Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation

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Aspects of neutrino physics (III)
Neutrino Masses, Mixing and Oscillations: Leptogenesis and Hierarchy problem

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The see-saw (continue)

2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu\right] v^2$$

example

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

δ<<1
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} << \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\bar{B}})}{s} \approx 6 \times 10^{-10}$$

Sakharov conditions met by the see-saw theory

1. (B-L) violation at high-temperature and (B+L) violation by pure SM interactions
2. C and CP violation by additional phases in see-saw Lagrangian (more on this later)
3. out-of-equilibrium condition

restrictions imposed by leptogenesis on neutrinos

here: thermal leptogenesis dominated by lightest ν^c no flavour effects]

active neutrinos should be light

out-of-equilibrium controlled by rate of RH neutrino decays

$$\frac{M_1}{8\pi}(y_\nu y_\nu^+)_{11} < \left.\frac{T^2}{M_{Pl}}\right|_{T\approx M_1}$$

Exercise 6: compute this

$$\frac{(y_\nu y_\nu^+)_{11} v^2}{M_1} \equiv \tilde{m}_1 < 10^{-3} \text{ eV}$$

more accurate estimate $m_i < 0.15$ eV

RH neutrinos should be heavy

$$\eta_B \approx 10^{-2}\,\varepsilon_1\,\eta$$

[efficiency factor ≤1 washout effects]

$$\varepsilon_1 = \frac{\Gamma(\nu_1^c \to l\Phi) - \Gamma(\nu_1^c \to \bar{l}\,\Phi^*)}{\Gamma(\nu_1^c \to l\Phi) + \Gamma(\nu_1^c \to \bar{l}\,\Phi^*)} = -\frac{3}{16\pi}\sum_{j=2,3}\frac{M_1}{M_j}\frac{\mathrm{Im}\{[(yy^+)_{1j}]^2\}}{(yy^+)_{11}} \approx 0.1 \times \frac{M_1 m_i}{v^2}$$

[Yukawas y in mass eigenstate basis for ν^c_i]

$M_1 > 6\times10^8$ GeV

more refined bound [Davidson and Ibarra 0202239]

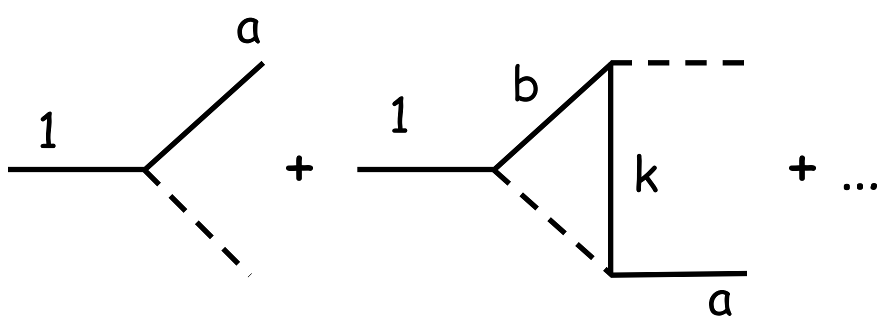
$$\left|\varepsilon_1^\infty\right| \le \varepsilon_1^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}(m_3 - m_1)$$

$$T_R \approx M_1 > (4\times 10^8 \div 2\times 10^9)\ GeV$$

in conflict with the bound on T_R in SUSY models to avoid overproduction of gravitinos

$$T_R^{SUSY} < 10^{7-9}\ GeV$$

Exercise 7: reconstruct the flavour structure of ε_1



$$\mathcal{A}(\nu_1^c \to l_a \Phi) \propto y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}$$

$$\mathcal{A}(\nu_1^c \to \bar{l}_a \Phi^*) \propto y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}$$

$$\varepsilon_1 \propto \frac{\left|y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}\right|^2 - \left|y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}\right|^2}{\left|y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}\right|^2 + \left|y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}\right|^2} \approx \frac{\mathrm{Im}(W)\,\mathrm{Im}\{[(yy^+)_{1k}]^2\}}{(yy^+)_{11}}$$

