# GW vs EM astronomy

- GWs interact very weakly with matter, strain h decays as 1/r
  → GWs visible to very high z, eg SMBHs with LISA, stochastic backgrounds

- Gravitational wavelength >~ source's size (because GWs generated by bulk motion of matter) vs EM wavelengths << source's size (because EM waves generated by moving charges, atomic processes, etc)

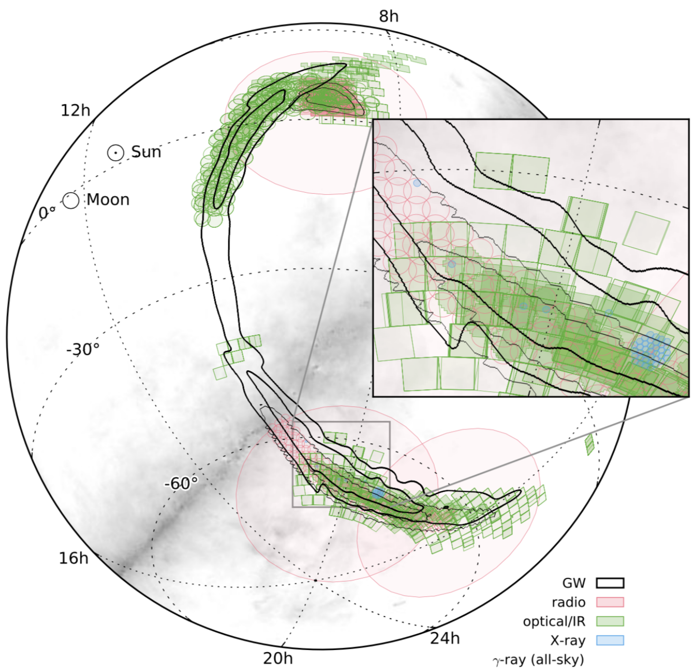
  → EM can be used for imaging, GWs do not have angular resolution (akin to sound)

  → EM surveys cover small areas, GWs cover whole sky

GW and EM waves are complementary tools for testing fundamental physics, astrophysics and cosmology

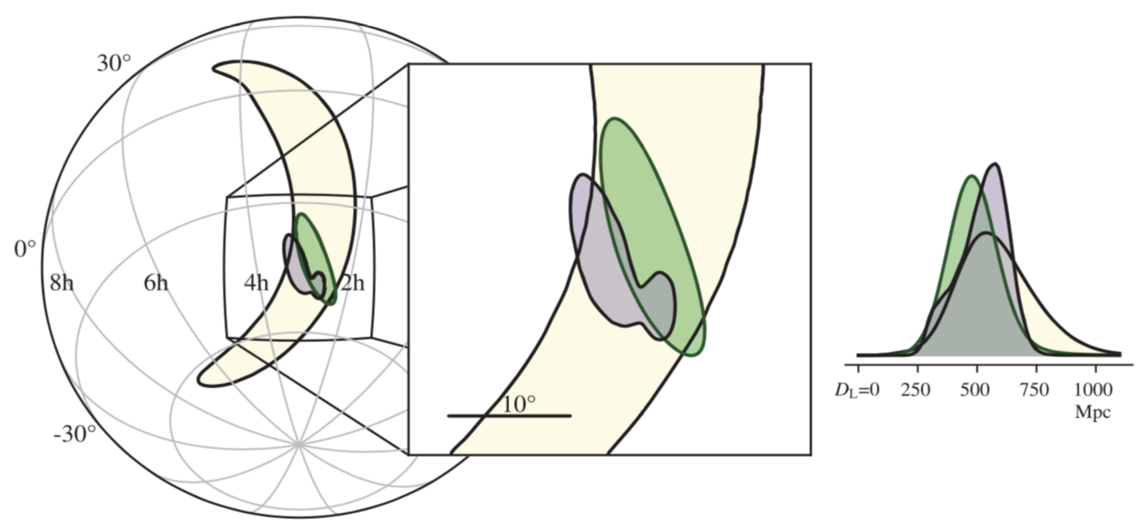
# GWs alone have poor sky localization

Need network of detectors/many pulsars (also to enhance detection confidence and minimize downtime)




GW150914

GW170814



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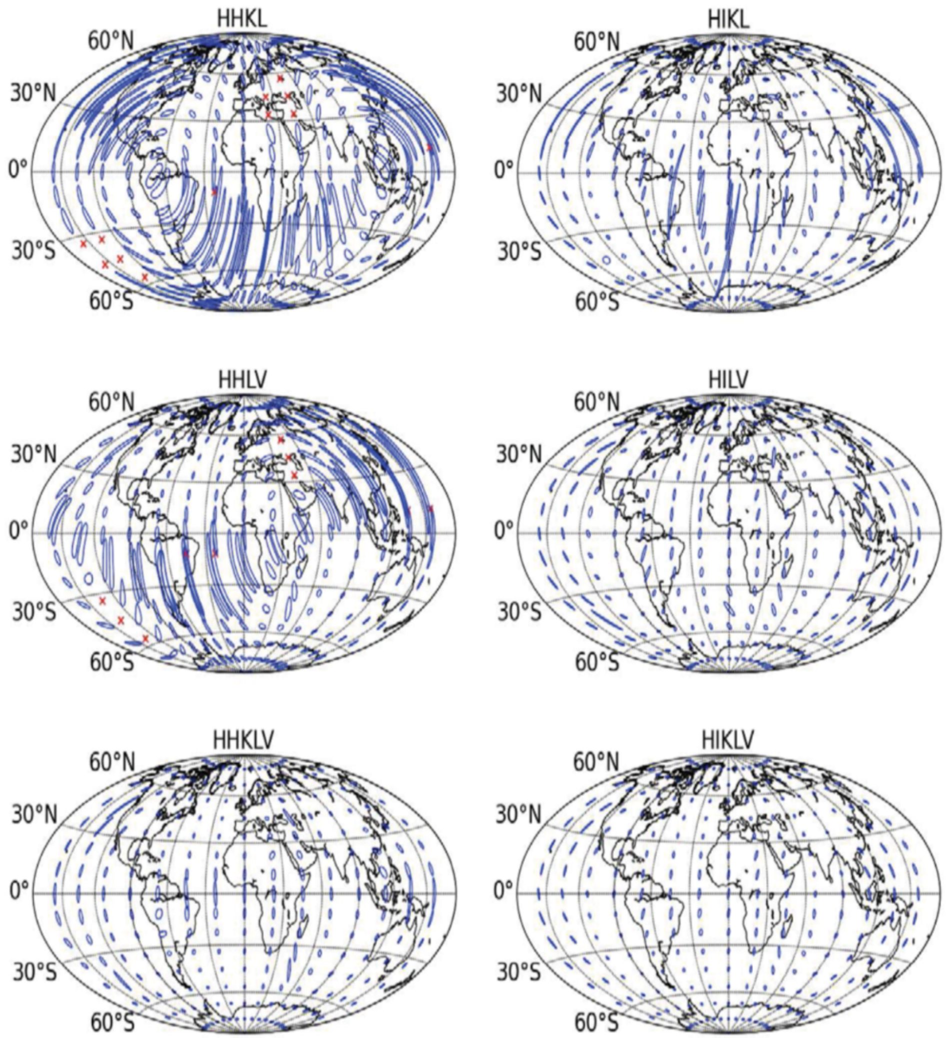
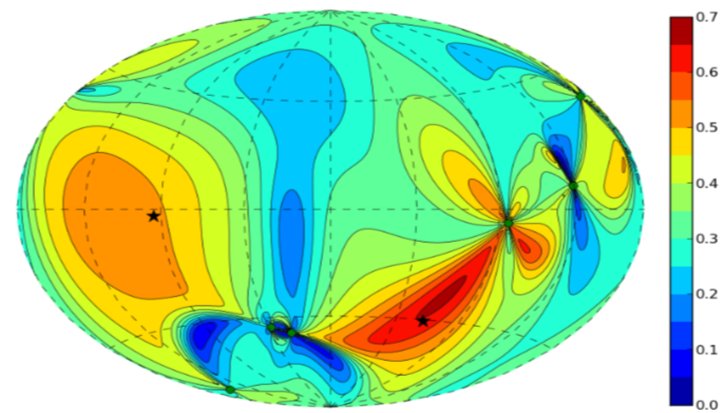
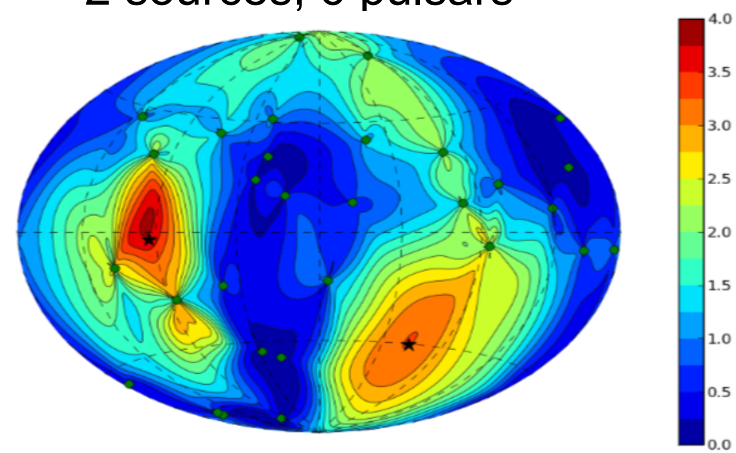



BH-NS face-on at 160 Mpc (Fairhurst 2014)




2 sources, 6 pulsars




2 sources, 30 pulsars

Babak & Sesana (2012)

PTAs: to estimate sky location of N sources, 3 x N pulsars are needed

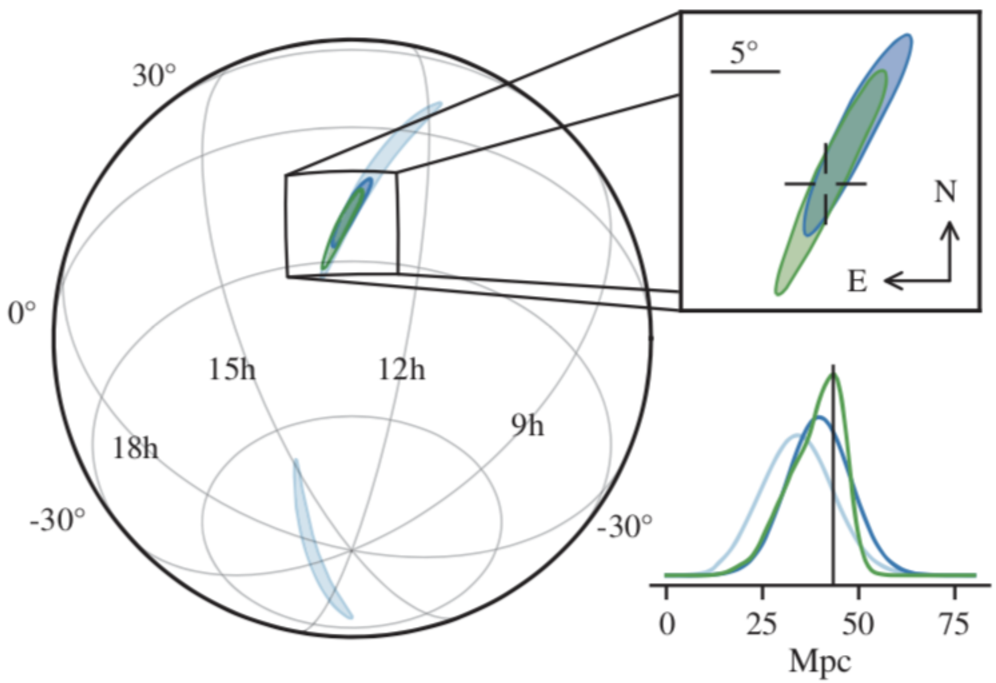
# EM counterparts to GW sources?

