 Dicembre 1959)

Summary. - The field generated by a charged article in uniform straight motion is expanded into a set of evanescent waves. The expansion is valid in any half-space with no points in common with the path of the particle. The evanescent waves may impinge on the surface of an optical diffraction grating and be diffracted. Some of the diffracted waves turn out to be ordinary plane waves, which carry energy away from the grating. It is possible in this way to explain the Smith and Purcell effect and to derive some quantitative conclusions.

Calculation of the S–P radiation from a metallic grating ($\sigma = 0$) – the work of van den Berg (1973) [3]

- method valid for arbitrary <u>metal</u> surface profile;
- concrete numbers for sinusoidal surface profile – possible benchmark
- travelling line charge (sheet current pulse) can be treated within a 2D model

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Smith–Purcell radiation from a line charge moving parallel to a reflection grating

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(Received 16 November 1972)

A rigorous solution is obtained for the problem of radiation from an electric line charge that moves, at a constant speed, parallel to an electrically perfectly conducting grating. The relevant vectorial electromagnetic problem is reduced to a two-dimensional scalar one. With the aid of a Green's-function formulation of the problem, an integral equation of the second kind for the surface current density on a single period of the grating surface is derived. This integral equation is solved numerically by a method of moments. Some numerical results pertaining to the radiation from a moving line charge above a sinusoidal grating are presented.

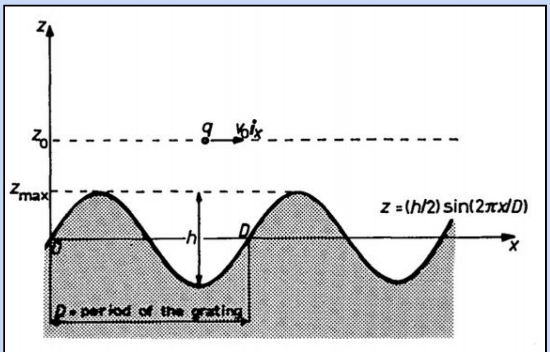
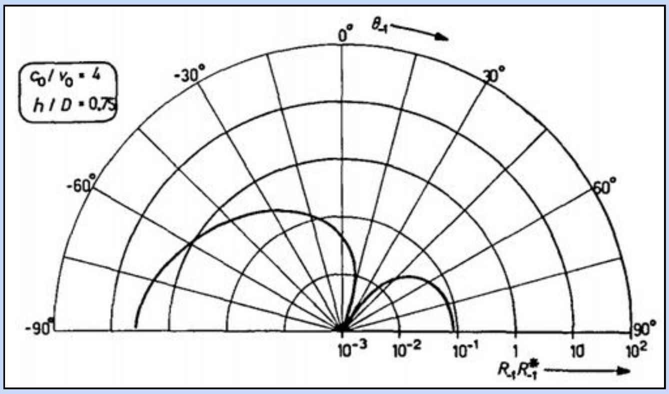


FIG. 4. Moving line charge above a sinusoidal grating.

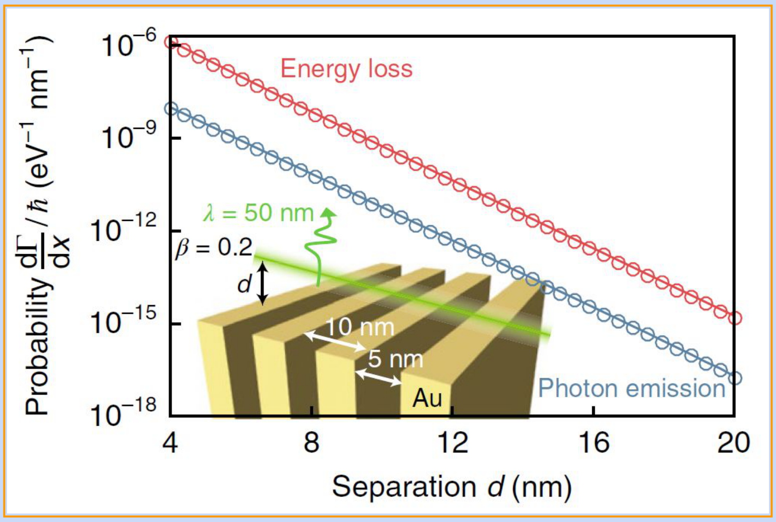


Another paper by van den Berg [4] similarly solves the problem for a travelling point charge.

