

# Optical Polarimetry for the Magnetic Birefringence of Vacuum

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# Summary

- Physics case
- Opto-polarimetric detection scheme
- PVLAS experimental set-up
- Latest results
- Prospects: the VMB@CERN proposal
- Conclusions

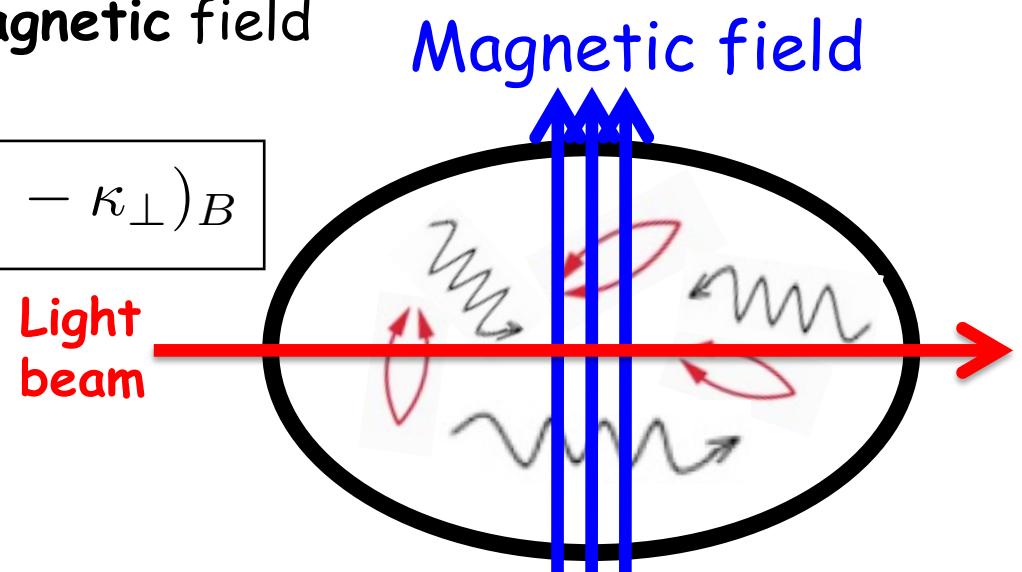
# Light propagation in an external field

- Experimental study of the structure and the nature of the quantum vacuum
- General method:
  - Perturb the vacuum with an external field
  - Probe the perturbed vacuum with a polarized light beam

Anisotropy of the index of refraction of vacuum induced by an external magnetic field

$$\Delta \tilde{n}_{\text{vacuum}} = (n_{\parallel} - n_{\perp})_B + i(\kappa_{\parallel} - \kappa_{\perp})_B$$

$$n_{\text{media}} = \frac{c}{v_{\text{light}}}$$



# Light by light scattering

H. Euler and B. Kockel (1935): an effective Lagrangian density describing electromagnetic interactions in the presence of the virtual electron-positron sea discussed a few years before by Dirac:

$$\begin{aligned}\mathcal{L}_{EK} = & \frac{1}{2\mu_0} \left( \frac{E^2}{c^2} - B^2 \right) + \\ & + \frac{A_e}{\mu_0} \left[ \left( \frac{E^2}{c^2} - B^2 \right)^2 + 7 \left( \frac{\mathbf{E}}{c} \cdot \mathbf{B} \right)^2 \right] + \dots\end{aligned}$$

$$A_e = \frac{2}{45\mu_0} \frac{\alpha^2 \hbar^3}{m_e^4 c^5} = 1.32 \times 10^{-24} \text{ T}^{-2}$$

H Euler and B Kockel, *Naturwissenschaften* **23**, 246 (1935)  
 W Heisenberg and H Euler, *Z. Phys.* **98**, 714 (1936)  
 H Euler, *Ann. Phys.* **26**, 398 (1936)  
 V Weisskopf, *Mat.-Fis. Med. Dan. Vidensk. Selsk.* **14**, 6 (1936)  
 See also: J. Schwinger, *Phys. Rev.*, **82**, 664 (1951)

Non-linear behaviour of Electromagnetism in vacuum

# Index of refraction

Linearly polarized light propagating through a transverse magnetic field

$$\mathbf{D} = \frac{\partial \mathcal{L}_{EK}}{\partial \mathbf{E}}$$

$$\mathbf{H} = \frac{\partial \mathcal{L}_{EK}}{\partial \mathbf{B}}$$



$$\mathbf{D} = \epsilon_0 \mathbf{E} + \epsilon_0 A_e \left[ 4 \left( \frac{E^2}{c^2} - B^2 \right) \mathbf{E} + 14 (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \right]$$

$$\mathbf{H} = \epsilon_0 \frac{\mathbf{B}}{\mu_0} + \frac{A_e}{\mu_0} \left[ 4 \left( \frac{E^2}{c^2} - B^2 \right) \mathbf{B} + 14 \left( \frac{\mathbf{E} \cdot \mathbf{B}}{c} \right) \frac{\mathbf{E}}{c} \right]$$

Light propagation is still described by Maxwell's equations in media but these are no longer linear due to Euler-Kockel correction.

The superposition principle no longer holds.

$$\epsilon_{\parallel}^{(EK)} = 1 + 10A_e B_{\text{ext}}^2$$

$$\epsilon_{\perp}^{(EK)} = 1 - 4A_e B_{\text{ext}}^2$$

$$\mu_{\parallel}^{(EK)} = 1 + 4A_e B_{\text{ext}}^2$$

$$\mu_{\perp}^{(EK)} = 1 + 12A_e B_{\text{ext}}^2$$

$$n_{\parallel}^{(EK)} = 1 + 7A_e B_{\text{ext}}^2$$

$$n_{\perp}^{(EK)} = 1 + 4A_e B_{\text{ext}}^2$$

# Vacuum magnetic birefringence

$$\begin{aligned} n_{\parallel, \perp} &> 1 \\ n_{\parallel} &\neq n_{\perp} \end{aligned}$$



$v \neq c$   
anisotropy



$A_e$  can be determined by  
measuring the magnetic  
birefringence of vacuum.

$$O(\alpha^2) : \quad \Delta n_B = 3A_e B_{\text{ext}}^2$$

$$O(\alpha^3) : \quad \Delta n_B = 3A_e B_{\text{ext}}^2 \left( 1 + \frac{25}{4\pi} \alpha \right)$$

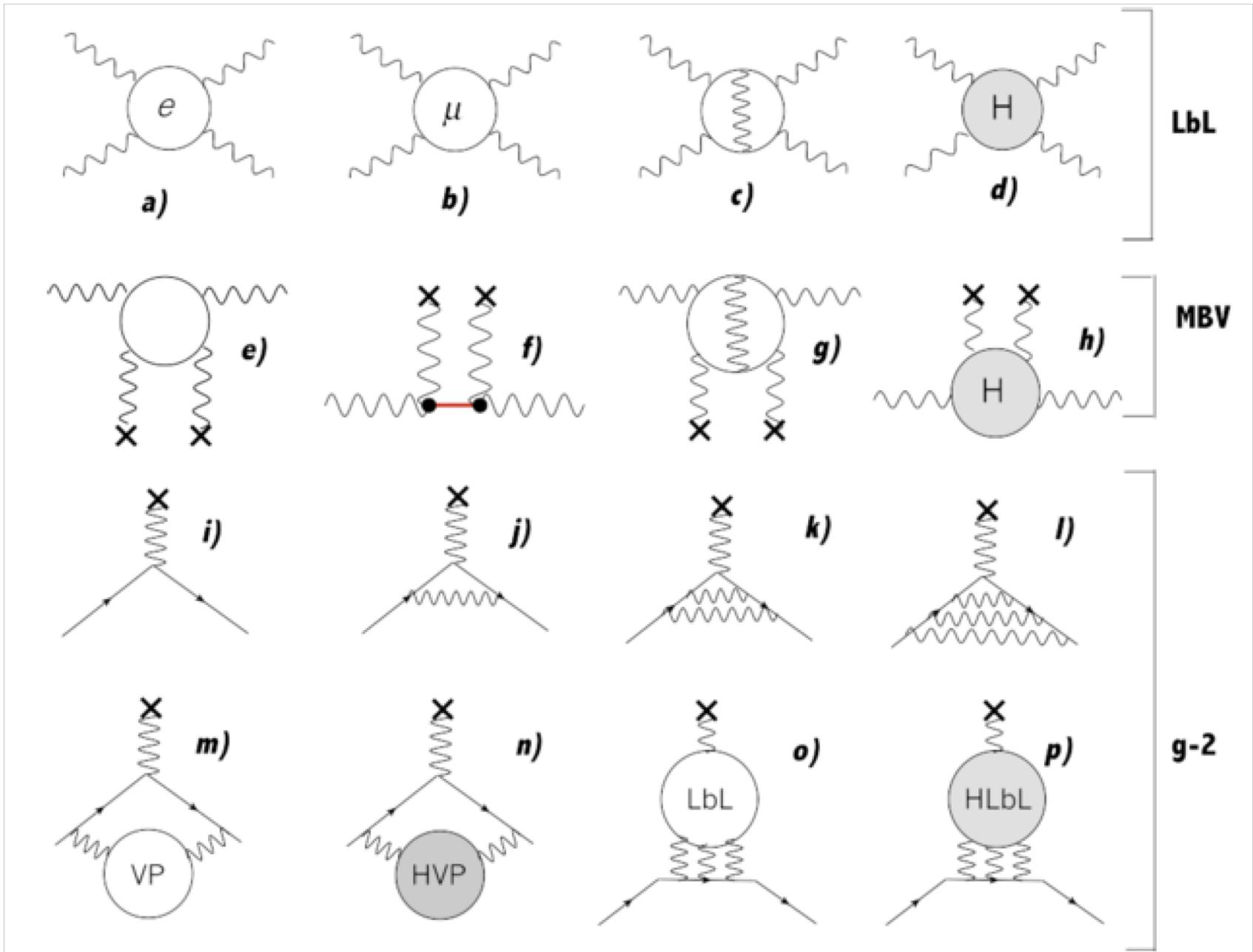
$$\Delta n_B = (4.031699 \pm 0.000002) \times 10^{-24} \left( \frac{B_{\text{ext}}}{1 \text{ T}} \right)^2$$

