Casimir effect in high-temperature superconductors

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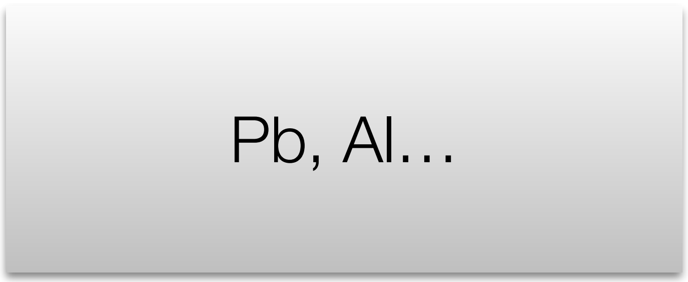
Conference: Vacuum Fluctuations at Nanoscale and Gravitation: theory and experiments

Orosei, 29.4.2019.

Motivation:

- understand the energetics of high-temperature superconductivity.
- Conventional superconductors limited to about Tc = 40 K
  - e.g. Pb, Al, Ti, Sn, Nb…
  - (more-or-less) homogeneous pieces of metal

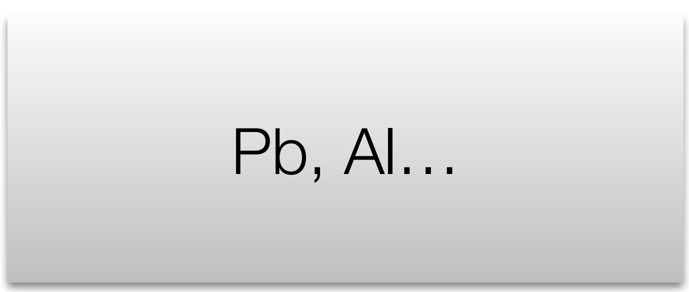


- BCS theory explains conventional superconductors,
- but, BCS phonon mediated electron-electron binding energy too small at higher temperatures.

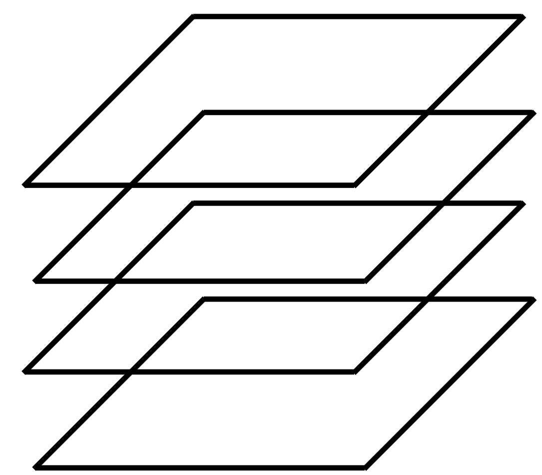
Motivation:

- understand the energetics of high-temperature superconductivity.
  - high-temperature superconductors go up to about Tc = 150 K
  - Y-Ba-Cu-O, Bi–Sr–Ca–Cu–O, La-Ba-Cu-O, etc.

conventional



high-temperature



YBCO, BSCCO,…

- But what explains the energetics of the high-temperature superconductors?

Proposal:

- Balance of energies may not be local
- Local vs. global energy trade-off

- The formation of each Cooper pair costs energy.

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Observations:

- high-temperature superconductors are layered structures
- above Tc the CuO(2) planes insulators, below Tc superconducting

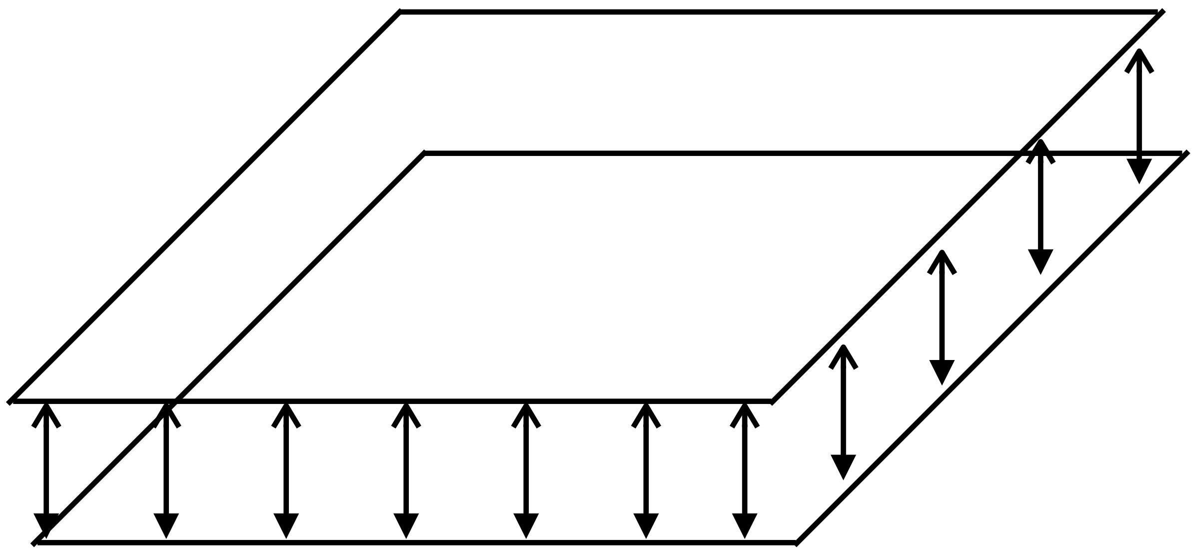
Idea (A. Kempf , 2004):

- what if Casimir effect plays a role?

A. Kempf, arXiv: gr-qc/0403112

A. Kempf, 2005, Proc. 10th Marcel Grossmann Meeting (Rio de Janeiro, 20–26 July 2003) ed. M. Novello, S. P. Bergliaffa and R. Ruffini (Singapore: World Scientific) p 2271

Quick review: Casimir effect



2 metal plates

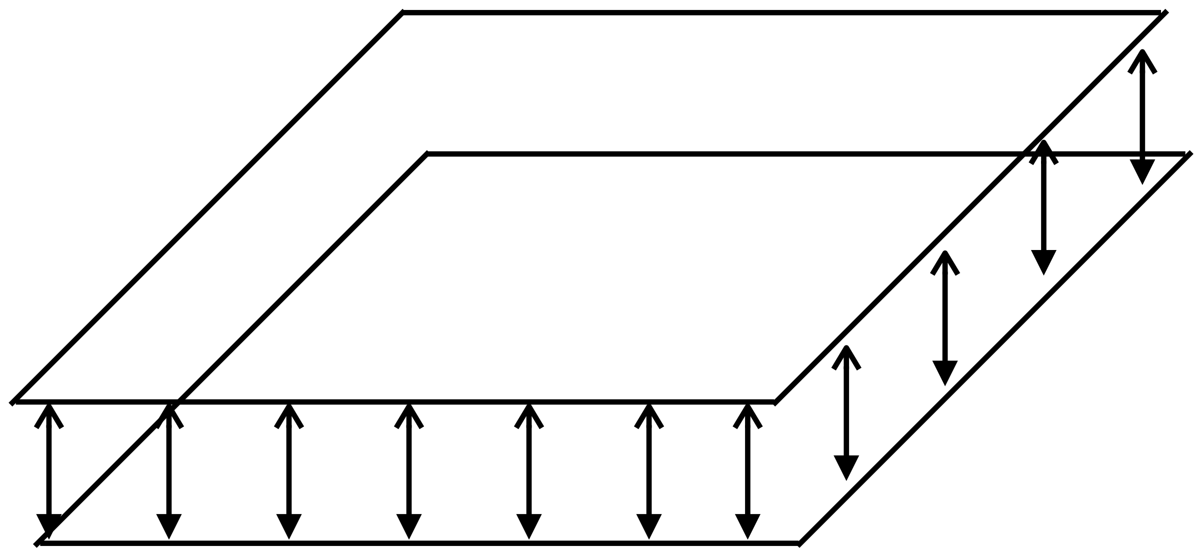
attract each other!

