CASIMIR EFFECT AND FREE FALL IN A SCHWARZSCHILD BLACK HOLE

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INTRODUCTION

- THE PRESENT WORK IS PART OF A TWO-FOLD PROJECT, CARRIED ON IN COLLABORATION WITH S. A. FULLING (@ TEXAS A&M UNIV.) AND J.H. WILSON (@ CALTECH).
- THE MAIN GOAL IS TO ANALYZE THE BEHAVIOUR OF A CASIMIR APPARATUS FALLING INTO A BH, TAKING INTO ACCOUNT
  - TIDAL EFFECTS
  - NON-LOCAL EFFECTS

INTRODUCTION (cont'd)

- TIDAL EFFECTS, DUE TO THE SPATIAL EXTENSION OF THE CASIMIR APPARATUS, ARE RESPONSIBLE FOR A NON-UNIFORM DISTRIBUTION OF THE VACUUM ENERGY DENSITY INSIDE THE CAVITY.
- SUCH EFFECTS WILL BE CONSIDERED IN DETAIL IN A FORTHCOMING PAPER (W, S & F), WHERE A 1+1D MODEL HAS BEEN TAKEN INTO ACCOUNT.

INTRODUCTION (cont'd)

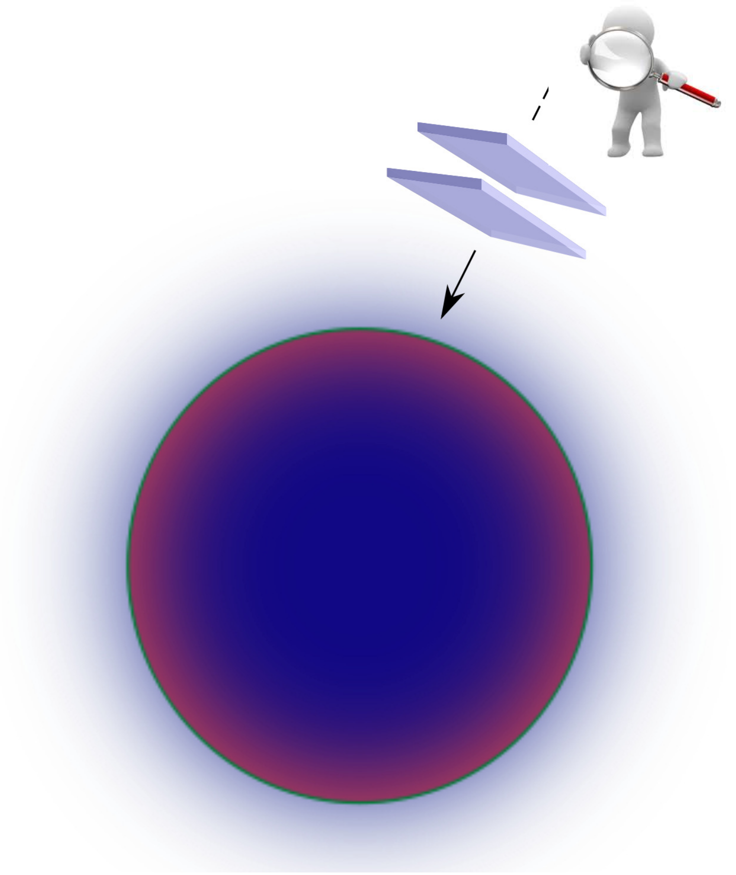
- NON-LOCAL (QUANTUM-MECHANICAL) EFFECTS ARE RELATED TO THE NON-LOCALITY OF THE THEORY DESCRIBING THE FIELD CONFINED TO THE CAVITY. IN SPITE OF THE INERTIAL FREELY FALLING MOTION, NON-LOCALITY ALLOWS FOR BOTH THE GLOBAL PROPERTIES OF THE BACKGROUND SPACETIME AND THE CAVITY MOTION TO INFLUENCE THE LOCAL MEASUREMENTS PERFORMED BY THE COMOVING OBSERVER.
- IN THIS TALK WE WILL FOCUS ON THIS LATTER ISSUE BY MEANS OF A 3+1D ANALYSIS (S & W).