- Allow for: - sky localization and detection confidence enhanced

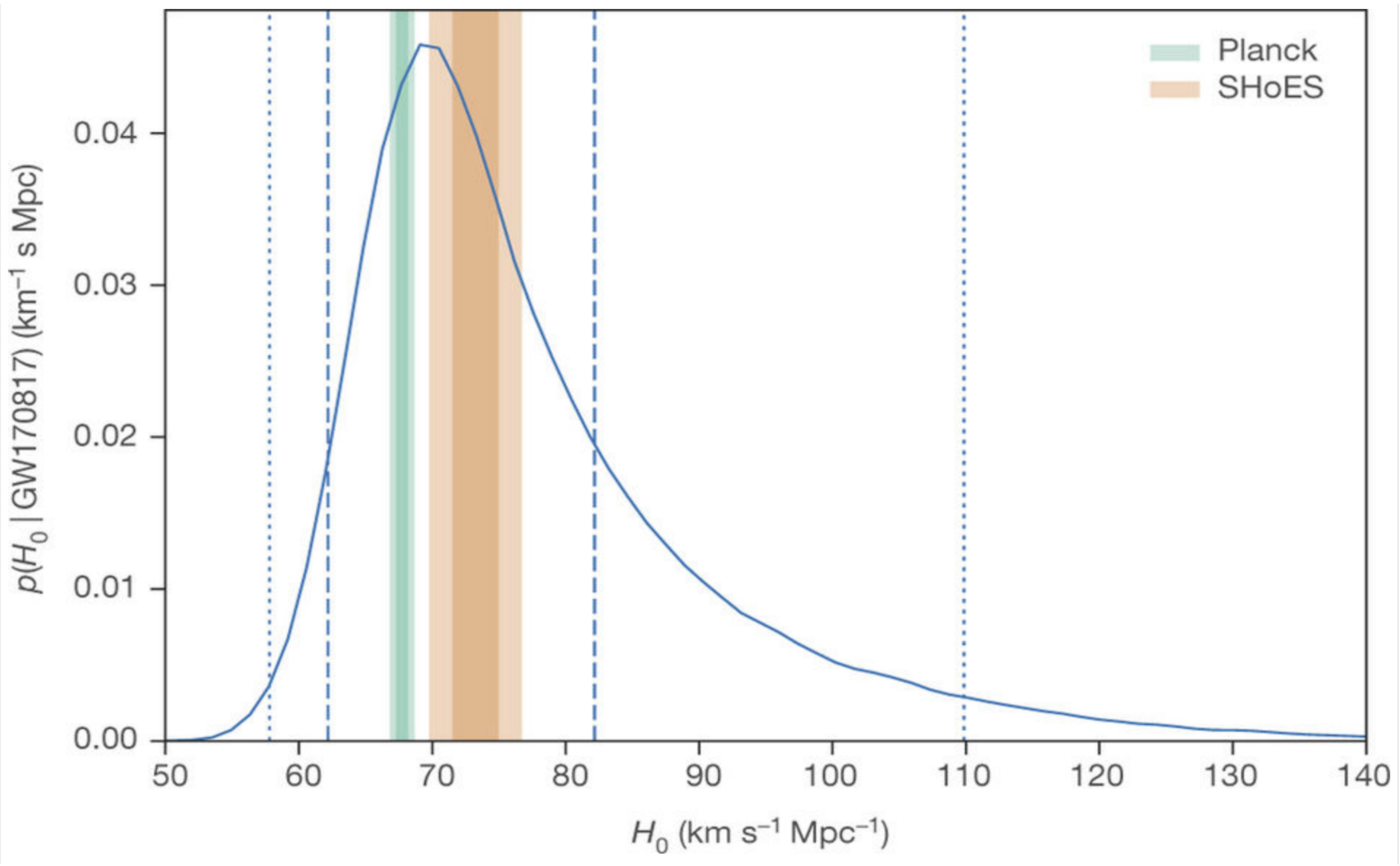
  - redshift measurement, unavailable with GWs alone (no intrinsic energy scale in GR)

- Goals: - GRB as triggers for GW searches

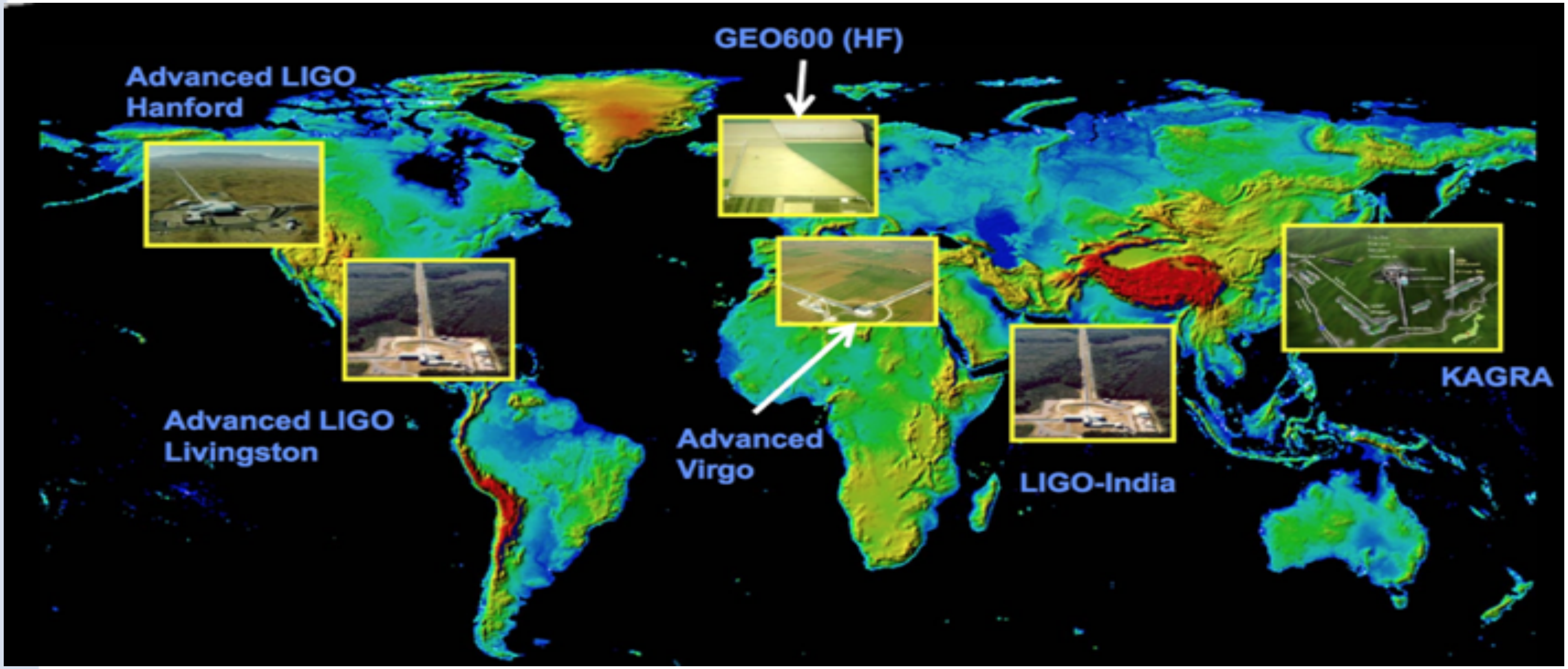
  - generate GW triggers to point telescopes in 10-100 sec to observe optical prompt emission, 100 sec-days for afterglow

