Quantum fluctuations at the superconducting transition in graphene (?), thin films, or layered materials

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Orosei 29.4.2019 Vacuum Fluctuations at Nanoscale and Gravitation- theory and experiments

superconductive phase in 3D:

φ⁴ Field theory with minimal coupling to em field (Ginzburg-Landau)

$$\bar{\Delta}(T) \sim 3.1\, k_B T_c^{1/2} (T_c - T)^{1/2} \qquad \xi(T) \sim \frac{\hbar v_F}{\pi \bar{\Delta}(0)} \left(\frac{T_c}{T_c - T} \right)^{1/2}$$

Thermal fluctuations at $T \sim T_c$

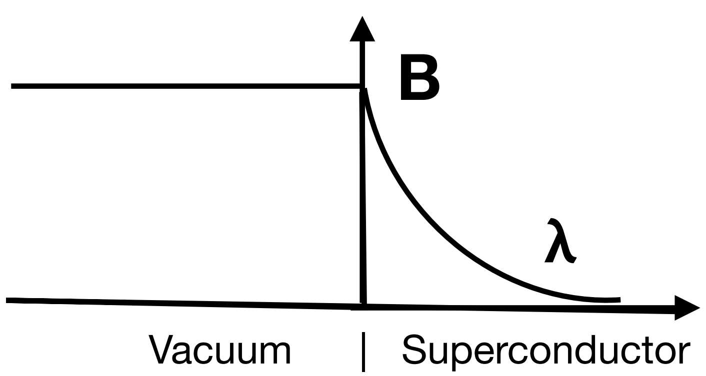
$$\frac{\left\langle (\Delta - \bar{\Delta})^2 \right\rangle}{\bar{\Delta}^2} \sim \left(\frac{T_c}{T_F} \right)^2 \left(\frac{T_c}{T - T_c} \right)^{1/2}$$

fluctuations at $T \sim 0$

$$\frac{\left\langle (\Delta - \bar{\Delta})^2 \right\rangle}{\bar{\Delta}^2} \sim \frac{T T_c}{T_F^2}$$

Anderson-Higgs mechanism adds a mass to the em field:

Meissner effect



$$\frac{1}{\lambda^2(T)} = \frac{4\pi n_s^{3D} e^2}{mc^2} \propto (T_c - T)$$

$\frac{\lambda}{\xi}$ = Ginzburg-Landau parameter

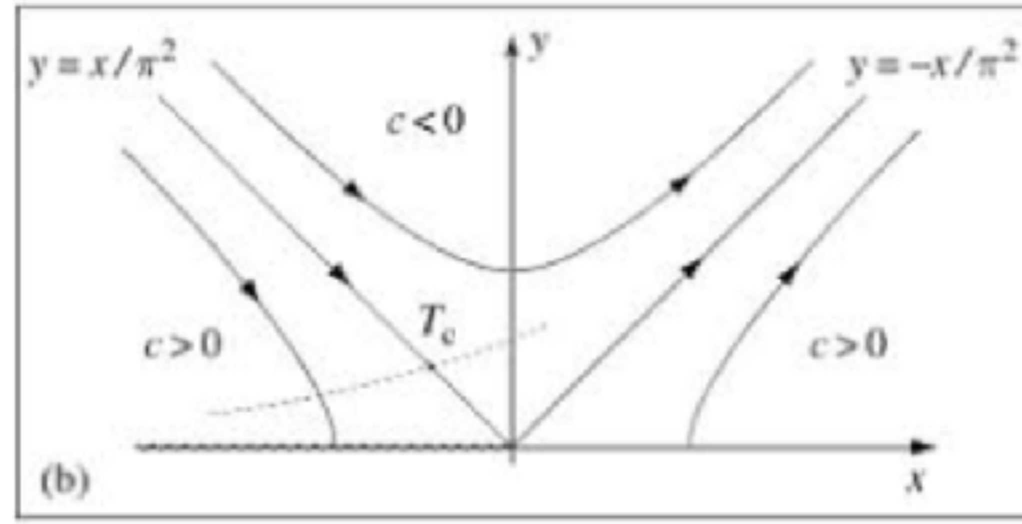
when $\frac{\lambda}{\xi}$ **>>1 vortices are stable excitations with a logarithmically decaying repulsive interaction**

order parameter decays algebraically with distance in 2D

$$\left\langle \vec{S}(0)\cdot\vec{S}(r)\right\rangle \sim r^{-\eta}, \quad \eta = \frac{k_B T}{2\pi J_s}$$

2D thin film

Berezinskii-Kosterlitz-Thouless transition

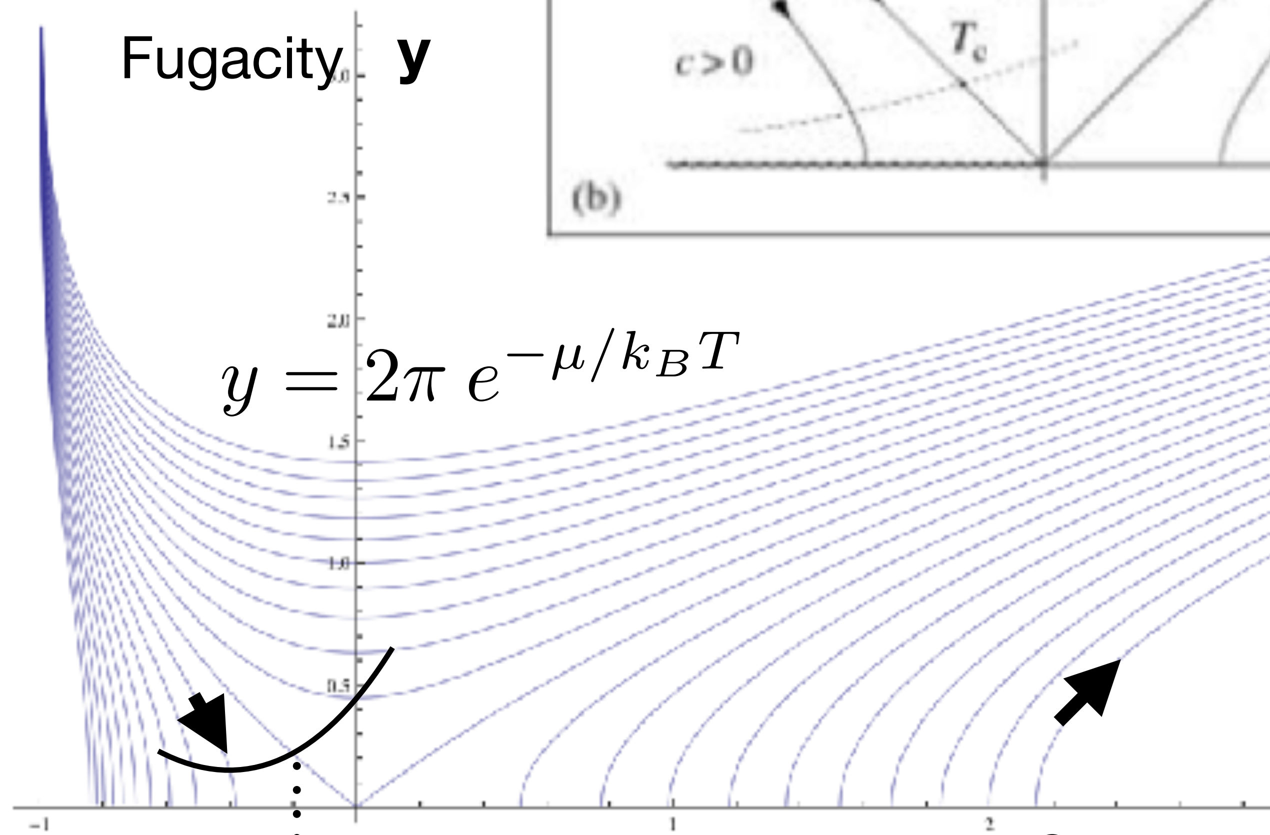


At the boundary of a free vortex:

$$\frac{1}{\lambda^2} = \frac{4\pi e^2 n_s}{m^* c^2}, \qquad A = \frac{\hbar c}{2e}\frac{1}{\lambda}$$

