ELBA 2019

Phenomenological analysis Extraction of partonic unpolarized TMDs and Sivers functions

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3DSPIN MAPPING THE PROTON IN 3D

24 June 2019

Outline

- Introduction to TMDs and phenomenology of TMDs
- Extraction of Sivers function
  - Relation between experimental observables and TMDs
  - Relation between unpolarized TMDs and Sivers distribution
  - Our choices for parametrization
  - Overview of experiments and data considered
  - Results and comparisons
- Outlook

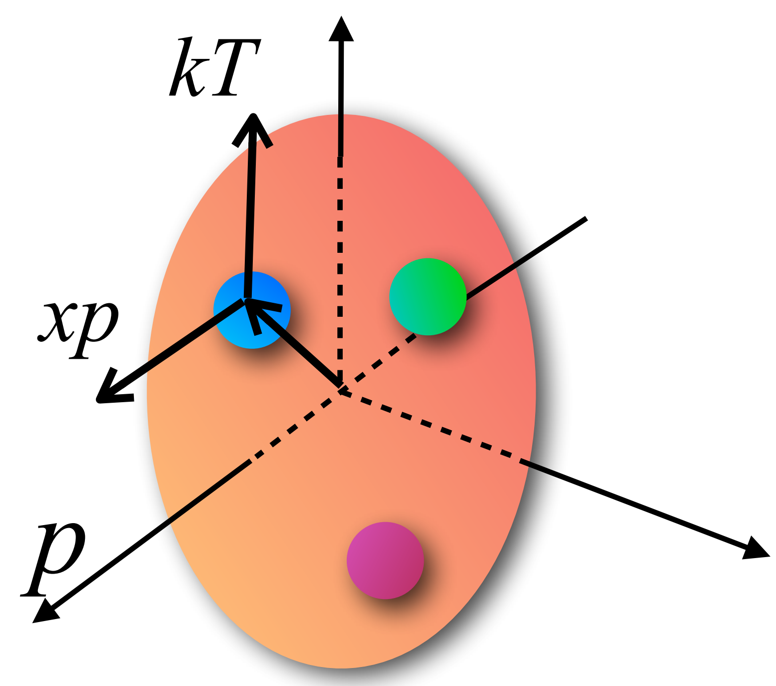
Multidimensional imaging of nucleon

Momentum and Position: how partons move inside the nucleon and distribution dependence on x

Flavor: how different flavors affect partonic distributions.

Spin: correlation between parton movement (OAM) and overall nucleon properties (missing spin budget).

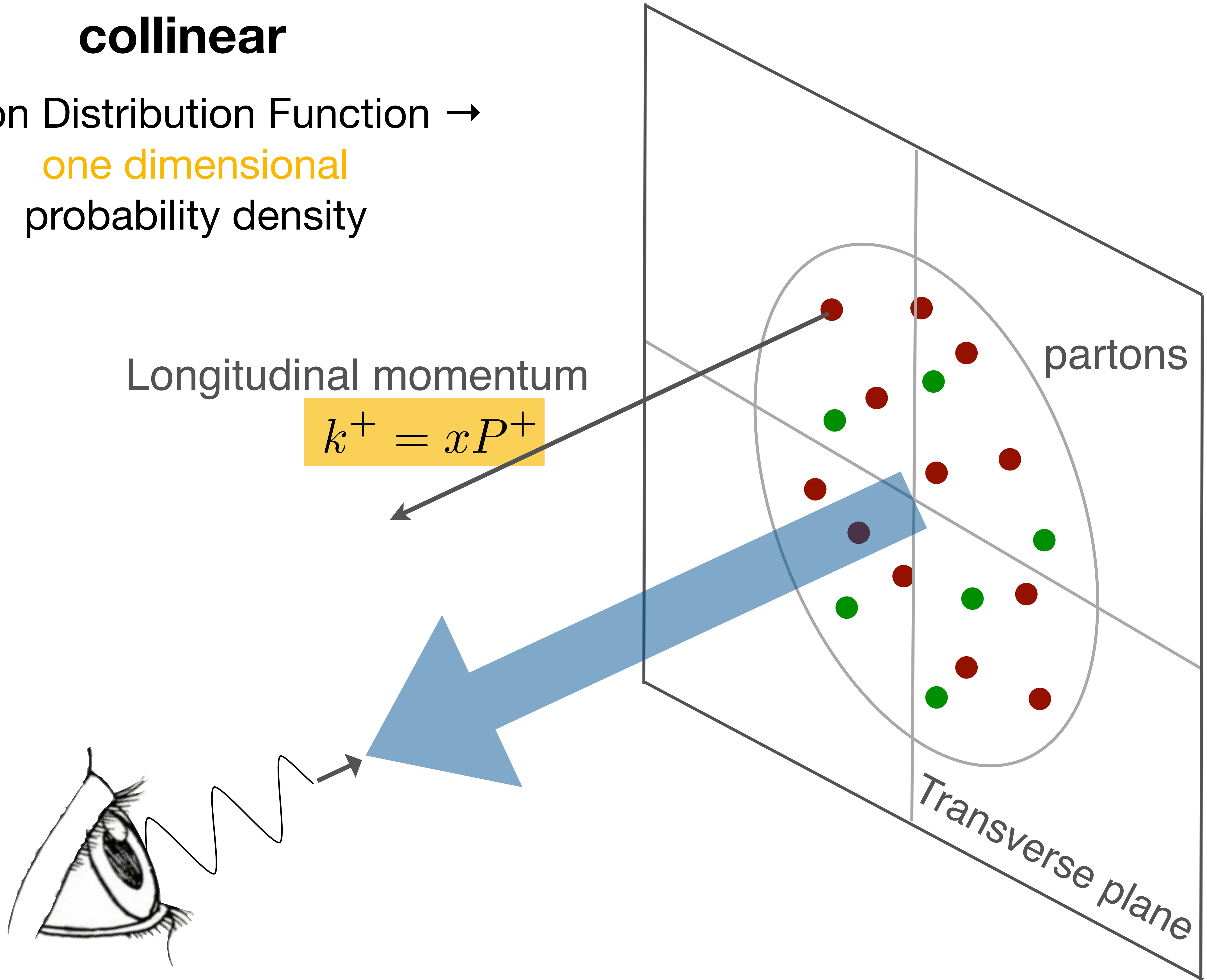
Information summarized as
Parton Distribution Function



1D picture of the nucleon: PDF

collinear

Parton Distribution Function →
one dimensional
probability density

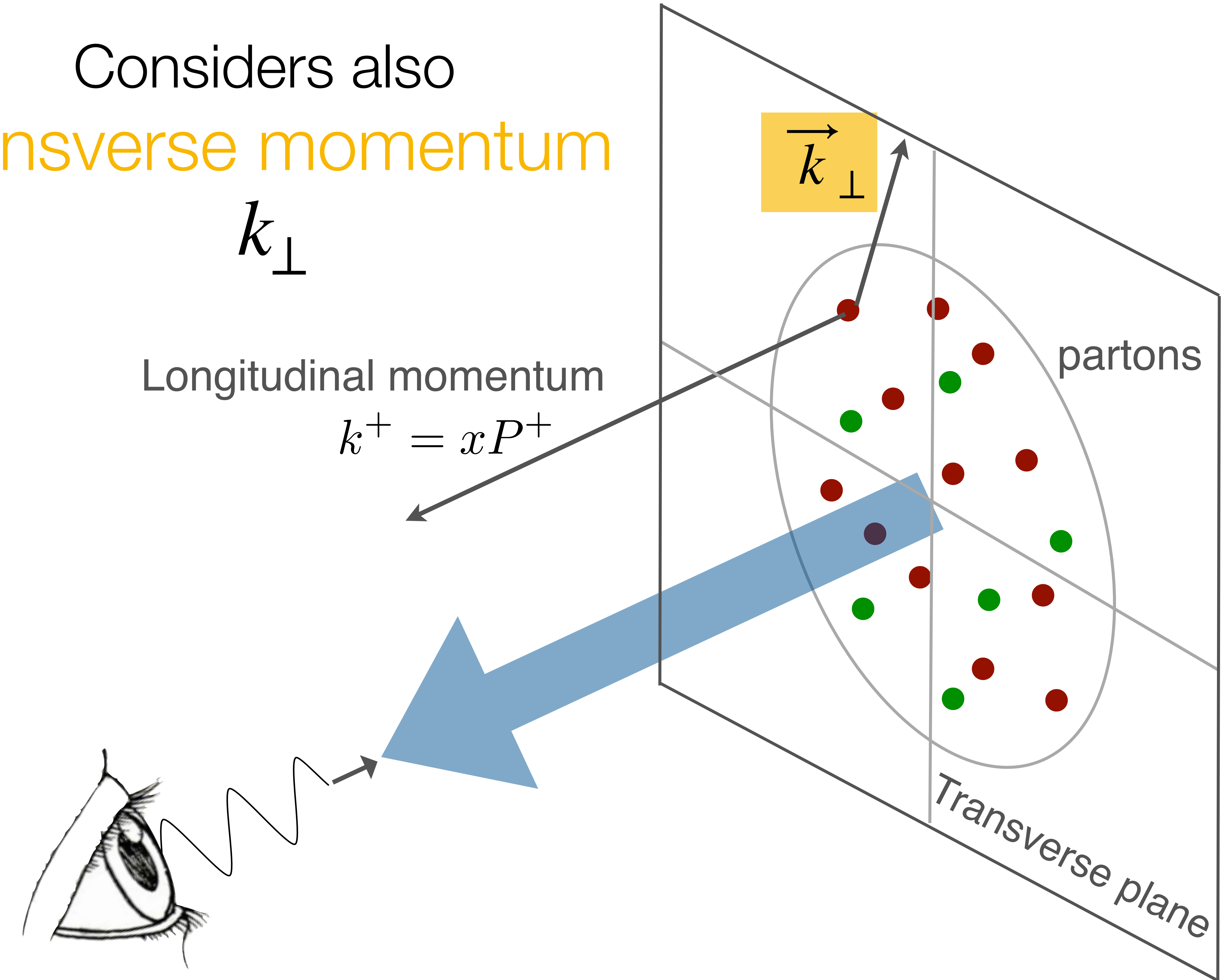


3Dimensional structure

Considers also

transverse momentum

$k_\perp$



Transverse Momentum Distributions

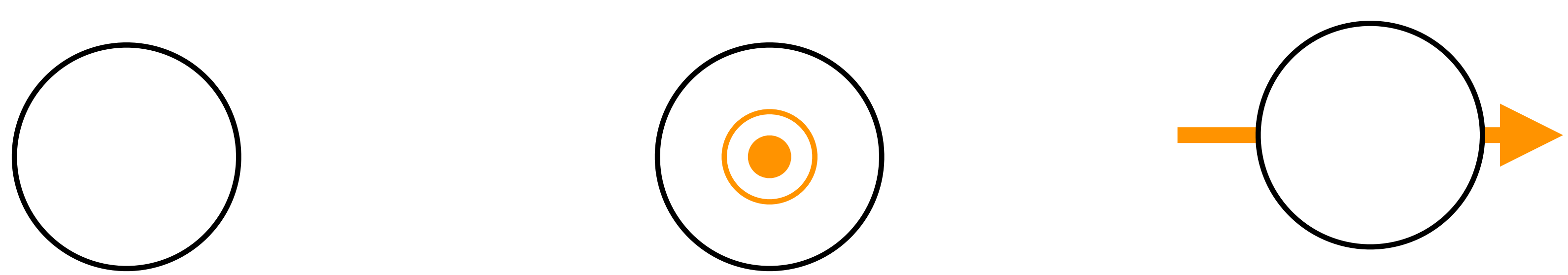
quark polarization

nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Unpolarized **Longitudinal** **Transverse**

Momentum direction
perp. screen



Transverse Momentum Distributions

Unpolarized

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$

Sivers function

Transverse Momentum Distributions: TMD PDF

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

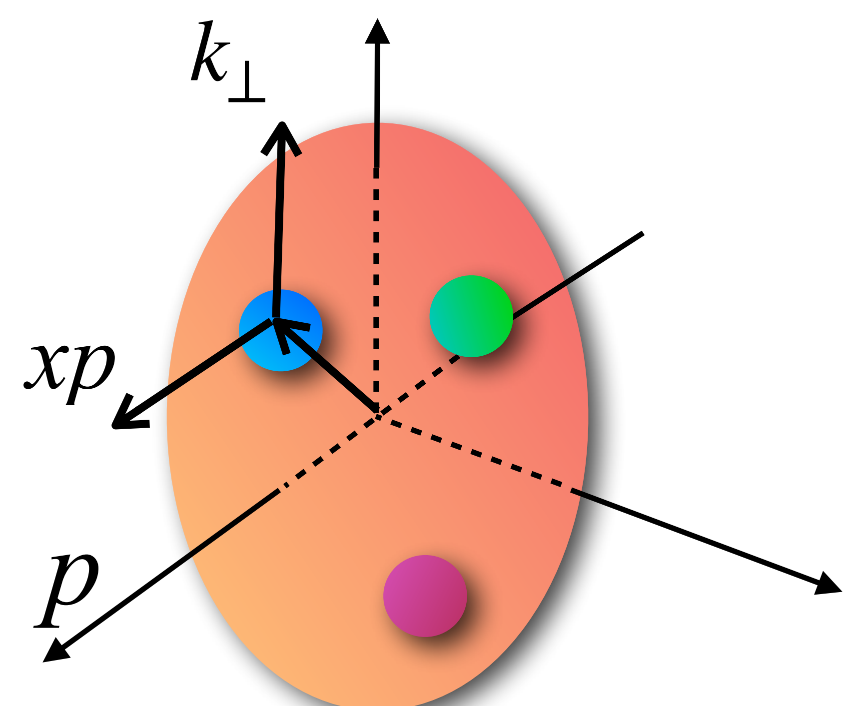
Sivers function

dependence on:

longitudinal momentum fraction x

transverse momentum $k_\perp$

energy scale



Phenomenology of polarized TMDs

⇒ presence of a non-zero Sivers function $f_{1T}^{\perp}$ will induce a dipole deformation of f_1