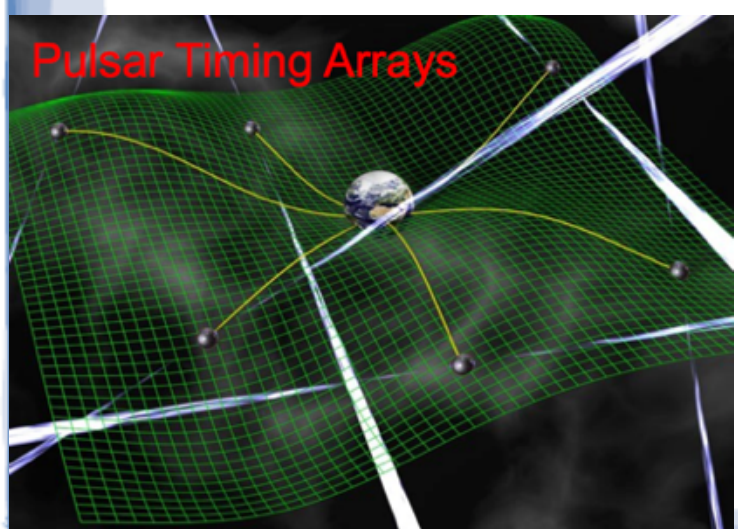


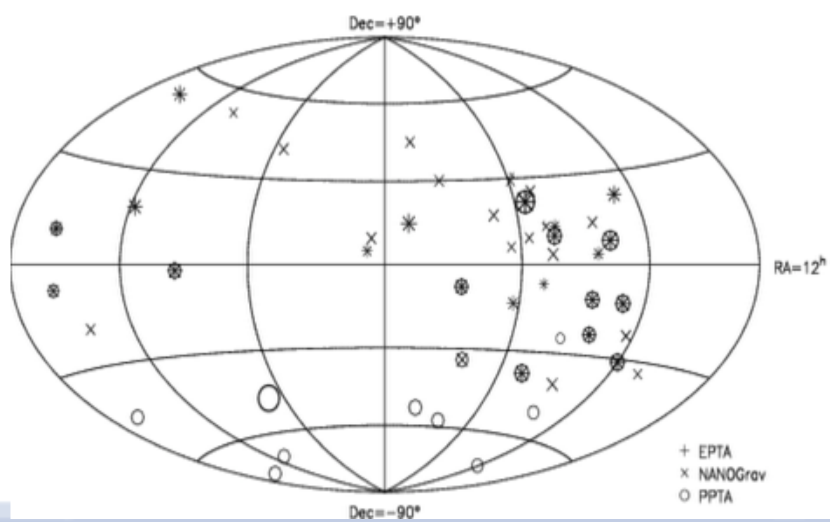
# GW-based distance ladder



# Existing detectors

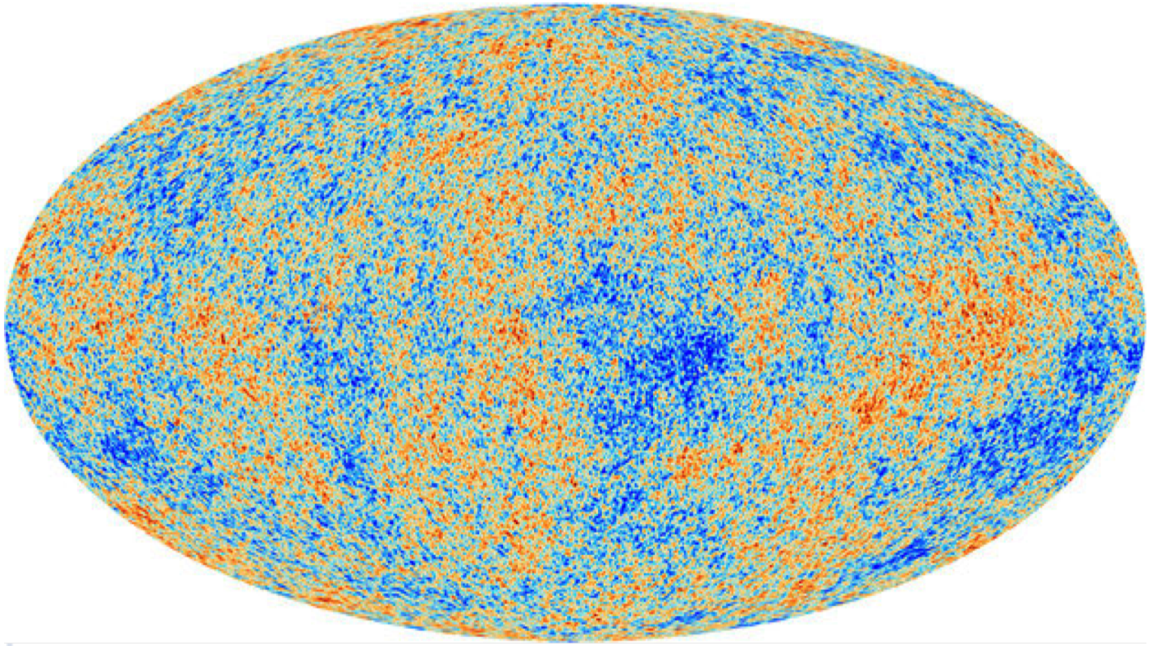


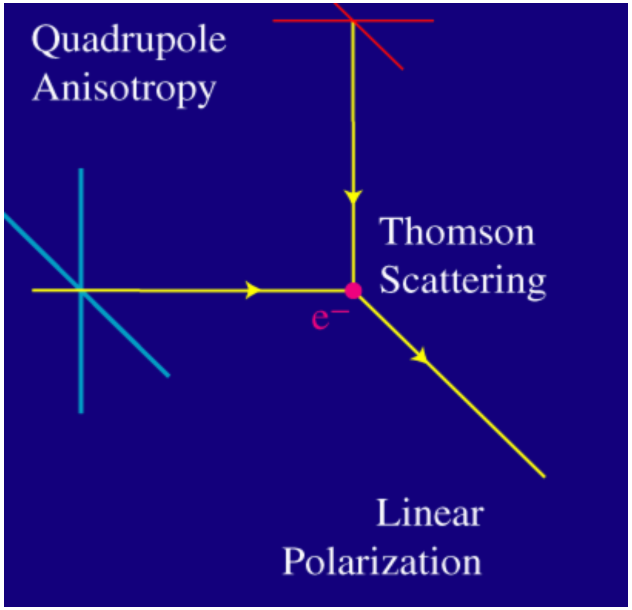



# Existing detectors

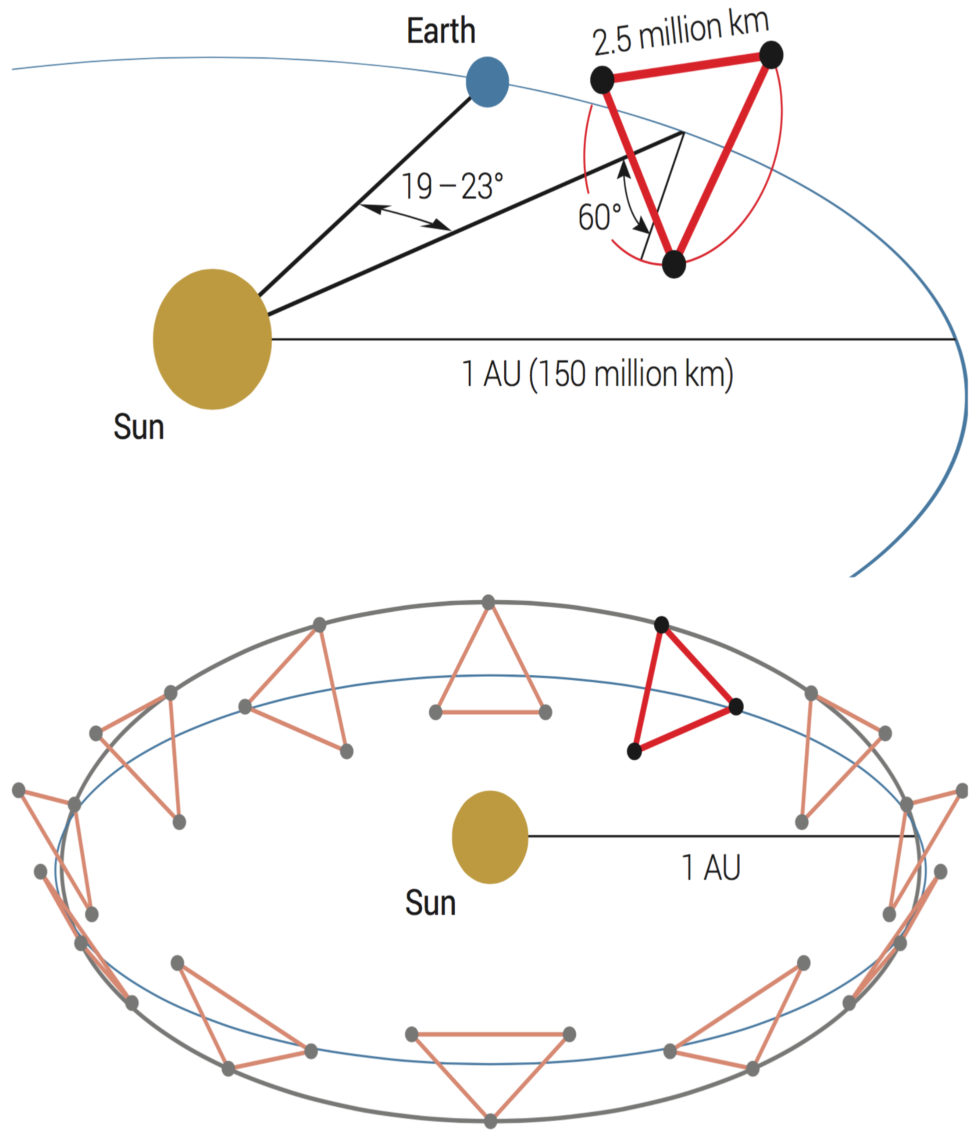
CMB B modes

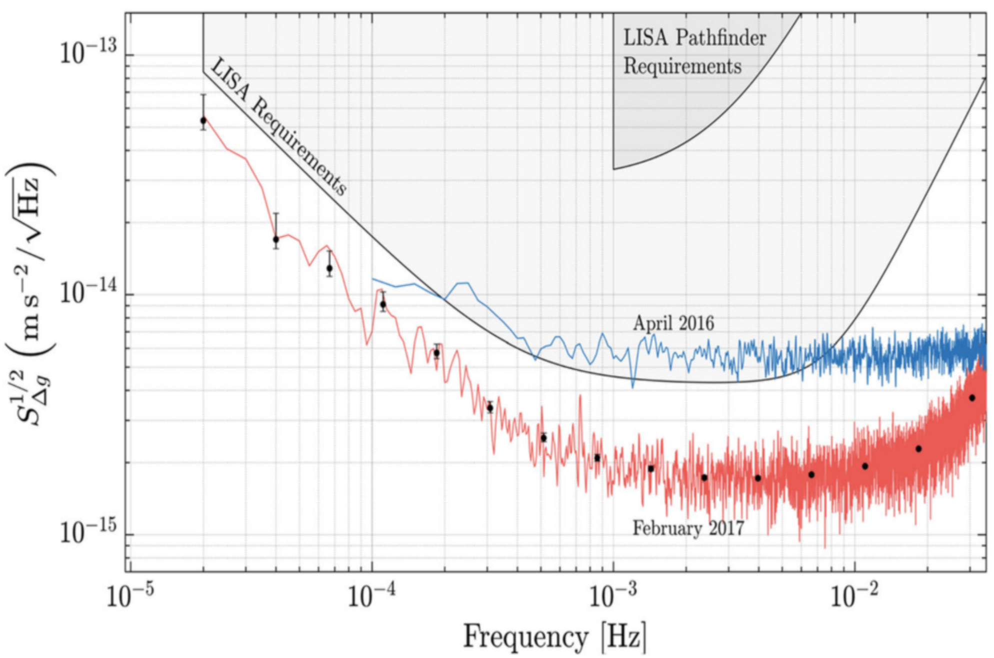





Animation from Hu 2001

# Next-generation detectors



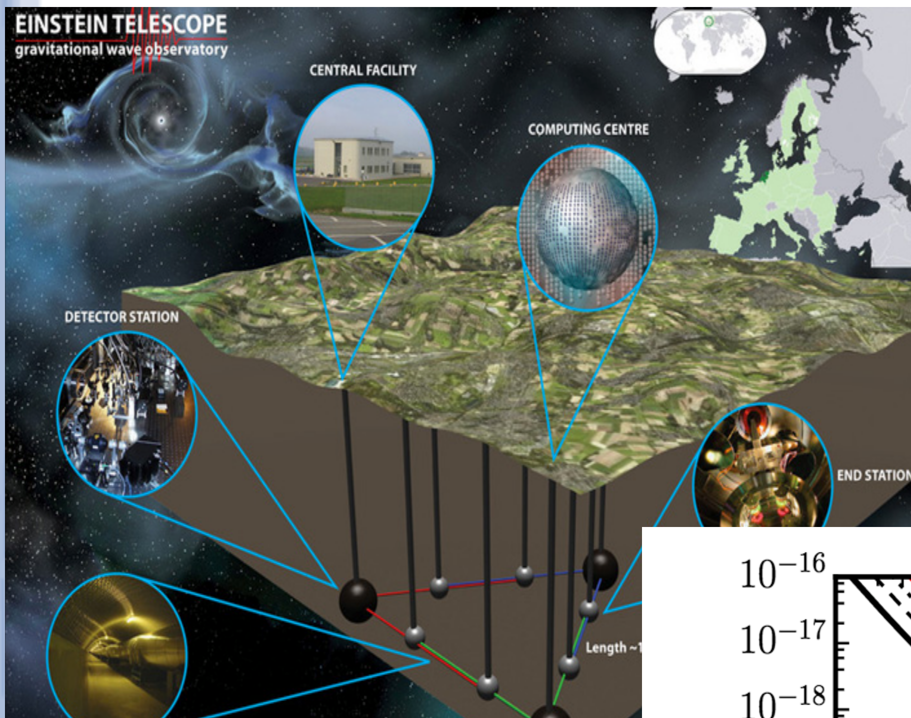




LISA: accepted for ESA's L3 launch Slot (Jan 2017)

Technology tested by LISA Pathfinder (2016)

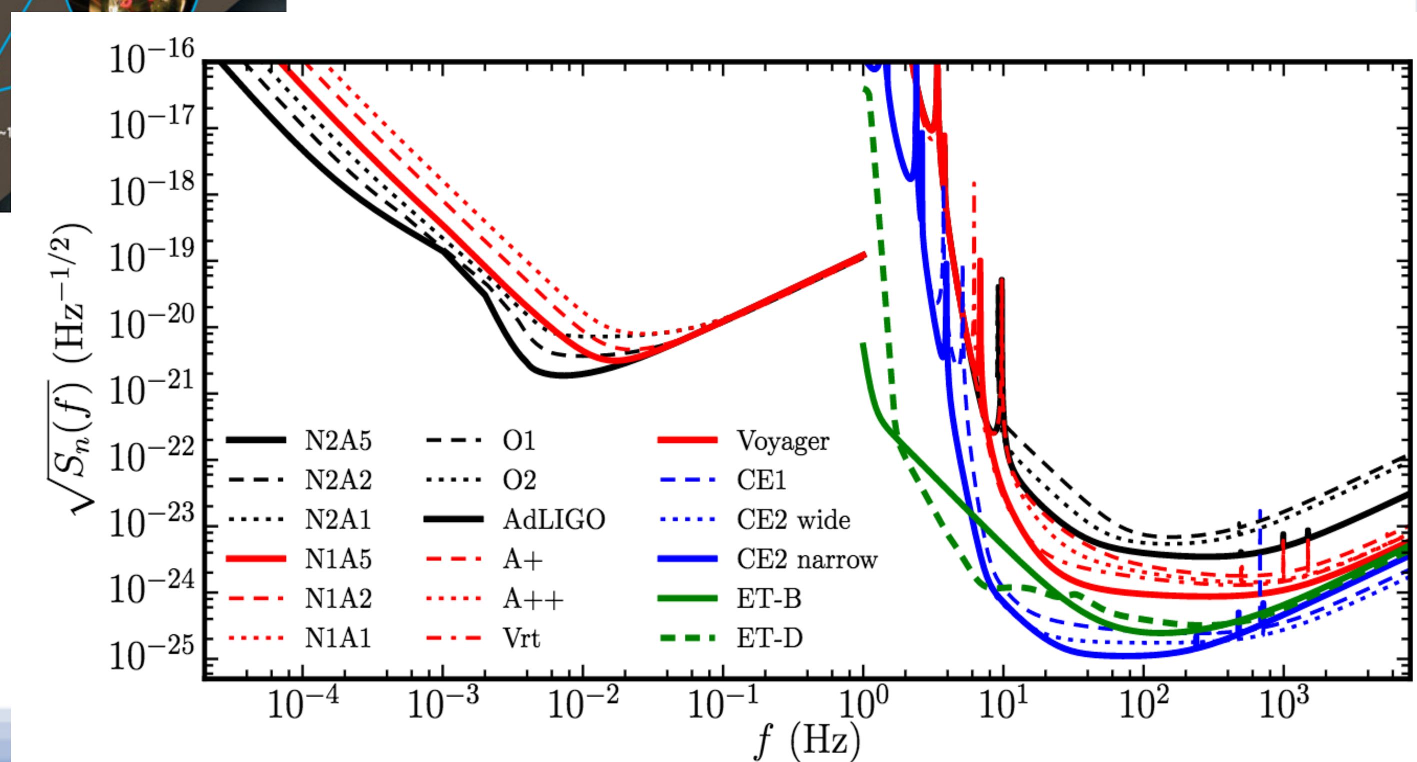
Launch in 2028-2030?

