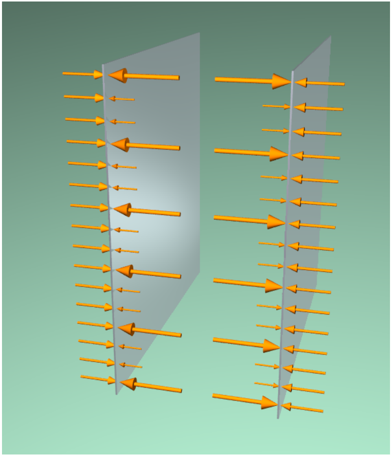

Casimir effect and dissipation

Gert-Ludwig Ingold
Universität Augsburg

- motivation
- scattering approach to the Casimir effect
- negative Casimir entropy
- numerical treatment of the plane-sphere geometry

- motivation
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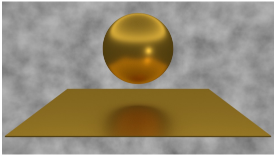
Casimir (1948)

- parallel planes
- perfect reflectors
- zero temperature

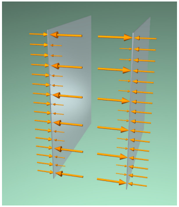
$$F = \frac{\hbar c \pi^2}{240 L^4} A$$



modern experiments

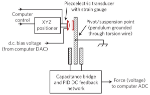
- more complicated geometries
- real materials
- quantum and thermal fluctuations
- surface roughness
- ...

Some experimental setups

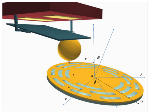


thy: Casimir (1948)
exp: Sparnaay (1958)
Bressi et al. (2002)

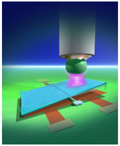
plane/plane



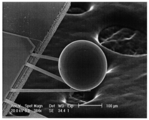
Lamoreaux group (2001)



Bimonte, Decca (2016)

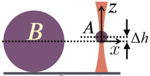


Capasso group (2001)

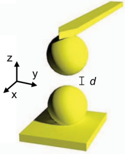


Mohideen group (1998)

plane/sphere

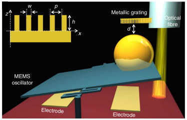


UFRJ group (2015)



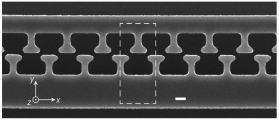
Munday group (2018)

sphere/sphere



López et al. (2013)

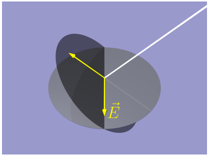
Chan group (2018)



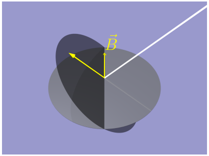
nanogratings

Drude and plasma model

two polarizations



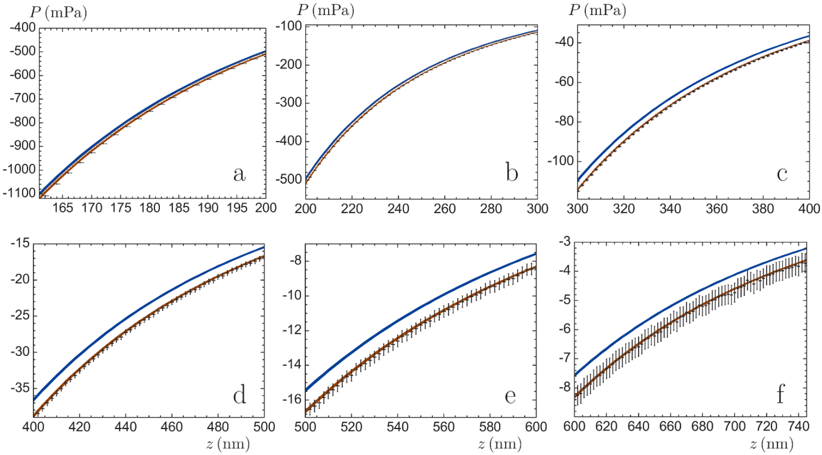
TE mode



TM mode

	plasma model	Drude model
permittivity	$\varepsilon(\omega) = 1 - \frac{\omega_P^2}{\omega^2}$	$\varepsilon(\omega) = 1 - \frac{\omega_P^2}{\omega^2 + i\gamma\omega}$
dc conductivity	∞	$\frac{\omega_P^2}{\gamma}$
Fresnel reflection coefficient at zero frequency		
TE mode	$\neq 0$	0
TM mode	-1	-1

