

Vacuum fluctuations and gravity

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The interplay between gravity and quantum mechanics can take place at widely different scales

- UV limit. GR non renormalizable. Need a UV completion (50+ yrs of work, string theory, ...)
- far IR limit (cosmology). Cosmological constant problem, dynamical DE,...
(more recent, largely stimulated by observation of DE)
- nanoscale and other intermediate scales ?
(Casimir, this conference)

at cosmological scales:

cosmological constant problem

- problem of naturalness, not a wrong prediction of QFT...
 - the current and future precision of cosmological data allows to test for alternatives
-
- why ρ_{vac} is not $O(M_{\text{Pl}}^4)$, or at least $(\text{TeV})^4$?
 - why ρ_{vac} is of order ρ_{matter} today ?
 - cosmological constant or dynamical DE? (modified GR?)

Degravitation

an interesting example of alternative proposals:
on cosmological scales, quantum vacuum fluctuations do
not gravitate

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

The idea (as I understand it now thanks to much subsequent work, in particular by our Geneva group) can be framed in terms of IR effects in the quantum effective action

we will see that further elaborations on this theme lead to
interesting modified gravity models

Nonlocality and the quantum effective action

At the fundamental level, the action in QFT is local

However, the quantum effective action is nonlocal

$$e^{iW[J]} \equiv \int D\varphi e^{iS[\varphi] + i \int J\varphi}$$

$$\frac{\delta W[J]}{\delta J(x)} = \langle 0 | \varphi(x) | 0 \rangle_J \equiv \phi[J]$$

quantum effective action: $\Gamma[\phi] \equiv W[J] - \int \phi J$

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = -J(x)$$

the quantum EA gives the exact eqs of motion for the vev, which include the quantum corrections

$$e^{i\Gamma[\phi]} = \int D\varphi e^{iS[\phi+\varphi] - i \int \frac{\delta\Gamma[\phi]}{\delta\phi} \varphi}$$

- We are 'integrating out' the quantum fluctuations, not the fields

It is not a Wilsonian effective action

- The regime of validity of the quantum EA is the same as that of the fundamental theory

- light particles \longleftrightarrow nonlocalities in the quantum effective action
these nonlocalities are well understood in the UV.

E.g. in QED

$$\Gamma_{\text{QED}}[A_\mu] = -\frac{1}{4} \int d^4x \left[F_{\mu\nu} \frac{1}{e^2(\Box)} F^{\mu\nu} + \mathcal{O}(F^4) \right] + \text{fermionic terms}$$

$$\frac{1}{e^2(\Box)} \simeq \frac{1}{e^2(\mu)} - \beta_0 \log \left(\frac{-\Box}{\mu^2} \right)$$

$$\log \left(\frac{-\Box}{\mu^2} \right) \equiv \int_0^\infty dm^2 \left[\frac{1}{m^2 + \mu^2} - \frac{1}{m^2 - \Box} \right]$$

it is just the running of the coupling constant in coordinate space

Note: we are not integrating out light particles from the theory!

The quantum effective action is especially useful in GR

$$e^{i\Gamma_m[g_{\mu\nu}]} = \int D\varphi e^{iS_m[g_{\mu\nu};\varphi]}$$

(vacuum quantum EA. We can also retain the vev's of the matter fields with the Legendre transform)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} \quad \langle 0|T_{\mu\nu}|0\rangle = \frac{2}{\sqrt{-g}} \frac{\delta \Gamma_m}{\delta g^{\mu\nu}}$$

It gives the exact Einstein eqs including quantum matter loops

$$G_{\mu\nu} = 8\pi G \langle 0_{\text{in}}|T_{\mu\nu}|0_{\text{in}}\rangle$$

$\Gamma = S_{\text{EH}} + \Gamma_m$ is an action that, used at tree level, give the eqs of motion that include the quantum effects

The quantum effective action in GR can be computed perturbatively in an expansion in the curvature using heat-kernel techniques

Barvinsky-Vilkovisky 1985,1987

$$\Gamma = \frac{m_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left[R k_R(\square) R + \frac{1}{2} C_{\mu\nu\rho\sigma} k_W(\square) C^{\mu\nu\rho\sigma} \right]$$

The form factors due to a matter field of mass m_s are known in closed form

Gorbar-Shapiro 2003

For $m_s \ll E$ $k(\square) \simeq k(\mu) - \beta_k \log \left(\frac{-\square}{\mu^2} \right) + c_1 \frac{m_s^2}{\square} + c_2 \frac{m_s^4}{\square^2} + \dots$

For $m_s \gg E$ $k(\square) \simeq O(\square/m_s^2)$

However, these corrections are only relevant in the UV (ie for quantum gravity) and not in the IR (cosmology):

$$R \log(-\square/m_s^2) R \ll m_{\text{Pl}}^2 R \quad \text{unless} \quad R \sim m_{\text{Pl}}^2$$

unavoidable, since these are one-loop corrections, and we pay a factor $1/m_{\text{Pl}}^2$

For application to cosmology, we rather need some strong IR effect

The techniques for computing the quantum EA are well understood in the UV, but much less in the IR

A typical IR effect is dynamical mass generation

- infrared divergences of massless fields in dS lead to dynamical mass generation,

$$m_{\text{dyn}}^2 \propto H^2 \sqrt{\lambda}$$

Starobinsky-Yokoyama 1994,
Riotto-Sloth 2008, Burgess et al 2010,
Rajaraman 2010,....

the graviton propagator has exactly the same IR divergences

Antoniadis and Mottola 1986,....

mass generation forbidden in GR forbidden by diff invariance ?

