Robert H. Dicke's varying vacuum index: A new paradigm for Cosmology?

From

X. Sarazin, F. Couchot, A. Djannati-Ataï, M. Urban,

Can the apparent expansion of the Universe be attributed to an increasing vacuum refractive index ?

Eur. Phys. J. C (2018) 78:444 (& arXiv:1805.03503)

Presented by F. Couchot **(most slides prepared by Xavier Sarazin) -** LAL, CNRS-IN2P3 / Univ. Paris-Saclay

Einstein's path toward General Relativity

➢ Einstein is the first one to note that the speed of light is modified in an accelerated frame, equivalent to a gravitation field *c* (*which allows to define a vacuum refractive index*):

- Einstein A., «*On the relativity principle and the conclusions drawn from it* », Jahrbuch fur radioact electronik 4, 411-462 (1907)
- Einstein A., «*Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes*', Annalen der Physik 35, 898-908 (1911)
- "*The constancy of the velocity of light can be maintained only insofar as one restricts oneself to spatio-temporal regions of constant gravitational potential*" (Einstein A., Ann. Physik 38 (1912) 1059)

https://einsteinpapers.press.princeton.edu/vol2-trans/290

Doc. 47

ON THE RELATIVITY PRINCIPLE AND THE CONCLUSIONS DRAWN FROM IT

by A. Einstein

[*Jahrbuch der Radioaktivität und Elektronik* 4 (1907): 411-462]

V. PRINCIPLE OF RELATIVITY AND GRAVITATION

§17. *Accelerated reference system and gravitational field*

So far we have applied the principle of relativity, i.e., the assumption that the physical laws are independent of the state of motion of the reference system, only to *nonaccelerated* reference systems. Is it conceivable that the principle of relativity also applies to systems that are accelerated relative to each other?

[…] These equations too have the same form as the corresponding equations of the nonaccelerated or gravitation-free space; however, c is here replaced by the value

$$c\left[1+\frac{\gamma\xi}{c^2}\right]=c\left[1+\frac{\Phi}{c^2}\right].$$

Einstein, A. Annalen der Physik 35, 898-908 (1911)

4. Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes; von A. Einstein.

Die Frage, ob die Ausbreitung des Lichtes durch die Schwere beinflußt wird, habe ich schon an einer vor 3 Jahren erschienenen Abhandlung zu beantworten gesucht.[1]) Ich komme auf dies Thema wieder zurück, weil mich meine damalige Darstellung des Gegenstandes nicht befriedigt, noch mehr aber, weil ich nun nachträglich einsehe, daß eine der wichtigsten Konsequenzen jener Betrachtung der experimentellen Prüfung zugänglich ist. Es ergibt sich nämlich, daß Lichtstrahlen, die in der Nähe der Sonne vorbeigehen, durch das Gravitationsfeld derselben nach der vorzubringenden Theorie eine Ablenkung erfahren, so daß eine scheinbare Vergrößerung des Winkelabstandes eines nahe an der Sonne erscheinenden Fixsternes von dieser im Betrage von fast einer Bogensekunde eintritt.

Es haben sich bei der Durchführung der Überlegungen auch noch weitere Resultate ergeben, die sich auf die Gravitation beziehen. Da aber die Darlegung der ganzen Betrachtung ziemlich unübersichtlich würde, sollen im folgenden nur einige ganz elementare Überlegungen gegeben werden, aus denen man sich bequem über die Voraussetzungen und den Gedankengang der Theorie orientieren kann. Die hier abgeleiteten Beziehungen sind, auch wenn die theoretische Grundlage zutrifft, nur in erster Näherung gültig.

§ 1. Hypothese über die physikalische Natur des Gravitationsfeldes.

In einem homogenen Schwerefeld (Schwerebeschleunigung γ) befinde sich ein ruhendes Koordinatensystem K, das so orientiert sei, daß die Kraftlinien des Schwerefeldes in Richtung

1) A. Einstein, Jahrb. f. Radioakt. u. Elektronik IV. 4.

Hieraus ergibt sich eine Konsequenz von für diese Theorie fundamentaler Bedeutung. Mißt man nämlich in dem beschleunigten, gravitationsfeldfreien System K' an verschiedenen Orten die Lichtgeschwindigkeit unter Benutzung gleich beschaffener Uhren U, so erhält man überall dieselbe Größe. Dasselbe gilt nach unserer Grundannahme auch für das System K. Nach dem soeben Gesagten müssen wir aber an Stellen verschiedenen Gravitationspotentials uns verschieden beschaffener Uhren zur Zeitmessung bedienen. Wir müssen zur Zeitmessung an einem Orte, der relativ zum Koordinatenursprung das Gravitationspotential Φ besitzt, eine Uhr verwenden, die — an den Koordinatenursprung versetzt — $(1 + \Phi/c^2)$ mal langsamer läuft als jene Uhr, mit welcher am Koordinatenursprung die Zeit gemessen wird. Nennen wir c_0 die Lichtgeschwindigkeit im Koordinatenanfangspunkt, so wird daher die Lichtgeschwindigkeit c in einem Orte vom Gravitationspotential Φ durch die Beziehung

$$c = c_0 \left(1 + \frac{\Phi}{c^2}\right) \tag{3}$$

gegeben sein. Das Prinzip der Konstanz der Lichtgeschwindigkeit gilt nach dieser Theorie nicht in derjenigen Fassung, wie es der gewöhnlichen Relativitätstheorie zugrunde gelegt zu werden pflegt.

§ 4. Krümmung der Lichtstrahlen im Gravitationsfeld.

Aus dem soeben bewiesenen Satze, daß die Lichtgeschwindigkeit im Schwerefelde eine Funktion des Ortes ist, läßt sich leicht mittels des Huygensschen Prinzipes schließen, daß quer zu einem Schwerefeld sich fortpflanzende Lichtstrahlen eine Krümmung erfahren müssen. Sei nämlich ε eine Ebene gleicher

Annalen der Physik - June 1911 -

Translated by Michael D. Godfrey
Information Systems Lab, Stanford University

On the Influence of Gravitation on the Propagation of Light

By A. Einstein.

[…] This has a consequence which is of fundamental importance for our theory. For if we measure the velocity of light at different locations in the accelerated, gravitation-free system K', employing clocks U of identical properties we obtain the same magnitude at all these locations. The same holds good, by our fundamental assumption, for the system K as well. But from what has just been said we must use clocks of unlike properties for measuring time at locations with differing gravitation potential. For measuring time at a location which, relative to the origin of the co-ordinates, has the gravitation potential Φ, we must employ a clock which – when transferred to the co-ordinate origin – goes $(1 + \Phi/c^2)$ times more slowly than the clock used for measuring time at the origin of co-ordinates. If we call the velocity of light at the origin of co-ordinates c_0, then the velocity of light c at a location with the gravitation potential Φ will be given by the relation

$$c = c_0\left(1 + \frac{\Phi}{c^2}\right). \tag{3}$$

The principle of the constancy of the velocity of light holds good according to this theory in a different form from that which usually underlies the ordinary theory of relativity.