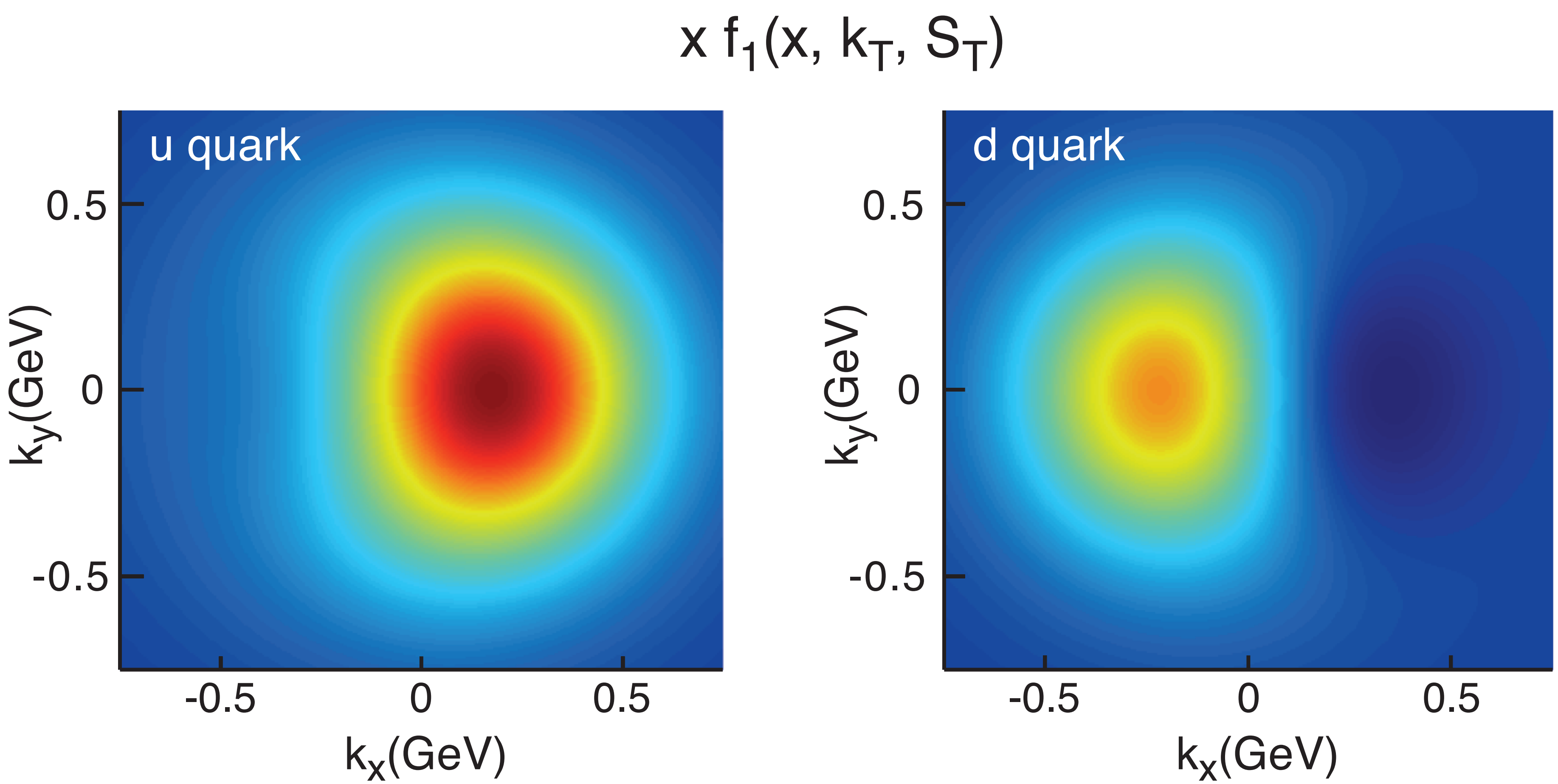
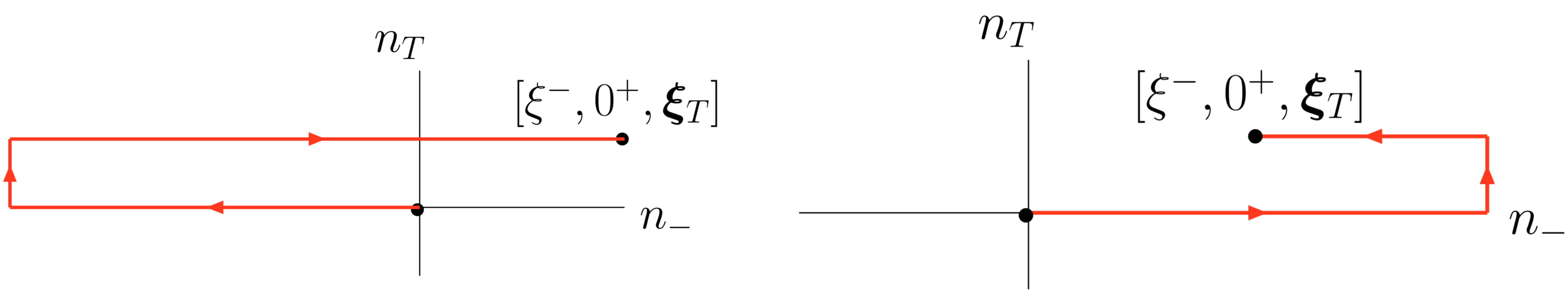


Figure 2.13: The density in the transverse-momentum plane for unpolarized quarks with $x = 0.1$ in a nucleon polarized along the $\hat{y}$ direction. The anisotropy due to the proton polarization is described by the Sivers function, for which the model of [77] is used. The deep red (blue) indicates large negative (positive) values for the Sivers function.

[EIC White Paper]

Sivers function sign change

vanishing Sivers function? → Final state interactions and Wilson lines to consider



DY
past-pointing

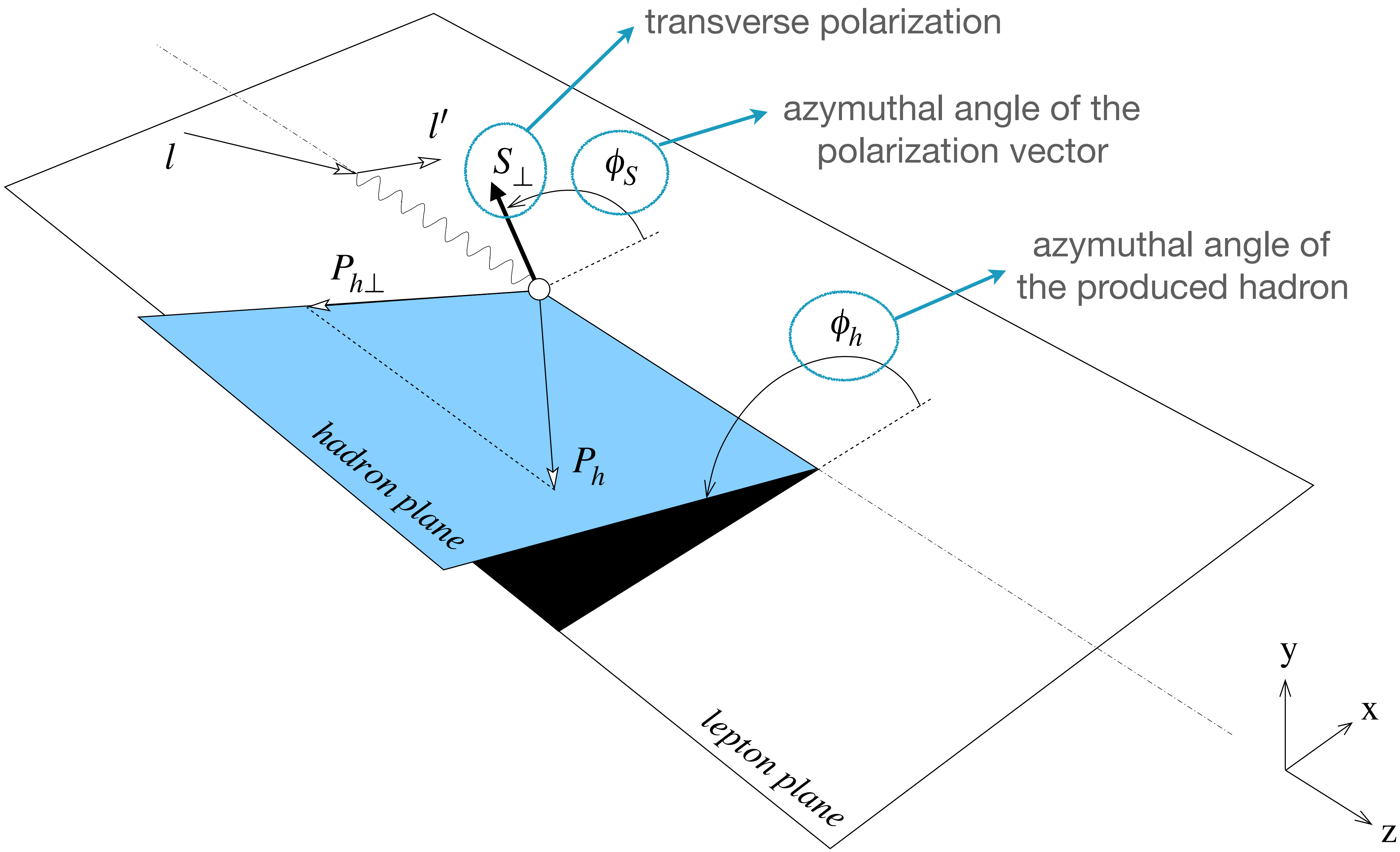
SIDIS
future-pointing

Sign change in Sivers function

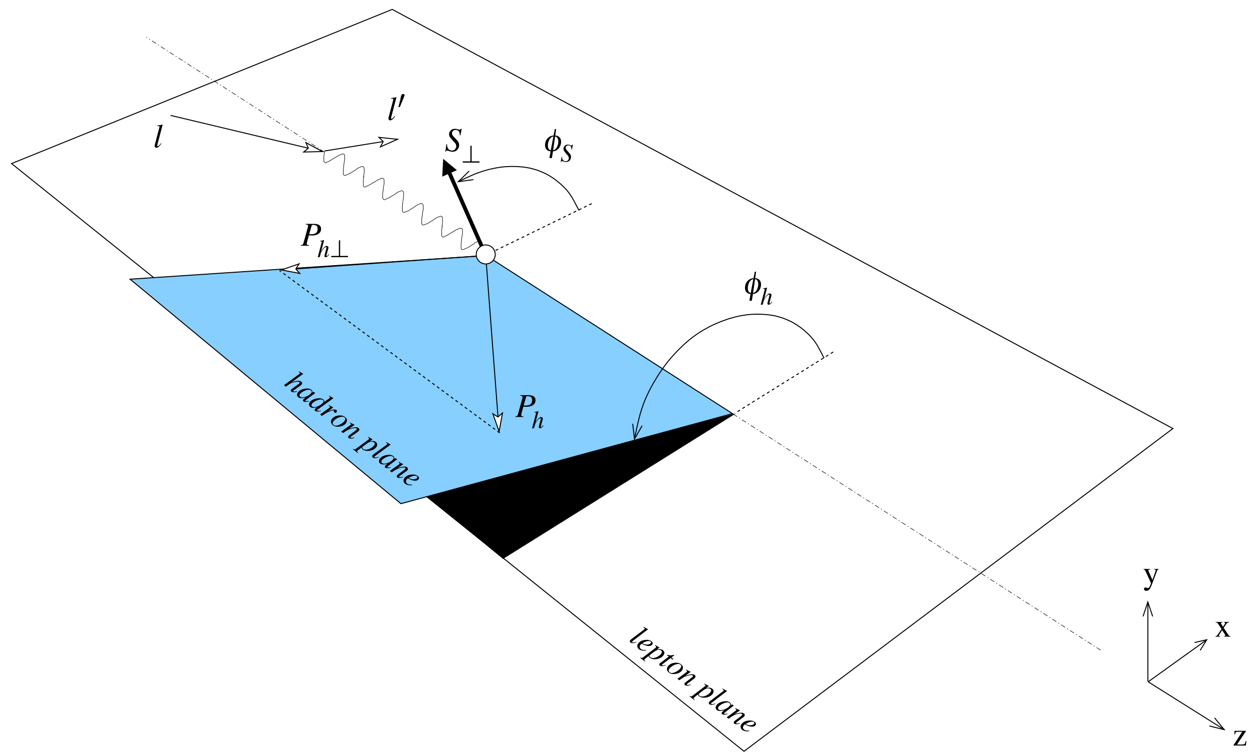
$$f_{1T,DIS}^{\perp} = -f_{1T,DY}^{\perp}$$

Extraction of Sivers Function

The Sivers function can be determined through its contributions to the cross section of the polarized SIDIS process.



Extraction of Sivers Function



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}$$

$$+\sin(\phi_h-\phi_S)\,|\boldsymbol{S}_T|\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right]+\ldots\Big\}$$

contributions from other spin structure functions

the spin structure function $F_{UT}^{\sin(\phi_h-\phi_S)}$ is a convolution of the Sivers function $f_{1T}^{\perp}$ with the unpolarized fragmentation function $D_{h/q}$

Extraction of Sivers Function

Isolating the terms relevant to the $\sin(\phi_h - \phi_S)$ modulation

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow - d\sigma^\downarrow]\, \sin(\phi_h - \phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow + d\sigma^\downarrow]}$$

in terms of structure functions

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}}{F_{UU,T} + \varepsilon F_{UU,L}}$$

we will consider only the terms at order α_S^0

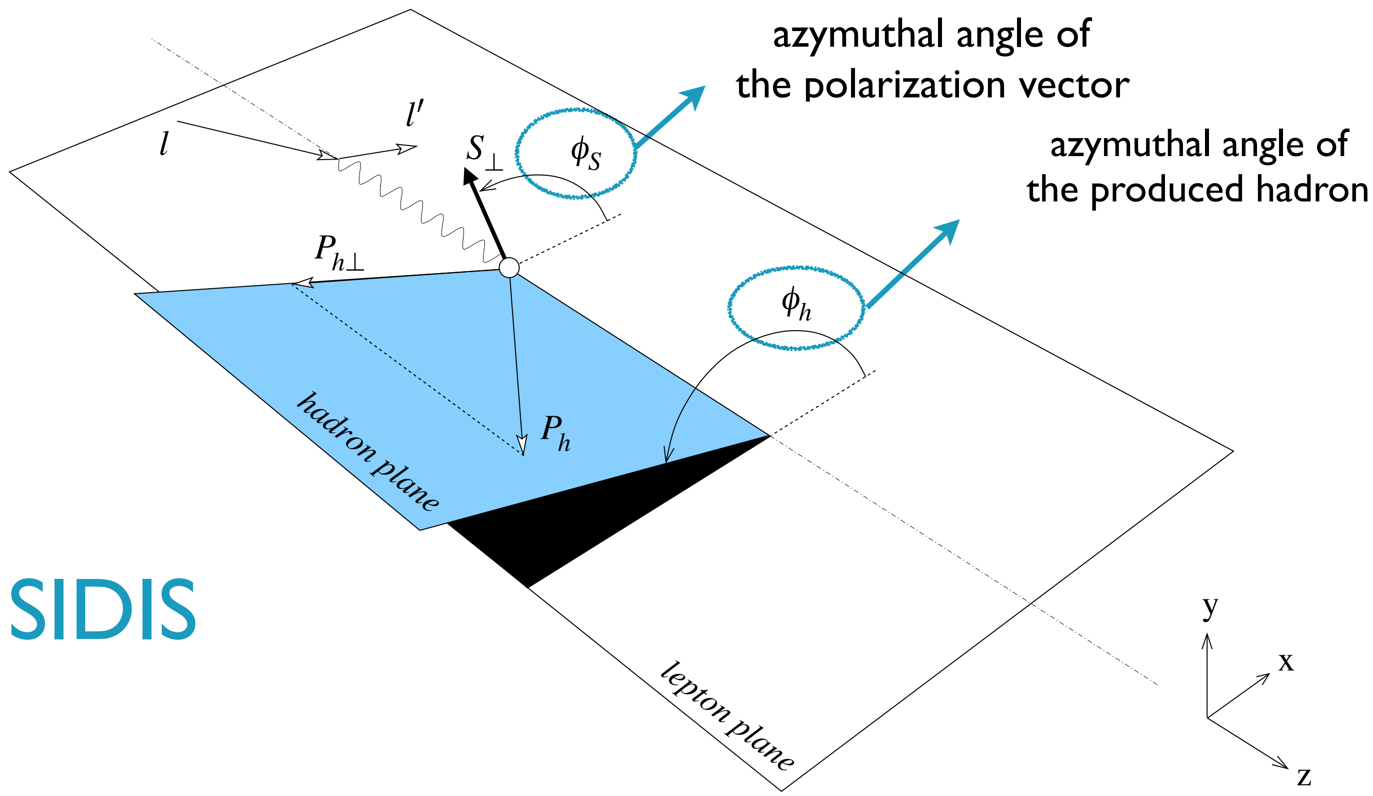
written in terms of convolutions of TMDs

