Spatial autocorrelation study for laser beam quality estimation

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Outline

• Motivation

• Analytical definition of spatial autocorrelation index

• Autocorrelation estimation and GPT electron beam emittance evaluation for:
  – Meshed beam
  – Real laser spots

• Conclusions
Motivation

- Motivation of this study: *High Brightness electron beam*

\[
B[A/m^2] = \frac{Ne}{V_{6D}} \propto \frac{Q}{\sigma_x \sigma_y \sigma_t \sigma_y}
\]

- Contributions to emittance degradations come from electromagnetic fields’ non-linearity which can be reduced using a **transversally and longitudinally uniform beam**.

- Aim of this work: *To find an additional parameter able to evaluate the transverse laser beam uniformity*
Analytical definition of spatial autocorrelation index

- Given a beam spot, represented by a matrix NXM, we can evaluate:

\[
\begin{pmatrix}
  a_{11} & \ldots & a_{1j} & \ldots & a_{1M} \\
  \ldots & \ldots & \ldots & \ldots & \ldots \\
  a_{i1} & a_{ij} & a_{iM} \\
  \ldots & \ldots & \ldots & \ldots & \ldots \\
  a_{N1} & a_{Nj} & a_{NM}
\end{pmatrix}
\]

main sample

Non uniformity ➔ Standard deviation \( \sigma_a \)

How non uniformity is distributed ➔ Index of spatial autocorrelation \( \Lambda \)
Analytical definition of spatial autocorrelation index

$a_{ijh}$ is the mean of the samples localized around the main sample $a_{ij}$:

$$a_{ijh} = \frac{1}{(2h+1)^2-1} \left[ \sum_{l=-h}^{h} \sum_{m=-h}^{h} a_{i+l,j+m} - a_{ij} \right]$$
Analytical definition of spatial autocorrelation index

- Non uniformity

\[ \text{var}(a) = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} (a_{ij} - \langle a \rangle)^2 \]

\[ \langle a \rangle = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{ij} \]

where \( T = NM \).

\[ \sigma_a = \sqrt{\text{var}(a)} \]

Standard deviation \( \sigma_a \) describes the contrast between spots in an image: \( \sigma_a > 0 \) means the image is uniform.
How non uniformity is distributed

The index $\Lambda$ of spatial autocorrelation is defined as:

$$
\Lambda(a, h) = \frac{\text{cov}(a, h)}{\sigma_a^2}
$$

with $-1 \leq \Lambda \leq 1$

where $\text{cov}(a, h)$ is the covariance matrix, defined as:

$$
\text{cov}(a, h) = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} (a_{ij} - \langle a \rangle) \cdot (a_{ijh} - \langle a \rangle)
$$

The quantity covariance answers the question whether a sample and its neighbour are at the same time different or not from the mean
Autocorrelation estimation of meshed beam
The charge distribution extracted from the cathode has been modelled as a sine and cosine function having a frequency $n$ and a charge intensity $\delta$.

$$\rho(i, j) = \rho_0 (1 + \delta \cos k_n i)(1 + \delta \cos k_n j)$$

where

$$k_n = \frac{2\pi n}{R}$$

with $R$ is the beam radius, $\rho_0$ is the normalization constant.
Autocorrelation estimation

Mean distance of the non homogeneity

\( h^* \times \text{pixel size} (\mu m) \)

Mean=0.25
\( \sigma=0.28 \ (n=10) \)

Mean=0.25
\( \sigma=0.28 \ (n=5) \)

Mean=0.30
\( \sigma=0.29 \ (n=1) \)

✓ Case \( n=1 \):

\( (h/R)^* = 0.5 \)

\( R = 78 \) pixel

Telecamera Pixel size=6.45 \( \mu m \)/pixel

\( h^*(\mu m) = 0.5 \times R \times (6.45 \mu m/\text{pixel}) = 256 \mu m \)

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GPT Parameters:

- $E_{RF} = 115$ MV/m
- Working RF phase = 30°
- Laser pulse length = 2 ps - rms (Gaussian profile)
- Laser radius = 500 μm (Flat top profile)
- $E = 5$ MeV - Electron beam energy
- Bunch charge = 50 pC
- $\epsilon_{intr} = 0.55$ μm/mm (normalized intrinsic emittance)
- $I_{picco} \approx 14.5$ A

Ideal laser spot
Electron beam emittance versus autocorrelation length (meshed beam)

\[ \varepsilon_0 = 0.55 \, \mu m/mm \] (value for the ideal laser spot image)

- From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface
Autocorrelation estimation of real laser spots
Real laser spots and autocorrelation estimation

Laser 1
Mean= 0.135
\( \sigma = 0.05 \)

Laser 2
Mean= 0.39
\( \sigma = 0.14 \)

Laser 3
Mean= 0.25
\( \sigma = 0.07 \)

Laser 4
Mean= 0.32
\( \sigma = 0.10 \)

Laser 5
Mean= 0.33
\( \sigma = 0.13 \)
GPT Parameters:
- $E_{RF} = 115$ MV/m
- Working RF phase = 30°
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- $E = 5$ MeV - Electron beam energy
- Bunch charge = 50 pC
- $\varepsilon_{intr} = 0.55$ μm/mm (normalized intrinsic emittance)
- $I_{picco} \approx 14.5$ A
Electron beam emittance versus autocorrelation length (real laser spots)

- $\varepsilon_0 = 0.55 \pm 0.02 \text{ $\mu$m/mm}$ (value for the ideal laser spot)

- From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface

<table>
<thead>
<tr>
<th>Real laser spot</th>
<th>$\varepsilon$ ($\mu$m)</th>
<th>$\varepsilon / \varepsilon_0$</th>
<th>$(h/R)^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser 1</td>
<td>0.62$\pm$0.02</td>
<td>1.13$\pm$0.06</td>
<td>0.218</td>
</tr>
<tr>
<td>Laser 2</td>
<td>0.59$\pm$0.02</td>
<td>1.08$\pm$0.06</td>
<td>0.166</td>
</tr>
<tr>
<td>Laser 3</td>
<td>0.58$\pm$0.02</td>
<td>1.04$\pm$0.06</td>
<td>0.168</td>
</tr>
<tr>
<td>Laser 4</td>
<td>0.58$\pm$0.02</td>
<td>1.06$\pm$0.06</td>
<td>0.166</td>
</tr>
<tr>
<td>Laser 5</td>
<td>0.59$\pm$0.02</td>
<td>1.08$\pm$0.06</td>
<td>0.166</td>
</tr>
</tbody>
</table>
Conclusions and to do list

• The standard deviation determines the contrast while the autocorrelation index determines how the non-uniformity are distributed

• They describe the laser beam quality, concerning the uniformity, and they give an idea of the emittance growth due to the laser beam degradation

• The parameter \((h/R)^*\) is a good estimator of the beam quality since it is strictly correlated with beam emittance at the emission!

Future directions:

- experimental emittance measurements with masks
- systematic study with larger laser dataset
Finally it’s over

Thank you for your attention