Proton rms-radius from (e,e): a serious oversight

Ingo Sick

Observation

large scatter of results for rms-radius R: $0.84 \div 0.92 fm$
occurs even when using same data
indicates model-dependence of fits

Observation

scatter particularly large for q-space parameterizations
fits done without consideration of $\rho(r)$
actually: most $G(q)$'s do not correspond to a density, have no Fourier transform!

To illustrate difficulties with $G(q)$-fits: discuss example

Fit of Bernauer data for $q < 2fm^{-1}$

includes all data sensitive to R

Parameterization [m/n]Pade: $G(q) = (1 + \sum^m a_i q^{2i})/(1 + \sum^n b_j q^{2j})$

successful in fits up to largest q_{max}: Kelly, Arrington, IS, ...

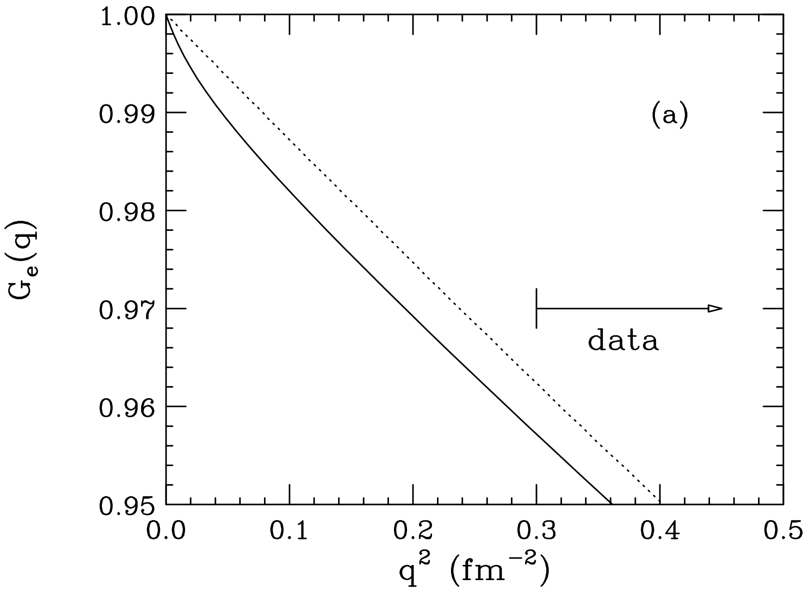
Pade with $m = 1$, $n = 3$

gives χ^2 as low as bestfit of Bernauer

has none of frequent diseases: poles, unphysical $q = \infty$ limit

Yields $R = 1.48 fm$!

Reason: curvature of $G(q)$ at *very* low q, below q_{min} of data



.... Standard fit, R∼0.88

——— Pade

Above $0.2 fm^{-2}$ Pade and standard fit differ by a constant 0.5%

note expanded scale

Pade and standard fit have same χ^2 as data floating

How does Pade generate $R = 1.48 fm$?

a_1 **and** b_1 **are coupled**

both large can produce behavior shown in figure

Is $R = 1.48 fm$ reasonable?

large coefficients cannot be excluded, parameters are not physical

some fits in literature have huge coefficients

[1/3]Pade as valid as any other q-space parameterization!

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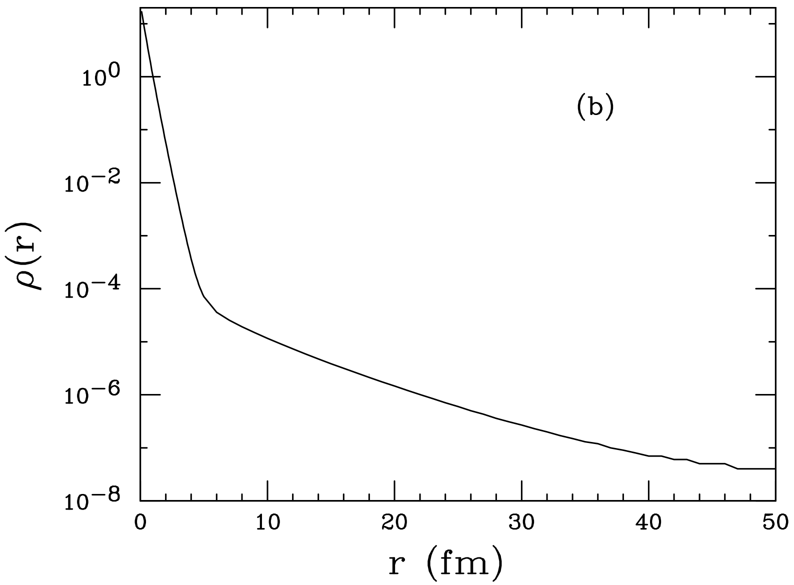
[1/3]Pade as valid as any other q-space parameterization!

