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Spatial autocorrelation study for laser beam quality estimation

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Outline

- Motivation
- Analytical definition of spatial autocorrelation index
- Autocorrelation estimation and GPT electron beam emittance evaluation for:
  - Meshed beam
  - Real laser spots
- Conclusions

Motivation

- Motivation of this study: ***High Brightness electron beam***

$$B[A/m^2] = \frac{Ne}{V_{6D}} \propto \frac{Q}{\varepsilon_{nx}\varepsilon_{ny}\sigma_t \sigma_\gamma}$$

→ High beam charge

→ Low emittance

- Contributions to emittance degradations come from electromagnetic fields' non-linearity which can be reduced using a transversally and longitudinally uniform beam.

- Aim of this work: ***To find an additional parameter able to evaluate the transverse laser beam uniformity***

Analytical definition of spatial autocorrelation index

➢ Given a beam spot, represented by a matrix NxM, we can evaluate:

<u>main sample</u>

$$\begin{Bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1M} \\ \dots & & \dots & & \dots \\ a_{i1} & & a_{ij} & & a_{1M} \\ \dots & & \dots & & \dots \\ a_{N1} & & a_{Nj} & & a_{NM} \end{Bmatrix}$$

Non uniformity → Standard deviation σ_a

How non uniformity is distributed → Index of spatial autocorrelation Λ

Analytical definition of spatial autocorrelation index

main sample

$$\left\{ \begin{matrix} a_{i-hj-h} & \dots & a_{i-hj} & \dots & a_{i-hj+h} \\ \dots & & \dots & & \dots \\ a_{ij-h} & & a_{ij} & & a_{ij+h} \\ \dots & h & \dots & & \dots \\ a_{i+h\,j-h} & & a_{i+hj} & & a_{i+hj+h} \end{matrix} \right\}$$

Samples at distance h around a_{ij}

a_{ijh} is the mean of the samples localized around the main sample a_{ij}:

$$a_{ijh} = \frac{1}{(2h+1)^2 - 1}\left[\sum_{l=-h}^{h}\sum_{m=-h}^{h} a_{i+l\;j+m} - a_{ij}\right]$$

Analytical definition of spatial autocorrelation index

- **Non uniformity**

$$\mathrm{var}(a) = \frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}(a_{ij} - \langle a \rangle)^2$$

variance

$$\langle a \rangle = \frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M} a_{ij}$$

mean

where T=NM.

$$\sigma_a = \sqrt{\mathrm{var}(a)}$$

Standard deviation

Standard deviation σ_a describes the contrast between spots in an image: $\sigma_a \to 0$ means the image is uniform

Analytical definition of spatial autocorrelation index

➢ How non uniformity is distributed

The index Λ of spatial autocorrelation is defined as:

$$\Lambda(a,h) = \frac{\mathrm{cov}(a,h)}{\sigma_a^2}$$

with $-1 \leq \Lambda \leq 1$

where cov(a,h) is the covariance matrix, defined as:

$$\mathrm{cov}(a,h) = \frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}(a_{ij} - \langle a \rangle)\cdot(a_{ijh} - \langle a \rangle)$$

The quantity covariance answers the question whether
a sample and its neighbour are at the same time different or not from the mean