$O(\alpha^4), O(\alpha^5)$ ? Also a theoretical challenge

$$\Delta n_B = 2.5 \times 10^{-23} \quad @ \quad 2.5 \text{ T}$$

# The QED bestiary

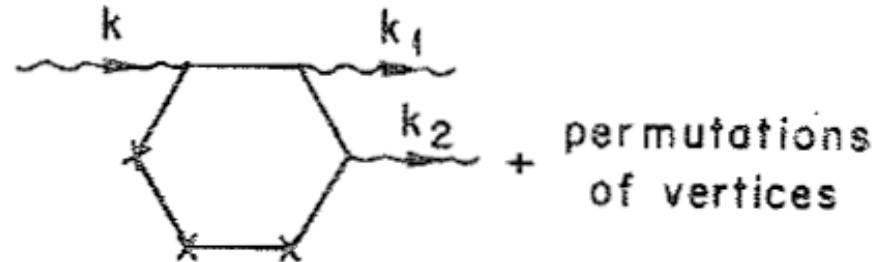
Feynman, Schwinger, Tomonaga 1946-1951



# Index of refraction: imaginary part

$$\tilde{n}_{\text{vacuum}} = n_B + i\kappa_B$$

S. Adler (1971) calculated the QED absorption related to **photon splitting**



$$\alpha_{\perp,\parallel} = \frac{4\pi}{\lambda} \kappa_{\perp,\parallel} = (0.51, 0.24) \left( \frac{\hbar\omega}{m_e c^2} \right)^5 \left( \frac{B}{B_{\text{cr}}} \right)^6 \text{ cm}^{-1}$$

$$B_{\text{cr}} = \frac{m_e^2 c^2}{e\hbar} = 4.41 \times 10^9 \text{ T}$$

$$\Delta\kappa_B = -2.5 \times 10^{-92} \left( \frac{1 \mu\text{m}}{\lambda} \right)^4 \left( \frac{B}{1 \text{ T}} \right)^6$$

Unmeasurably small

# Other QED tests:

- Microscopic tests
  - QED tests in bound systems - Lamb shift, Delbrück scattering
  - QED tests with charged particles - ( $g-2$ )
  - High energy light-by-light scattering (ATLAS, this workshop)

⋮

- Macroscopic tests
  - Casimir effect (photon zero point fluctuations)
  - MBV of magnetars (Mignani et al, this workshop)
    - Recent proposals:
  - Refraction of light by light (Sarazin et al, this workshop)
  - Direct light-by-light scattering (King and Heinzl, this workshop)

⋮

D Bernard et al, EPJD **10**, 141 (2000)  
Lundstrom, Tommasini...

QED laboratory tests with only photons in the initial and final states are still missing

# Axion-like particles (ALP)

Extra Lagrangian density terms to include contributions from hypothetical neutral light particles weakly interacting with two photons

$g_a, g_s$  coupling constants

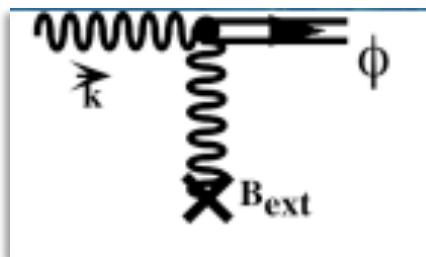
pseudoscalar

$$L_a = g_a \phi_a (\vec{E}_\gamma \cdot \vec{B}_{ext})$$

scalar (e.g. chameleon)

$$L_s = g_s \phi_s (\vec{B}_\gamma \cdot \vec{B}_{ext})$$

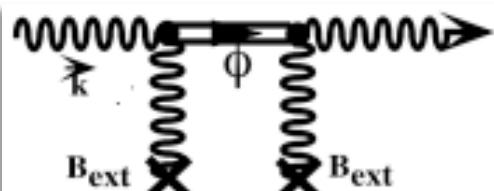
both interactions are polarization dependent



Absorption → rotation

$$x = \frac{Lm_{a,s}^2}{4\omega}$$

$$|\Delta\kappa^{(ALP)}| = \kappa_{||}^a = \kappa_{\perp}^s = \frac{2}{\omega L} \left( \frac{g_{a,s} B_{ext} L}{4} \right)^2 \left( \frac{\sin x}{x} \right)^2$$



Dispersion → ellipticity

$$|\Delta n^{(ALP)}| = n_{||}^a - 1 = n_{\perp}^s - 1 = \frac{g_{a,s}^2 B_{ext}^2}{2m_{a,s}^2} \left( 1 - \frac{\sin 2x}{2x} \right)$$

Maiani L, Petronzio R, Zavattini E, Phys. Lett B **173**, 359 (1986)  
Raffelt G and Stodolsky L Phys. Rev. D **37**, 1237 (1988)

# Linear birefringence

- The index of refraction (real part) is different for two orthogonal directions

$$\Delta n = n_{\parallel} - n_{\perp} \neq 0$$

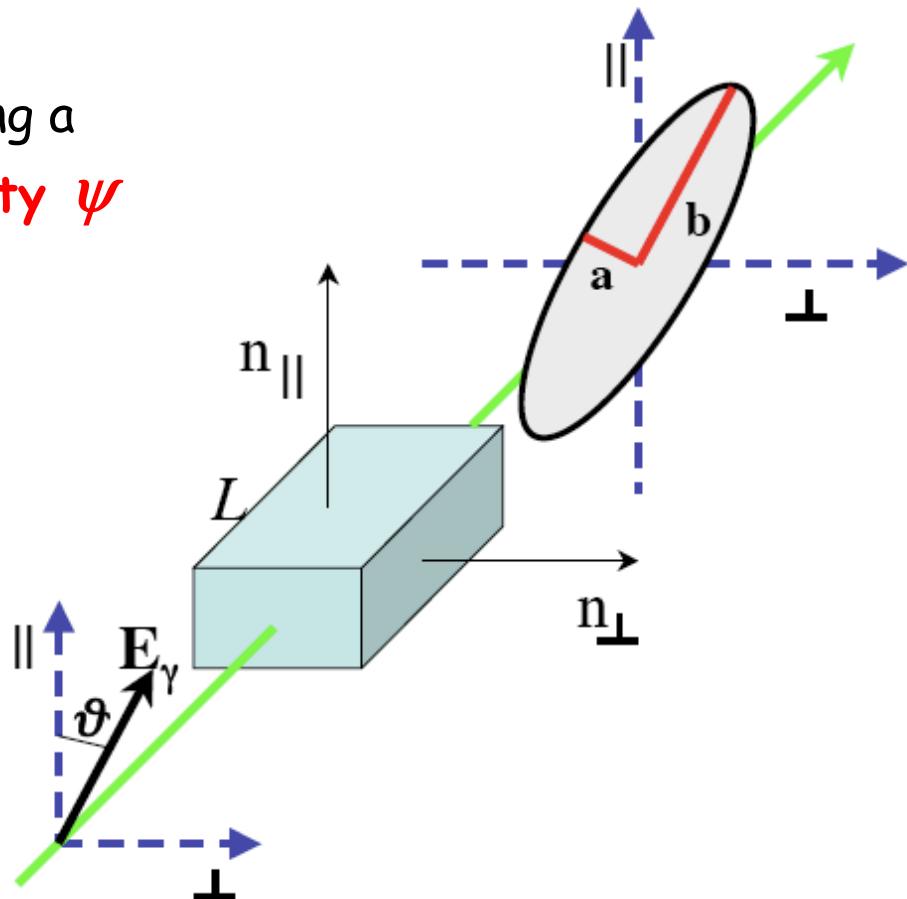
- A linearly polarized light beam traversing a birefringent medium **acquires an ellipticity  $\psi$**

$$\psi = \pm \frac{a}{b} = \pi \frac{L}{\lambda} \Delta n \sin 2\vartheta$$

QED vacuum magnetic birefringence  
 $L = 1.64 \text{ m}$ ,  $\lambda = 1064 \text{ nm}$ ,  $B = 2.5 \text{ T}$

$$\Delta n_{\text{QED}} = 2.5 \times 10^{-23}$$

$$\psi_{\text{QED}} = 1.2 \times 10^{-16}$$



# Linear dichroism

- The **extinction coefficient** is different for two orthogonal directions

$$\tilde{n} = n + i\kappa$$

$$\Delta\kappa = \kappa_{\parallel} - \kappa_{\perp} \neq 0$$

Absorption  
coefficient

$$\alpha = 4\pi \frac{\kappa}{\lambda}$$

A linearly polarised light beam traversing a dichroic medium **is rotated by an angle  $\varepsilon$**

