Neutrino Physics Tutorials - GGI 2019

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1. Neutrino oscillation in vacuum

a) Derive the general expression for the probability of neutrino oscillation in vacuum, namely

$$\begin{aligned} P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<k} \mathrm{Re}\left(U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\right) \sin^2\left(\varphi_{ik}\right) \\ + 2 \sum_{i<k} \mathrm{Im}\left(U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\right) \sin(2\,\varphi_{ik})\,, \end{aligned} \tag{1}$$

where $\varphi_{ik} = \Delta m_{ik}^2 L/(4\,E)$, $\Delta m_{ik}^2 = m_{\nu_i}^2 - m_{\nu_k}^2$ and L is the traveled distance.

b) Show that the above expression is invariant under the transformation

$$U_{\alpha k} \to e^{i\phi_\alpha}\, U_{\alpha k}\, e^{i\theta_k}\,, \tag{2}$$

where ϕ_α and θ_k are generic angles.

c) Derive the expressions for P_{ee}, $P_{\mu\mu}$ and $P_{e\mu}$ in the limit $\Delta m_{21}^2 \approx 0$, for neutrino oscillation in vacuum.

d) Derive the P_{ee} expression for the KamLAND experiment by keeping the full dependence on Δm_{21}^2 and by averaging on the oscillations driven by Δm_{32}^2. Neglect matter effects and keep U_{e3} non-vanishing.

e) Derive the $P_{\mu e}$ expression relevant to T2K and MINOS by expanding the general expression (1) at first order in $\Delta m_{12}^2/\Delta m_{13}^2$. Neglect matter effects.

2. Z-boson decays

Do neutrinos produced in the decay $Z^0 \to \nu\bar{\nu}$ oscillate? If so, design an experiment in which these oscillations could be observed.

3. Gauge anomalies

Check that B and L_i are anomalous in the SM, while the three combinations $B/3 - L_i$ are anomaly free.

4. Lepton flavor violation

Let us consider the SM extended with Dirac neutrino masses.

a) Draw the Feynman diagrams contributing to $\mu \to e\gamma$ in the unitary gauge.

b) Show that the most general amplitude for this process, consistent with gauge and Lorentz symmetries, can be written as

$$\mathcal{M}(\mu \to e\gamma) = \epsilon^{\alpha} \cdot \bar{u}_e(p-q)\Big[\sigma_{\alpha\beta}(A + B\,\gamma_5)q^{\beta}\Big]u_{\mu}(p)\,, \tag{3}$$

where p and q denote respectively the muon and photon 4-momentum vectors, and A and B are Lorentz invariant quantities.

c) Write down the lowest-order effective operator contributing to this process with the SM particle content.

d) Without computing the loop-diagrams, argue that the coefficients given above take the form

$$A = B \propto \frac{e\,g_2^2}{16\pi^2}\frac{m_\mu}{m_W^2}\sum_i U_{\mu i}^{*}U_{ei}\frac{m_i^2}{m_W^2}\,, \tag{4}$$

where U is the PMNS matrix. Estimate $\mathcal{B}(\mu \to e\gamma)$ from this amplitude.

e) How would the above expression change if we assume that neutrinos have Majorana masses?

5. Type-I see-saw

a) Derive the Weinberg operator in the type-I see-saw framework by integrating-out the right-handed neutrinos.

b) Derive the expression for the NLO dimension-six operator appearing in this scenario.

6. Baryon number violation

List the lowest order operators with the SM particle content that violate baryon number.