LO - NLL

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathscr{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp}{M} f_{1T}^\perp D_1\right] \qquad F_{UU,T} = \mathscr{C}\left[f_1 D_1\right]$$

$$F_{UU,L}^{\sin(\phi_h-\phi_S)} = 0 \qquad F_{UU,L} = \mathcal{O}\left(M^2/Q^2, P_{hT}^2/Q^2\right) = 0$$

Extraction of Sivers Function



SIDIS

$$A_{UT}^{\sin(\phi_h - \phi_S)} \equiv \langle \sin(\phi_h - \phi_S) \rangle \sim \frac{f_{1T}^{\perp} \otimes D_1^{a \to h}}{f_1^a \otimes D_1^{a \to h}}$$

universality

first Sivers extraction with unpolarised TMDs extracted from data

TMDs in coordinate space

TMD
Fragmentation Function

$$F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right) \simeq \sum_a x \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;Q^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;Q^2)$$
$$\times\, \delta^2(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

TMD Parton
Distribution Function

Parametrization defined through previous global fit

Global fit of unpolarized TMDs

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

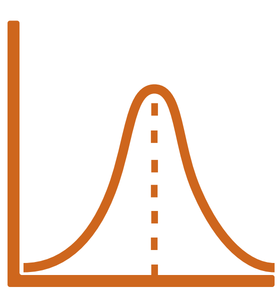
published in
[JHEP06(2017)081]

Summary of results

Total number of data points: 8059

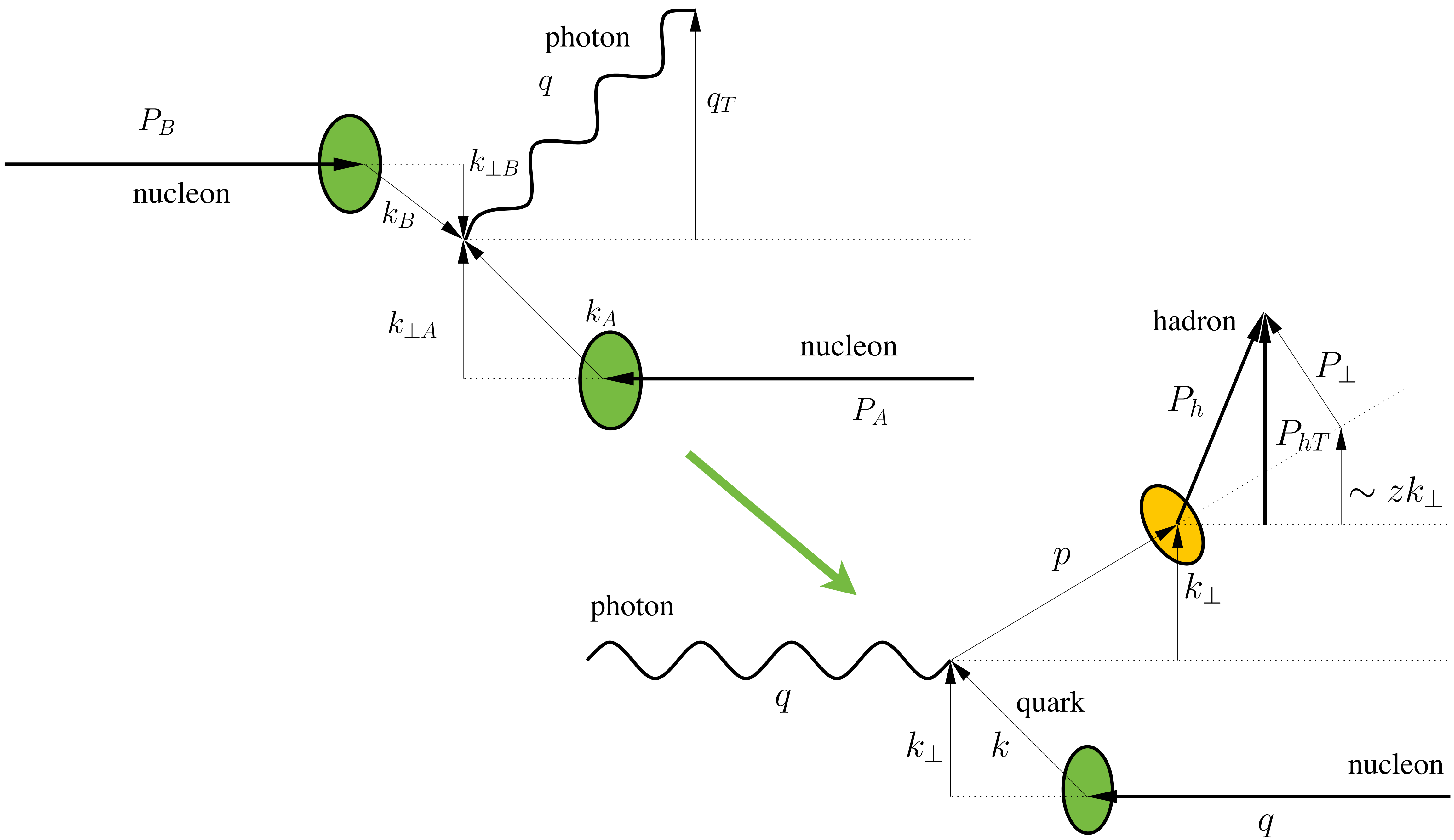
Total number of free parameters: 11

➛ 4 for TMD PDFs ➛ 6 for TMD FFs
➛ 1 for TMD evolution



$$\chi^2/d.o.f. = 1.55 \pm 0.05$$

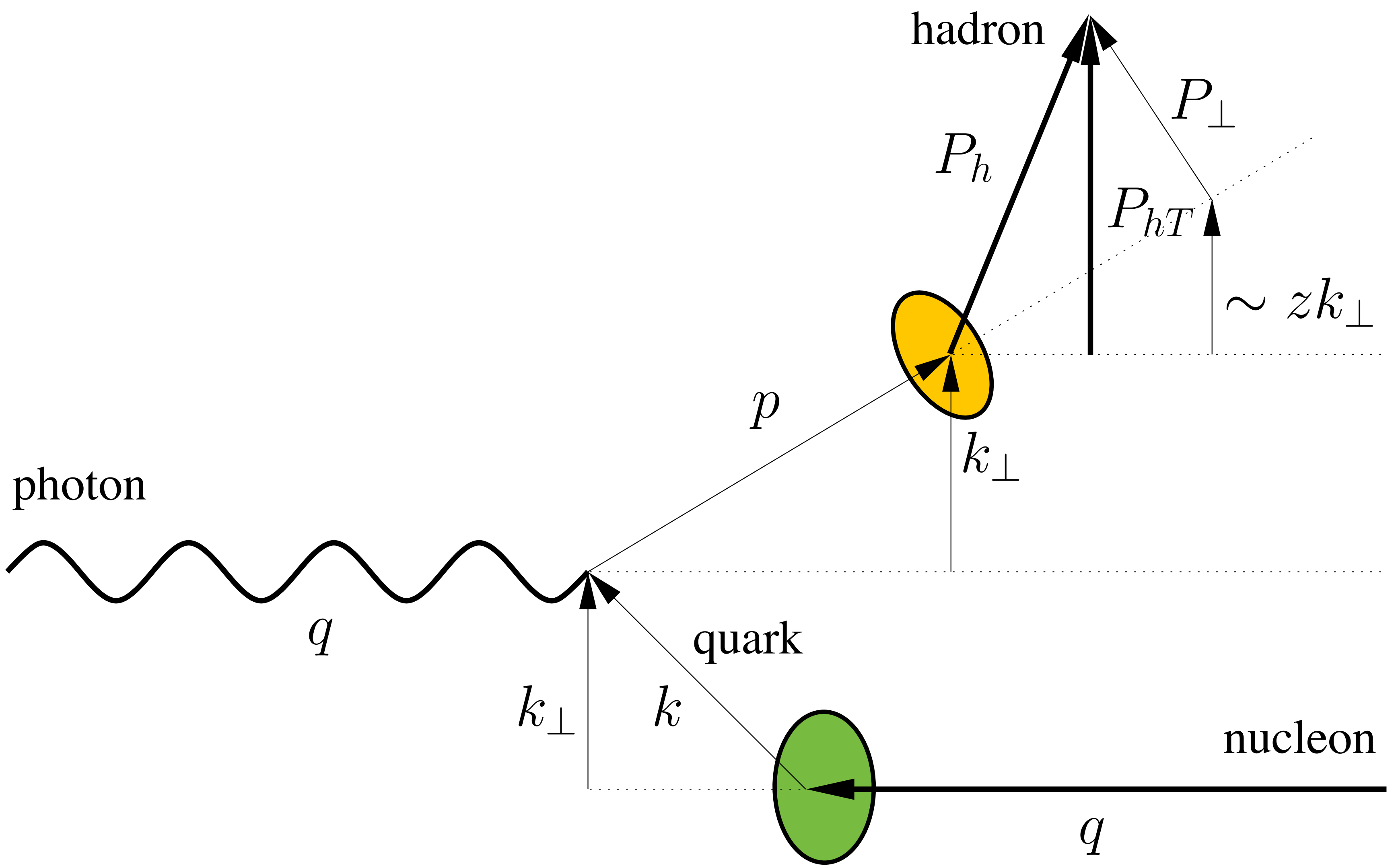
universality



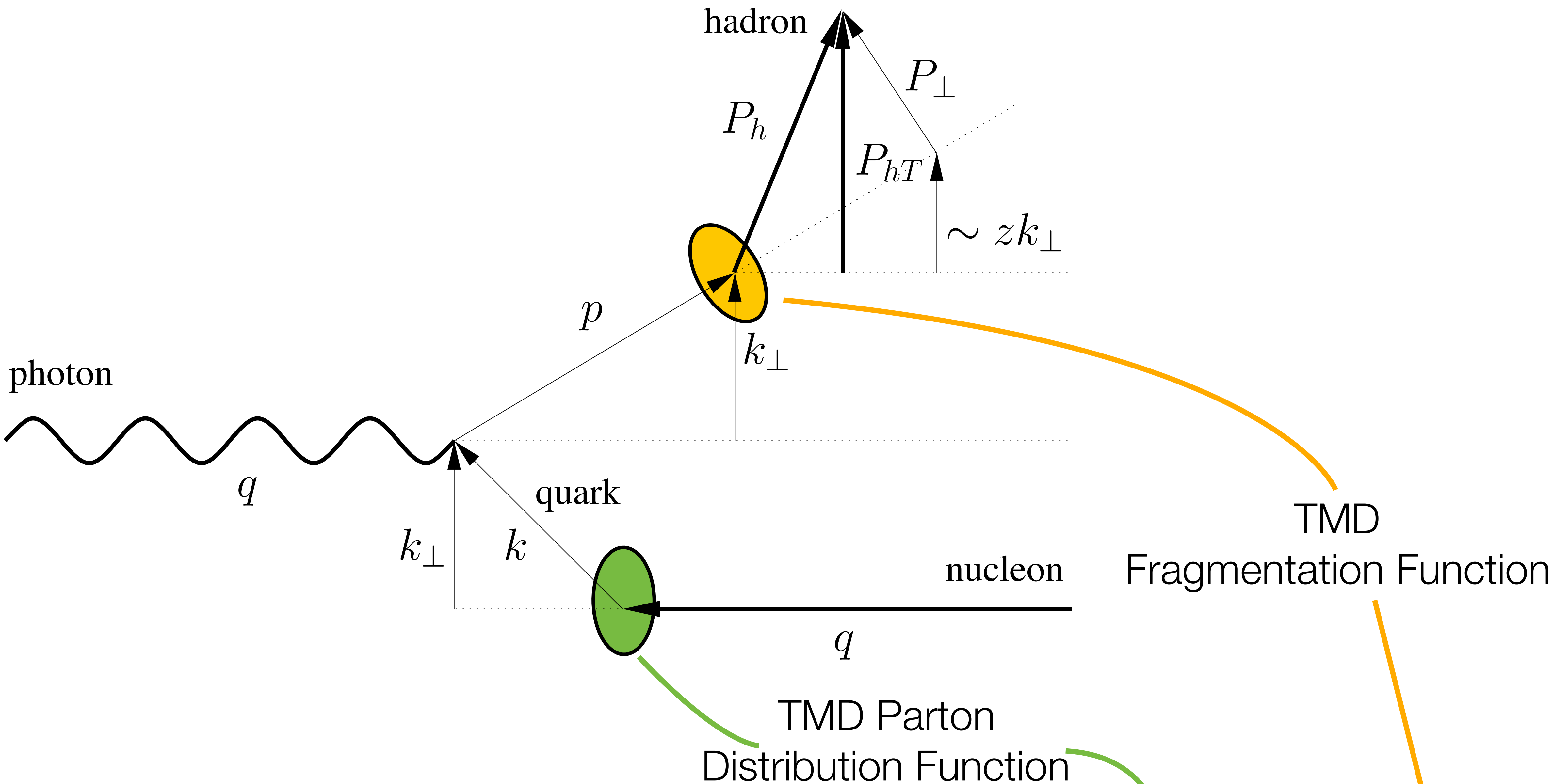
Structure functions and TMDs: SIDIS

multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dxdzd\boldsymbol{P}_{hT}^2dQ^2\right)}{d\sigma_{DIS}/\left(dxdQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



