Parton distribution functions from lattice QCD

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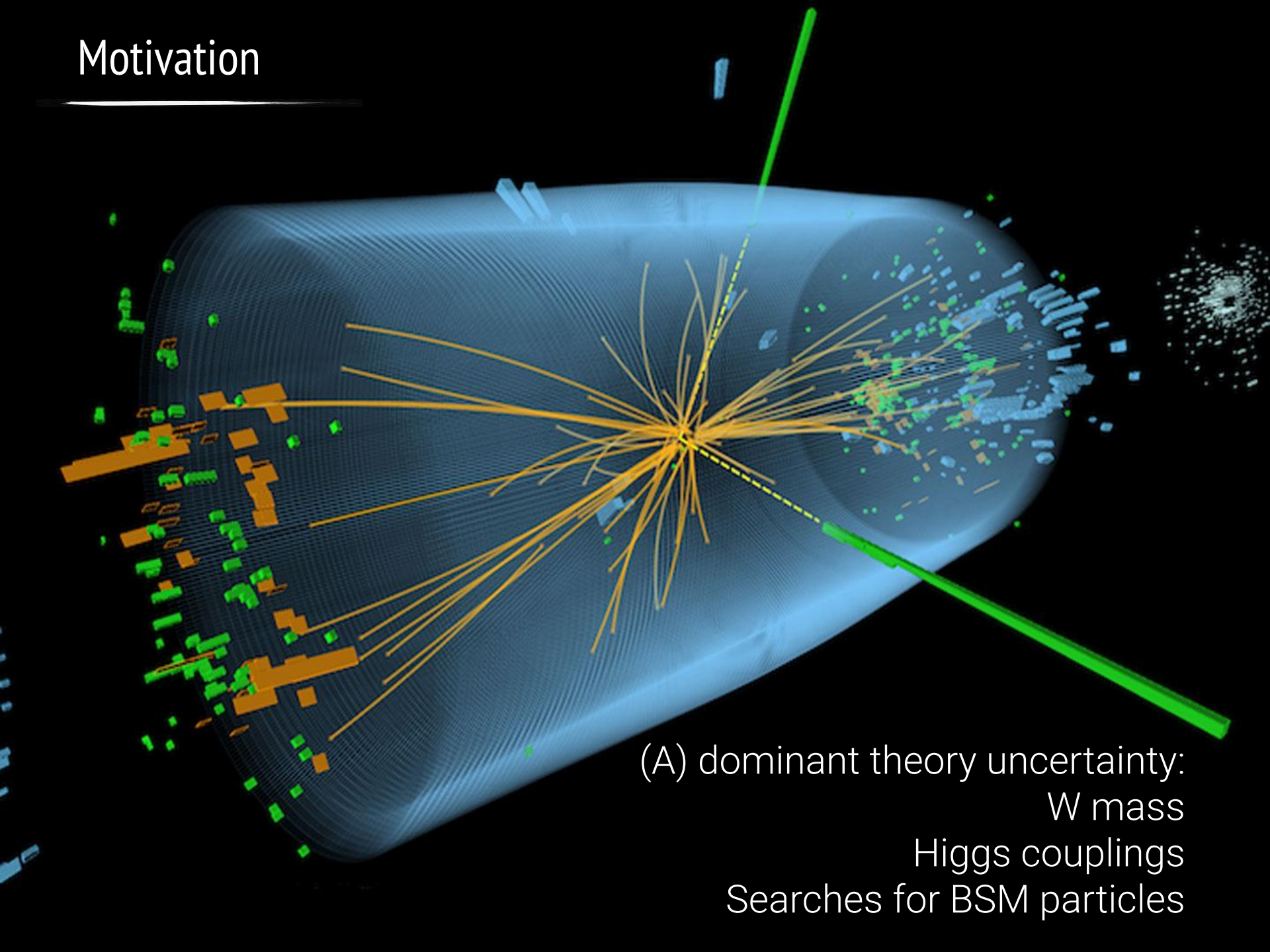
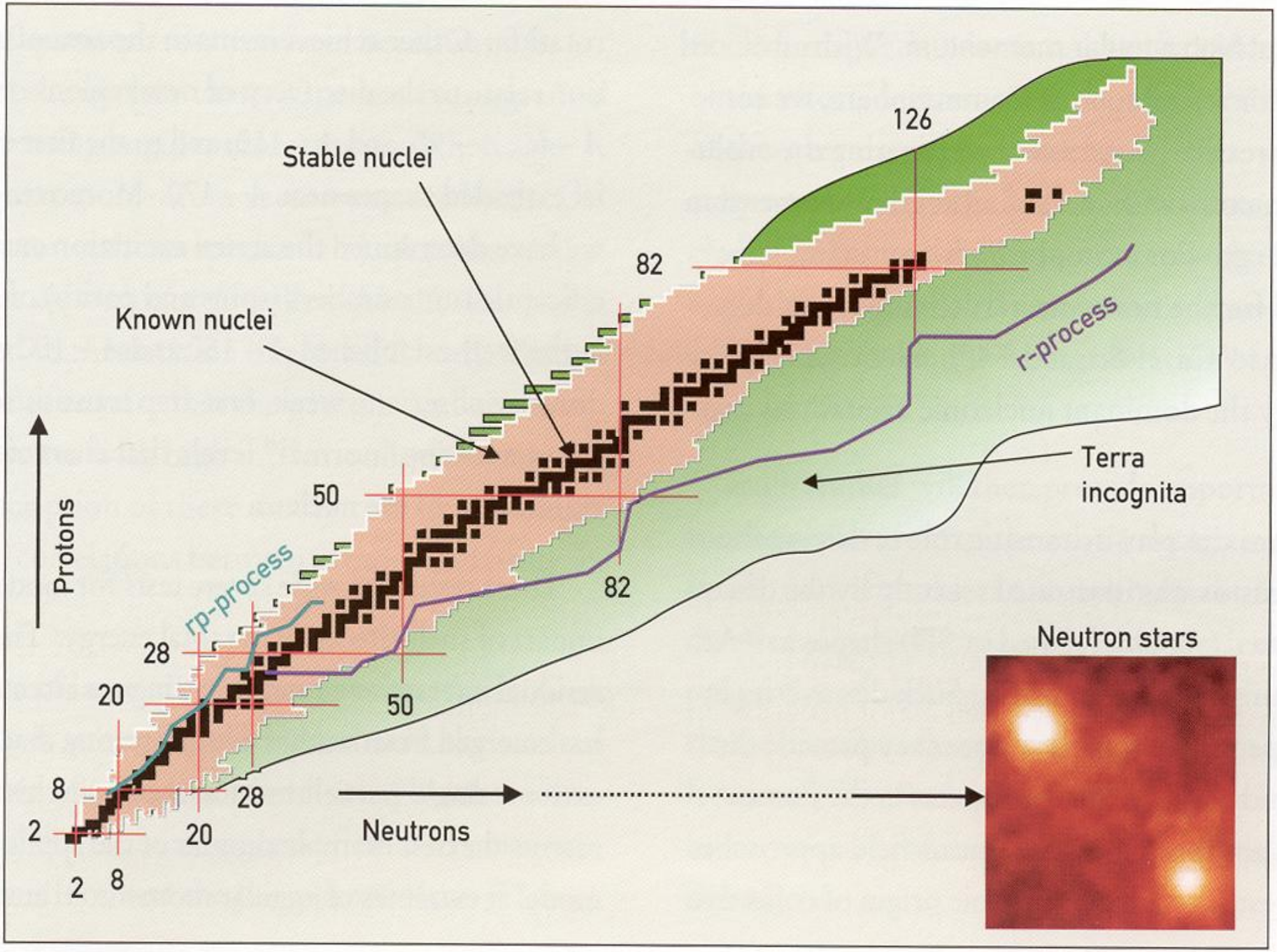
Motivation