Drude-plasma puzzle



Drude model

plasma model

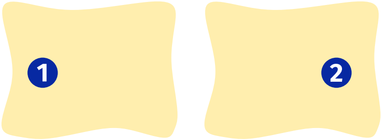
R. Decca et al., Phys. Rev. D **75**, 077101 (2007)

- motivation
- ► scattering approach to the Casimir effect
- negative Casimir entropy
- numerical treatment of the plane-sphere geometry

Scatterers in the electromagnetic vacuum

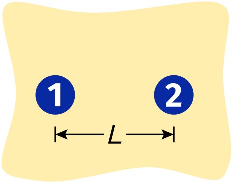
$$E_{\text{vac}}^{(0)} = \frac{1}{2}\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}}$$

infinite vacuum energy



$$\Delta E_{\text{vac}}^{(1,2)} = E_{\text{vac}}^{(1,2)} - E_{\text{vac}}^{(0)}$$

change of the vacuum energy due to a scatterer, still infinite

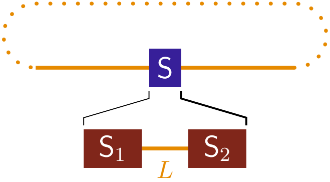


$$E_{\text{Cas}}(L) = \Delta E_{\text{vac}} - \Delta E_{\text{vac}}^{(1)} - \Delta E_{\text{vac}}^{(2)}$$

finite distance dependent change of vacuum energy due to two scatterers

Scattering theoretical approach in one dimension

Universität Augsburg University



change of vacuum energy due to a scatterer

$$\Delta E_{\text{vac}} = \frac{\mathrm{i}\hbar c}{4\pi}\int_0^\infty \mathrm{d}k \ln\left(\det(\mathsf{S})\right)$$

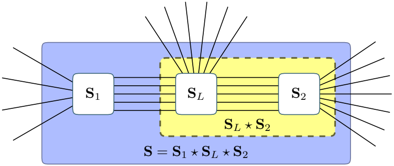
$$\det(\mathsf{S}) = \det(\mathsf{S}_1)\det(\mathsf{S}_2)\frac{1-\left[\bar{r}_1 r_2 \mathrm{e}^{2\mathrm{i}kL}\right]^*}{1-\left[\bar{r}_1 r_2 \mathrm{e}^{2\mathrm{i}kL}\right]}$$

$$\Delta E_{\text{vac}} = \Delta E_{\text{vac}}^{(1)} + \Delta E_{\text{vac}}^{(2)} + E_{\text{Cas}}(L)$$

Casimir energy

$$E_{\text{Cas}}(L) = \Delta E_{\text{vac}} - \Delta E_{\text{vac}}^{(1)} - \Delta E_{\text{vac}}^{(2)} = \frac{\hbar c}{2\pi}\operatorname{Im}\int_0^\infty \mathrm{d}k \, \ln\left[1-\bar{r}_1 r_2 \mathrm{e}^{2\mathrm{i}kL}\right]$$

for a pedagogical presentation see GLI, A. Lambrecht, Am. J. Phys. **83**, 156 (2015)



$$\det \mathbf{S} = \det(\mathbf{S}_1)\det(\mathbf{S}_2)\det(\mathbf{S}_L)\frac{\det(\mathcal{D}_{21})}{\det(\mathcal{D}_{21})^*}$$

with $\quad \mathcal{D}_{21} = \left(1 - \mathbf{S}_1^{ii}\mathbf{T}_{12}^{ii}\mathbf{S}_2^{ii}\mathbf{T}_{21}^{ii}\right)^{-1}$