[sums understood]

$$\mathrm{Im}(W) \approx \frac{M_1}{M_k}$$

Exercise 8: count the number of physical parameters in the type I see-saw model distinguish between moduli and phases

y_e, y_ν and M depend on (18+18+12)=48 parameters, 24 moduli and 24 phases

we are free to choose any basis leaving the kinetic terms canonical
(and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad \nu^c \to \Omega_{\nu^c} \nu^c \qquad l \to \Omega_l l \qquad [U(3)^3]$$

these transformations contain 27 parameters (9 angles and 18 phases)
and effectively modify y_e, y_ν and M

$$y_e \to \Omega^T_{e^c} y_e \Omega_l \qquad y_\nu \to \Omega^T_{\nu^c} y_\nu \Omega_l \qquad M \to \Omega^T_{\nu^c} M \Omega_{\nu^c}$$

so that we can remove 27 parameters from y_e, y_ν and M

we remain with 21 parameters: 15 moduli and 6 phases
the moduli are 9 physical masses and 6 mixing angles

the same count in the quark sector would give a total of 9 moduli
(6 masses amd 3 mixing angles) and 0 phases <- wrong
how the above argument should be modified, in general?

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2}\nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles and 3 phases, as in lecture 1

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is

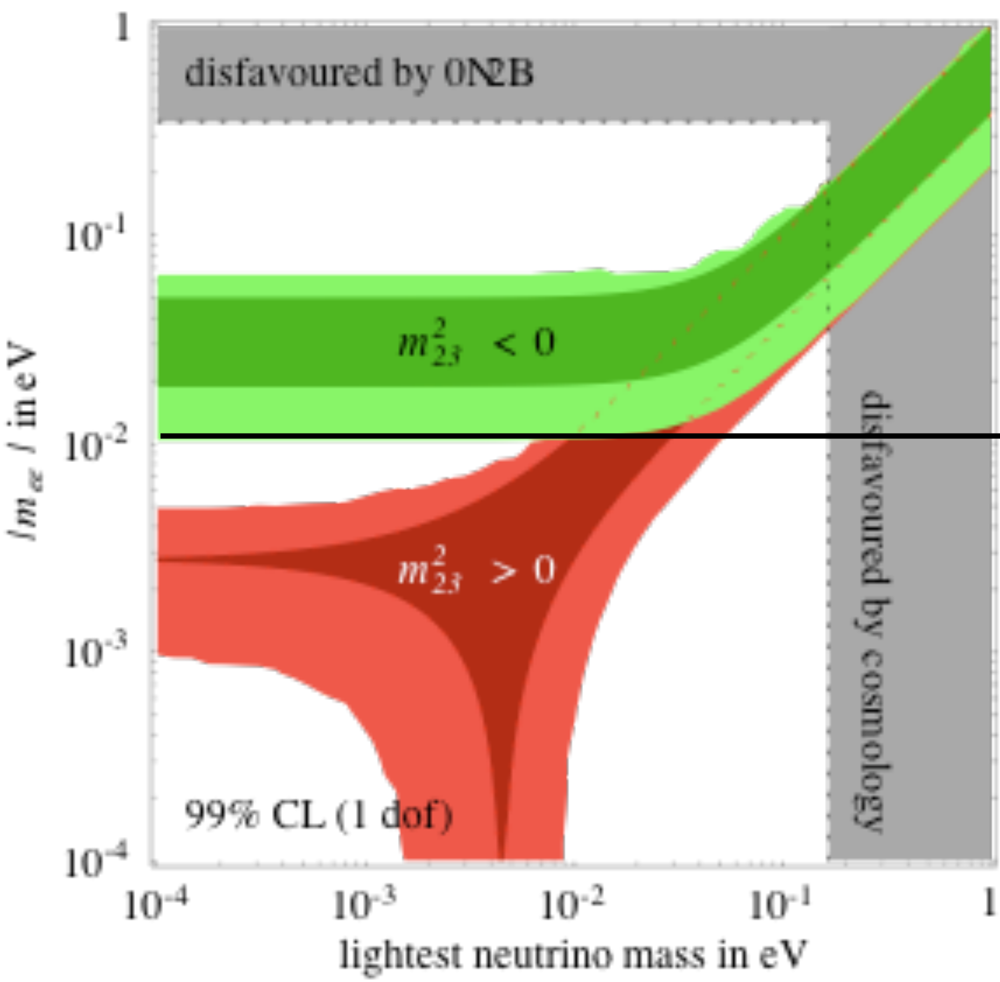
$0\nu\beta\beta$ decay: $(A,Z) \to (A,Z+2)+2e^-$