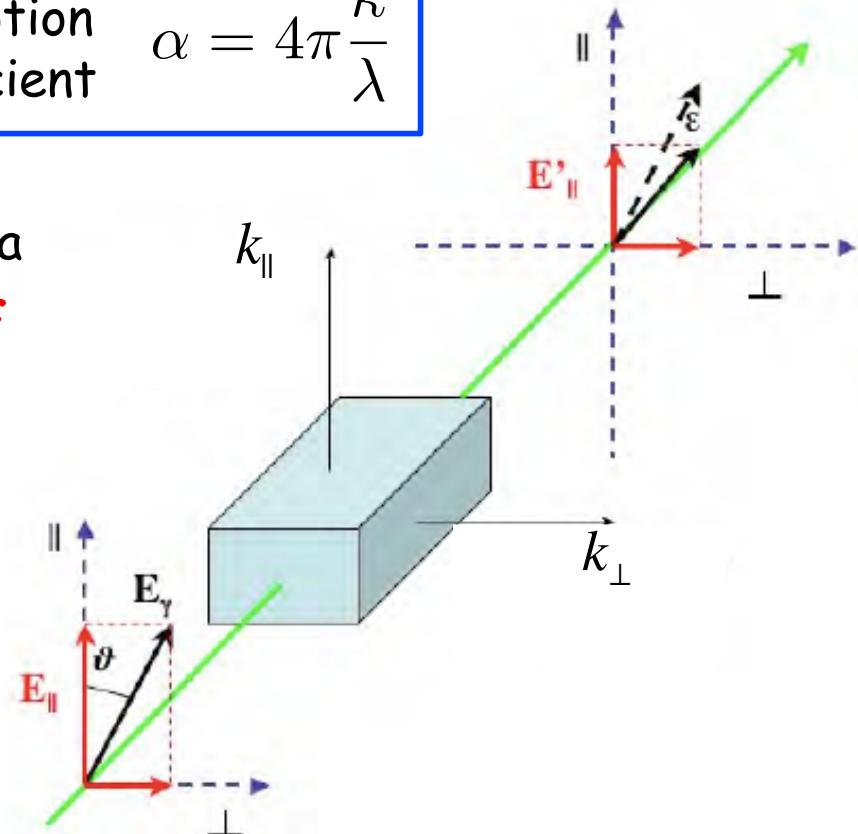
$$\varepsilon = \pi \frac{L}{\lambda} \Delta\kappa \sin 2\vartheta$$

QED vacuum magnetic photon splitting

$$L = 1.64 \text{ m}, \lambda = 1064 \text{ nm}, B = 2.5 \text{ T}$$

$$\Delta k_{\text{QED}} = -5 \times 10^{-91}$$

$$\varepsilon_{\text{QED}} = -2 \times 10^{-83}$$



Larger effects might come from axion-like particles

# Sensitive magnetic polarimetry

Volume 85B, number 1

PHYSICS LETTERS

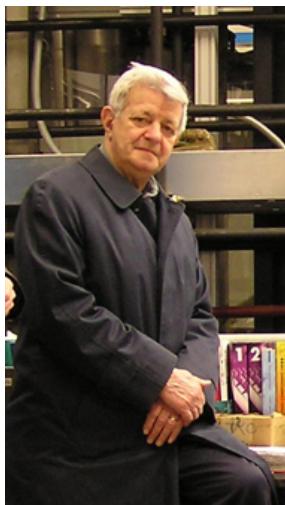
30 July 1979

## EXPERIMENTAL METHOD TO DETECT THE VACUUM BIREFRINGENCE INDUCED BY A MAGNETIC FIELD

E. IACOPINI and E. ZAVATTINI

CERN, Geneva, Switzerland

In this letter a method of measuring the birefringence induced in vacuum by a magnetic field is described: this effect is evaluated using the non-linear Euler–Heisenberg–Weisskopf lagrangian. The optical apparatus discussed here may detect an induced ellipticity on a laser beam down to  $10^{-11}$ .



Emilio Zavattini  
(1927 -2007)

- signal modulation; beat with a known effect for linearization
- high magnetic field  $B$
- longest possible optical path  $L$

# Experimental strategies

- **Signal modulation**

**Periodic change of the effect:** modulate either field **intensity** (BFRT) or field **direction** (PVLAS, Q & A).

Add a modulated ellipticity: **heterodyne detection**

**Pulsed magnets:** (BMV, OVAL)

Beat with a static effect: **homodyne detection**

- **High magnetic field  $B$**

**Superconductive magnets:** (BFRT, PVLAS LNL)

**Electromagnets:** (BMV, OVAL)

**Dipole permanent magnets:** (PVLAS Ferrara, Q & A)

long duty cycle; high frequency rotation (PVLAS reached **23 Hz**)

- **Longest possible optical path  $L$**

**Multi-pass cavity:** (BFRT)

**High-Q Fabry-Perot resonator:** (BMV, OVAL, PVLAS, Q & A) largest optical path-length multiplication factor  $\approx 5 \times 10^5$  (PVLAS Ferrara)

BFRT: R Cameron et al, PRD **47**, 3707 (1993)

PVLAS LNL: E Zavattini et al, PRD **77**, 032006 (2008)  
M Bregant et al, PRD **78**, 032006 (2008)

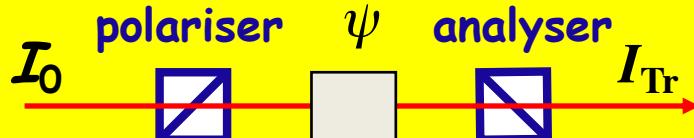
Q & A: H-H Mei et al, MPLA **25**, 983 (2010)

BMV: A Cadène et al, EPJD **68**, 16 (2014)

OVAL: X Fan et al, EPJD **71**, 308 (2017)

PVLAS Ferrara: F Della Valle et al, EPJC **76**, 24 (2016)  
G Zavattini et al, EPJC **78**, 585 (2018)

