Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation

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Aspects of neutrino physics (II)
Neutrino Masses, Mixing and Oscillations: Implication for Physics BSM

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Lecture 1
Neutrino Masses

Summary of data

$m_\nu < 2.2\, eV \quad (95\%\, CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

Parameter	Ordering	Best fit	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO	7.34	2.2
	IO	7.34	2.2
$\sin^2\theta_{12}$	NO	3.04	4.4
	IO	3.03	4.4
$\sin^2\theta_{13}/10^{-2}$	NO	2.14	3.8
	IO	2.18	3.7
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.455	1.4
	IO	2.441	1.4
$\sin^2\theta_{23}/10^{-1}$	NO	5.51	5.2
	IO	5.57	4.8
δ/π	NO	1.32	14.6
	IO	1.52	9.3

[Capozzi, Lisi, Marrone, Palazzo 1804.09678]

violation of individual lepton number implied by neutrino oscillations

Summary of unkowns

absolute neutrino mass scale is unknown [but well-constrained!]

sign $[\Delta m^2_{atm}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

α, β unkown

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established

Beyond the Standard Model

a non-vanishing neutrino mass is the first evidence of the incompleteness of the Standard Model [SM]

in the SM neutrinos belong to SU(2) doublets with hypercharge Y=-1/2
they have only two helicities (not four, as the other charged fermions)

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2)$$

the requirement of invariance under the gauge group G=SU(3)xSU(2)xU(1) forbids pure fermion mass terms in the lagrangian. Charged fermion masses arise, after electroweak symmetry breaking, through gauge-invariant Yukawa interactions

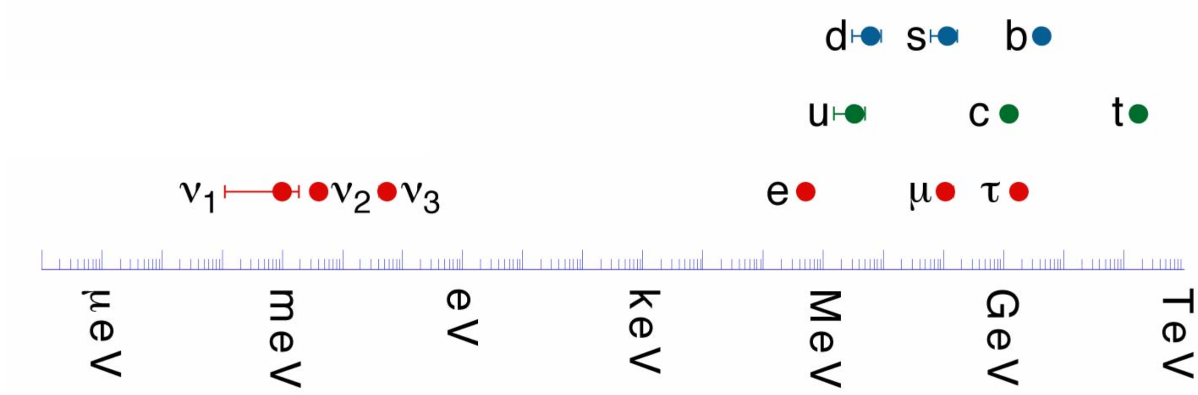
$$\Phi \quad \underbrace{\Psi \Psi'}_{\text{same helicity}}$$

not even this term is allowed for SM neutrinos, by gauge invariance

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS}=\begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$$\lambda \approx 0.22$$

How to modify the SM?

the SM, as a consistent QFT, is completely specified by

0. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

1. particle content — three copies of (q, u^c, d^c, l, e^c)

   one Higgs doublet Φ

2. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(0.+1.+2.) leads to the SM Lagrangian, L_{SM}, possessing an additional, accidental, global symmetry: (B-L) -> EXERCISE

0. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!
   We could extend G, but, to allow for neutrino masses, we need to modify 1. (and/or 2.) anyway...