OUTLINE

- THE SCENARIO
- BASIC ASSUMPTIONS
- SCHWARZSCHILD SPACETIME IN LEMAITRE COORDINATES
- PROPER-TIME SCHWINGER APPROACH AND THE EFFECTIVE ACTION
- STATIC CASIMIR EFFECT
- PARTICLE CREATION
- CONCLUSIONS

THE SCENARIO

- A SCHWARZSCHILD BH
- A CASIMIR APPARATUS, FREELY FALLING IN THE BH GRAVITATIONAL FIELD
- A COMOVING OBSERVER (W.R.T. THE CASIMIR CAVITY)

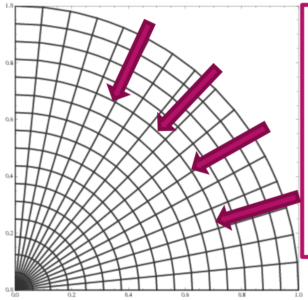


BASIC ASSUMPTIONS

- CASIMIR CAVITY FALLING FROM SPATIAL INFINITY WITH ZERO INITIAL VELOCITY AND ANGULAR MOMENTUM
- TYPICAL CAVITY SIZE MUCH SMALLER THAN THE BH GRAVITATIONAL RADIUS: $L << r_g$ (L = PLATE SEPARATION)
- RIGIDITY ASSUMPTION: CAVITY SIZE & SHAPE DO NOT SUFFER ANY DISTORTION (i.e., NO TIDAL EFFECTS ARE TAKEN INTO ACCOUNT)
- TRUE GEODESIC MOTION: OTHER NON-GRAVITATIONAL EFFECTS ARE NEGLECTED.

SCHWARZSCHILD SPACETIME IN LEMAITRE COORDINATES

- SCHWARZSCHILD METRIC IN LEMAITRE COORDINATES $\{\tau, \rho, \theta, \phi\}$ IS:
  [G. Lemaitre, Ann. Soc. Sci. I A53, 51 (1933)]



Coordinate grid comoving with a freely falling test body

$$ds^2 = d\tau^2 - \frac{r_g}{r(\tau)}d\rho^2 - r^2(\tau)d\Omega^2, \qquad r(\tau) = r_g\left(1 - \frac{3\tau}{2r_g}\right)^{2/3}$$

- WE ASSUME THE PROPER TIME $\tau = 0$ WHEN THE CAVITY CROSSES THE BH HORIZON:

$$\rho = \rho_0 = \frac{2}{3}r_g$$

$$\Rightarrow \quad -\infty < \tau \leq 0.$$

SCHWARZSCHILD SPACETIME IN LEMAITRE COORDINATES

- WE INTRODUCE A **TETRAD FRAME** ADAPTED TO THE COMOVING OBSERVER
- RECALL THAT

$$r = r(\tau) = r_g \left(1 - \frac{3\tau}{2r_g}\right)^{2/3}.$$

$$e_\tau = \partial_\tau$$
$$e_x = \sqrt{\frac{r}{r_g}}\partial_\rho$$
$$e_y = \frac{1}{r}\partial_\theta$$
$$e_z = \frac{1}{r\sin\theta}\partial_\phi.$$

tetrad as a "rigid" frame

PROPER-TIME SCHWINGER APPROACH – step 1

- WE FOCUS ON A MASSLESS SCALAR FIELD φ CONFINED TO THE CAVITY.
- IN THE COMOVING TETRAD FRAME, THE K-G EQUATION READS:

$$\left[\Box + \frac{1}{4}\frac{\xi^2}{(1-\xi\tau)^2}\right]\varphi = 0, \qquad \xi = \frac{3}{2r_g}.$$

- AS USUAL, REQUIRE DIRICHLET B.C. AT THE CAVITY PLATES.

PROPER-TIME SCHWINGER APPROACH – step 2

- K-G EQUATION CAN BE SOLVED. WE FIND

$$\varphi(x^a) \sim e^{i\vec{k}_\perp \cdot \vec{x}_\perp} \sin\left(\frac{n\pi}{L}x\right)\chi(\tau),$$

Tidal effects neglected

$$n \in N$$
$$\vec{k}_\perp \equiv (k_y, k_z)$$

- WHERE THE FIELD MODES ARE

$$\chi_k(\tau) = \frac{1}{2}\sqrt{\frac{\pi}{\xi}(1-\xi\tau)}H_0^{(1)}\left(\frac{\omega_k}{\xi}(1-\xi\tau)\right),$$

$$\omega_k^2 = k_\perp^2 + \left(\frac{n\pi}{L}\right)^2$$

PROPER-TIME SCHWINGER APPROACH – step 3

- WRITE THE PROPER-TIME HAMILTONIAN AS

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \partial_\tau^2 - \vec{\nabla}^2 \equiv -\hat{p}_0^2 + \hat{\vec{p}}^2$$