§ 4. Bending of Light-Rays in the Gravitational Field

From the proposition which has just been proved, that the velocity of light in the gravitational field is a function of the location, we may easily infer, by means of Huygens's principle, that light-rays propagated across a gravitational field undergo

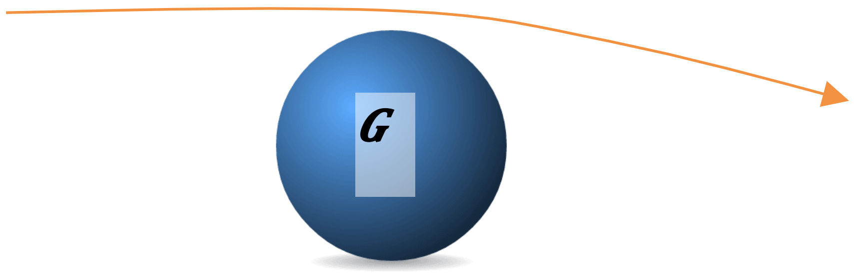
Path to GR

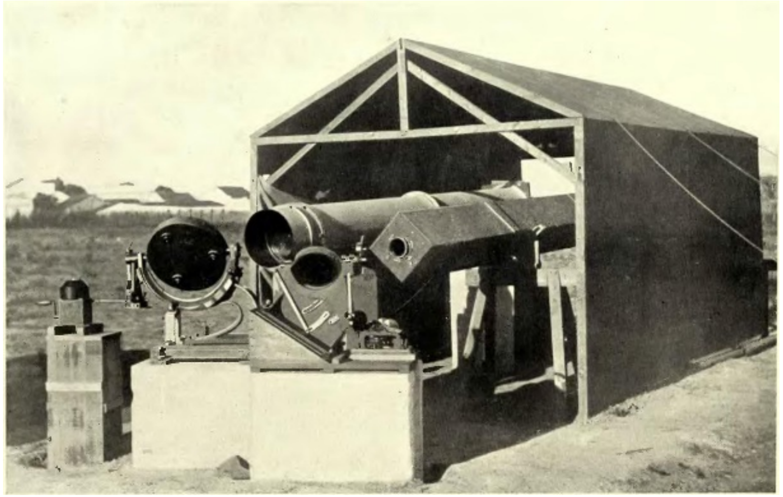
- So, Einstein first used the notion of vacuum refractive index and thought c is affected by gravitation.
- But in the end he generalized the « » relativity principle thanks to the introduction of a ***curved spacetime metric***

$\Rightarrow$ General Relativity is a « *geo-metric* » theory

$\Rightarrow$ Vacuum has no physical role anymore

➡ Deflection of light first observed by Eddington in 1919, in agreement with GR





Path to GR

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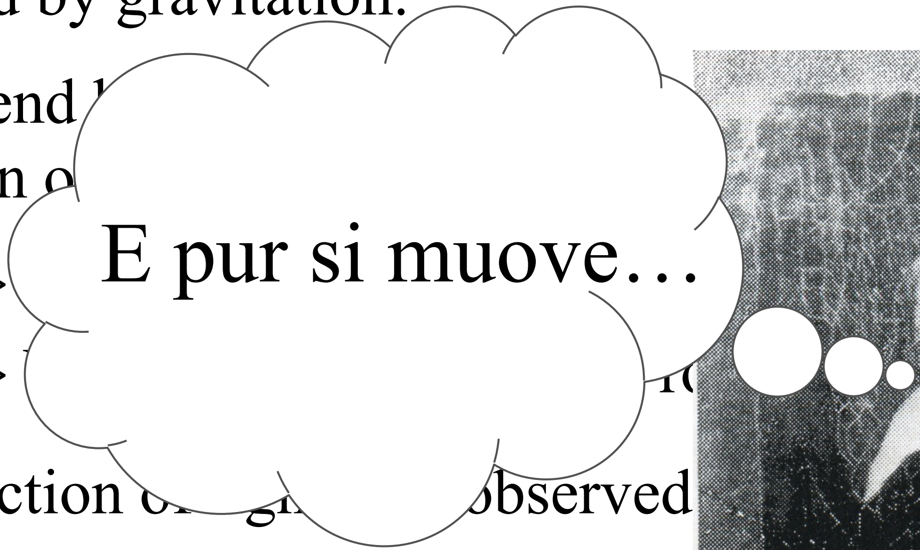

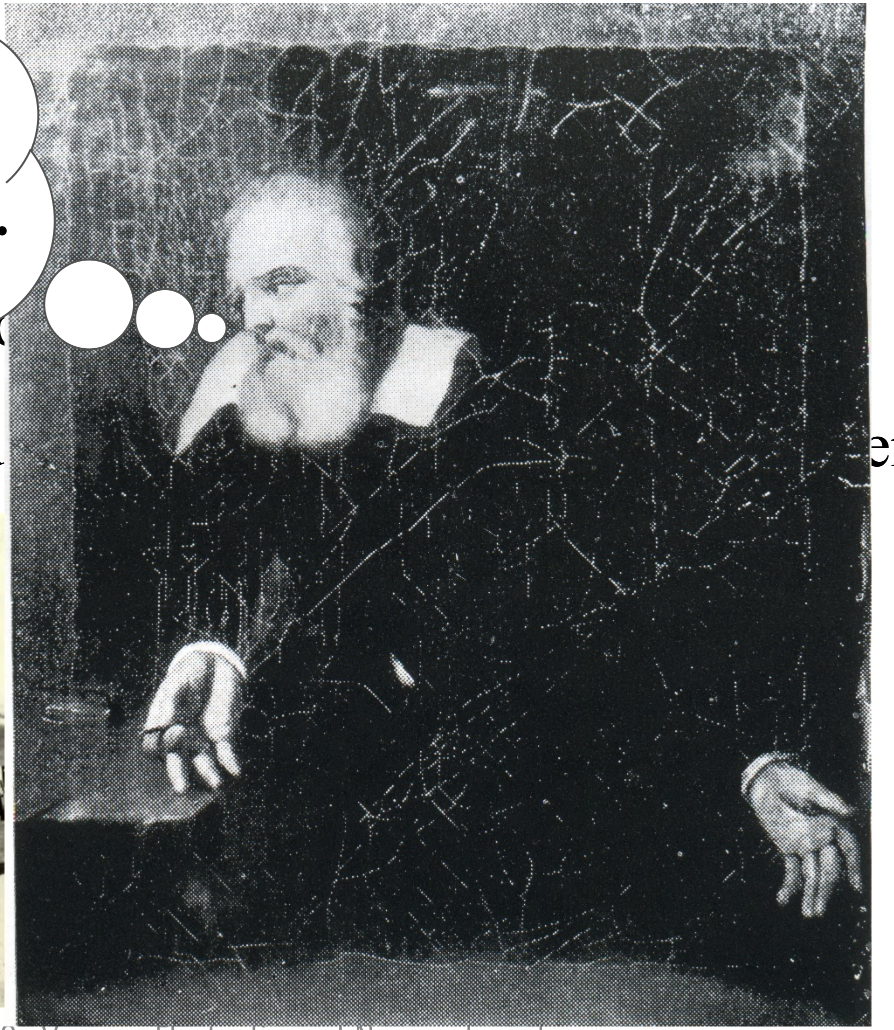

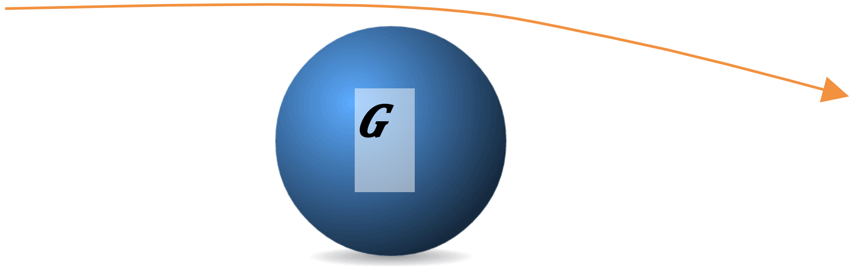

1rst Step

Back to Euclidian space-time and vacuum index

- An empirical approach initially proposed by Wilson (1921), who inspired Dicke (1957) (who inspired this work ☺)
  - ✓ **Euclidean flat metric**
  - ✓ **Spatial change of ε_0 and μ_0** induced by the gravitational potential

    $\Rightarrow$ Modification of the vacuum optical index and inertial test mass

H.A. Wilson, Phys. Rev. ***17****, 54 (1921)*

AN ELECTROMAGNETIC THEORY OF GRAVITATION.