Structure functions and TMDs

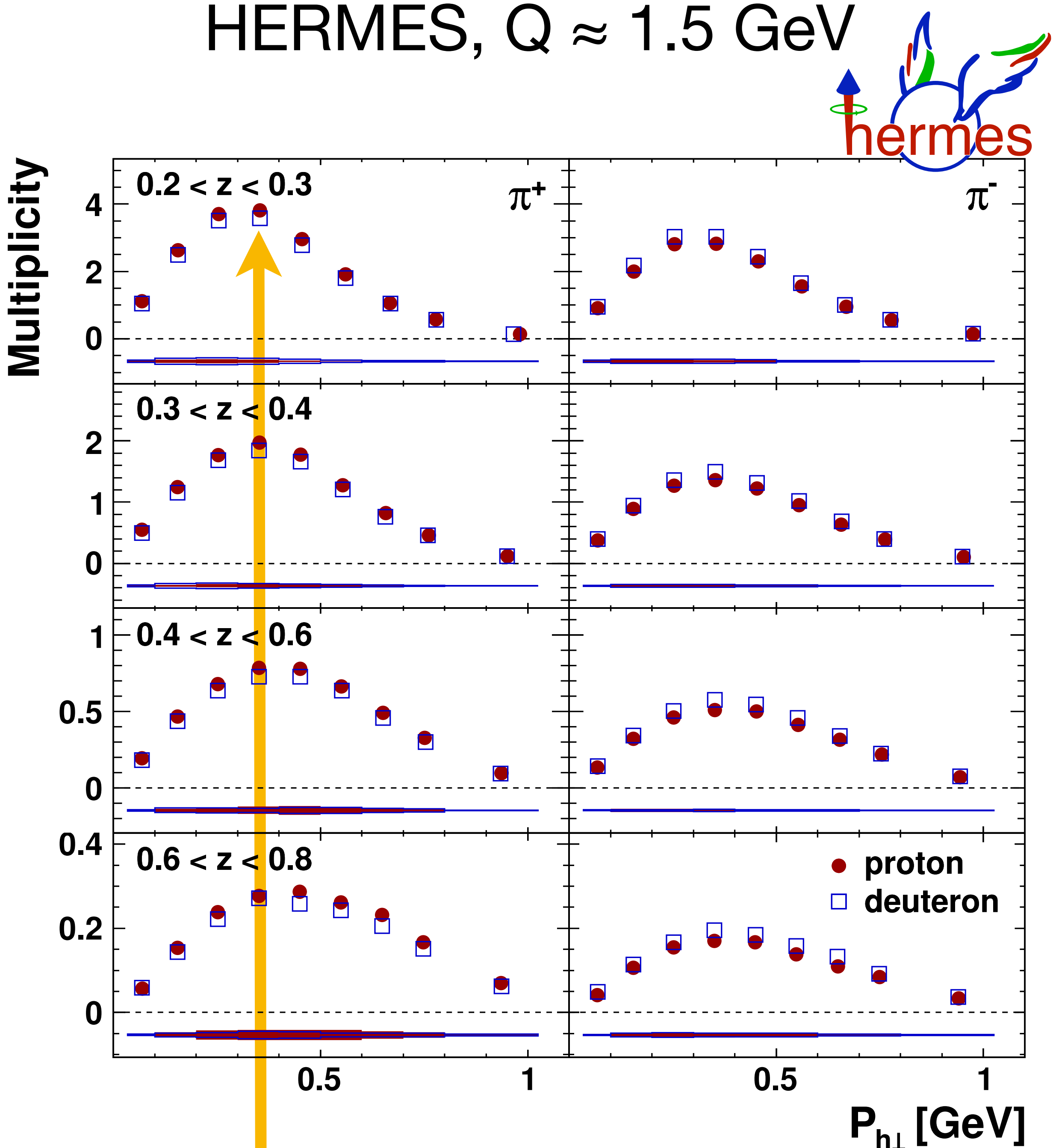


$$F_{UU,T}\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right) \simeq \sum_a x \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; Q^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2)$$

$$\times \, \delta^2(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

TMD Evolution

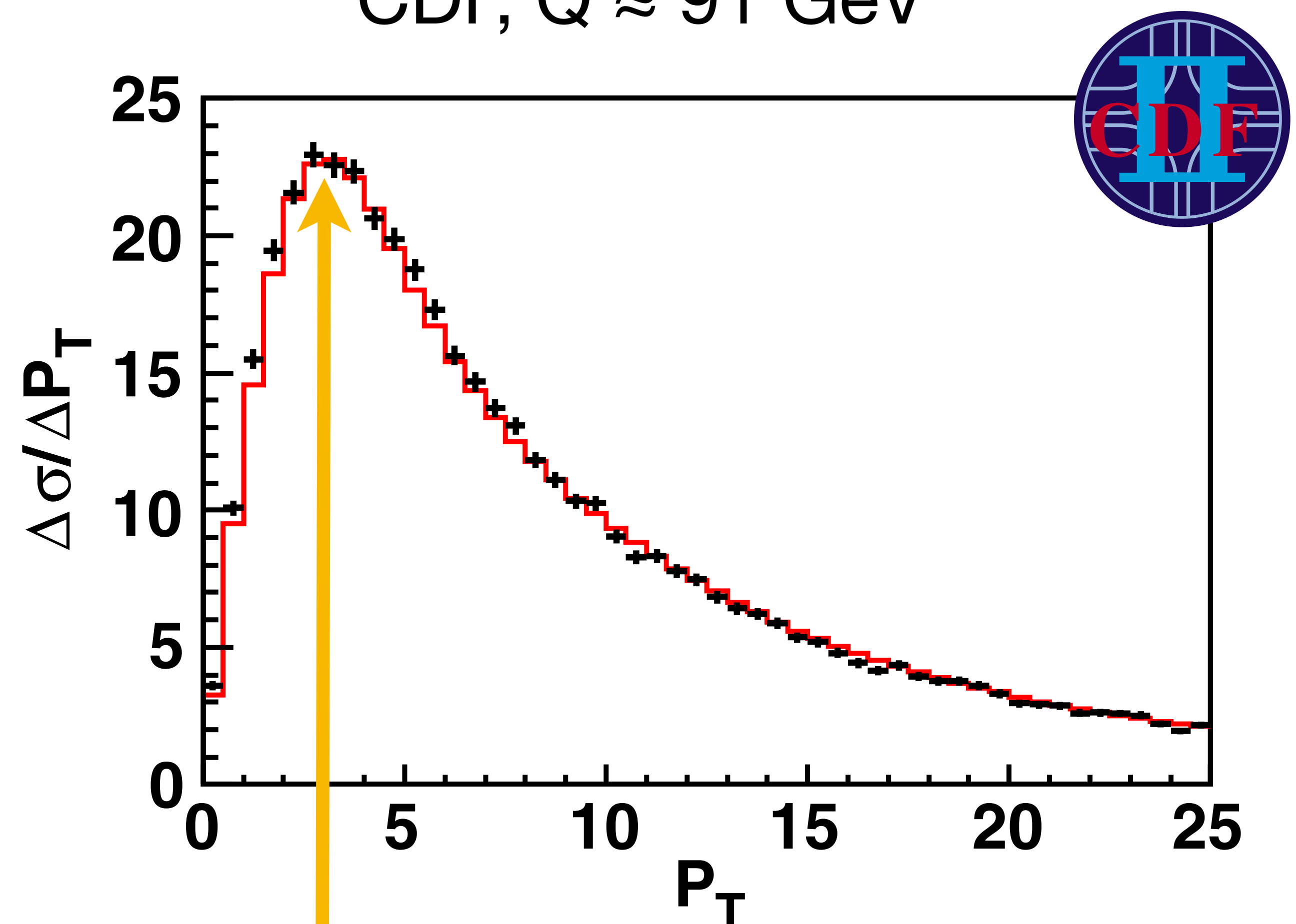
HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

reproduce shift of
TMD peak with energy scale

CDF, Q ≈ 91 GeV



Aaltonen et al., PRD86 (2012)

Width of TMDs changes of one order of magnitude → EVOLUTION

Evolved TMDs

Fourier transform: ξ_T space

alternative notation: b_T

nonperturbative part of evolution

$$\tilde{f}_1^a\left(x, \xi_T; \mu^2\right) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x, \bar{\xi}_*; \mu_b\right) e^{\tilde{S}(\bar{\xi}_*;\mu_b,\mu)} e^{g_K(\xi_T)\ln(\mu/\mu_0)} \hat{f}_{NP}^a\left(x, \xi_T\right)$$

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

nonperturbative part of TMD

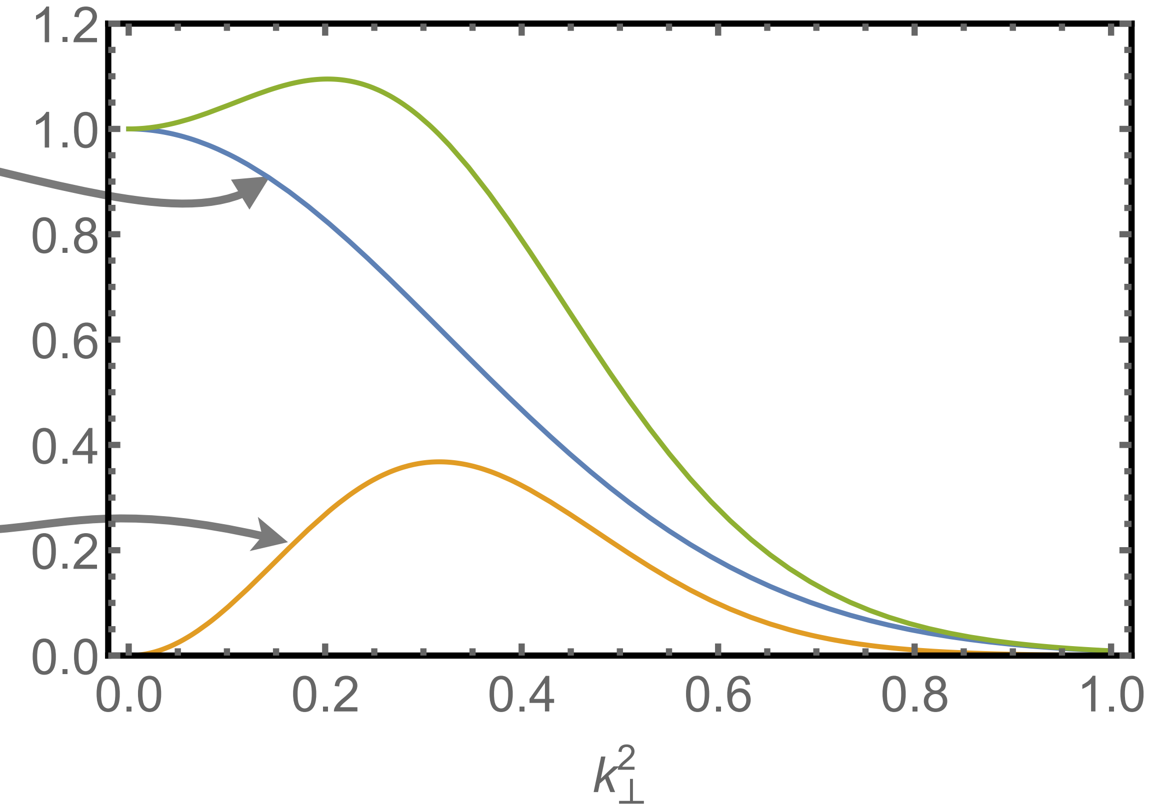
Non-perturbative contributions have to be extracted from experimental data, after parametrization

Model: non perturbative elements

input TMD PDF @ $Q^2=1\,\mathrm{GeV}^2$

$$\tilde{f}^a_{NP} = \mathcal{F}.\mathcal{T}.\ \text{of}$$

$$\left(\exp\left(\frac{-k_\perp^2}{g_1}\right) + \lambda k_\perp \exp\left(\frac{-k_\perp^2}{g_1}\right)\right)$$



sum of two different gaussians
dependent on transverse momenta

$$g_1(x) = N_1 \frac{(1-x)^\alpha\, x^\sigma}{(1-\hat{x})^\alpha\, \hat{x}^\sigma}$$

where

$$N_1 \equiv g_1(\hat{x})$$
$$\hat{x} = 0.1$$

for the FF we use two different variances: $g_3(z), g_4(z)$

Sivers in coordinate space

ξ_T space

to apply
CSS formalism for evolution

Sivers distribution function

$$\boxed{\tilde{f}_{1T}^{\perp(n)a}(x,\xi_T^2;Q^2)} = n!\left(-\frac{-2}{M^2}\partial_{\xi_T^2}\right)^n \tilde{f}_{1T}^{\perp a}(x,\xi_T^2;Q^2) = \frac{n!}{(M^2)^n}\int_0^\infty d|\boldsymbol{k}_\perp||\boldsymbol{k}_\perp|\left(\frac{|\boldsymbol{k}_\perp|}{\xi_T}\right)^n J_n(\xi_T|\boldsymbol{k}_\perp|)\tilde{f}_{1T}^{\perp a}(x,\xi_T^2;Q^2)$$

first moment

$$\boxed{\tilde{f}_{1T}^{\perp(1)a}(x,\xi_T^2;Q^2)} = \frac{1}{M^2}\int_0^\infty d|\boldsymbol{k}_\perp||\boldsymbol{k}_\perp|\left(\frac{|\boldsymbol{k}_\perp|}{\xi_T}\right) J_1(\xi_T|\boldsymbol{k}_\perp|)\tilde{f}_{1T}^{\perp a}(x,\xi_T^2;Q^2)$$

Parametrization of Sivers function

Sivers function can be parametrized in terms of its first moment

$$f_{1T}^{\perp}(x,k_{\perp}^2) = f_{1T}^{\perp(1)}(x)\,f_{1TNP}^{\perp}(x,k_{\perp}^2)$$

Its nonperturbative part is arbitrary, but constrained by the positivity bound.

$$f_{1TNP}^{\perp}(x,k_{\perp}^2) = \frac{1}{\pi K_f}\frac{1}{F_{max}}\frac{(1+\lambda_S k_{\perp}^2)}{(M_1^2+\lambda_S M_1^4)}e^{-k_{\perp}^2/M_1^2}f_{1NP}(x,k_{\perp}^2)$$

following the definition to the nonperturbative part of the unpolarized TMD distribution

$$f_{1NP}(x,k_{\perp}^2) = \frac{1}{\pi}\frac{(1+\lambda k_{\perp}^2)}{(g_{1a}+\lambda g_{1a}^2)}e^{-k_{\perp}^2/g_{1a}}$$

Free parameters $\lambda_S\,, M_1$

Parametrization of Sivers function

Sivers function can be parametrized in terms of its first moment

$$f_{1T}^{\perp}(x,k_\perp^2) = f_{1T}^{\perp(1)}(x)\, f_{1TNP}^{\perp}(x,k_\perp^2)$$

Its nonperturbative part is arbitrary, but constrained by the positivity bound.

$$f_{1TNP}^{\perp}(x,k_\perp^2) = \frac{1}{\pi K_f}\frac{1}{F_{max}}\frac{(1+\lambda_S k_\perp^2)}{(M_1^2+\lambda_S M_1^4)}e^{-k_\perp^2/M_1^2} f_{1NP}(x,k_\perp^2)$$

normalization factor $K_f \equiv \int d^2k_\perp \frac{k_\perp^2}{2M^2} f_{1TNP}^{\perp}$

following the definition to the nonperturbative part of the unpolarized TMD distribution

$$f_{1NP}(x,k_\perp^2) = \frac{1}{\pi}\frac{(1+\lambda k_\perp^2)}{(g_{1a}+\lambda g_{1a}^2)}e^{-k_\perp^2/g_{1a}}$$

Free parameters $\lambda_S\,, M_1$

Parametrization of Sivers function

normalization
(abs.value <1)

$T_n(x)$ Chebyshev polynomials

$$f_{1T}^{\perp(1)}(x) = \frac{N_{Siv}^{a}}{G_{max}^{a}} x^{\alpha_a}(1-x)^{\beta_a}\Big(1 + A_a T_1(x) + B_a T_2(x)\Big)\, f_1(x, Q^2)$$

maximum value
of the function

Radici [Phys. Rev. Lett., 120(19):192001, 2018]

Free parameters $N_{Siv}^{a}, \alpha_a, \beta_a, A_a, B_a$

Flavor dependent: distinct for up, down, sea

Evolution of Sivers

We simply assume that $f_{1T}^{\perp(1)}$ evolves in the same way as unpolarized f_1

Difference in the Wilson coefficients: $C^i \rightarrow C^{Siv}$

At our accuracy level (LO): $C^{Siv(0)} = \delta(1-x)\delta^{ai}$

The evolved Sivers function first moment becomes

$$\tilde{f}_{1T}^{\perp(1)a}(x,\xi_T^2;Q^2) = f_1^a(x;\mu_b^2)\ e^{S(\mu_b^2,Q^2)}\ e^{g_K(\xi_T)\ln(Q^2/Q_0^2)}\ \tilde{f}_{1T\mathrm{NP}}^{\perp(1)a}(x,\xi_T^2)$$

same choices used for evolved unpolarized TMDs

Experimental data

proton [H]

95
data points

neutron [^{3}He]

6
data points

deuteron [^{6}LiD]

88
data points

Proton [NH_3]

