Exotic quarkonia from effective field theories of QCD

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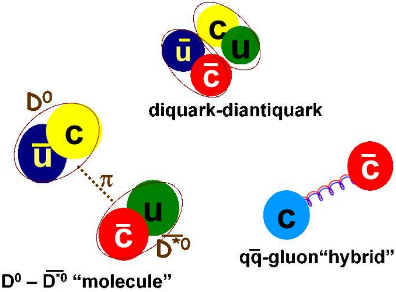
Exotic Quarkonia

- Quarkonium, $\bar{q}q'$ states, $q, q' = u, d, s, c, b$
- Exotic, non-trivial quark or/and gluon content beyond the constituent $\bar{q}q'$
  - May lead to exotic J^{PC}: 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, ...



Godfrey, Olsen, 2008

Exotic Heavy Quarkonia

- We shall restrict ourself to $q, q' = Q, Q' = c, b$
- Motivation:
  - Undestanding the XYZ states from QCD:
    - Hidden charm (hidden bottom)
    - Above open charm (bottom) threshold
    - Do not fit usual potential model expectations
  - Tools:
    - Effective Field Theories
    - Lattice QCD inputs

XYZ with $c\bar{c}$ (Olsen, 15)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K + (J/\psi\, \pi^+\pi^-)$
				$p\bar{p} \to (J/\psi\, \pi^+\pi^-) + \ldots$
				$B \to K + (J/\psi\, \pi^+\pi^-\pi^0)$
				$B \to K + (D^0 \bar{D}^0 \pi^0)$
				$B \to K + (J/\psi\, \gamma)$
				$B \to K + (\psi'\, \gamma)$
				$pp \to (J/\psi\, \pi^+\pi^-) + \ldots$
$X(3915)$	3917.4 ± 2.7	$28^{+10}_{-\ 9}$	0^{++}	$B \to K + (J/\psi\, \omega)$
				$e^+e^- \to e^+e^- + (J/\psi\, \omega)$
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$0(?)^{-(?)+}$	$e^+e^- \to J/\psi + (D^* \bar{D})$
				$e^+e^- \to J/\psi + (\ldots)$
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+e^- \to \gamma + (D\bar{D})$
$Y(4008)$	$4008^{+121}_{-\ 49}$	226 ± 97	1^{--}	$e^+e^- \to \gamma + (J/\psi\, \pi^+\pi^-)$
$Y(4140)$	4144 ± 3	17 ± 9	$?^{?+}$	$B \to K + (J/\psi\, \phi)$
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$0(?)^{-(?)+}$	$e^+e^- \to J/\psi + (D^* \bar{D})$

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$Y(4260)$	4263^{+8}_{-9}	95 ± 14	1^{--}	$e^+e^- \to \gamma + (J/\psi\,\pi^+\pi^-)$
				$e^+e^- \to (J/\psi\,\pi^+\pi^-)$
				$e^+e^- \to (J/\psi\,\pi^0\pi^0)$
$Y(4360)$	4361 ± 13	74 ± 18	1^{--}	$e^+e^- \to \gamma + (\psi'\,\pi^+\pi^-)$
$X(4630)$	$4634^{+\ 9}_{-11}$	92^{+41}_{-32}	1^{--}	$e^+e^- \to \gamma\,(\Lambda_c^+\Lambda_c^-)$
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^- \to \gamma + (\psi'\,\pi^+\pi^-)$
$Z_c^+(3900)$	3890 ± 3	33 ± 10	1^{+-}	$Y(4260) \to \pi^- + (J/\psi\,\pi^+)$
				$Y(4260) \to \pi^- + (D\bar{D}^*)^+$
$Z_c^+(4020)$	4024 ± 2	10 ± 3	$1(?)^{+(?)-}$	$Y(4260) \to \pi^- + (h_c\,\pi^+)$
				$Y(4260) \to \pi^- + (D^*\bar{D}^*)^+$
$Z_c^0(4020)$	4024 ± 4	10 ± 3	$1(?)^{+(?)-}$	$Y(4260) \to \pi^0 + (h_c\,\pi^0)$
$Z_1^+(4050)$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$B \to K + (\chi_{c1}\,\pi^+)$
$Z^+(4200)$	4196^{+35}_{-32}	370^{+99}_{-149}	1^{+-}	$B \to K + (J/\psi\,\pi^+)$
$Z_2^+(4250)$	$4248^{+185}_{-\ 45}$	$177^{+321}_{-\ 72}$	$?^{?+}$	$B \to K + (\chi_{c1}\,\pi^+)$
$Z^+(4430)$	4477 ± 20	181 ± 31	1^{+-}	$B \to K + (\psi'\,\pi^+)$
				$B \to K + (J\psi\,\pi^+)$

XYZ with $b\bar{b}$ (Olsen, 15)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$Y_b(10890)$	10888.4±3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\Upsilon(nS)\,\pi^+\pi^-)$
$Z_b^+(10610)$	10607.2±2.0	18.4±2.4	1^{+-}	$\Upsilon(5S) \to \pi^- + (\Upsilon(1,2,3S)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (h_b(1,2P)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (B\bar{B}^*)^+$
$Z_b^0(10610)$	10609± 6		1^{+-}	$\Upsilon(5S) \to \pi^0 + (\Upsilon(1,2,3S)\,\pi^0)$
$Z_b^+(10650)$	10652.2±1.5	11.5±2.2	1^{+-}	$\Upsilon(5S) \to \pi^- + (\Upsilon(1,2,3S)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (h_b(1,2P)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (B^*\bar{B}^*)^+$

Heavy Quarkonium Hybrids

We shall further restrict ourself to the case of hybrids

- Hybrids: have a non-trivial gluon content

Pioneering works:

- Exotics: MIT bag model (Jaffe, Johnson, 76)
- Heavy Hybrids:
  - String model (Giles, Tye, 77; Horn, Mandula, 78)
  - Born-Oppenheimer approximation (Hasenfratz, Horgan, Kuti, Richard, 80)
  - Lattice potentials (Griffiths, Rakow, Michael, 83)

Outline

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q >> \Lambda_{QCD}$, $\alpha_s(m_Q) << 1$

- Heavy quarks move slowly $v << 1$
- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q >> m_Q v$ (Relative momentum)
  - $m_Q v >> m_Q v^2$ (Binding energy)
  - $m_Q >> \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))
  - NRQCD: $m_Q \gg m_Q v$, $m_Q v^2$, Λ_{QCD} (W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986))
  - pNRQCD (weak coupling): $m_Q v \gg m_Q v^2$, Λ_{QCD} (A. Pineda, JS, Nucl.Phys.Proc.Suppl.64:428-432,1998)
  - pNRQCD (strong coupling): $m_Q v$, $\Lambda_{QCD} \gg m_Q v^2$ (N. Brambilla, A. Pineda, JS, A. Vairo, Nucl.Phys.B566:275,2000)

