Inhomogeneous cosmology and expansion of the Universe: a fractal exact solution

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based on Cosmai, Fanizza, Sylos Labini, Pietronero, Tedesco
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Outline

- Standard Cosmological Model: assumptions, solutions and issues
- Inhomogeneous Cosmologies: an overview
- Inhomogeneous Cosmologies and Cosmological principle
- Analytical fractal model

Standard Cosmological Model

Basic Assumptions

- General Relativity holds on largest scales
- No special points in the Universe
- On larger scales sources are perfect fluids with pressure and density related by an equation of state

$$p = w\,\rho$$

Standard Cosmological Model
Direct Successes

- Expansion already known since the 20's of XX century
- Blackbody spectrum of Cosmic Microwave Background (CMB) measured in 60's

Standard Cosmological Model
Problems and Solutions

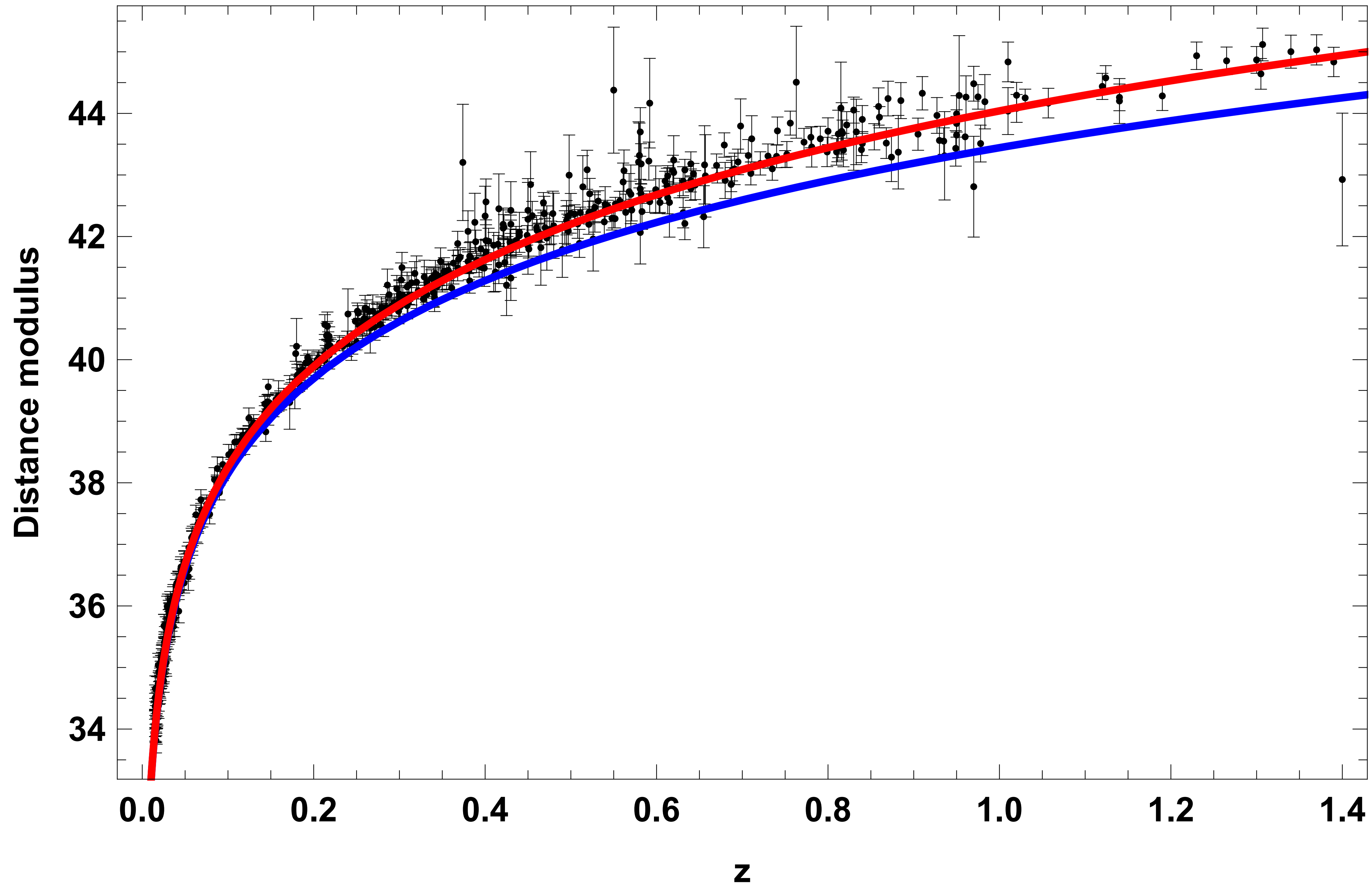
- CMB is too isotropic on larger angular scales: Inflation
- The observed matter is not enough for Structure Formation: Dark Matter
- Local sources are more far than expected: Dark Energy