Autocorrelation estimation of meshed beam

Cosine-like distribution of spots model

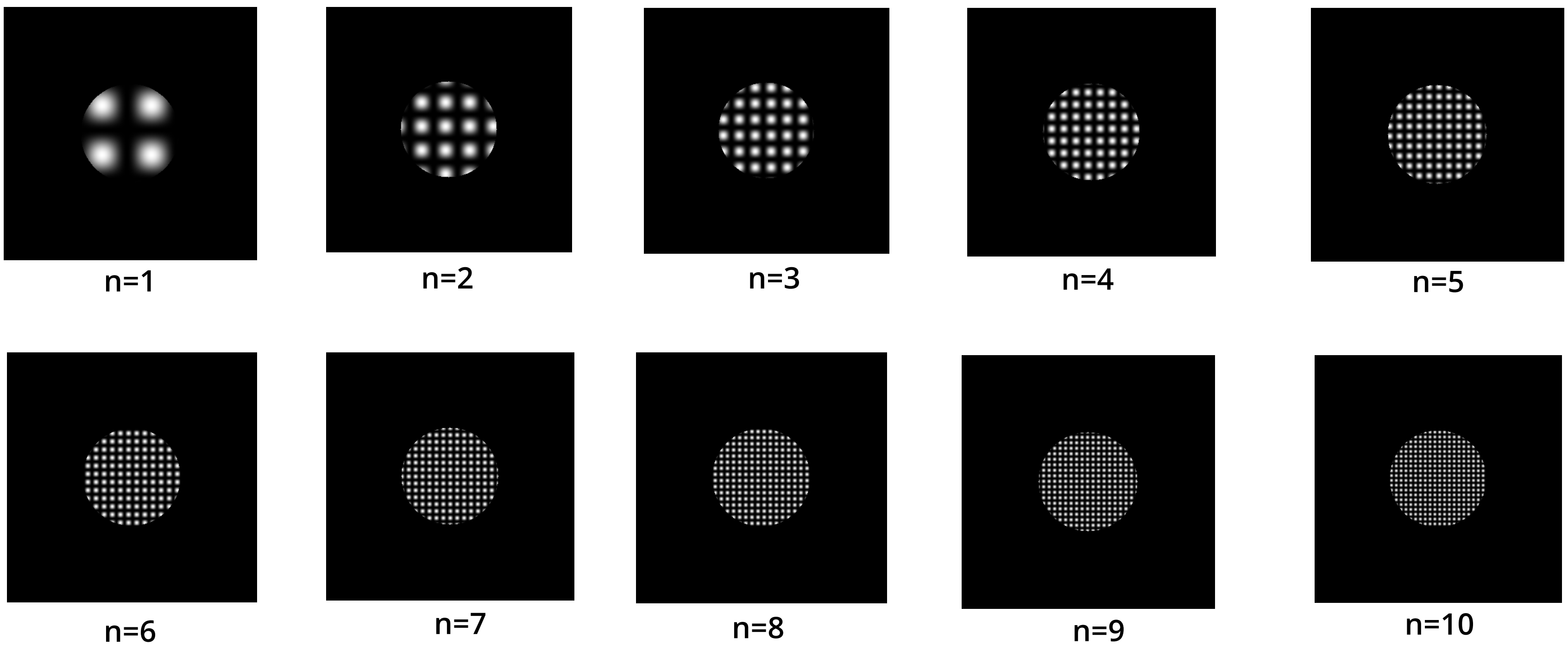
➢ The charge distribution extracted from the cathode has been modelled as a sine and cosine function having a frequency *n* and a charge intensity δ

$$\rho(i,j) = \rho_0(1+\delta\cos k_n i)(1+\delta\cos k_n j)$$

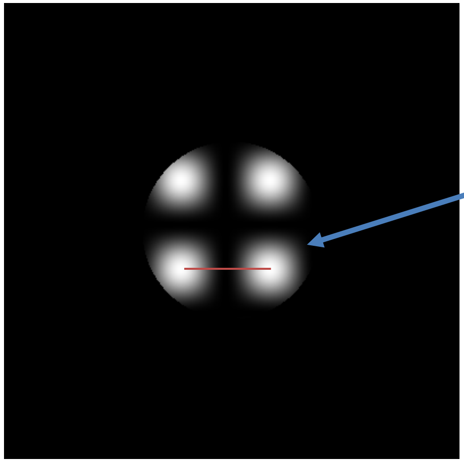
where

$$k_n = \frac{2\pi n}{R}$$

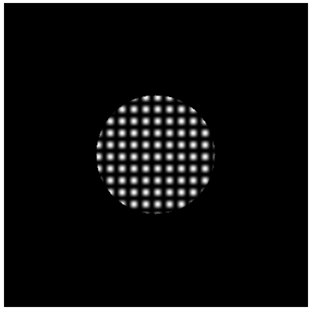
with *R* is the beam radius, ρ_0 is the normalization constant.