Flat space-time Hamiltonian

$$\hat{V} = \frac{1}{4}\frac{\xi^2}{(1-\xi\tau)^2}$$

Small time-dependent perturbation

PROPER-TIME SCHWINGER APPROACH – step 4

- ACCORDING TO SCHWINGER, THE EFFECTIVE ACTION IS:

$$W = \lim_{\nu \to 0} W(\nu),$$

- WHERE

$$W(\nu) = -\frac{i}{2} \int_0^\infty ds\, s^{\nu-1} \mathrm{Tr}\, e^{-is\hat{H}}$$

- TRACE HAS TO BE EVALUATED ALL OVER THE CONTINUOUS AS WELL THE DISCRETE DEGREES OF FREEDOM, THOSE OF SPACETIME INCLUDED

[see, e.g., J. Schwinger, Lett. Math. Phys. 24, 59 (1992)].

THE EFFECTIVE ACTION – step 1

- USING THE FIELD MODES FOUND ABOVE WE FIND, AFTER A QUITE LONG ALGEBRA (involving a lot of special function properties):

$$W(\nu) = -\frac{iA}{32\pi^{5/2}} \int_0^\infty ds \int_{-\infty}^{T} d\tau \sum_n \frac{s^{\nu-3/2-1}}{\beta^{1/2}} e^{-is(n\pi/L)^2}$$

$$\times \left[\pi^{3/2} e^{-i/(2\beta)} H_0^{(1)}(1/(2\beta)) + 2G_{23}^{31}\left(-\frac{i}{\beta} \middle| \begin{matrix} 0 & 1/2 & \\ 0 & 0 & 0 \end{matrix} \right) \right]$$

Hankel function

Meijer G-function

Adimensional parameter

$$\beta = \frac{s\xi^2}{(1-\xi\tau)^2}$$

THE EFFECTIVE ACTION – step 2

- CAREFUL INSPECTION OF $W(\nu)$ SHOWS US TWO CONTRIBUTIONS, ONE REAL :

$$\Re\mathrm{e}\, W = \lim_{\nu\to 0} W_H(\nu)$$

$$W_H(\nu) \stackrel{\text{def}}{=} -\frac{iA}{32\pi^{5/2}} \int_0^\infty ds \int_{-\infty}^{T} d\tau \sum_n \frac{s^{\nu-3/2-1}}{\beta^{1/2}}$$

$$\times e^{-is(n\pi/L)^2} \left[\pi^{3/2} e^{-i/(2\beta)} H_0^{(1)}(1/(2\beta))\right],$$

THE EFFECTIVE ACTION – step 3

- THE OTHER CONTRIBUTION IS IMAGINARY:

$$\Im\mathrm{m}\, W = \lim_{\nu \to 0} W_G(\nu)$$

$$iW_G(\nu) \overset{\text{def}}{=} -\frac{iA}{16\pi^{5/2}} \int_0^\infty ds \int_{-\infty}^{T} d\tau \sum_n \frac{s^{\nu-3/2-1}}{\beta^{1/2}}$$

$$\times e^{-is(n\pi/L)^2} G_{23}^{31}\left(-\frac{i}{\beta} \,\middle|\, \begin{matrix} 0 & 1/2 & \\ 0 & 0 & 0 \end{matrix} \right).$$

THE EFFECTIVE ACTION – VACUUM POLARIZATION

- EXPANDING IN POWERS OF THE ADIMENSIONAL PARAMETER β, WE GET THE PART RESPONSIBLE FOR VACUUM POLARIZATION
  (VACUUM ENERGY DENSITY → STATIC CASIMIR EFFECT)
- FOLLOWING THE STANDARD PRESCRIPTION WE HAVE:

$$\langle \epsilon_{Cas} \rangle = - \lim_{\nu \to 0} \frac{1}{AL} \frac{\partial}{\partial \tau} \Re\mathrm{e}\, W(\nu)$$

$$= - \frac{\pi^{3/2}}{16 L^4} \sum_{k=0}^{\infty} \frac{2^k \xi^{2k} a_k}{(1 - \xi\tau)^{2k}} \left(\frac{L}{\pi} \right)^{2k} \Gamma\left(-\frac{3}{2} + k \right) \zeta(-3 + 2k).$$

STATIC CASIMIR EFFECT

- TAKING THE LEADING ($k = 0$) AND THE NEXT TO LEADING ORDER ($k = 1$) TERMS, YIELDS

$$\langle \epsilon_{Cas} \rangle = -\frac{\pi^2}{1440 L^4} + \frac{1}{384 L^2} \frac{\xi^2}{(1 - \xi\tau)^2} + O(\xi^4).$$

- AT THE BH HORIZON:

$$\langle \epsilon_{Cas} \rangle_{hor} = -\frac{\pi^2}{1440 L^4} \left[1 - \frac{135}{(4\pi)^2} \left(\frac{L}{r_g} \right)^2 \right].$$

Not a tidal effect!