this would discriminate L_5 from other possibilities, such as Example 1.

The decay in 0νββ rates depend on the combination $|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12} e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13} e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



from the current knowledge of $(\Delta m^2_{ij}, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

future expected sensitivity on $|m_{ee}|$

10 meV

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

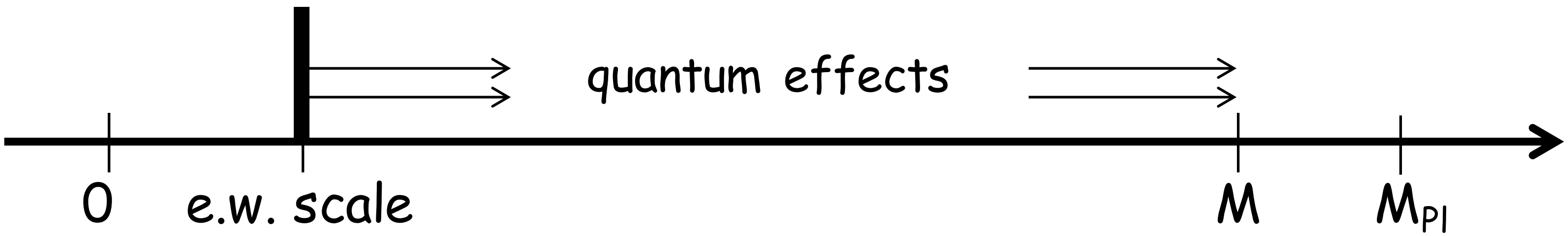
Neutrinos and the Higgs boson

1. neutrinos and the hierarchy problem
2. neutrinos and the stability of the electroweak vacuum

1. Why

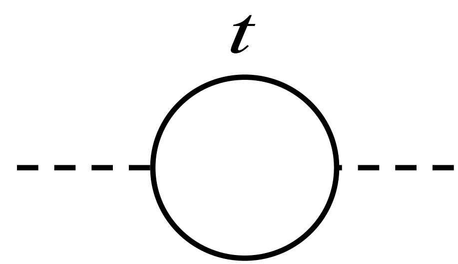
any new particle threshold: $M_{GUT}, \ldots$

e.w. scale $\ll$ …, M_{Pl} ? $\Leftrightarrow$ sensitivity of m_h to UV physics



often discussed in terms of quadratic divergences

$$\delta m_h^2 \propto \frac{y_t^2}{16\pi^2}\Lambda^2$$



but

-- what represents exactly Λ ? Any evidence from experiment?