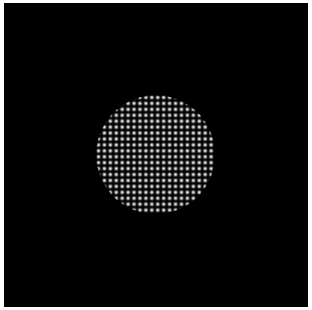
Autocorrelation estimation



mean distance of the non homogeneity
h*×pixel size(μm)

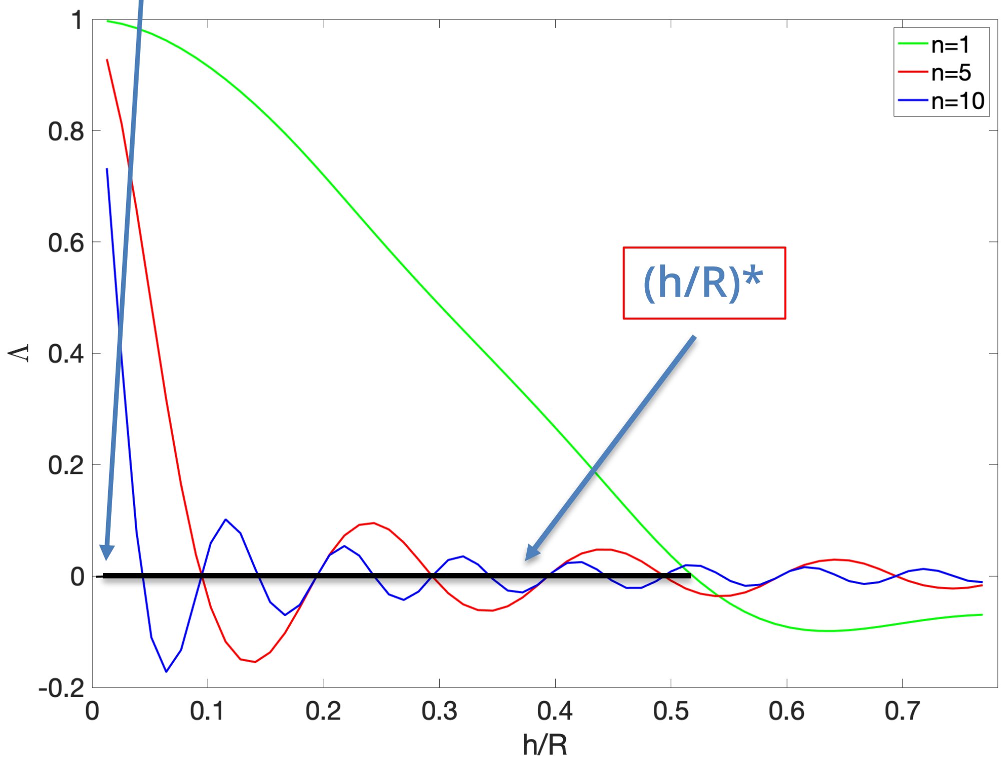


Mean=0.25
σ=0.28 (n=5)



Mean=0.25
σ=0.28 (n=10)

Mean=0.30
σ=0.29 (n=1)