$$j_s = e n_s \frac{p}{m^*} = n_s \frac{d}{\lambda}\frac{e^2}{m^* c} A$$

$$p \sim \frac{\hbar}{\Lambda} \sim \frac{\hbar}{2}\frac{d}{\lambda^2}$$



$$y = 2\pi\, e^{-\mu/k_B T}$$

Θ_{BKT}

$$\Theta = 2 - \pi\frac{J_s(T)}{T}$$

Λ = Pearls length of the free vortex

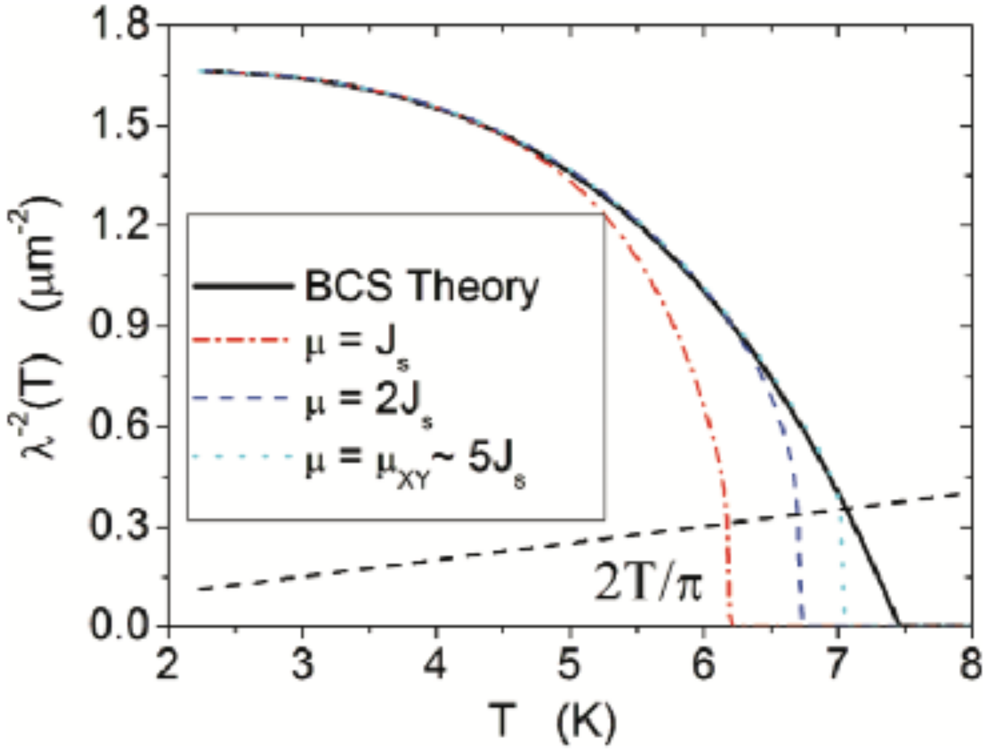
J_S = stiffness of the "ordered" phase

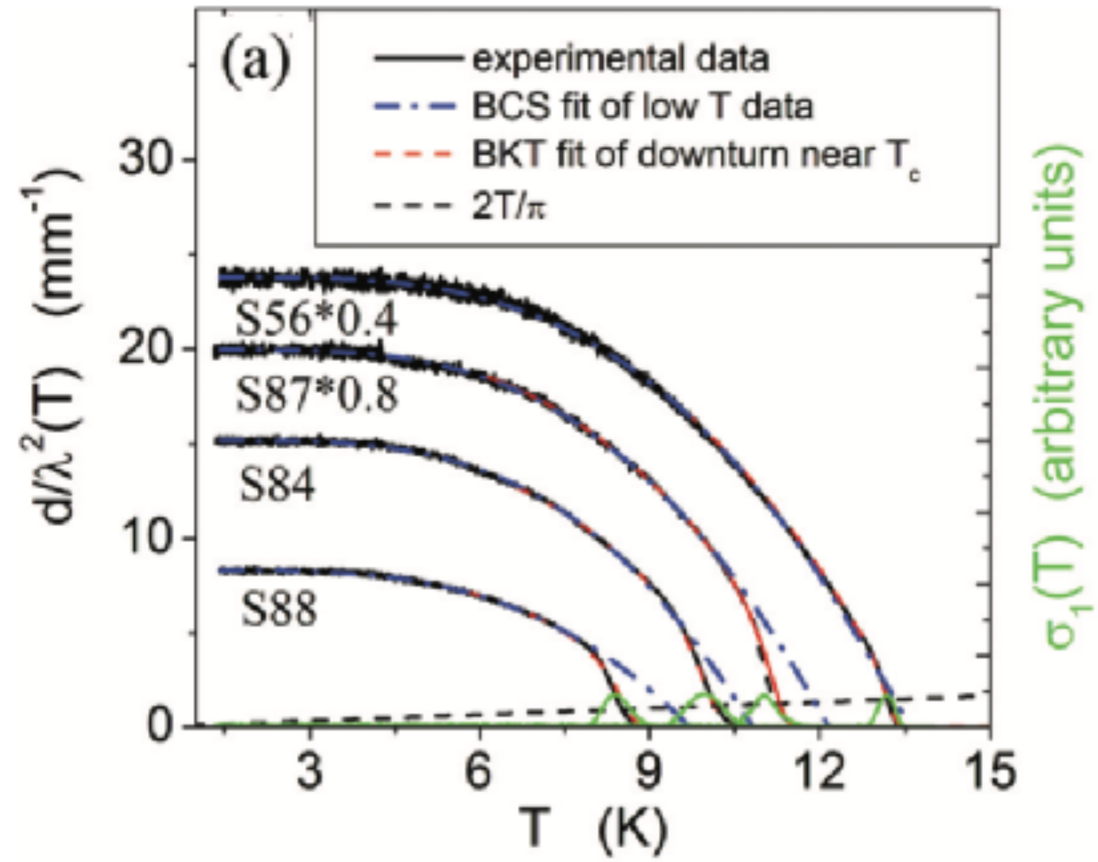
$$J_s \sim \frac{\hbar^2 n_{s\square}}{4m^*} \sim \frac{1}{2\pi}\left(\frac{\hbar c}{2e}\right)^2 \frac{d}{2\lambda^2}$$

$$J_S \sim \frac{d}{2\lambda^2} \text{ jumps from } \frac{2\,T_{BKT}}{\pi} \searrow 0$$

NbN film

YONG, LEMBERGER, BENFATTO, ILIN, AND SIEGEL PRB (2013)



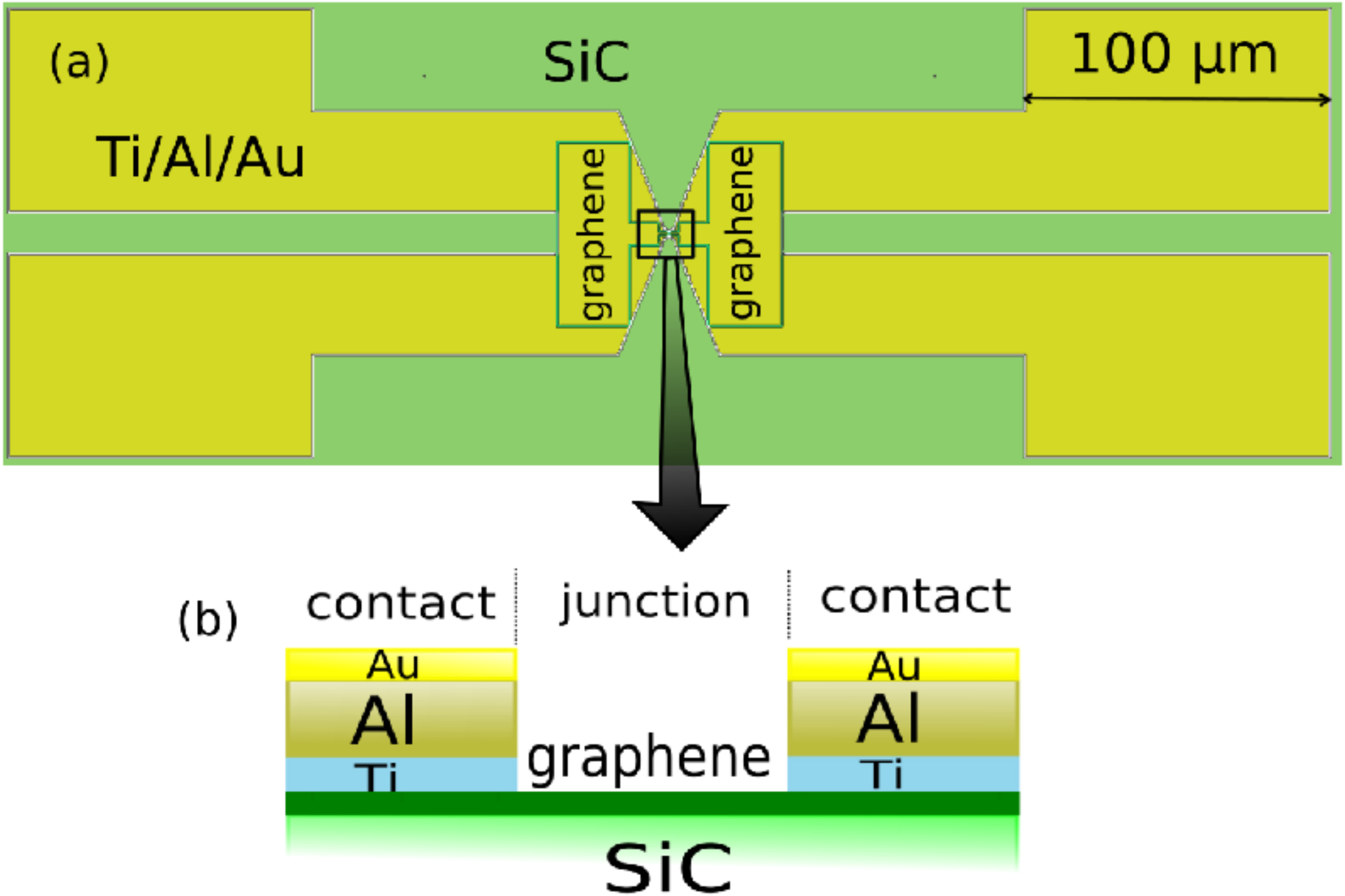


μ **:** energy required to create a vortex core

comparison **J_S - μ :**

The larger is the ratio μ/J_S the harder is to spot the BKT transition

μ rules, not **J_S**



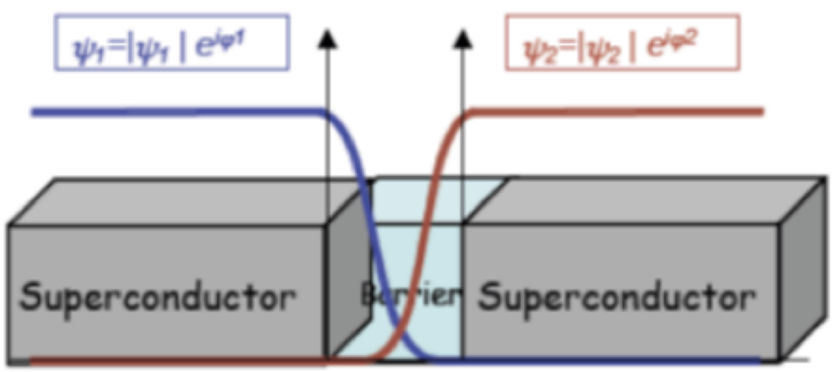
Graphene/SiC : CRHEA-CNRS, Valbonne and CNRS, Montpellier

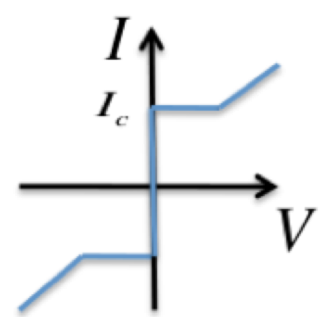
Junction: Chalmers, Göteborg , Measurement: CNR Naples

D.Massarotti et al. PRB ***94****, 054525 (2016)*

Superconducting junctions

✓Josephson junctions





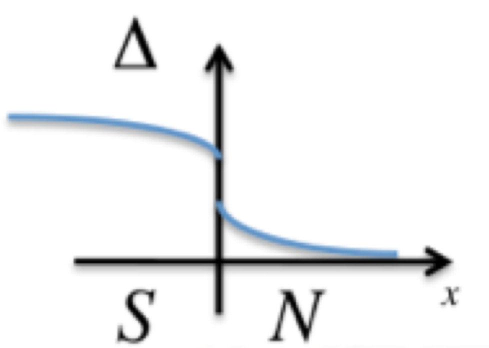
✓Josephson equations

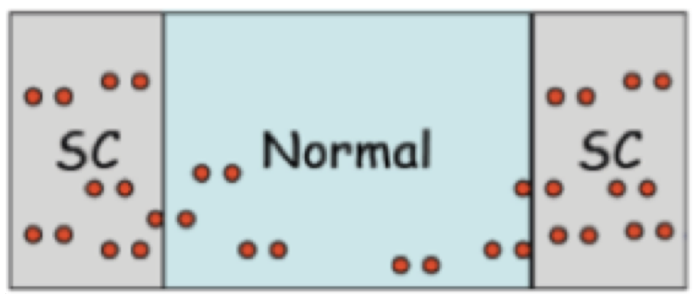
$$I_c = I\sin(\varphi)$$

$$\frac{\partial\varphi}{\partial t} = \frac{2eV}{\hbar} = \frac{2\pi V}{\Phi_0}$$

B.D. Josephson: Physics Letters 1, 251 ('62)

✓Proximity effect based on Josephson Junctions





$$I_c = I_o\left(1-\frac{T}{T_c}\right)^2 \frac{L/\xi_n}{\sinh(L/\xi_n)}$$

$$\xi_N = \sqrt{\frac{\hbar D}{2\pi k_B T}}$$

→ A few 10 nm

De Gennes: Rev Mod Physics 36, 225('64)