Standard Cosmological Model

Status of Art

- Given these above-mentioned facts, the Standard Cosmological Model fits data with the aim of a small number of parameters to be tuned with data

- Among all of them, just 3 parameters are needed to study local sources: $\Omega_{\Lambda 0}$ Ω_{m0} H_0



557 SnIa from UNION2 catalog

Dark energy + Dark matter $\Omega_{\Lambda 0} = 0.685$ **and** $\Omega_{m0} = 0.315$

Dark matter

However…

- The parameters we have introduced so far have been determined just by a data fitting

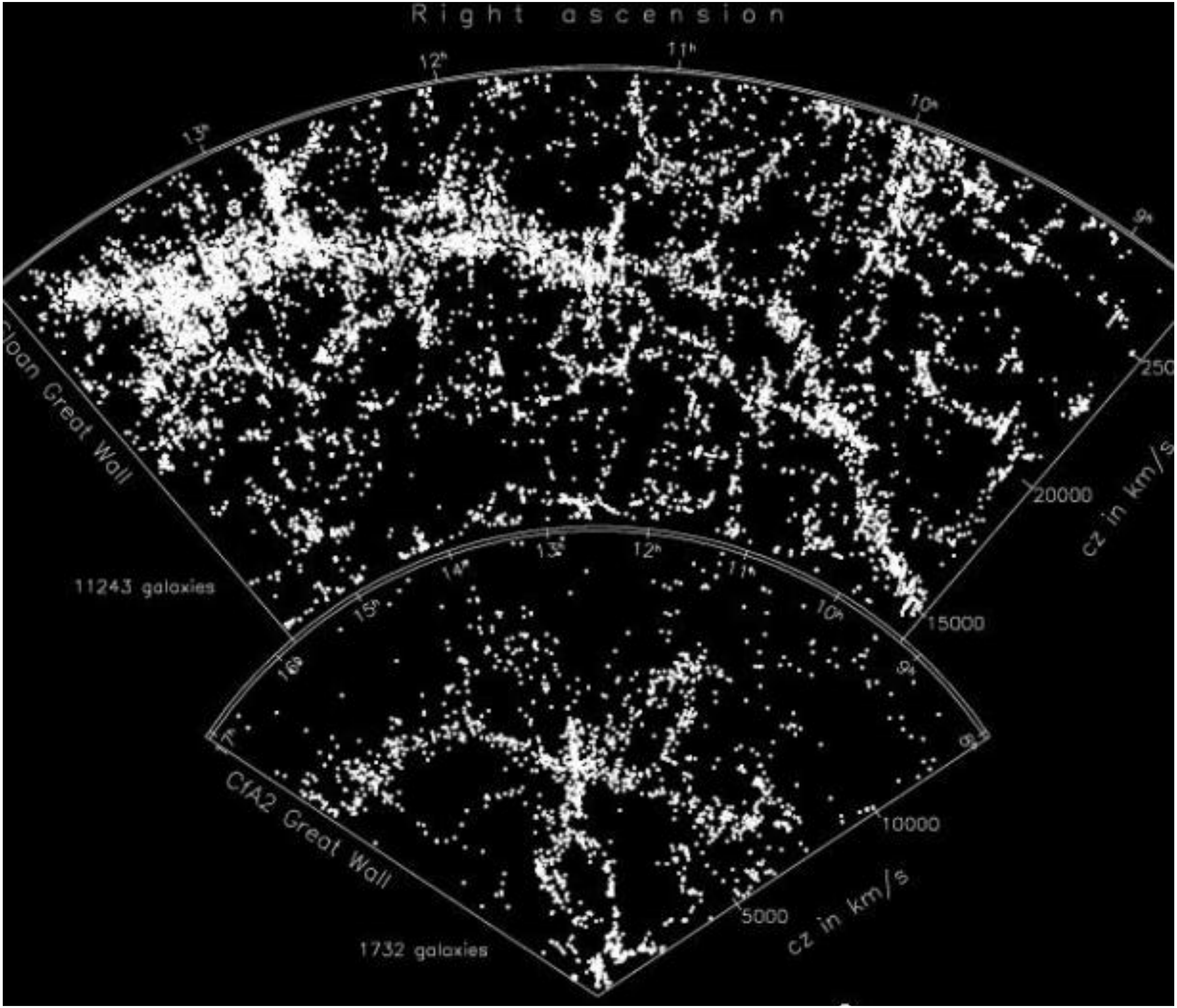
- We do not have idea about what Dark Energy and Dark Matter are made of