By H. A. Wilson.

Synopsis.

An Electromagnetic Theory of Gravitation.—An electric system in a medium whose specific inductive capacity k varies from point to point tends to move in the direction of increasing k. It is suggested that if we *assume the specific inductive capacity of the ether to vary near matter*, gravitation may be explained as a result of this tendency. In a medium in which at a distance r from a mass m, $k = 1 + m/r$, it is shown that a rigid electrostatic system would be acted on by a force directed toward m and equal to mm'/r^2, where m' is the electromagnetic mass of the system. But in order to *explain the observed deflection of light by the sun* we must have $k = 1 + 2m/r$; and this will not give the force mm'/r^2 unless the system contracts in the ratio of $1 : 1 - m/r$. A physical explanation of this assumed contraction is suggested. If the system with the mass m' is also supposed to modify k, it is necessary to take into account the energy changes in m and in the ether. The *effect of gravitation on the frequency of the light emitted by an atom*, which was predicted by Einstein, can be easily deduced from the present theory.

Dicke, Rev. Mod. Phys. ***29****, 363 (1957)*

REVIEWS OF MODERN PHYSICS VOLUME 29, NUMBER 3 JULY, 1957

Gravitation without a Principle of Equivalence

R. H. Dicke

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The previous article has considered the observational and experimental facts and has concluded that there is no substantial evidence to support the belief that the coupling constants of the weak interactions are independent of time or place. Consequently, it is possible that the principle of equivalence may be satisfied, if at all, only when the contributions to the binding energy of a system having their origin in the weak interactions are neglected. This article considers a form which a theory of gravitation may take when the principle of equivalence is satisfied in a weakened form only.

Jordan has previously considered a similar problem,* and Fierz has made a critical analysis of Jordan's theory.†

The great difficulty with constructing a theory of gravitation is the paucity of experimental evidence. After 40 years there are still only the four famous observational checks of the theory of relativity. Of these only two have any real accuracy. With so few experimental facts to guide one, any number of *ad hoc* theories can be constructed. To choose between them, standards going beyond the observational evidence must be introduced. The danger of judging a theory on the basis of elegance, simplicity, or perfection is obvious.

While "elegance" may not be a valid criterion for judging a theory, there are a few rules for the construction of a formalism which if followed should improve the prospects for later agreement with observation. First, it should be noted that there is much experimental evidence on the validity of the Lorentz invariance of

nates, the curvatures of a space are modified. With a proper redefinition of units making them dependent upon coordinates and orientation of an infinitesimal interval a curved space can be converted into a flat one and vice versa. Rosen[1] has shown how to formulate general relativity within the framework of a flat metric.

To illustrate the arbitrary character of the choice of metric tensor, consider the following physical example. Twelve identical rods can be normally assembled into a hexagonal pattern with 6 rods joining at the center. If this assembly is carried out in a suitable gravitational field, the 6 rods no longer join at the center. There are at least two geometrical explanations for this result. The conventional one is that the rods have not changed but are now in a curved space which "causes" a gap to open at the center of the geometrical figure. Another possible explanation is that the gravitational field has shortened the radial rods relative to the circumferential ones. The two explanations are equivalent in the sense that they both agree as to the existence of the gap in the geometrical figure constructed out of real atoms.

It has been argued[2] that space is "really" curved and that the rods do not change their "real" length. Without splitting hairs over the meaning of the word "really" this argument is based on the assumption that such changes are presumably independent of the material out of which the rods are constructed. However, all rods are constructed from electrons, protons, and neutrons held together almost completely by the strong interactions. They have a common structure and could vary in length in a common way.

Robert Dicke: a flat metric gravitation model linked to vacuum fluctuations

[…] simplest of several alternatives should be chosen. The theory to be described accepts Mach's principle, the cosmological principle, and is generally covariant. Also as much of the principle of equivalence as is supported by the Eötvös experiment is accepted.

The general features of a theory of gravitation without a principle of equivalence are easily outlined. The motivation for introducing a Riemannian metric into the geometry of space and time is now largely absent, as there is no single universal gravitational acceleration at a given space-time point. Simply by redefining units of length and time as functions of space-time coordinates, the curvatures of a space are modified. With a proper redefinition of units making them dependent upon coordinates and orientation of an infinitesimal interval a curved space can be converted into a flat one and vice versa. Rosen[1] has shown how to formulate general relativity within the framework of a flat metric.

[…] By analogy with electromagnetic or meson force fields it is reasonable to expect that when viewed sufficiently closely the gravitational effects would be quantum in nature. Stated more exactly, it might be expected that the gravitational force acting on an elementary particle would have its origin in a local interaction of the particle with virtual particles present in the vacuum.

Remembering the virtual electron-positron pairs reputed to be present in the vacuum as a result of zero point fluctuations, it becomes interesting to inquire whether the gravitational effect can be linked to these particles already present. If so, it should be possible eventually to construct a theory of particles and obtain the gravitational interaction as a weak effect connected with more primitive strong interactions. A less ambitious approach would be to start in the middle of the problem, to ignore the quantum aspects of the interaction of a particle with a bath of virtual particles, and to treat this interaction as a classical field.

Gravitation and Vacuum

- Another empirical approach initially proposed by Wilson (1921) and Dicke (1957)
  - ✓ **Euclidean flat metric**
  - ✓ **Spatial change of ε_0 and μ_0** by the gravitational potential

    $\Rightarrow$ Modification of the vacuum optical index and inertial test mass

- **Exemple : Static spherical gravitational field** *(Wilson-Dicke Analogy)*

$$\begin{cases} n(r) \cong 1 + \dfrac{2GM}{rc_\infty^2} \\ m(r) = m_\infty \times n^{3/2}(r) \end{cases}$$

(to preserve the equivalence principle) $\rightarrow M\,c^2$ scales like an energy

$$\begin{cases} \varepsilon_0(r) = n(r) \times \varepsilon_{0,\infty} \\ \mu_0(r) = n(r) \times \mu_{0,\infty} \\ c(r) = n^{-1}(r) \times c_\infty \\ E_{atom}(r) = n^{-1/2}(r) \times E_{atom,\infty} \\ L(r) = n^{-1/2} \times L_\infty \qquad (L \equiv L_{\text{ruler}}) \end{cases}$$

$e, \hbar$ are constant $\rightarrow T(r) = n^{1/2} \times T_\infty \quad (T \equiv T_{\text{clock}})$

$\Rightarrow \alpha = \dfrac{e^2}{4\pi\varepsilon_0\hbar c}$ is constant

Gravitation and Vacuum

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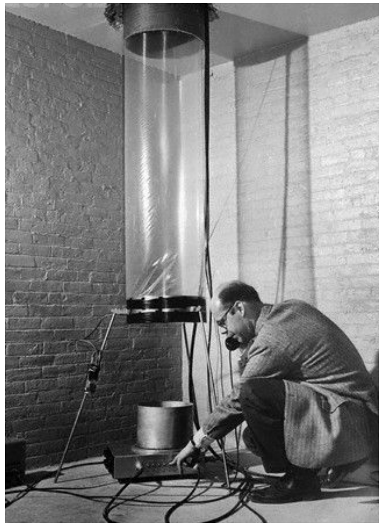
(to preserve the equivalence principle)