—> Casimir effect

$$E_{Cas}(a) = -\frac{\pi^2 \hbar c A}{720 a^3}$$

(for ideal conductors)

Quick review: Casimir effect



2 metal plates

attract each other!

—> Casimir effect

$$E_{Cas}(a) = -\frac{\pi^2 \hbar c A}{720 a^3}$$

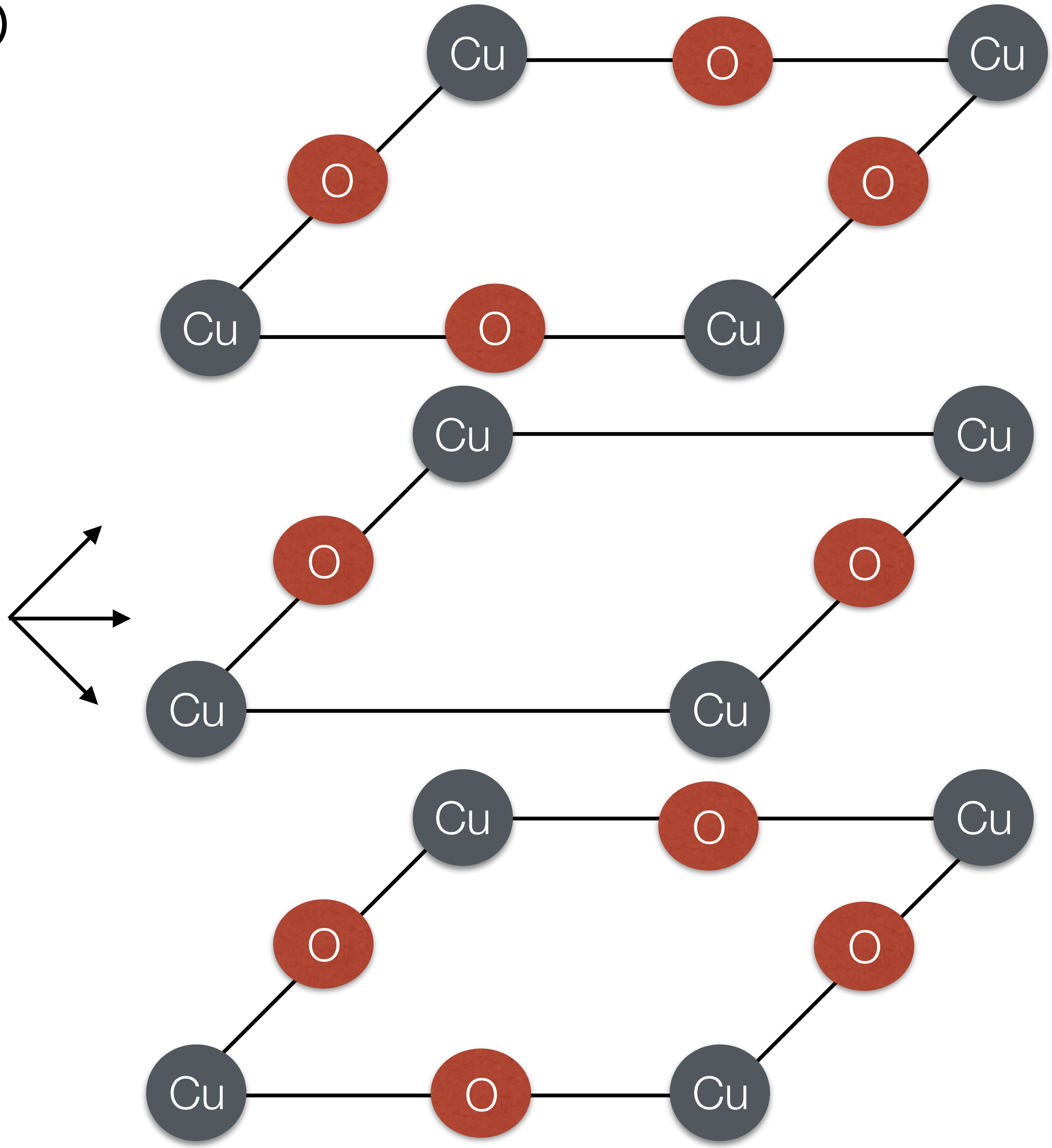
(for ideal conductors)

negative!