NRQCD

W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986)
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125

$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\begin{aligned}\mathcal{L}_\psi \quad = \quad & \psi^\dagger \Bigg\{ iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 + \frac{1}{8m_Q^3}\mathbf{D}^4 + \frac{c_F}{2m_Q}\boldsymbol{\sigma}.g\mathbf{B} + \\ & + \frac{c_D}{8m_Q^2}\left(\mathbf{D}.g\mathbf{E} - g\mathbf{E}.\mathbf{D}\right) + i\,\frac{c_S}{8m_Q^2}\boldsymbol{\sigma}.\left(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D}\right)\Bigg\}\psi\end{aligned}$$

c_F, c_D and c_S are short distance matching coefficients calculable from QCD in powers of $\alpha_{\rm s}$. They depend on m_Q and μ (factorization scale) but not on the lower energy scales.

pNRQCD weak coupling regime $\Lambda_{QCD} \lesssim m_Q v^2$

$$\mathcal{L}_{\rm pNRQCD} = \int d^3\mathbf{r}\ {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - h_s(\mathbf{r},\mathbf{p},\mathbf{P_R},\mathbf{S}_1,\mathbf{S}_2,\mu)\right) {\rm S} + \right.$$

$$\left. + {\rm O}^\dagger \left(iD_0 - h_o(\mathbf{r},\mathbf{p},\mathbf{P_R},\mathbf{S}_1,\mathbf{S}_2,\mu)\right) {\rm O} \right\}$$

$$+ V_A(r,\mu) {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} \right\} +$$

$$+ \frac{V_B(r,\mu)}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r}\cdot g\mathbf{E} \right\} + \mathcal{O}(\mathbf{r}^2, \frac{1}{m_Q})$$

- $h_{s,o} = \frac{\mathbf{p}^2}{m_Q} + V_{s,o}(r,\mu) + \mathcal{O}(\frac{1}{m_Q})$, quantum mechanical hamiltonians with scale dependent potentials calculable in pertubation theory in $\alpha_{\rm s}(m_Q v)$ and $1/m_Q$
- Spin symmetry holds in $h_{s,o}$ up to $\mathcal{O}(\frac{1}{m_Q^2})$
- S=S($\mathbf{r}$, $\mathbf{R}$, t), O=O($\mathbf{r}$, $\mathbf{R}$, t) are the color singlet/octet wave function fields
- $\mathbf{E}$=$\mathbf{E}$($\mathbf{R}$, t) is the chromoelectric field

pNRQCD strong coupling regime $\Lambda_{QCD} \lesssim mv$

$$L_{\rm pNRQCD} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \; S^\dagger \big(i\partial_0 - h_s(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)\big) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \cdots ,$$

All V_ss can be, and most of them have been, calculated on the lattice

- $V_s^{(0)}$ and $V_s^{(1)}$ are central Spin Symmetry holds
- $V_s^{(2)}$ contains spin and velocity dependent terms

pNRQCD strong coupling regime at LO

- Matching to NRQCD in the static limit $\Rightarrow$ $V_s^{(0)}$ is the ground state energy of two static color sources separated at a distance r
- Can be extracted from lattice calculations of the Wilson loop

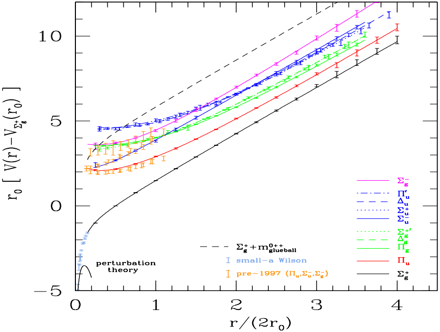


Figure: Meyer, Swanson, 2015

- Well fitted by the Cornell potential

$$V_s^{(0)} = V_{\Sigma_g^+}(r) \approx -\frac{k_g}{r} + \kappa r + E_g^{Q\bar{Q}} \quad , \; k_g = 0.489 \quad , \kappa = 0.187\, GeV^2$$

pNRQCD strong coupling regime at LO

Charmonium spin averages

$$m_c = 1.47\,GeV \quad , \quad E_g^{c\bar{c}} = -0.242\,GeV$$

nL	$M_{c\bar{c}}$	$M_{c\bar{c}\,EXP}$	$S=0$	$S=1$
1S	3068 (0)	3068	0^{-+}	1^{--}
1P	3494 (-31)	3525	1^{+-}	$(0,1,2)^{++}$
2S	3678 (4)	3674	0^{-+}	1^{--}
1D	3793 (20)	3773	2^{-+}	$(1,2,3)^{--}$
2P	3968 (41)	3927	1^{+-}	$(0,1,2)^{++}$
3S	4130 (91)	4039	0^{-+}	1^{--}
2D	4210 (57)	4153	2^{-+}	$(1,2,3)^{--}$
4S	4517 (96)	4421	0^{-+}	1^{--}

pNRQCD strong coupling regime at LO

Bottomonium spin averages

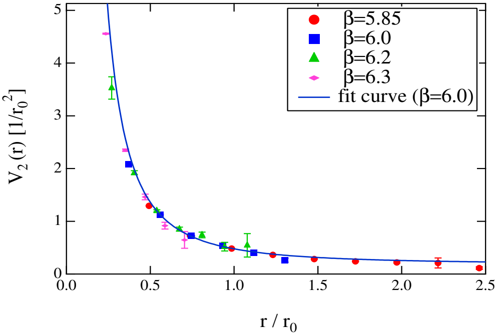
$$m_b = 4.88\,GeV \quad , \quad E_g^{b\bar{b}} = -0.228\,GeV$$

nL	$M_{b\bar{b}}$	$M_{b\bar{b}EXP}$	$S=0$	$S=1$
1S	9442 (-3)	9445	0^{-+}	1^{--}
1P	9908 (8)	10017	0^{-+}	1^{--}
1D	10155 (-9)	10164	2^{-+}	$(1,2,3)^{--}$
2P	10265 (5)	10260	$1+-$	$(0,1,2)^{++}$
3S	10356 (1)	10355	0^{-+}	1^{--}
4S	10638 (59)	10579	0^{-+}	1^{--}
5S	10885 (9)	10876	0^{-+}	1^{--}
6S	11110 (91)	11019	0^{-+}	1^{--}

pNRQCD strong coupling regime beyond LO

An example at $\mathcal{O}(1/m_Q^2)$: the $V_{L_2S_1}^{(1,1)}$ spin-orbit potential