# Heterodyne detection - ellipticity



$$I_{tr} = I_0 [\sigma^2 + \psi^2]$$

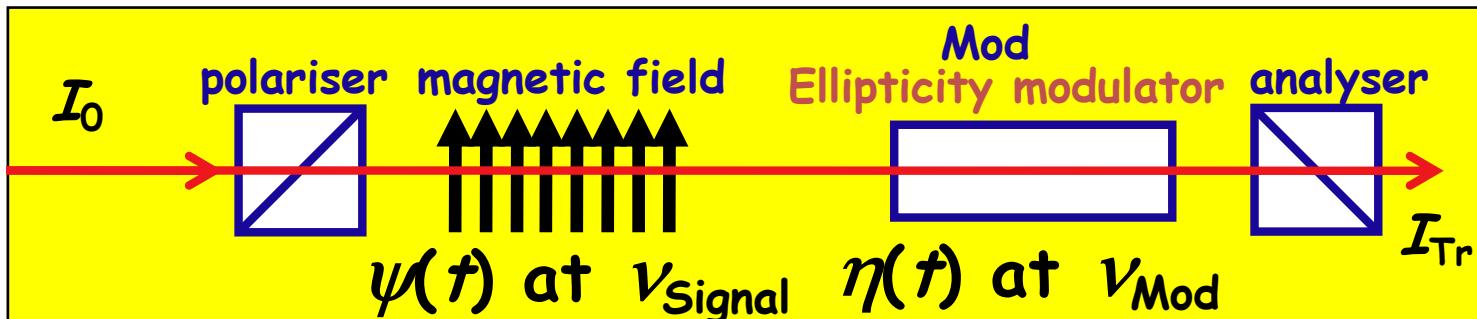
$$\psi_{QED} = 1.2 \times 10^{-16}$$

$$\sigma^2 \approx 10^{-7} - 10^{-8}$$

static detection excluded

Signal modulated in time. Beat with a calibrated effect

- Signal linear in the ellipticity
- Smaller 1/f noise



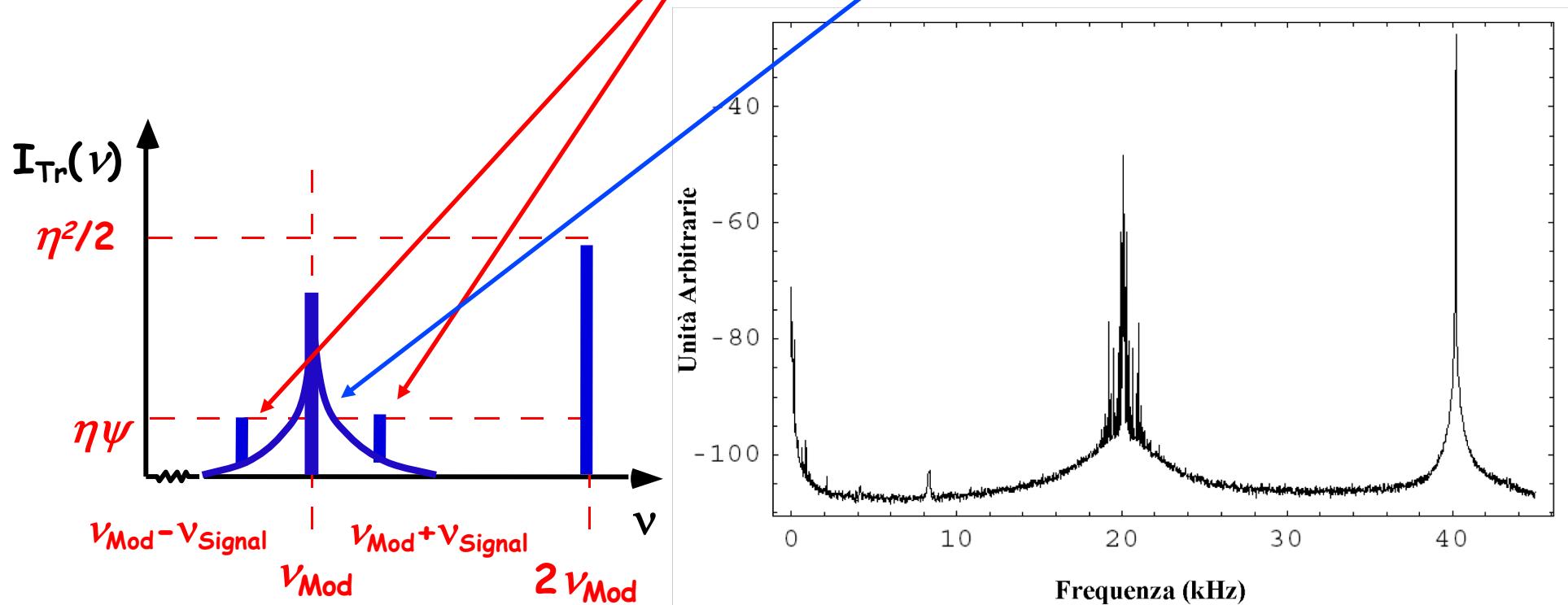
$$I_{tr} = I_0 [\sigma^2 + (\psi(t) + \eta(t))^2] = I_0 [\sigma^2 + (\psi(t)^2 + \eta(t)^2 + 2\psi(t)\eta(t))]$$

Main frequency components at  $\nu_{Mod} \pm \nu_{Signal}$  (and  $2\nu_{Mod}$ )

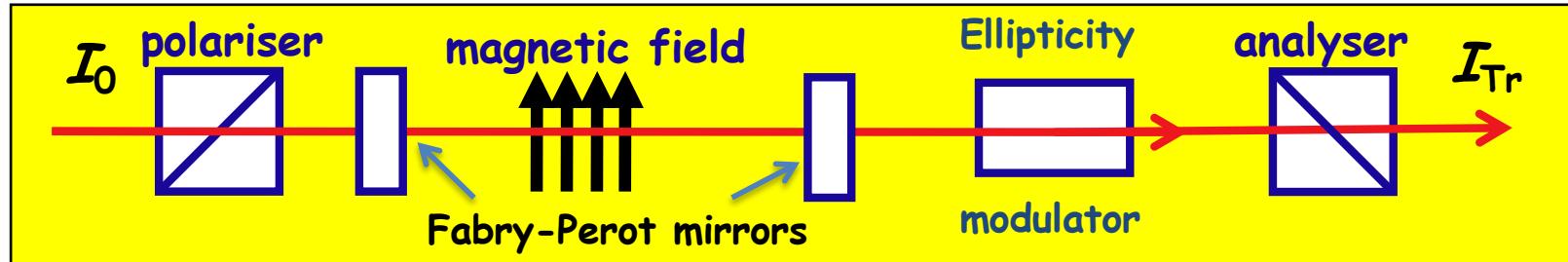
# Signal frequency layout

Nearly static birefringences  $\alpha_s(t)$  generate a  $1/f$  noise centred at the carrier modulation frequency  $\nu_{\text{Mod}}$

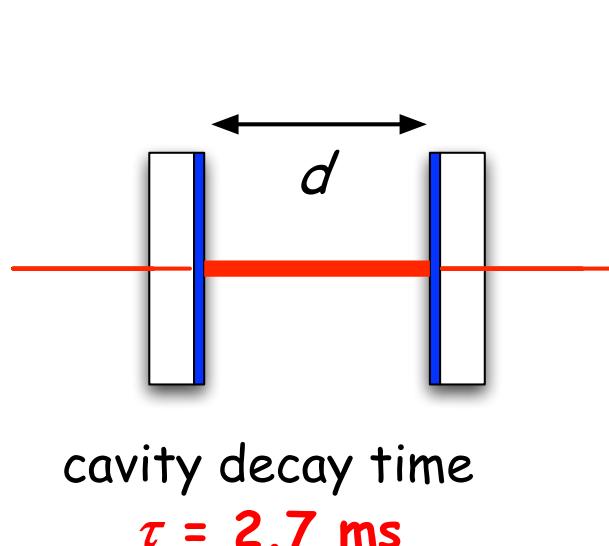
$$\begin{aligned} I_{Tr} &= I_0 \left[ \sigma^2 + (\psi(t) + \eta(t) + \alpha_s(t))^2 \right] \\ &= I_0 \left[ \sigma^2 + (\eta(t)^2 + 2\psi(t)\eta(t) + 2\alpha_s(t)\eta(t) + \dots) \right] \end{aligned}$$



# Signal amplification: Fabry Perot cavity



Fabry-Perot: resonant optical cavity increasing the effective optical path.  
 Made of two mirror placed at a separation  $d$  which is an integer multiple of  $\lambda/2$ .  
 The laser is frequency-locked to the cavity using a feedback circuit.



## Finesse

$$\mathcal{F} = \frac{\pi C\tau}{d}$$

Vacuum magnetic birefringence:  
 $L = 1.64 \text{ m}$ ,  $\lambda = 1064 \text{ nm}$ ,  $B = 2.5 \text{ T}$   
 $N = 445000$

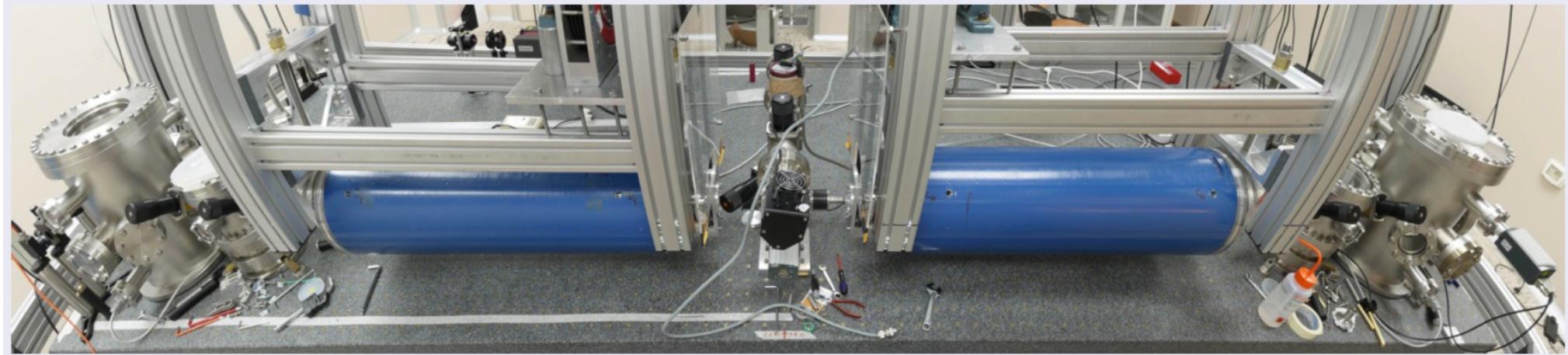
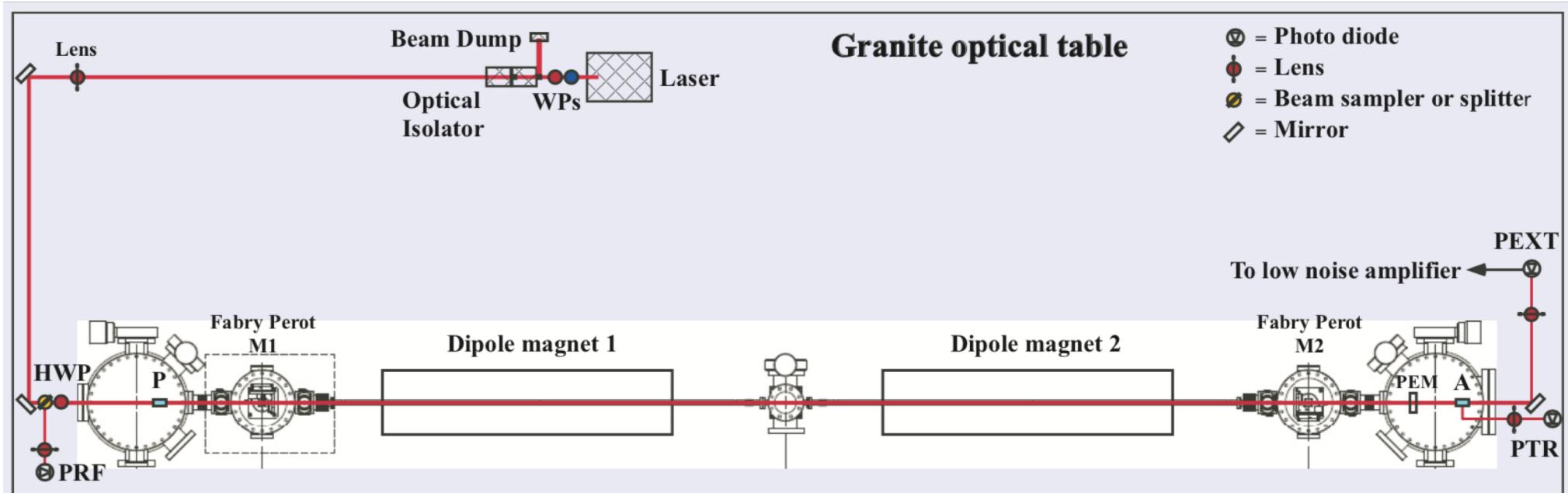
## Amplification