# Next-generation detectors

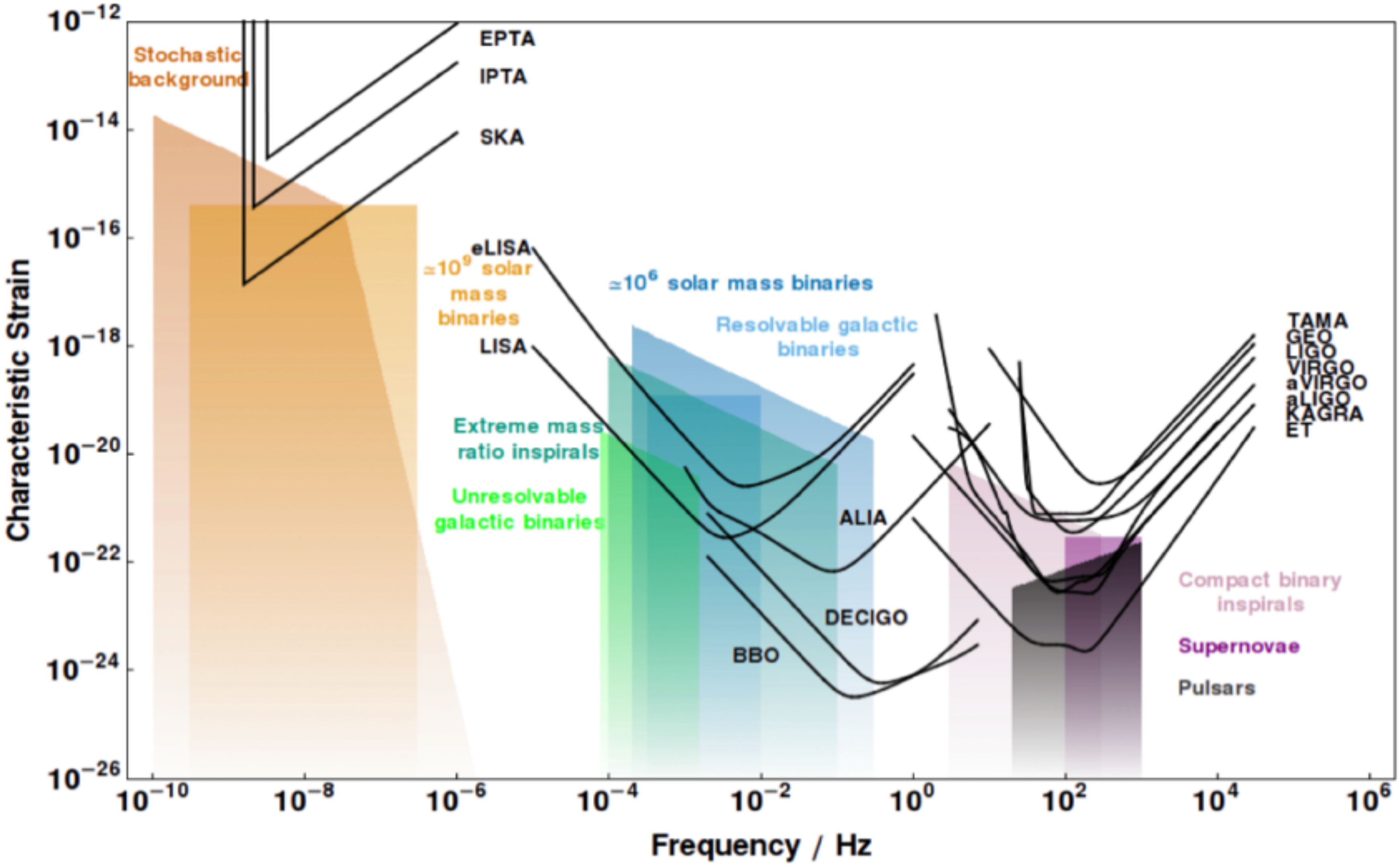



3rd generation ground detectors in Europe and US (ET, CE, Voyager...)

Longer arms (10-40 km?), underground, cryonigenic, squeezed laser states, etc



# Frequency windows



# GWs from binary systems

From quadrupole formula, GW frequency is twice orbital one

$$f_{\rm GW} = \frac{6\times10^4}{\tilde{m}\tilde{R}^{3/2}}\,{\rm Hz} \qquad \tilde{R} = R/m \qquad \tilde{m} = m/M_\odot$$

aLIGO/aVirgo:

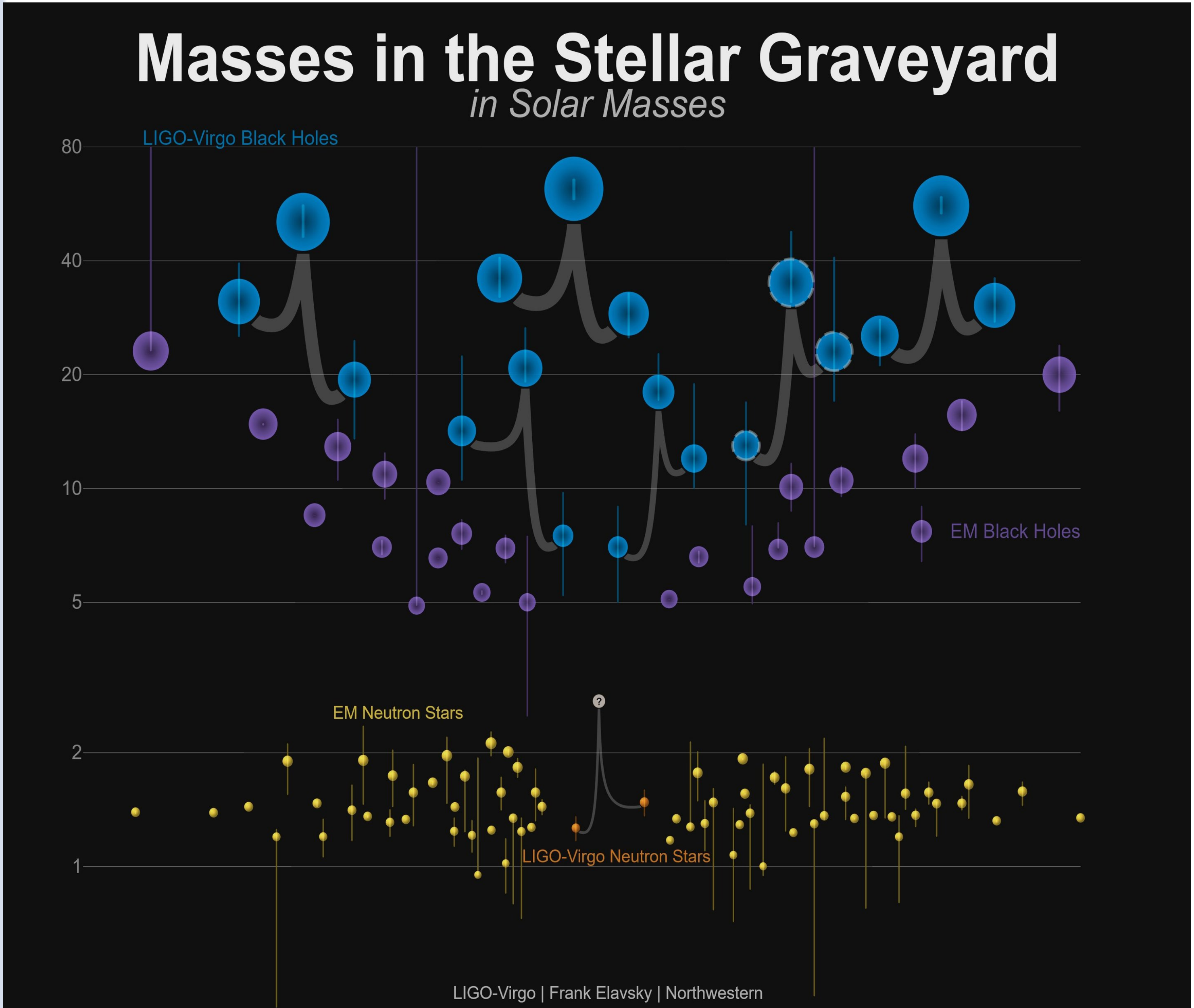
1) BH-BH late inspiral and merger, with masses up to 60-70 $M_{sun}$

2) NS-NS and possibly BH-NS: from few to hundreds of events per year

Binary pulsars observed with masses ~ 1.4 $M_{sun}$, but isolated NS can have masses  2 $M_{sun}$

3) If intermediate mass BHs exists, IMBH-BH/NS/WD and IMBH-IMBH observable with third generation ground detectors

# LIGO/Virgo detections




Why important?

- First direct detection of GWs (indirect evidence from binary pulsars)
- Opens up era of multi-band EM+GW astronomy
- Evidence that sGRB=NS+NS
- High BH masses imply formation in weak-wind/low-metallicity environment
- Test GR for the first time in strong-field ($U{\sim}c^2$) and highly relativistic ($v{\sim}c$) regime

# GWs from binary systems

LISA:

Supermassive BHs observed in center of galaxies with masses ~ $10^5$ – $10^9$ $M_{sun}$; believed to merge when galaxies merge (cf double AGNs)

1) Inspiral and merger of SMBH-SMBH (with masses ~ $10^5$ – $10^6$ $M_{sun}$): from a few to hundreds per year

2) Inspiral and merger of SMBH – BH/NS/WD (aka Extreme Mass Ratio Inspirals, EMRIs): rates uncertain, from a few to hundreds/thousands per year