The big difference

[1/3]Pade *does* correspond to a density

simple look at $\rho(r)$ eliminates fit as unphysical

note radial scale: $0 - 50 fm$



Elimination only possible because Pade does have FT

without considering $\rho(r)$ would not know about disease

Could published $G(q)$'s *without* look at $\rho(r)$ have similar problems?

(they do! see below)

Origin of problem

$\rho(r)$ ignored in most analyses

R not obtained from $\int \rho(r)\ r^4\ dr$

R **obtained from slope of** $G(q=0)$ **despite obvious problems:**

– $q = 0$ not measurable

– must extrapolate (always dangerous)

– doubly dangerous as need *slope* of extrapolated $G(q)$

– near $q = 0$ finite size effect $q^2R^2/6$ *very* small $\rightarrow$ problems with syst. errors

Why use only $G(q)$**?**

$G(q) = \mathrm{FT}(\rho(r))$ strictly valid for non-relativistic recoil only

would need relativistic corrections

Hope: can ignore this 'complication' if restrict attention to $q = 0$

.... which is an illusion as must extrapolate from *finite* q

Must reconsider approach, only $\rho(r)$ can exhibit diseases of fit

Do relativistic corrections exclude consideration of $\rho(r)$?

not at all!

relativistic corrections have been calculated

Licht70, Mitra77, Ji91, Holzwarth96

consequences are understood

1. Electron sees moving proton

must describe scattering in Breit frame

can be taken into account by using in FT $\tilde{q}$ instead of q

$$\tilde{q} = q/\sqrt{1 + q^2/4M^2}$$

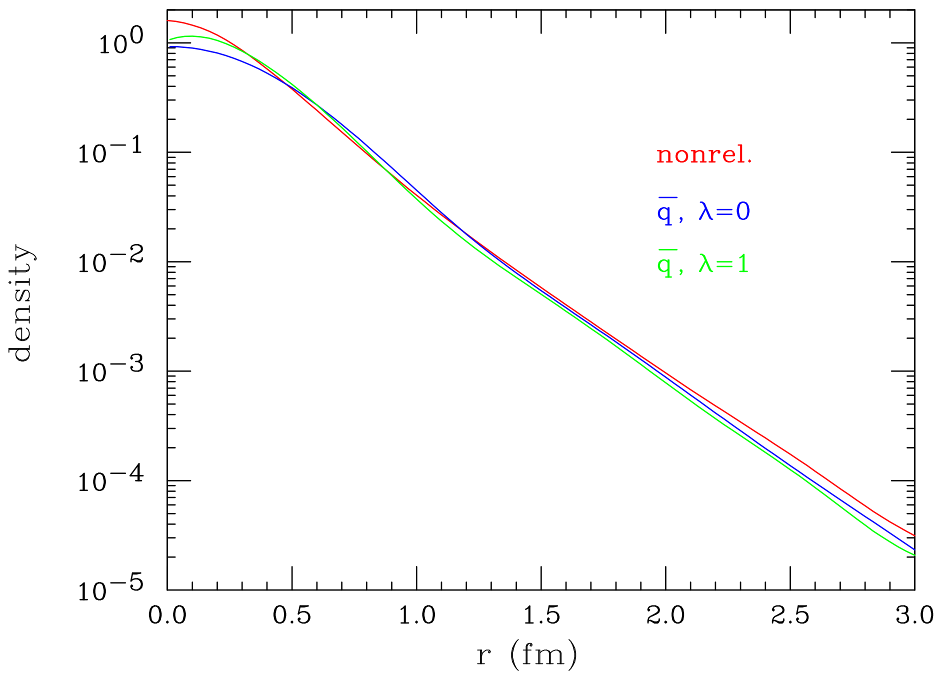
2. For composite relativistic systems additional correction

$$G \to \tilde{G} = G(1 + q^2/4M^2)^\lambda$$

different theories give, for charge-form factor, $\lambda = 0$ or 1

Numerical effect: start from Pade fit of *world* data $q < 10 fm^{-1}$, calculate

- $\rho(r)$ non-relativistically
- $\rho(r)$ using $\tilde{q}$
- $\rho(r)$ using $\tilde{q}$ and $\lambda = 1$



Result

significant change at $r \sim 0$, minor change of shape of $\rho(r > 1 fm)$
irrelevant ambiguity due to $\lambda = 0, 1$
no effect upon R and $q = 0$ slope

Despite relativistic corrections shape at large r remains well-defined

What do we know about $\rho(r)$ at large r?