111
data points

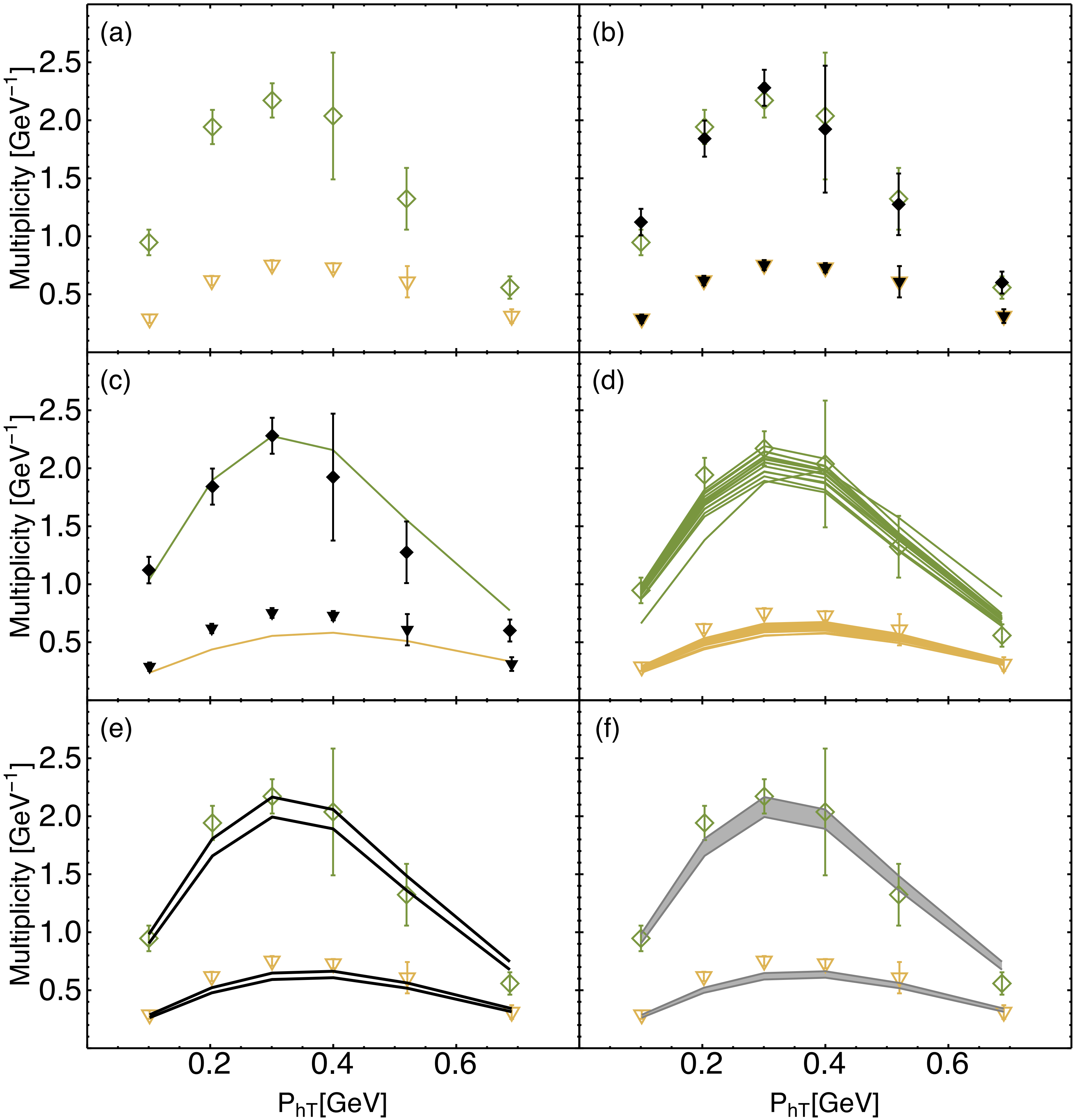
COMPASS 2009

COMPASS 2017

Same kinematic cuts applied to unpolarized

x, z, P_{hT} data projections

Replica Methodology



a) Example of original data (two bins)

b) Data are replicated with Gaussian noise

c) The fit is performed on the replicated data

d) The procedure is repeated 200 times

e) For each point a 68% confidence level is identified

f) These point connects to create a 68% C.L. band

Results

LO - NLL

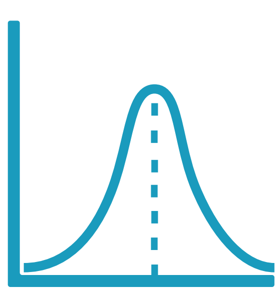
Replica method

Summary of results

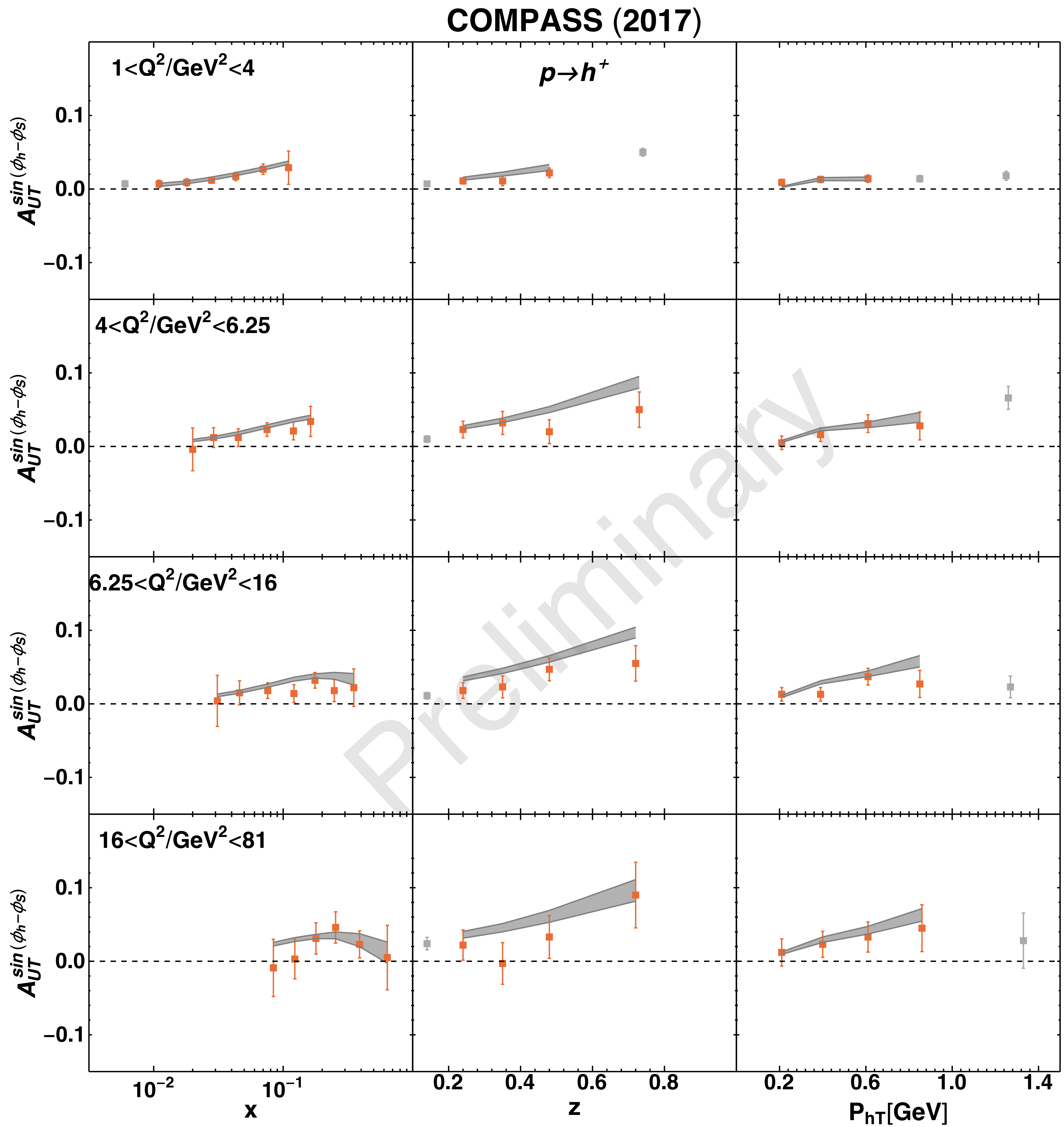
Total number of data points: 118

Total number of free parameters: 14

➛ for 3 different flavors



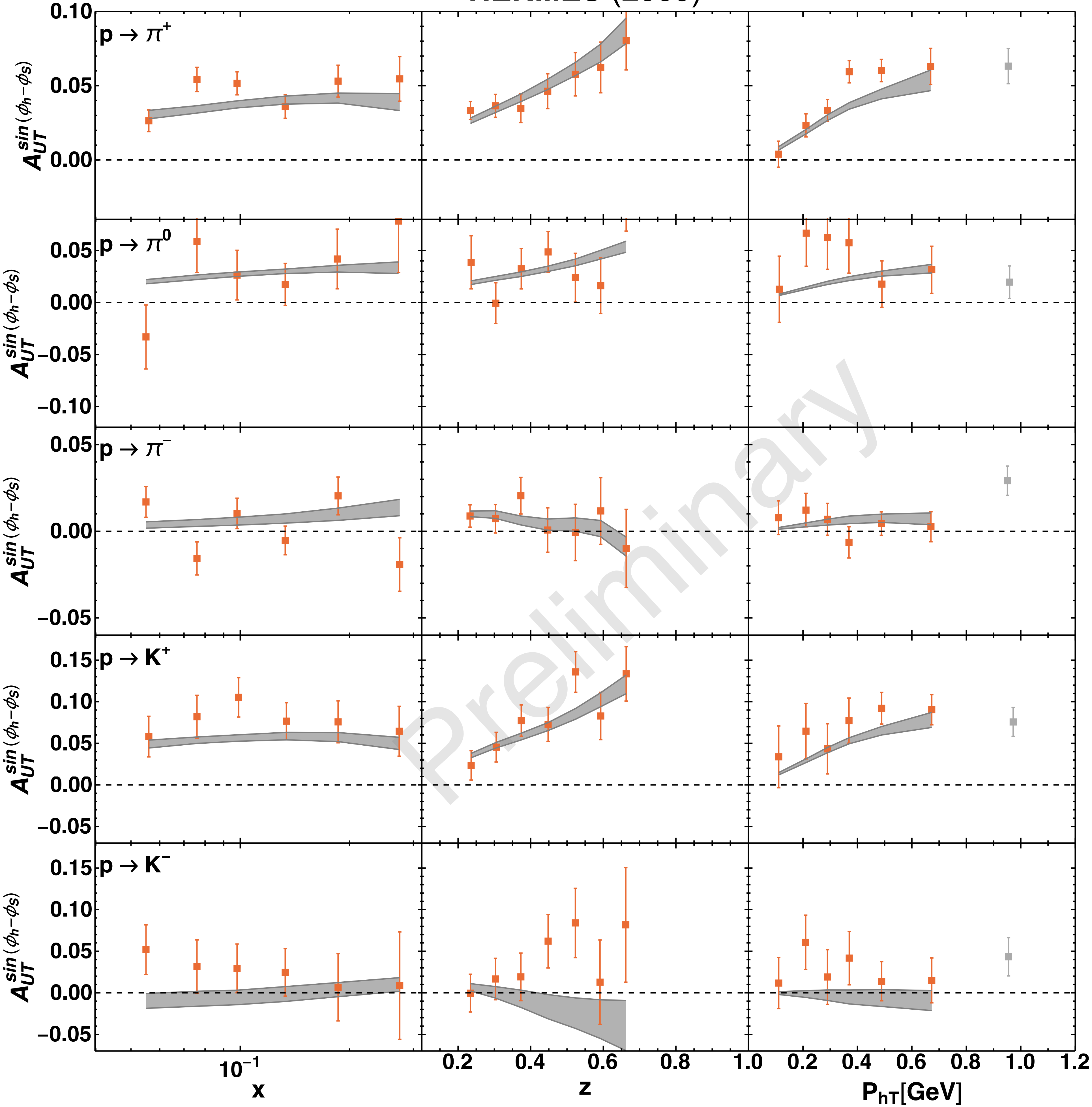
$$\chi^2/d.o.f = 1.22 \pm 0.20$$

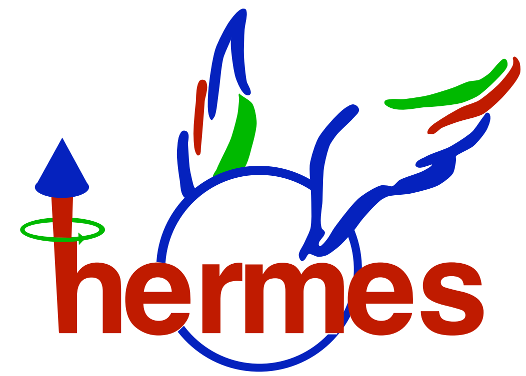


proton

positive hadron

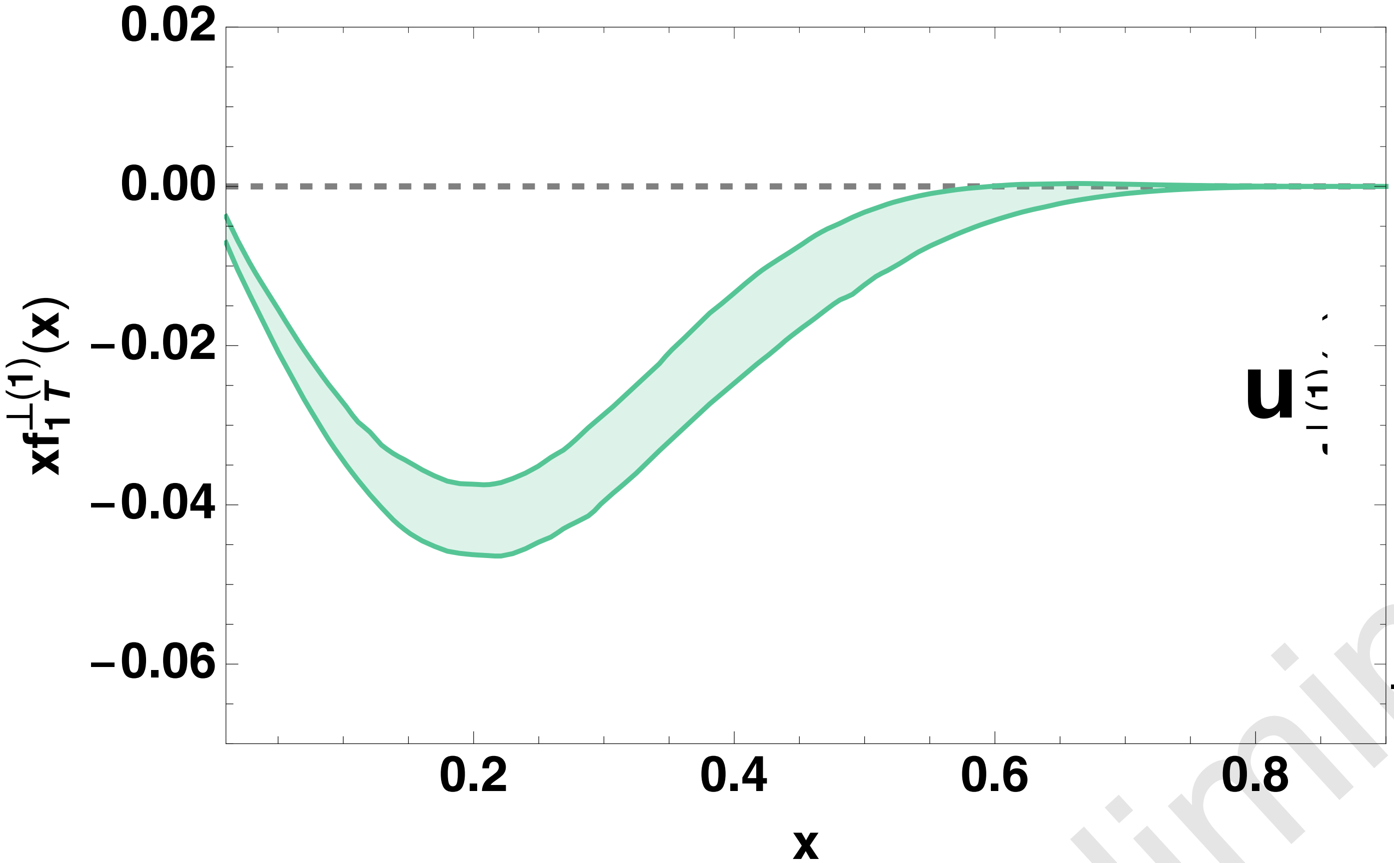
HERMES (2009)