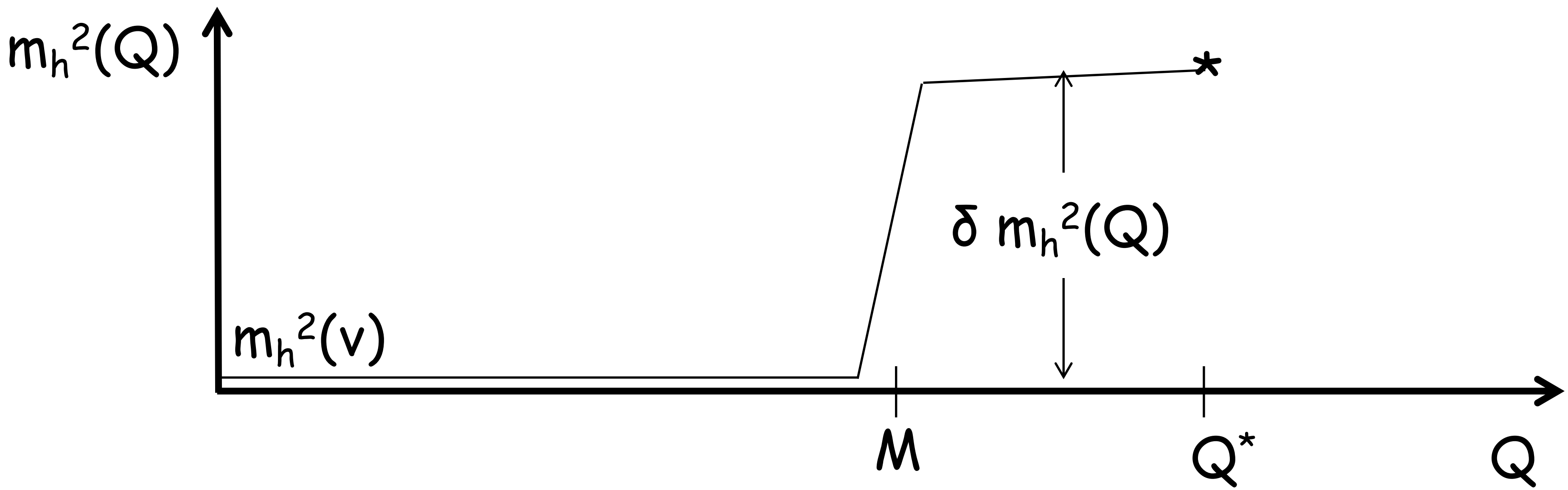
-- can we get rid of Λ in some suitable scheme ?

-- technical aspect obscure physics

hierarchy problem can be formulated entirely in terms of renormalized quantities with no reference to regulators

assumption: coupling y of Higgs particle to an heavy state of mass M

running Higgs mass $$\delta m_h^2(Q) \approx \frac{y^2}{16\pi^2} M^2 \log\frac{Q}{M} \qquad Q > M$$



fine-tune the initial conditions at Q^* such that

$$m_h^2(v) \approx m_h^2(Q^*) - \frac{y^2}{16\pi^2} M^2 \log\frac{Q^*}{M}$$

consider type I see-saw

heavy state ν^c	mass M
Yukawa coupling	y_ν

we will see in a moment

$$\delta m_h^2(Q) \approx -\frac{y_\nu^2}{4\pi^2} M^2 \log\frac{Q}{M} \qquad Q > M$$

by using $m_\nu \approx \frac{y_\nu^2 v^2}{M}$ to eliminate the y^2 dependence

$$\left|\delta m_h^2(Q)\right| \approx \frac{1}{4\pi^2}\frac{m_\nu M^3}{v^2}\log\frac{Q}{M} < v^2$$

$$\boxed{M < 1.4\times 10^7 \quad \text{GeV}}$$

$$\left[\begin{array}{c} \log\frac{Q}{M} \approx 1 \\ m_\nu \approx 0.05\,\text{eV} \end{array}\right]$$

$$\boxed{y_\nu \approx \sqrt{\frac{m_\nu M}{v^2}} < 10^{-4}}$$

too small for thermal leptogenesis ?