1. Cloudy bag-type models

$r < 1 fm$ complicated quark/gluon structure

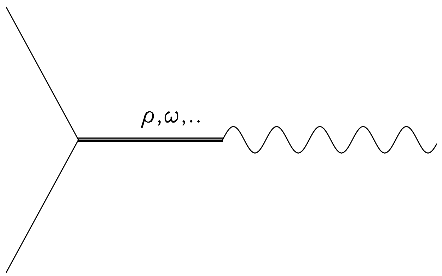
$r > 1 fm$ dominated by Fock component with lowest separation energy: n+π^+
asymptotic wave function of pion given by $W_{-\eta,3/2}(2\kappa r)/r$
can be used to calculate *shape* of $\rho(r)$

only input: quantum-mechanics, π-separation energy

used extensively for $A \geq 2$

2. Vector Dominance Model

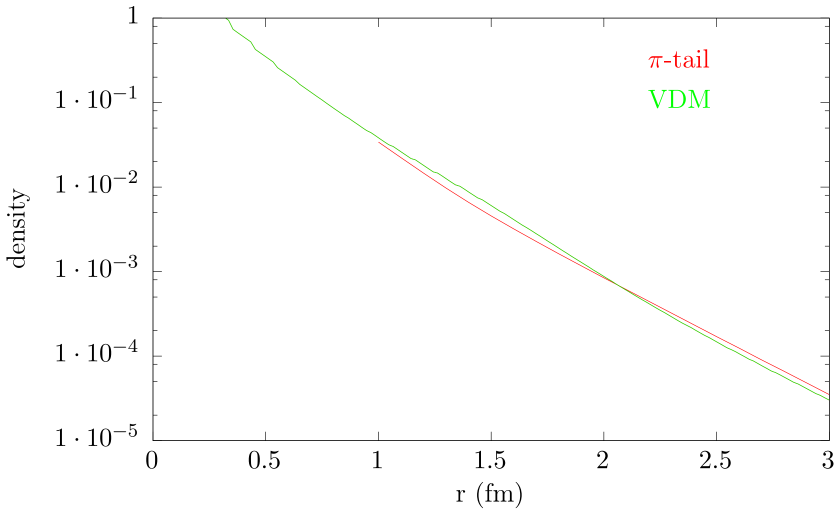
basic assumption of VDM



using known vector mesons and coupling constants

using dispersion relations to calculate 2π *ect* contributions (longest range)
Ina Lorenz, Bonn group

Comparison



shape of large-r densities very similar

given by *understood physics*

not affected by rel. corrections

shape should be $\pm$ respected in fits of data

Shape-constraint most helpful as $r > 1fm$ contributes ∼50% to R

see review in *Atoms* 6 (2018) 2

Here: consider *much more elementary constraint*

$\rho(r > 3.5fm) \sim 0$ for practical purposes, $\qquad \rho < 10^{-5}\rho(0)$

this minimal (common sense) constraint *is* important when aiming at R!

Take seriously as rel.corr. do not generate/remove (apparent) contribution at $r > 3.5fm$

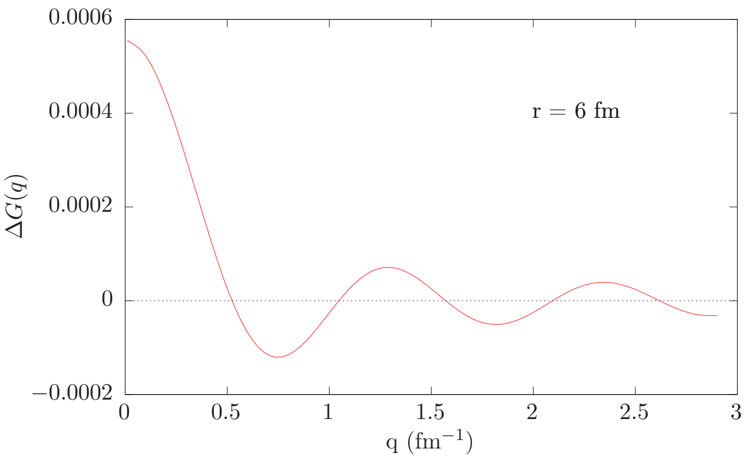
Why is $\rho(r)$ at large r so important?

large r have large weight in calculation of R

give largest contribution at *small* q: $G(q) = \int sin(qr)/qr \; \rho(r) \; r^2 \; dr$

Example: $\Delta G(q)$ for charge ΔQ at $r = 6fm$ producing $\Delta R = 1\%$

biggest contribution at $q < 0.5 fm^{-1}$, region is not covered by data



effect upon $G(q)$ at $q > 0.5 fm^{-1}$

$\Delta G(q) < 0.0001$

$\Delta\sigma/\sigma < 0.0002$ not measurable by far!

5 times larger $\Delta\sigma$ not measurable either

$\longrightarrow$ amplitude of $sin(qr)/qr$ term for large r not determined

contributions from $r > 3.5 fm$ add noise (model dependence) to R-determination

Important question: do published fits respect $\rho(r > 3.5fm) = 0$?

Parameterizations that do correspond to a density

$\rho(r > 3.5fm) = 0$ easy to enforce/verify

can discard fit if $\rho(r > 3.5) \neq 0$

What about all these $G(q)$'s that do *not* correspond to a density?