Exercise 1: anomalies of B and L_i

the anomaly of the baryonic current and the individual leptonic currents are proportional to tr[Q {T^A,T^B}] and tr[Q {Y,Y}] where Q=(B,L_i) and (T^A,Y) are the generators of the electroweak gauge group
compute these traces in the SM with 3 fermion generations

$$\frac{1}{2}\mathrm{tr}[B\{T^A,T^B\}]=3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{4}(up)+\frac{1}{4}(down)\right]\delta^{AB}=\frac{3}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{T^A,T^B\}]=1(L_i)\times\left[\frac{1}{4}(nu)+\frac{1}{4}(e)\right]\delta^{AB}=\frac{1}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[B\{Y,Y\}]=3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{18}(Doubl)-\frac{10}{18}(Singl)\right]=-\frac{3}{2}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{Y,Y\}]=1(L_i)\times\left[\frac{1}{2}(Doubl)-1(Singl)\right]=-\frac{1}{2}$$

(B+L) is anomalous, (B/3-L_i) [and (B-L)] are anomaly-free

First possibility: modify (1), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)

ask for (global) invariance under B-L
(no more automatically conserved as in the SM)

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = -d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) - e^c y_e (\Phi^+ l) - \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

a generic problem of this approach

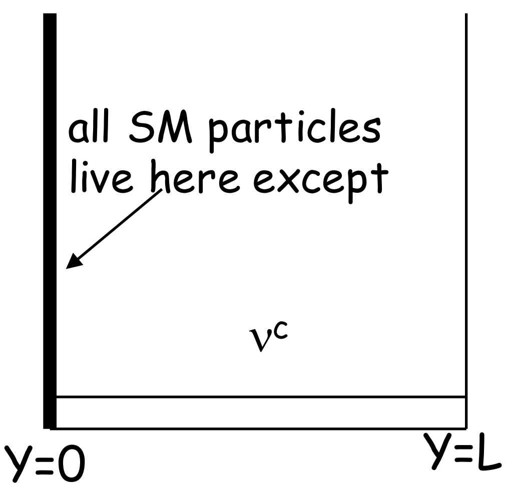
the particle content can be modified in several different ways
in order to account for non-vanishing neutrino masses
(additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,...). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \leq 10^{-12}$$

Quite a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$
$$= \frac{1}{\sqrt{L}} \nu_0^c(\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale)
then neutrino Yukawa coupling is suppressed

Second possibility: abandon (2) renormalizability

A disaster?

$$L = L_{d\le 4}^{SM} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

a new scale Λ enters the theory. The new (gauge invariant!) operators L_5, L_6,... contribute to amplitudes for physical processes with terms of the type

$$\frac{L_5}{\Lambda} \to \frac{E}{\Lambda} \qquad \frac{L_6}{\Lambda^2} \to \left(\frac{E}{\Lambda}\right)^2 \qquad \ldots$$

the theory cannot be extrapolated beyond a certain energy scale $E \approx \Lambda$.
[at variance with a renormalizable (asymptotically free) QFT]

If $E \ll \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \approx 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will *look like* a renormalizable theory!

$$\frac{E}{\Lambda} \approx \frac{10^2\,GeV}{10^{15}\,GeV} = 10^{-13}$$

an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top}!

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{\left(\tilde{\Phi}^+ l\right)\left(\tilde{\Phi}^+ l\right)}{\Lambda} =$$

$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a unique operator!
[up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ) with respect to the neutrino mass term of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$

full singlet under G=SU(3)xSU(2)xU(1)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = -\nu^c y_\nu (\tilde{\Phi}^+ l) - \frac{1}{2}\nu^c M \nu^c + h.c.$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out" the field ν^c

$$L_{eff}(l) = \frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + ...$$

terms suppressed by more powers of M^{-1}

this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.

Exercise 2

derive the see-saw relation by integrating out the fields ν^c through their e.o.m. in the heavy M limit. Compute the 1st order corrections in p/M