Exercise 9: derive the threshold corrections to $m_\sigma^2(Q)$ in the toy model

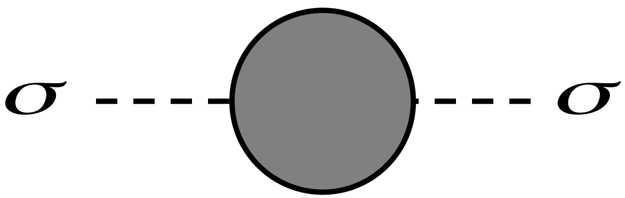
$$L = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + i\bar{\xi}\bar{\sigma}^\mu \partial_\mu \xi - \frac{1}{2}\left[\xi^T \mathcal{M} \xi + h.c.\right]$$

$$\xi = \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$$

$$\mathcal{M}(\sigma) = \begin{pmatrix} 0 & y(\sigma+v) \\ y(\sigma+v) & M \end{pmatrix}$$

assume $m_\sigma^2(0) = 0$

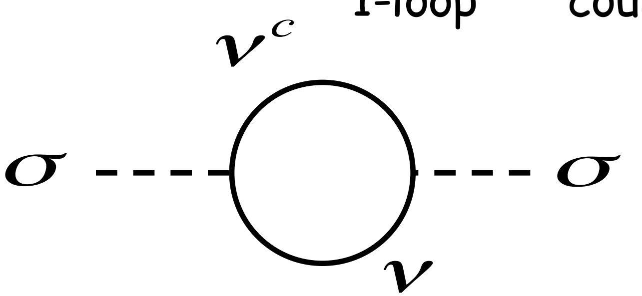
1. start from the 1-loop renormalized self-energy



$= i\left[Q^2 - \Pi_f(Q^2)\right]$

$$\Pi_f(Q^2) = \underbrace{\Pi(Q^2)}_{\text{1-loop}} - \underbrace{\Pi(0) - Q^2\Pi'(0)}_{\text{counterterms}}$$

OS scheme $\begin{cases} \Pi_f(0) = 0 \\ \Pi'_f(0) = 0 \end{cases}$



$= -i\,\Pi(Q^2)$

2. evaluate 1-loop diagram $-i\,\Pi(Q^2)$ in the limit $0 \approx m_1 << m_2 \approx M$

$$m_{1,2} = \frac{1}{2}\left(M \pm \sqrt{M^2 + 4y^2v^2}\right) \approx \begin{cases} -y^2v^2/M \\ M + y^2v^2/M \end{cases}$$

in dimensional regularization

$$\boxed{\Pi(Q^2) = \frac{y^2}{2\pi^2}\int_0^1 dx\left[(D - \log\Omega)(2\Omega - Q^2x(1-x)) + \Omega\right]}$$

$$D = \frac{2}{\varepsilon} - \gamma + \log 4\pi$$

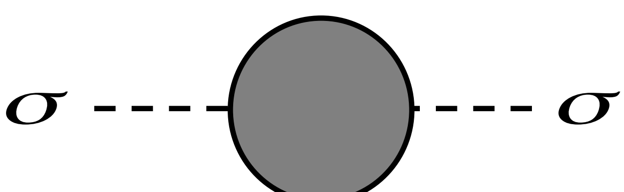
$$\Omega = -Q^2x(1-x) + M^2x$$

3. compute $\Pi_f(Q^2)$

$$\Pi_f(Q^2) = \frac{y^2}{2\pi^2}\int_0^1 dx\left[-2Q^2x(1-x) - (2M^2x - 3Q^2x(1-x))\log\frac{\Omega}{M^2x}\right]$$ finite

relevant limits $Q^2 << M^2$ $\quad \Pi_f(Q^2) = -\frac{y^2}{12\pi^2}\frac{Q^4}{M^2} + \ldots$