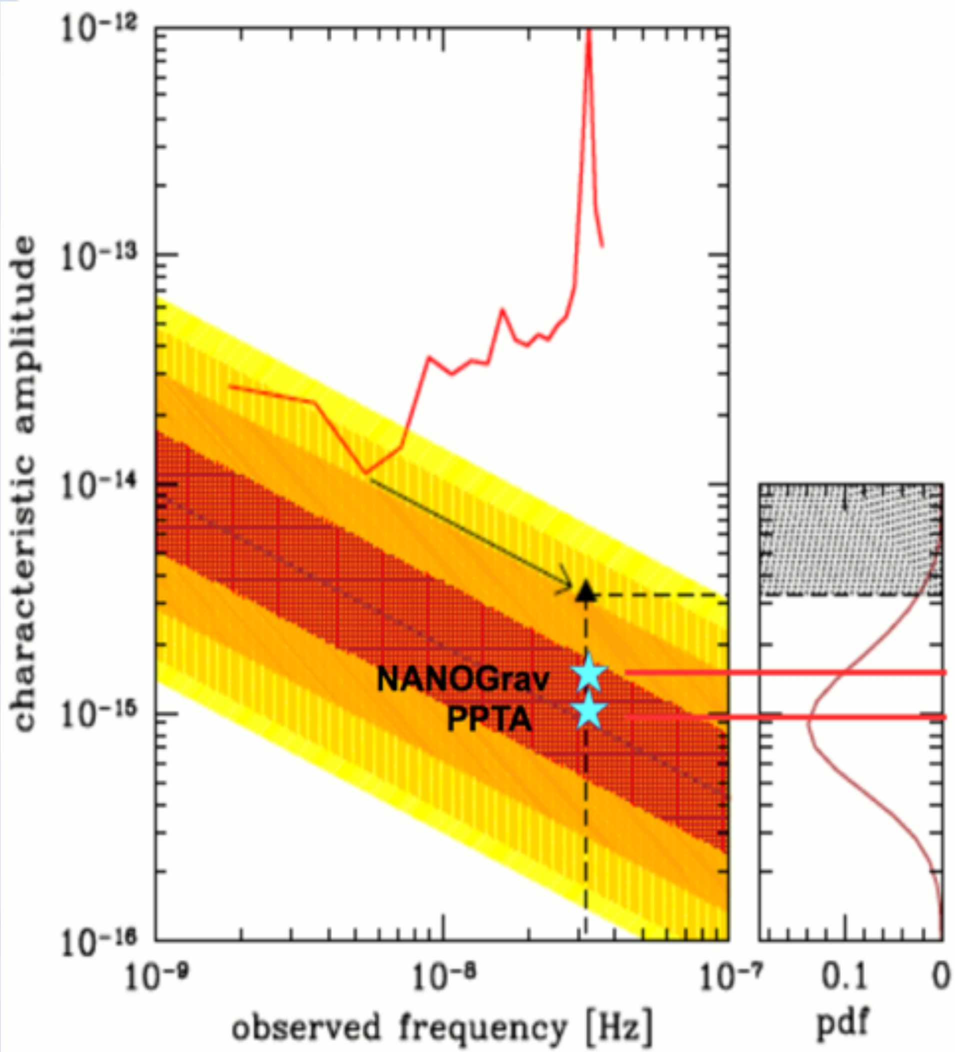
3) IMBH-SMBH: rates uncertain

4) WD-WD at separations of a few star radii (~ $10^5$ km): thousands of resolved sources, a few guaranteed sources in the Galaxy

Pulsar timing array:

SMBH-SMBH at $0.2 < z < 1.5$, with masses л 5 x $10^8$ $M_{sun}$ and separations of hundreds gravitational radii

# PTA limits on stochastic background from SMBH binaries



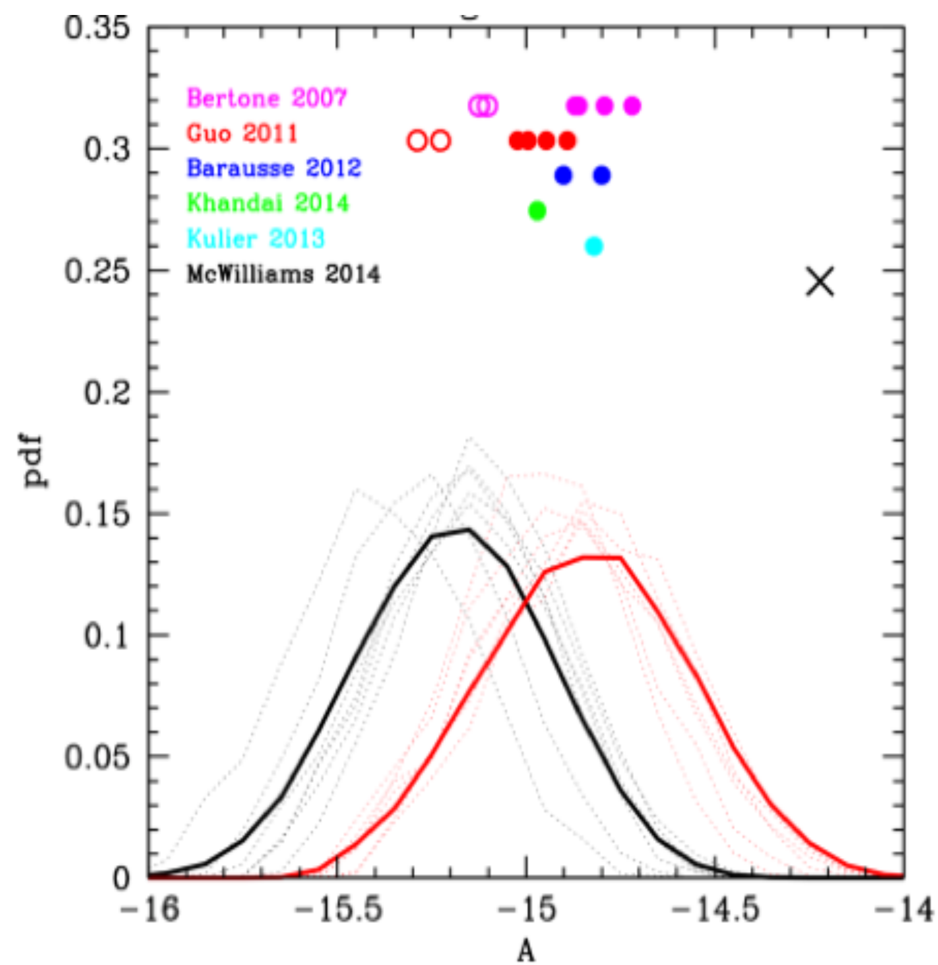




Figure courtesy of A. Sesana

# GWs from isolated systems

- Rotating axisymmetric star/spherical collapse do not emit
- Core collapse supernovae (type II) produce bursts of GWs if instabilities develop due to high rotational velocities, or if asymmetries are present:

  possible sources for LIGO/Virgo/Einstein telescope
- Rotating pulsar can radiate monochromatically if rotation deviates from axisymmetry: possible sources for LIGO/Virgo/Einstein telescope but no good model for $\epsilon$

$$h \sim \frac{G}{c^4}\frac{I f^2 \epsilon}{r} \qquad \epsilon = (I_{xx} - I_{yy})/I$$

LIGO/Virgo will constrain $\epsilon < 10^{-7}$

# Stochastic backgrounds

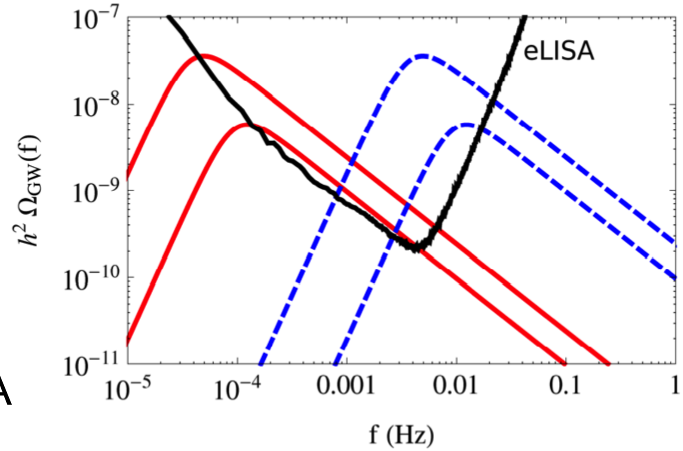
- Astrophysical origin: superposition of many unresolved GW signals (eg from WD-WD binaries for LISA, or SMBH binaries for PTAs)
- Cosmological origin, eg inflationary or due to phase transitions
- Isotropic and homogenous (cosmological origin) or approximately so (astrophysical origin)
- Look like noise by can be detected by cross-correlating detectors
- Inflationary GWs depend on energy scale of inflation

$$\Omega_{gw}(f) \propto (E_{inflation}/M_P)^4 \approx \text{constant} \quad \Rightarrow \quad E_{inflation} < 1.9 \times 10^{16}\,\text{GeV}$$

- GWs produced by phase transitions have peaked spectrum

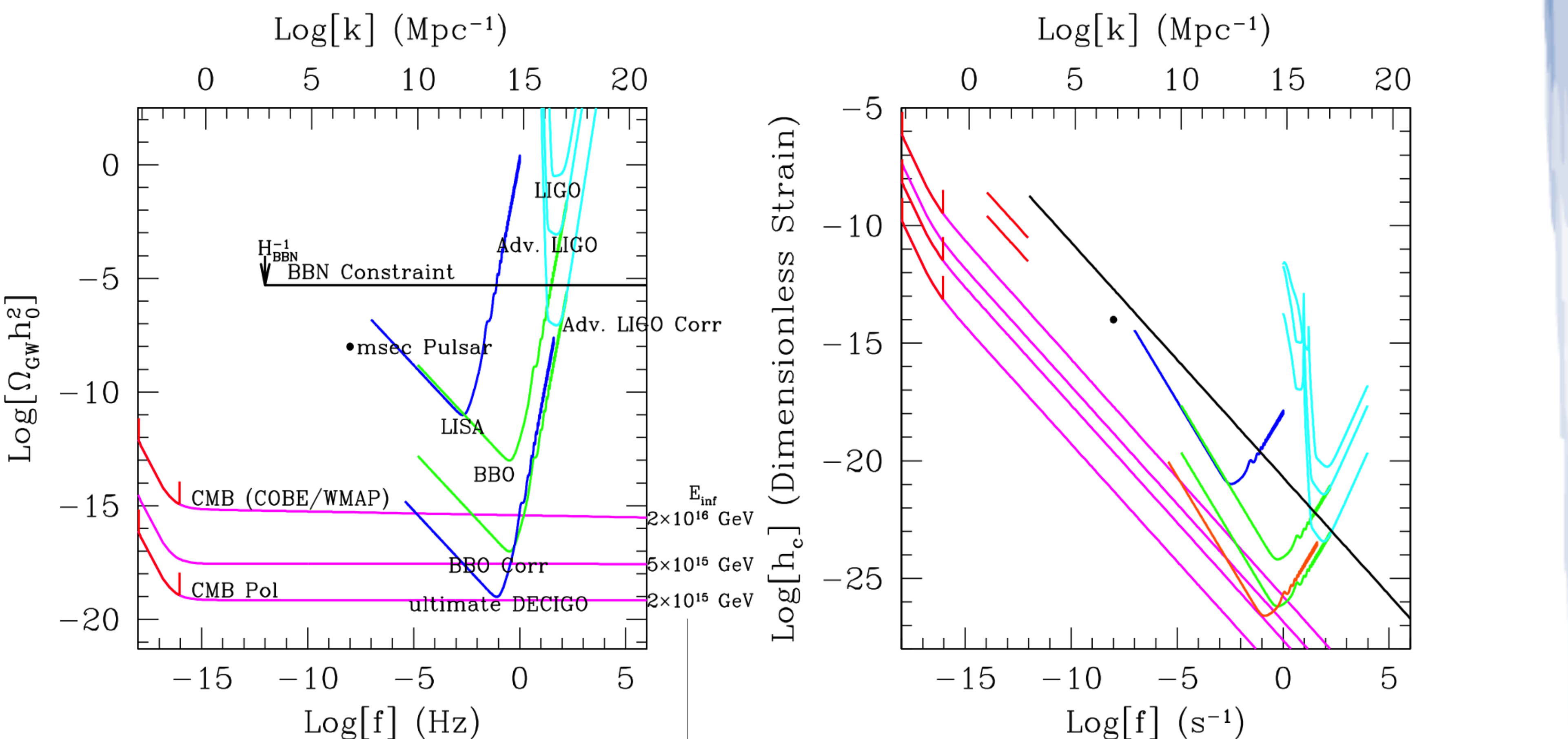
$$f_{\text{peak}} \sim 100\,\text{Hz}\left(\frac{T}{10^5\,\text{TeV}}\right)$$

E.g. some exotic models (eg extra dimensions, cosmic strings) could produce phase transitions observable by LISA (Dufaux 2012)