High-temperature superconductors

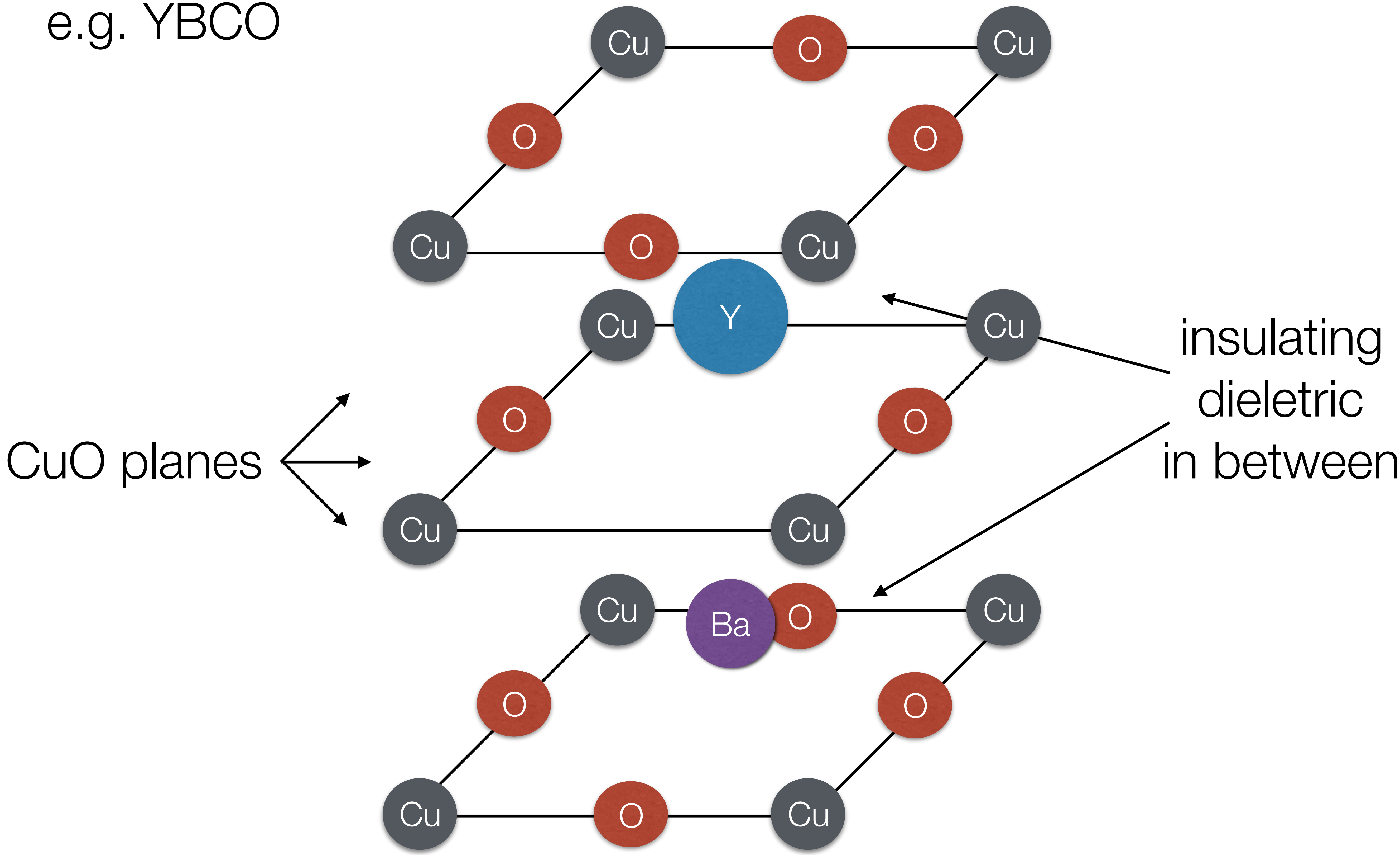
e.g. YBCO

CuO planes



High-temperature superconductors

e.g. YBCO



How could it work concretely?

- Calculate Casimir energy of superconducting layers separated by a dielectric medium, and compare to condensation energy!
- A simple model:
    - parallel plasma sheets separated by vacuum, with realistic layer distance and electron density

A. Kempf, arXiv:cond-mat/0603318
A. Kempf, arXiv:0711.1009
A. Kempf 2008 J. Phys. A: Math. Theor. 41 164038

- Casimir energy of parallel plasma sheets:

$$E_c(a) = -5 \times 10^{-3} \hbar c A \sqrt{\frac{nq^2}{2mc^2\epsilon_0}} a^{-5/2}$$ (Bordag, 2006)

- Casimir energy is spent on the condensation into superconducting state:

$$E_c(a) = E_{cond}$$

- Condensation energy is related to the transition temperature:

$$E_{cond} = -D(\epsilon_F)\Delta^2(0)/2$$

$$T_c = \Delta(0)/\eta k_B$$

Bordag M 2006 J. Phys. A: Math. Gen. 39 6173

- Realistic layer distance and electron density:

$$a = 1 \text{ nm}$$

$$n = 10^{14} \ (\text{cm})^{-2}$$

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$$\boxed{T_c = 125 \text{ K}}$$

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$$a = 1 \text{ nm}$$

$$n = 10^{14} \text{ (cm)}^{-2}$$

- Transition temperature:

$$\boxed{T_c = 125 \text{ K}}$$

- the right order of magnitude!

- Compare:

$$T_c = 125\ \text{K}$$

- HTSCs:

Formula	T_c (K)
$YBa_2Cu_3O_7$	92
$Bi_2Sr_2CuO_6$	20
$Bi_2Sr_2CaCu_2O_8$	85
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110
$Tl_2Ba_2CuO_6$	80
$Tl_2Ba_2CaCu_2O_8$	108
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125
$TlBa_2Ca_3Cu_4O_{11}$	122
$HgBa_2CuO_4$	94
$HgBa_2CaCu_2O_6$	128
$HgBa_2Ca_2Cu_3O_8$	134

(Wiki)

- Compare:

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(Wiki)

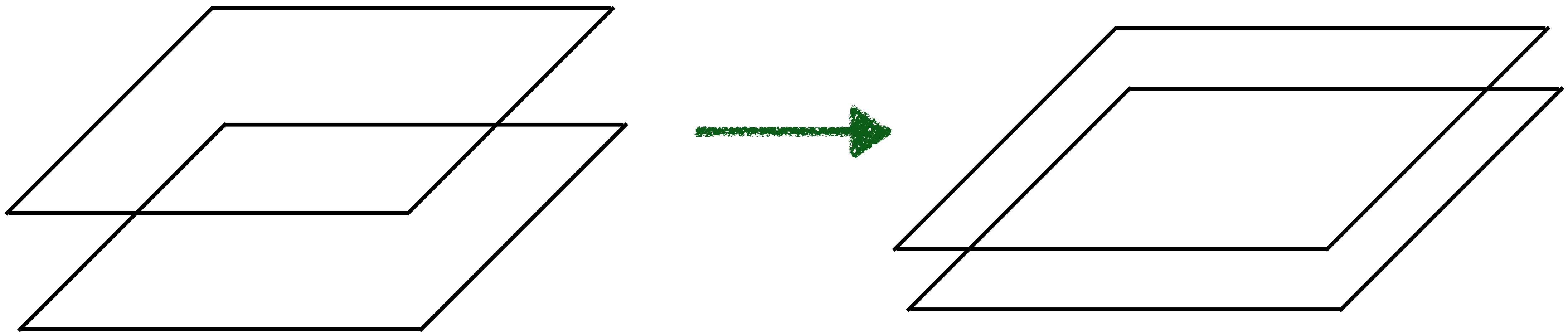
- Note: superconductors are worse conductors than ideal conductors
- If layers were ideal conductors: $T_c \approx 3000\ \text{K}$

Other indications that Casimir effect plays a role:

- smaller layer distance -> higher Tc (Li et al, Lowndes et al)

$$E_c(a) = E_{cond}$$

$$E_{cond} \propto T_c^2$$



$$T_c \quad < \quad T_c'$$

Li Q, Xi X X, Wu X D, Inam A, Vadlamannati S and McLean W L 1990 Phys. Rev. Lett. 64 3086
Lowndes D H, Norton D P and Budai J D 1990 Phys. Rev. Lett. 65 1160