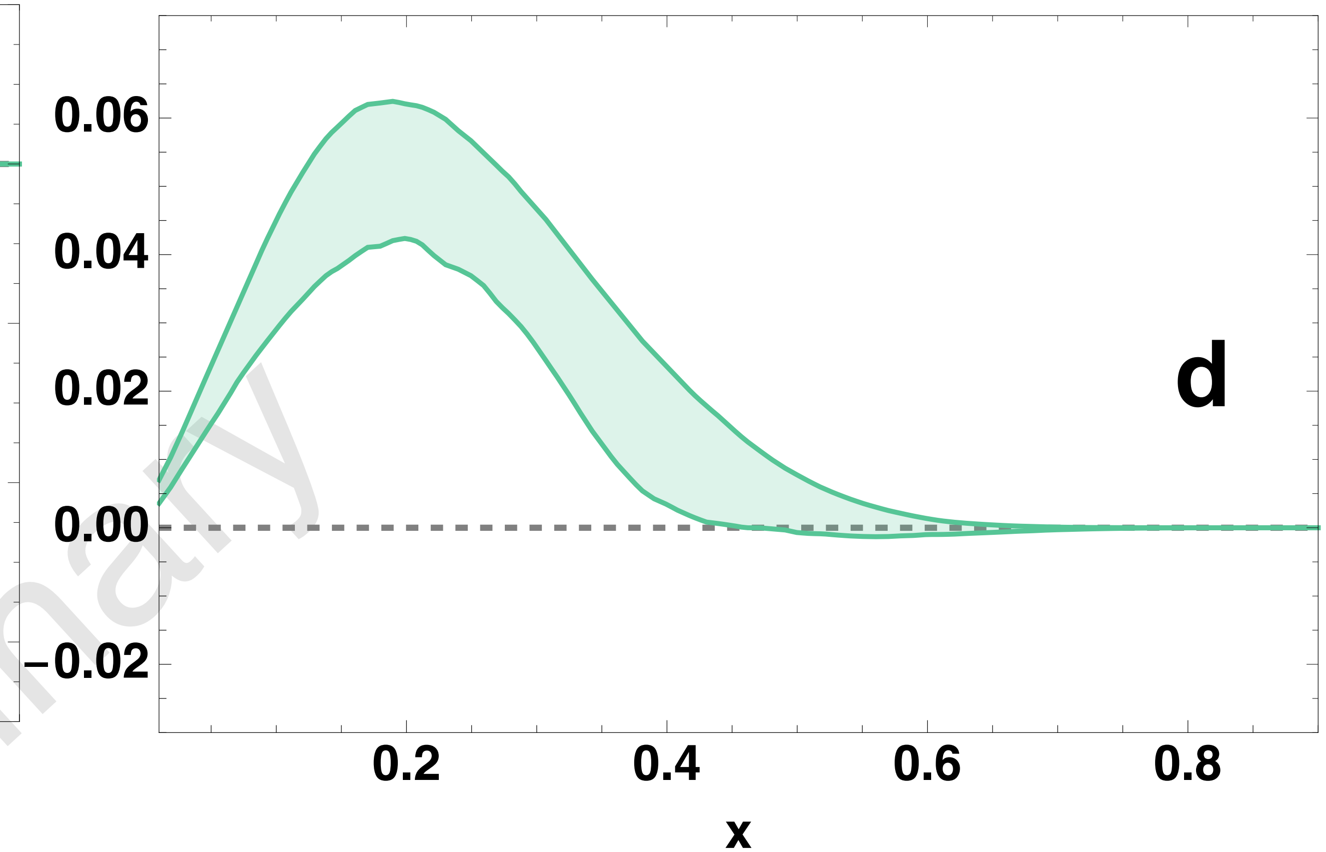


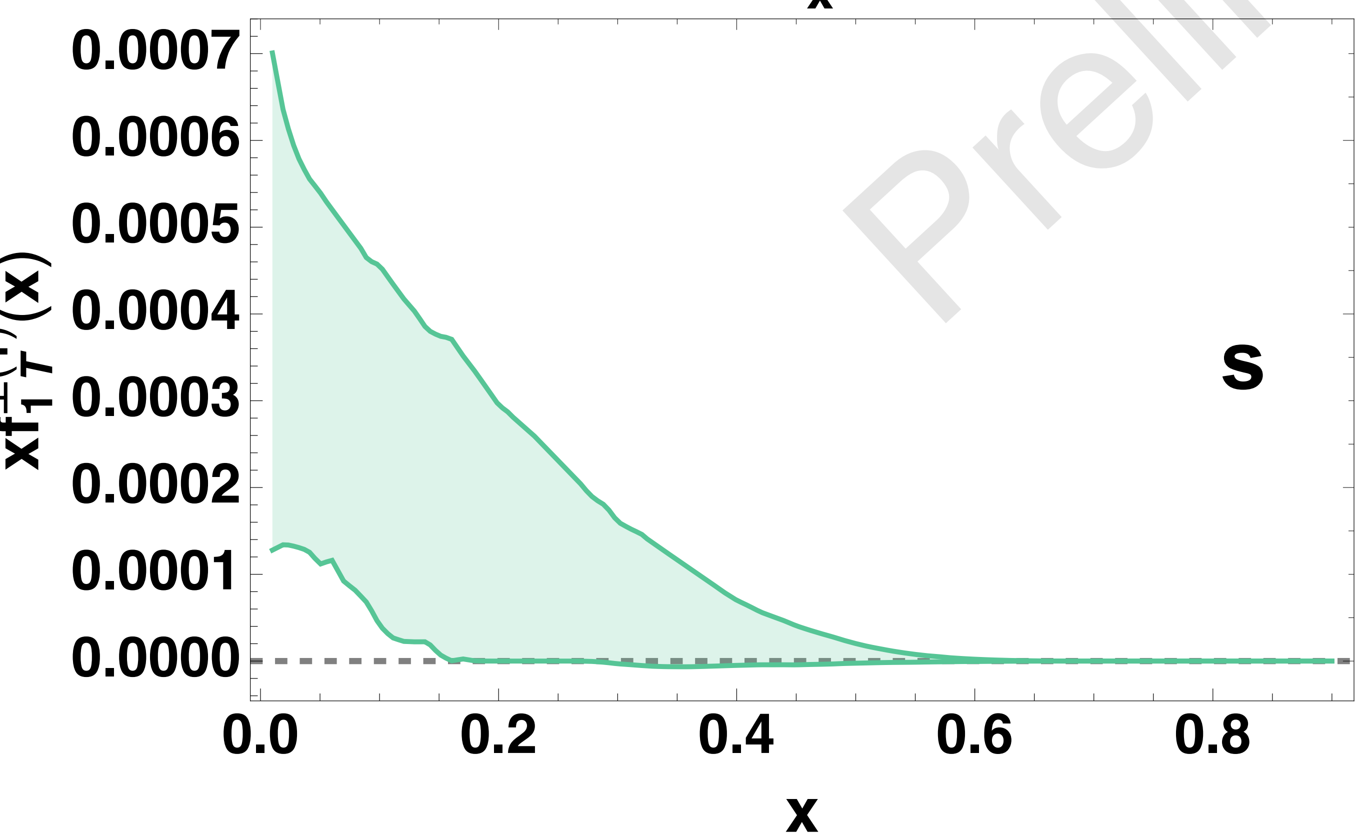


proton

Sivers function first moment

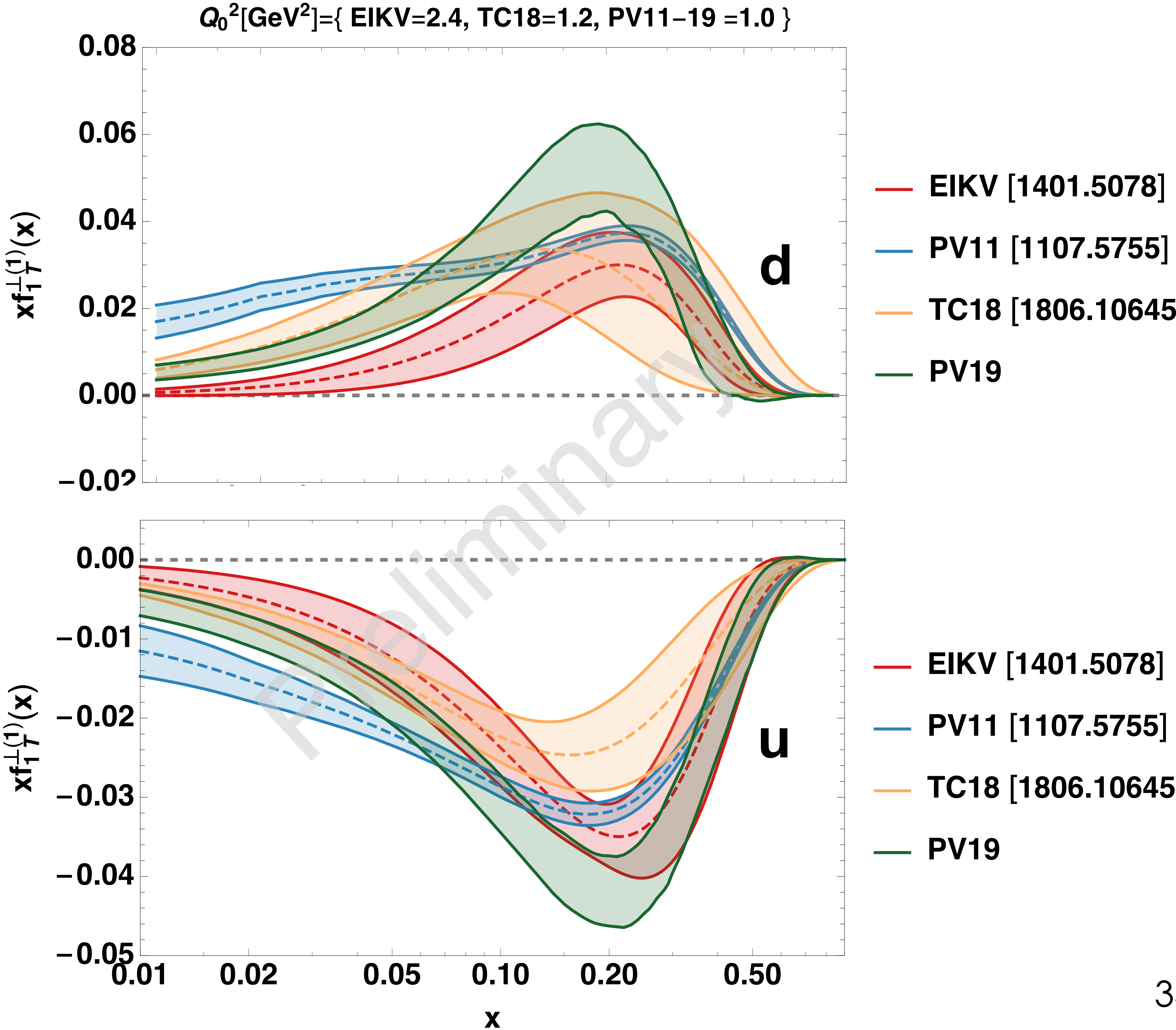




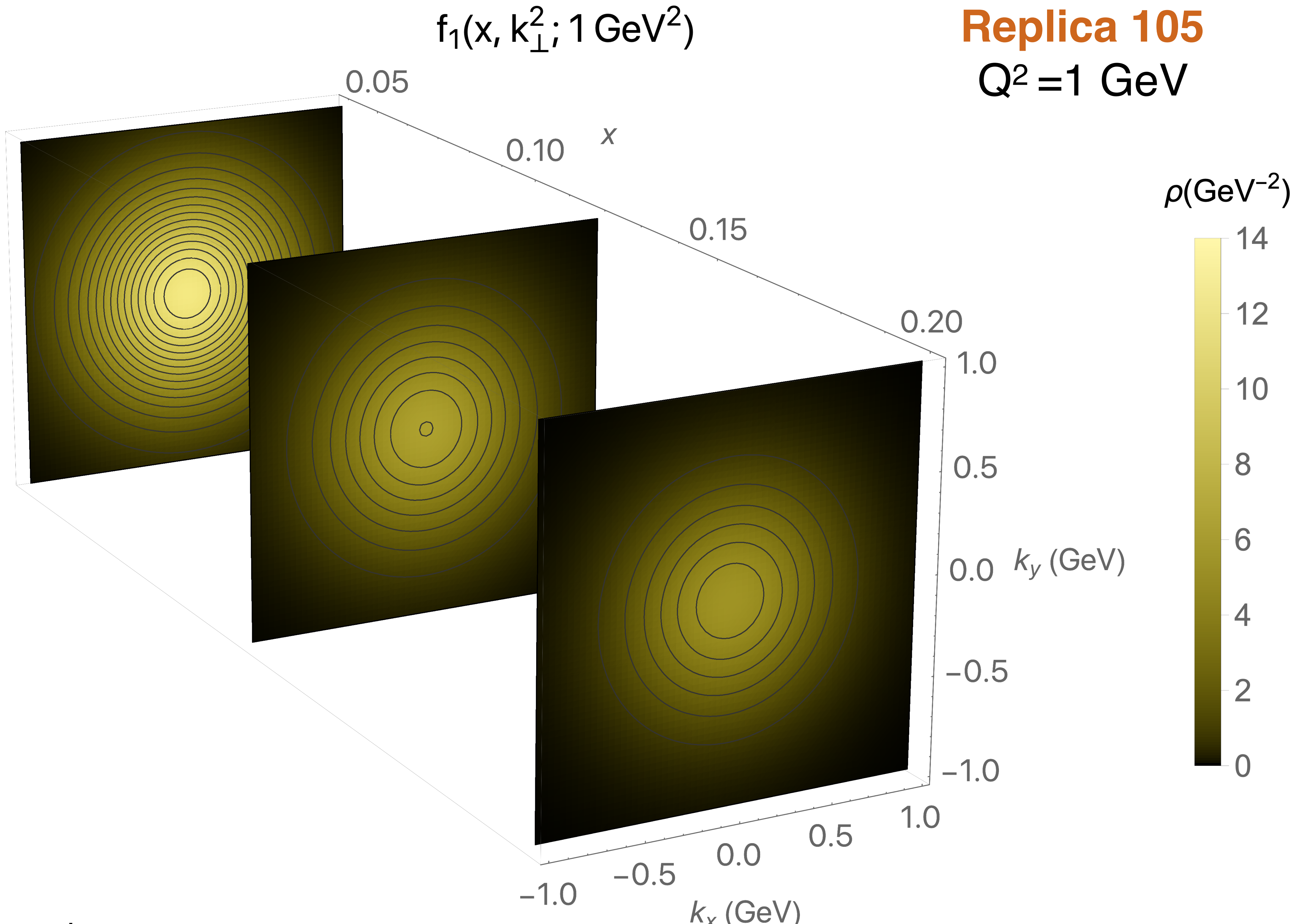


logarithmic plots of 68% C.L bands for
first moment of Sivers function
for down, up and s quarks