..... the vast majority of published $G(q)$'s

could imply $sin(qr)/qr$ components corresponding to $r > 3.5fm$!

would give unphysical contributions to R

which would be poorly constrained by data

How can be verified?

how can make sure that *most elementary property* $\rho(r > 3.5fm) = 0$ is respected?

Can be done by borrowing old idea from F. Lenz Z. Physik 222 (1969) 491

who studied model-independent information determined by (e,e) data

Model-independent information from (e,e):

Is contained in first moment function

$$T(Q) = \int_0^Q r(Q')dQ'$$ with Q = integrated charge between radii 0 and r

All $\rho(r)$ with same $T(Q) \pm \delta T(Q)$ give same $\sigma \pm \delta\sigma$

Convenient representation

Sum of delta-functions

$$\rho(r) = \sum \frac{p_i}{r_i^2}\, \delta(r - r_i) \qquad \longrightarrow \qquad T(Q_j) = \sum_{i=1}^{j} p_i\, r_i$$

With enough δ-functions at $0 < r_i < r_{max}$ can represent $T(Q)$ to any accuracy desired

Consequence: can represent $G(q)$ with $\sum p_i\ sin(qr_i)/(qr_i)$

Basic idea

decompose $G(q)$ into $sin(qr_i)/qr_i$ - components

allows localization of charge in r *without* need for FT

Test of published $G(q)$'s

use $q_{max} = 1.5 fm^{-1}$ (covers range sensitive to R)

generate pseudo-data from $G(q)$

fit with $\sum p_i \; sin(qr_i)/qr_i$

select r_i's uniformly distributed over range 0 ... $7fm$

- have tried several r-ranges for $r > 3.5$ contributions
- range 3.5...$7fm$ covers most relevant region

goal: gives $r > 3.5$ significant overall-contribution to R?

to avoid over-fitting with correlated p_i:

- constrain $p(r_i > 3.5fm)$ to either > 0 or < 0

check $\sum p_i$ for $r_i > 3.5fm$

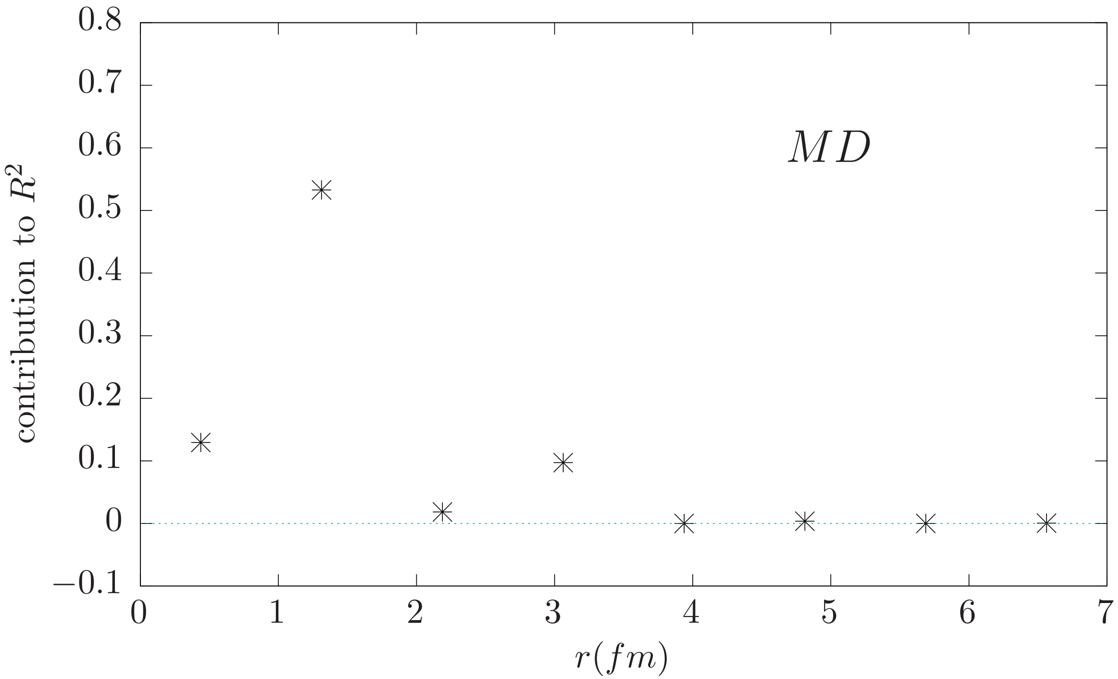
Results

1. For G's corresponding to density with $\rho(r > 3.5fm) \sim 0$

fits used: MD, Pade, Laguerre, Borisyuk, VDM Mergel, Graczyk

find contribution to R^2 of $|\sum p_i|$'s for $r > 3.5fm$ typically 0.8% (discretization noise)