✓ Case n=1:
(h/R)*=0.5
R=78 pixel
Camera Pixel size=6.45 μm/pixel

h*(μm)=0.5xRx(6.45μm/pixel)=256 μm

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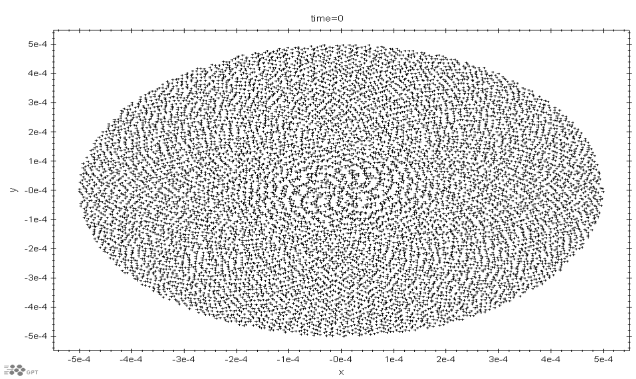
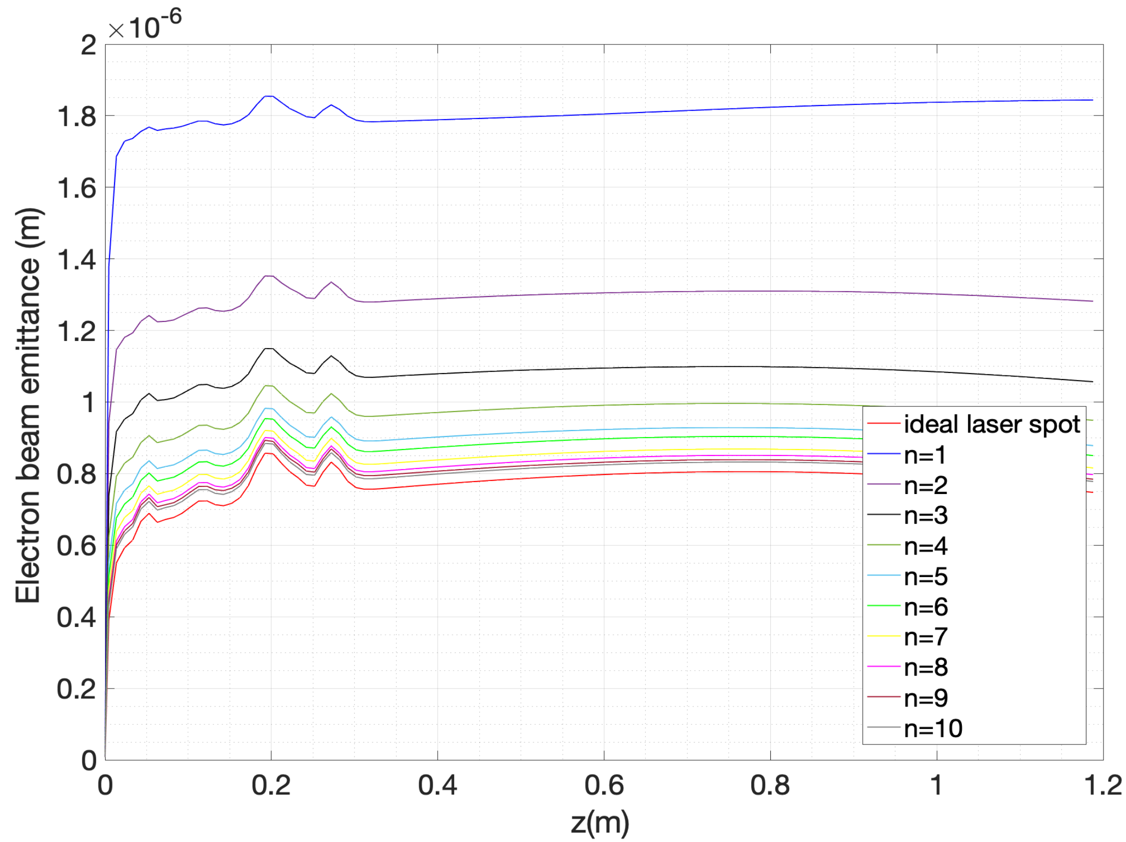
GPT simulation with meshed beam

GPT Parameters:

- E_{RF}= 115MV/m
- Working RF phase=30°
- Laser pulse length=2ps - rms (Gaussian profile)
- Laser radius =500 μm (Flat top profile)
- E= 5MeV - Electron beam energy
- Bunch charge = 50pC
- ε_{intr} = 0.55 μm/mm (normalized intrinsic emittance)
- $I_{picco} \simeq 14.5$ A
- Particles number=50000
- Mesh number: $N_x=N_y=80$, $N_z=50$

