When loss destroys the Hilbert space: Purcell factors and strong coupling in plasmonic systems

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What to take from this talk

- Every band structure is complex
- Material dispersion and loss let conventional theory fail
- Ignoring imaginary parts (e.g. frequency or wave number) is dangerous

Manipulating emission dynamics for:

• Single-photon sources



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[Science Adv. 2, e1501168 (2016)]

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- Strongly coupled graphene or van-der-Waals materials







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- For continuous spectra: $\rho(\omega) \simeq \frac{\partial \alpha}{\partial \omega}.$

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S. M. Barnett et al. J. Phys. B 29, 3763 (1996)

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- Replace scalar product with bilinear form:

$$(\phi,\psi) = \int_{\mathrm{WSC}} \mathrm{d}^3 r \begin{pmatrix} \vec{H}_{\phi} \\ E_{\phi} \end{pmatrix} \mathcal{W}(\vec{r}) \begin{pmatrix} \vec{H}_{\psi} \\ E_{\psi} \end{pmatrix}.$$

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Define adjoints of operator *L* and modes via (*L*[‡]φ[‡], ψ) = (φ[‡], *L*ψ).
For canonical weight function: ψ[‡](*r*, *k*, ω) = ψ(*r*, -*k*, -ω).

• Group velocity along \hat{z} :

$$v_{z} = \frac{\int_{\rm WSC} d^{3}r \ \hat{z} \cdot (\vec{E^{\dagger}} \times \vec{H} + \vec{E} \times \vec{H^{\dagger}})}{\int_{\rm WSC} d^{3}r \ (\vec{E^{\dagger}} \frac{\partial(\omega\varepsilon)}{\partial\omega} \vec{E} + \vec{H^{\dagger}} \frac{\partial(\omega\mu)}{\partial\omega} \vec{H})}.$$

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• Local density of states connected to Green function:

$$\rho(\vec{r},\omega) = -\frac{2\omega}{\pi c^2} \Im \left\{ \mathcal{G}_{\omega}(\vec{r},\vec{r},\omega) \right\}$$

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$$\mathcal{G}(\vec{r},\vec{r}',\omega) = \frac{1}{V_{\rm BZ}} \sum_{n} \int_{\rm BZ} \mathrm{d}^{3}k \; \frac{\Psi_{n\vec{k}}(\vec{r}) \otimes \Psi_{n\vec{k}}^{\dagger}(\vec{r}')}{\omega - \omega_{n\vec{k}}}$$

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• With dispersion: Maxwell's equations form nonlinear eigenvalue problem:

$$\mathcal{L}(\omega)\Psi(\vec{r}) = \omega\Psi(\vec{r}).$$

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• Fix: Auxiliary frequency-like eigenvalue λ :

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• Generalized Green function with dispersion (ω fixed):

$$\mathcal{G}_{\omega}(\vec{r},\vec{r}',\lambda) = \frac{1}{V_{\mathrm{BZ}}} \sum_{n} \int_{\mathrm{BZ}} \mathrm{d}^{3}k \; \frac{\Psi_{n,\vec{k},\omega}(\vec{r}) \otimes \Psi^{\dagger}_{n,\vec{k},\omega}(\vec{r}')}{\lambda - \lambda_{n,\vec{k},\omega}}$$

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