The orbital parameters of GW170817 and GW150914

1 Figure 1 shows the frequency-time maps produced by the two LIGO detectors and by Virgo for the binary neutron star event GW170817 [1].

1a Extract[1] a set of data $(f_i, \Delta t_i)$ from one of the panels (LIGO-Livingston provides the clearest signal), and given the analytic relation:

$$f_{\rm GW}^{-8/3} = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_0 - t)\ , \tag{1}$$

fit the observed points to obtain the chirp mass of the binary $\mathcal{M}$.

1b Compute the value of the chirp mass for different values of the frequency using eq. (1). Is $\mathcal{M}$ constant during the inspiral? In which range?

1c Knowing that $\mathcal{M} = \nu^{3/5}M$, where $\nu = m_1 m_2/M^2$, and $M = m_1 + m_2$, find a lower bound on the total mass. [hint: plot $M(\mathcal{M}, m_i)$ vs one of the two masses m_i.]

1d Assuming circular motion, compute the separation and the orbital velocity of the two bodies during the inspiral.

2 The gravitational strain measured by LIGO for the first binary black hole event is shown in Fig. 2. The 6 points on the ascissa correspond to $t = (0, 0.009, 0.02, 0.0275, 0.034, 0.04)$.

2a Estimate the GW frequency at $t_{\rm AB} = (t_{\rm A}+t_{\rm B})/2$ and $t_{\rm DE} = (t_{\rm D}+t_{\rm E})/2$, and assuming eq. (1) compute the chirp mass of the system [hint: remember you also need to derive t_0].

2b As done for the points 1c and 2d, derive the lower mass of the binary, and assuming a Newtonian evolution compute an upper bound on the compactness of the 2 coalescing objects, and their velocity at the largest frequency.

[1] You can use one of the free codes available online, like `GrapiClick` or `WebPlotDigitizer`.

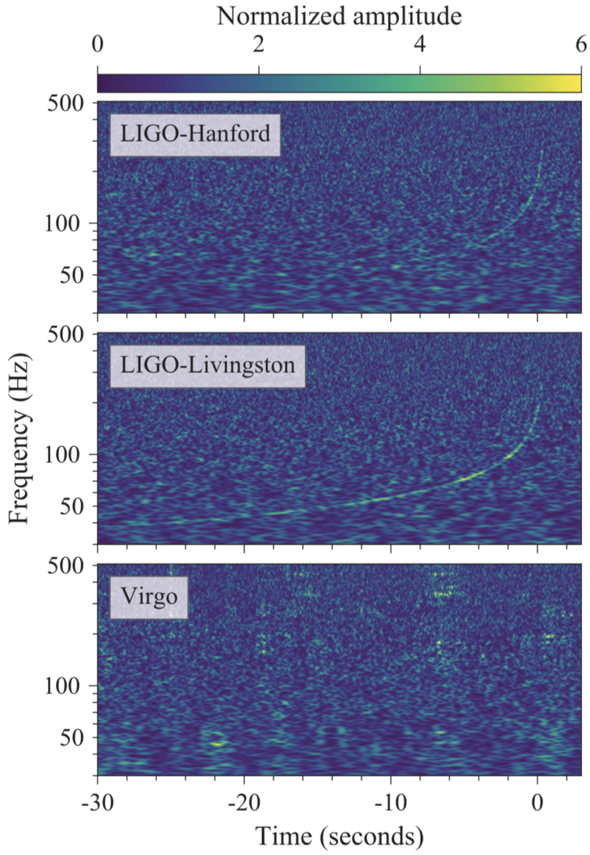

Figure 1: GW time-frequency maps for the GW170817 event, obtained from the 3 ground based interferometers LIGO/Virgo. Taken from [1].

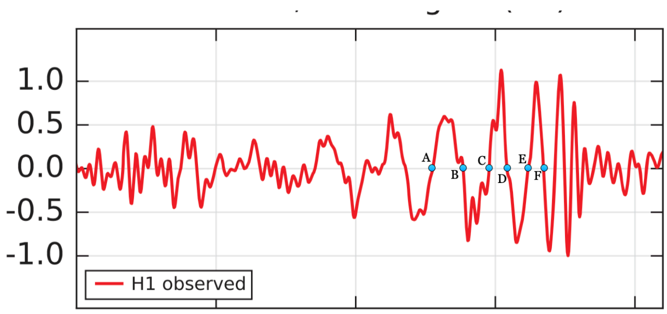

Figure 2: Gravitational wave strain measured for the GW150914 event by the LIGO Hanford interferometer. Adapted from [2].

Bibliography

[1] B. Abbott et al. GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. *Phys. Rev. Lett.*, 119(16):161101, 2017.

[2] B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. *Phys. Rev. Lett.*, 116(6):061102, 2016.