- The previous fit does not take into account the presence of structures on cosmological scales

Adding structures in the model

- Indeed we know that structures are not only present but also understood as the growth of the seeds of quantum fluctuations during the inflation

- Homogeneity and isotropy are meant to be valid only in an averaged sense


Right ascension
13h
12h
11h
10h
9h
Sloan Great Wall
11243 galaxies
250
20000
15000
cz in km/s
14h
13h
12h
11h
10h
15h
16h
9h
CfA2 Great Wall
1732 galaxies
10000
5000
cz in km/s

1) What do we mean by “averaged”?

- Observational approach: the averages are taken over the observables of interest
- Geometrical approach: the averages are performed over the metric tensor
- In the latter case, given the second-order non-linear behavior of the Einstein equations, the evolution of an averaged metric tensor is not the same as the average of the evolved metric tensor (Buchert-Ehlers commutation rules and their generalization)

2) How do we treat these deviations from homogeneity?

- Perturbative framework: the observational approach indicates that these perturbations are not enough to change our view on $\Omega_{\Lambda 0}$ (Ben-Dayan, Gasperini, Marozzi, Nugier, Veneziano, 2013)

- Perturbative framework: some still debated results about the geometrical approach indicates that the average over inhomogeneities is not able to mimic the effect of $\Omega_{\Lambda 0}$ (Green, Wald)

- What about exact GR solutions for inhomogeneous models?

Exact inhomogeneous solvable models

- There are very few classes of inhomogeneous analytical solutions of cosmological interest (Szekeres, Bianchi)
- The ones we are about to talk about refer to the so-called Lemaitre-Tolman-Bondi (LTB) models

LTB models (1)

- These models consider a generalization of FRW where the metric tensor is radially inhomogeneous

$$ds^2 = -\,dt^2 + X^2(t,r)\,dr^2 + A^2(t,r)\,d\Omega^2$$

- Moreover the energy-tensor $T_{\mu\nu}$ is still considered to be a perfect fluid, where pressure and density may be radially inhomogeneous as well

- However this assumption allows only matter radial inhomogeneities, thanks to the Bianchi identity $w\,\rho'(t,r) = 0$

LTB models (2)

- In this framework Einstein equations provide the evolution of the metric given by

$$\frac{\dot{A}^2 + k}{A^2} + \frac{2\dot{A}\dot{A}' + k'}{AA'} = 8\pi G\, T_{tt}$$

$$\frac{\dot{A}^2 + 2A\ddot{A} + k}{A^2} = 8\pi G\, T_{rr}$$

$$X = \frac{A'}{\sqrt{1 - k(r)}}$$

-

What do we get for these models?

- The solutions of the field equations for a pure matter energy tensor depend on the choice of the free functions

$$A_0(r) \qquad M(r) \qquad k(r)$$

- The interesting thing is that these solutions can fit the SnIa data (Alnes, Amarzguioui, Celerier, Enqvist)

- However they require that the observer must live too close to the center of a local giant void to fit properly also the CMB data (Alnes, Amarzguioui)

Intermediate summary

- Perturbative approaches rely on the choice of a maximally symmetric background: perturbations are defined with respect to it

- Exact solutions seem to be highly constrained by observations and provide fine-tuning conditions which look unnatural

LTB models vs Standard Cosmological Model

- General Relativity holds on largest scales: true in both descriptions

- No special points in the Universe: true only in Standard Cosmological Model

- On larger scales sources are perfect fluids: true in both descriptions

Restoring the Cosmological Principle for inhomogeneous models

- The appearance of a privileged observer is a strong constraint on the interpretation of these models when I try to describe local structures surrounding the observer

- However what happens if we try to restore the cosmological principle in a statistical sense but we relax the hypothesis of a perfect fluids?