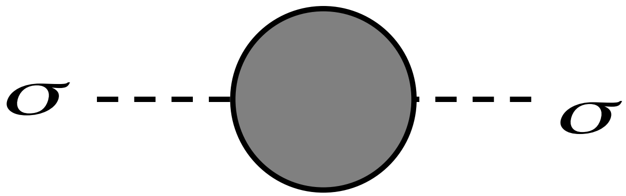
decoupling



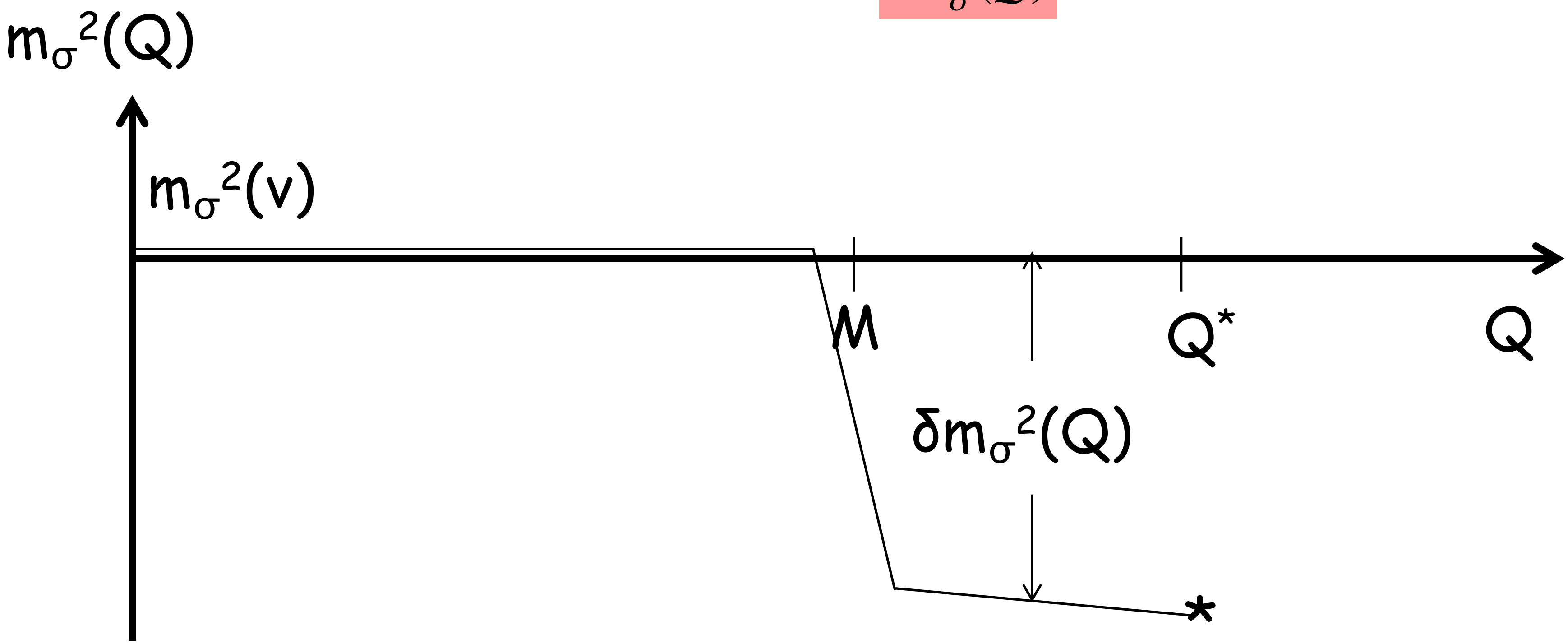
$$= iQ^2\left[1 + \frac{y^2}{12\pi^2}\frac{Q^2}{M^2} + \ldots\right]$$

$$m_\sigma^2(Q) = 0$$

$Q^2 >> M^2$ $\qquad \Pi_f(Q^2) = \frac{y^2}{2\pi^2}\left[Q^2\left(-\frac{3}{4}+\frac{1}{4}\log\frac{-Q^2}{M^2}\right) - M^2\log\frac{-Q^2}{M^2}\right]+\dots$



$$= i\left(Q^2 + \underbrace{\frac{y^2}{2\pi^2}M^2\log\frac{-Q^2}{M^2}}_{-m_\sigma^2(Q)}\right)\left(1+O(y^2)\right)$$



similar conclusions in type II and type III see-saw where threshold corrections are dominated by 2-loop gauge interactions