Example: MD

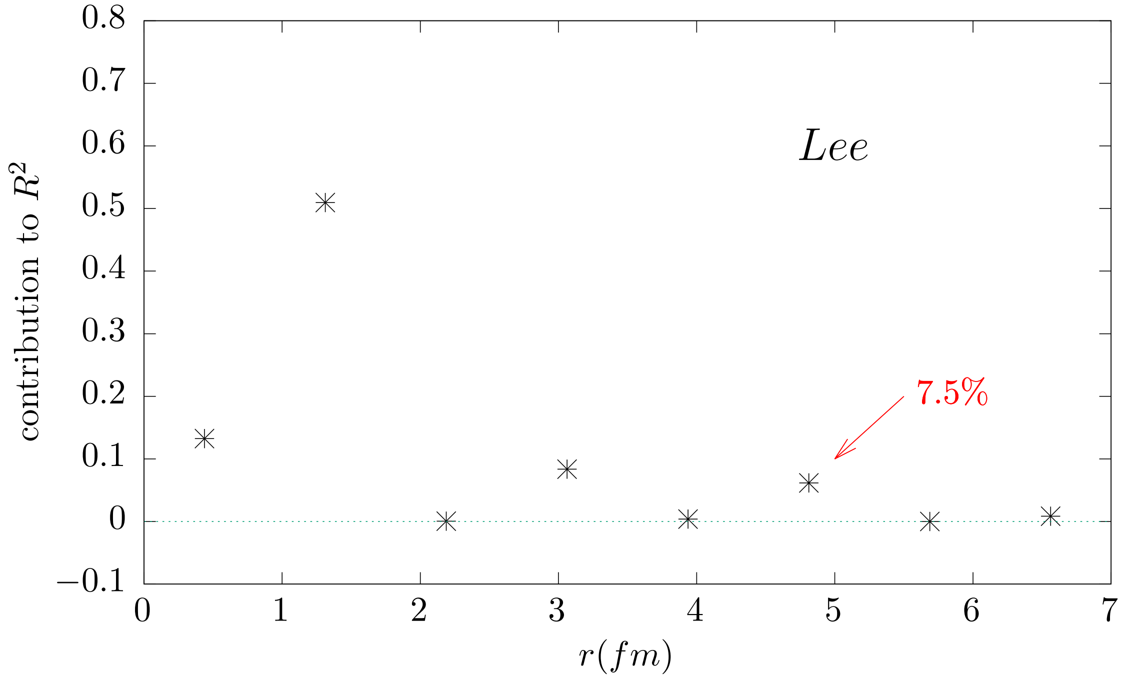


2. For fits $G(q)$ *not* corresponding to $\rho(r)$

fits used: Lee, Paz, HH, polynomial Bernauer, pol. Griffioen, inv. polynomial

contribution to R^2 of $r > 3.5 fm$ up to 20%, typical contribution 10%!

Example: Lee+Arrington, $R = 0.916 \pm 0.024 fm$



these contributions are unphysical, and model dependent as not constrained by data