- Let us try to describe the distribution of matter in the Universe as a discrete collection of masses rather than a perfect fluid

- 1) In a more rigorous way, I impose that the distribution of matter in the Universe can be described by a discrete matter source field as

$$\rho(\vec{r}) = \sum_i m_i \delta(\vec{r} - \vec{r}_i) \approx < n(r) >$$

- 2) The observer will occupy one of these points

- 3) From any point of the distribution, the observer experiences an averaged density decay

Physical hints for this model

- A similar behavior on smaller scales has been observed in the statistical analysis of galaxy three dimensional surveys (Antal, Sylos Labini, Vasilyev, Barishev, 2009)

$$< n(r) > \sim r^{-\gamma} = r^{D-3}$$

where $\gamma = 0.9 \pm 0.1$ **for** $0.1 < r < 20\, Mpc/h$

and $\gamma = 0.2 \pm 0.1$ **for** $20 < r < 100\, Mpc/h$

- This behavior seems to hold up to the physical scale given by the volume of the survey

- For r >100 Mpc/h it is still debated whether a transition to homogeneity exhibits

Hybrid model
Starting equations

- Having this in mind, we speculate that this averaged behavior extends on arbitrarily large scales

- Hence, in order to provide an analytical solution within the GR framework, we adopt < n(r) > as the density source for the Einstein equations

$$\frac{\dot{A}^2 + k}{A^2} + \frac{2\dot{A}\dot{A}' + k'}{AA'} = 8\pi G < n(r) >$$

$$\frac{\dot{A}^2 + 2A\ddot{A} + k}{A^2} = 0$$

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Hybrid model
Analytical solution

- Assuming no spatial curvature, we have can get an analytical solution for the non-analytical distribution of matter

$$A(t,r) = A_0(r)\left(1 + \frac{3}{2}\sqrt{\frac{M(r)}{2G\,A_0^3(r)}}\,t\right)^{2/3}$$

- The two free functions can be set as

$$A_0(r) = r$$

- $$M(r) \equiv 4\pi \int_{S_P^3(r)} < n(r) > A'A^2 dr = \Phi\, r^D$$

Hybrid model
Interpretation

- The choice of the free functions is motivated by these assumptions

- 1) $k(r) = 0$ implies that we are assuming no spatial curvature, no matter how deeply inhomogeneous it may be

- 2) $A_0(r) = r$ can be chosen thanks to the freedom that I have in redefining the LTB radial coordinate at a given time. Just a rescaling of distances

- 3) $D = 3 - \gamma$ takes into account the possible deviation from the homogeneous distribution (D = 3), allowing for a fractal behavior of matter on larger scales

Hybrid model
From solutions to observations (1)

- Provided that an analytical solution for the proposed fractal distribution exists, we can analyze the local data for SnIa

- In this regards, it is well known that LTB models allows to write exactly the luminosity-distance as