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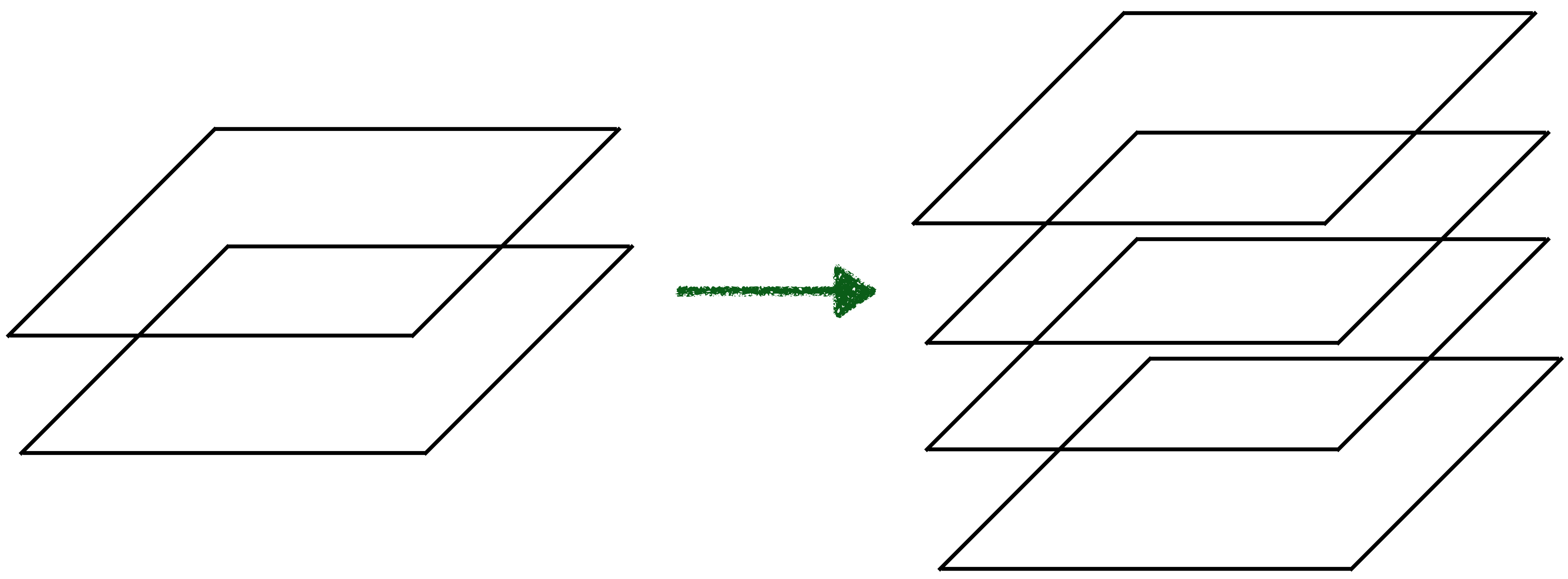
- more layers -> higher Tc (Li et al, Lowndes et al)



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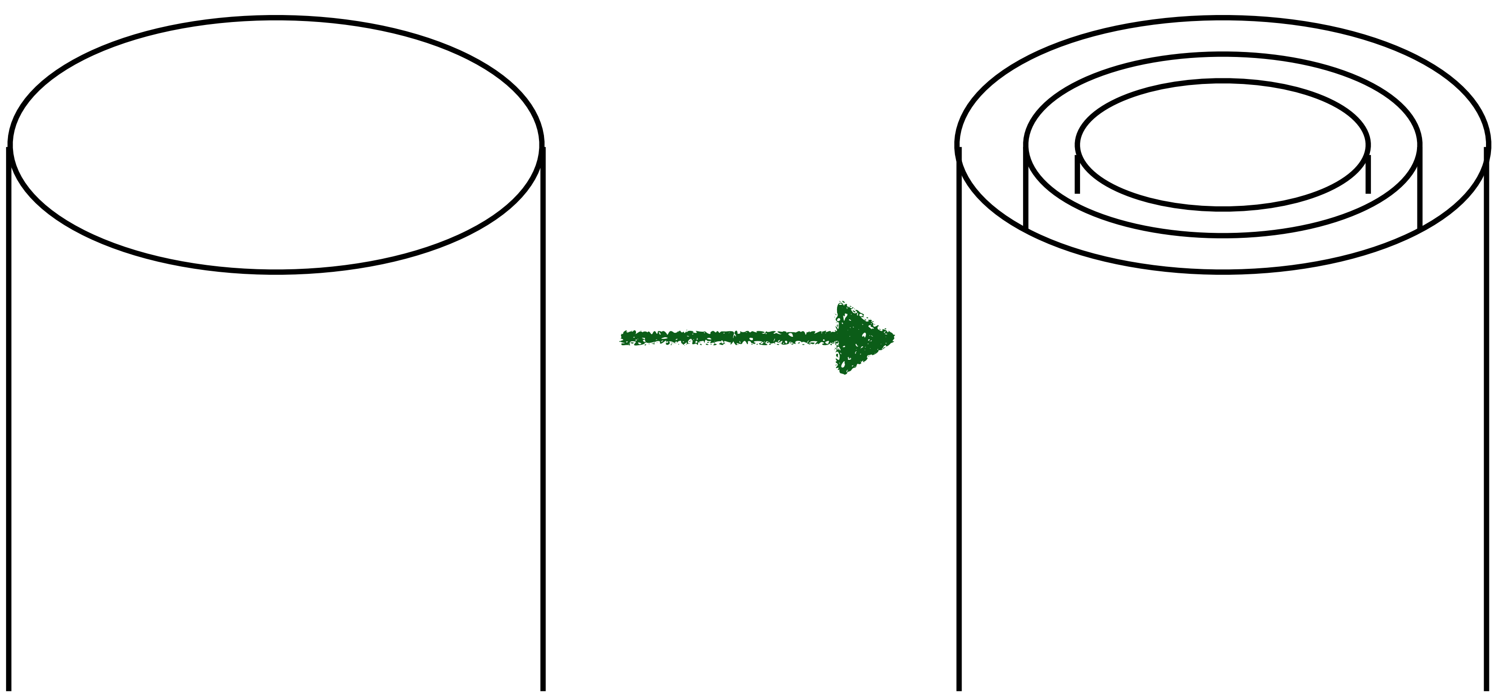
Other indications that Casimir effect plays a role:

- smaller layer distance -> higher Tc (Li et al, Lowndes et al)
- more layers -> higher Tc (Li et al, Lowndes et al)
- more layers of carbon nanotubes -> higher Tc (Takesue, 2006)