THE EFFECTIVE ACTION – VACUUM PERSISTENCE AMPLITUDE

- THE IMAGINARY PART OF $W(\nu)$ IS RELATED TO PARTICLE CREATION. IN THE IN-OUT FORMALISM WE HAVE

$$|\langle 0\,\mathrm{out}|0\,\mathrm{in}\rangle|^2 = e^{2i\Im\mathrm{m}W}$$

- AFTER SOME ALGEBRA WE OBTAIN

$$\Im\mathrm{m}W = \frac{A}{24\pi^3 L^2}\left[-\frac{\xi^2 L^2}{(1-\xi\tau)^2}\zeta(1) + \frac{2\xi^4 L^4}{15(1-\xi\tau)^4} + \cdots\right]$$

DIVERGENT !

THE EFFECTIVE ACTION – VACUUM PERSISTENCE AMPLITUDE

- IN SPITE OF THE DIVERGENT TERM, WE HAVE

$$\xi L = \frac{3L}{2r_g} \to 0 \Longrightarrow \Im\mathrm{m}\, W \to 0$$

- HENCE, IN THAT LIMIT, WE HAVE NO PARTICLE CREATION, AS EXPECTED.
- FURTHERMORE, A DIVERGENT PARTICLE NUMBER DENSITY DOESN'T IMPLIES A DIVERGENT ENERGY DENSITY IN CREATED PARTICLES.

$$\langle n \rangle \simeq \frac{2\, \Im\mathrm{m}\, W}{AL}$$

PARTICLE CREATION – step 1

- WE TRY TO GET RID OF THE ABOVE DIVERGENCE FOLLOWING THE BOGOLUBOV APPROACH.
- WRITE THE FIELD MODES AS

$$\chi_k(\eta) = \frac{1}{2}\sqrt{\frac{\pi}{\xi}}\eta H_0^{(1)}\left(\frac{\omega_k}{\xi}\eta\right), \qquad \eta = 1 - \xi\tau,$$

- THE ABOVE MODES HAVE THE REQUIRED MINKOWSKIAN BEHAVIOUR AT $\eta \to \infty$ (i.e. $\tau \to -\infty$), WHEN THE CAVITY IS AT THE SPATIAL INFINITY (→ FLAT SPACETIME REGION)

PARTICLE CREATION – step 2

- THE ABOVE MODES SATISFY THE BUNCH-DAVIES REQUIREMENTS

$$\left.\begin{array}{l} \chi_k(\eta) \to \frac{1}{\sqrt{2\omega_k}} e^{i\frac{\omega_k}{\xi}\eta} \sim \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k \tau} \\ \frac{\dot{\chi}_k(\eta)}{\chi_k(\eta)} \to i\frac{\omega_k}{\xi} \end{array}\right\} \qquad \eta \to \infty.$$

- IN THE FAR PAST, MODE SOLUTIONS HAVE THE USUAL PLANE WAVE FORM

$$\chi_k(\tau) = \frac{\alpha}{\sqrt{2\omega_k}} e^{-i\omega_k \tau} + \frac{\beta}{\sqrt{2\omega_k}} e^{i\omega_k \tau}$$

PARTICLE CREATION – step 3

- WE MATCH THE TWO SOLUTIONS NEAR THE BH HORIZON $(\eta = 1 - \xi\tau \geq 1)$

$$\frac{\alpha}{\sqrt{2\omega_k}}e^{-i\omega_k\tau} + \frac{\beta}{\sqrt{2\omega_k}}e^{i\omega_k\tau} = \frac{1}{2}\sqrt{\frac{\pi}{\xi}(1-\xi\tau)}H_0^{(1)}\left(\frac{\omega_k}{\xi}(1-\xi\tau)\right),$$

$$\frac{-i\omega_k\alpha}{\sqrt{2\omega_k}}e^{-i\omega_k\tau} + \frac{i\omega_k\beta}{\sqrt{2\omega_k}}e^{i\omega_k\tau} = \frac{1}{2}\sqrt{\frac{\pi}{\xi}}\left[\frac{-\xi}{2\sqrt{(1-\xi\tau)}}H_0^{(1)}\left(\frac{\omega_k}{\xi}(1-\xi\tau)\right) + \sqrt{1-\xi\tau}\omega_k H_1^{(1)}\left(\frac{\omega_k}{\xi}(1-\xi\tau)\right)\right]$$