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t\, \langle\langle g\mathbf{B}(t, \mathbf{r}/2) \times g\mathbf{E}(0, -\mathbf{r}/2) \rangle\rangle$$



$V_2 = V_{L_2S_1}^{(1,1)}/c_F$, Koma, Koma, 09

pNRQCD strong coupling regime beyond LO

An example at $\mathcal{O}(1/m_Q^2)$: the $V_{L_2S_1}^{(1,1)}$ spin-orbit potential

- Short distance constraint: it must coincide with the perturbative evaluation,

$$V_{L_2S_1}^{(1,1)}(r) \sim c_F \frac{C_F \alpha_s}{r^3} \quad , r \to 0$$

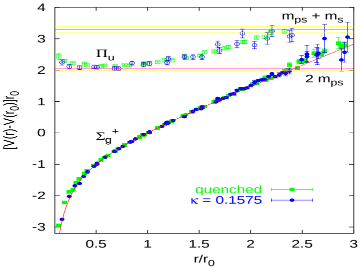
(Gupta, Radford, 81)

- Long distance constraint: it must coincide with the QCD effective string theory result

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F g^2 \Lambda^2 \Lambda'}{\kappa\, r^2} \quad , r \to \infty$$

(Perez-Nadal, JS, 08)

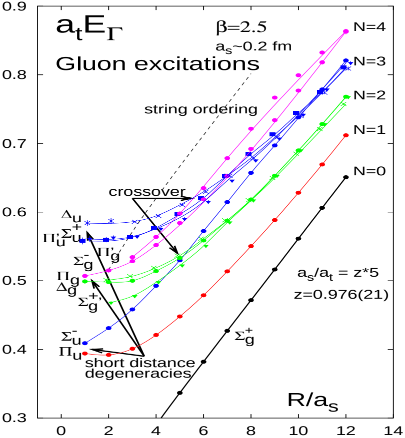
Exotic Heavy Quarkonia: The Static Limit



G.S. Bali at al. (TXL Collaboration), Phys. Rev. **D62**,(2000):054503

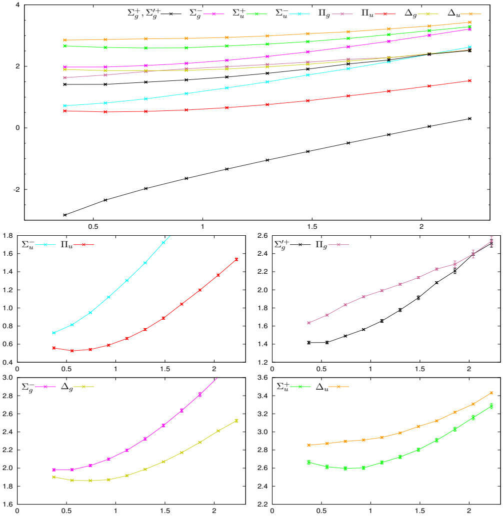
- Heavy Quarkonium: states built on the Σ_g^+ potential (color singlet)
- Heavy Hybrids: states built on the Π_u, ... potentials (color octet)
- Heavy Tetraquarks, Pentaquarks, ...: analogous to Hybrids but using operators that involve light quarks
- Molecular states: states built on $D\bar{D}$ or $B\bar{B}$
- ...

The Static Limit



Juge, Kuti, Morningstar (2002)

The Static Limit



Capitani, Philipsen, Reisinger, Riehl, Wagner (2018)

$D_{h\infty}$

- The symmetry group of a diatomic molecule (two equal atoms separated at a distance r)
- The generators are
  - Rotations around the z-axis, labeled by $|L| = 0, 1, 2, \cdots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Reflections about the xz plain, labeled by $\pm$ (only important for Σ states)
  - Parity, labeled by g (positive) and u (negative). In the case of a $Q\bar{Q}$ pair is replaced by CP.
- When $r \to 0$ reduces to $O(3)$ (plus C in the case of $Q\bar{Q}$), which implies short distance degeneracies

$$(\Sigma_u^-, \Pi_u) \quad , \quad (\Sigma_g^-, \Pi_g, \Delta_g) \quad , \quad (\Sigma_g^{+\prime}, \Pi_g^\prime) \quad , \quad (\Sigma_u^+, \Pi_u^\prime, \Delta_u) \quad , \quad \ldots$$

- When $r \to \infty$ the degeneracies of the QCD string must be reproduced

$$(\Pi_u) \quad , \quad (\Sigma_g^{+\prime}, \Pi_g, \Delta_g) \quad , \quad (\Sigma_u^+, \Sigma_u^-, \Pi_u^\prime, \Pi_u^{\prime\prime}, \Delta_u, \Phi_u) \quad , \quad \ldots$$

Heavy Hybrids (Oncala, JS, 17)

- The Hybrid potentials have a minimum at $r \sim 1/\Lambda_{QCD}$
- The energy fluctuations about the minimum $E \sim \sqrt{\Lambda_{QCD}^3/m_Q} \ll \Lambda_{QCD}$
- The energy scale Λ_{QCD} can be integrated out
- An EFT can be built for the short distance multiplets, in a way analogous to the strong coupling regime of pNRQCD
- We focus on the lowest lying doublet
  - At short distances it is described by $\mathrm{tr}(\mathrm{O}\mathbf{B})$ in weak coupling pNRQCD
  - We introduce a wave function field **H** with the same symmetry properties as $\mathrm{tr}(\mathrm{O}\mathbf{B})$ [$\mathrm{tr}(\mathrm{O}\mathbf{B}) \sim \bar{Q}\mathbf{B}Q$ in QCD]
  - The potentials for **H** are obtained from fitting the static energies of Juge, Kuti, Morningstar (2002), and imposing:
    - ⋆ Weak coupling pNRQCD constraints at short distances, $V_{\Lambda_\eta^\sigma} \sim \frac{\alpha_s}{2Nc}\frac{1}{r} + \cdots$
    - ⋆ QCD string constraints at long distances, $V_{\Lambda_\eta^\sigma} \sim \kappa r + \cdots$

Short distance operators

Table: Examples of short distance gluonic and light-quark operators for quarkonium hybrids and tetraquarks respectively, $\boldsymbol{q} = (u, d)$ and τ^a are isospin Pauli matrices.