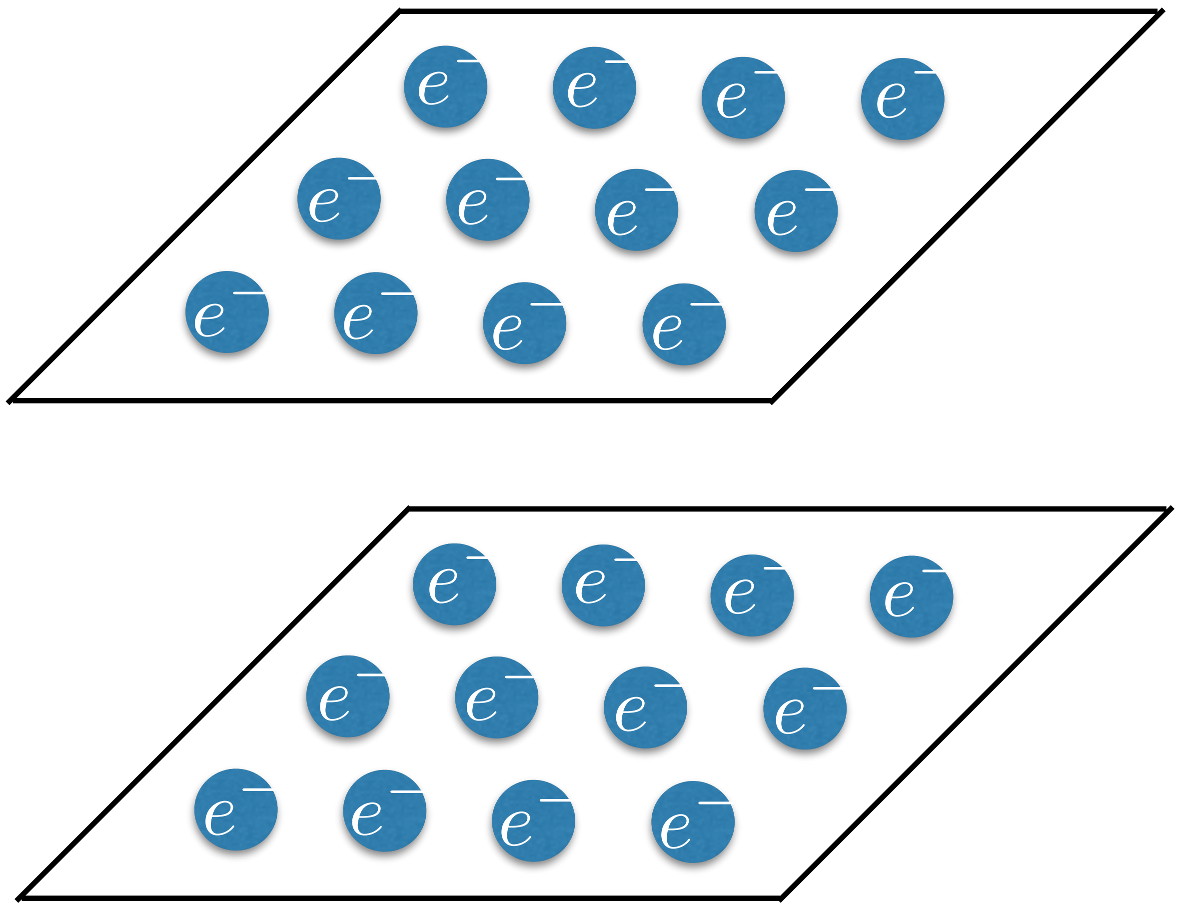


$$T_c \quad < \quad T_c'$$

Takesue I et al 2006 Phys. Rev. Lett. 96 057001

Overall picture:

above Tc

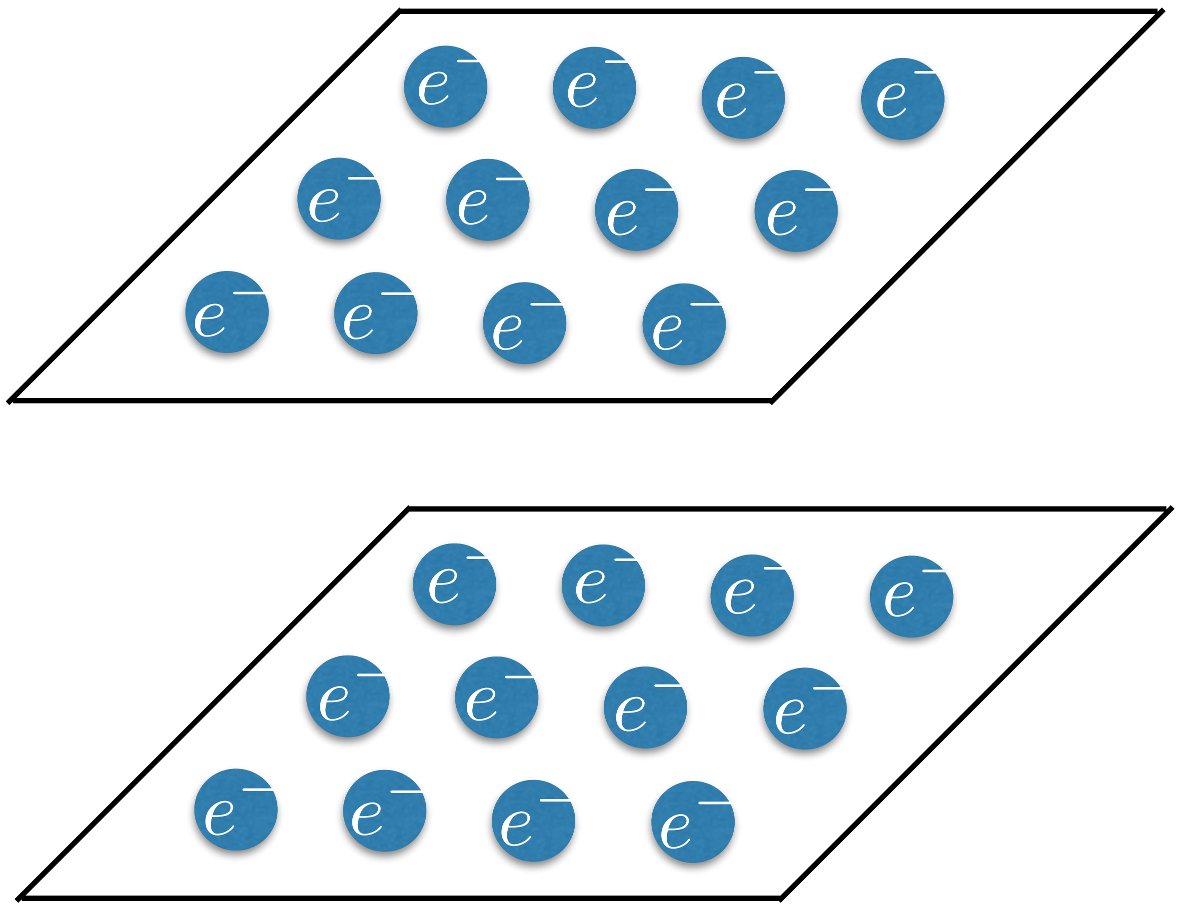


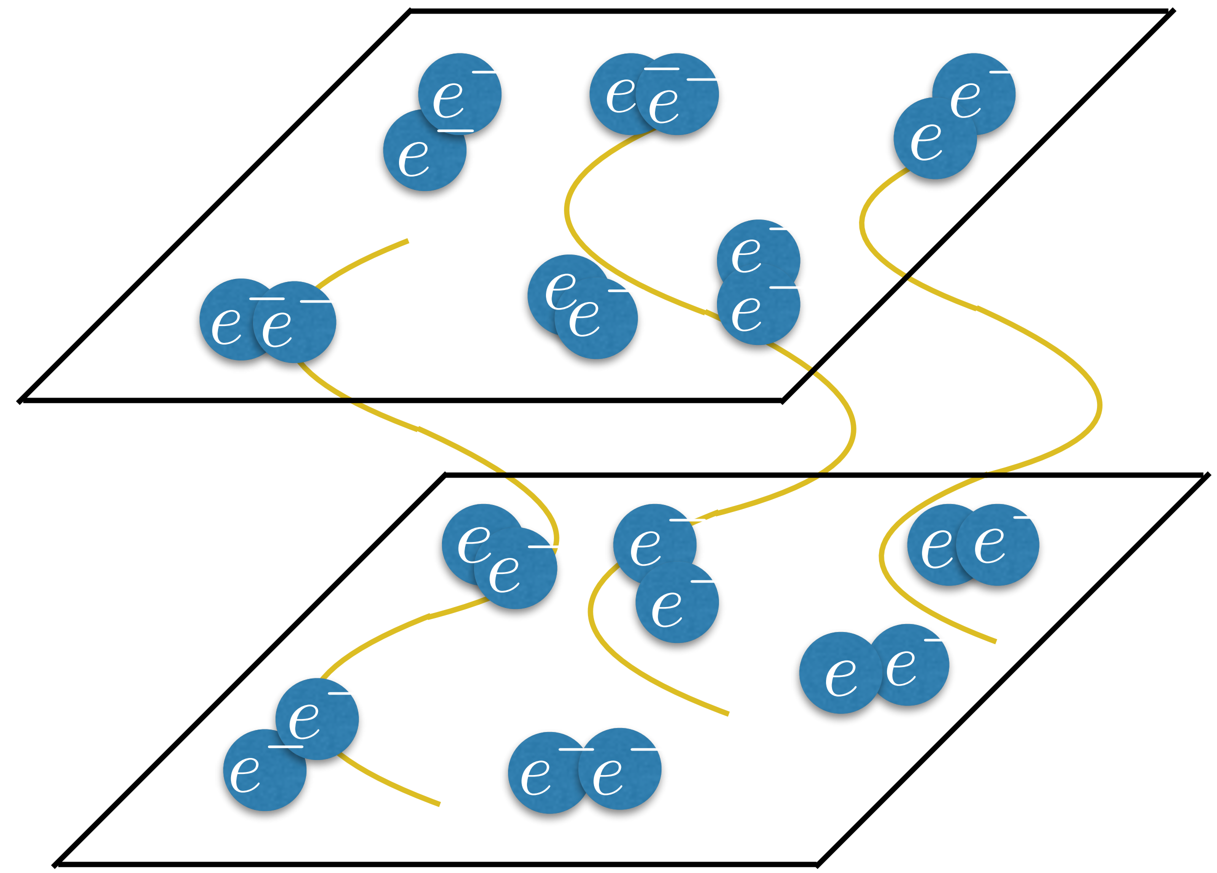
- Coulomb repulsion
- low conductivity
- low Casimir effect

Overall picture:

above Tc

below Tc

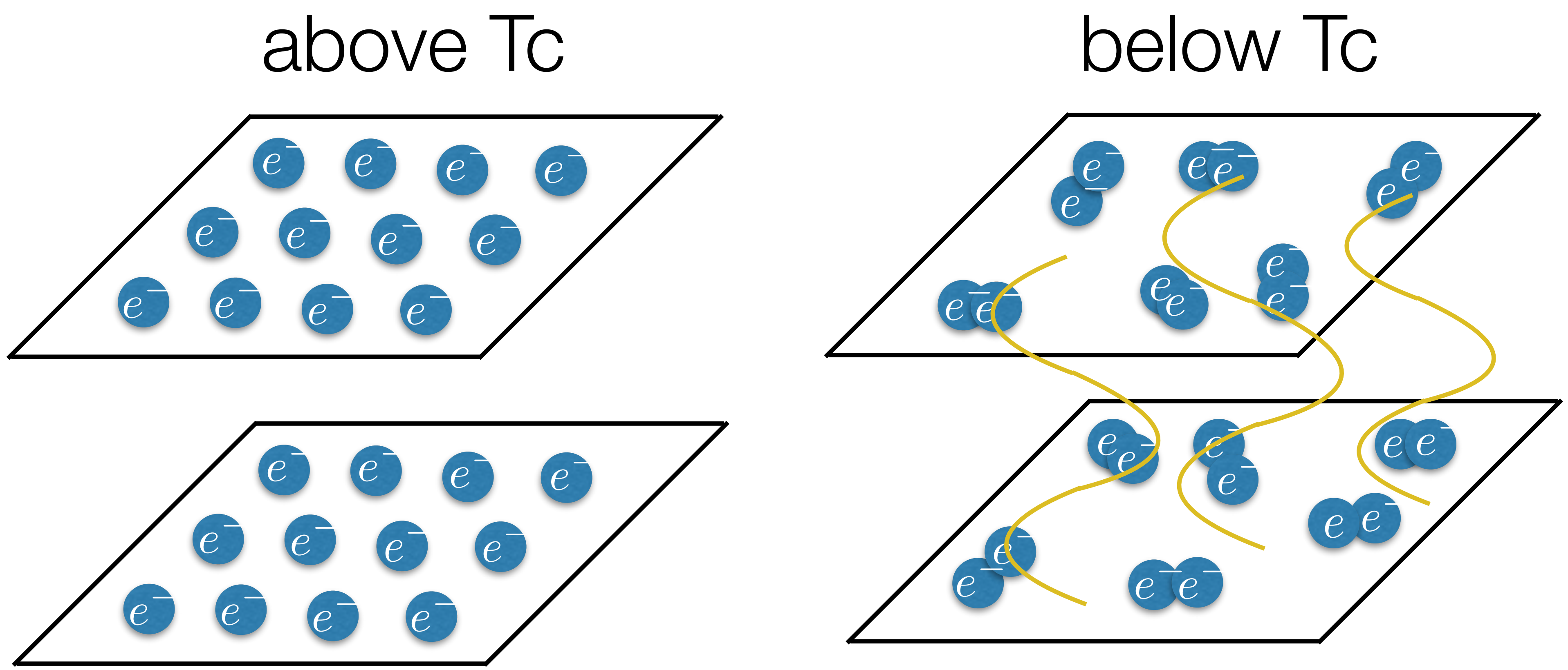




- Coulomb repulsion
- low conductivity
- low Casimir effect

- Cooper pairs formed
- superconductivity
- strong Casimir effect

Overall picture:

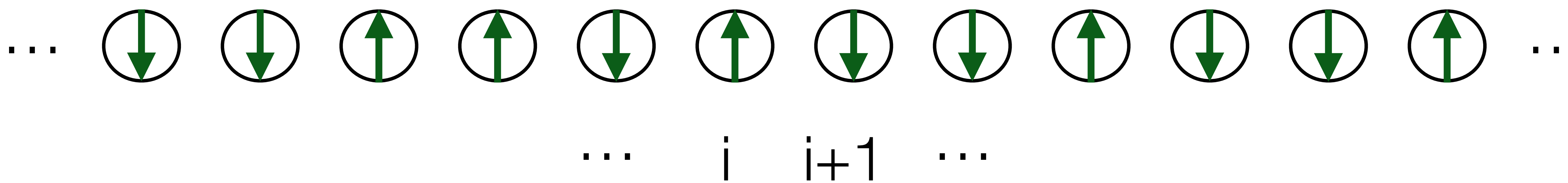


- At Tc Cooper pairs form - high conductivity sets in
- There is a strong coordination between the currents in layers (depicted by wavy lines)

The onset of Casimir effect pays the energy price for Cooper pair formation

A toy model: **local** energy investment **vs. global** payoff

- N two-level systems:



- they can be in up state: $|\uparrow\rangle$ = “Cooper pair has formed,

  and down state: $|\downarrow\rangle$ = “Cooper pair did not form”