S-P radiation from dielectric gratings in Yang *et al.* (2018) [5]

This is another possible benchmark. They use both 3D models with a point charge and 2D models with a line charge.

Yang et al. (2018) [5],
Fig. 1e. (3D model) →→→→



Analysis of units:

Quantity	SI unit	Interpretation
$\frac{d\Gamma}{dx}/\hbar$	[Meter^-1 * Joule^-1]	? (any ideas?)
$\frac{d\Gamma}{dx}$	[Second * Meter^-1]	Spectral probability of photon emission per electron per unit travelled length
$\hbar\omega\frac{d\Gamma}{dx}$	[Joule * Second * Meter^-1]	Energy emitted by the electron per unit travelled length per unit of angular frequency
$\hbar\omega\frac{d\Gamma}{dx}\Delta\omega$	[Joule * Meter^-1]	Energy emitted by the electron per unit travelled length in the frequency range $(\omega,\ \omega+\Delta\omega)$
$\int\hbar\omega\frac{d\Gamma}{dx}d\omega$	[Joule * Meter^-1]	Energy emitted by the electron per unit travelled length

dx		emission per electron per unit travelled length
$\hbar\omega \frac{d\Gamma}{dx}$	[Joule * Second * Meter^-1]	Energy emitted by the electron per unit travelled length per unit of angular frequency
$\hbar\omega \frac{d\Gamma}{dx} \Delta\omega$	[Joule * Meter^-1]	Energy emitted by the electron per unit travelled length in the frequency range $(\omega,\ \omega + \Delta\omega)$
$\int \hbar\omega \frac{d\Gamma}{dx} d\omega$	[Joule * Meter^-1]	Energy emitted by the electron per unit travelled length

$\left[\frac{J}{m}\right]$

$\Delta\omega \cdot P_{sim} \cdot \frac{2\pi}{\Delta\omega} \cdot \frac{1}{\lambda_p} \frac{1}{\Delta\omega} \Big\} \frac{J}{m \cdot \frac{1}{s}}$

$\int_{\omega_1}^{\omega_2} [\,] d\omega$ → $\frac{J}{m}$

$W \cdot s^2/m \sim \frac{J}{s\, m} = \frac{W}{m}$

$\frac{P_{sim}}{\Delta\omega} \cdot 2\pi$

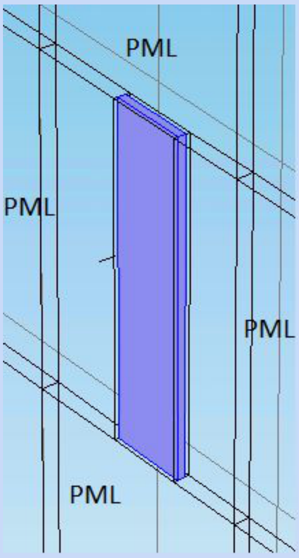
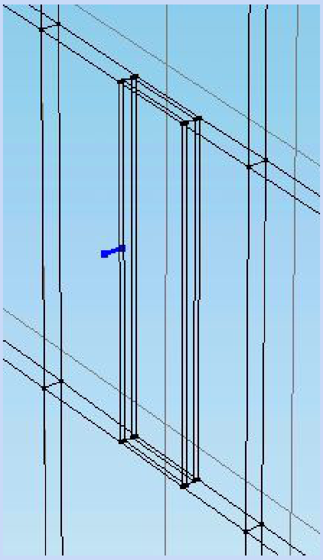
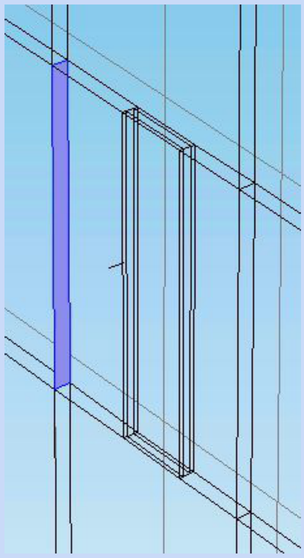
$\omega_2 = \omega_1 + \Delta\omega$