Λ_η^σ	κ	H	$H = H^a T^a (I = 0, I = 1)$
Σ_g^+	0^{++}	1	$\bar{\boldsymbol{q}}\, T^a (1, \boldsymbol{\tau}) \boldsymbol{q}$
Σ_u^-	1^{+-}	$\hat{\mathbf{r}} \cdot \mathbf{B}$	$\bar{\boldsymbol{q}}\, [(\hat{\boldsymbol{r}} \times \gamma) \cdot, \gamma]\, T^a (1, \boldsymbol{\tau}) \boldsymbol{q}$
Π_u	1^{+-}	$\hat{\mathbf{r}} \times \mathbf{B}$	$\bar{\boldsymbol{q}}\, [\hat{\boldsymbol{r}} \cdot \gamma, \gamma]\, T^a (1, \boldsymbol{\tau}) \boldsymbol{q}$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\mathbf{r}} \cdot \mathbf{E}$	$\bar{\boldsymbol{q}}\, (\hat{\boldsymbol{r}} \cdot \gamma) T^a (1, \boldsymbol{\tau}) \boldsymbol{q}$
Π_g	1^{--}	$\hat{\mathbf{r}} \times \mathbf{E}$	$\bar{\boldsymbol{q}}\, (\hat{\boldsymbol{r}} \times \gamma) T^a (1, \boldsymbol{\tau}) \boldsymbol{q}$

Tarrús Castellà, 19

For the tetraquark case:

- No lattice data
- Unkown Effective String theory at long distances, if it exists at all
- Only short distance weak coupling pNRQCD constraints can be implemented

Spectrum

$$\mathcal{L} = H^{i\dagger} \left(\delta_{ij} i\partial_0 - h_{H\,ij}\right) H^j$$

$$h_{H\,ij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r)\right)\delta_{ij} + (\delta_{ij} - \hat{r}_i\hat{r}_j) V_q(r)$$

$$V_q(r) = V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)$$

$$\left[-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \begin{pmatrix} \frac{(J-1)J}{m_Q r^2} + V_q(r)\frac{J+1}{2J+1} & V_q(r)\frac{\sqrt{(J+1)J}}{2J+1} \\ V_q(r)\frac{\sqrt{(J+1)J}}{2J+1} & \frac{(J+1)(J+2)}{m_Q r^2} + V_q(r)\frac{J}{2J+1} \end{pmatrix} + V_{\Sigma_u^-}(r)\right] \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix} = E \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix}$$

$$\left(-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \frac{J(J+1)}{m_Q r^2} + V_{\Pi_u}(r)\right) P_J^0(r) = E P_J^0(r)$$

- $\mathbf{J} = \mathbf{L} + \mathbf{J}_g$
- $\mathbf{L}$= orbital angular momentum of the heavy quarks
- $\mathbf{J}_g$ = total angular momentum of the gluons, $|\mathbf{J}_g| = 1$
- Heavy quark spin independence

Results for charm

NL_J	w-f	$c\bar{c}$	Hybrid	$\mathcal{J}^{PC}$ $(S=0)$	$\mathcal{J}^{PC}$ $(S=1)$	Λ_η^ϵ
$1s$	S	3068		0^{-+}	1^{--}	Σ_g^+
$2s$	S	3678		0^{-+}	1^{--}	Σ_g^+
$3s$	S	4131		0^{-+}	1^{--}	Σ_g^+
$1p_0$, (H_3)	P^+		4486	0^{++}	1^{+-}	Σ_u^-
$4s$	S	4512		0^{-+}	1^{--}	Σ_g^+
$2p_0$	P^+		4920	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		5299	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		5642	0^{++}	1^{+-}	Σ_u^-
$1p$	S	3494		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p$	S	3968		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$, (H_1)	$P^\pm$		4011	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$, (H_2)	P^0		4145	1^{++}	$(0,1,2)^{+-}$	Π_u
$2(s/d)_1$	$P^\pm$		4355	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p$	S	4369		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p_1$	P^0		4511	1^{++}	$(0,1,2)^{+-}$	Π_u
$3(s/d)_1$	$P^\pm$		4692	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	$P^\pm$		4718	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4p$	S	4727		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3p_1$	P^0		4863	1^{++}	$(0,1,2)^{+-}$	Π_u
$5(s/d)_1$	$P^\pm$		5043	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$5p$	S	5055		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1d$	S	3793		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d$	S	4210		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$, (H_4)	$P^\pm$		4231	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$, (H_5)	P^0		4334	2^{--}	$(1,2,3)^{-+}$	Π_u
$2(p/f)_2$	$P^\pm$		4563	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d$	S	4579		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d_2$	P^0		4693	2^{--}	$(1,2,3)^{-+}$	Π_u
$3(p/f)_2$	$P^\pm$		4886	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$4d$	S	4916		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$4(p/f)_2$	$P^\pm$		4923	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d_2$	P^0		5036	2^{--}	$(1,2,3)^{-+}$	Π_u

Results for bottom

NL_J	w-f	$b\bar{b}$	Hybrid	$\mathcal{J}^{PC}$ $(S=0)$	$\mathcal{J}^{PC}$ $(S=1)$	$\Lambda^{\epsilon}_{\eta}$
1s	S	9442		0^{-+}	1^{--}	Σ_g^+
2s	S	10009		0^{-+}	1^{--}	Σ_g^+
3s	S	10356		0^{-+}	1^{--}	Σ_g^+
4s	S	10638		0^{-+}	1^{--}	Σ_g^+
$1p_0$, (H_3)	P^+		11011	0^{++}	1^{+-}	Σ_u^-
$2p_0$	P^+		11299	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		11551	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		11779	0^{++}	1^{+-}	Σ_u^-
1p	S	9908		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
2p	S	10265		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
3p	S	10553		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$, (H_1)	$P^{\pm}$		10690	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$, (H_2)	P^0		10761	1^{++}	$(0,1,2)^{+-}$	Π_u
4p	S	10806		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2(s/d)_1$	$P^{\pm}$		10885	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$2p_1$	P^0		10970	1^{++}	$(0,1,2)^{+-}$	Π_u
5p	S	11035		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3(s/d)_1$	$P^{\pm}$		11084	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	$P^{\pm}$		11156	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p_1$	P^0		11175	1^{++}	$(0,1,2)^{+-}$	Π_u
6p	S	11247		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$5(s/d)_1$	$P^{\pm}$		11284	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
1d	S	10155		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
2d	S	10454		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
3d	S	10712		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$, (H_4)	$P^{\pm}$		10819	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$, (H_5)	P^0		10870	2^{--}	$(1,2,3)^{-+}$	Π_u
$4d$	S	10947		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2(p/f)_2$	$P^{\pm}$		11005	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$2d_2$	P^0		11074	2^{--}	$(1,2,3)^{-+}$	Π_u
5d	S	11163		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$3(p/f)_2$	$P^{\pm}$		11197	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d_2$	P^0		11275	2^{--}	$(1,2,3)^{-+}$	Π_u
$4(p/f)_2$	$P^{\pm}$		11291	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$

Comments

- Only inputs:
  - Lattice potential
  - Heavy Quarkonium spectrum
  - $m_c = 1.47$ GeV, $m_b = 4.88$ GeV
- Braaten, Langmack, Hudson Smith, 2014
  - Neglect $\Pi_u - \Sigma_u^-$ coupled channel
  - Fix the lowest lying state for each potential to the Hadron Spectrum Collaboration, 2012 (HSC)
- Berwin, Brambilla, Tarrús Castellà, Vairo, 2015
  - Based on the weak coupling regime of pNRQCD
  - Equivalent equations, different potentials and normalization
  - Our results are consistent with the lower end of their error bars
- The hierarchy of the lower lying hybrid multiplets agrees with the lattice HSC 2016 for charmonium but our numbers are about 380-150 MeV lower

Hybrids in XYZ?