Ideal laser spot





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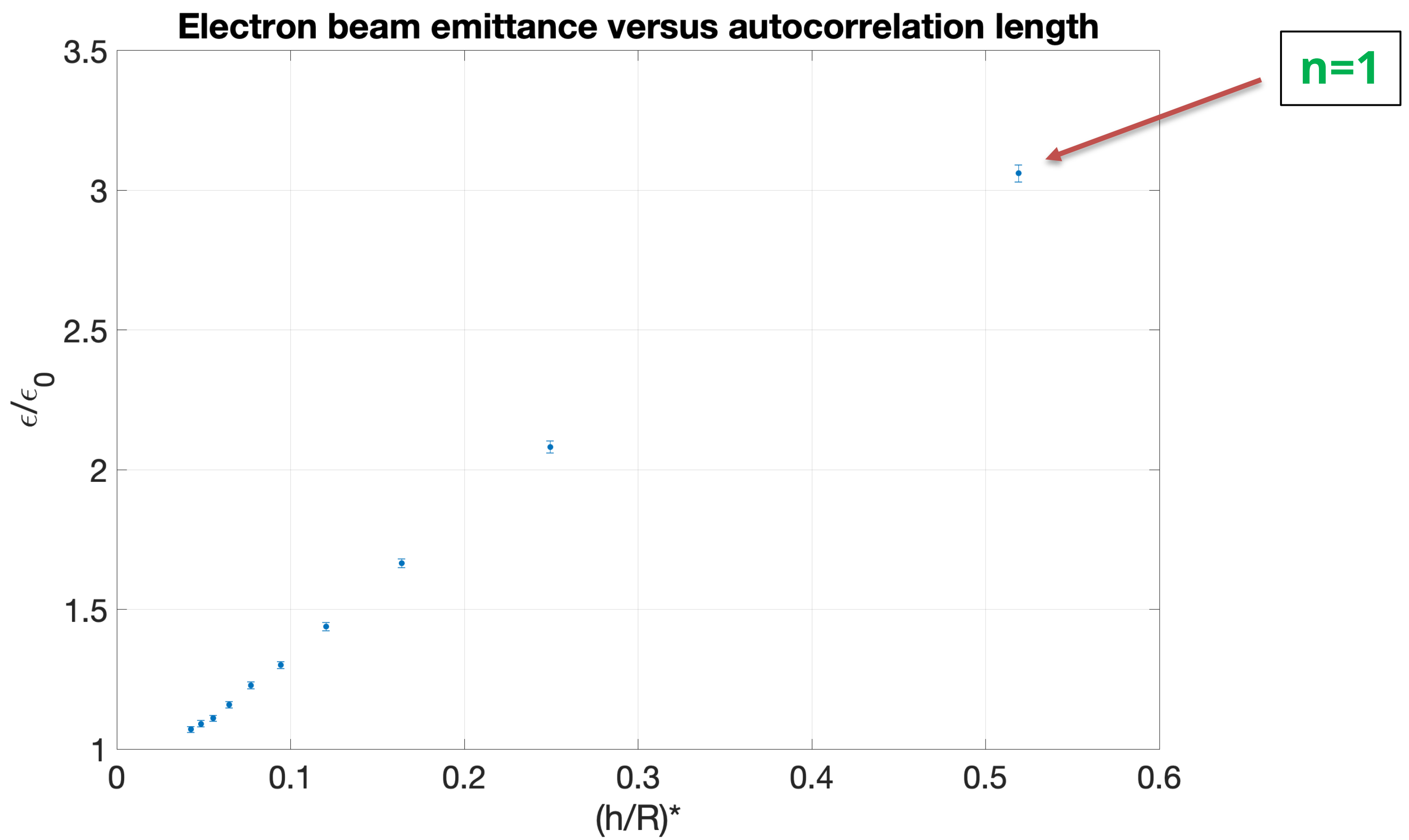
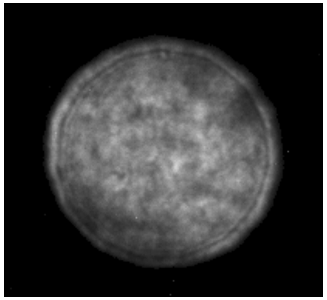
Electron beam emittance versus autocorrelation length (meshed beam)

- ✓ ε_0= 0.55 μm/mm (value for the ideal laser spot image)
- ✓ From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface

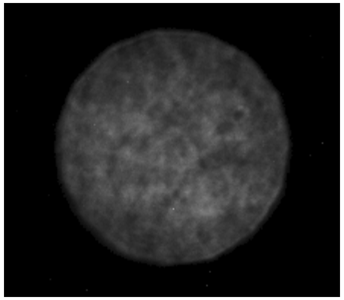


Autocorrelation estimation of real laser spots

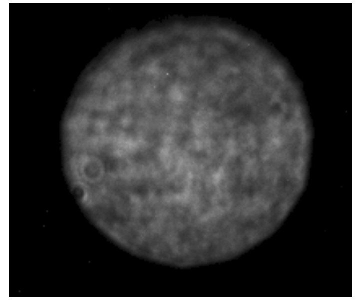
Real laser spots and autocorrelation estimation