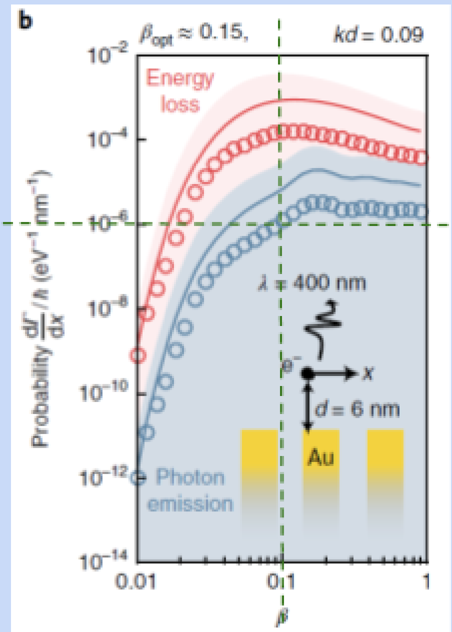
4

Frequency domain calculation of *S*–P radiation

Constructing a 3D Comsol model

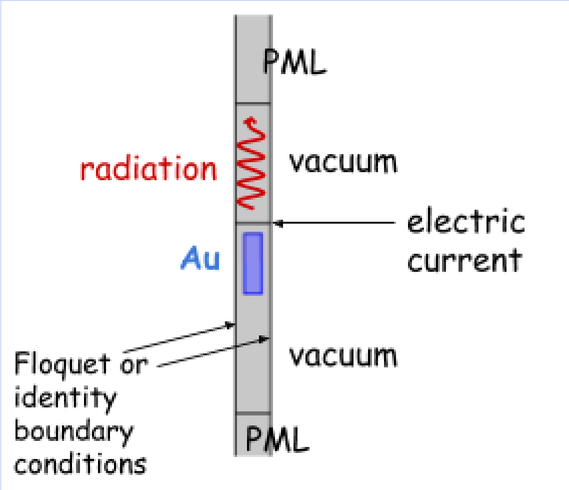
A point charge, forming a line current, requires a 3D model

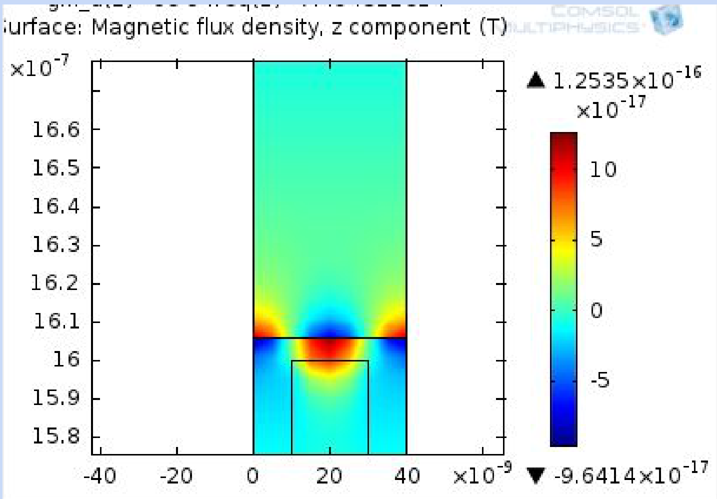
Au slab inside the unit cell	Line current in frequency domain [A] $I_0 = \frac{1}{2\pi} e \exp(-j\omega z/v)\ \Delta\omega$ $[\times \exp(j\omega t)]$	Radiated energy [W] through a surface → convert to [J/m]
		