State	M	J^{PC}	XYZ	M_{exp}	Γ_{exp}	J^{PC}_{exp}
$1(s/d)_1$	4011	1^{--},$(0,1,2)^{-+}$	Y(4008)	4008^{+121}_{-49}	226 ± 97	1^{--}
$1p_1$	4145	1^{++},$(0,1,2)^{+-}$	Y(4140) X(4160)	$4144,5 \pm 2,6$ 4156^{+29}_{-25}	15^{+11}_{-7} 139^{+113}_{-65}	1^{++} $?^{?+}$
$2(s/d)_1$	4355	1^{--},$(0,1,2)^{-+}$	X(4320) X(4350) Y(4360) Y(4390)	4320 ± 17 4351 ± 5 4361 ± 13 4391 ± 6	101 ± 30 13^{+18}_{-10} 74 ± 18 139 ± 16	1^{--} $?^{?+}$ 1^{--} 1^{--}
$1p_0$	4486	0^{++},1^{+-}	X(4500)	4506^{+16}_{-19}	92^{+30}_{-29}	0^{++}
$3(s/d)_1$	4692	1^{--},$(0,1,2)^{-+}$	Y(4660) X(4630)	4664 ± 12 4634^{+9}_{-11}	48 ± 15 92^{+41}_{-32}	1^{--} 1^{--}
$2(s/d)_1$	10885	1^{--},$(0,1,2)^{-+}$	$Y_b(10890)$	$10888,4 \pm 3$	$30,7^{+8,9}_{-7,7}$	1^{--}

- C-parity implies that only spin zero hybrids would have been observed, except for $X(4350)$.
- Decays to vector heavy quarkonium states have been observed for all spin zero 1^{--} states above, except for $X(4630)$ and $Y(4390)$, which disfavors the hybrid interpretation due to spin symmetry (Braaten, Langmack, Hudson Smith, 14; Berwin, Brambilla, Tarrús Castellà, Vairo, 15).

Mixing with Heavy Quarkonium (Oncala, JS, 17)

- States with the same quantum numbers may mix
  - Mixing is a $1/m_Q$ suppressed effect
  - If the energy gap $\Delta E \lesssim \Lambda_{QCD}^2/m_Q$ it becomes next to leading order
  - For $\Lambda_{QCD} = 400$ MeV: $\Delta E|_c \sim 100$ MeV, $\Delta E|_b \sim 30$ MeV

NL_J	w-f	$c\bar{c}$	Hybrid	$\mathcal{J}^{PC}\,(S=0)$	$\mathcal{J}^{PC}\,(S=1)$	Λ_η^ϵ
$3s$	S	4131		0^{-+}	1^{--}	Σ_g^+
$1p_0$	P^+		4486	0^{++}	1^{+-}	Σ_u^-
$2p$	S	3968		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$	$P^\pm$		4011	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$	H^0		4145	1^{++}	$(0,1,2)^{+-}$	Π_u
$2(s/d)_1$	$P^\pm$		4355	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p$	S	4369		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2d$	S	4210		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$	$P^\pm$		4231	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$

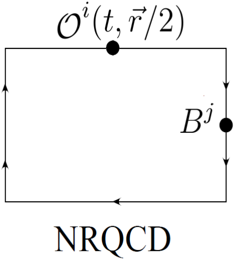
NL_J	w-f	$b\bar{b}$	Hybrid	$\mathcal{J}^{PC}\,(S=0)$	$\mathcal{J}^{PC}\,(S=1)$	Λ_η^ϵ
$2(s/d)_1$	$P^\pm$		10885	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4d$	S	10947		2^{-+}	$(1,2,3)^{--}$	Σ_g^+

Mixing with Heavy Quarkonium

- Symmetries imply

$$\mathcal{L}_{\rm mixing} = {\rm tr}\left[S^\dagger V_S^{ij} \left\{ \sigma^i \, , H^j \right\} + {\rm h.c.} \right]$$

- Matching to NRQCD at O($1/m_Q$) implies $V_S^{ij}(\mathbf{r}) \sim \frac{1}{m_Q} \times$



- $\mathcal{O}^i \sim B^i$

This term mixes $S = 0$ Hybrids with $S = 1$ Quarkonium, and may explain spin symmetry violating decays of the 1^{--} Y states

Mixing with Heavy Quarkonium

- $V_S^{ij} = (\delta^{ij} - \hat{r}^i\hat{r}^j)V_S^{\Pi_u} + \hat{r}^i\hat{r}^j V_S^{\Sigma_u^-}$
- No lattice evaluation of $V_S^{\Pi_u}$, $V_S^{\Sigma_u^-}$, so far
- Short distance constraints: pNRQCD at weak coupling implies for $r \to 0$

$$V_S^{\Pi_u}(r) \sim V_S^{\Sigma_u^-}(r) \to \pm\frac{\lambda^2}{m_Q} = const.$$

- Long distance constraints: QCD string (EST) implies for $r \to \infty$

$$V_S^{\Sigma_u^-}(r) \to -\frac{2\pi^2\Lambda'''}{m_Q\kappa r^3}\,, \quad V_S^{\Pi_u}(r) \to \sqrt{\frac{\pi^3}{\kappa}}\frac{\Lambda'}{m_Q r^2}$$

  - κ, $\Lambda^{'}$ and $\Lambda^{'''}$ also appear in the spin dependent potentials (Perez-Nadal, JS, 2011; Brambilla, Groher, Martinez, Vairo, 2014)
  - Can be extracted from available lattice results of the long distance potentials (Koma, Koma, 2007; 2010)

$$\kappa \sim 0.187\mathrm{GeV}^2 \quad , \quad \Lambda' \sim -59\mathrm{MeV} \quad , \quad \Lambda''' \sim \pm 230\mathrm{MeV}$$