Gauge-invariant (or diff-invariant) mass terms can be obtained with nonlocal operators

eg massive electrodynamics

Dvali 2006

$$\Gamma = -\frac{1}{4} \int d^4x \left(F_{\mu\nu} F^{\mu\nu} - m_\gamma^2 F_{\mu\nu} \frac{1}{\square} F^{\mu\nu} \right)$$

in the gauge $\partial_\mu A^\mu = 0$ we have

$$\frac{1}{4} m_\gamma^2 F_{\mu\nu} \frac{1}{\square} F^{\mu\nu} = \frac{1}{2} m_\gamma^2 A_\mu A^\mu$$

it is a nonlocal but gauge-inv photon mass term!

equivalently,
$$\left(1 - \frac{m_\gamma^2}{\square} \right) \partial_\mu F^{\mu\nu} = 0 \quad \rightarrow \quad \begin{cases} \partial_\mu A^\mu = 0 \\ (\square - m_\gamma^2) A^\mu = 0 \end{cases}$$

- Numerical results on the gluon propagator from lattice QCD and OPE are reproduced by adding to the quantum effective action a term

$$\frac{m_g^2}{2} \text{Tr} \int d^4x F_{\mu\nu} \frac{1}{\square} F^{\mu\nu}$$

$$F_{\mu\nu} = F_{\mu\nu}^a T^a, \quad \square^{ab} = D_\mu^{ac} D^{\mu,cb}, \quad D_\mu^{ab} = \delta^{ab} \partial_\mu - g f^{abc} A_\mu^c$$

(Boucaud et al 2001, Capri et al 2005, Dudal et al 2008)

it is a nonlocal but gauge invariant mass term for the gluons,
generated dynamically by strong IR effects

The degeneration idea is based on a phenomenological modification of Einstein equation

- massive photon: can be described by replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \rightarrow \quad \left(1 - \frac{m^2}{\square}\right) \partial_\mu F^{\mu\nu} = j^\nu \quad (\text{Dvali 2006})$$

- a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \rightarrow \quad \left(1 - \frac{m^2}{\square_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

assumed to emerge from IR quantum effects

(causality ok if interpreted as the eq of motion for the in-in expectation values)

inverting $\left(1 - \frac{m^2}{\square}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$

we get $G_{\mu\nu} = 8\pi G \frac{\square}{\square - m^2} T_{\mu\nu}$

the contribution from $8\pi G T_{\mu\nu} = \Lambda g_{\mu\nu}$ is zero!

The contribution of modes with $\omega \gg m$ is as in GR

Taking $m \approx H_0$, GR is recovered at short distances but the cosmological constant is degravitated down to the observed value

This model gives an explicit example of a non-trivial interplay between vacuum fluctuations and gravity

Unfortunately, the model is not really viable:

$$\left(1 - \frac{m^2}{\square}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \nabla^\mu (\square_g^{-1} G_{\mu\nu}) \neq 0$$

we lose energy-momentum conservation

However, this was the starting point for further investigations by our group

The logic of our approach: we will postulate some nonlocal term, that depends on a mass scale, and is supposed to catch IR effects in GR

- phenomenological approach. Identify a non-local modification of GR that works well
- attempt at a more fundamental understanding

- to preserve energy-momentum conservation:

(Jaccard,MM,
Mitsou, 2013)

$$S_{\mu\nu} = S_{\mu\nu}^T + \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$$

$$G_{\mu\nu} - m^2(\square^{-1}G_{\mu\nu})^T = 8\pi GT_{\mu\nu}$$

(Foffa,MM,
Mitsou, 2013)

however, instabilities in the cosmological evolution

- $G_{\mu\nu} - m^2(g_{\mu\nu}\square^{-1}R)^T = 8\pi GT_{\mu\nu}$ “RT model” (MM 2013)

to date, still the only fully viable model, among many possibilities
that we have studied (physical meaning: mass for the conformal mode)

- a related model:

(MM and M.Mancarella, 2014)

$$\Gamma_{\text{RR}} = \frac{m_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{6} R \frac{1}{\square^2} R \right]$$