✓Likharev's Model for SNS junctions

$$I_c = 4\frac{\Delta^2(T)Le^{(-L/\xi_n)}}{R_n\pi e k_B T_c \xi_n}$$

K.K. Likharev: Sov Tech. Phys Lett. 2,12 ('76)

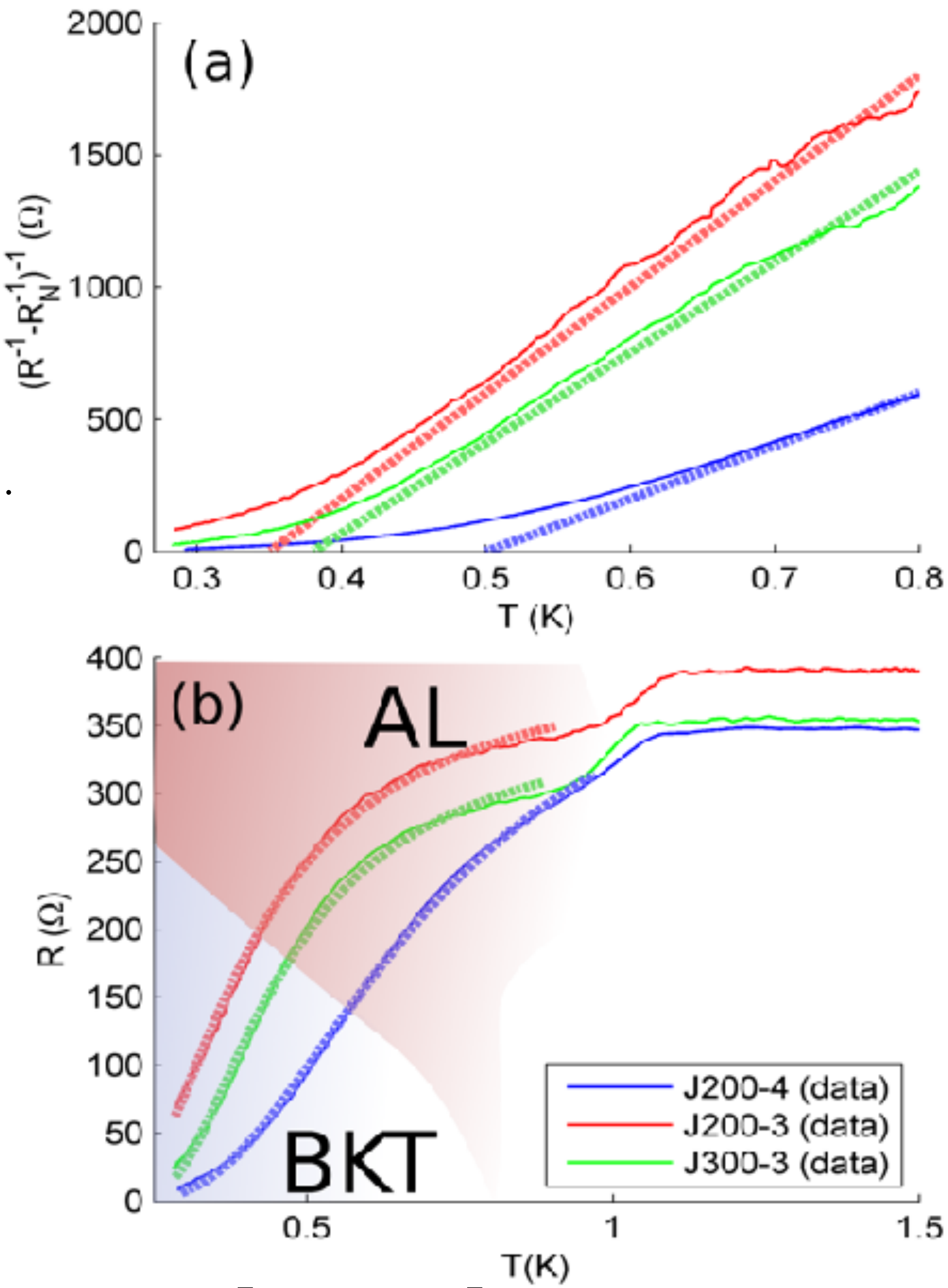
Extrapolation for T_{c0}

$$\delta\sigma(T) \approx \ln^{-1}(T/T_{c0})$$

$$\frac{1}{R} - \frac{1}{R_N} = \frac{1}{R_0}\,\frac{T_{co}}{T - T_{co}}; \qquad T > T_{co}.$$

Aslamazov-Larkin fluctuation-enhanced conductivity in two dimensions

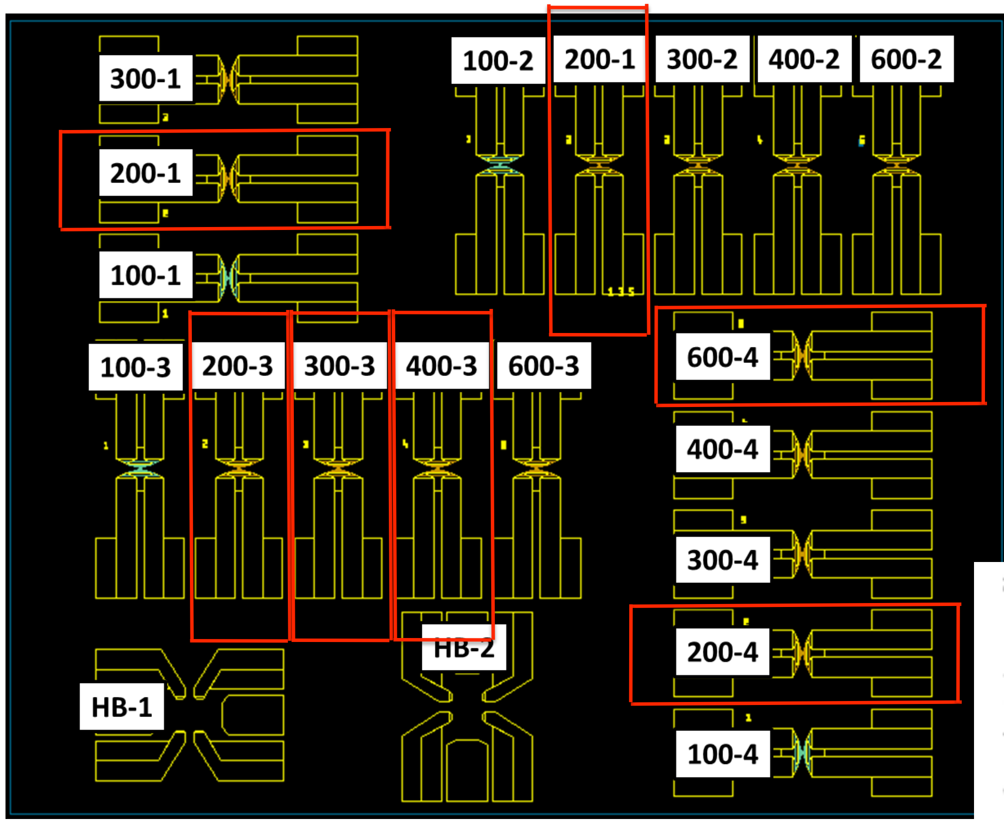
$$R_0 = 16\,\hbar/e^2 \sim 65\ \mathrm{k}\Omega$$



B. I. Halperin and D. R. Nelson, Jou.Low Temp. Phys. 36,599 (1979)

$$R(T) = \frac{0.37}{b}\,R_N \sinh^2\left[\left(\frac{bt_c}{t}\right)^{1/2}\right], \qquad T_{c0} > T > T_{BKT}$$

$$t_c = (T_{c0} - T_{BKT})/T_{BKT}, \quad t = (T - T_{BKT})/T_{BKT}$$



Red rectangles indicate the measured junctions, the first number (for instance indicates the distance between the Al electrodes

$$\mu/J_s = \pi^2\sqrt{b}/4$$

$$\mu_{XY} = \frac{\pi^2}{2}J_s$$

here $\mu/J_s \sim 6 \div 7$

Name	L nm	R_N Ω	I_c nA	R_0 kΩ	T_{c0} K	T_{BKT} mK	b
J200-1	200	720	4	8.5	0.23		
J200-2	200	425	5				
J200-3	200	410	10	1.4	0.35	130	6.1
J200-4	200	470	50	1.0	0.5	135	8.6
J300-3	300	370	30	1.3	0.38	175	7.2
J400-1	400	650	0	16.0	0.285		
J600-1	600	440	0				

Large *b*, large T_{c0} - T_{BKT}

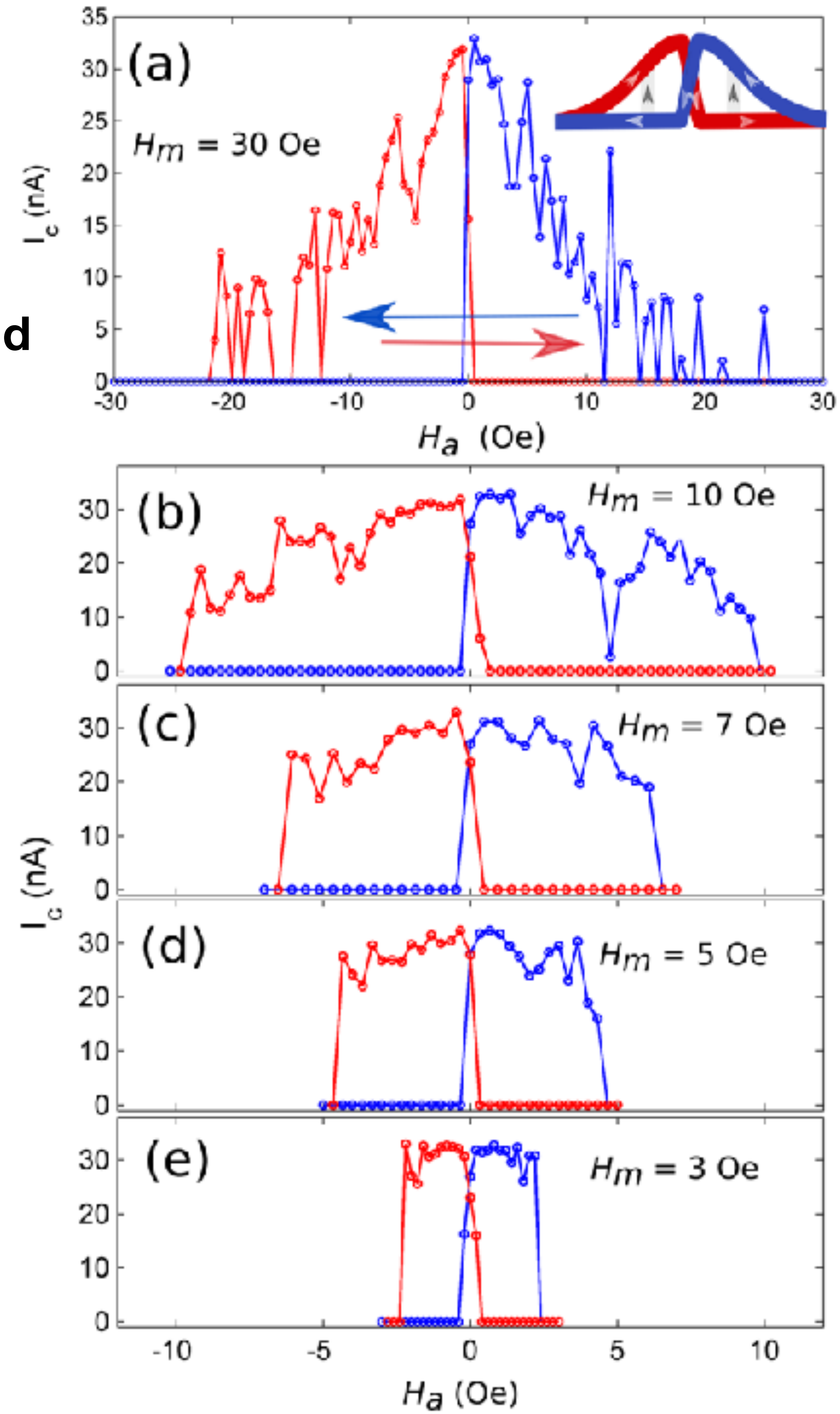
Condensation energy loss in vortex core
>> Superconducting stiffness