Modeling the mixing potential

$$V_S^{\Pi}[\pm-](r) = \frac{\lambda^2}{m_Q}\left(\frac{\pm 1 - (\frac{r}{r_\Pi})^2}{1+(\frac{r}{r_\Pi})^4}\right), \quad r_\Pi = (\frac{|\Lambda'|\pi^{\frac{3}{2}}}{2\lambda^2\kappa^{\frac{1}{2}}})^{\frac{1}{2}}$$

$$V_S^{\Sigma}[\pm\pm](r) = \frac{\lambda^2}{m_Q}\left(\frac{\pm 1 \pm (\frac{r}{r_\Sigma})^2}{1+(\frac{r}{r_\Sigma})^5}\right), \quad r_\Sigma = (\frac{|\Lambda'''|\pi^2}{\lambda^2\kappa})^{\frac{1}{3}}$$

- Simple interpolation that allows for a sign flip between short and long distance behavior
- We focus on charm and scan $\lambda = 100, 300, 600$ MeV
- $V_S^{\Pi}[+-]$ with $V_S^{\Sigma}[++]$ and $\lambda = 600$ MeV produce the maximum mixing
- The spin symmetry violating decays of $Y(4008)$ (29% S=1), $Y(4360)$ (35% S=1) and $Y(4660)$ (17% S=1) are qualitatively explained
- It would be important that lattice calculations confirm the signs and size of the mixing potentials

Hybrid-Quarkonium coupled equations

- The $S = 0$ case:
  - $P^{L}_{0\,\mathcal{J}\mathcal{M}}(r)$, $J = \mathcal{J}$, $L = J \pm 1\,(\pm)\,, J\,(0)$
  - $S^{L}_{1\,\mathcal{J}\mathcal{M}}(r)$, $L = \mathcal{J} \pm 1\,(\pm)\,, \mathcal{J}\,(0)$
  - 2 coupled equations for $S^{0}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{0}_{0\,\mathcal{J}\mathcal{M}}(r)$
  - 4 coupled equations for $S^{+}_{1\,\mathcal{J}\mathcal{M}}(r)\,, S^{-}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{+}_{0\,\mathcal{J}\mathcal{M}}(r)\,, P^{-}_{0\,\mathcal{J}\mathcal{M}}(r)$
- The $S = 1$ case:
  - $P^{L\,J}_{1\,\mathcal{J}\mathcal{M}}(r)$, $J = \mathcal{J} \pm 1\,(\pm)\,, \mathcal{J}(0)$, $L = J \pm 1(\pm)\,, J\,(0)$
  - $S_{0\,\mathcal{J}\mathcal{M}}(r)$, $L = \mathcal{J}$
  - 2 coupled equations $P^{+0}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{-0}_{1\,\mathcal{J}\mathcal{M}}(r)$
  - 6 coupled equations
    $S_{0\,\mathcal{J}\mathcal{M}}(r)\,, P^{++}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{-+}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{+-}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{--}_{1\,\mathcal{J}\mathcal{M}}(r)\,, P^{00}_{1\,\mathcal{J}\mathcal{M}}(r)$

Results

Resonance	J^{PC}	Assignement	Mass (MeV)	Observations
X(3823)	2^{--}	$1d$	3792	
X(3860)	0 or 2^{++}	$2p$	3968	
X(3872)	1^{++}	$2p$	3967	
X(3915)	0 or 2^{++}	$2p$	3968	
X(3940)	$?^{??}$	$2p$	3968	
Y(4008)	1^{--}	$1(s/d)_1$	4004	mixing
X(4140)	1^{++}	??	??	$1p_1$ does not decay to quarkonium
X(4160)	$?^{??}$	$1p_1$	4146	
Y(4220)	1^{--}	$2d$	4180	$Y(4260) \to Y(4220)$, mixing
X(4230)	1^{--}	$2d$	4180	$X(4230) = Y(4220)$, mixing
X(4350)	$?^{?+}$	$2(s/d)_1$ or 3p	4355 or 4369	
Y(4320)	1^{--}	$2(s/d)_1$	4366	mixing
Y(4360)	1^{--}	$2(s/d)_1$	4366	$Y(4360) = Y(4320)$?
X(4390)	1^{--}	$2(s/d)_1$	4366	$Y(4390) = Y(4360)$?
X(4500)	0^{++}	$1p_0$	4566	not enough mixing
Y(4630)	1^{--}	$3d$	4559	
Y(4660)	1^{--}	$3(s/d)_1$	4711	mixing
X(4700)	0^{++}	$4p$	4703	
Υ(10860)	1^{--}	$5s$	10881	mixing
Y_b(10890)	1^{--}	$2(s/d)_1$	10890	mixing
Υ(11020)	1^{--}	$4d$	10942	

- A recent combined fit to $e^+e^- \to \omega\chi_{c0}$, $\pi^+\pi^- h_c$, $\pi^+\pi^- J/\psi$, $\pi^+\pi^-\psi(3686)$ only needs $Y(4220)$, $Y(4390)$ and $Y(4660)$ in order to describe data
- Implies $Y(4220) = X(4230)$ and $Y(4390) = Y(4360) = Y(4320)$, consistent with our spectrum

Zhang, Yuan, Wang, 18

1^{--} charmonium spectrum

- $Y(4008)$ has not been confirmed by LHCb and BESIII. If it is not there:

NL_J	$\lambda = 0.6$	Hybrid %	PDG
$1s$	3.001	4	J/ψ
$2s$	3.628	14	$\psi(2S)$
$1d$	3.687	12	$\psi(3773)$
$1(s/d)_1$	4.014	71	$\psi(4040)$
$3s$	4.107	10	$\psi(4160)$
$2d$	4.180	79	$X(4230) = X(4260) = Y(4220)$
$2(s/d)_1$	4.366	65	$X(4360) = Y(4390)$
$4s$	4.497	0	$\psi(4415)$
$3d$	4.559	8	$Y(4630)$
$3(s/d)_1$	4.711	83	$X(4660)$

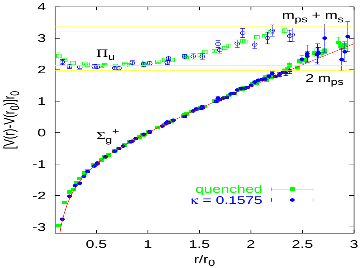
- Decays to h_c should be observed for $\psi(4040)$, $X(4230)/X(4260)$, $X(4360)$ and $X(4660)$