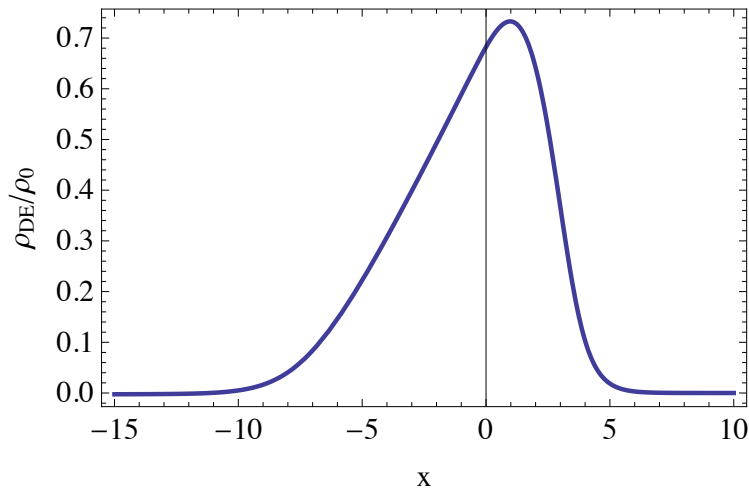
now ruled out by LLR

(Belgacem,Finke,Frassino,MM, 2019)

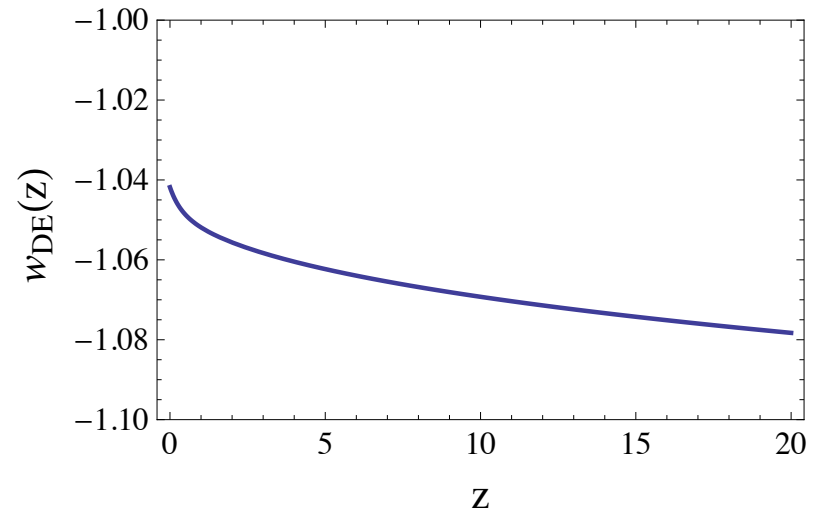
Features of the RT model

- predicts a dynamical DE (with a phantom EoS)

MM 2013



$$x \equiv \log a \quad (x_{\text{eq}} \simeq -8.1)$$



$$\dot{\rho}_{\text{DE}} + 3(1 + w_{\text{DE}})H\rho_{\text{DE}} = 0$$

Cosmological perturbations

- well-behaved? YES

Dirian, Foffa, Khosravi, Kunz, MM
JCAP 2014

this step is already non-trivial, see e.g. DGP or bigravity

- consistent with data? YES

- comparison with Λ CDM

Dirian, Foffa, Kunz, MM, Pettorino,
JCAP 2015, 2016

Dirian, 2017

Belgacem, Dirian, Foffa, MM 2018

implement the perturbations in a Boltzmann code

compute likelihood, χ^2 , perform parameter estimation

- We test the non-local models against
 - Planck 2015 TT, TE, EE and lensing data,
 - isotropic and anisotropic BAO data,
 - JLA supernovae,
 - local H_0 measurements,
 - growth rate data

and we perform Bayesian parameter estimation and model comparison

- we vary $\omega_b = \Omega_b h_0^2$, $\omega_c = \Omega_c h_0^2$, H_0 , A_s , n_s , z_{re}
we have the same free parameters as in Λ CDM

the model turns out to fit the data at a level statistically
equivalent to Λ CDM (actually slightly better)

Potentially distinguishable from Λ CDM with future cosmological
observations (EUCLID, SKA, DESI ...)

predicts higher value of H_0

(RT: $H_0 = 68.9$; Λ CDM: $H_0 = 67.9$ with our CMB+BAO+SNe datasets)

- the model passes solar system tests (including LLR)
- predicts GW propagation with $c_{\text{GW}}=c$
(ok with GW170817)
- and predicts an interesting and novel effect in GW propagation leading to a GW luminosity distance, testable with ET and LISA

Belgacem, Dirian, Foffa, MM, PRD 2018a,2018b
Belgacem et al (LISA CosmoWG), to appear

See Enis' talk

Conclusions

- In cosmology, it is conceivable that vacuum fluctuations might have a non-trivial interplay with gravity.
The idea leads to interesting and testable modified gravity models
- the relevance of these considerations for vacuum fluctuations at the nanoscale is not obvious.

However, experimental input is lacking and would certainly be very valuable

Thank you!