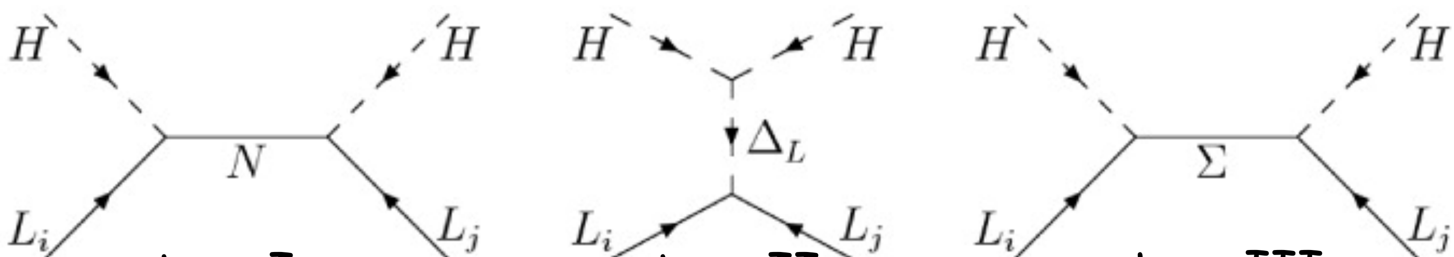
equations of motion of ν^c

$$\begin{pmatrix} \nu^c \\ \bar{\nu}^c \end{pmatrix} = \begin{pmatrix} i\bar{\sigma}^\mu \partial_\mu & -M^+ \\ -M & i\sigma^\mu \partial_\mu \end{pmatrix}^{-1} \begin{pmatrix} y_\nu^* \bar{\omega} \\ y_\nu \omega \end{pmatrix} = \begin{pmatrix} -M^{-1} y_\nu \omega \\ -M^{*-1} y_\nu^* \bar{\omega} \end{pmatrix} + \ldots \qquad \omega \equiv (\tilde{\Phi}^+ l)$$

$$L_{eff} = i\bar{l}\,\bar{\sigma}^\mu \partial_\mu l + \frac{1}{2}\underbrace{\left[\omega(y_\nu^T M^{-1} y_\nu)\omega + h.c.\right]}_{\text{d-=5}} + \underbrace{i\bar{\omega}(y_\nu^+ M^{+-1} M^{-1} y_\nu)\bar{\sigma}^\mu \partial_\mu \omega}_{\text{d-=6}} + O(M^{-3})$$

renormalizes the KE of ν by v^2/M^2

there are 3 types of see-saw depending on the particle we integrate out
they all give rise to the same d=5 operator



type I | type II | type III

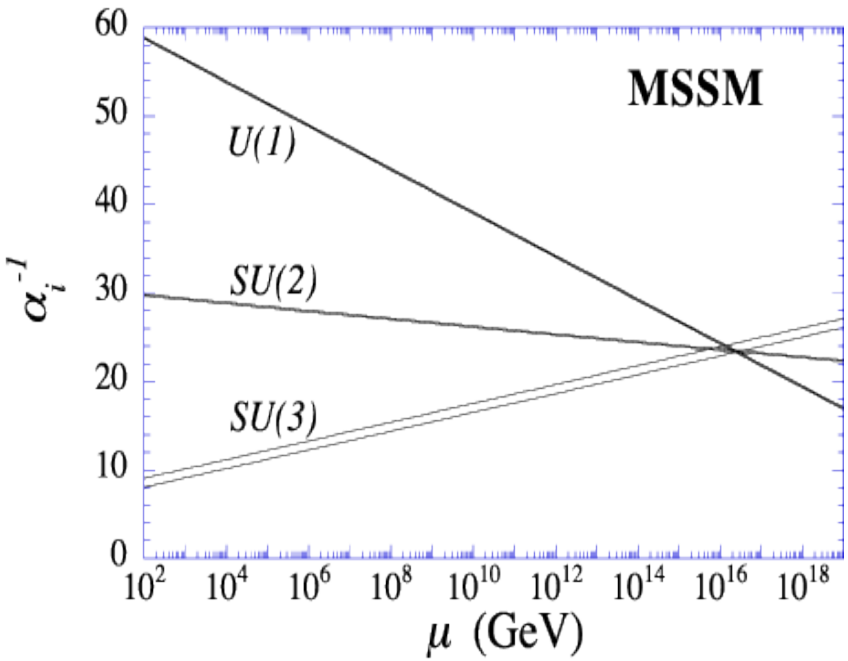
$$y_N^T (M_N)^{-1} y_N \qquad y_\Delta \frac{\mu}{M_\Delta^2} \qquad y_\Sigma^T (M_\Sigma)^{-1} y_\Sigma$$

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs):
the SM gauge group G is embedded into a simple group such as SU(5), SO(10),...