Results comparison

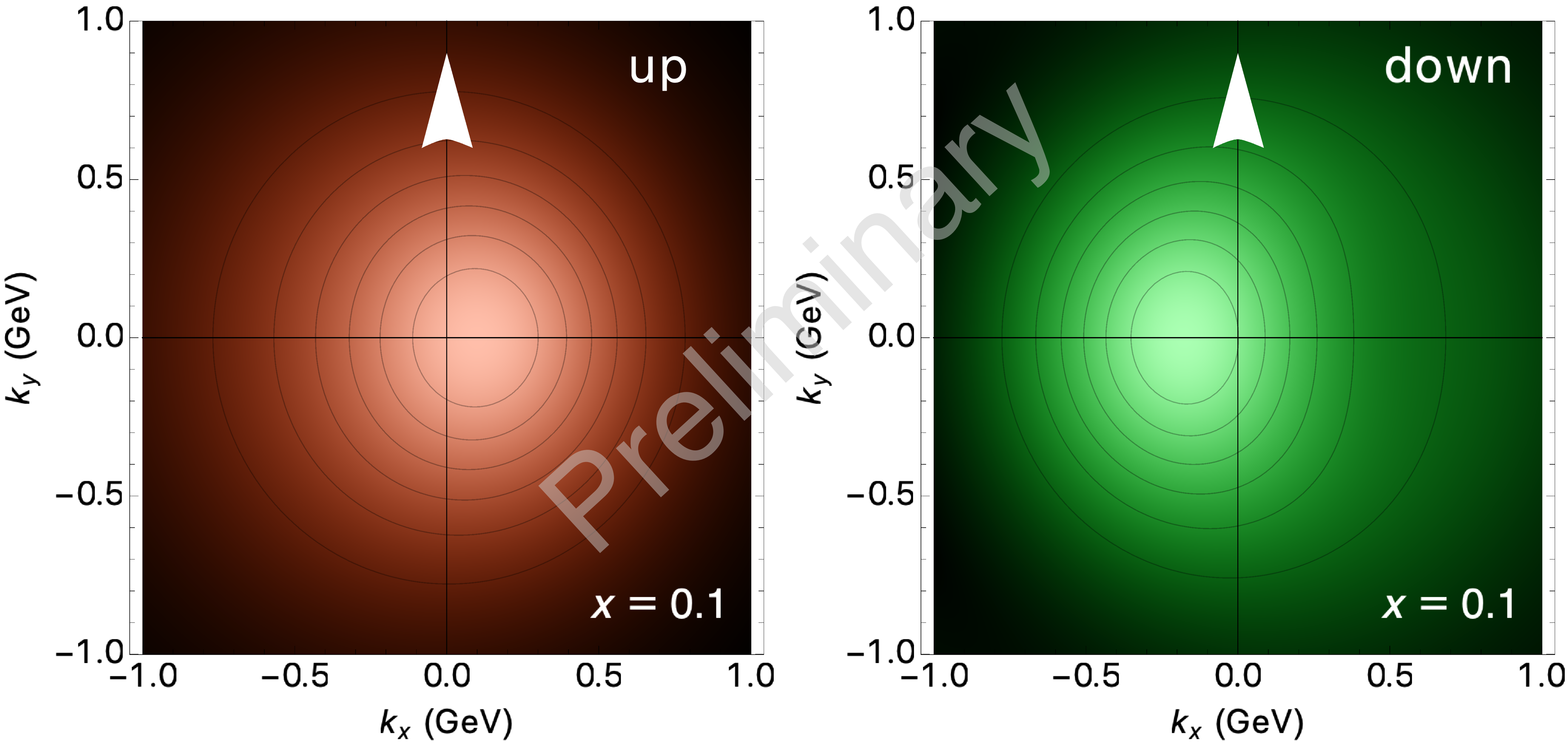


Visualization of TMDs: PDF 3D structure



Momentum space

Visualization of TMDs: structure deformation



$$xf_1(x, k_\perp^2; Q^2) - xf_{1T}^\perp(x, k_\perp^2; Q^2)$$

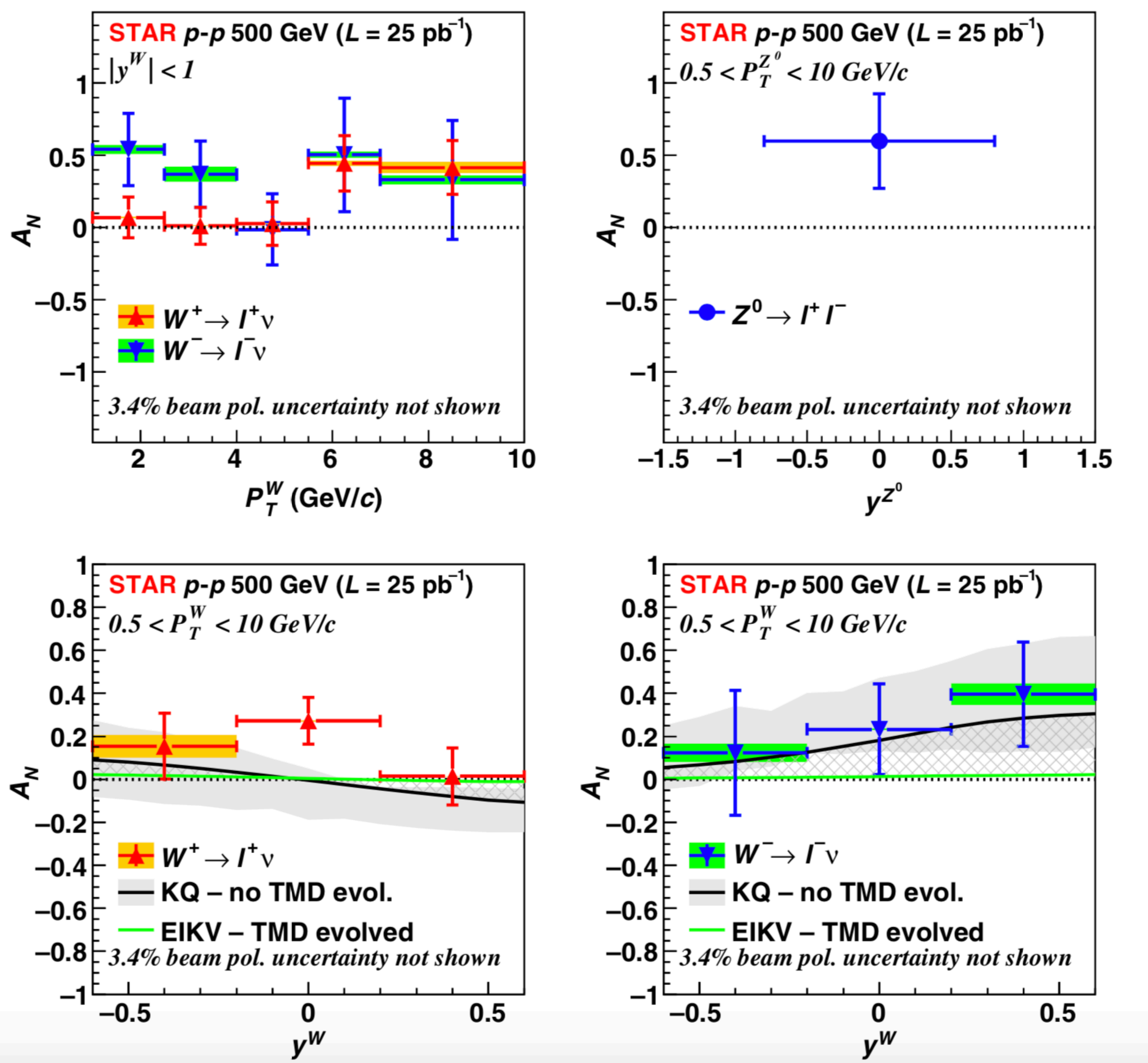
Conclusions

We extracted a functional form for Sivers distribution function, able to describe SIDIS data, even for different projections

For the first time the determination of A_{UT} included unpolarized TMDs extracted directly from data. Moreover, the analysis included the full formalism for QCD evolution

We are able to observe a deformation of the internal nucleon structure using our parametrization.

Future outlooks: Sivers



Anomalous magnetic moment
(testing Pavia2011 hypothesis)

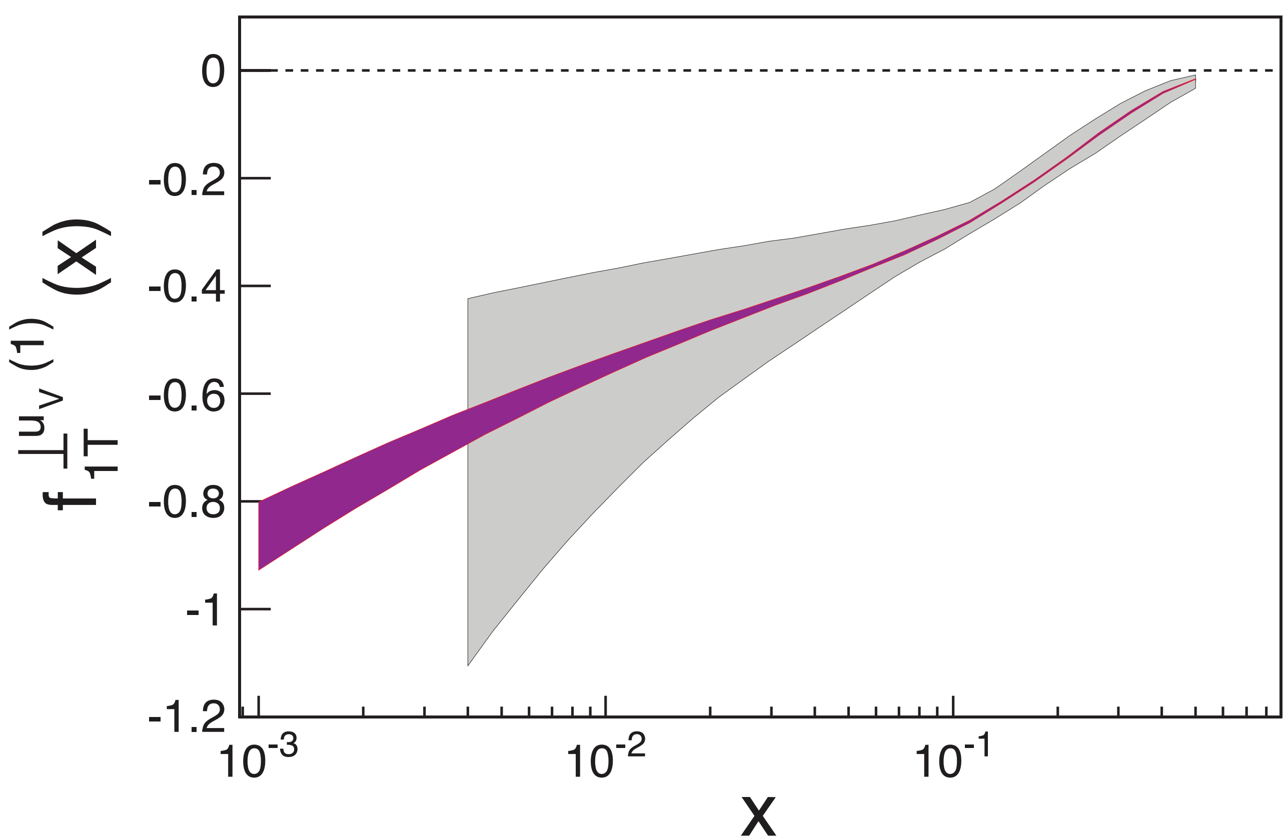
$$J^a(Q^2) = \frac{1}{2}\int_0^1 dxx[H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)].$$

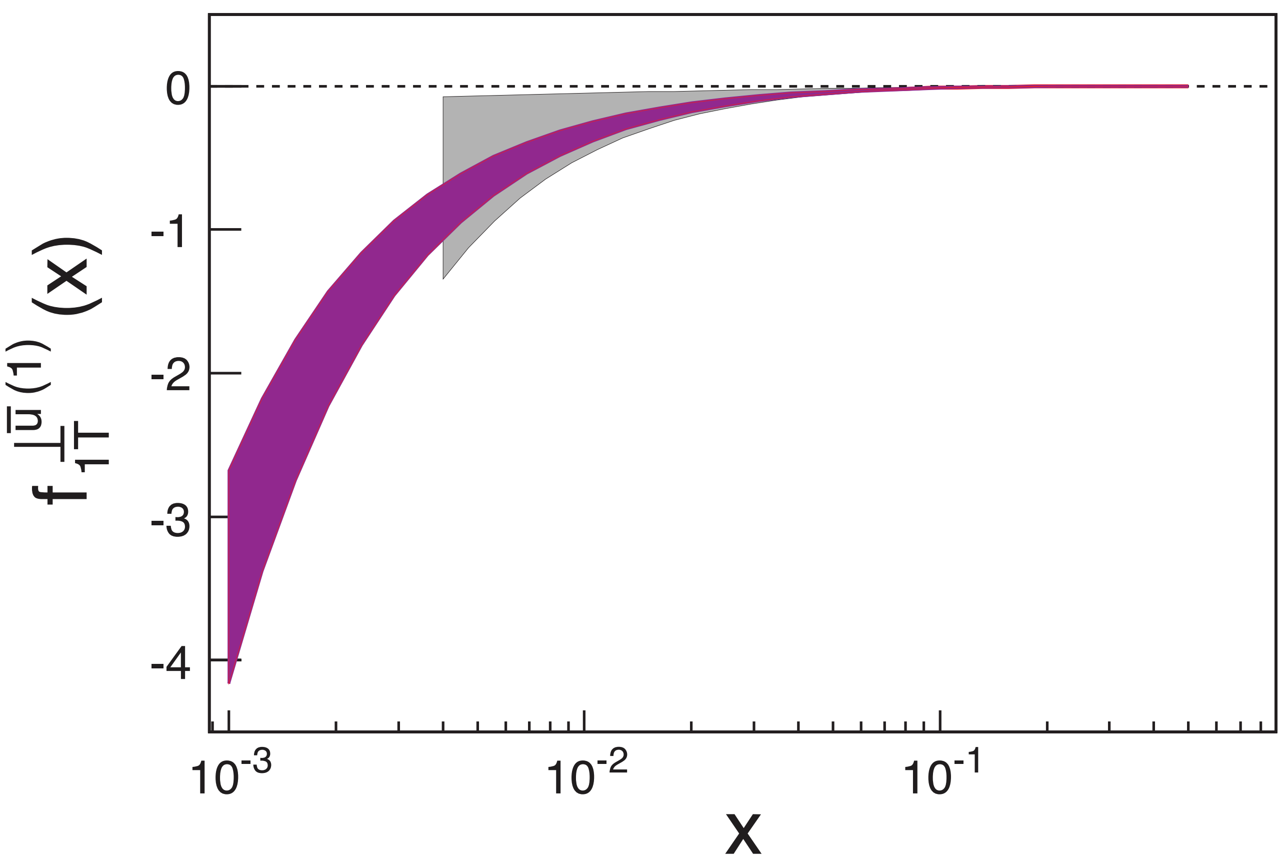
Higher accuracy
(after unpol. TMD improved fit)