Casimir energy at zero temperature ($\xi = -i\omega$)

$$E_{\mathrm{Cas}}(L) = \hbar\int_0^\infty \frac{d\xi}{2\pi}\log\det\left[\mathbf{1} - \mathbf{S}_1^{ii}\mathbf{T}_{12}^{ii}\mathbf{S}_2^{ii}\mathbf{T}_{21}^{ii}(\xi)\right]$$

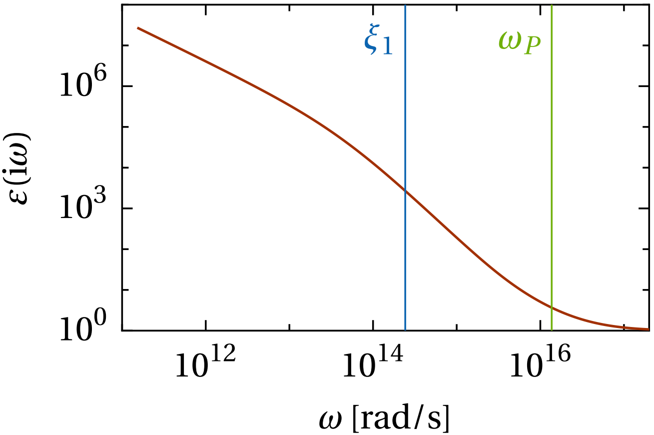
R. Guérout, GLI, A. Lambrecht, S. Reynaud, Symmetry **10**, 37 (2018)

Casimir free energy at finite temperature

$$\mathcal{F}(L) = \frac{k_B T}{2}\sum_{n=-\infty}^{\infty}\log\det\left[\mathbf{1} - \mathbf{S}_1^{ii}\mathbf{T}_{12}^{ii}\mathbf{S}_2^{ii}\mathbf{T}_{21}^{ii}(|\xi_n|)\right] \qquad \xi_n = \frac{2\pi\hbar n}{k_B T}$$

Optical properties of gold

Universität Augsburg University



E. D. Palik, *Handbook of Optical Constants of Solids*

A. Lambrecht, S. Reynaud, Eur. Phys. J. D **8**, 309 (2000) motivation

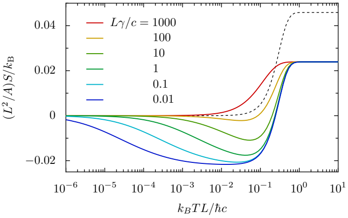
scattering approach to the Casimir effect

- negative Casimir entropy

numerical treatment of the plane-sphere geometry

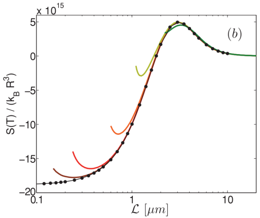
Negative Casimir entropy

Universität Augsburg University



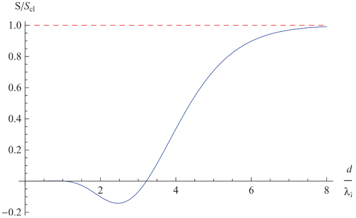
plane/plane, Drude metal

Bezerra, Klimchitskaya, Mostepanenko (2002)
Høye, Brevik, Aarseth, Milton (2003)
Boström, Sernelius (2004)



plane/sphere, perfect reflector

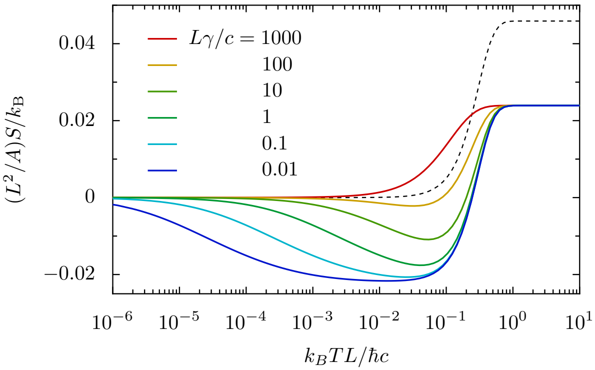
Canaguier-Durand, Maia-Neto, Lambrecht, Reynaud (2010)



sphere/sphere, perfect reflector

Rodriguez-Lopez (2011)