1^{--} bottomonium spectrum

NL_J	$\lambda = 0.6$	Hybrid %	PDG
$1s$	9.441	0	$\Upsilon(1S)$
$2s$	10.000	2	$\Upsilon(2S)$
$1d$	10.133	2	$\Upsilon(1D)$
$3s$	10.352	0	$\Upsilon(3S)$
$2d$	10.440	2	$\Upsilon(10.520)$? (Belle, 19)
$4s$	10.635	1	$\Upsilon(4S)$
$1(s/d)_1$	10.688	79	??
$3d$	10.713	56	$\Upsilon(10.750)$ (Belle, 19)
$5s$	10.881	17	$\Upsilon(10860)$
$2(s/d)_1$	10.886	75	$Y_b(10890)$
$4d$	10.942	11	$\Upsilon(11020)$

- The 17% hybrid component of $\Upsilon(10860)$ may explain the observed spin symmetry violating decays to h_b

Decay to lowest lying Heavy Quarkonium



- If the energy gap ΔE to lowest lying heavy quarkonium is $\Delta E \gtrsim 1\text{GeV}$ $\Rightarrow$ a perturbative estimate makes sense.
- If $\Delta E \left\langle H|r|Q\bar{Q}\right\rangle \ll 1$ $\Rightarrow$ weak coupling pNRQCD can be used

$$\Gamma(H_m \to S_n) = \frac{4}{3}\frac{\alpha_s T_F}{N_c}\left\langle H_m|r^i|S_n\right\rangle\left\langle S_n|r^i|H_m\right\rangle(\Delta E_n)^3$$

Results

- Hybrids with $L = J$ do not decay to Heavy Quarkonium

Charm

$NL_J \to nL$	ΔE (MeV)	$\langle r \rangle_{nm}$ (GeV^{-1})	$\lvert \Delta E \langle r \rangle_{nm} \rvert$	Γ (MeV)
$1p_0 \to 2s$	808	0.40	0.32	6.1
$2(s/d)_1 \to 1p$	861	0.63	0.54	19

Bottom

$NL_J \to nL$	ΔE (MeV)	$\langle r \rangle_{nm}$ (GeV^{-1})	$\lvert \Delta E \langle r \rangle_{nm} \rvert$	Γ (MeV)
$1p_0 \to 1s$	1569	-0.416	0.65	31
$1p_0 \to 2s$	1002	0.432	0.43	8.7
$2p_0 \to 1s$	1857	-0.422	0.78	53
$2p_0 \to 2s$	1290	-0.137	0.18	1.9
$2p_0 \to 3s$	943	0.462	0.44	8.3
$2(s/d)_1 \to 1p$	977	0.470	0.46	9.6

Hyperfine Splittings

Solé; JS, 17

- They appear at $\mathcal{O}(1/m_Q)$ ($\mathcal{O}(1/m_Q^2)$) in hybrids (quarkonium)
- They are controlled by a single operator

$$i\epsilon^{ijk} V^S(r) tr\left(H^{i\dagger}\left[\sigma^k, H^j\right]\right)$$

- It leads to the following mass formulae

$$\frac{M_{1\,J+1} - M_{0\,J}}{M_{1\,J} - M_{0\,J}} = -J \qquad \frac{M_{1\,J-1} - M_{0\,J}}{M_{1\,J} - M_{0\,J}} = J+1$$

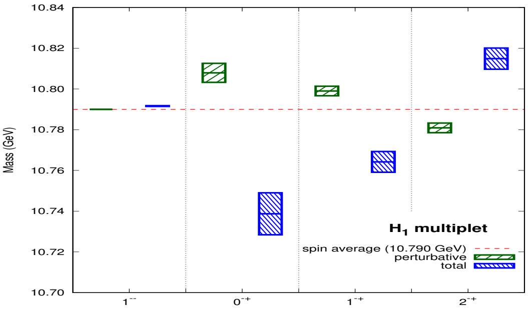
$$\begin{aligned} (s/d)_1: \quad & M_{2^{-+}} + M_{0^{-+}} &= M_{1^{-+}} + M_{1^{--}} \\ p_1: \quad & M_{2^{+-}} + M_{0^{+-}} &= M_{1^{+-}} + M_{1^{++}} \\ (p/f)_2: \quad & M_{3^{+-}} + M_{1^{+-}} &= M_{2^{+-}} + M_{2^{++}} \\ d_2: \quad & M_{3^{-+}} + M_{1^{-+}} &= M_{2^{-+}} + M_{2^{--}} \end{aligned}$$

- Consistent with the values of the lattice HSC
- Induces mixing between different hybrid states

Hyperfine Splittings

(Brambilla, Kin Lai, Segovia, Tarrús Castellà, Vairo, 18)

- $V^S(r) \to$ const. when $r \to 0$
- Calculate the relevant $1/m_Q^2$ potentials in the above limit
- Fit the unkown constants to reproduce the splitting of the HSC for charmonium
- Predict the hyperfine splittings for bottomonium



Conclusions

- Exotic Hadrons containing two heavy quarks can be studied in an EFT framework
  - NRQCD holds
  - Given the light degrees of freedom, an EFT similar to pNRQCD in the strong coupling regime can be built.
    - The LO is nothing but the Born-Oppenheimer approximation.
    - Lattice inputs are needed
- Heavy Hybrids containing $c\bar{c}$ or $b\bar{b}$ have been studied from QCD in a largely model independent way (EFT+lattice inputs)
  - The lower lying states have been calculated at LO in the $1/m_Q$ expansion of the potentials, including the $\Sigma_u^- - \Pi_u$ mixing.
  - The mixing with Heavy Quarkonium states has been addressed
    - It is an important source of spin symmetry violations that explains the decays of certain spin zero Hybrids to spin one Quarkonia.

Conclusions

- Heavy Hybrids containing $c\bar{c}$ or $b\bar{b}$ have been studied from QCD in a largely model independent way (EFT+lattice inputs) (Cont.)
  - The decay width to lower lying Heavy Quarkonium states has been estimated
    - ★ The decays of $J = L$ states are forbidden
  - Spin and velocity dependent terms in the hybrid potentials enter at $O(1/m_Q)$
    - ★ Model independent formulas for the hyperfine splittings have been produced
  - A number of them can be identified with XYZ states ($Y(4008)$, $X(4160)$, $X(4350)$, $Y(4320)/Y(4360)/Y(4390)$, $Y(4660)$, $Y_b(10890)$)