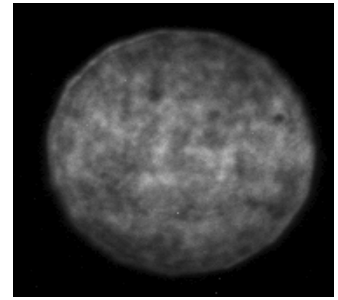
Laser 2
Mean= 0.39
σ=0.14



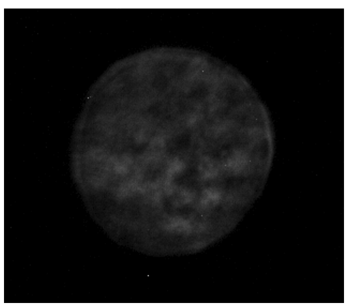
Laser 3
Mean= 0.25
σ=0.07



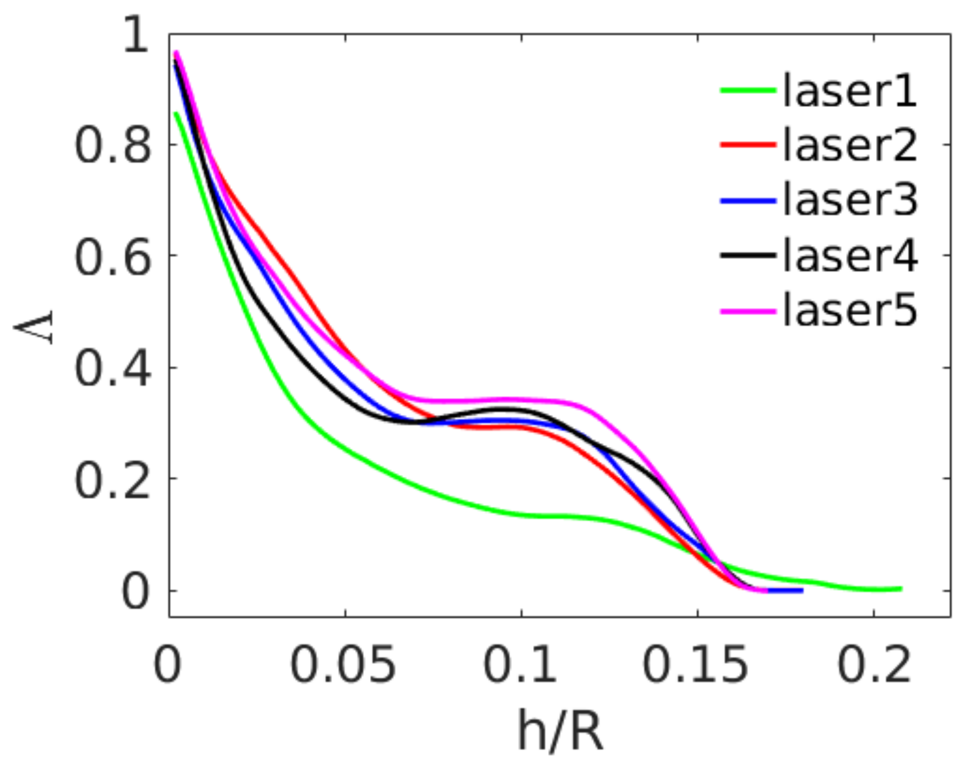
Laser 4
Mean= 0.32
σ=0.10



Laser 5
Mean=0.33
σ=0.13



Laser 1
Mean= 0.135
σ= 0.05



GPT simulation with real laser spots

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- E= 5MeV - Electron beam energy
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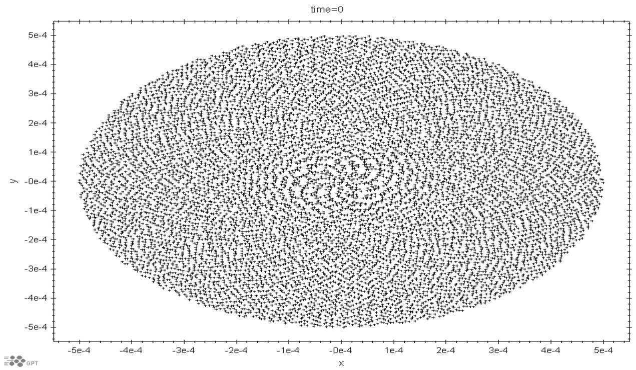
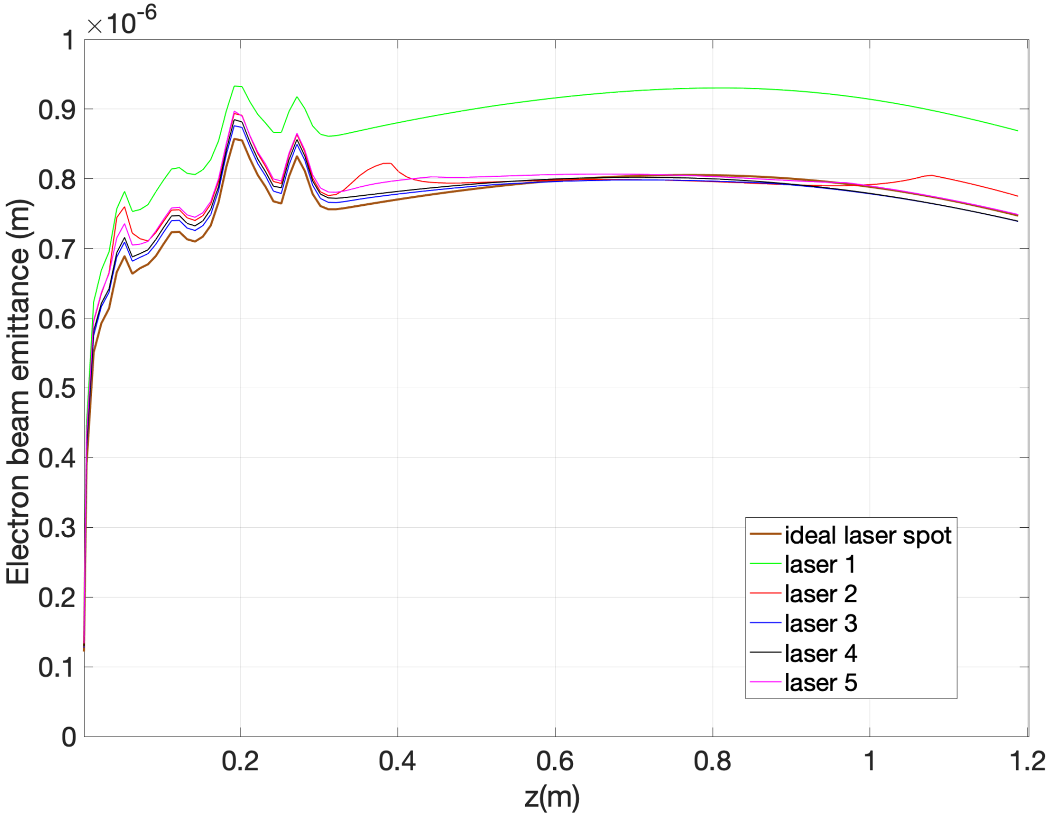
- $I_{picco} \simeq 14.5$ A
- Particles number=50000
- Mesh number: $N_x=N_y=80$, $N_z=50$

Ideal laser spot





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Electron beam emittance versus autocorrelation length (real laser spots)

- ε_0= 0.55±0.02 μm/mm (value for the ideal laser spot)
- From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface

Real laser spot	ε (μm)	$\varepsilon / \varepsilon_0$	(h/R)*
Laser 1	0.62±0.02	1.13±0.06	0.218
Laser 2	0.59±0.02	1.08±0.06	0.166
Laser 3	0.58±0.02	1.04±0.06	0.168
Laser 4	0.58±0.02	1.06±0.06	0.166
Laser 5	0.59±0.02	1.08±0.06	0.166

Conclusions and to do list

• The standard deviation determines the contrast while the autocorrelation index determines how the non-uniformity are distributed

• They describe the laser beam quality, concerning the uniformity, and they give an idea of the emittance growth due to the laser beam degradation

• The parameter (h/R)* is a good estimator of the beam quality since it is strictly correlated with beam emittance at the emission!

➢ Future directions:

- experimental emittance measurements with masks
- systematic study with larger laser dataset

Finally it's over

Thank you for your attention