Berezinskii-Kosterlitz-Thouless (BKT)
as in 2D-XY model

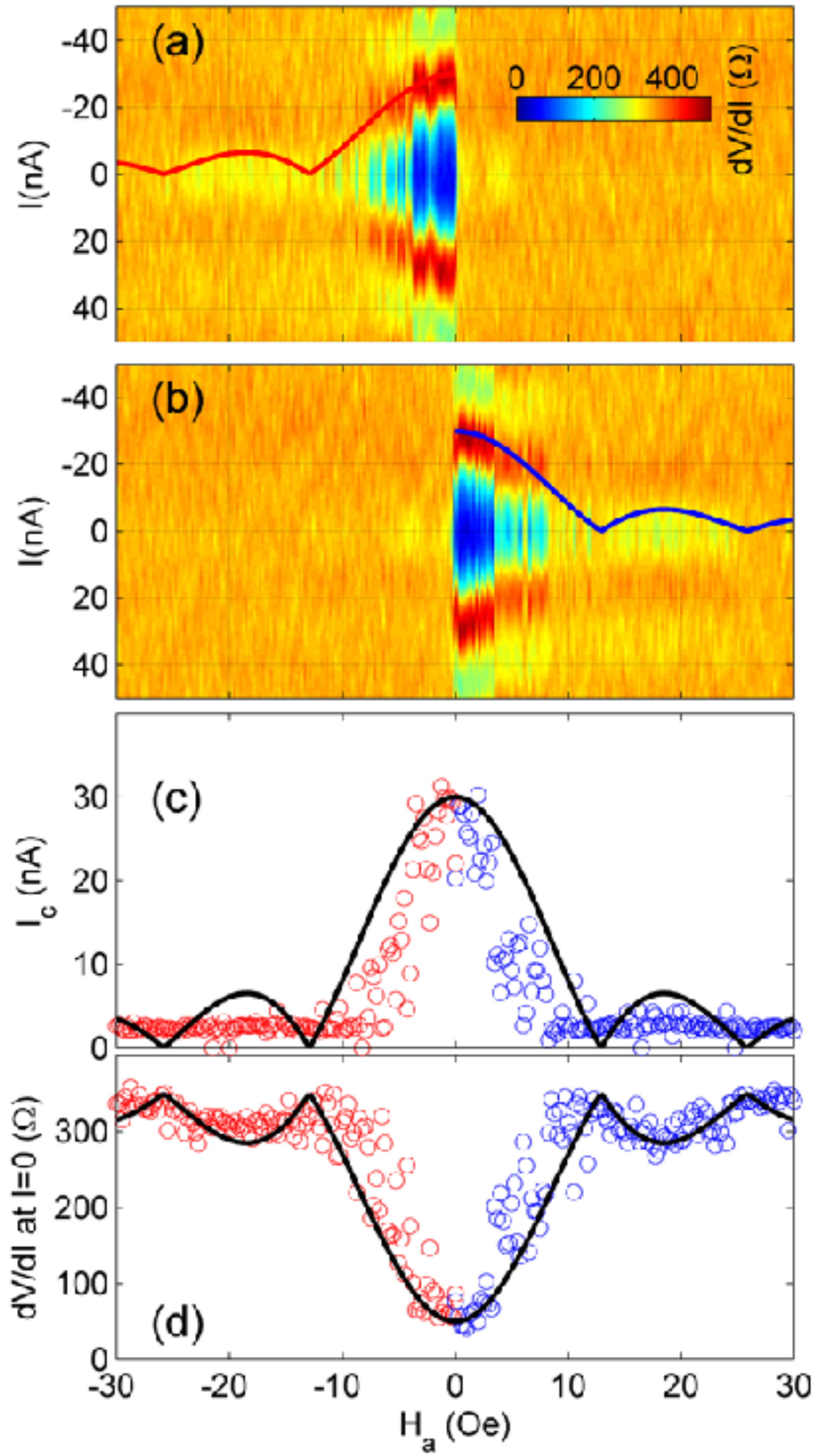
!

Vortex dynamics in magnetic field

→ **hysteresis of the critical current**

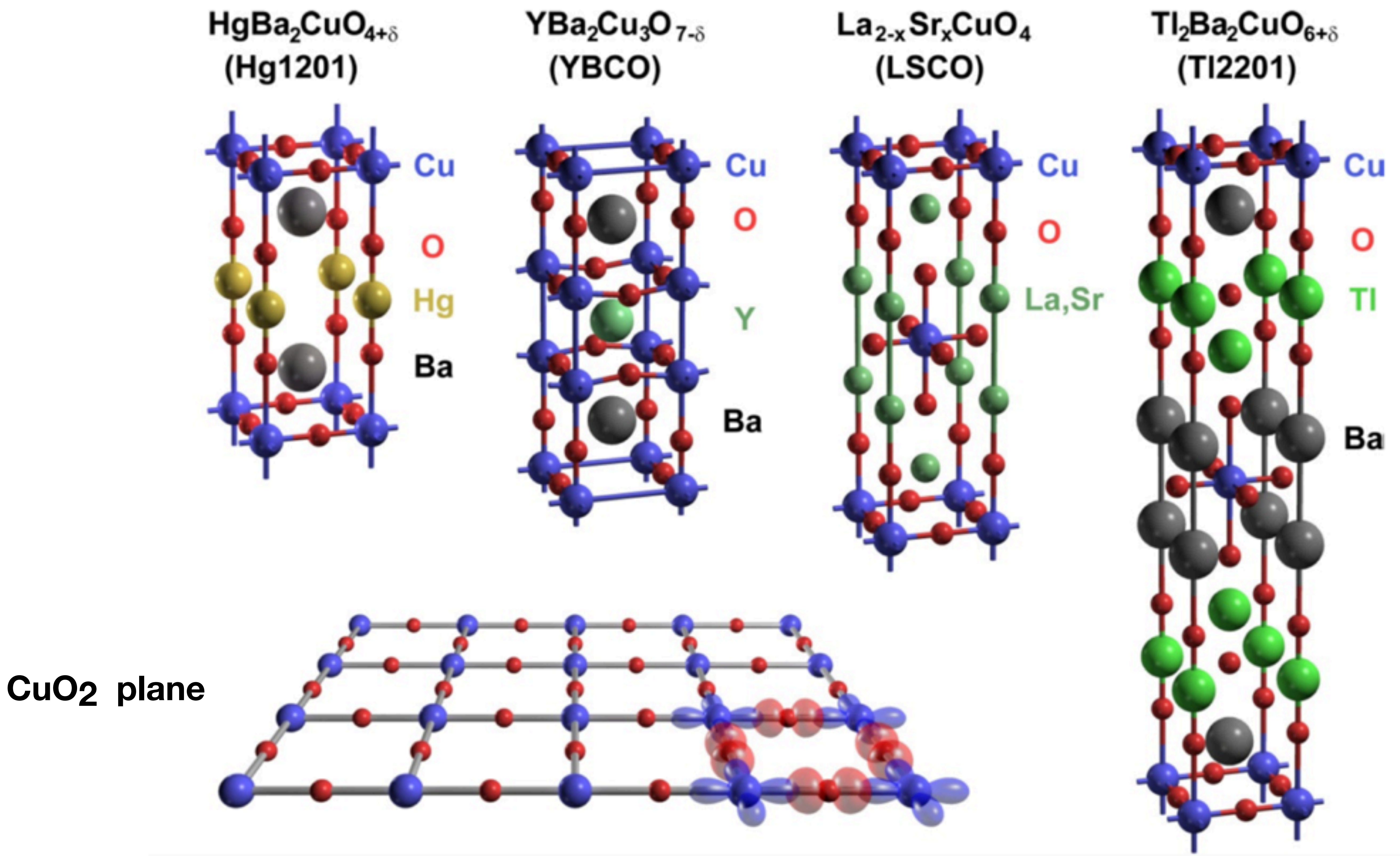


Increasing magnetic field Fraunhofer diffraction only for one direction sweep



Possible view:

at temperatures very near to the critical one the 3D fluctuation regime takes place. Here the size of the Cooper pairs along the c-axis is so large that the peculiarities of the layered structure do not play any more role



observable signatures of KT physics in a layered three-dimensional (3D) system?