their contribution explains scatter of results for R

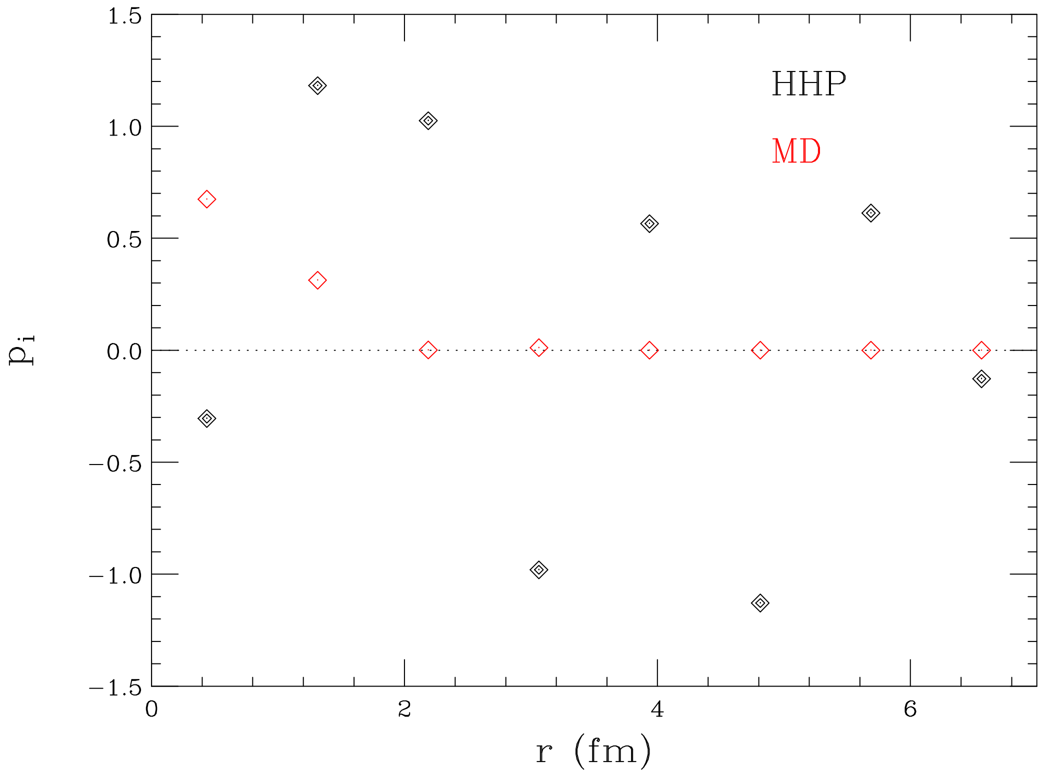
3. Special case: polynomials in q^2

get good fit only when at least one $p_i < 0$ for $r > 3.5 fm$
confirms old insight of disease of $\sum p_i q^{2i}$

4. Very special case: Horbatsch, Hessels, Pineda

extremely small $\langle r^4 \rangle$, disagrees with data (and common sense)

can only be fit with weird combination of p_i's



	$r^4/(r^2)^2$	$r^6/(r^2)^3$
Dipole	2.50	11.6
VDM Lorenz	2.62	13.5
HHP	1.25	14.5

..... but HHP can anyway be ignored due to very poor χ^2

Conclusion

Parameterizations *without* $\rho(r)$ often imply $sin(qr)/qr$ contributions from $r > 3.5fm$

they give significant contributions to R which

– depend on model used to parameterize $G(q)$

– are not constrained by data as main effect occurs at $q < q_{min}$

These unphysical contributions add 'noise' to R-determinations, *i.e.* model-dependence

can be avoided by parameterizing $\rho(r)$ instead of $G(q)$

and imposing $\rho(r > 3.5fm) = 0$

...... fixes problem of unphysical contributions to R even if $\rho(r)$ is of no interest

...... fixes the major disease of R-determinations from standard $G(q)$-parameterizations