- $\boldsymbol{n(r)}$ **formally identical to $\boldsymbol{g_{00}}$ in General Relativity**

  $\Rightarrow$ *See Landau & Lifshitz (1975) : "A static gravitational field is formally identical to a medium with electric and magnetic permeabilities* $\varepsilon_0 = \mu_0 = 1/\sqrt{g_{00}}$*"*

Redshift "*à la* Dicke" viewed from the Energy side

Rebka & Pound 1959



E_γ = Constant

$$E_{\rm level}(r+h) = \frac{E^{\infty}_{\rm level}}{\sqrt{n(r+h)}} \qquad n(r+h) = 1 + \frac{2G_N M}{(r+h)c^2_\infty}$$

- The photon energy keeps constant during its propagation
- The atomic energy levels are really modified

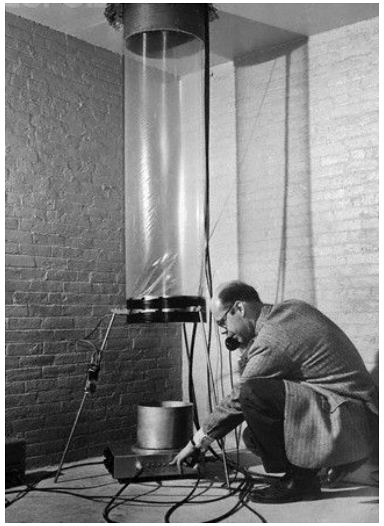
$$E_{\rm level}(r) = \frac{E^{\infty}_{\rm level}}{\sqrt{n(r)}} < E_{\rm level}(r+h)$$

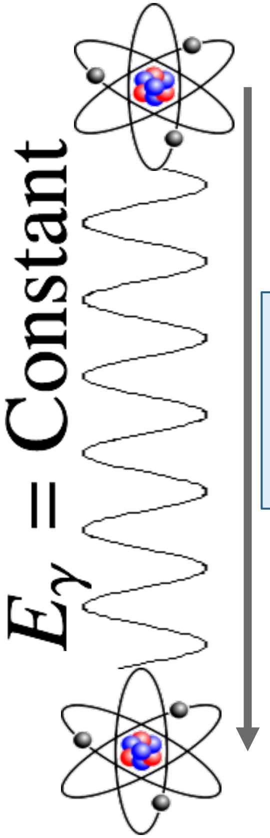
$$\Delta E_{\rm level} \approx E_{\rm level}(r) \times \frac{G_N M}{r^2 c^2_\infty} \times h$$

in agreement with G.R.

Redshift “*à la* Dicke” viewed from the wavelength side

Rebka & Pound 1959





$$L_{\text{ruler}}(r+h) = \frac{L^{\infty}_{\text{ruler}}}{\sqrt{n(r+h)}}$$

- The photon energy keeps constant during its propagation
- The ruler lengths are really modified

$$L_{\text{ruler}}(r) = \frac{L^{\infty}_{\text{ruler}}}{\sqrt{n(r)}} < L_{\text{ruler}}(r+h)$$

$$E_\gamma = \text{Constant} = \frac{hc}{\lambda} \rightarrow \lambda \propto c \rightarrow \lambda \propto 1/n$$

So the gamma ray is well seen as blueshifted on the lower side

Landau & Lifshitz, The Classical Theory of Fields, 3rd revised English version (1971)

PROBLEM

Write the Maxwell equations in a given gravitational field in three-dimensional form (in the three-dimensional space with metric $\gamma_{\alpha\beta}$), introducing the three-vectors **E**, **D** and the antisymmetric three-tensors $B_{\alpha\beta}$ and $H_{\alpha\beta}$ according to the definitions:

$$E_\alpha = F_{0\alpha}, \qquad B_{\alpha\beta} = F_{\alpha\beta},$$
$$D^\alpha = -\sqrt{g_{00}}\, F^{0\alpha}, \qquad H^{\alpha\beta} = \sqrt{g_{00}}\, F^{\alpha\beta}. \tag{1}$$

Solution: The quantities introduced above are not independent. Writing out the equations

$$F_{0\alpha} = g_{0l} g_{\alpha m} F^{lm}, \qquad F^{\alpha\beta} = g^{\alpha l} g^{\beta m} F_{lm},$$

and introducing the three-dimensional metric tensor $\gamma_{\alpha\beta} = -g_{\alpha\beta} + h g_\alpha g_\beta$ [with g and h from (88.11)], and using formulas (84.9) and 84.12), we get:

$$D_\alpha = \frac{E_\alpha}{\sqrt{h}} + g^\beta H_{\alpha\beta}, \qquad B^{\alpha\beta} = \frac{H^{\alpha\beta}}{\sqrt{h}} + g^\beta E^\alpha - g^\alpha E^\beta. \tag{2}$$

We introduce the vectors **B** and **H**, dual to the tensors $B_{\alpha\beta}$ and $H_{\alpha\beta}$, in accordance with the definition:

$$B^\alpha = -\frac{1}{2\sqrt{\gamma}} e^{\alpha\beta\gamma} B_{\beta\gamma}, \qquad H_\alpha = -\frac{1}{2}\sqrt{\gamma}\, e_{\alpha\beta\gamma} H^{\beta\gamma} \tag{3}$$

(see the footnote on p. 252; the minus sign is introduced so that in galilean coordinates the vector: **H** and **B** coincide with the ordinary magnetic field intensity). Then (2) can be written in the forms

$$\mathbf{D} = \frac{\mathbf{E}}{\sqrt{h}} + \mathbf{H}\times\mathbf{g}, \qquad \mathbf{B} = \frac{\mathbf{H}}{\sqrt{h}} + \mathbf{g}\times\mathbf{E}. \tag{4}$$

Introducing definition (1) in (90.2), we get the equations:

$$\frac{\partial B_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial B_{\gamma\alpha}}{\partial x^\beta} + \frac{\partial B_{\beta\gamma}}{\partial x^\alpha} = 0,$$
$$\frac{\partial B_{\alpha\beta}}{\partial x^0} + \frac{\partial E_\alpha}{\partial x^\beta} - \frac{\partial E_\beta}{\partial x^\alpha} = 0,$$

or, changing to the dual quantities (3):

$$\operatorname{div}\mathbf{B} = 0, \qquad \operatorname{curl}\mathbf{E} = -\frac{1}{c\sqrt{\gamma}}\frac{\partial}{\partial t}(\sqrt{\gamma}\,\mathbf{B}) \tag{5}$$

($x^0 = ct$; the definitions of the operations div and curl are given in the footnote on p. 252). Similarly we find from (90.6) the equations

$$\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial x^\alpha}(\sqrt{\gamma} D^\alpha) = 4\pi\varrho,$$
$$\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial x^\beta}(\sqrt{\gamma}\, H^{\alpha\beta}) + \frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial x^0}(\sqrt{\gamma}\, D^\alpha) = -4\pi\varrho\frac{dx^\alpha}{dx^0},$$

or, in three-dimensional notation:

$$\operatorname{div}\mathbf{D} = 4\pi\varrho, \qquad \operatorname{curl}\mathbf{H} = \frac{1}{c\sqrt{\gamma}}\frac{\partial}{\partial t}(\sqrt{\gamma}\,\mathbf{D}) + \frac{4\pi}{c}\mathbf{s}, \tag{6}$$

where **s** is the vector with components $s^\alpha = \varrho\, dx^\alpha/dt$.