$$N = \frac{2\mathcal{F}}{\pi} \approx 5 \times 10^5$$

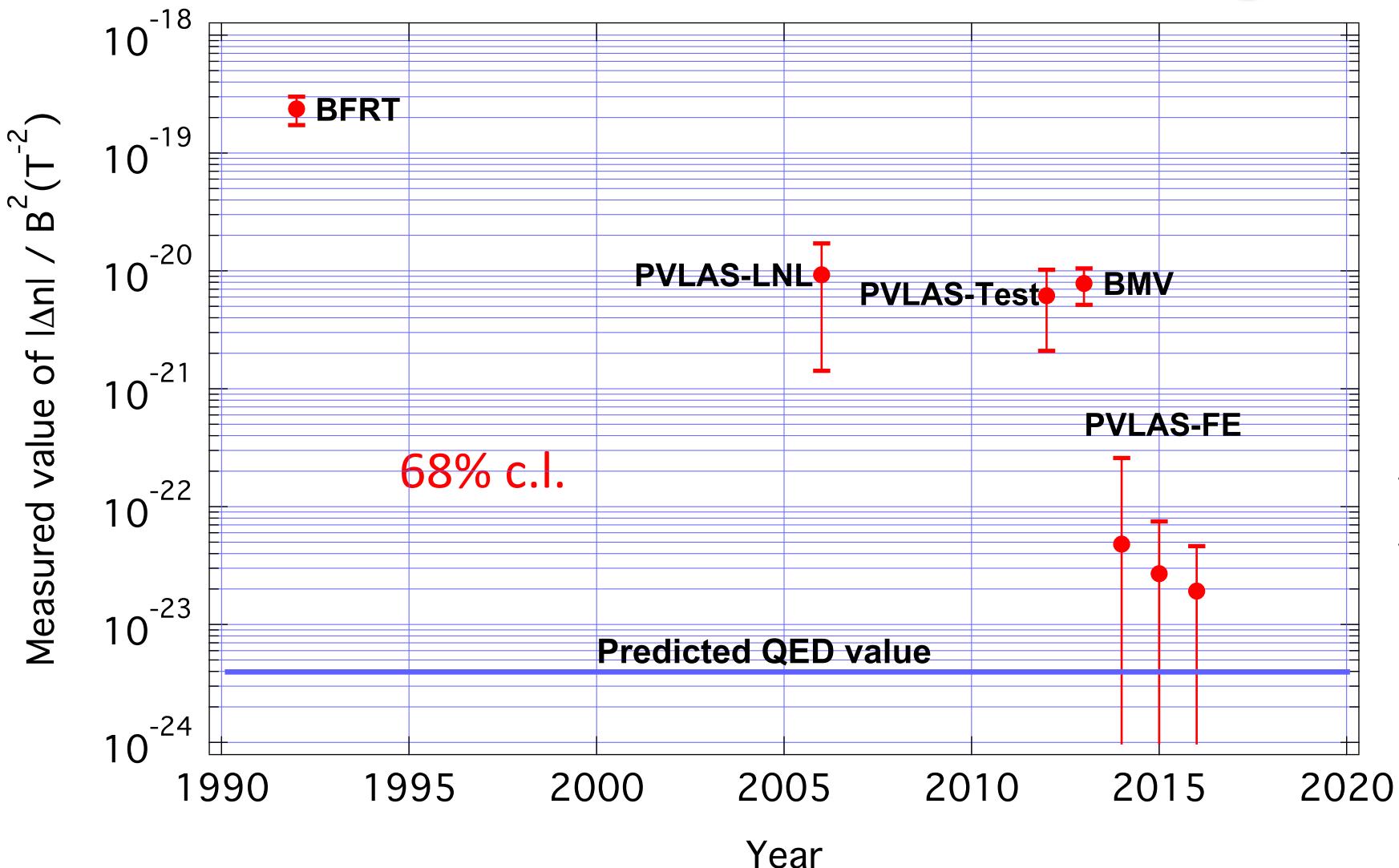
$$\Delta n_{QED} = 2.5 \times 10^{-23}$$

$$Ny_{QED} = 5.4 \times 10^{-11}$$

# The PVLAS apparatus

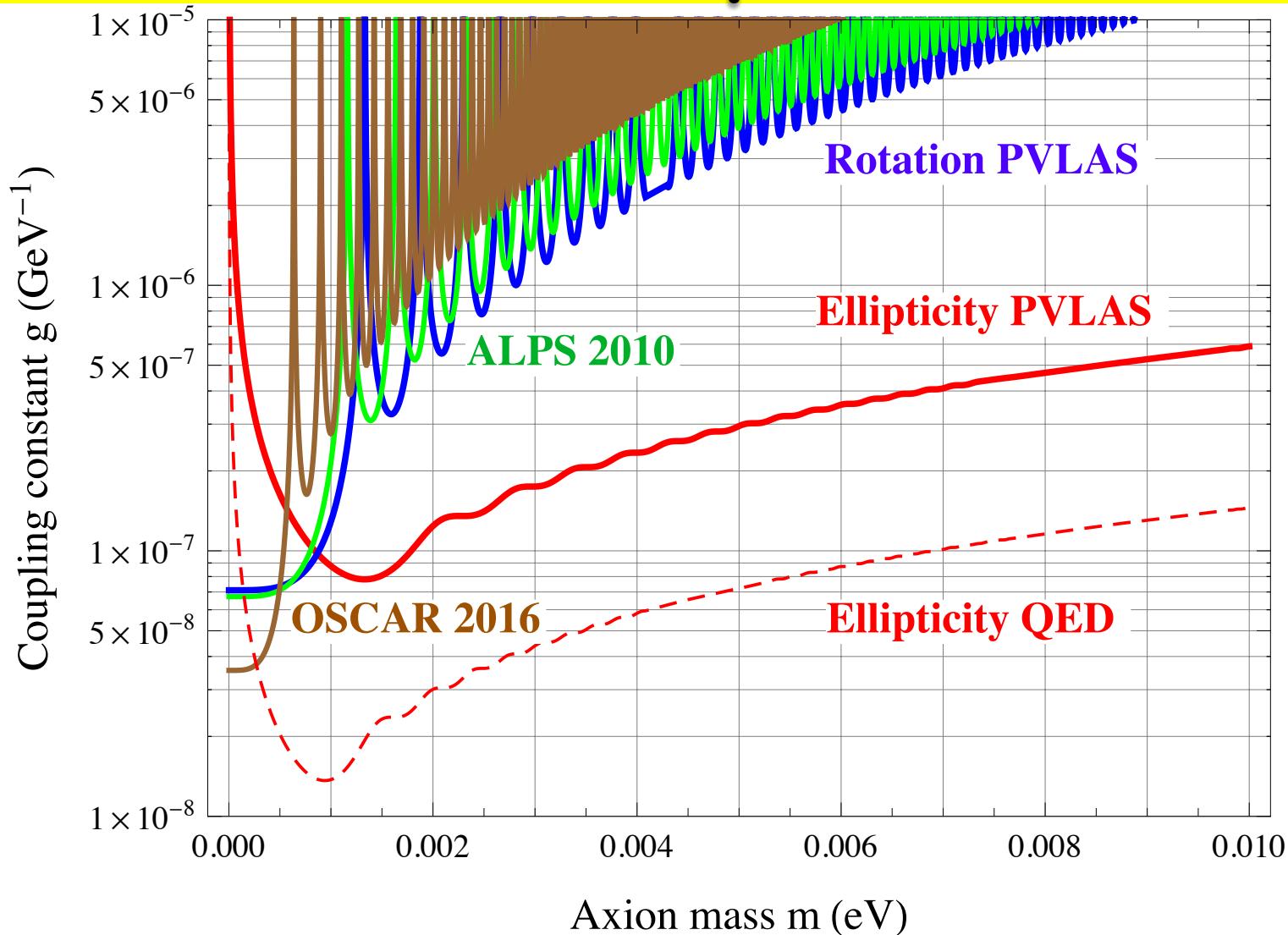


# Timeline of vacuum birefringence



$\Delta n_{\text{PVLAS-2014}} = 4.0 \pm 20 \times 10^{-23} T^{-2}$	$T_{2014} \approx 1 \times 10^6 \text{ s}$	F Della Valle et al, PRD <b>90</b> , 092003 (2014)
$\Delta n_{\text{PVLAS-2015}} = -2.4 \pm 4.8 \times 10^{-23} T^{-2}$	$T_{2015} \approx 3 \times 10^6 \text{ s}$	F Della Valle et al, EPJC <b>76</b> , 24 (2016)
$\Delta n_{\text{PVLAS-2016}} = 3.8 \pm 3.2 \times 10^{-23} T^{-2}$	$T_{2016} \approx 5 \times 10^6 \text{ s}$	A Ejlli, PhD Thesis, 2017, unpublished