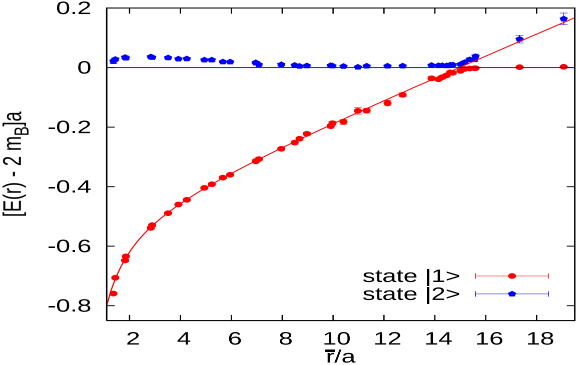
$D_{h\infty}$

- The symmetry group of a diatomic molecule (two equal atoms separated at a distance r)
- The generators are
  - Rotations around the z-axis, labeled by $|L| = 0, 1, 2, \cdots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Reflections about the xz plain, labeled by $\pm$ (only important for Σ states)
  - Parity, labeled by g (positive) and u (negative). In the case of a $Q\bar{Q}$ pair is replaced by CP.
- When $r \to 0$ reduces to $O(3)$ (plus C in the case of $Q\bar{Q}$)
  - Implies short distance degeneracies

$$(\Sigma_u^-, \Pi_u) \quad , \quad (\Sigma_g^-, \Pi_g, \Delta_g) \quad , \quad (\Sigma_g^{+\prime}, \Pi_g^\prime) \quad , \quad (\Sigma_u^+, \Pi_u^\prime, \Delta_u) \quad , \quad \ldots$$

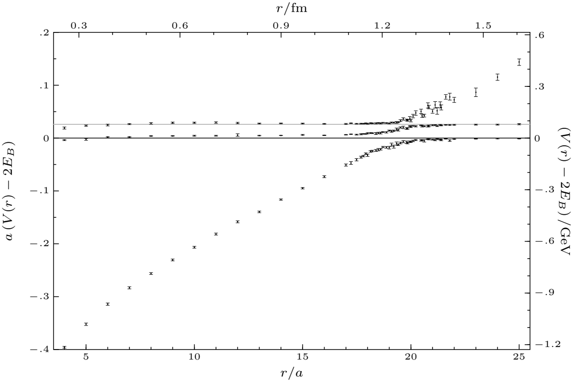
(Brambilla, Pineda, JS, Vairo, 99)

String breaking



Bali, Neff, Duessel, Lippert, Schilling, 2005

String breaking



Bulava, Hörz, Knechtli, Koch, Moir, Morningstar, Peardon, 2019

XYZ with $c\bar{c}$ (Olsen, 15)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K + (J/\psi\, \pi^+\pi^-)$
				$p\bar{p} \to (J/\psi\, \pi^+\pi^-) + \ldots$
				$B \to K + (J/\psi\, \pi^+\pi^-\pi^0)$
				$B \to K + (D^0 \bar{D}^0 \pi^0)$
				$B \to K + (J/\psi\, \gamma)$
				$B \to K + (\psi'\, \gamma)$
				$pp \to (J/\psi\, \pi^+\pi^-) + \ldots$
$X(3915)$	3917.4 ± 2.7	$28^{+10}_{-\ 9}$	0^{++}	$B \to K + (J/\psi\, \omega)$
				$e^+e^- \to e^+e^- + (J/\psi\, \omega)$
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$0(?)^{-(?)+}$	$e^+e^- \to J/\psi + (D^* \bar{D})$
				$e^+e^- \to J/\psi + (\ldots)$
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+e^- \to \gamma + (D\bar{D})$
$Y(4008)$	$4008^{+121}_{-\ 49}$	226 ± 97	1^{--}	$e^+e^- \to \gamma + (J/\psi\, \pi^+\pi^-)$
$Y(4140)$	4144 ± 3	17 ± 9	$?^{?+}$	$B \to K + (J/\psi\, \phi)$
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$0(?)^{-(?)+}$	$e^+e^- \to J/\psi + (D^* \bar{D})$

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$Y(4260)$	4263^{+8}_{-9}	95 ± 14	1^{--}	$e^+e^- \to \gamma + (J/\psi\, \pi^+\pi^-)$
				$e^+e^- \to (J/\psi\, \pi^+\pi^-)$
				$e^+e^- \to (J/\psi\, \pi^0\pi^0)$
$Y(4360)$	4361 ± 13	74 ± 18	1^{--}	$e^+e^- \to \gamma + (\psi'\, \pi^+\pi^-)$
$X(4630)$	$4634^{+\ 9}_{-11}$	92^{+41}_{-32}	1^{--}	$e^+e^- \to \gamma\, (\Lambda_c^+\Lambda_c^-)$
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^- \to \gamma + (\psi'\, \pi^+\pi^-)$
$Z_c^+(3900)$	3890 ± 3	33 ± 10	1^{+-}	$Y(4260) \to \pi^- + (J/\psi\, \pi^+)$
				$Y(4260) \to \pi^- + (D\bar{D}^*)^+$
$Z_c^+(4020)$	4024 ± 2	10 ± 3	$1(?)^{+(?)-}$	$Y(4260) \to \pi^- + (h_c\, \pi^+)$
				$Y(4260) \to \pi^- + (D^*\bar{D}^*)^+$
$Z_c^0(4020)$	4024 ± 4	10 ± 3	$1(?)^{+(?)-}$	$Y(4260) \to \pi^0 + (h_c\, \pi^0)$
$Z_1^+(4050)$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$B \to K + (\chi_{c1}\, \pi^+)$
$Z^+(4200)$	4196^{+35}_{-32}	370^{+99}_{-149}	1^{+-}	$B \to K + (J/\psi\, \pi^+)$
$Z_2^+(4250)$	$4248^{+185}_{-\ 45}$	$177^{+321}_{-\ 72}$	$?^{?+}$	$B \to K + (\chi_{c1}\, \pi^+)$
$Z^+(4430)$	4477 ± 20	181 ± 31	1^{+-}	$B \to K + (\psi'\, \pi^+)$
				$B \to K + (J\psi\, \pi^+)$

XYZ with $b\bar{b}$ (Olsen, 15)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$Y_b(10890)$	10888.4±3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\Upsilon(nS)\,\pi^+\pi^-)$
$Z_b^+(10610)$	10607.2±2.0	18.4±2.4	1^{+-}	$\Upsilon(5S) \to \pi^- + (\Upsilon(1,2,3S)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (h_b(1,2P)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (B\bar{B}^*)^+$
$Z_b^0(10610)$	10609± 6		1^{+-}	$\Upsilon(5S) \to \pi^0 + (\Upsilon(1,2,3S)\,\pi^0)$
$Z_b^+(10650)$	10652.2±1.5	11.5±2.2	1^{+-}	$\Upsilon(5S) \to \pi^- + (\Upsilon(1,2,3S)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (h_b(1,2P)\,\pi^+)$
				$\Upsilon(5S) \to \pi^- + (B^*\bar{B}^*)^+$