New Analytical derivation of Group Velocity in TW accelerating structures

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Abstract

Ultra high-gradient accelerating structures are needed for the next generation of compact light sources. In the framework of the Compact Light XLS project, we are studying a high harmonic traveling-wave accelerating structure operating at a frequency of 35.982 GHz, in order to linearize the longitudinal space phase. In this paper, we propose a new analytical approach for the estimation of the group velocity in the structure and we compare it with numerical electromagnetic simulations that are carried out by using the code HFSS in the frequency domain.

Introduction

- The next generation of linear accelerators require unprecedented accelerating gradients for high energy physics and compact light sources.
- One of the main limitations to achieve ultra-high gradients, today around 100 MV/m [1], the RF breakdown rate (BDR) which is defined as the number of breakdowns in the structure per unit time and length.
- Recently, an electromagnetic quantity called the modified Poynting vector has been demonstrated to be the main predictor for the BDR. It is defined as Sc=Re[S]+1/6 Im[S] [2], where S is the Poynting vector describing the RF power flow through the traveling wave accelerating structure. As a consequence, the BDR is strictly related to the group velocity ν_g which is proportional to S [3, 4]. Therefore, the group velocity represents a crucial parameter to be characterized for each accelerating cavity [5].
- We propose here an innovative analytical approach to the estimation of the group velocity which can be used before extensive 3D simulations. Moreover, the scaling laws of the group velocity with frequency and cavity dimensions are derived analytically, allowing to make an initial practical and useful choice of the main cavity parameters in order to design a structure to linearize the phase space for the Compact Light XLS project [6].
- We apply the Bethe's theory [7] to a traveling wave accelerating structure. In Fig. 1 we show one cell with on-axis coupling through a circular aperture (iris). This theory states that the aperture is equivalent to electric or/and magnetic dipole moments. These dipole moments are proportional to the normal electric and tangential magnetic fields of the incident wave, respectively.
- More details about the RF characterization of the accelerating structure is found in [8].

Group velocity derivation

From Slater perturbation theorem and using the definition of group velocity [9],

$$\nu_g = \frac{d\omega}{dk_z} = K_1 c (\frac{a}{\lambda})^3 \sin(\psi) e^{-\alpha h} \tag{1}$$

By considering the operating wavelength of the TM_{01} mode ($\lambda = 2.61\ b$, where b is the cavity radius) we obtain,

$$v_g = K_2 c (\frac{a}{b})^3 \sin(\psi) e^{-\alpha h} \tag{2}$$

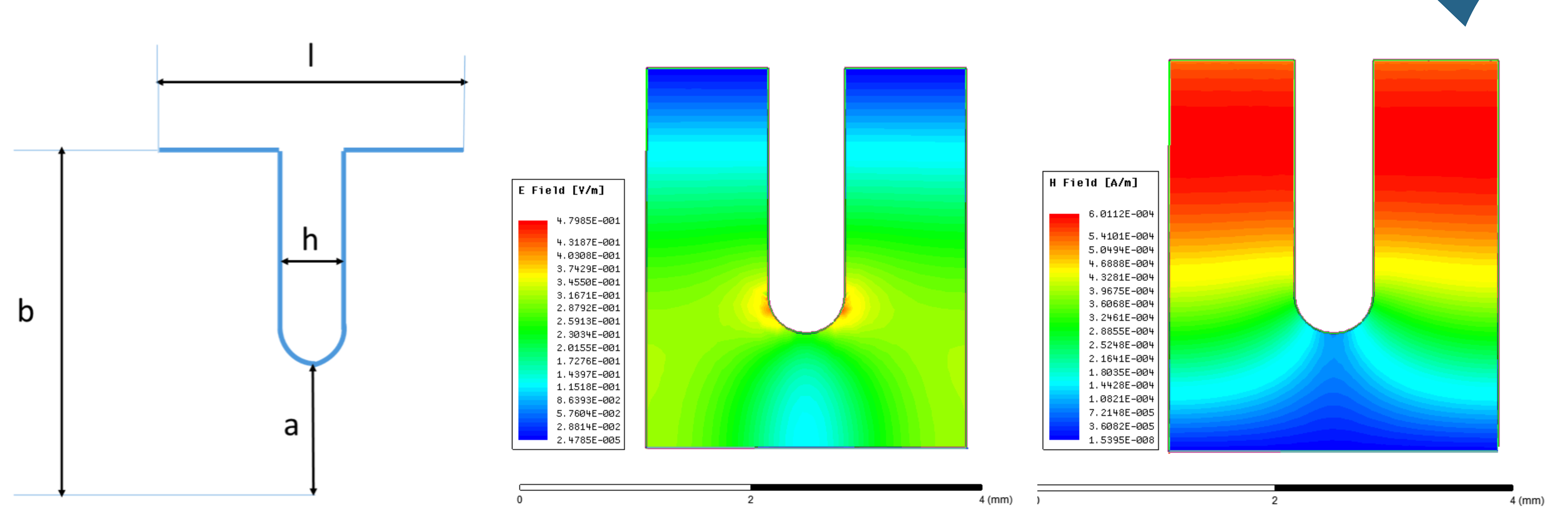
where ψ is the phase advance per cell, h is the iris thickness, and K_1 and K_2, depending on the operating mode and the cavity geometry, are equal to 33.72 and 1.895, respectively. Replacing $\alpha \approx \frac{2.405}{a}$, expanding the exponential function with a Taylor series and considering $h = 0.08\lambda$ for a practical periodic accelerating structures, Eqs. (1) and (2) can be written as [10, 11],