type III $\delta m_h^2(Q) \approx -\frac{72 g^4}{(4\pi)^4} M^2 \log\frac{Q}{M}$ $\quad Q > M$ $\quad M < 940$ GeV

type II $\quad M < 200$ GeV

ways out

the initial conditions at the scale Q^* are fine-tuned to an accuracy of order (e.w. scale)/M

the threshold correction at the scale M is almost cancelled by an other contribution, as e.g. in supersymmetry with a splitting between neutrinos and sneutrinos of order 4π x (e.w. scale)

the Higgs is not an elementary particle and dissolves above a compositness scale ~ TeV
gap between the e.w. scale and the compositeness scale if the Higgs is a PGB

2. neutrinos and the stability of the electroweak vacuum

for the current values

$$m_h = (125.66 \pm 0.34) \quad \text{GeV}$$
$$m_t = (173.2 \pm 0.9) \quad \text{GeV}$$
$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

the Higgs potential develops an instability at

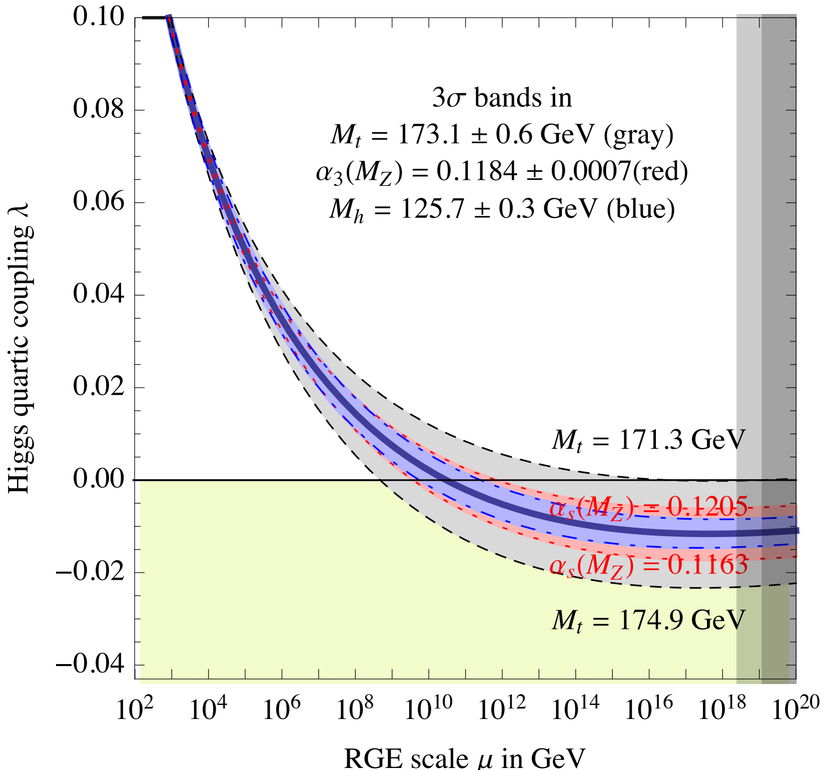
$$10^9\,\text{GeV} < \Lambda < 10^{15}\,\text{GeV}$$

assumption: only SM all the way up to the scale Λ

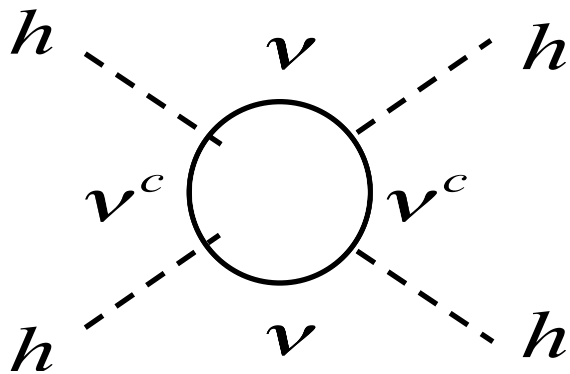
for large values of the field h

$$V(h) \approx \frac{\lambda}{4} h^4$$

$$(4\pi)^2 \frac{d\lambda}{dt} = -6y_t^4 + \frac{3}{8}[2g^4 + (g^2 + g'^2)] + \underbrace{12\lambda y_t^2 - 3\lambda(g^2 + 3g'^2)}_{O(\lambda)} + \underbrace{24\lambda^2}_{O(\lambda^2)} + \ldots$$



above the scale M a new contribution to β_λ arises from neutrino Yukawa couplings