We also write the continuity equation (90.5) in three-dimensional form:

$$\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial t}(\sqrt{\gamma}\,\varrho) + \operatorname{div}\mathbf{s} = 0. \tag{7}$$

The reader should note the analogy (purely formal, of course) of equations (5) and (6) to the Maxwell equations for the electromagnetic field in material media. In particular, in a static gravitational field the quantity $\sqrt{\gamma}$ drops out of the terms containing time derivatives, and relation (4) reduces to $\mathbf{D} = \mathbf{E}/\sqrt{h}$, $\mathbf{B} = \mathbf{H}/\sqrt{h}$. We may say that with respect to its effect on the electromagnetic field a static gravitational field plays the role of a medium with electric and magnetic permeabilities $\varepsilon = \mu = 1/\sqrt{h}$.

Landau & Lifshitz, The Classical Theory of Fields, 3rd revised English version (1971)

Coherent and in agreement with GR for static cases

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Write the Maxwell equations in a given gravitational field in three-dimensional form (in the three-dimensional space with metric $\gamma_{\alpha\beta}$), introducing the three-vectors **E**, **D** and the antisymmetric three-tensors $B_{\alpha\beta}$ and $H_{\alpha\beta}$ according to the definitions:

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$$\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial x^\alpha}(\sqrt{\gamma} D^\alpha) = 4\pi\varrho,$$
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$\varepsilon = \mu = 1/\sqrt{h}$.

2^{nd} Step

Cosmology with a vacuum index increasing with time

- **1st Dicke's remark**

$$n(r) = 1 + \frac{2GM}{rc_\infty^2}$$

?

Dicke's idea: $1 = n(t=0) = \int^{R_{\mathrm{Max}}(t)} \frac{2G(r)4\pi\rho r^2}{rc^2(r)} dr$

$\Rightarrow n(t)$ increases with time

- Linked to the Mach's principle, developped in:
  - Sciama, D. W. 1953. On the origin of inertia. Monthly Notices of the Royal Astronomical Society 113: 34-42
  - Sciama, D. W. 1964. The Physical Structure of General Relativity. Reviews of Modern Physics 36: 463-469
- Cited in: Peebles, P.J.E. Robert Dicke and the naissance of experimental gravity physics, 1957-1967, EPJ H (2017) 42: 177.

Cosmology with a vacuum index increasing with time

- **2nd Dicke's remark**

e.m. waves propagating through a medium with a uniform index varying in time have the following property:

→ Frequency ν (Energy $h\nu$) varies with time like $\nu(t)=\nu_0/n(t)$

(simply from Maxwell equations)

$$\mathbf{D} = \epsilon \mathbf{E} \,, \quad \mathbf{B} = \mu \mathbf{H} \qquad \nabla \cdot \mathbf{B} = 0 \,, \quad \nabla \cdot \mathbf{D} = 0$$

$$\epsilon = n\epsilon_0 \,, \quad \mu = n\mu_0 \qquad \nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times H = \frac{\partial D}{\partial t}$$

Plane wave polarized along *x* and propagating along *z*, in a uniform time varying index *n(t)*

Propagation equation

$$\frac{\partial^2 D_x}{\partial z^2} - \left(\frac{n}{c}\right)^2 \frac{\partial^2 D_x}{\partial t^2} - \frac{n\dot{n}}{c^2}\frac{\partial D_x}{\partial t} = 0$$

Exact solution

$$D_x(z,t) = D^0 \; e^{-i \; k\left[c \int_{t_0}^{t} \frac{d\tau}{n(\tau)} - z\right]}$$

Cosmology with a vacuum index increasing with time

We assume :

- Flat and static metric (x,y,z,t) $\rightarrow$ **There is no expansion of the metric**
- The metric is defined by the speed of light today $c_0 = c(t=0)$

$$n(t=0) = 1 \qquad \text{and} \qquad dt^2 = 1/c_0^2 \times (dx^2 + dy^2 + dz^2)$$

- $\boldsymbol{n(t)}$ **increases with time**
- The relative index variation is time-independent (at least for recent epoch of the Univers)

$$dn(t)/n(t) = \text{constant} \Rightarrow n(t) = \exp(-t/\tau_0)$$

- **A photon propagates in vacuum with $\boldsymbol{\lambda}$ = constant, and $\boldsymbol{\nu(t) = \nu_0/n(t)}$**
- **Spacetime metric expansion is replaced by an increase with time of $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$**

$$\begin{cases} \varepsilon_0(t) = n(t) \times \varepsilon_{0,0} \\ \mu_0(t) = n(t) \times \mu_{0,0} \\ c(t) = n^{-1}(t) \times c_0 \\ E_{atom}(t) = n^{-1/2}(t) \times E_{atom,0} \\ \mathrm{m}(t) = n^{3/2}(t) \times m_0 \end{cases}$$

$e, \hbar$ are constant $\Rightarrow \alpha$ constant

$$T(t) = n^{1/2}(t) \times T_0 \qquad (T \equiv T_{\text{clock}})$$

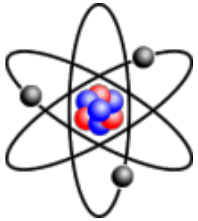
$$L(t) = n^{-1/2}(t) \times L_0 \qquad (L \equiv L_{\text{ruler}})$$

Origin of the cosmological redshift

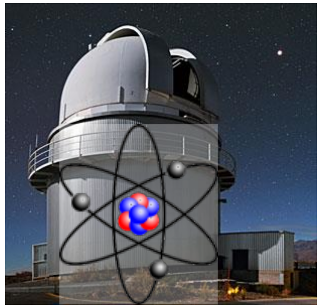
Supernova* *or any other distant light source — t<0

Telescope — t = 0

Cosmological time



$\lambda = \text{constant} = \lambda_{emission}(t)$



$E_{atom}(t)$

$E_{atom,0}(\text{reference}) = \sqrt{n(t)} \times E_{atom}(t) < E_{atom}(t)$

$$\left.\begin{aligned}\lambda_{observed} = \lambda_{emission} &= \frac{hc(t)}{E_{atom}(t)} \\ \lambda_{reference} = \lambda_0 &= \frac{hc_0}{E_{atom,0}}\end{aligned}\right\} \Rightarrow 1 + z = \frac{\lambda_{observed}}{\lambda_{reference}} = \frac{c(t)}{c_0}\frac{E_{atom,0}}{E_{atom}(t)} = \frac{1}{n(t)}\sqrt{n(t)} = \frac{1}{\sqrt{n(t)}} > 1 \qquad (t < 0)$$