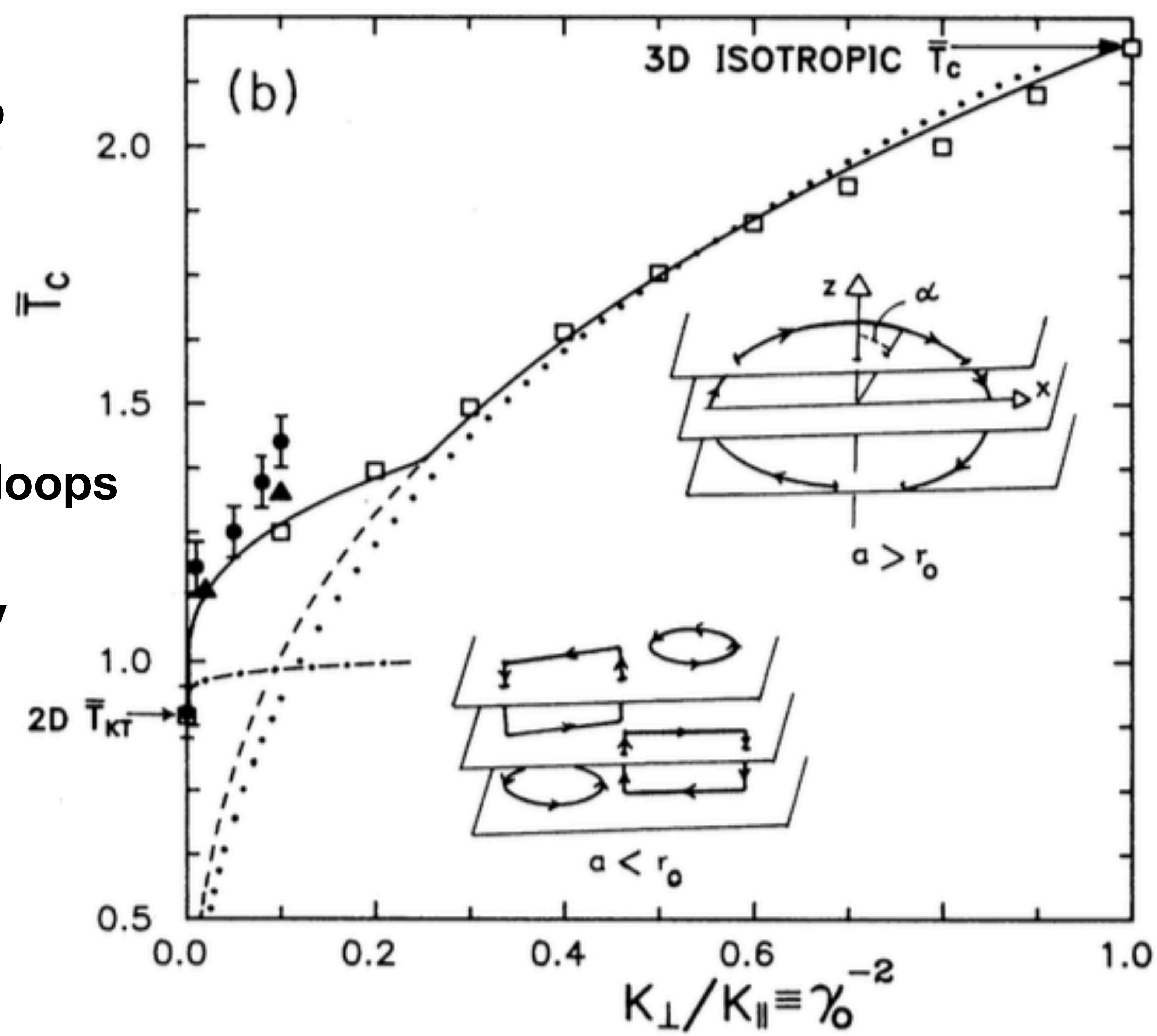
3-d XY model

$$H = -\sum_{\mu=\hat{x},\hat{y},\hat{z}} \sum_i \mathcal{K}_\mu \left[\cos \nabla_\mu \theta_i - 1\right]$$

MC results:
increasing K ratio T_C tends to 3D isotropic XY model

a: average diameter of vortex loops

S.R.Shenoy &Chattopadhhyay PRB 51 (1995)



BKT correlation length $$\xi \sim a \exp\left(\pi/\sqrt{T-T_K}\right)$$

When 3rd dimension matters $$\mathcal{K}_\perp \left(\frac{\xi}{a}\right)^2 \sim \mathcal{K}_\parallel \qquad \frac{\mathcal{K}_\perp}{\mathcal{K}_\parallel} \equiv \delta$$

2D scaling still feasible until $$\frac{\xi}{a} \sim \frac{1}{\sqrt{\delta}}$$ **(interaction is still logarithmic)**

or $$T_c = T_K + \left(\frac{\pi}{\ln 1/\sqrt{\delta}}\right)^2$$

Define $\frac{J_s(T)}{T} \sim \mathcal{K}_{eff}(T)$ **such that** $T_c = T_K \left[1 + (\pi \mathcal{K}_{eff} - 2)^2\right]$

gives $$\mathcal{K}_{eff} = \frac{2}{\pi} + \frac{1}{\ln \frac{1}{\sqrt{\delta}}}$$

S.Hikami & T.Tsuneto. Progress of Theoretical Physics 63 ,387 (1980)

Alternative view: Lawrence-Doniach model

In-plane: free vortex dynamics; Interlayer: Cooper pair hopping $\rho_c >> \rho_{ab}$

Duality vortex -Cooper pair

$$\begin{cases} v \propto j \quad F \propto V & \text{Cooper pair} \\ v \propto V \quad F \propto j & \text{vortex} \end{cases}$$

Vortex: $E = \dfrac{\phi_0}{c} n_V v_V$ **Cooper pair :** $E = \dfrac{2e\, nv}{\sigma}$

if $nv \approx n_V v_V$ **:** $\dfrac{2e}{\sigma} \approx \dfrac{\phi_0}{c}$ $\sigma \approx \dfrac{(2e)^2}{h}$

Layer Ginzburg-Landau condensate and Josephson coupling between layers

$$\mathcal{L}[u_n, A] = \sum_{n=1}^{N} \int_{\square} \left[\frac{1}{2} \left| \nabla' - iA'_n \right) u_n \right|^2 + \frac{1}{4\epsilon^2} \left(|u_n|^2 - 1 \right)^2 \right] dx\, dy$$

$$+ \sum_{n=1}^{N} \int_{\square} \left[\frac{1}{2\lambda^2} \left| u_n - u_{n-1}\, e^{i \int_{z_{n-1}}^{z_n} A_z(x,y,z)\, dz} \right|^2 \right] dxdy + \frac{1}{8\pi} \int_{\mathcal{V}} |\nabla \times A - h_{ext}|^2 \; dxdydz$$

At nano-level

$$\mathcal{H}_J = \sum_i \left\{ \frac{Q_i^2}{2C} - E_J \left[\cos\left(\theta_i - \theta_{i-1}\right) - 1\right] \right\}$$

$$\left[\frac{Q_j}{2e}, e^{i\varphi_j}\right] = e^{i\varphi_j}, \qquad \varphi_j = \theta_{j+1} - \theta_j$$

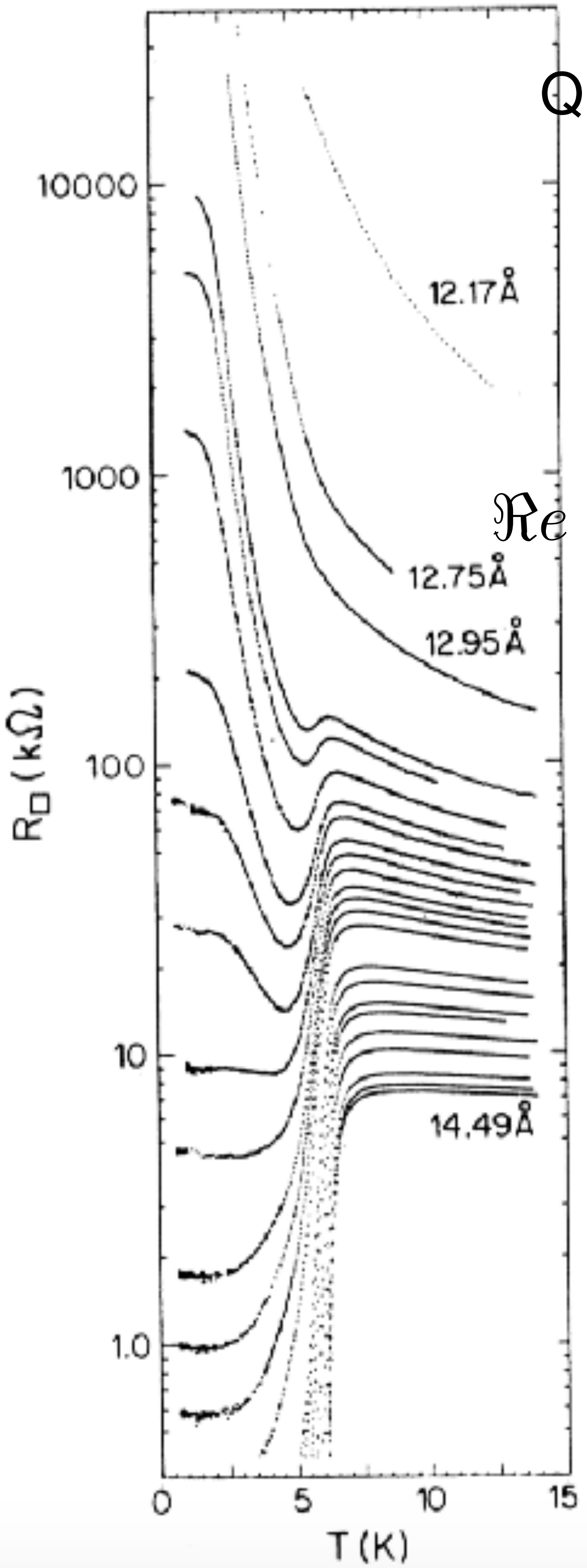
$$\left\langle (\varphi_j - \bar{\varphi}_j)^2 \right\rangle_{osc} = 4\left(\frac{E_C}{E_J}\right)^{1/2} \qquad E_C = \frac{(2e)^2}{2C}$$

Phase slip $\varphi(\tau) = 4\ \arctan\ e^{\pm\omega_J \tau}$ $\qquad \omega_J = \left(\frac{2eI_{c0}}{\hbar c}\right)^{1/2}$

Coherent tunneling of phase slips provides a quantum resistance:

$$R_0 = \frac{h}{e^2}\pi^{1/2}\left(\frac{8E_J}{E_C}\right)^{1/4} e^{-S^{inst}/\hbar} \qquad S^{inst}/\hbar = 8\left(\frac{E_J}{E_C}\right)^{1/2}$$

Also in : Amorphous 2D Ga Films



Quantum provides classical Langevin approach with noise current

$$C\ddot{\varphi} + v'(\varphi) + \int_{-\infty}^{t} dt'\, Y(t-t')\dot{\varphi}(t') = \frac{e}{\hbar} I_N(t)$$

$$\Re e\left\{Y(\omega + i0^+)\right\} = \frac{\pi}{2}\left(\frac{e^2}{\hbar c}\right)^2 \sum_\alpha \frac{C_\alpha^2}{m_\alpha}\left[\delta(\omega-\omega_\alpha) + \delta(\omega+\omega_\alpha)\right]$$

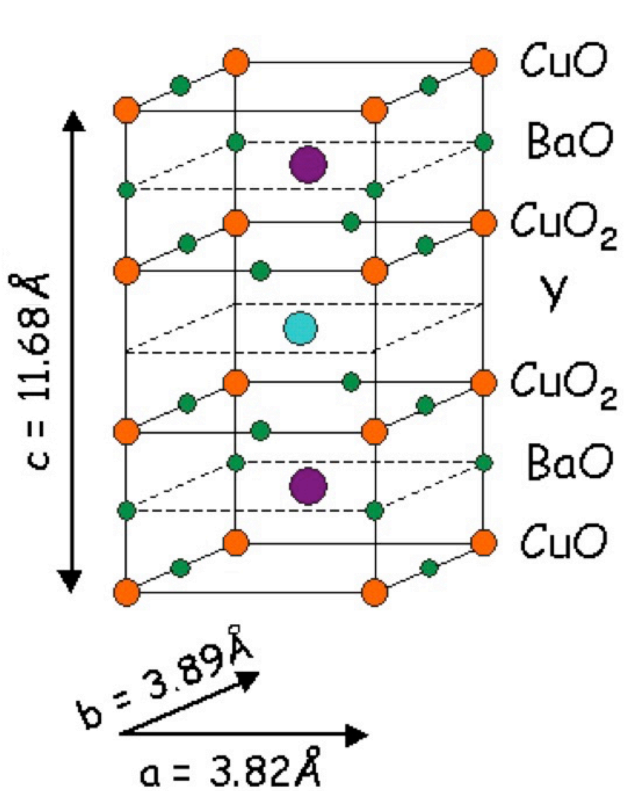
$$R_\square \propto \frac{(2e)^2}{\hbar}$$