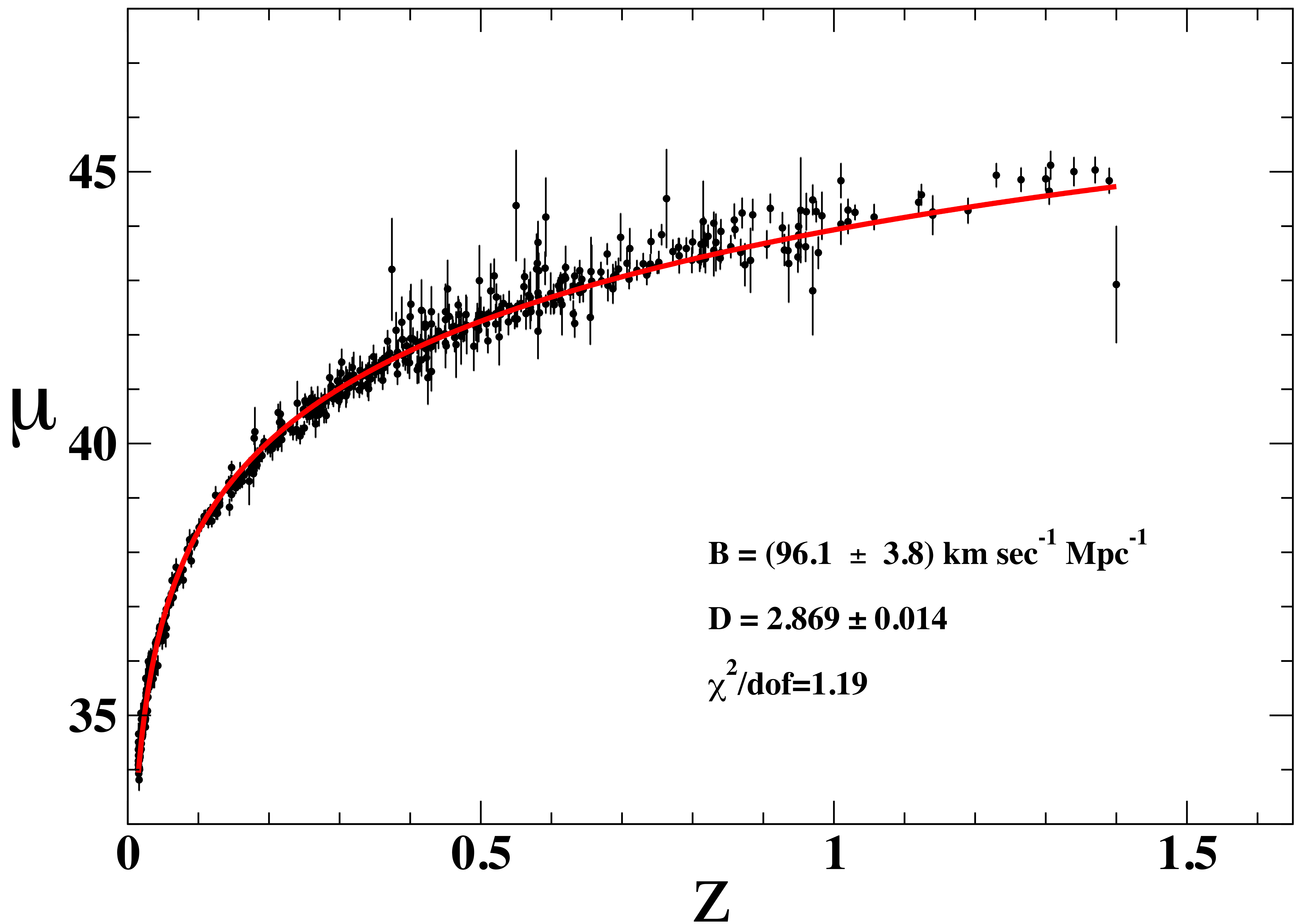
$$d_L(t, r) = (1 + z)^2 A(t, r)$$

Hybrid model
From solutions to observations (2)

- In order to directly compare with data, last step to do is solving the geodesic light-like equations to express the luminosity-distance/redshift relation

$$\frac{dt}{dz} = -\frac{A'(t(z), r(z))}{(1+z)\dot{A}'(t(z), r(z))}$$
$$\frac{dr}{dz} = \frac{1}{(1+z)\dot{A}'(t(z), r(z))}$$

- which can be solved only numerically



45
40
35
μ
0
0.5
1
1.5
Z
B = (96.1 ± 3.8) km sec-1 Mpc-1
D = 2.869 ± 0.014
χ2/dof=1.19

Hybrid model
Interpretation (1)

- The analytical solution we found fits well the local SnIa data (from UNION2 catalog)
- The value for the fractal dimension is not so far (but significantly different) from the homogeneity
- The number of parameters that we have used is the same as the one in the Standard Cosmological model
- The related value for γ from the best-fit analysis gives
  $\gamma = 3 - D = 0.13 \pm 0.02$

Hybrid model
Interpretation (2)

- Despite the fact that we adopt an analytical inhomogeneous model, no special points are present in our description, differently from the local void interpretation of LTB literature

- More interesting, we have assumed no dark energy component in our model: it has been enough to allow a little deviation from perfect homogeneity

Hybrid model
Next steps (1)

- The proposed solution seems to work very good to be an oversimplified model
- More refined suggestions can be done by allowing a transition-to-homogeneity scale in the model. This will add one more parameter but a study about whether and where this transition might happen will be interesting to be done

Hybrid model
Next steps (2)

- Other kinds of cosmological probes must be tested but the road seems to be promising: the stronger constraint from the CMB dipole does not apply here because we do not need for a displacement from the center