Negative Casimir entropy in the plane-plane geometry



entropy difference between Drude and plasma model depends on $\dfrac{\hbar\gamma}{k_\mathrm{B}T}$

$\longrightarrow$ limit of zero damping: plasma model at high temperatures

GLI, A. Lambrecht, S. Reynaud, Phys. Rev. E **80**, 041113 (2009)

Two sources of negative Casimir entropy

round-trip operator $\mathcal{M}(\xi) = \mathcal{R}_1(\xi)\mathcal{T}_{12}(\xi)\mathcal{R}_2(\xi)\mathcal{T}_{21}(\xi)$

free energy at high temperatures

$$\mathcal{F} = \frac{k_B T}{2} \ln\left[\det\left(1 - \mathcal{M}(0)\right)\right]$$

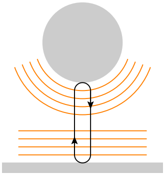
entropy

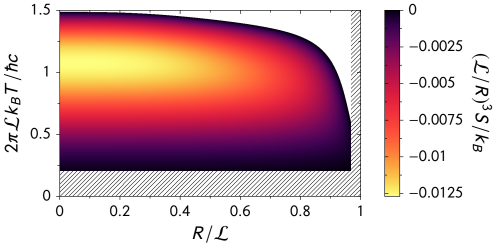
$$\mathcal{S} = -\frac{\partial \mathcal{F}}{\partial T}$$

Two reasons why the roundtrip operator can vanish at zero frequency:

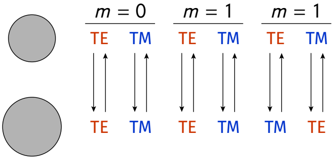
1. reflection operator $\mathcal{R}$
   electromagnetic response of scattering object
   ✗ *Drude metal*
2. translation operator $\mathcal{T}$
   polarization mixing
   ✗ *geometry of scattering objects*

Negative entropy induced by geometry





- only one round-trip $\mathcal{R}_1\mathcal{T}_{12}\mathcal{R}_2\mathcal{T}_{21}$
- only dipole scattering ($\ell = 1$)

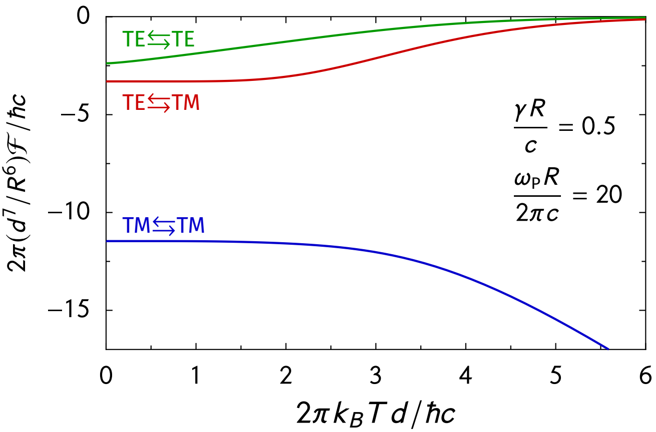


GLI, S. Umrath, M. Hartmann, R. Guérout, A. Lambrecht, S. Reynaud, K. A. Milton, Phys. Rev. E **91**, 033203 (2015)