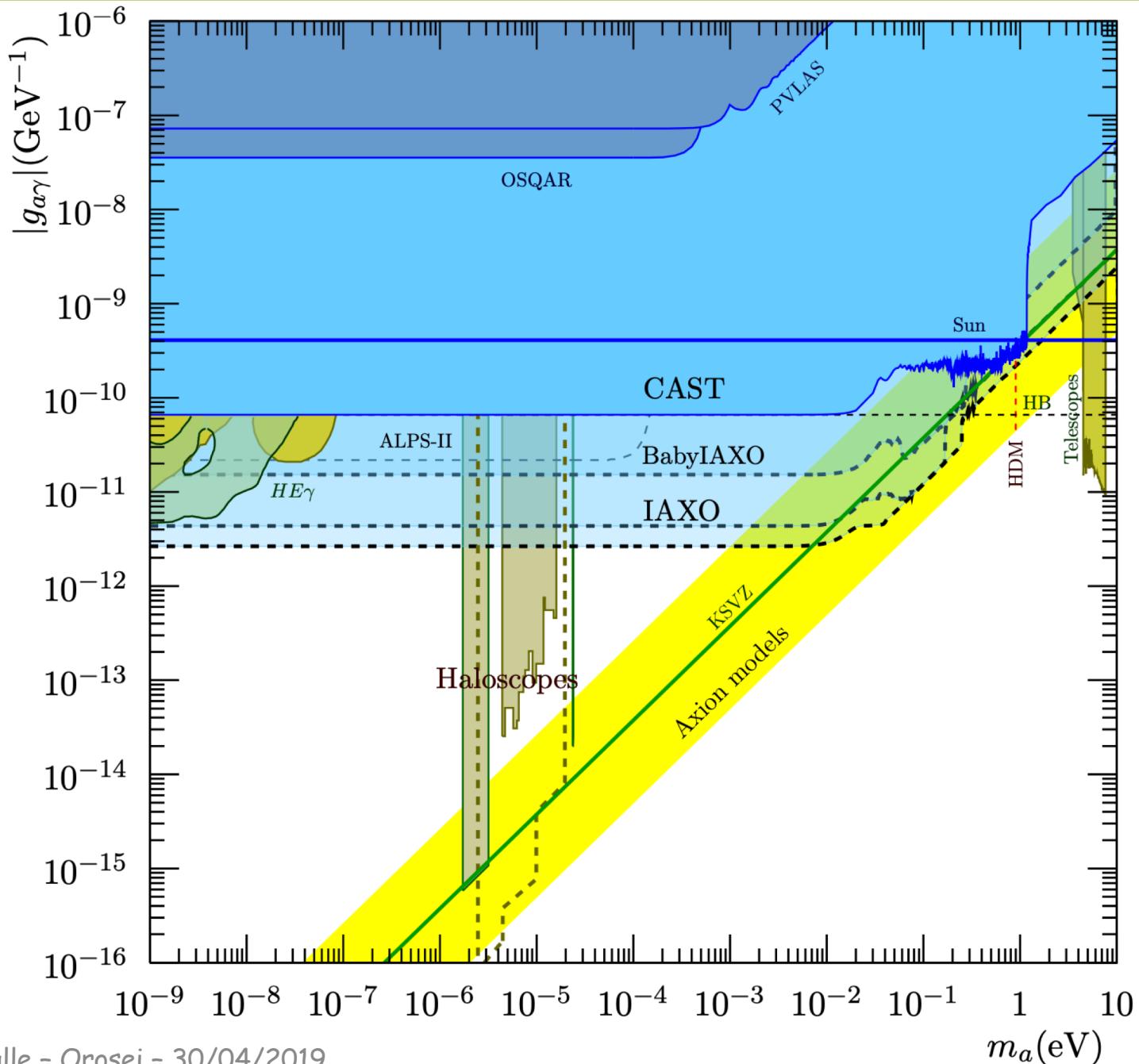
# Axion-like particles



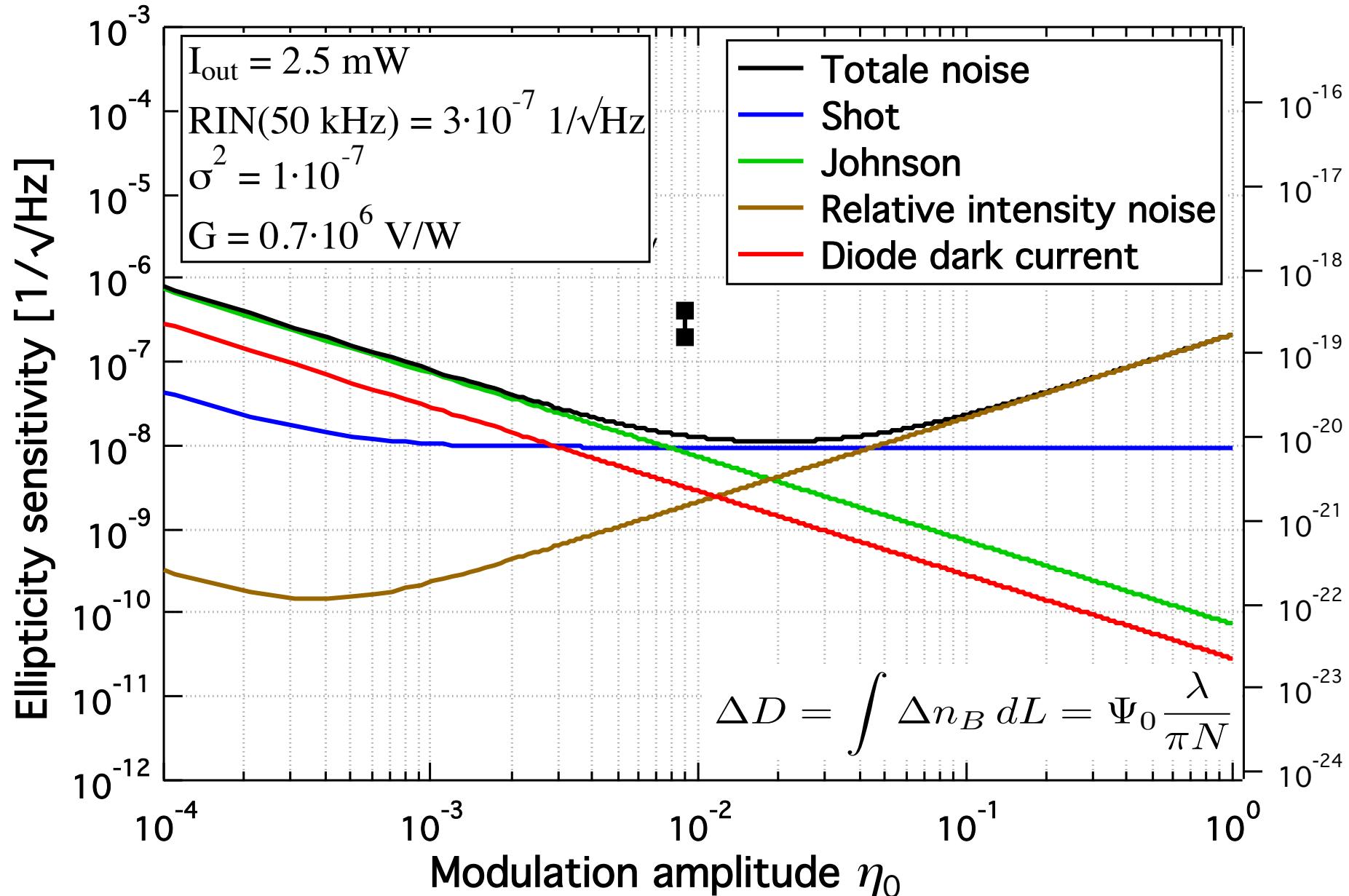
$\Delta n^{(\text{PVLAS})} = (-19 \pm 20) \times 10^{-23}$	@ $B = 2.5 \text{ T.}$
$\Delta \kappa^{(\text{PVLAS})} = (-24 \pm 30) \times 10^{-23}$	@ $B = 2.5 \text{ T}$