2D model:
Reading values from Fig 2b. of Yang *et al.* (2018)

Constructing a 2D Comsol model



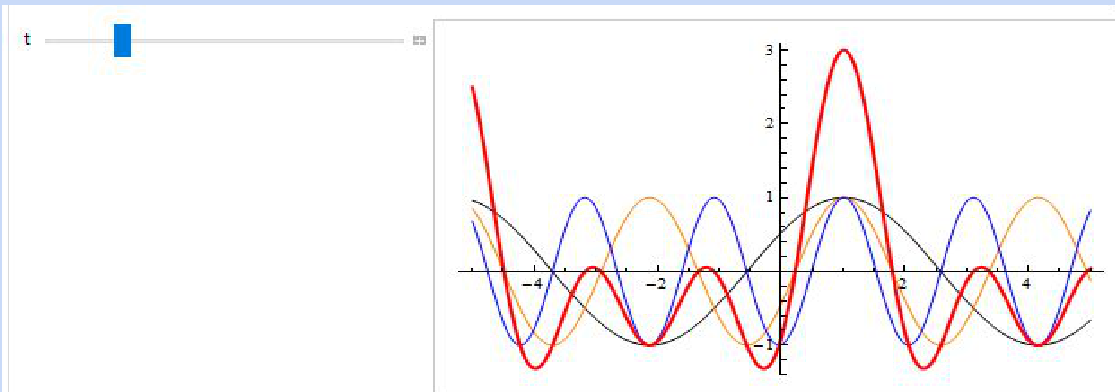


Expressions for current density J_z :

	Delta pulse	Gaussian pulse, duration σ_t
Time domain	$J_z(\vec{r},t) = (-e)\,\delta(x)\,\delta(y)\,\delta(t - z/v_0)$ [A/m^2]	$J_z(\vec{r},t) = (-e)\,\delta(x)\,\delta(y)\,\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_t}\exp[-(t - z/v_0)^2/2\sigma_t^2]$ [A/m^2]
Frequency domain	$J_z(\vec{r},\omega) = \frac{1}{2\pi}(-e)\,\delta(x)\,\delta(y)\,\exp[-j\,(\omega/v_0)\,z]$ [C/m^2]	$J_z(\vec{r},\omega) = \frac{1}{2\pi}(-e)\,\delta(x)\,\delta(y)\,\exp[-j\,(\omega/v_0)\,z]\,\exp[-\sigma_t^2\omega^2/2]$ [C/m^2]

Note different units of $J_z(\vec{r},t)$ and $J_z(\vec{r},\omega)$.

Decomposition of a travelling charge into harmonic charge oscillations ■ ■ ■



Problem – a paradox: although an electron (localized charge density) radiates a finite amount of joules per unit travelled length [per unit frequency or total], a single Fourier component of this charge distribution appears to radiate with power which is constant in time (as FD software appears to tells us), meaning an infinite amount of joules per unit cell per unit frequency $\Delta\omega$)

Resolution of the paradox: harmonics from the frequency range $(\omega,\ \omega + \Delta\omega)$ add up coherently only over a finite z range, so the energy in this range is finite → see **Calculations**

```
Integrate[(Sinc[Δω t / 2])^2, {t, -∞, ∞}]
```

$$\frac{2\pi}{\Delta\omega}$$

Set the sheet current:

Surface current density:

J_{s0}	e_const/(2*pi)*exp(-i*2*pi*f0/(bm_beta*c_const)*x)/1[nm]*(2*pi/1[s])	x	A/m
	0	y	
	0	z	

Integrate the component of the Poynting vector S_y (emw.Poavy)
Calculate $S_y \cdot \frac{2\pi}{\Delta\omega}/a/\Delta\omega$ → see **Calculations**