# Frequency ranges

Figure from A. Cooray, astro-ph/0503118

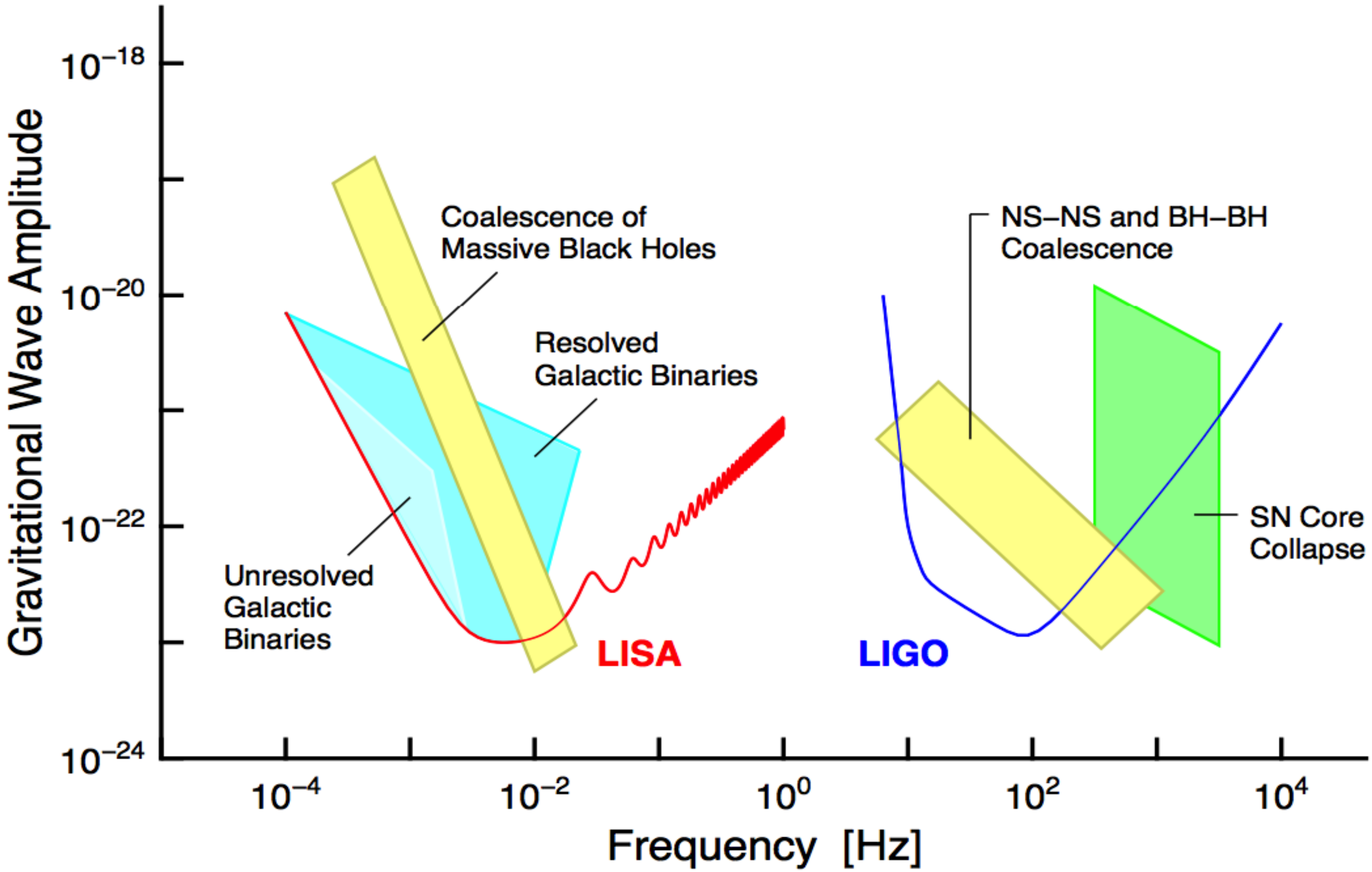



$$\Omega_{\rm gw}(f) = \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm gw}}{d\ln f}$$

Energy scale of inflation

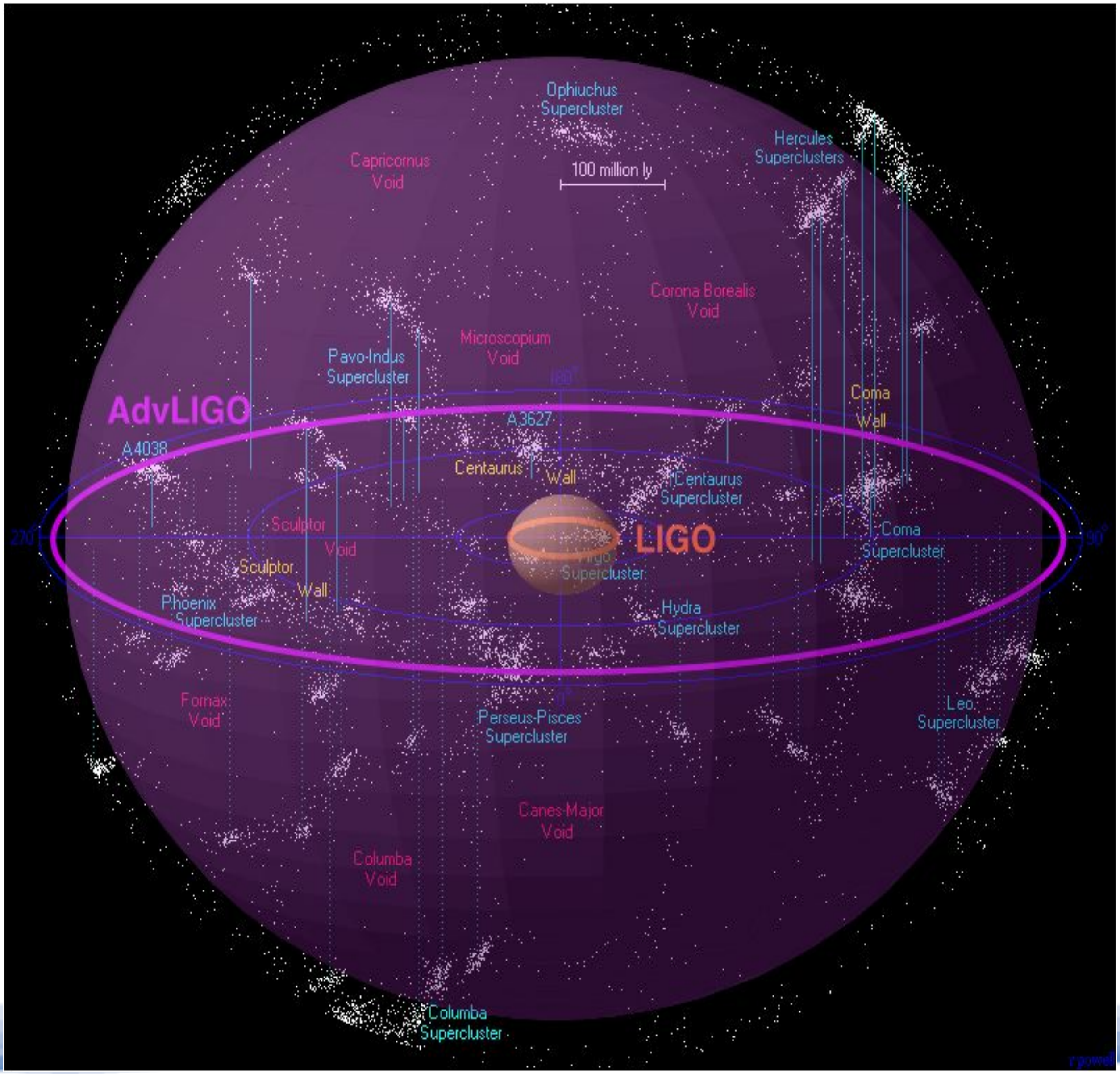
$$\Omega_{\rm gw}(f) = 2\pi^2 \left[f h_c(f)\right]^2 / (3H_0^2)$$

# LISA vs LIGO/Virgo



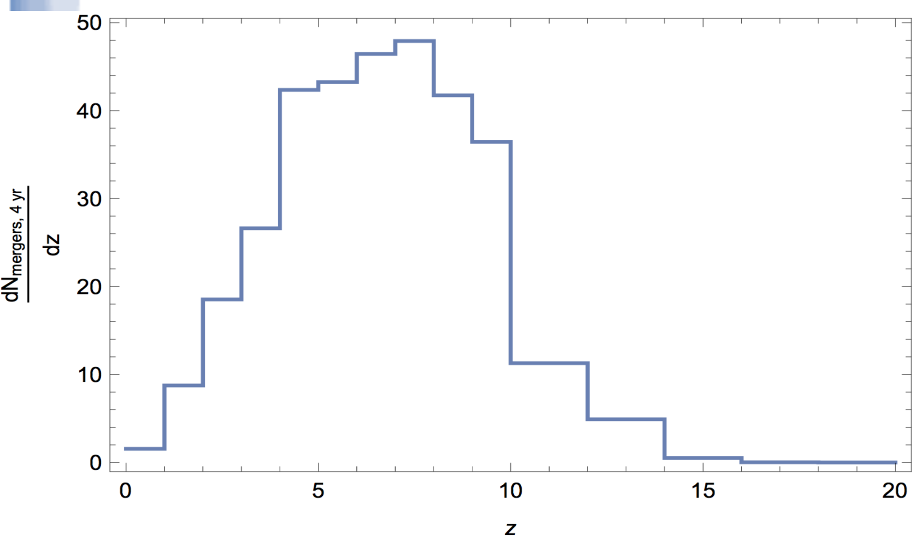
# LISA vs LIGO/Virgo

Range depends on sources, but is at most z~0.1 for LIGO/Virgo...

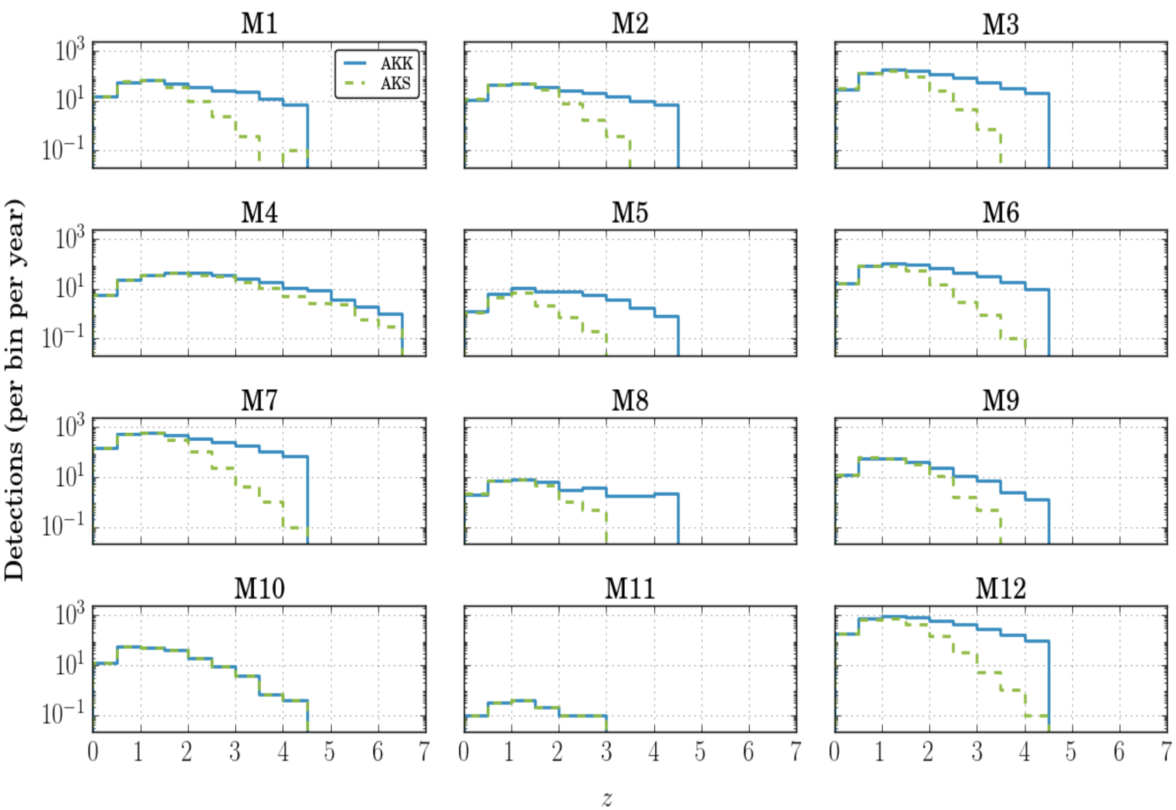


# LISA vs LIGO/Virgo

... vs z>10 for LISA (for SMBH binaries)