# PVLAS is model independent

Armengaud et al, arXiv:1904.09155v1

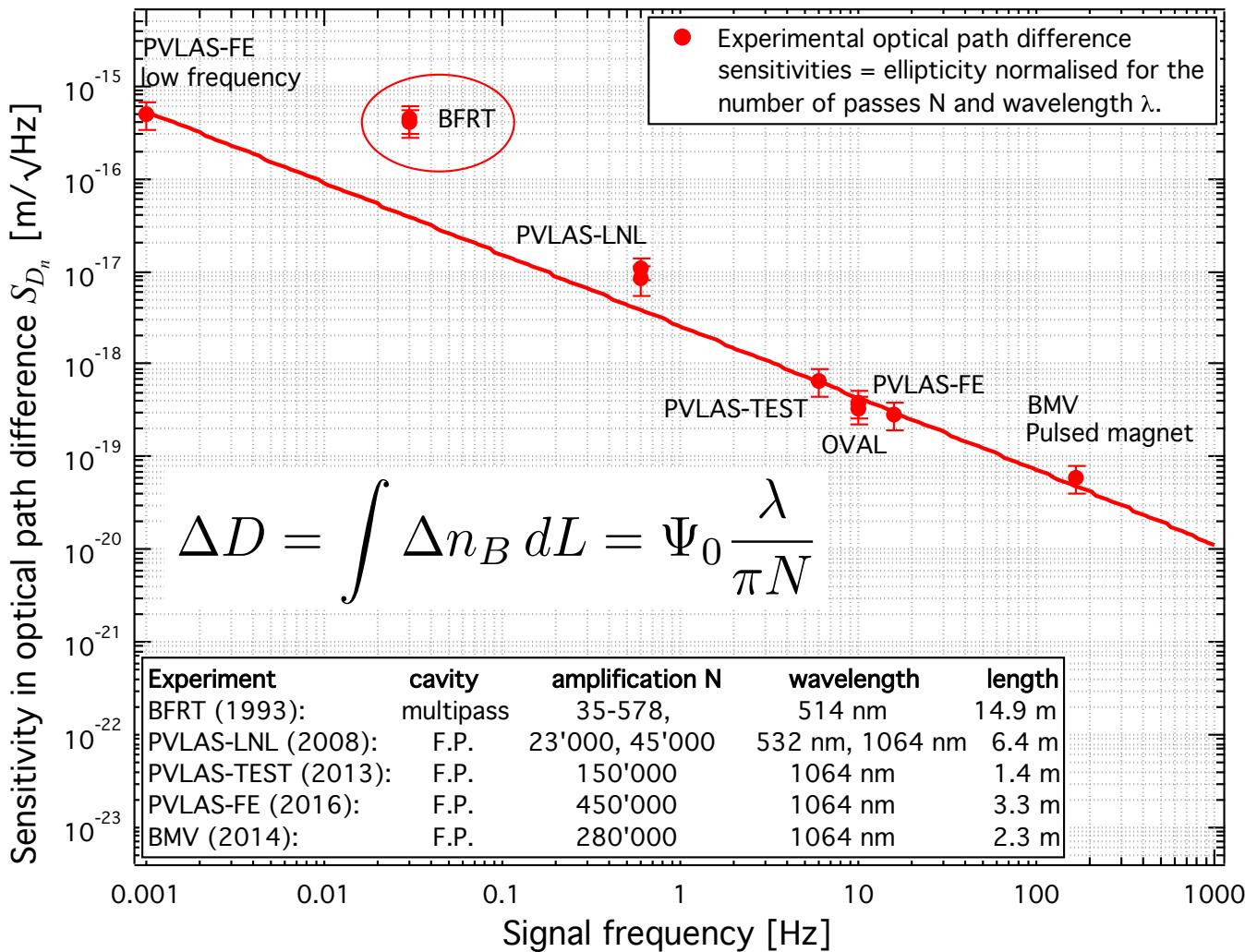


# The sensitivity problem



# Intrinsic noise?

Sensitivity in optical path difference  $\Delta D$  between two perpendicular polarizations



Updated graph from G. Zavattini et al. EPJC 76, 294 (2016)

Sensitivity in  $\Delta D$  does not depend on finesse

BFRT: R Cameron et al, PRD **47**, 3707 (1993)

PVLAS-LNL: E Zavattini et al, PRD **77**, 032006 (2008)

M Bregant et al, PRD **78**, 032006 (2008)

PVLAS-TEST: F Della Valle et al, NJP **15**, 053026 (2013)

BMW: A Cadène et al, EPJD **68**, 16 (2014)

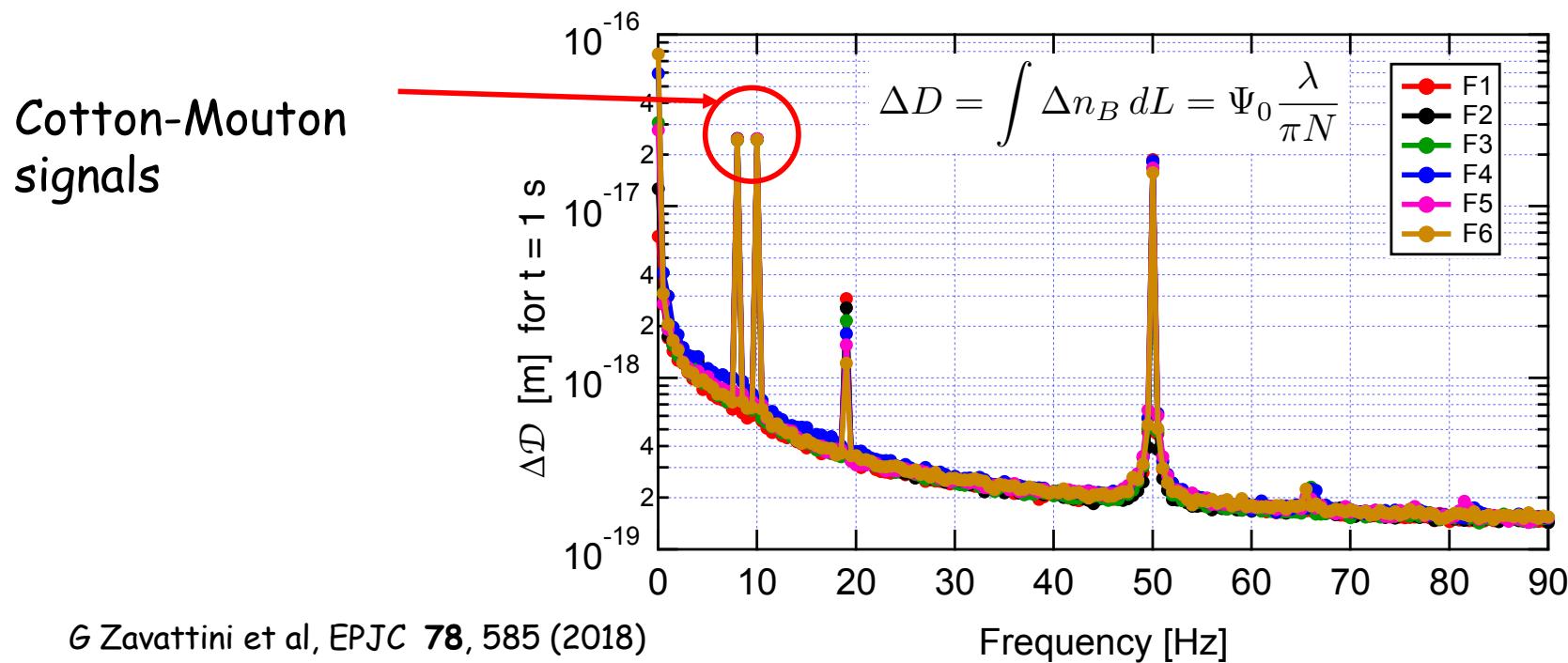
OVAL: X Fan et al, EPJD **71**, 308 (2017)

PVLAS-FE: F Della Valle et al, EPJC **76**, 24 (2016)

G Zavattini et al, EPJC **78**, 585 (2018)

# Intrinsic noise

- Measured ellipticity noise and Cotton-Mouton signal as a function of the finesse
- Introduced controlled extra losses  $p \approx 10^{-5}$  in the cavity by clipping the beam
- Finesse range (F1 - F6): 250'000 - 690'000



G Zavattini et al, EPJC 78, 585 (2018)

Noise and Cotton-Mouton  $\Delta D$  signals are independent of the finesse

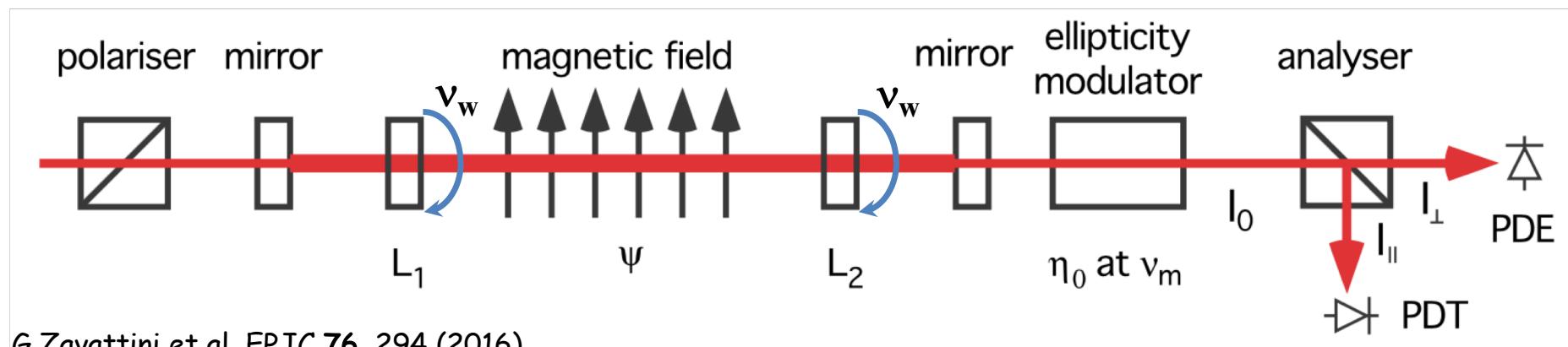
# How to beat the noise

- Increase the frequency of the signal by rotating faster
  - $S_{\Delta D} \propto \nu^\alpha$  with  $\alpha \approx -0.8$
  - Maybe improve by a factor 2 with the PVLAS apparatus
- Increase the signal:  $B^2L$  of magnet
  - Only real option is to use superconducting static magnets
  - One LHC magnet has  $B^2L = 1200 \text{ T}^2\text{m}$ . At present we have  $10 \text{ T}^2\text{m}$ .
  - Superconductor magnets cannot be modulated at  $\approx 10 \text{ Hz}$
- Change origin of modulation    G Zavattini et al, EPJC 76, 294 (2016)
  - Rotate the polarization inside the field
  - ... But must be kept fixed on the mirrors.