R < $R_\square$ → **transition to superconductor**

R > $R_\square$ → **transition to insulator**

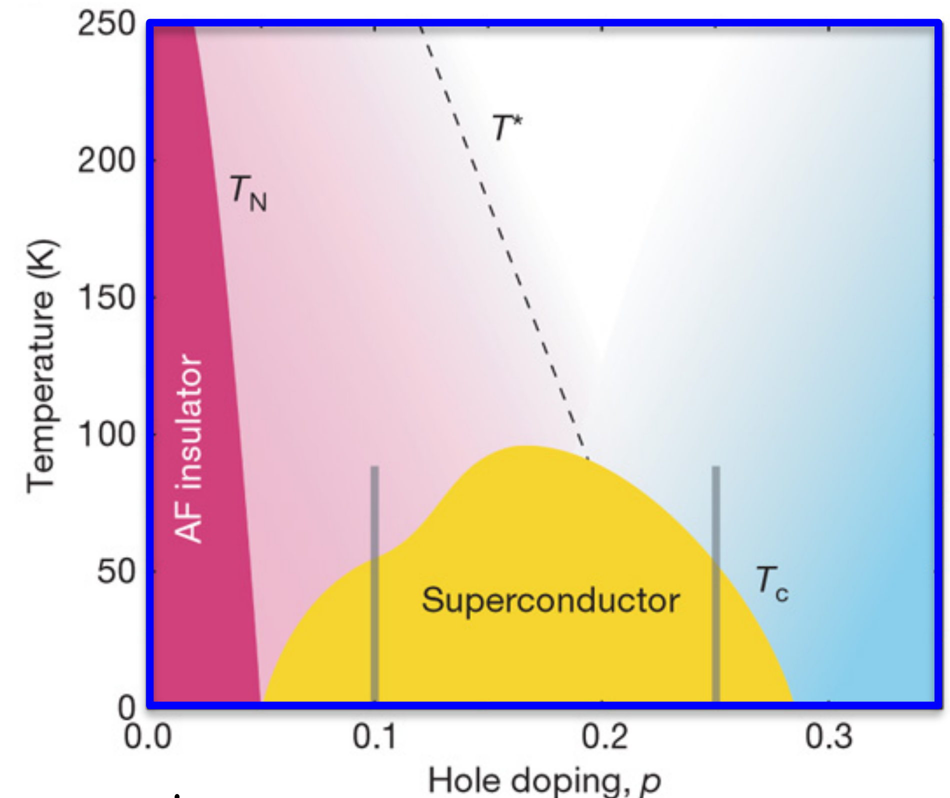
Jaeger et al PRB34,4920 (1986)

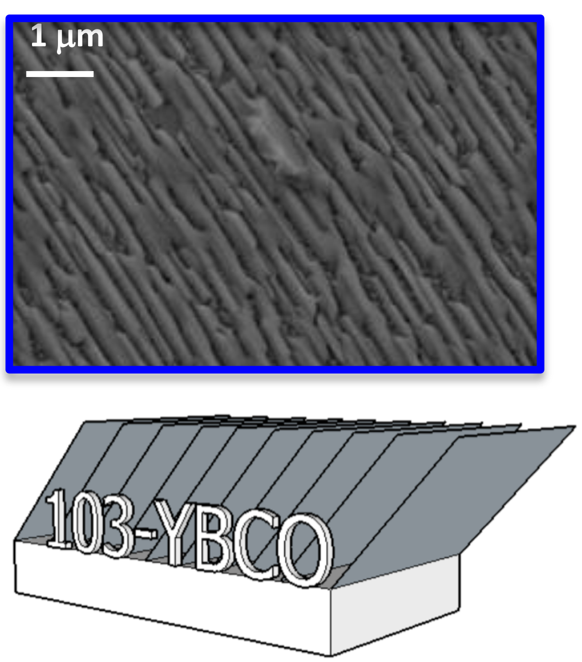
High Critical Temperature Superconductor: YBCO

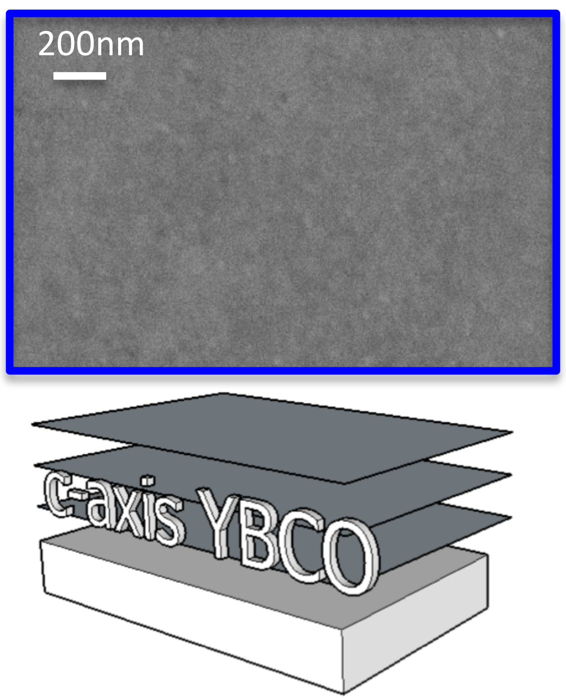


YBCO details:

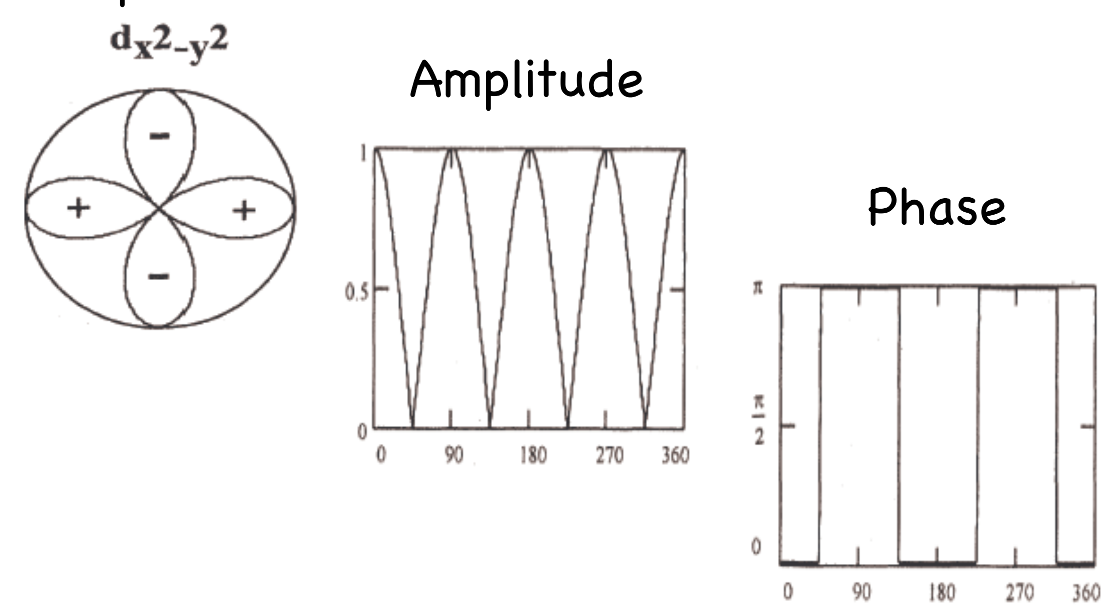
T_C=92K
$\Delta(0)$=20-25meV
λ^a_L (nm)=(150-300) nm;
$\lambda^b_L=\lambda^a_L$ /2; λ^c_L=1000nm
$\xi_a=\xi_b$=(1-3)nm; ξ_c=0.24nm
$H^a_{C2}=H^b_{C2}$=250T;
H^c_{C2}=120T