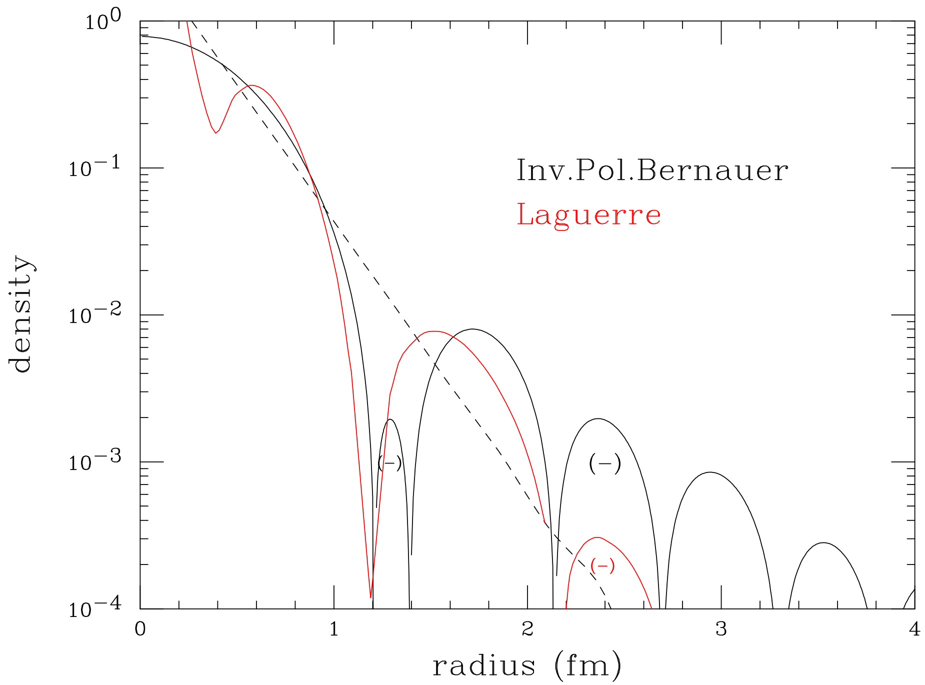
As an aside

look at $\rho(r)$ also useful for

- evaluating plausibility of fit
- locating potential problems with data

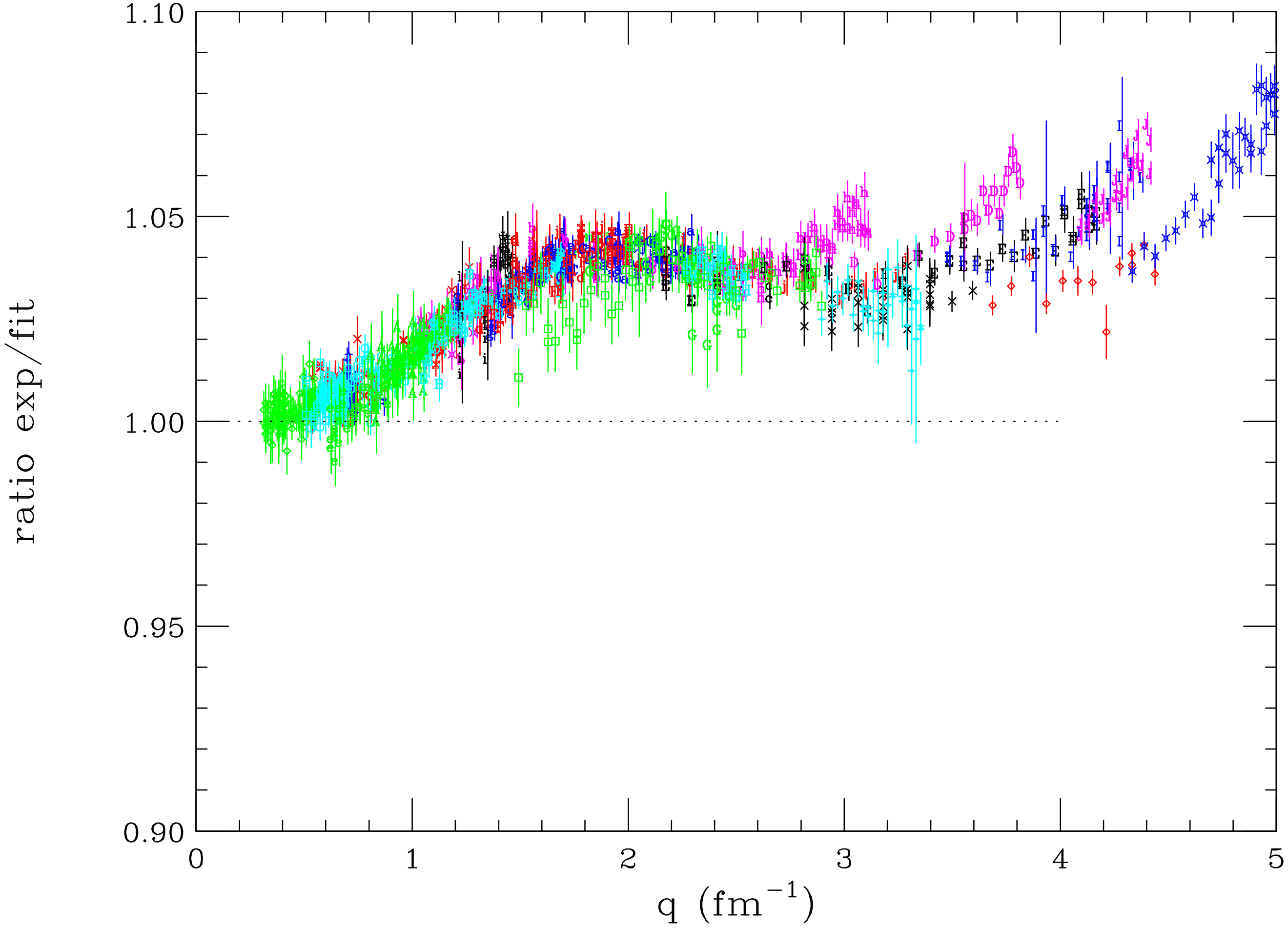
Example: Bernauer data $\rightarrow$ important dip at $r \sim 1.3 fm$:

contradicts physics understanding



Origin of dip: difference data world ↔ Bernauer

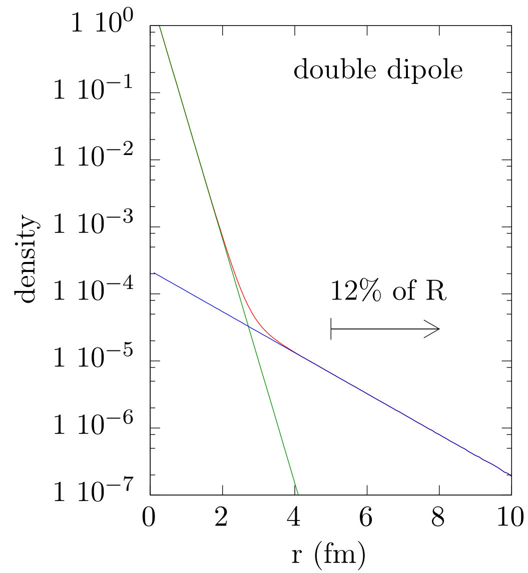
Ratio of cross section Bernauer/(Fit world) using Laguerre fit



One more parameterization of $G(q)$ *without* consideration of ρ

double dipole fit of Bernauer

big contribution from unphysical r's



Can be avoided using *physics understanding*

Hierarchy of physics constraints

1. Enforce $\rho(r > 3.5 fm)$=0

   avoids unphysical contributions

   reduces model dependence

2. Ensure $\pm$exponential large-r fall-off

   $\sim W(r)/r$ behavior of *any* bound-state wave function

   easily done using MD, Laguerre, ... with coefficient from physics

   approximately incorporates physical behavior in parameterization

   $\rightarrow$ reduces δR

3. Add large-r shape of $\rho(r)$ from *physics*

   shape from VDM, or π^++n tail

   get smallest δR, safest R

Popular parameterization $G(q) = \sum a_i \langle r^{2i} \rangle q^{2i}$

Claim: yields model-independent moments. Illusion!