- NEXT, WE SOLVE FOR THE α AND β BOGOLUBOV COEFFICIENTS

PARTICLE CREATION – step 4

- BOGOLUBOV COEFFICIENTS ARE

$$|\alpha_k|^2 = 1 + \frac{\xi^2}{16\omega_k^2(1-\xi\tau)^2}, \qquad |\beta_k|^2 = \frac{\xi^2}{16\omega_k^2(1-\xi\tau)^2},$$

- NUMBER DENSITY OF CREATED FIELD QUANTA IS

$$\langle n \rangle = \frac{1}{AL}\left[\frac{A}{(2\pi)^2}\sum_n \int d^2k_\perp \frac{\xi^2}{16\omega_k^2|\eta|^2}\right] = \frac{\xi^2}{64\pi L}\zeta(1)$$

DIVERGENT !

- THIS BASICALLY AGREES WITH OUR PREVIOUS RESULT, BASED UPON IMAGINARY PART OF EFFECTIVE ACTION.

PARTICLE CREATION – step 5

- HOWEVER, ENERGY DENSITY OF CREATED QUANTA IS FINITE

$$\langle \epsilon_{\rm dyn} \rangle = \frac{1}{AL}\left[\frac{A}{(2\pi)^2}\sum_n \int d^2k_\perp \frac{\xi^2}{16\omega_k^2|\eta|^2}\omega_k\right] = -\frac{\xi^2}{32L^2|\eta|^2}\zeta(-1) = \boxed{\frac{\xi^2}{384L^2(1-\xi\tau)^2}}.$$

- QUITE INTERESTINGLY THIS IS JUST THE SAME ENERGY AMOUNT WE FOUND LOOKING AT THE CORRECTION TO STATIC CASIMIR ENERGY

$$\langle \epsilon_{Cas} \rangle = -\frac{\pi^2}{1440L^4} + \frac{1}{384L^2}\frac{\xi^2}{(1-\xi\tau)^2}.$$

- A MERE COINCIDENCE?

CONCLUSIONS

- ONE COULD WONDER THAT CORRECTIONS TO THE STATIC CASIMIR EFFECT AS WELL AS PARTICLE CREATION ARE DETECTED BY AN OBSERVER IN A FREELY FALLING INERTIAL FRAME (where has the Equivalence Principle gone?)
- ACTUALLY, WHEN QUANTUM FIELDS ARE TAKEN INTO ACCOUNT, THE NON-LOCAL CHARACTER OF OF THE UNDERLYING QUANTUM THEORY CONFLICTS WITH THE EP, CAUSING THE LATTER TO BE NOT STRAIGHTFORWARDLY APPLICABLE

[see, e.g., M.O. Scully, S.A. Fulling, D. Lee, D. Page, W. Schleich & A. Svidzinsky, arXiv: quant-ph 1709.00481v2 (2017)].

CONCLUSIONS (cont'd)

- WHEN EVALUATING THE CASIMIR ENERGY, THE ENCOUNTERED DIVERGENCES ARE USUALLY CURED BY MEANS OF VARIOUS TECHNIQUES.
- ANY RENORMALIZATION PROCEDURE IMPLIES - MORE OR LESS EXPLICITLY - A MODE CUTOFF, SO THAT THE RENORMALIZED STRESS-ENERGY TENSOR $T^{\text{ren}}_{\mu\nu}$ IS DETERMINED BY THE LOW-ENERGY CONTRIBUTION OF THE FULL $T_{\mu\nu}$, THUS PROBING THE GLOBAL STRUCTURE OF THE BACKGROUND GEOMETRY (→GRAVITO-INERTIAL CONTRIBUTIONS) – [see, e.g., Birrell & Davies, QFCS].

CONCLUSIONS (cont'd)

- THE ABOVE RESULTS SEEM CLOSELY RELATED TO THE NUMBER OF INVOLVED SPATIAL DIMENSIONS. ACTUALLY, WE FOUND NO EFFECT IN A 1+1D CASIMIR CAVITY (priv. comm.).
- IS MAY BE THAT SOME (UNPHYSICAL) DIVERGENCES FOUND IN THE ABOVE ANALYSIS DO DISAPPEAR WHEN CONSIDERING A MORE PHYSICAL FINITE 3D APPARATUS.
- FINALLY, THE NEXT STEP WILL BE TO CONSIDER ALSO TIDAL EFFECTS IN A FULL 3D MODEL (WORK IN PROGRESS WITH H.J. WILSON & S. A. FULLING).

THANK YOU VERY MUCH

F. SORGE