$$\Rightarrow \quad n(t) = \frac{1}{(1+z)^2} \qquad \sqrt{n(t)} \equiv a(t)$$

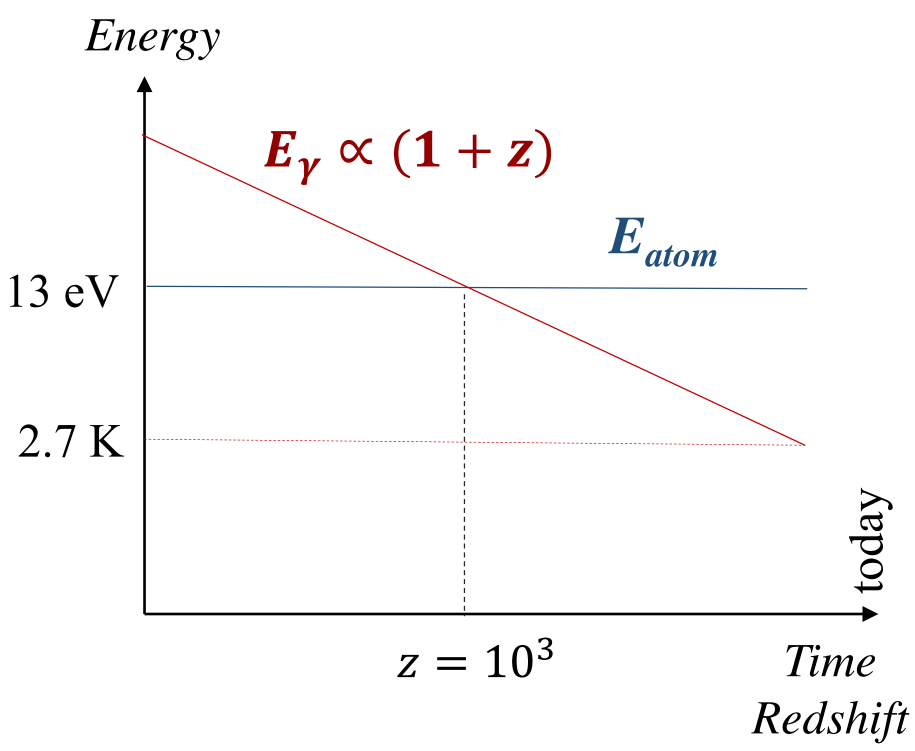
Refractive index increasing with time ⇒ Redshift
(speed of light decreasing with time)

Dicke's vaccum index cosmology

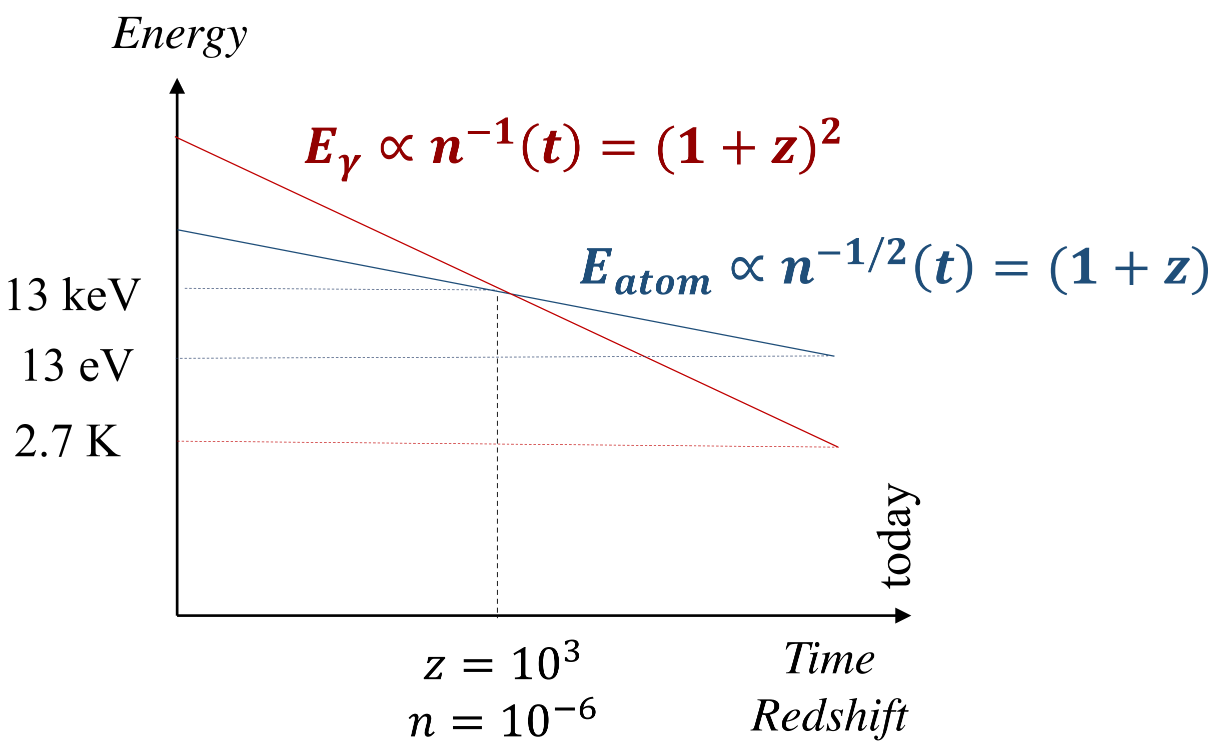
- Of course this vision misses RG, Einstein equations and all that
  - No model for *n(t)* except from Mach's principle:
  - Test solution $n(t)=e^{(t/\tau 0)}$, $t<0$ in the past
- But interesting to confront it with some standard cosmological probes
  - CMB
  - SN1a
  - …

Evolution of the CMB

Standard Cosmology



Vacuum index model



$E_\gamma / E_{atom} \propto (1+z)$ in both models

Evolution of the CMB

The energy density $\mathcal{E}_\gamma$ of the CMB radiation is $\mathcal{E}_\gamma(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$

- **Standard cosmology:**

Energy γ : $E_\gamma \propto (1+z)$

Energy mass of baryons $E_b =$ constante

$\mathcal{E}_\gamma = n_\gamma \times E_\gamma \propto (1+z)^3 \times (1+z) = (1+z)^4$

$$\mathcal{E}_\gamma = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4 \quad \left.\begin{array}{r} \Rightarrow T = 1+z \\ \nu = 1+z \end{array}\right\} \Rightarrow \text{CMB black body shape is preserved}$$

- **Cosmology with increasing index:**

$$\left.\begin{array}{l} E_\gamma \propto n^{-1}(t) = (1+z)^2 \\ E_b = mc^2 \propto n^{-1/2}(t) = (1+z) \end{array}\right\} \Rightarrow \text{Apparent energy } \gamma\text{, relatively to baryon, decreases as } n^{-1/2}(t) = (1+z)$$

In a volume defined with physical rods, $n_\gamma \propto n^{-3/2}(t) = (1+z)^3$

⇒ The energy density $\mathcal{E}_\gamma$, relatively to baryons, decreases as $(1+z)^4$, as in standard cosmology

⇒ If k_B is constant (as $\hbar$), then the temperature (relatively to physical temp.°) $T \propto n^{-1/2}(t) = 1+z$, as in standard cosmology, and the black body spectral shape is preserved

Also n_γ/n_b is constant with time

Fit Supernovae Type Ia

Hubble diagram: Distance modulus μ_{mes} vs redshift z

$$\mu_{mes} = m_b - M_b + \alpha X - \beta C = 5 log_{10}\left(\frac{d_L}{10\text{ pc}}\right)$$

- X = stretch factor
- C = color-band factor
- α and β : global nuisance parameters

m_b = magnitude at peak $= -2.5\log(\mathcal{F}) + M_b$ $\quad M_b = -19.25$ (*Richardson, AJ, 2014*)

$\mathcal{F}$ = obs. flux in the SNIa rest frame (at emission) $= \dfrac{\mathcal{L}}{4\pi d_L^2 (1+z)^2}$