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

Unity of All Elementary-Particle Forces
Phys. Rev. Lett. 32, (1974) 438
Howard Georgi and S. L. Glashow

Georgi, H.; Quinn, H.R. and Weinberg, S.
Hierarchy of interactions in unified gauge theories.
Phys. Rev. Lett. 33 (1974) 451

Exercise 3: gauge coupling unification

0th order approximation

justify this
$$\sqrt{\frac{5}{3}}g_Y = g_2 = g_3 \qquad \sin^2\vartheta_W = \frac{g_Y^2}{g_Y^2+g_2^2} = \frac{3}{8} \approx 0.375$$

include 1-loop running

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(m_Z)} + \frac{b_i}{2\pi}\log\frac{Q}{m_Z}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{MSSM} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{SM} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}$$

knowledge of b.c. M_{GUT} and $\alpha_U=\alpha(M_{GUT})$ would allow to predict $\alpha_i(m_Z)$
in practice, we use as inputs

$$\alpha_{em}^{-1}(m_Z)\Big|_{\overline{MS}} = 127.934 \qquad \sin^2\vartheta(m_Z)\Big|_{\overline{MS}} = 0.231$$

to predict
[MSSM]

$$\alpha_3(m_Z)\Big|_{\overline{MS}} = \frac{7\alpha_{em}(m_Z)}{15\sin^2\vartheta(m_Z)-3} \approx 0.118$$

$$\alpha_U = \frac{28\alpha_{em}(m_Z)}{36\sin^2\vartheta(m_Z)-3} \approx \frac{1}{25}$$

$$\log\left(\frac{M_{GUT}}{m_Z}\right) = \pi\frac{3-8\sin^2\vartheta(m_Z)}{14\alpha_{em}(m_Z)} \quad\Rightarrow\quad M_{GUT} \approx 2\times 10^{16}\,\text{GeV}$$

[corrections from 2-loop RGE,
threshold corrections at M_{SUSY},
threshold corrections at M_{GUT}]

Exercise 4: effective lagrangian for nucleon decay

recognize that, the with the SM particle content, the lowest dimensional operators violating B occur at d=6. Make a list of them

$$\frac{1}{\Lambda_B^2} \times \begin{cases} q q u^{c+} e^{c+} & q q q l \\ q l u^{c+} d^{c+} & u^c u^c d^c e^c \end{cases}$$

color and SU(2) indices contracted

notice that they respect $\Delta B = \Delta L$: nucleon decay into antileptons e.g. $p \to e^+ \pi^0$, $n \to e^+ \pi^-$ [$n \to e^- \pi^+$ suppressed by further powers of Λ_B]

naïve estimate

$$\tau_p \approx \frac{\Lambda_B^4}{m_p^5}$$

assuming

$$\tau_p(p \to e^+ \pi^0) > 1.4 \times 10^{34}\ ys$$

[SK]

we get

$$\Lambda_B > 2.6 \times 10^{16} \quad \text{GeV}$$

in GUTs Λ_B is related to the scale M_{GUT} at which the grand unified symmetry is broken down to SM gauge group
the observed proton stability is guaranteed by the largeness of M_{GUT}

In SUSY extensions of the SM the lowest dimensional operators violating B occur at d=5: why?

flavor puzzle made simpler in SU(5) ?

$\bar{5} = (l, d^c) \qquad 10 = (q, u^c, e^c) \qquad 1 = \nu^c$

Higgs

$\Phi_5 = (\Phi_D, \Phi_T)$

$$L_Y = -10\, y_u 10\, \Phi_5 - \bar{5}\, y_d 10\, \Phi_5^+ - 1\, y_\nu \bar{5}\, \Phi_5 - \frac{1}{2} 1 M 1 + h.c.$$

$y_d = y_e^T$

$m_b = m_\tau$ O.K.

$m_s = m_\mu$, $m_d = m_e$: wrong, but not by orders of magnitude; can be fixed with additional Higgs

$m_s \approx m_\mu / 3$

$m_d \approx 3\, m_e$

suppose that y_u, y_e, y_ν and M/Λ are anarchical matrices [O(1) matrix elements] and that the observed hierarchy is due to the wave function renormalization of matter multiplets (we will see how later on)

$$\begin{aligned} 10 &\to F_{10}\, 10 \\ \bar{5} &\to F_{\bar{5}}\, \bar{5} \\ 1 &\to F_1\, 1 \end{aligned} \qquad F_X = \begin{pmatrix} \lambda^{Q_{X_1}} & 0 & 0 \\ 0 & \lambda^{Q_{X_2}} & 0 \\ 0 & 0 & \lambda^{Q_{X_3}} \end{pmatrix}$$