Consider low-q data fixing 2 independent parameters:

2-parameter Fermi: $\rho(r) = 1/(1 + e^{(r-c)/z})$, with $\langle r^2 \rangle, \langle r^4 \rangle$ = function of c, z
power expansion: $G(q) = 1 - q^2 \langle r^2 \rangle /6 + q^4 \langle r^4 \rangle /120$

Both parameterizations make strong assumptions on $\langle r^n \rangle$ for $n > 4$

Fermi density: $\langle r^n \rangle$ fixed by analytical shape
Power series: $\langle r^n \rangle = 0$

The main difference:

for 2pFermi: moments could be sensible
for q^n series: moments guaranteed to be wrong
there is NO density that has $\langle r^2 \rangle, \langle r^4 \rangle \neq 0$ and all higher moments $= 0$!
formally: power series has diverging Fourier transform

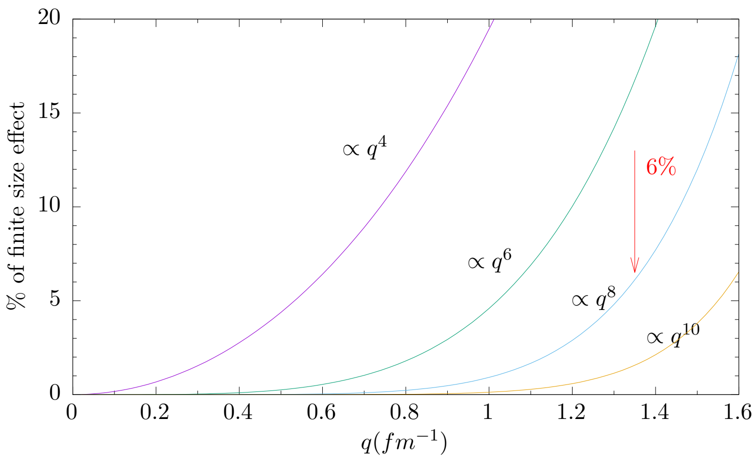
Consequence: power series expansion has worst possible model assumptions!!

Robust extraction of δR

Yan etal: models $\rightarrow$ pseudo-data $q < 1.35 fm^{-1}$, error bars $\delta G/G \leq 1\% \sim$ PRAD
find that [1/1]Pade gives good χ^2, reproduces input-R, yields $\delta R/R \sim 1\%$

- according to fit 2 parameters enough
- how many are needed according to *knowledge* on G?

Contributions of $\langle r^{2n} \rangle$ to finite size effect FSE $= 1 - G(q)$



calculated using $\langle r^{2n} \rangle$ of Bernauer
fit up to $5 fm^{-1}$
at $1.35 fm^{-1}$ contribution $\langle r^8 \rangle \sim 6\%$

For $\delta R/R \sim 1\%$ need δFSE/FSE$\sim 2\%$ $\rightarrow$ need terms up to $\langle r^8 \rangle$ $\rightarrow$ 4 free parameters

[1/1]Pade has only 2 free parameters, remainder fixed by Pade shape
$\rightarrow$ model dependence, vast underestimate of δR

Apparent low dispersion of R from Pade fits of models:
due to low dispersion of $\langle r^{6,8} \rangle$ as models have been $\pm$ fit to (e,e)

Default of strategy easily explained using power-expansion fit

Above procedure equivalent to: choose q_{max}, fit with $\langle r^2 \rangle$, $\langle r^4 \rangle$

fix $\langle r^{n>4} \rangle$ using "independent evidence" (or model-$G(q)$)

get small δR

More extreme, but in same vein:

fix $\langle r^{n>2} \rangle$ using "independent evidence", subtract

fit with $\langle r^2 \rangle$, get δR

$\rightarrow \delta R/R$ small as given by $\delta G/G$ near q_{max}

obvious cheat as R should be given by $G(q)$ near $q = 0$!!

Should have been obvious

First step:

choose q_{max} such that $\langle q^4 \rangle$ contribution negligible

fit with $\langle r^2 \rangle$

find large δR for data of realistic δG

Second step:

choose q_{max} such that $\langle r^6 \rangle$ contribution negligible

fit with $\langle r^2 \rangle$, $\langle r^4 \rangle$

find similar δR as additional data used to fix $\langle r^4 \rangle$

Third step: behaviour repeats for all higher moments