$\mathcal{L}$ = peak luminosity

d_L = luminosity distance $\quad d_L = \int_t^0 c(t')dt' = c_0 \int_t^0 \dfrac{dt'}{n(t')}$

$n(t) = \exp(t/\tau_0)$ (t<0) $\Longrightarrow$ $d_L = c_0\tau_0(n^{-1}(t) - 1) = c_0\tau_0\left((1+z)^2 - 1\right)$

$$\mu_p = 5 log_{10}\left((1+z)^2 - 1\right) + 5 log_{10}\left(\frac{c_0\tau_0}{10\text{ pc}}\right)$$

Fit Supernovae Type Ia

Data from the joint analysis SDSS-II and SNLS
(*Betoule et al., A&A, 2014*)

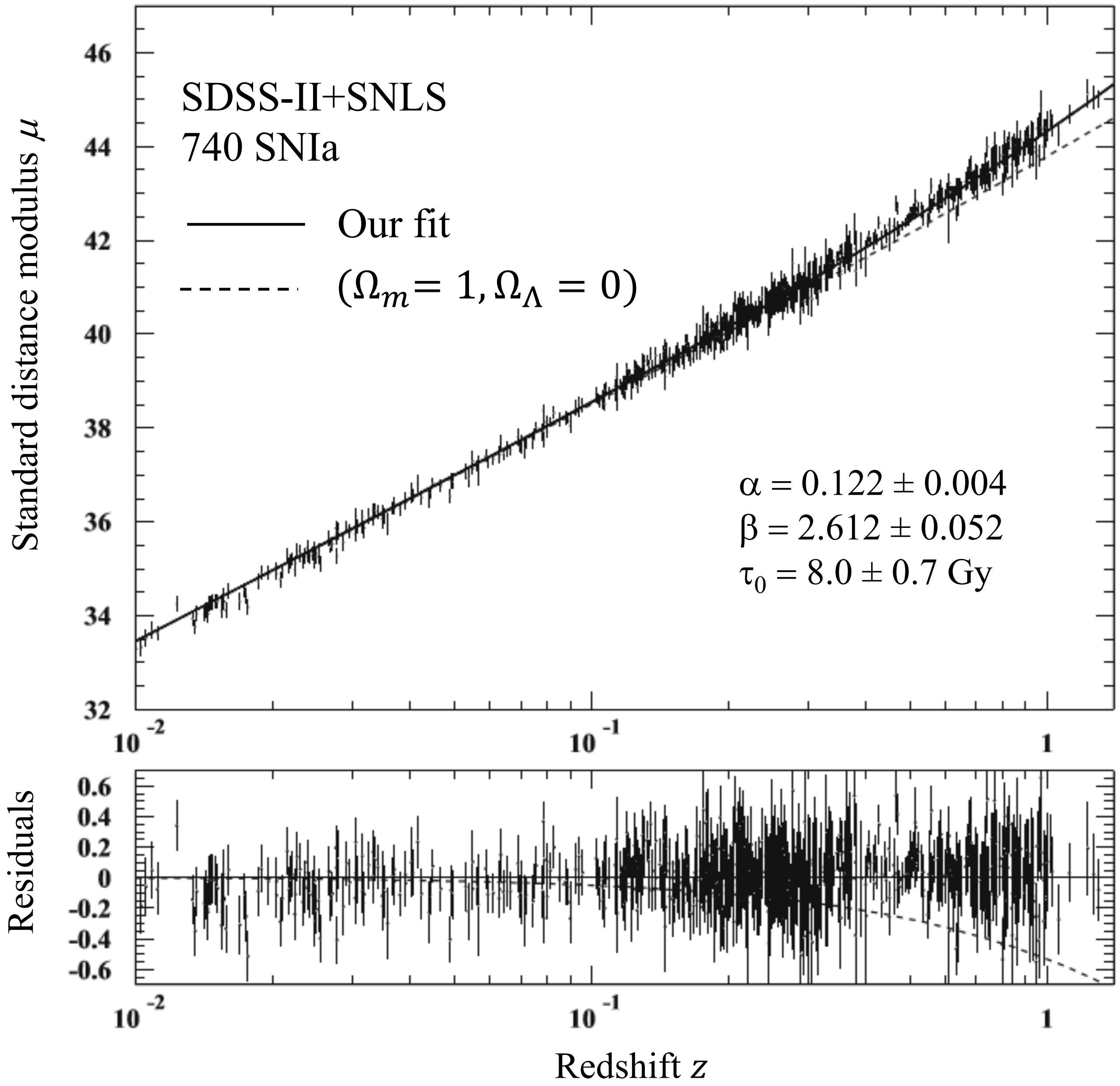
$$\chi^2(\alpha,\beta,\tau_0) = \sum_i \frac{\left(\mu_{mes,i}(\alpha,\beta) - \mu_{p,i}(z,\tau_0)\right)^2}{\sigma_{\mu,i}^2}$$

$$\mu_p = 5log_{10}\left((1+z)^2-1\right) + 5log_{10}\left(\frac{c_0\tau_0}{10\ \text{pc}}\right)$$

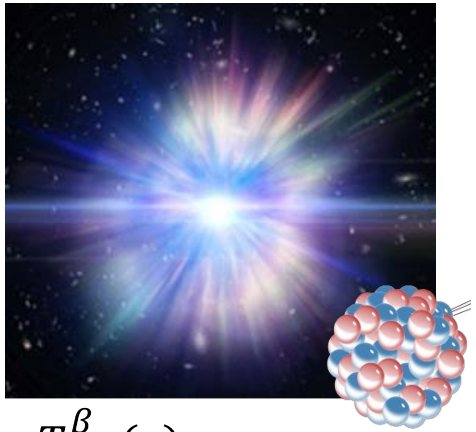
$n(t) = \exp(t/\tau_0)$ (t<0)

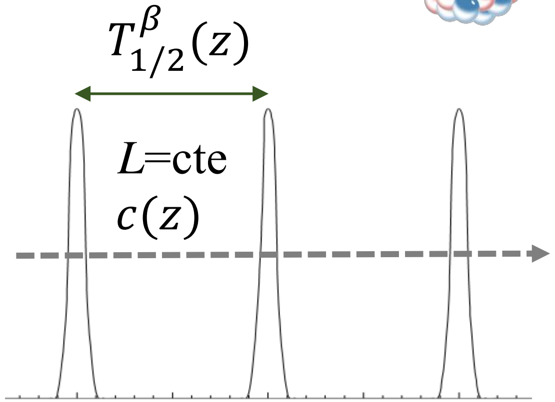
$\tau_0 = 8.0 \pm 0.7$ Gy

$\Rightarrow \frac{\Delta n}{n} = 4\ 10^{-18}\ \text{s}^{-1}$

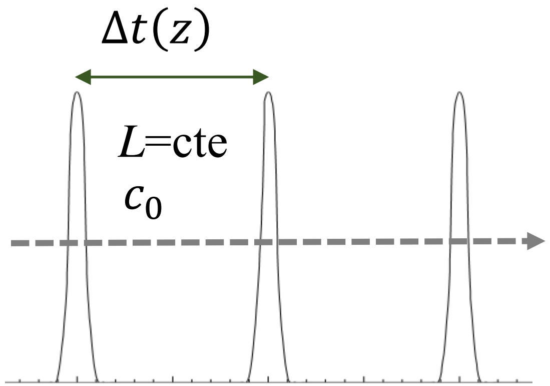


Cosmological time dilatation in SN-Ia





$c(z)$ L=cte c_0



$$L = T_{1/2}^{\beta}(z) \times c(z)$$

$$\begin{cases} \Delta t(z) = \dfrac{L}{c_0} = T_{1/2}^{\beta}(z) \times \dfrac{c(z)}{c_0} = T_{1/2}^{\beta}(z) \times n^{-1}(z) \\ T_{1/2}^{\beta}(z) = T_{1/2}^{\beta}(z=0) \times n^{1/2}(z) \end{cases}$$

$$\Rightarrow \quad \boxed{\Delta t(z) = \Delta t(z=0) \times (1+z)}$$

$$T_{1/2}^{\beta} \propto 1/(\text{Rest Energy})$$

Local apparent expansion ?