Predictions of
A_N asymmetries
for W/Z production

Long term outlooks

Current knowledge of Sivers function (both valence and sea quarks) can be greatly improved thanks to the high luminosity measurements at EIC





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Results comparison: Pavia 2011

Constraining Quark Angular Momentum through
Semi-Inclusive Measurements

Angular momentum

GPD

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx x[H^a(x, 0, 0; Q^2) + E^a(x, 0, 0; Q^2)].$$

$f_1^a(x, Q^2)$

no corresponding collinear pdf

$$\sum_q e_{q_v} \int_0^1 dx E^{q_v}(x, 0, 0) = \kappa,$$

→

[Bacchetta, Radici - PRL 107, 212001 (2011)

Results comparison: Pavia 2011

Constraining Quark Angular Momentum through
Semi-Inclusive Measurements

Angular momentum

GPD

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx x[H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)].$$

$f_1^a(x, Q^2)$

no corresponding collinear pdf

$$\sum_q e_{q_v} \int_0^1 dx E^{q_v}(x,0,0) = \kappa,$$

..from theoretical consideration and spectator model results:

Lensing function

$$\rightarrow \quad f_{1T}^{\perp(0)a}(x; Q_L^2) = -L(x) E^a(x,0,0;Q_L^2),$$

$$L(x) = \frac{K}{(1-x)^\eta}$$

[Bacchetta, Radici - PRL 107, 212001 (2011)

Results comparison: Pavia 2011

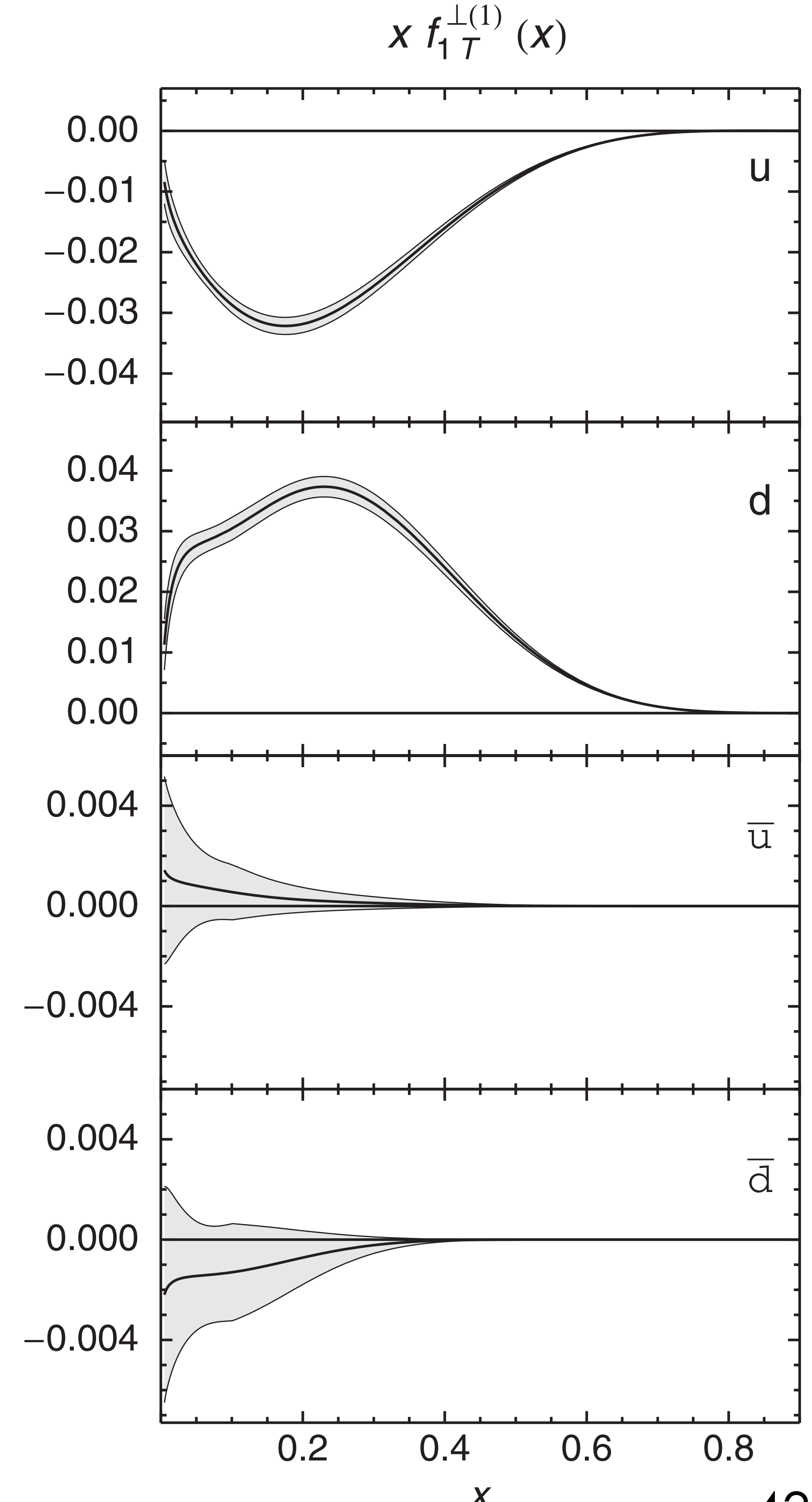
Azimuthal asymmetries

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,z,P_T^2,Q^2)$$
$$= -\frac{M_1^2(M_1^2+\langle k_\perp^2\rangle)}{\langle P_{\rm Siv}^2\rangle^2}\frac{zP_T}{M}\left(z^2+\frac{\langle P_\perp^2\rangle}{\langle k_\perp^2\rangle}\right)^3 e^{-z^2P_T^2/\langle P_{\rm Siv}^2\rangle}$$
$$\times\frac{\sum_a e_a^2 f_{1T}^{\perp(0)a}(x;Q^2)D_1^a(z;Q^2)}{\sum_a e_a^2 f_1^a(x;Q^2)D_1^a(z;Q^2)},$$

Hermes, Compass, Jlab data

TABLE I. Best-fit values of the 8 free parameters for the case $C^{s_v}=C^{\bar{s}}=0$. The final $\boxed{\chi^2/\text{d.o.f. is } 1.323.}$ The errors are statistical and correspond to $\Delta\chi^2=1$

C^{u_v}	C^{d_v}	$C^{\bar{u}}$	$C^{\bar{d}}$
-0.229 ± 0.002	1.591 ± 0.009	0.054 ± 0.107	-0.083 ± 0.122
M_1 (GeV)	K (GeV)	η	α^{u_v}
0.346 ± 0.015	1.888 ± 0.009	0.392 ± 0.040	0.783 ± 0.001

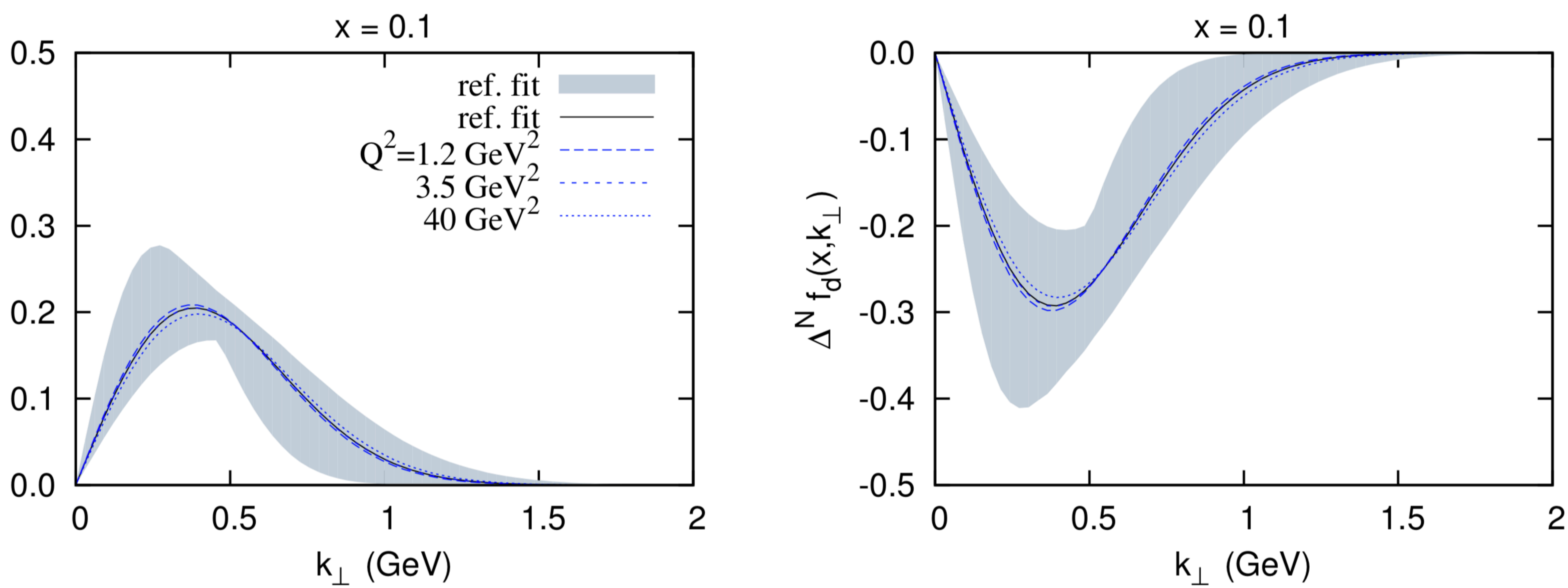


Results comparison: TO - CA group

Same selection of data, considering all projections

$$A_{UT}^{sin(\phi_h - \phi_S)}$$

3 cases for evolution: no evolution, collinear twist-3, TMD-like evolution



$$\chi^2/dof \sim 0.94$$

[Eur. Phys. J., A39:89– 100, 2009]

Results comparison: EIKV

Global fit of the HERMES, COMPASS and JLab experimental data on polarized reactions to extract the Sivers functions.

➔Hermes, Compass, Jlab data

➔using CSS evolution

➔relating the first moment of the Sivers function to the twist-three Qiu-Sterman quark-gluon correlation function

$$f_{1T,\text{SIDIS}}^{\perp q(\alpha)}(x,b;Q) = \left(\frac{ib^{\alpha}}{2}\right) T_{q,F}(x,x,c/b_*) \exp\left\{-\int_{c/b_*}^{Q} \frac{d\mu}{\mu}\left(A \ln\frac{Q^2}{\mu^2} + B\right)\right\}$$

$$\times \exp\left\{-b^2\left(g_1^{\text{sivers}} + \frac{g_2}{2} \ln\frac{Q}{Q_0}\right)\right\}$$

$$T_{q,F}(x,x,\mu) = N_q \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta q}} x^{\alpha_q}(1-x)^{\beta_q} f_{q/A}(x,\mu)$$

[Echevarria et al. - Phys. Rev. D.89.074013 (2014)]

Results comparison: EIKV

$T_{qF}(x, x, \mu)$ ➔ “collinear counterpart" of the Sivers function

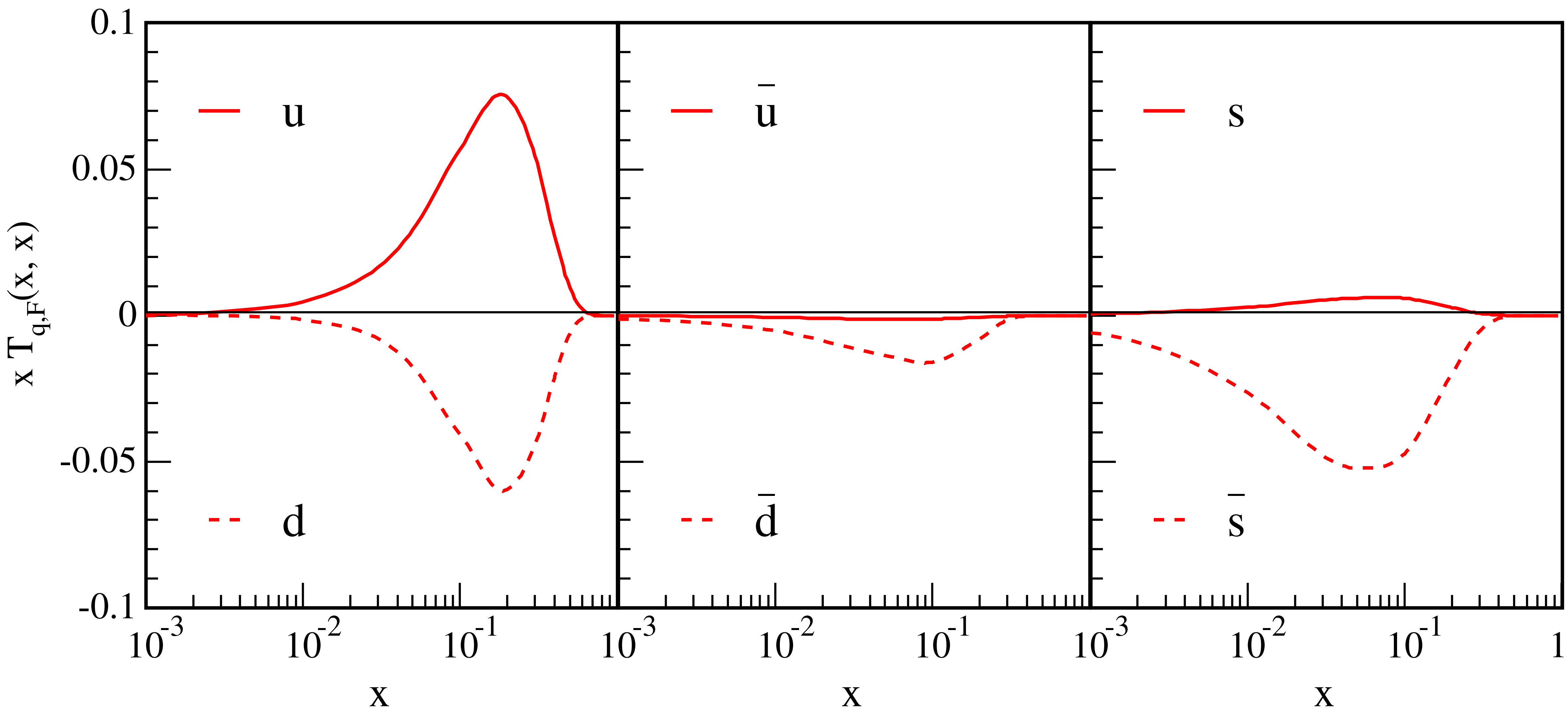


FIG. 11 (color online). Qiu-Sterman function $T_{q,F}(x, x, Q)$ for u, d and s flavors at a scale $Q^2 = 2.4$ GeV^2, as extracted by our simultaneous fit of JLab, HERMES and COMPASS data.

[Echevarria et al. - Phys. Rev. D.89.074013 (2014)]