$\lambda \approx 0.22$

$Q_{X_1} \geq Q_{X_2} \geq Q_{X_3}$

F_1 dependence cancels in m_ν

$\mathcal{Y}_u = F_{10} y_u F_{10} \qquad \mathcal{Y}_d = F_{\bar{5}} y_d F_{10} \qquad \mathcal{Y}_e = F_{10} y_d^T F_{\bar{5}} \qquad m_\nu \propto F_{\bar{5}} y_\nu^T M^{-1} y_\nu F_{\bar{5}}$

large mixing in lepton sector suggests $F_{\bar{5}} \approx \mathrm{diag}(1,1,1)$

hierarchy mostly due to F_{10} $\quad m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$

large l mixing corresponds to a large d^c mixing: unobservable in weak int. of quarks

how can a wave function renormalization (effectively) arise?

several possibilities

here (Exercise 5): bulk fermions in a compact extra dimension S^1/Z_2

$$\mathcal{L} = i\bar{\Psi}_1\Gamma^M\partial_M\Psi_1 + i\bar{\Psi}_2\Gamma^M\partial_M\Psi_2 - m_1\varepsilon(y)\bar{\Psi}_1\Psi_1 + m_2\varepsilon(y)\bar{\Psi}_2\Psi_2 - \left[\delta(y)\frac{y}{\Lambda}\bar{f}_1(h+v)f_2 + h.c.\right]$$

$$\Psi_1 = \begin{pmatrix} E_1 \\ \bar{f}_1 \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_2 \\ \bar{E}_2 \end{pmatrix}$$

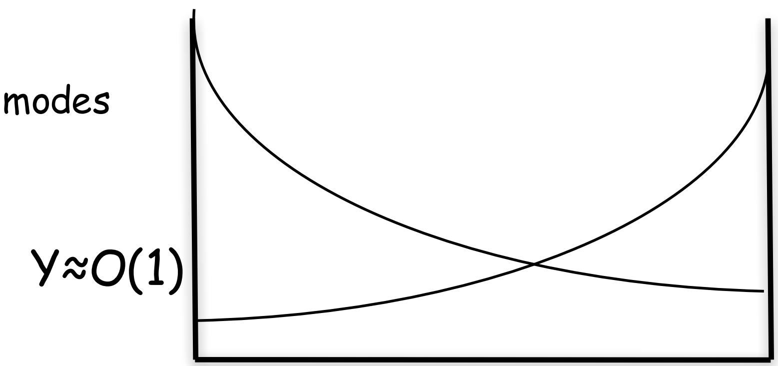
solve the e.o.m. for the fermion zero modes with the b.c.