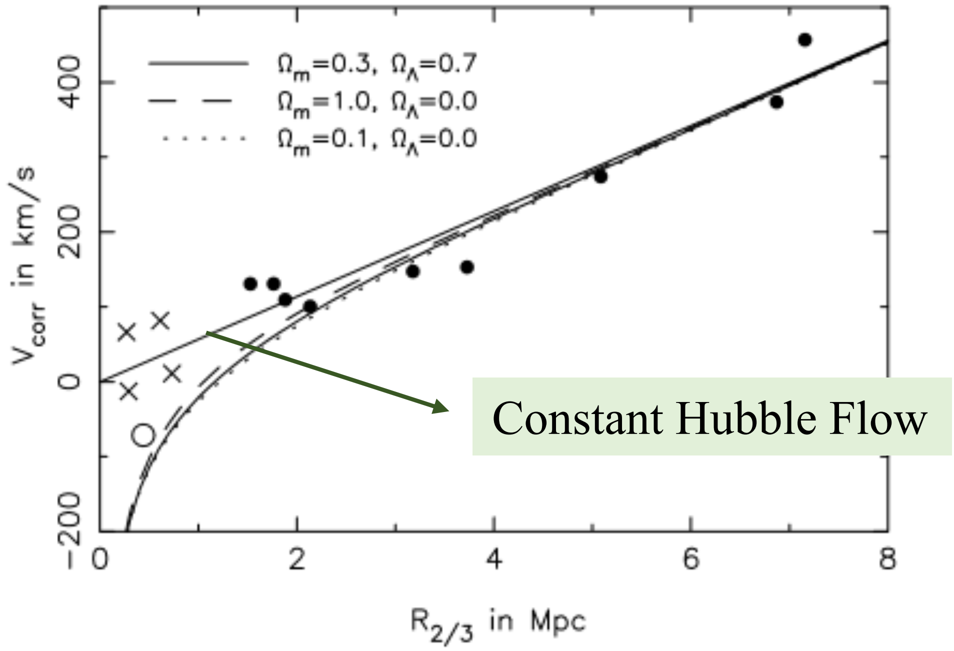
Increase of n with time

$\Delta n/n = 4\,10^{-18}\ \text{s}^{-1}$

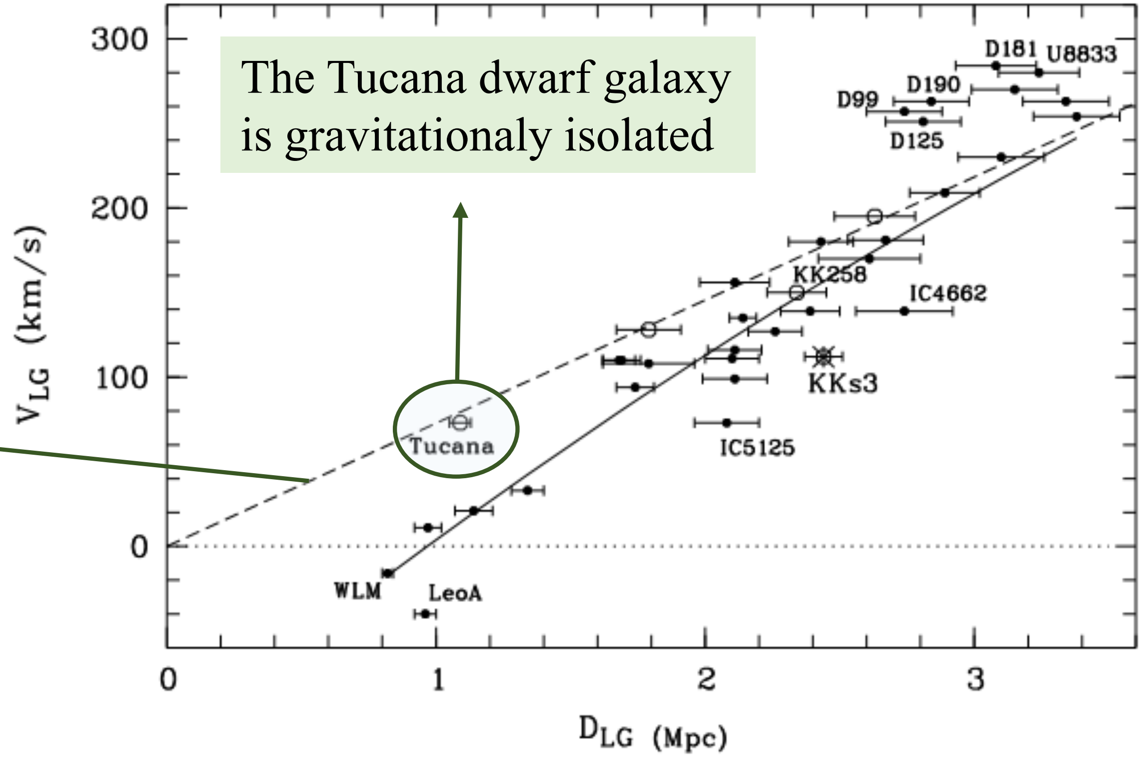
Decrease of E_{atom} with time

$\Delta E_{atom}/E_{atom} = -\,2\,10^{-18}\ \text{s}^{-1} \cong H_0$

Hubble flow at small scale (inside the galaxy cluster, solar system ?)



Ekholm et al., A&A 368, L17 (2001)



Karachentsev et al. Astron. Nachr. 366, 7, 707 (2015)

Local apparent expansion - Studies possible on close by objects

- See for instance P. J. E. Peebles, Dynamics of the Local Group: the Outer Galactic Globular Star Clusters, arXiv:1708.04542v1
    - He developps a complete model of local matter from the measured objects between 50 kpc and 2.6 Mpc
    - Tracks back in time several globular clusters
    - Looks like their come from the Hubble flow

Conclusion

- ✓ Cosmology with static Euclidean metric + vacuum index increasing with time
  - Cosmological redshift of the SN-Ia well fitted by $n(t) = \exp(-t/\tau_0)$
  - Cosmological dilatation of clocks as (1+z)
  - Evolution of the CMB consistent with the standard cosmology
  - Despite the static metric, the universe is not stationary: early universe is also hot with radiative period…
  - This framework is different to « tired light » models and VSL

- ✓ This study is obviously not complete. Other cosmological probes as CMB anisotropies must be studied

  ⇒ Possible variation of G with time

- ✓ The observed flateness of the Univers does not require any fine-tuning since the metric is Euclidean

  ⇒ Dark energy is not required…

- ✓ If $n(t) = \exp(-t/\tau_0)$ is true at the highest redshift, then absence of begining (t=0) of the Universe

  ⇒ two given location is space were causally connected in past, which solve the horizon problem

- ✓ Local apparent expansion is a possible but challenging experimental test

This work has been published in *Eur. Phys. J. C (2018) 78:444 (arXiv:1805.03503)*