K. A. Milton, R. Guérout, GLI, A. Lambrecht, S. Reynaud, J. Phys.: Condens. Matter **27**, 214003 (2015)

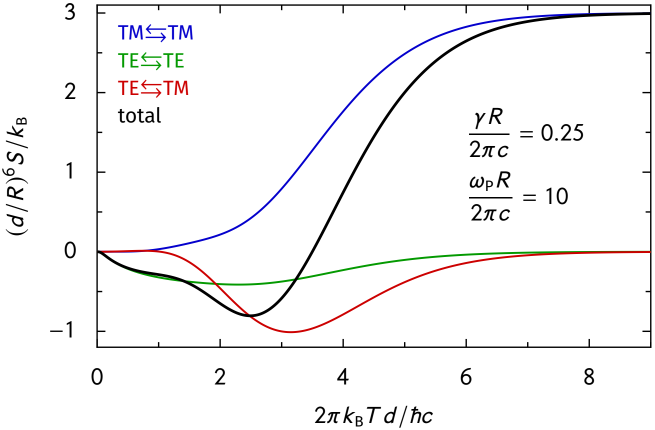
Free energy at weak damping

two spheres with equal radii



S. Umrath, M. Hartmann, GLI, P. A. Maia Neto, Phys. Rev. E **92**, 042125 (2015)

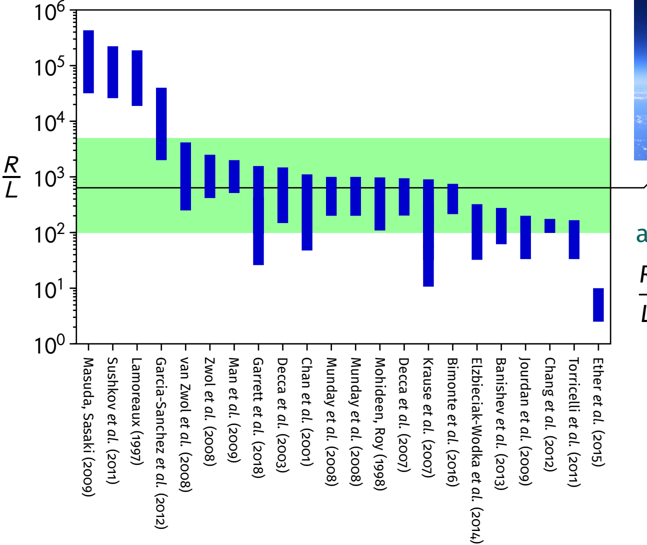
Channel analysis for weak damping



S. Umrath, M. Hartmann, GLI, P. A. Maia Neto, Phys. Rev. E **92**, 042125 (2015)

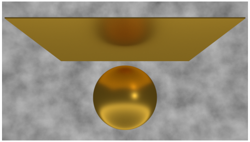
- motivation
- scattering approach to the Casimir effect
- negative Casimir entropy
- ► numerical treatment of the plane-sphere geometry

Experimental aspect ratios

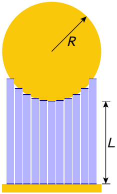


aspect ratio

$\frac{R}{L}$ $R \longleftarrow$ sphere radius, $L \longleftarrow$ distance plane-sphere



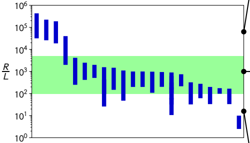
Numerics for the sphere-plane geometry



proximity force approximation (PFA)

- Casimir force is non-additive
- PFA from semiclassics in k space

  B. Spreng, M. Hartmann, V. Henning, GLI,
  P. A. Maia Neto, Phys. Rev. A **97**, 062504 (2018)



intermediate aspect ratios

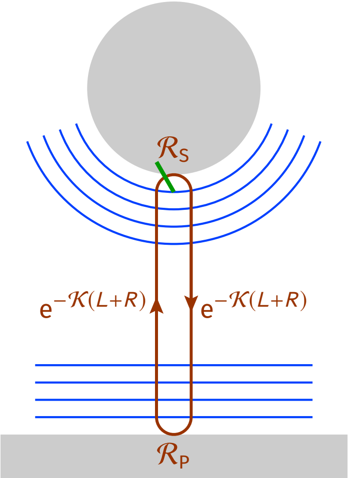
- accurate description of experiments
- theoretical understanding of corrections to PFA

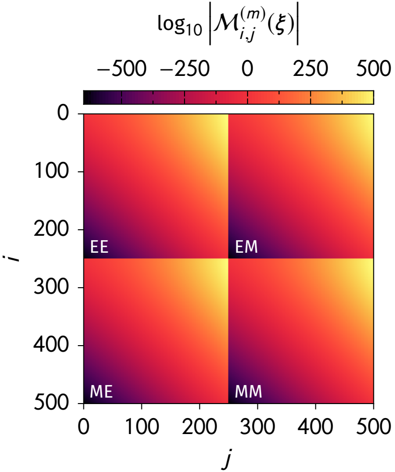
multipole expansion

- ℓ_{max} increases linearly with aspect ratio
- numerics for larger aspect ratios becomes demanding