Detectable SMBH binaries in a popIII seed formation model




Detectable EMRIs

# Detector noise

- *s(t)=h(t)+n(t)*
- By central theorem limit, noise n(t) should be close to a Gaussian process, i.e. noise should be uncorrelated in Fourier but not in time domain

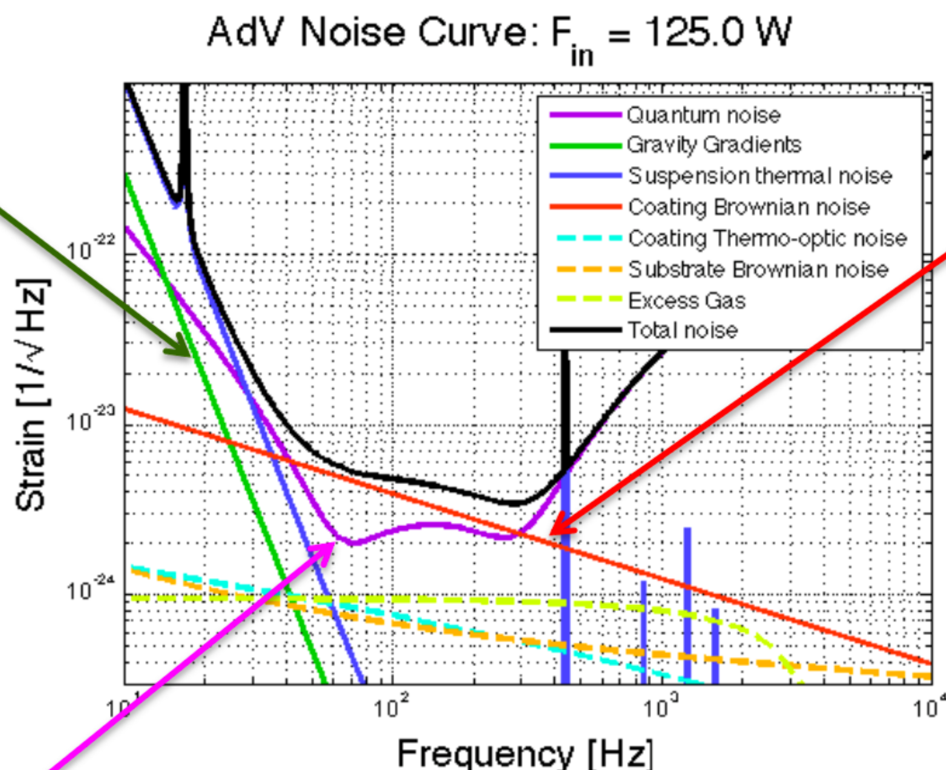
$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \frac{1}{2}S_n(f)\delta(f-f')$$