$$\frac{\nu_g}{c} = K_1\ \sin(\psi)[(\frac{a}{\lambda})^3 - 0.19(\frac{a}{\lambda})^2 + 0.0185(\frac{a}{\lambda}) - 0.0012 + 0.000057(\frac{\lambda}{a})] \tag{3}$$

$$\frac{\nu_g}{c} = K_2\ \sin(\psi)[(\frac{a}{b})^3 - 0.502(\frac{a}{b})^2 + 0.126(\frac{a}{b}) - 0.0211 + 0.0026(\frac{b}{a})] \tag{4}$$

we observe that the above equations are independent from the operating frequency and paving the way a fast and accurate way to estimate the group velocity as a function of the geometry of the structure.

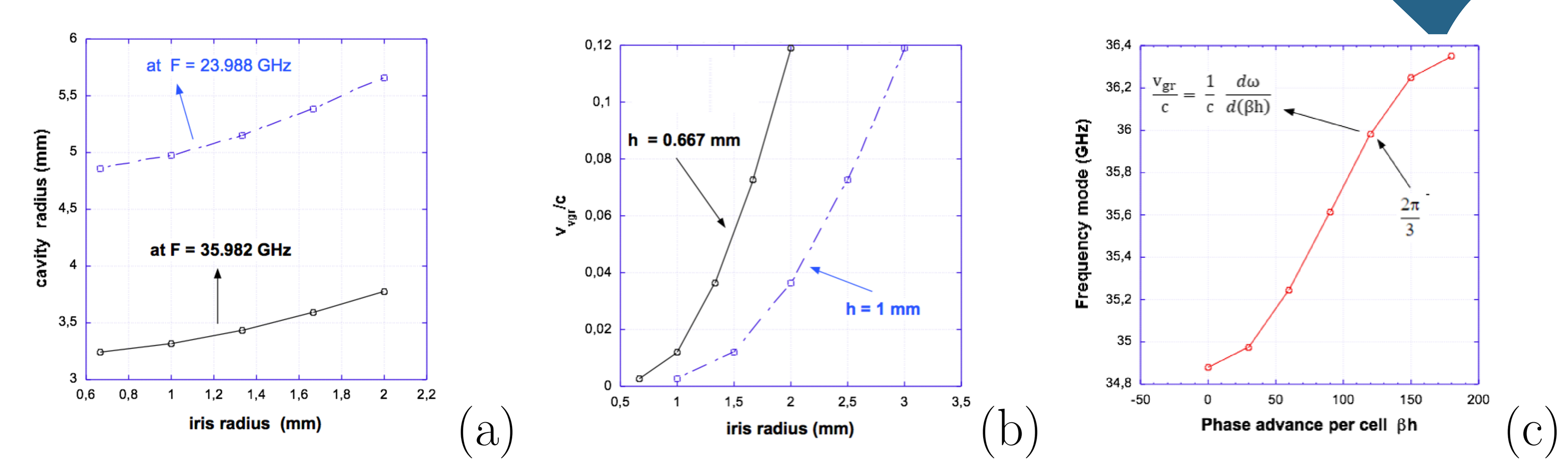
Electric and magnetic field magnitudes



TW cavity shape for the $2\pi/3$ mode. b, a, h and l are cavity radius, iris aperture radius, iris thickness and 1/3 of the cell length, respectively. Electric and magnetic field magnitudes for the TM_{01} mode of the high accelerating periodic structure. Minimum value of the electric and maximum value of the magnetic fields are near the outer surface of the cavity.

The RF power is fed to the periodic structure and the electromagnetic mode is excited with 120° phase advance per cell. By applying proper boundary conditions, it can be avoided to simulate the entire structure because the code HFSS [12] allows to simulate periodic structures using only one cell.