Round-trip matrix

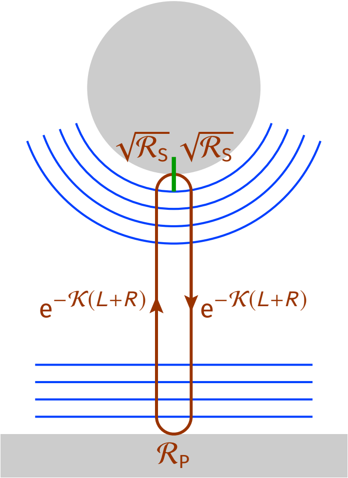
$$\mathcal{M}(\xi) = \mathrm{e}^{-\mathcal{K}(L+R)}\mathcal{R}_\mathrm{P}\mathrm{e}^{-\mathcal{K}(L+R)}\mathcal{R}_\mathrm{S}$$

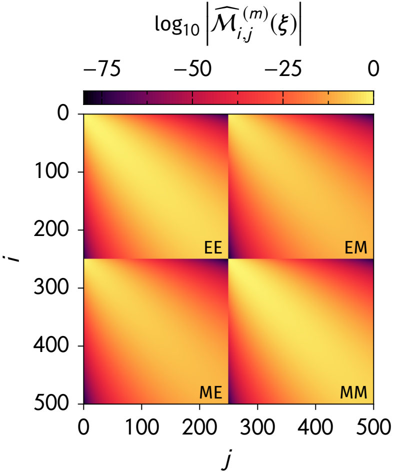




Symmetrized round-trip matrix

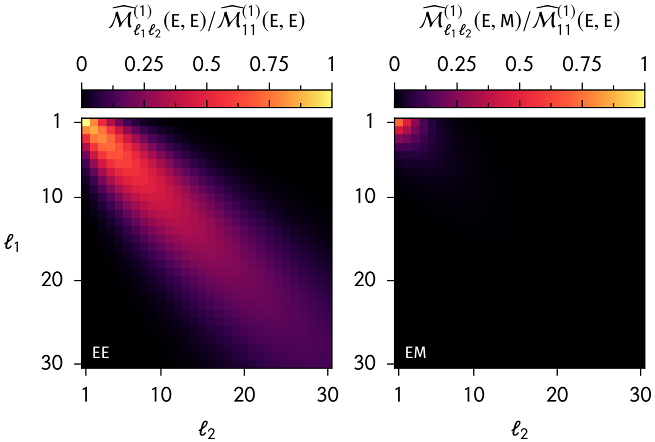
$$\widehat{\mathcal{M}}(\xi) = \sqrt{\mathcal{R}_S}\,e^{-\mathcal{K}(L+R)}\mathcal{R}_P\,e^{-\mathcal{K}(L+R)}\sqrt{\mathcal{R}_S}$$





M. Hartmann, GLI, P. A. Maia Neto, Phys. Rev. Lett. **119**, 043901 (2017)

Symmetrized round-trip matrix



polarization conserving block

polarization mixing block

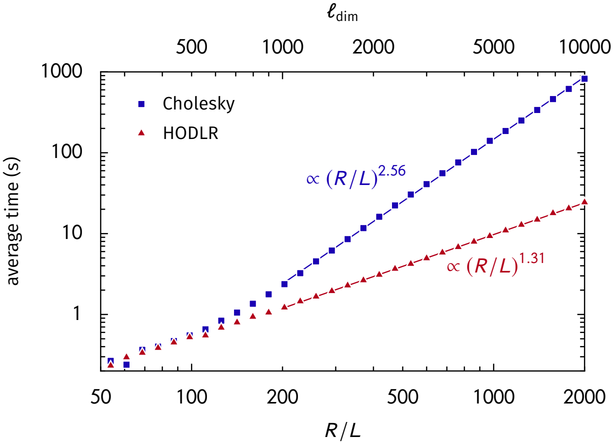
$m = 1$

$\frac{R}{L} = 50$

$\xi = \frac{c}{L+R}$

perfect refl.