A toy model: **local** energy investment **vs. global** payoff

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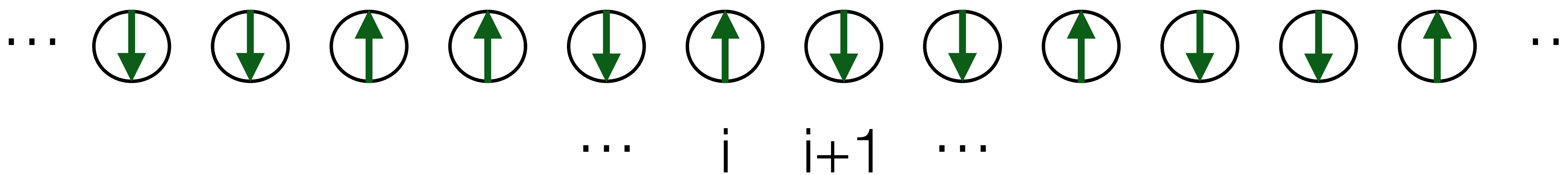

- Hamiltonian:

local

$$H = a\sum_i \bar{\sigma}_i$$

$$\bar{\sigma}_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\mathcal{H}_i}$$

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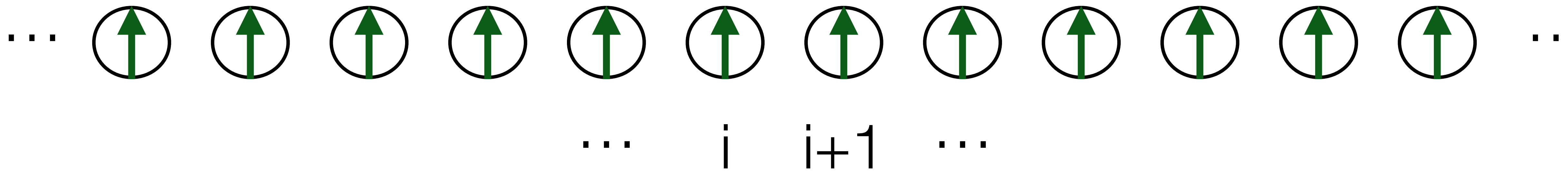

- Hamiltonian:

global

$$H = \qquad - b\, N \prod_i \bar{\sigma}_i$$

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A toy model: **local** energy investment **vs. global** payoff

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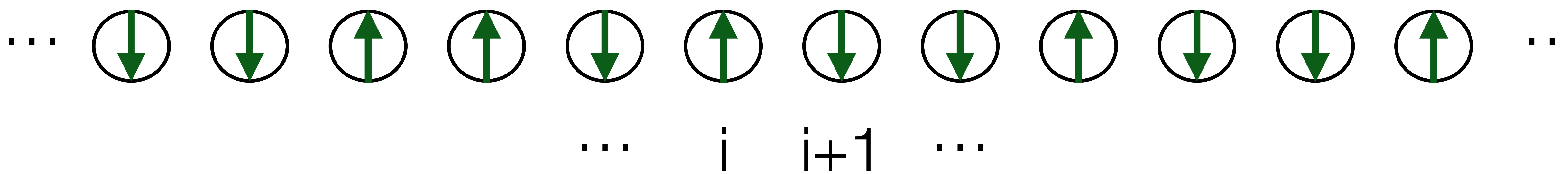


- Hamiltonian:

local + global

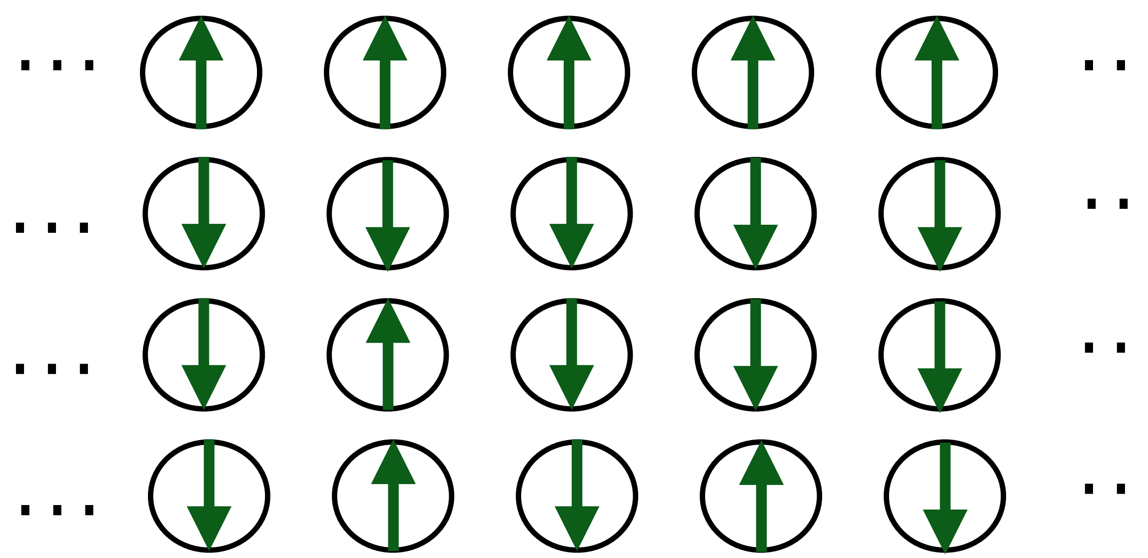
$$H = a \sum_i \bar{\sigma}_i \; - b \; N \prod_i \bar{\sigma}_i$$

$$\bar{\sigma}_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\mathcal{H}_i}$$

A toy model: **local** energy investment **with global** payoff

- Thermodynamical properties of this system:

$$E_0 = -(b-a)N$$
$$E_1 = 0$$
$$E_2 = a$$
$$E_3 = 2a$$
$$\cdots$$

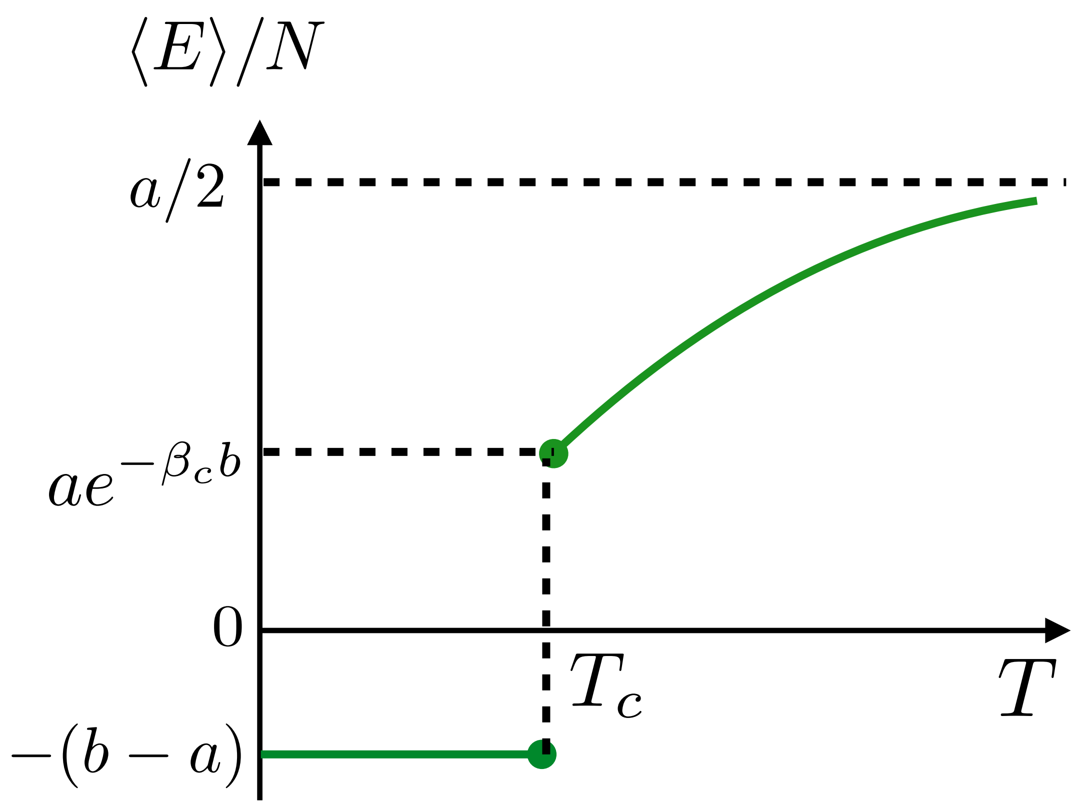


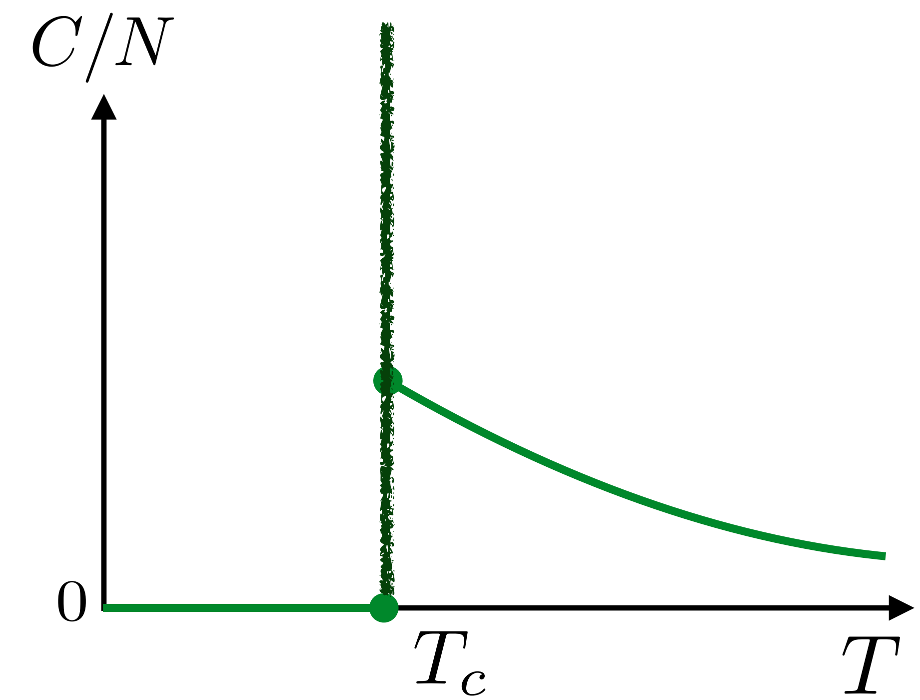
$$Z = e^{\beta(b-a)N} + \left(1 + e^{-\beta a}\right)^N - e^{-\beta a N}$$

$$\langle E \rangle = -\frac{\partial \log Z}{\partial \beta} \qquad C = \frac{\partial \langle E \rangle}{\partial T}$$

A toy model: **local** energy investment **vs. global** payoff

- Thermodynamical properties of this system:

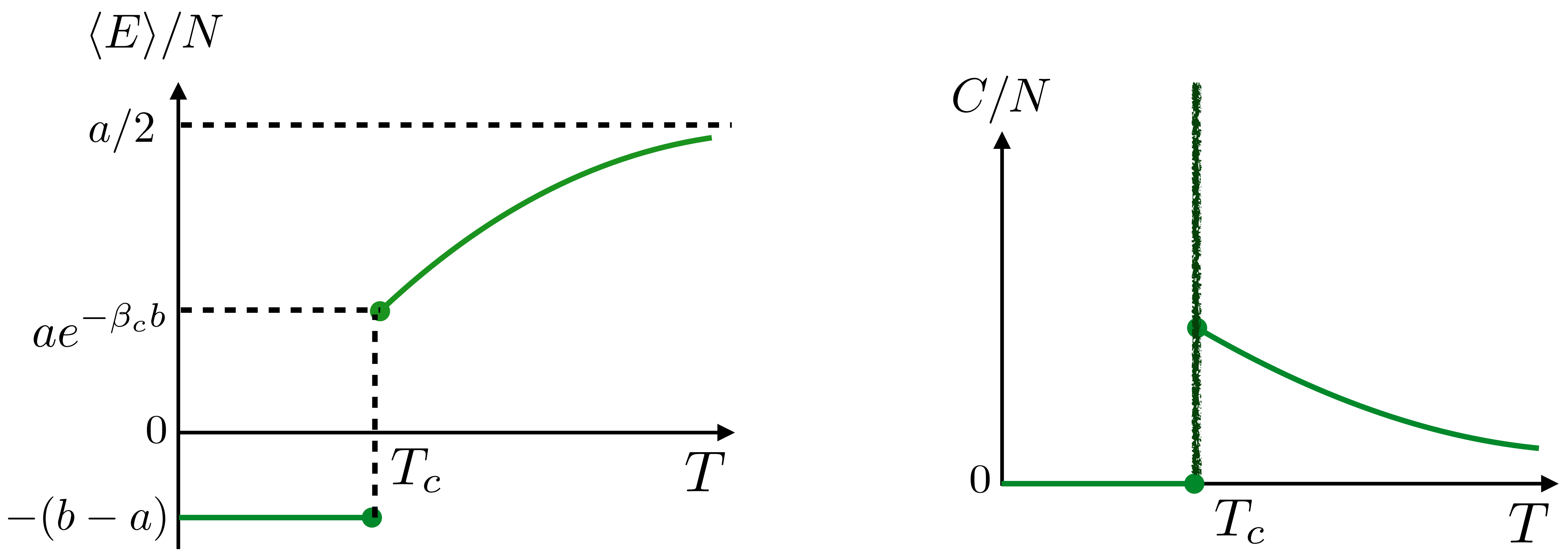




- phase transition at: $k_B T_c = \dfrac{b-a}{\log\left(1 + e^{-\frac{a}{k_B T_c}}\right)}$

A toy model: **local** energy investment **vs. global** payoff

- Thermodynamical properties of this system:



- This is a simple model that reproduces the main idea of Casimir effect in superconductors.

Outlook for Casimir effect in high-temperature superconductors:

- look for the experimental signature of Casimir effect in high-temperature superconductors:
    - observe small squeezing of the high-temperature superconductor at Tc
- this model suggests where to look for new materials:
    - need high contrast in conductivity between normal and superconducting state
    - small layer spacing

Outlook for local vs. global energy trade-off:

This simple model clearly demonstrates these effects are possible!

- Challenge 1: beyond high-temperature superconductivity, are there other phenomena where local energy expense is offset by global benefit?
- Challenge 2: are there different theoretical models that can show this property?

Experimental challenge:

- Challenge 3: can we make an experimental realization of the proposed model or a similar model?

Thank you for your attention!