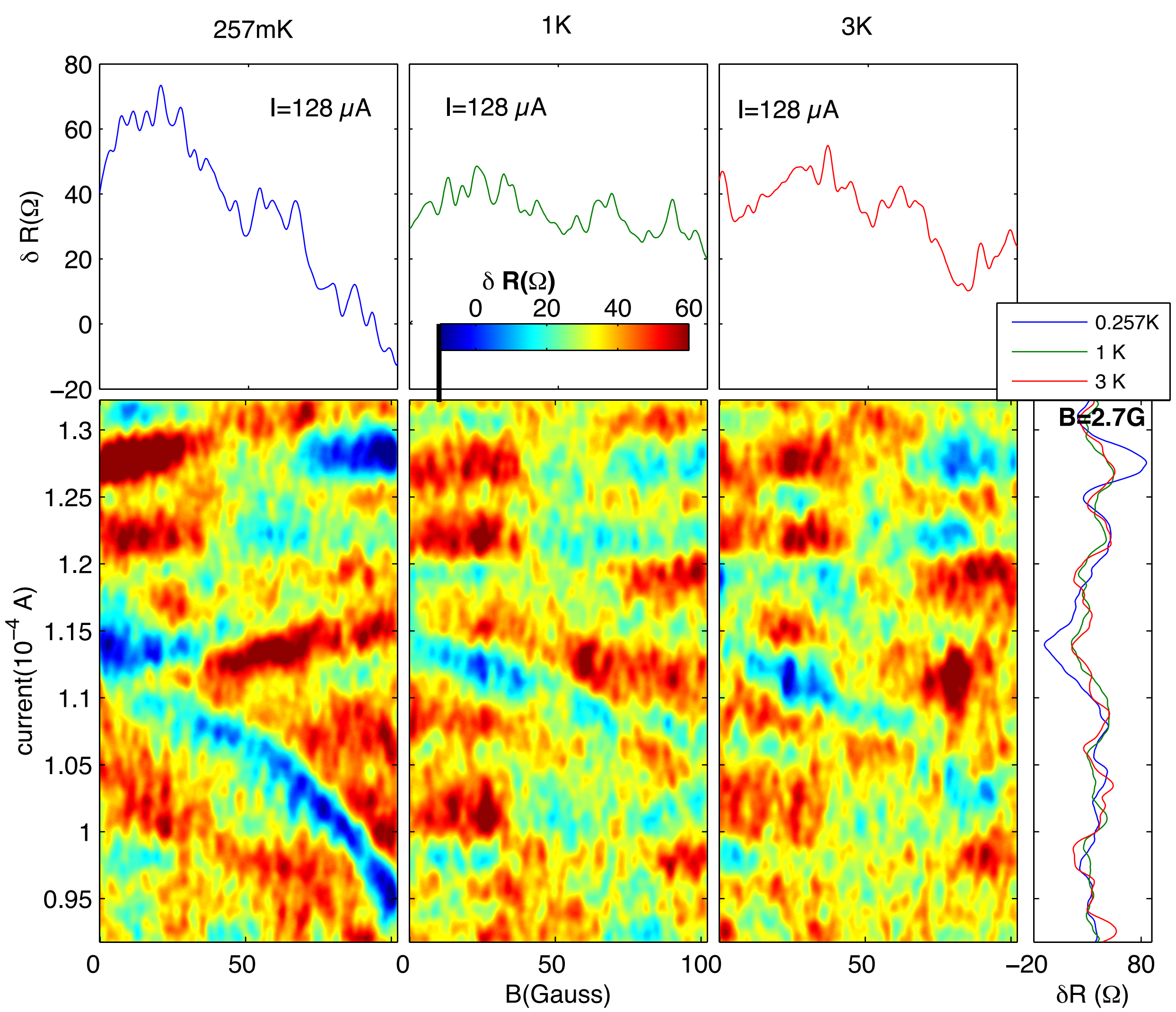


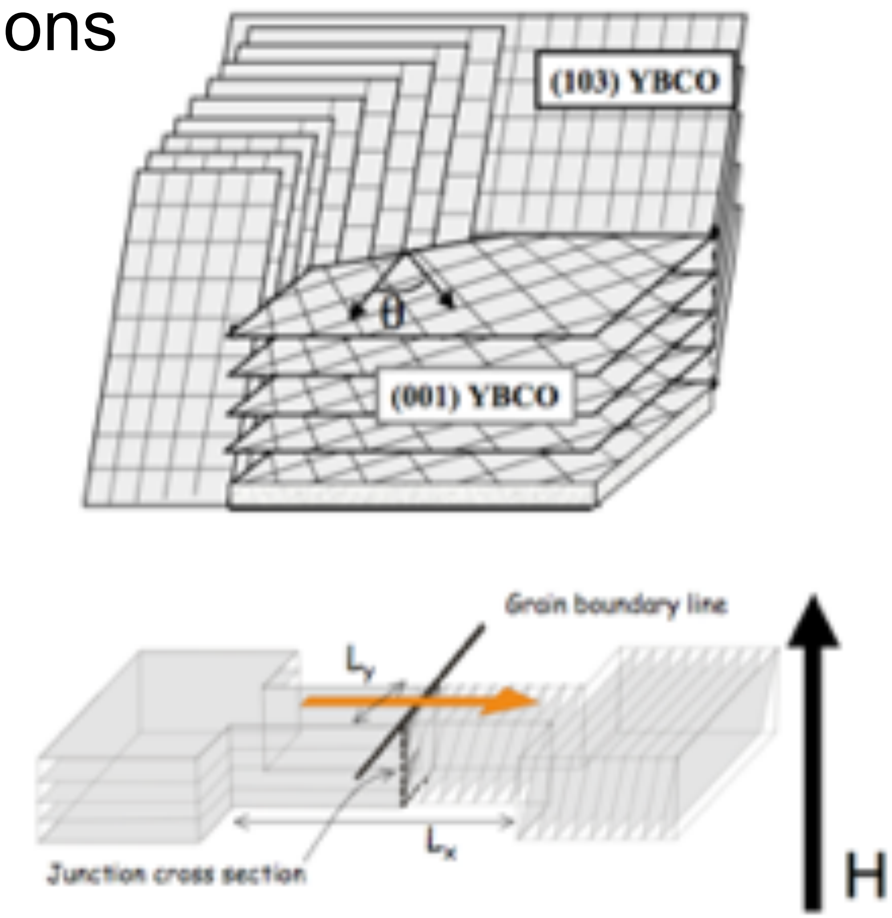


Order parameter



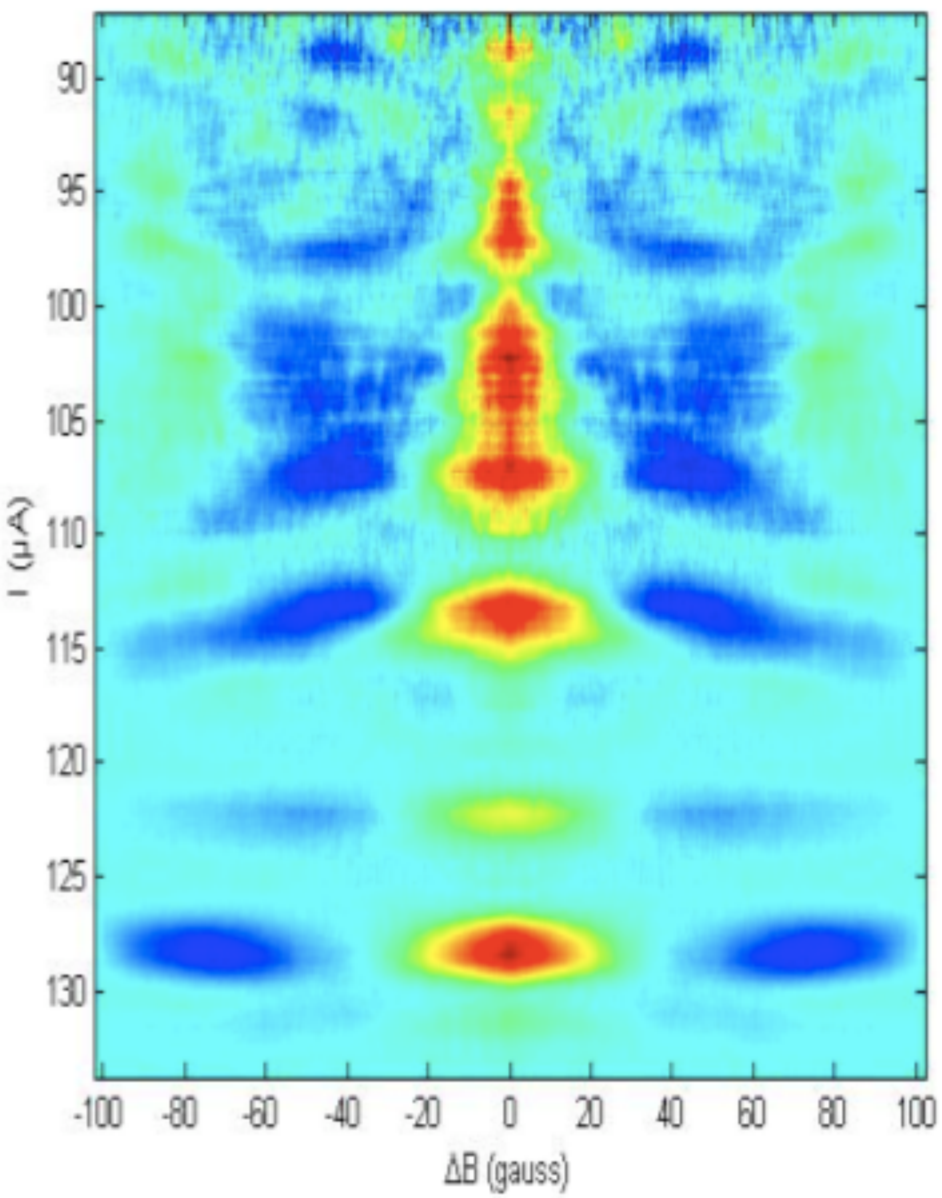
YBCO grain boundary Josephson Junctions

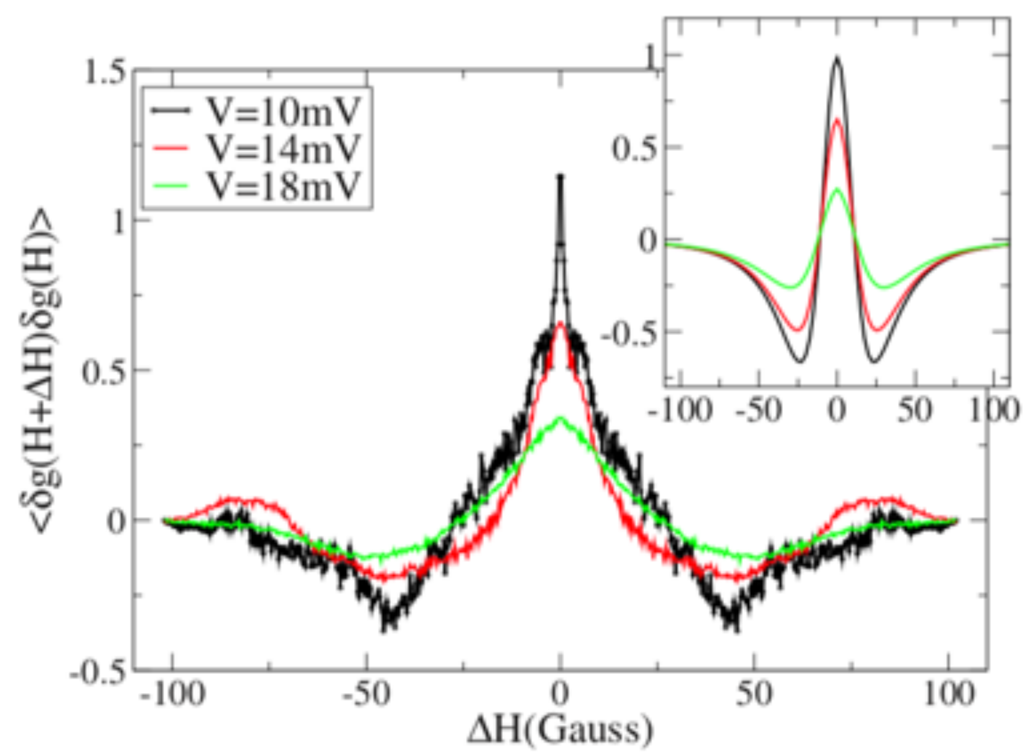




Universal Conductance fluctuations

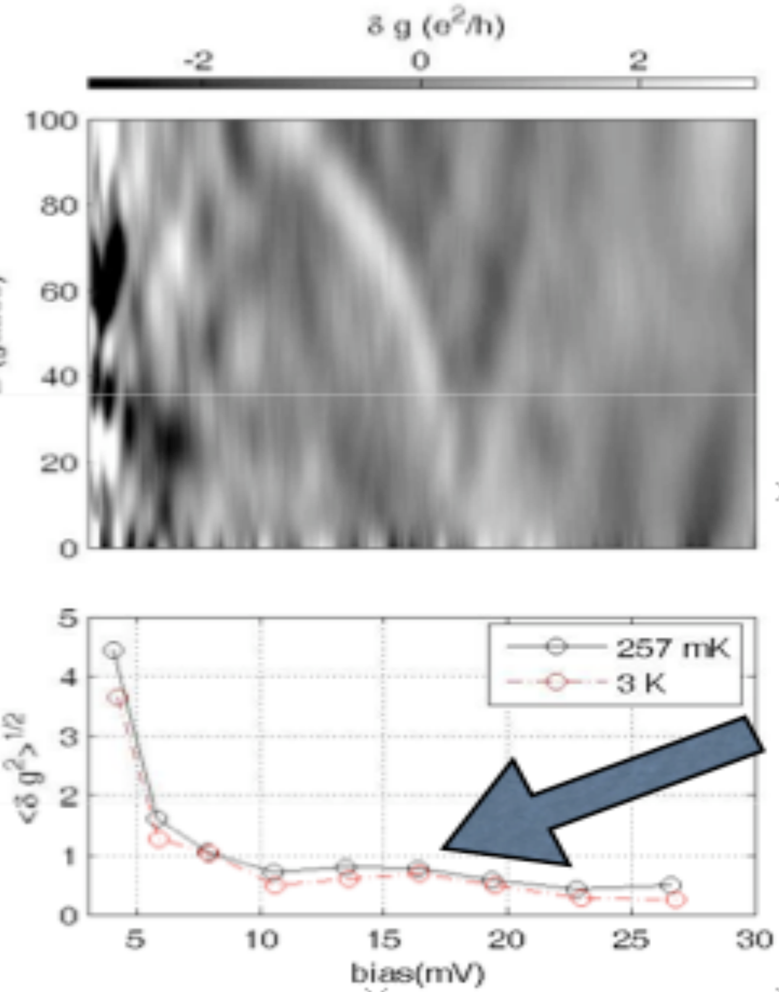
A.T. et al, Phys. Rev. B 79, 024501(2009)