Performance comparison for log-det



M. Hartmann, GLI, P. A. Maia Neto, Phys. Scr. **93**, 114003 (2018)

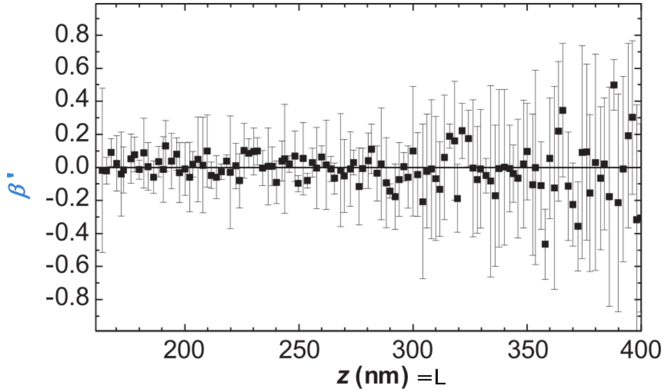
HODLR = hierarchical off-diagonal low-rank

S. Ambikasaran, HODLR: Fast direct solver and determinant computation for dense linear systems
`https://github.com/sivaramambikasaran/HODLR`

S. Ambikasaran, E. Darve, J. Sci. Comp. **57**, 477 (2013)

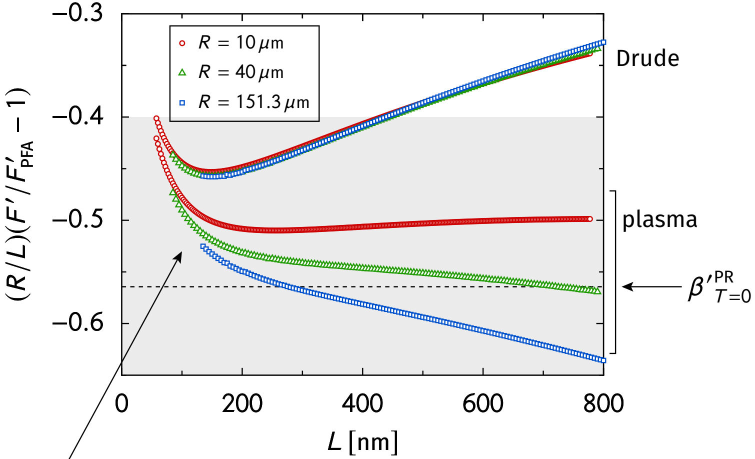
Limit on corrections beyond PFA

$$P^{\text{eff}}(L,R) = -\frac{1}{2\pi R}\frac{\mathrm{d}F}{\mathrm{d}L} = P^{pp}(L)\left[1 + \beta' \frac{L}{R} + \dots\right]$$



Krause, Decca, López, Fischbach, Phys. Rev. Lett. **98**, 050403 (2007)

Corrections to the force gradient



excluded by Krause, Decca, López, Fischbach, Phys. Rev. Lett. **98**, 050403 (2007)

- experimental bounds for β' violated for Drude and plasma model
- violation for plasma model more significant than for Drude model

Conclusions

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- Casimir measurements can be precise enough to depend on material properties
- despite a finite dc conductivity of the material, the plasma model often fits better the experimental results
- for vanishing damping, the Drude model turns into the plasma model at high temperatures
- a scattering channel analysis reveals dissipation and polarization mixing as possible origins of negative Casimir entropy
- Casimir free energy can be determined numerically for experimentally relevant parameters by symmetrization of the round-trip matrix

Thanks to ...

Antoine Canaguier-Durand
Romain Guérout
Astrid Lambrecht
Serge Reynaud
Laboratoire Kastler Brossel, Paris

Marc-Thierry Jaekel
Laboratoire de Physique Théorique,
ENS Paris

Kimball A. Milton
University of Oklahoma, Norman

Vinicius Henning
Paulo A. Maia Neto
Universidade Federal do
Rio de Janeiro

Michael Hartmann
Benjamin Spreng
Stefan Umrath
Universität Augsburg