$$\langle n^2(t)\rangle = \int_0^\infty S_n(f)df$$

- $S_n(f)$ is called (single sided) noise spectral density

# Detector noise

**Seismic and gravity gradient noise**
Geophysics

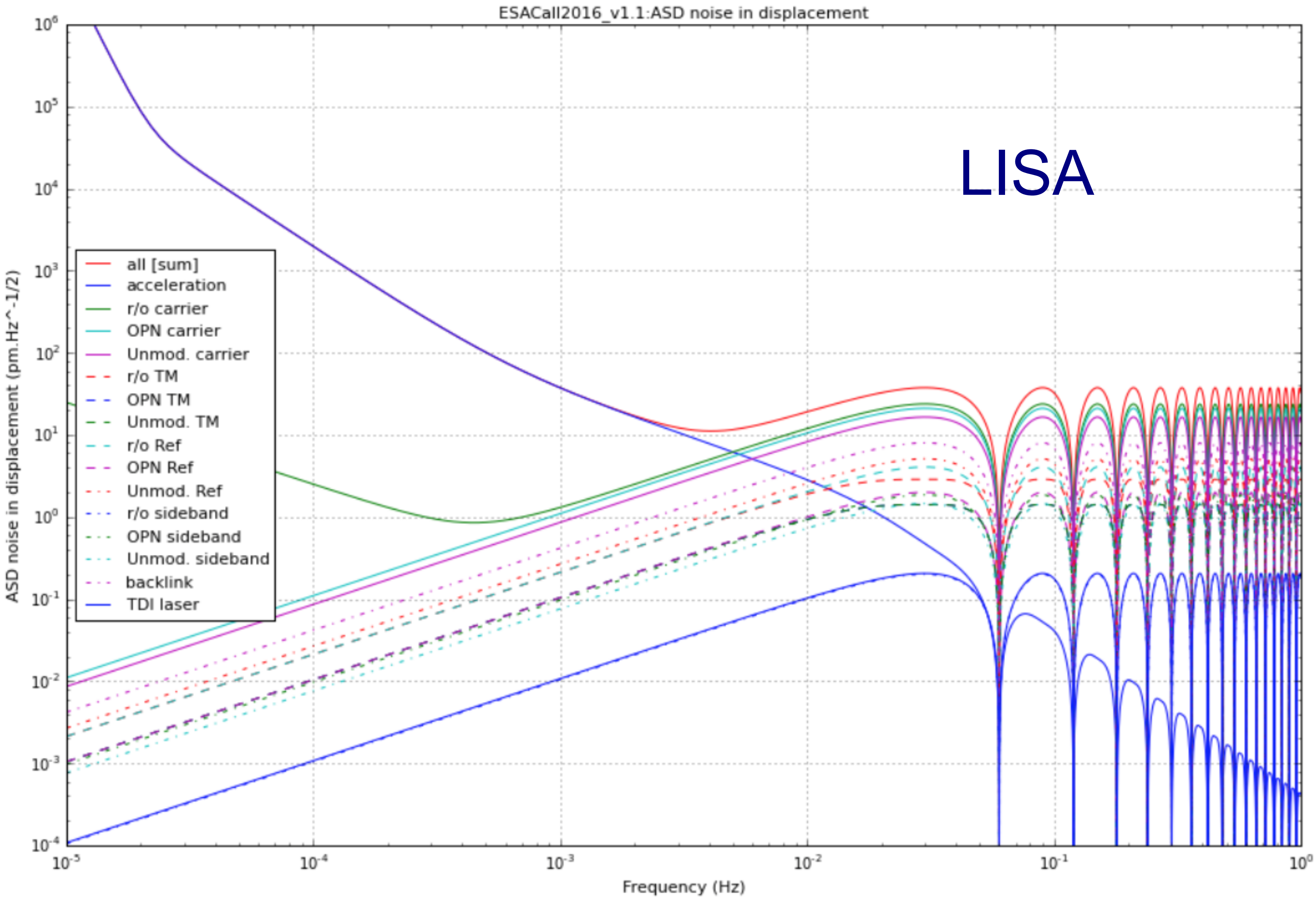



**Thermal noise**
Thermodynamics

**Quantum noise**
Quantum mechanics

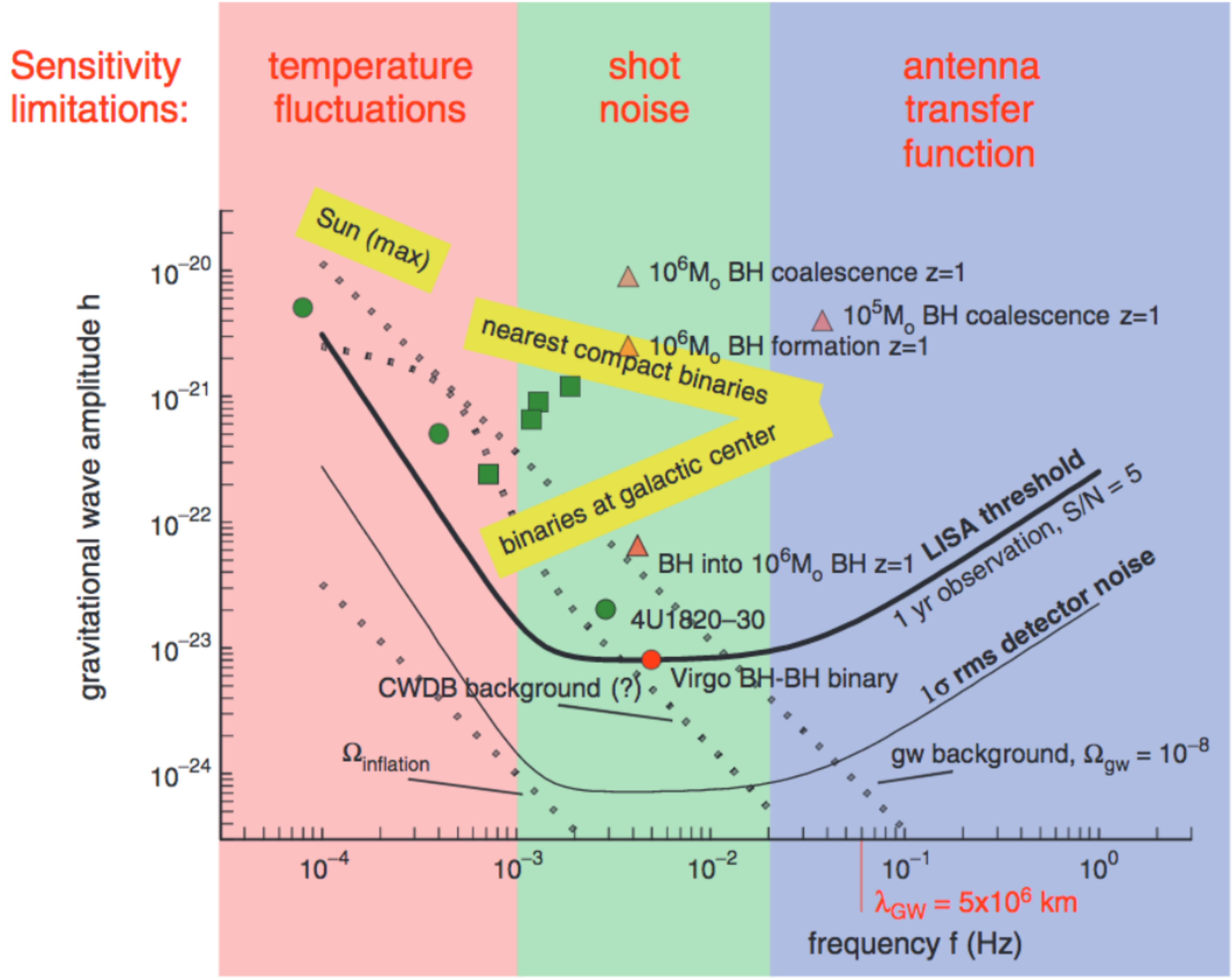
Figure courtesy Matteo Barsuglia

# Detector noise



# Detector noise

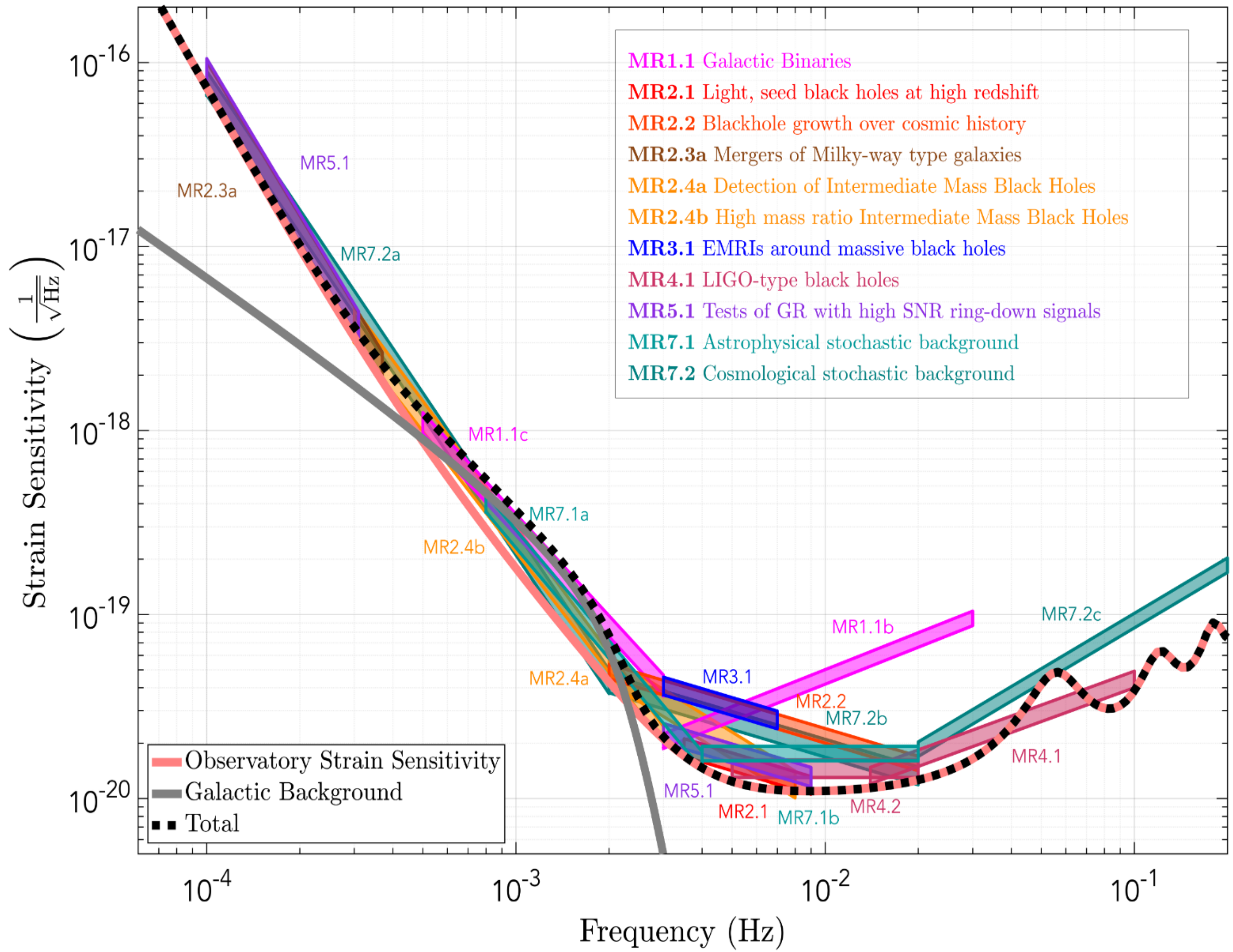
LISA




Rudiger et al 2008

# Detector noise

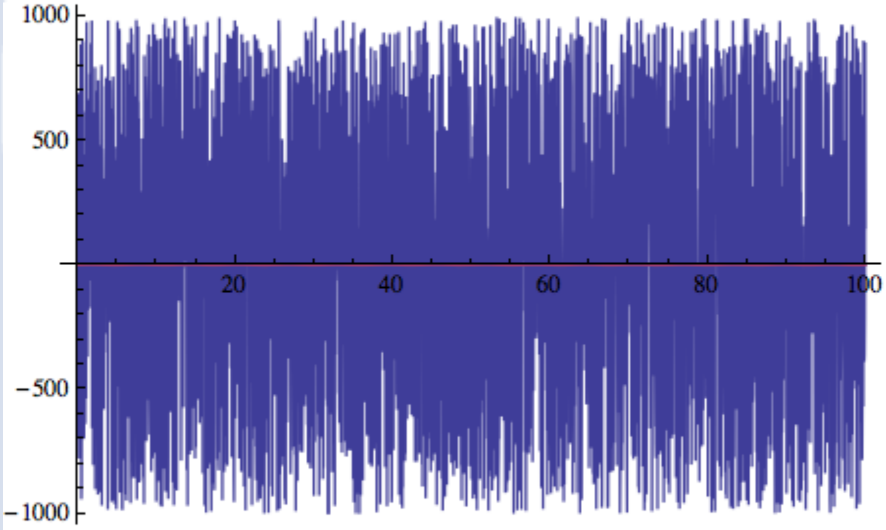
LISA

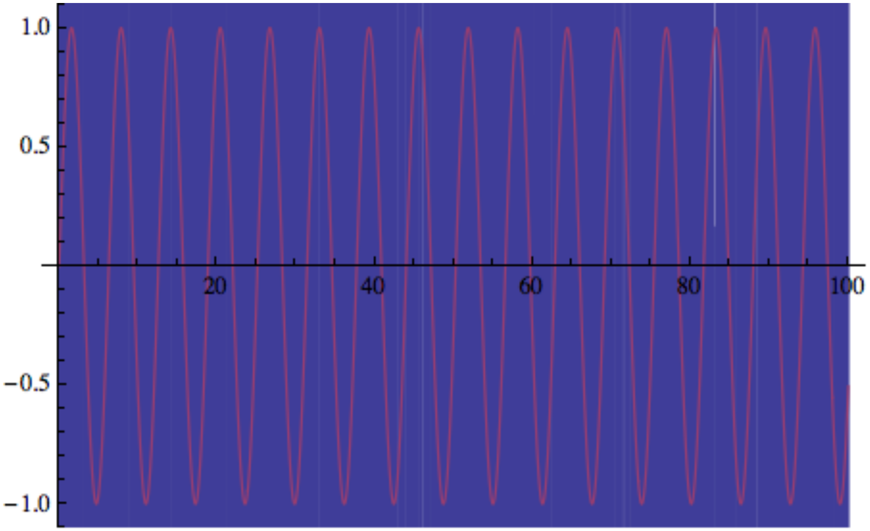


# How to extract signal from noise?

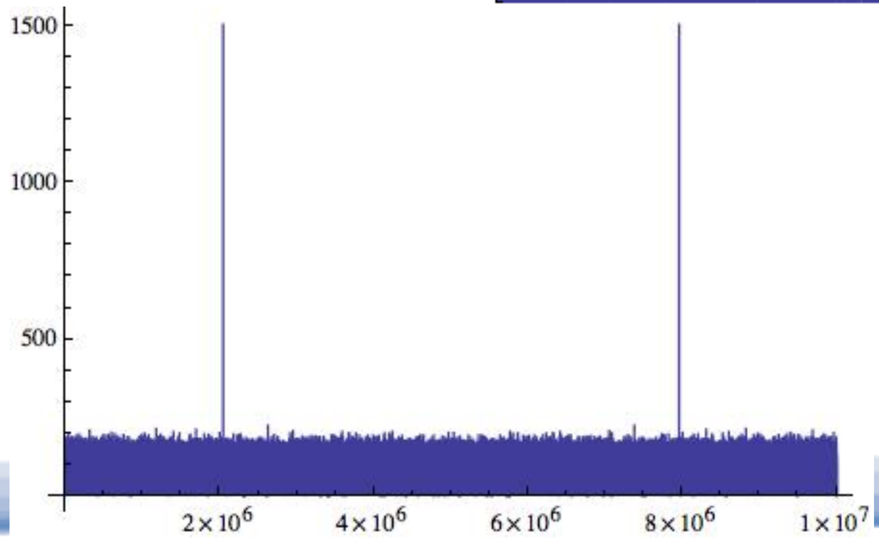
Can we extract GW signal even if it is much smaller than the instrumental noise?

Elementary example




Discrete FT



# The matched filtering theorem

- *s(t)=h(t)+n(t)*

- Define filter $\hat{s} \equiv \int_{-\infty}^{+\infty} s(t)K(t)dt$

- Maximum signal-to-noise ratio S/N, with $S \equiv \hat{s}(h \neq 0), \; N \equiv \hat{s}(h = 0)$, is given by optimal filter

$$\tilde{K}(f) \propto \tilde{h}(f)/S_n(f) \quad \Longrightarrow \quad \left(\frac{S}{N}\right)^2 = 4\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)} df$$

(proof on the blackboard, c.f. also Maggiore's book)

- *h(f)* is called template

# SNR for compact binaries

- From quadrupole + pattern functions formula, $h(t) = F_+ h_+(t) + F_\times h_\times(t)$
- Use Newtonian dynamics (i.e. Kepler's law) and energy conservation
- Compute Fourier transform via stationary phase approximation, and account for propagation in cosmological background by replacing distance with luminosity distance and masses by redshifted masses

$$\tilde{h}(f) = \sqrt{\frac{5}{6}} \frac{\mathcal{M}^{5/6} f^{-7/6}}{2\pi^{2/3} D_L} e^{i\psi} \frac{2Q}{2} . \qquad Q = \frac{1+\cos^2 \iota}{2} F_+ + i \cos\iota F_\times \qquad \mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \times (1+z)$$

- If sky and orientation averaged, $\langle (1+\cos^2\iota)^2 F_+^2 + 4\cos^2\iota F_\times^2 \rangle^{1/2} = \frac{4}{5}$

$$\Rightarrow \quad \tilde{h}(f) = \sqrt{\frac{5}{6}} \frac{\mathcal{M}^{5/6} f^{-7/6}}{2\pi^{2/3} D_L} e^{i\psi} \frac{2}{5} = \frac{1}{\sqrt{30}} \frac{\mathcal{M}^{5/6} f^{-7/6}}{\pi^{2/3} D_L} e^{i\psi}$$

$$\left(\frac{S}{N}\right)^2 = 4 \int_{f_{\rm in}}^{f_{\rm out}} \frac{|\tilde{h}(f)|^2}{S_n(f)} df$$

# SNR for quasi-monochromatic sources

$$h(t) = \sqrt{2} h_0 \cos\left[\phi(t)\right] \qquad \phi(t) = 2\pi[f + \dot{f}(t - t_0) + ...](t - t0)$$

$$\tilde{h}(f) \simeq \frac{h_0}{\sqrt{2\dot{f}}}$$

$$\left(\frac{S}{N}\right)^2 = 4\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)} df = \frac{2h_0^2 T_{\rm obs}}{S_n(f)}$$

For long-lived sources, SNR grows with sqrt of observation time

# Parameter estimation

- With what accuracy can observations estimate the source parameters?
- Assuming Gaussian stationary noise,

$$p(n_0) \propto \exp\left[-\frac{1}{2}(n_0|n_0)\right], \qquad (A|B) \equiv 4\mathrm{Re}\int_0^\infty df\, \frac{\tilde{A}^*(f)\tilde{B}(f)}{S_n(f)}, \rightarrow$$

$$s(t) = h(t;\boldsymbol{\theta}_t) + n_0(t) \qquad h_t \equiv h(\boldsymbol{\theta}_t)$$

- Extracted parameters maximize the likelihood $\Lambda(s|\theta_t) \propto \exp\left[(h_t|s) - \frac{1}{2}(h_t|h_t)\right]$

$$\rightarrow (\partial_i h_t|s) - (\partial_i h_t|h_t) = 0 \quad \partial_i \equiv \partial/\partial\theta_t^i$$

- Expanding to quadratic order near true parameters, and assuming large SNR

$$\Lambda(s|\theta) \propto \exp\left[-\frac{1}{2}\Gamma_{ij}\Delta\theta^i\Delta\theta^j\right] \qquad \theta_t^i = \hat{\theta}^i + \Delta\theta^i \qquad \Gamma_{ij} = (\partial_i h|\partial_j h)$$

Fisher Information Matrix=
Inverse of covariance matrix

- Errors on parameters:

$$\sqrt{\langle(\Delta\theta^i)^2\rangle} = \sqrt{(\Gamma^{-1})_{ii}}$$

- More advanced tecniques (MCMC) used to sample likelihood