Simulation results



(a) (b) (c)

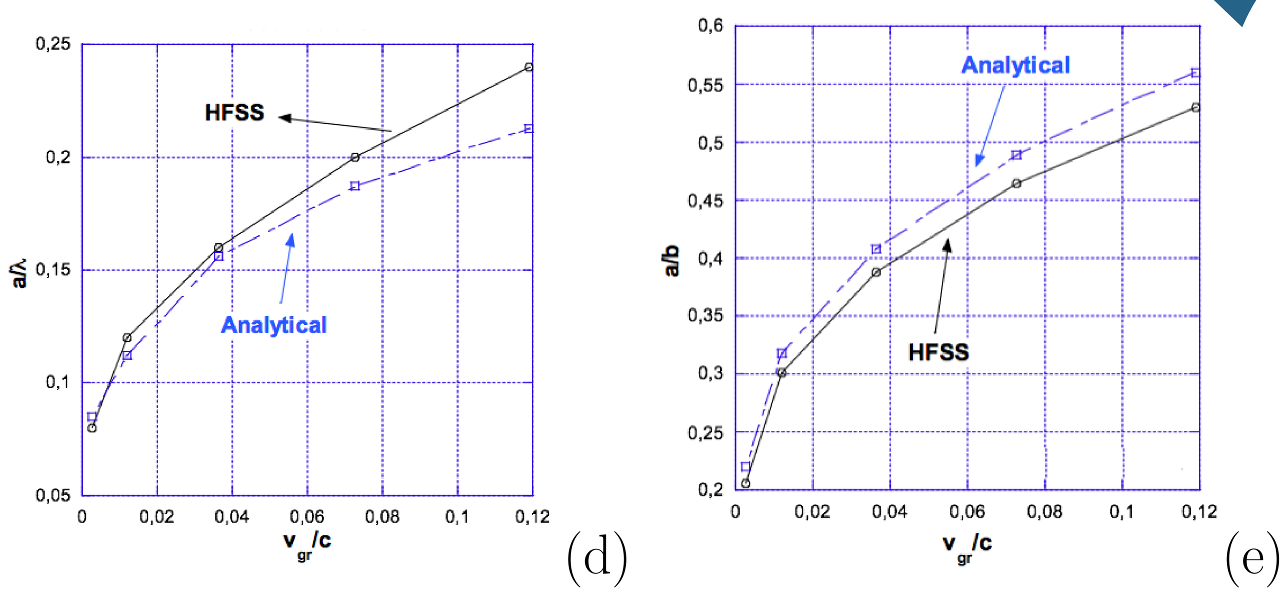
(a) Cavity radius as a function of the iris radius at 23.988 GHz and 35.982 GHz. (b) Group velocity (v_g/c) as a function of the iris radius at 23.988 GHz (solid line) and 35.982 GHz (dashed line). (c) Frequency mode as a function of the phase advance of the TW structure for 35.982 GHz. The iris radius, iris thickness, cavity radius are 1.3333 mm, 0.6667 and 3.4345 mm, respectively.

In the following equation, in order to calculate the group velocity, we use the slope of the curve (i.e. frequency shift per phase shift):

$$\frac{\nu_g}{c} = \frac{2\pi h}{c}\frac{df}{d\phi} \tag{5}$$

where ϕ is the phase advance per cell.

Comparison between analytical and numerical results



(d) (e)

(d) The ratio of a/λ as a function of the group velocity v_g/c. Comparison between the HFSS and Analytical estimations. The iris radius and RF wavelength are a and λ, respectively. (e) The ratio of a/b as a function of the group velocity v_g/c. Comparison between the HFSS and Analytical estimations. The iris radius and cavity radius are a and b, respectively.

It should be noted that increasing the iris radii, the errors between the analytical approach and HFSS are amplified in absolute values while they are constant in relative ones. One physical reason is that we used an electric polarization coefficient for electric moment which is $\alpha = -2/3a^3$. The assumption for using this coefficient is that the holes should be small compared with the wavelength. It is possible to extend the model to bigger size by considering a factor e^{ika} in the normal electric field in which the variation in the Green's function must be considered and this correction can be of the order of $(ka)^2$ rather than ka [7]. New ideas were investigated in order to solve this problem [13, 14, 15]. The author of [15] solved the problem of diffraction of arbitrary electromagnetic field by a circular perfectly conducting disk using a series representation in powers of k using the results of generalized Babinet's principle [16] and considering that the disk problem and the aperture problem are equivalent. Taking two terms instead of the whole equation we have, $P_z = \frac{4}{3}a^3\epsilon_0 E_z^0(1 - \frac{3}{10}(ka)^2)$. The change in the stored energy would be less than that we obtained from electric-dipole moment based on Bethe's theory. In this case, the energy change causes a smaller perturbation, which leads to less frequency shift and consequently gives a smaller group velocity variation, bringing our prediction and simulations results closer.

Conclusions

- We applied the Bethe's theory to the circular aperture of coupled cavities that can be approximated as an electric dipole for the TM_{01} mode and we observed that the perturbation due to the interaction energy of these dipoles leads to the variation of stored energy.
- We demonstrated that the group velocity can be obtained from these variations using different polarization coefficients.
- We compared the analytical and numerical results and we have observed that group velocity shows a good agreement when the holes are small compared with the wavelength applying the electric dipole moment obtained with the Bethe's theory.
- Furthermore, when the irises are comparable in size with the wavelength, we suggested to use an electric dipole moment considering the variation of the electric field in the Green's function adding a term of the order of $(ka)^2$ as a correction factor.

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