New Analytical derivation of Group Velocity in TW accelerating structures

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Abstract

Ultra high-gradient accelerating structures are needed for the next generation of compact light sources. In the framework of the Compact Light XLS project, we are studying a high gain traveling wave accelerating structure operating at a frequency of 35.982 GHz, in order to investigate the longitudinal space phase. In this paper, we propose a new analytical approach for the estimation of the group velocity in the structure and compare it with numerical electromagnetic simulations that are carried out by using the code HFSS in the frequency domain.

Introduction

• The next generation of linear accelerators require unprecedented accelerating gradients for high energy physics and compact light sources.
• One of the main limitations to achieve ultra-high gradients, today around 100 MV/m [4], is the RF breakdown rate (BDR) which is defined as the number of breakdowns in the structure per unit time and length.
• Recently, an electromagnetic quantity called the modified Poynting vector has been demonstrated to be the main predictor for the BDR. It is defined as $\mathbf{S} = \mathbf{E} \times \mathbf{B}$, where $\mathbf{S}$ is the Poynting vector describing the RF power flow through the traveling wave accelerating structure. As a consequence, the BDR is strictly related to the group velocity $\alpha$, which is proportional to $\frac{1}{\lambda}$ [4, 5]. Therefore, the group velocity represents a crucial parameter to be characterized for each accelerating cavity [5].
• We propose here an innovative analytical approach to the estimation of the group velocity which can be used before extensive 3D simulations. Moreover, the scaling laws of the group velocity with frequency and cavity dimensions are derived analytically, allowing to make an initial practical and useful choice of the main cavity parameters in order to design a structure to linearize the phase space for the Compact Light XLS project [6].

• We apply the Bethe’s theory [7] to a traveling wave accelerating structure. In Fig. 1 we show one cell with on-axis coupling through a circular aperture (thin). This theory states that the aperture is equivalent to electric or magnetic dipole moments. These dipole moments are proportional to the normal electric and tangential magnetic fields of the incident wave, respectively.
• More details about the RF-characterization of the accelerating structure are found in [8].

Simulation results

Figure 2 shows that the TW cavity has the 2π/3 mode. By applying proper boundary conditions, it can be avoided to simulate the entire structure because the accelerating periodic structure. Minimum value of the electric and maximum value of the magnetic fields are near the order surface of the cavity.

The RF power is fed to the periodic structure and the electromagnetic mode is excited with $120^\circ$ phase advance per cell. By applying proper boundary conditions, it can be avoided to simulate the entire structure because the accelerating periodic structure. Minimum value of the electric and maximum value of the magnetic fields are near the order surface of the cavity.

In the following equation, in order to calculate the group velocity, we use the slope of the curve (i.e. frequency shift per phase shift):

$$\nu_c = \frac{d\nu}{d\phi}$$

where $\phi$ is the phase advance per cell.

Conclusions

• We applied the Bethe’s theory to a circular aperture of coupled apertures that can be approximated as an electric dipole for the TM0 mode and we observed that the perturbation due to the interaction of these dipoles leads to the variation of the group velocity.
• We demonstrated that the group velocity can be obtained from these variations using different polarization coefficients.
• We compared the analytical and numerical results and we have observed that group velocity shows a good agreement when the holes are small compared with the wavelength applying the electric dipole moment obtained with the Bethe’s theory.
• Furthermore, when the times are comparable in size with the wavelength, we suggested to use an electric dipole moment considering the variation of the electric field in the Green's function adding a term of the order $\lambda^2$ as a correction factor.

Comparison between analytical and numerical results

(a) Cavity radius as a function of the iris radius at 23.988 GHz and 35.982 GHz. (b) Group velocity ($\nu_c$) as a function of the iris radius at 23.988 GHz (solid line) and 35.982 GHz (dashed line). (c) Frequency mode as a function of the phase advance of the TW structure for 35.982 GHz. The iris radius, iris thickness, cavity radius are 1.3333 mm, 0.6667 and 3.4345 mm, respectively.

References