# Separate magnet from modulation

## Polarization modulation scheme

- Insert two half wave plates co-rotating @  $\nu_w$  with a fixed relative angle  $\Delta\phi$
- Rotate polarization inside the magnetic field
- Fix polarization on mirrors to avoid mirror birefringence signal
  - Total losses  $\leq 0.4\%$  (commercial). Maybe 10 times lower is possible
  - Maximum finesse  $\approx 10000$  (with  $\leq 0.04\%$  losses)



G Zavattini et al, EPJC 76, 294 (2016)

# Signal and possible problems

$$I(t) = I_{\text{out}} \left\{ \eta(t)^2 + 2\eta(t)N \left[ \psi_0 \sin(4\phi(t)) + \alpha_1 \sin 2\phi(t) + \alpha_2 \sin(2\phi(t) + 2\Delta\phi) \right] \right\}$$

Signal appears at the 4<sup>th</sup> harmonic of  $\nu_{\text{waveplate}}$

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Wave-plate defects  $\alpha_{1,2}$

$$\alpha_{1,2} = \alpha_{1,2}^{(0)} + \alpha_{1,2}^{(1)} \cos \phi + \alpha_{1,2}^{(2)} \cos 2\phi + \dots$$

- $\alpha_{1,2}^{(0)} \approx 10^{-3}$  (from manufacturer): appears @ 2<sup>nd</sup> harmonic
- $\alpha_{1,2}^{(1)} \approx 10^{-6}$  (wedge of wave-plate): appears @ 1<sup>st</sup> and 3<sup>rd</sup> harmonic
- $\alpha_{1,2}^{(2)} \Rightarrow$  appears @ 4<sup>th</sup> harmonic
- Condition is that  $\alpha_{1,2}^{(2)} < \psi_0$  with  $\psi_0 \approx 10^{-14}$ . Must be tested.

- Signal

$$\Delta D = 3A_e B^2 L = 4 \times 10^{-24} \left( \frac{B}{1 \text{ T}} \right)^2 \left( \frac{L}{1 \text{ m}} \right) \text{ m}$$


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- Intrinsic noise

$$S_{\Delta D}^{(\text{intrinsic})} = 2.6 \times 10^{-18} \left( \frac{\nu}{1 \text{ Hz}} \right)^{-0.77} \frac{\text{m}}{\sqrt{\text{Hz}}}$$


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- Shot-noise

$$S_{\Delta D}^{(\text{shot})} = \sqrt{\frac{e}{I_0 q}} \frac{\lambda}{\pi N} \frac{\text{m}}{\sqrt{\text{Hz}}}$$


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- Maximum measurement time

$$T = \left( \frac{S_{\Delta D}}{\Delta D} \right)^2 \lesssim 10^6 \text{ s}$$


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- LHC example:

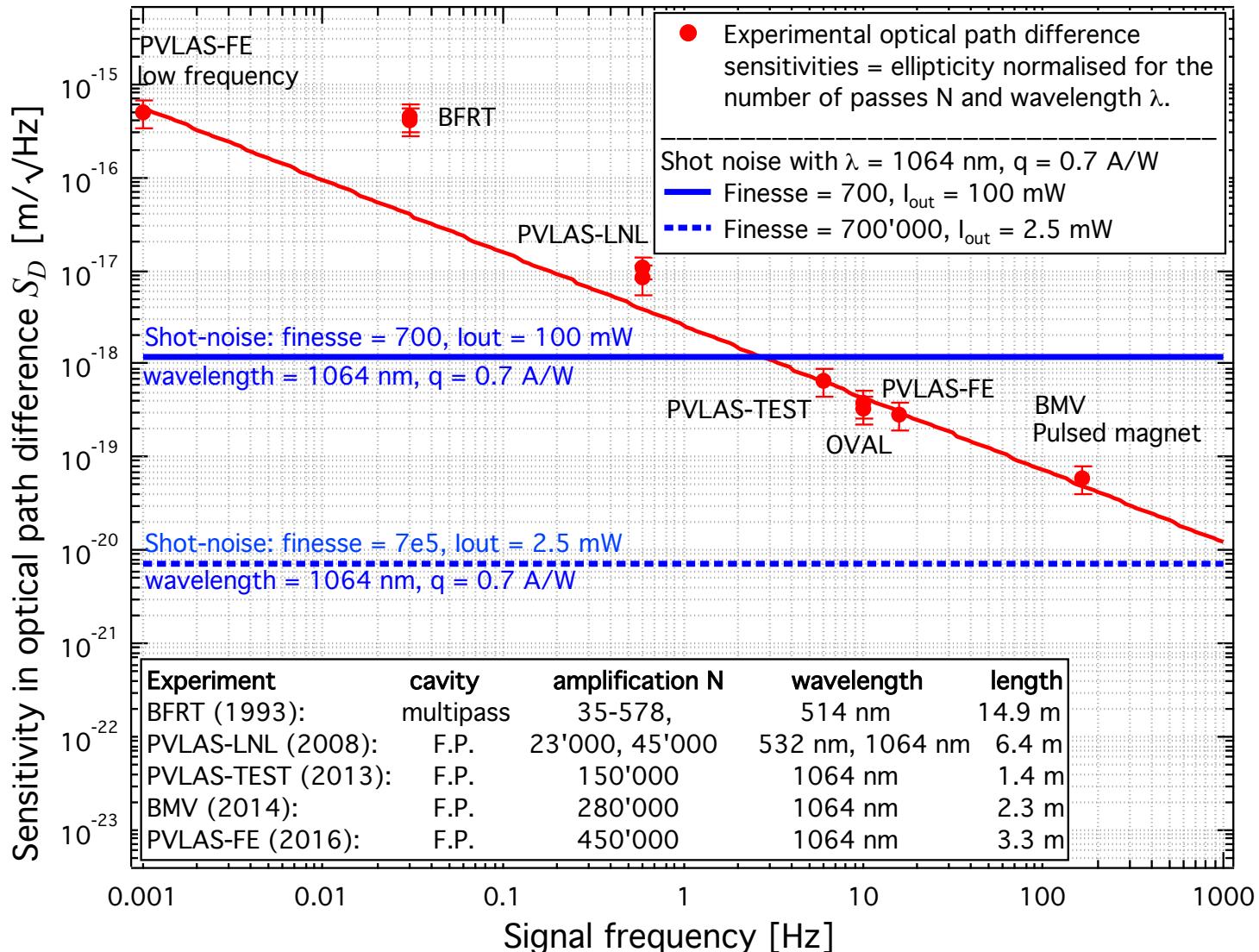
$$B^2 L = 1200 \text{ T}^2 \text{m}$$

$$S_{\Delta D} = 10^{-18} \frac{\text{m}}{\sqrt{\text{Hz}}} \quad @ \quad 3 \text{ Hz}$$

$$\Rightarrow T = 12 \text{ h}$$

# What sensitivity could be reached?

Sensitivity in optical path difference  $\Delta D$  between two perpendicular polarizations



Updated graph from G. Zavattini et al. EPJC 76, 294 (2016)

$S_{\Delta D} \approx 10^{-18}$  m/ $\sqrt{\text{Hz}}$  goal sensitivity

# Conclusions

- I have presented a brief updated report of the **opto-polarimetric method** to measure vacuum magnetic birefringence
- PVLAS, the most advanced experimental effort in this field is unable to reach the goal due to insufficient sensitivity. PVLAS achievements:
  - integrated signal is less than one order of magnitude from the QED signal
  - best laboratory limits for axion-like particles with  $m > 1 \text{ meV}$
- **New idea:** separate magnet and modulation
  - employ a **LHC magnet** ( $B^2L = 1200 \text{ T}^2\text{m}$ )
  - modulate the effect by **inserting two co-rotating half-wave plates inside the optical cavity**  
(polarization fixed on the surface of the mirrors)

# Optical Polarimetry for the Magnetic Birefringence of Vacuum

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