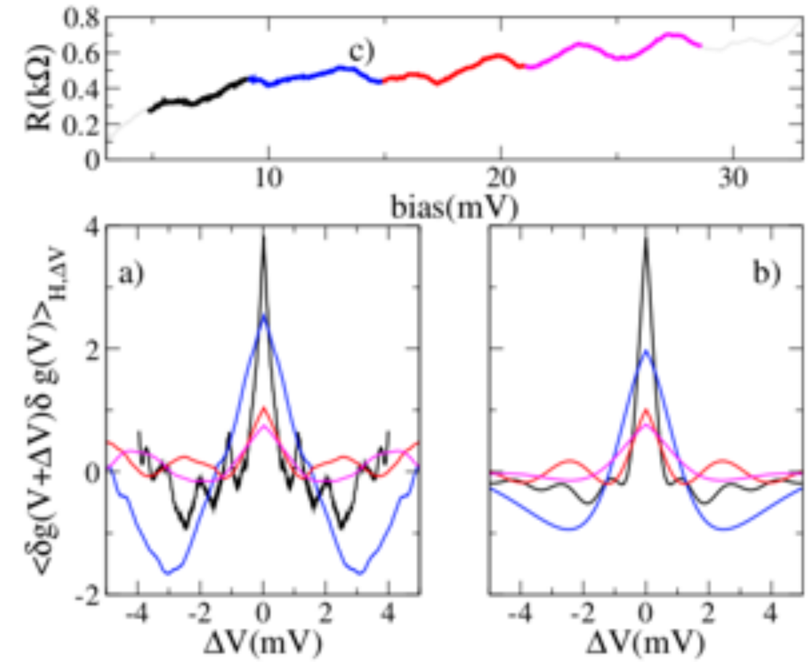


Amplitude (color) of the correlation in the (I, ΔB) plane

Conductance fluctuations



$$\left\langle (\delta G)^2 \right\rangle \sim \left(\frac{2e^2}{h} \right)^2$$



2D Square lattice : lattice spacing = 1 band structure

$$\epsilon_k = -2t\left(\cos k_x + \cos k_y\right) = -2t \cos\frac{k_x+k_y}{2}\cos\frac{k_x-k_y}{2}$$

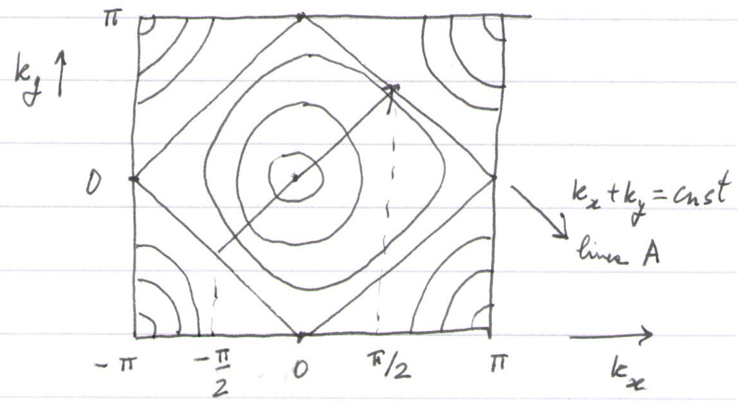
Brillouin zone is a square

& constant energy level curves : at the Fermi energy :

lungo A : $k_x + k_y = \pi$

direzione ortogonale $k_x - k_y = \text{const}$

$$\nabla \epsilon_k \Big|_{k_x+k_y=\pi} = 0$$



densità di stati :

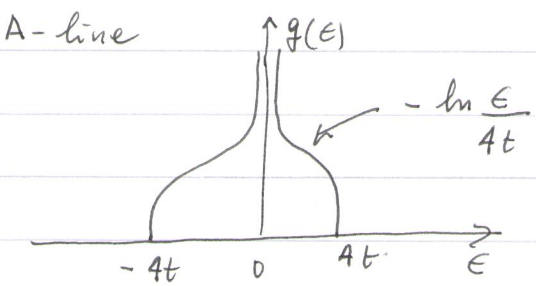
$$g(\epsilon) = \frac{2}{V}\sum_{\vec{k}} \delta(\epsilon - \epsilon_{\vec{k}}) \quad : \quad d\# = \frac{V}{(2\pi)^3}\int dk \times \begin{cases} 1 & \epsilon \le \epsilon(\vec{k}) \le \epsilon + d\epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$g(\epsilon)\, d\epsilon = \int_{\Gamma(\epsilon)} \frac{\delta k(\vec{k})}{\delta \epsilon} \frac{dl}{4\pi^3}\, \delta\epsilon$$

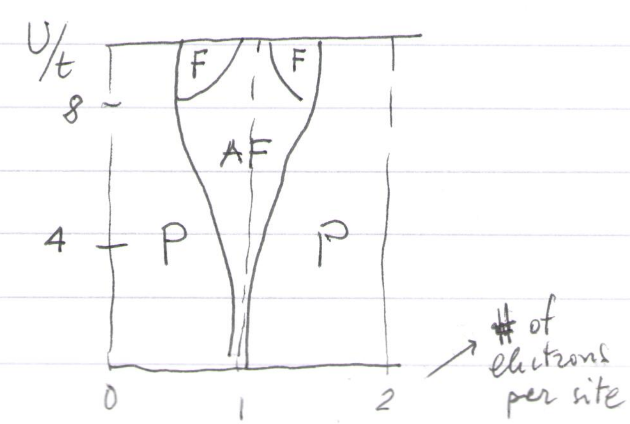
δk is $\perp$ $\Gamma(\epsilon)$ (the boundary of $S = \text{const}$

$$\Rightarrow \quad g(\epsilon) = \int_{\Gamma(\epsilon)} \frac{dl}{4\pi^3} \frac{1}{|\vec{\nabla}_k \epsilon_k|}$$

diverges along the A-line



Penn's diagram (the result of HF) :



Fe : pnictides $T_c \sim 40°K$

$LaFeAsO_{1-x}F_x$

Ce

Sm

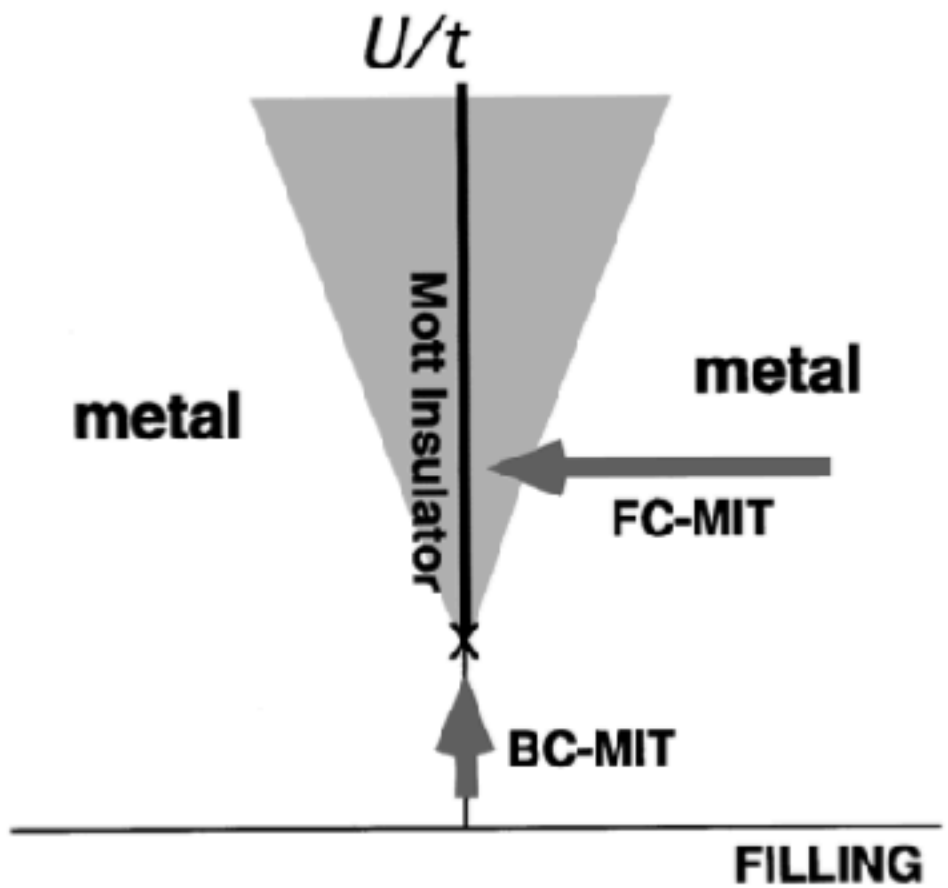


FIG. 1. Metal-insulator phase diagram based on the Hubbard model in the plane of U/t and filling n. The shaded area is in principle metallic but under the strong influence of the metal-insulator transition, in which carriers are easily localized by extrinsic forces such as randomness and electron-lattice coupling. Two routes for the MIT (metal-insulator transition) are shown: the FC-MIT (filling-control MIT) and the BC-MIT (bandwidth-control MIT).

Imada, Fujimori, and Tokura: Metal-insulator transitions

Conclusion

- In superconductors non perturbative coupling between matter and radiation
- Low dimensionality enhances the role of quantum fluctuations
- Vacuum of 2D thin superconducting films include vortex anti.vortex pairs as quantum fluctuations
- Same in layered structures as HTc superconductors.
- Quantum properties of HTc superconductors can be seen in transport as universal conductance fluctuations
- HTc materials are strongly correlated electron systems and additional fluctuations can be present when doping and moving away from the parent Mott insulator