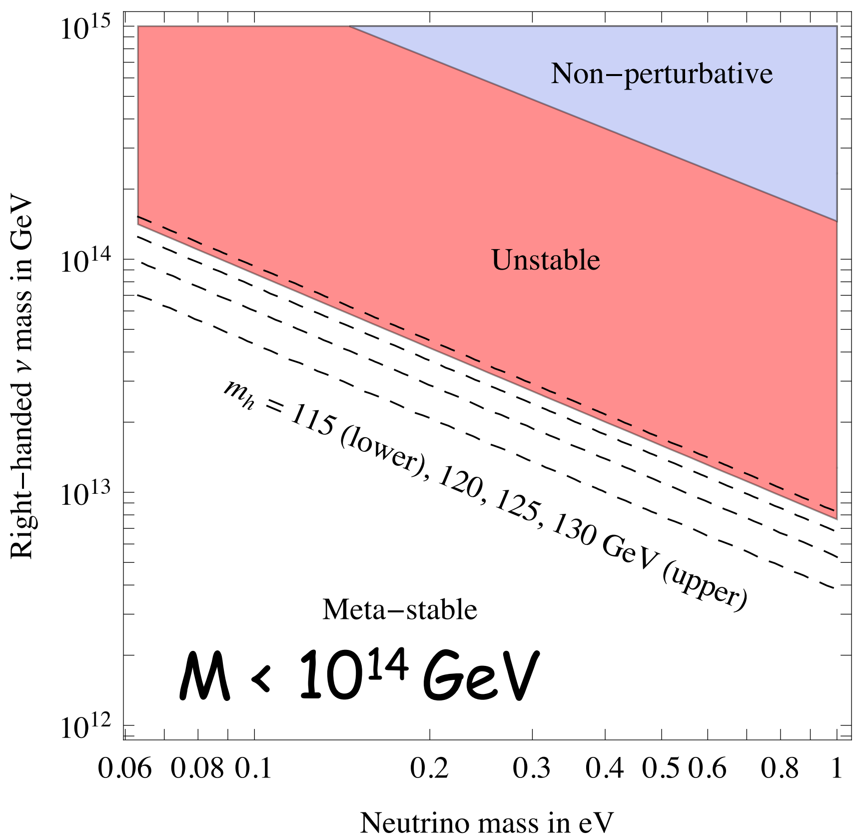


$$\delta\beta_\lambda = -2\,\mathrm{tr}(y_\nu y_\nu^+ y_\nu y_\nu^+) < 0$$

contributes to instability above M

the larger M,
the larger the contribution

$$y_\nu \approx \sqrt{\frac{m_\nu M}{v^2}}$$



the bound applies only to the portion of SM parameter space that guarantees a stable vacuum in the limit $y_\nu=0$
(m_t on the lower side
α_S on the higher side)

Back up slides

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Type-III see-saw at LHC

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Abstract

Neutrino masses can be generated by fermion triplets with TeV-scale mass, that would manifest at LHC as production of two leptons together with two heavy SM vectors or higgs, giving rise to final states such as $2\ell + 4j$ (that can violate lepton number and/or lepton flavor) or $\ell + 4j + \not{E}_T$. We devise cuts to suppress the SM backgrounds to these signatures. Furthermore, for most of the mass range suggested by neutrino data, triplet decays are detectably displaced from the production point, allowing to infer the neutrino mass parameters. We compare with LHC signals of type-I and type-II see-saw.