$$-\gamma_5\partial_y\Psi^0_{1,2} \pm m_{1,2}\,\varepsilon(y)\Psi^0_{1,2} = 0$$

$$\Psi_1(-y) = +\gamma_5\Psi_1(y)$$
$$\Psi_2(-y) = -\gamma_5\Psi_2(y)$$

$$\boxed{f_i^0(y) = \sqrt{\frac{2m_i}{1-e^{-2m_i\pi R}}}e^{-m_i y}}$$

vanishing zero-modes for $(E_1, \bar{E}_2)$

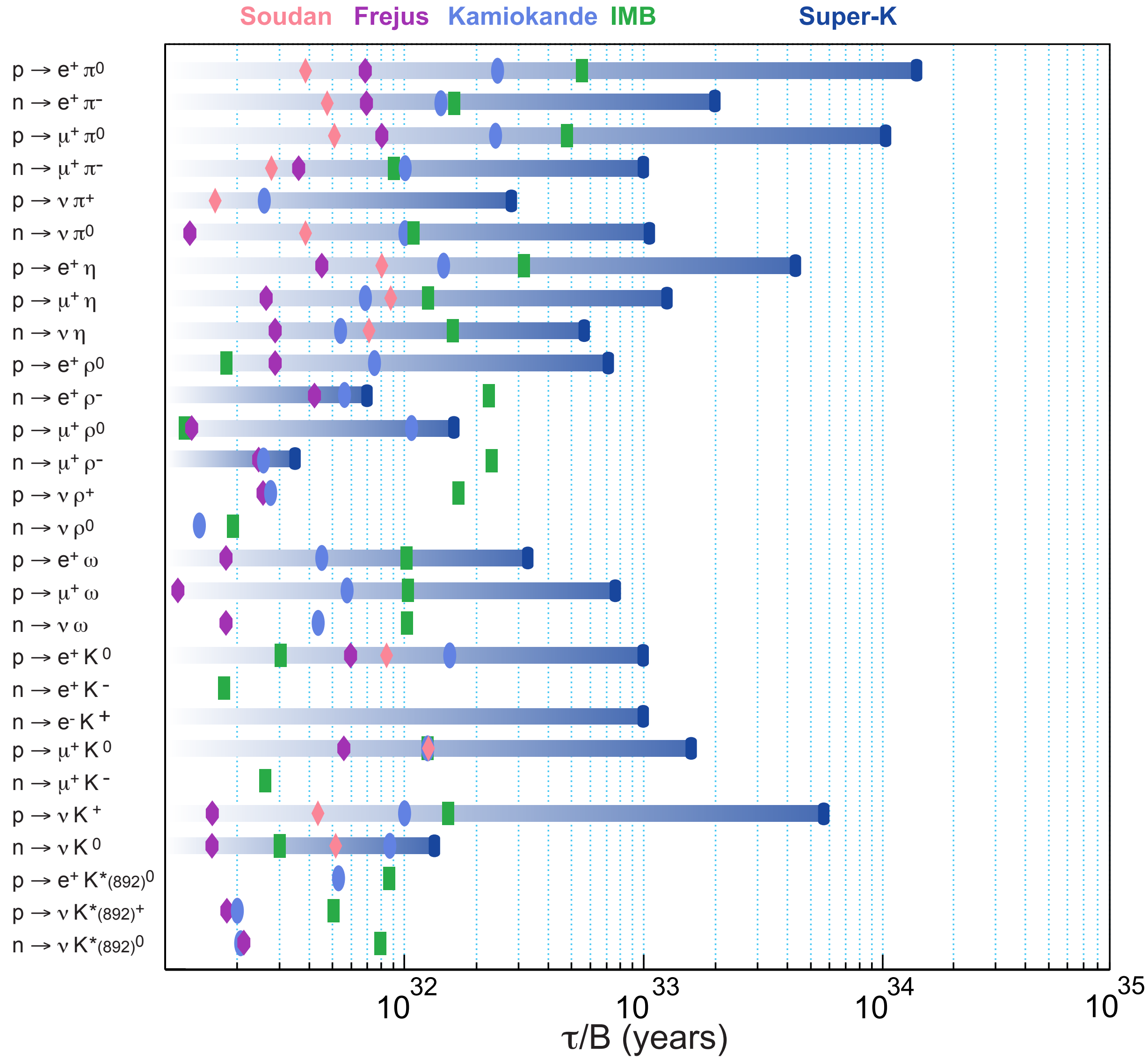


$$\boxed{\mathcal{L}_Y = -\frac{1}{\Lambda\pi R}\bar{f}_1(F_1 Y F_2)(h+v)f_2}$$

$$F_i = \sqrt{\frac{x_i}{1-e^{-x_i}}} \approx \begin{cases} e^{-x_i/2} & x_i >> 1 \\ 1 & x_i \approx 0 \\ \sqrt{-x_i} & x_i << -1 \end{cases}$$

Back up slides

Antilepton + meson two-body modes



Flavor symmetries I (the hierarchy puzzle)

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

call ξ_i the generic small parameter. A modern approach to understand why ξ_i<<1 consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When ξ_i=0 the theory becomes invariant under a flavour symmetry F

Example: why y_e<<y_{top}? Assume F=U(1)$_F$

F(t)=F(t^c)=F(h)=0 $\qquad y_{top}(h+v)t^c t \qquad$ allowed

F(e^c)=p>0 F(e)=q>0 $\qquad y_e(h+v)e^c e \qquad$ breaks U(1)$_F$ by (p+q) units

if ξ=<φ>/Λ<1 breaks U(1) by one negative unit $\qquad y_e \approx O(\xi^{p+q}) << y_{top} \approx O(1)$

provides a qualitative picture of the existing hierarchies in the fermion spectrum