Expression:

```
(((emw.Poavy*1[nm])*(2*pi)/(2*pi/1[s])/gm_a)/(2*pi/1[s])/photon_energy/hbar_const)*(1[nm]*1[eV])
```

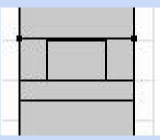
Moderately successful benchmarking ;-)

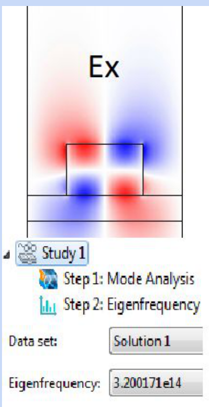
Yang et al. (2018), Fig 2b	This work
$\sim 1 \cdot 10^{-6}$ /nm/eV	$20 \cdot 10^{-6}$ /nm/eV

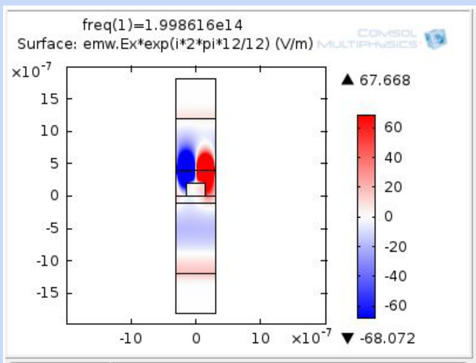
Close in orders of magnitude, but 20-fold discrepancy. Any ideas for improvement?

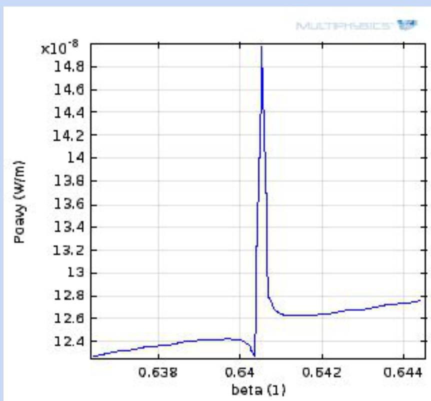
Resonant enhancement of S-P radiation (see [5])

At certain ω, the field from the source current matches the eigenmode









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```
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2π/Δω
```

Set the sheet current:

Surface current density:

J_{s0}	e_const/(2*pi)*exp(-i*2*pi*f0/(bm_beta*c_const)*x)/1[nm]*(2*pi/1[s])	x	A/m
	0	y	
	0	z	

$j(\omega, z)$; $\Delta\omega$

Integrate the component of the Poynting vector S_y (emw.Poavy)

Calculate $S_y \cdot \frac{2\pi}{\Delta\omega}/a/\Delta\omega$ → see **Calculations**

Expression:

```
(((emw.Poavy*1[nm])*(2*pi)/(2*pi/1[s])/gm_a)/(2*pi/1[s])/photon_energy/hbar_const)*(1[nm]*1[eV])
```

[W] ; sheet current ; a ; Δω ; Yang et al. 2018

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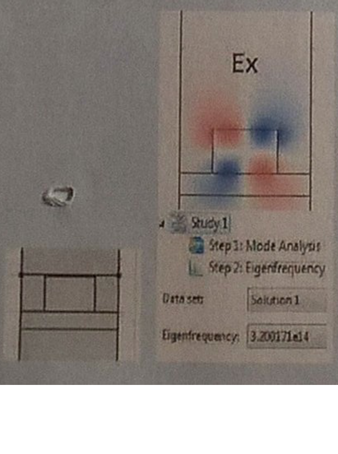
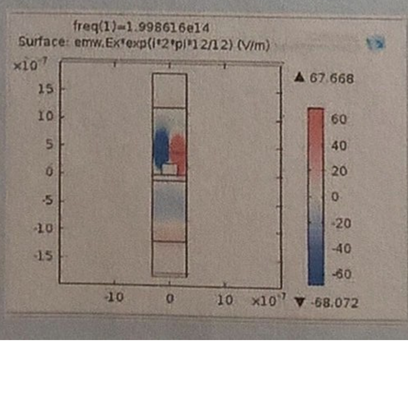
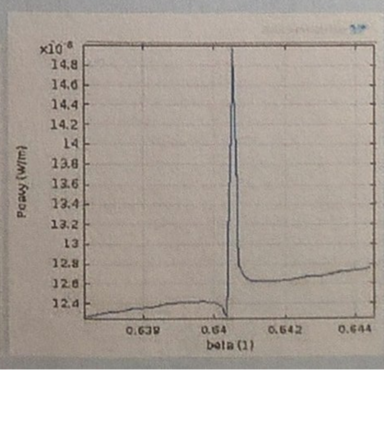




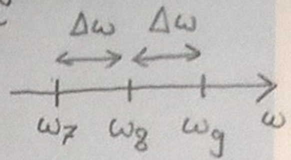


Calculations

superposition of pure harmonic oscillations in time

current pulse $= j_z = e \cdot \delta\left(\frac{z}{v_0} - t\right) = \int_{-\infty}^{\infty} d\omega \cdot e \cdot e^{-j\frac{z}{v_0}\omega} \cdot e^{j\omega t}$

divide the frequency range into $\Delta\omega$ intervals



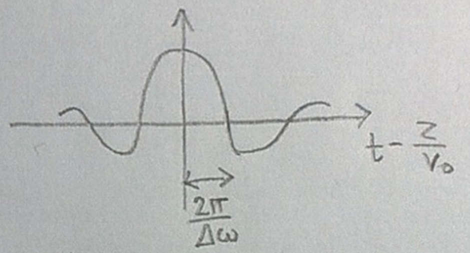
$$j_z = \sum_i \int_{\omega_i}^{\omega_i + \Delta\omega} d\omega \, e^{j\omega\left(t - \frac{z}{v_0}\right)} \cdot e = j_z(z, t)$$

$$= \sum_i e \cdot e^{j\left(\omega_i + \frac{\Delta\omega}{2}\right)\left(t - \frac{z}{v_0}\right)} (\cdot \Delta\omega) \cdot \mathrm{sinc}\left[\frac{\Delta\omega}{2}\left(t - \frac{z}{v_0}\right)\right]$$

discrete superposition of harmonic oscillations in time

modulated by amplitude slowly varying in time

freq. domain code will consider one such harmonic



energy from a (Comsol) f.d. calculation

$E \sim \int_{time} \left(j_z^2 \cdot \Delta\omega\right) dt$ is arbitrarily large, depending on time of integration

necessary correction factor for the time-varying amplitude

$$\int_{-\infty}^{\infty} \mathrm{sinc}^2\left[\frac{\Delta\omega}{2}\left(t - \frac{z}{v_0}\right)\right] dt = \frac{2\pi}{\Delta\omega}$$

now energy emitted per unit frequency is finite even for time integration over infinite time $\int_{-\infty}^{\infty} dt$

Acknowledgements

I am grateful to Joel England, Levi Schächter and Avraham Gover for helpful comments and inspiring discussions.

Literature

[1] S.J. Smith and E.P. Purcell, *Visible light from localized surface charges moving across a grating*, Phys. Rev. 92, 1069 (1953).

[2] Toraldo di Francia, On the theory of some Čerenkovian effects, Nouvo Cimento 16, 61 (1960).

[3] P.M. van den Berg, *Smith-Purcell radiation from a line charge moving parallel to a reflection grating*, JOSA 63, 689 (1973).

[4] P.M. van den Berg, *Smith-Purcell radiation from a point charge moving parallel to a reflection grating*, JOSA 63, 1588 (1973).

[5] Y. Yang *et al*, Maximal spontaneous photon emission and energy loss from free electrons, Nature Physics 14, 894 (2018).