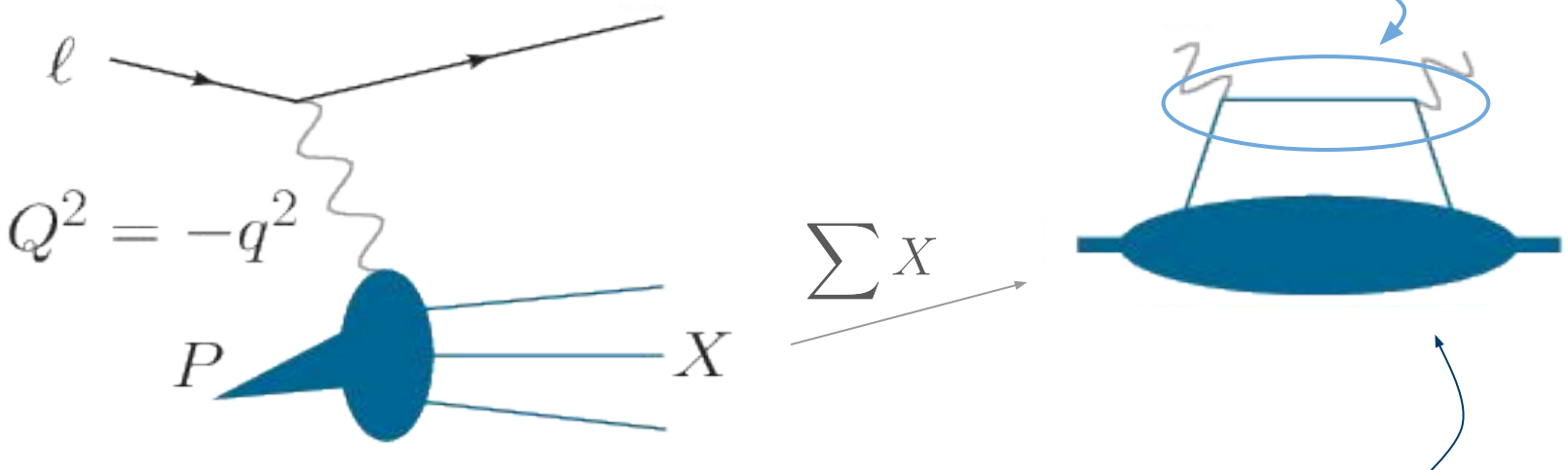


(A) dominant theory uncertainty:
W mass
Higgs couplings
Searches for BSM particles


Stable nuclei
Known nuclei
126
82
50
82
50
28
20
28
20
8
2
8
2
r-process
rp-process
Terra
incognita
Protons
Neutrons
Neutron stars

Deep inelastic scattering (DIS)

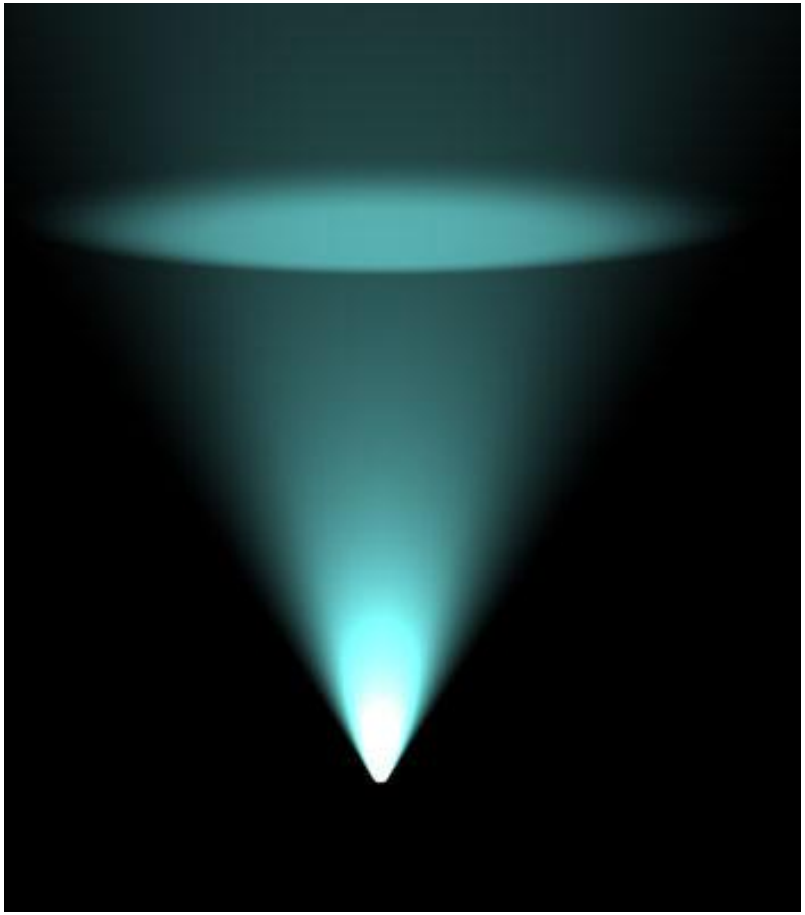
perturbative Wilson coefficient

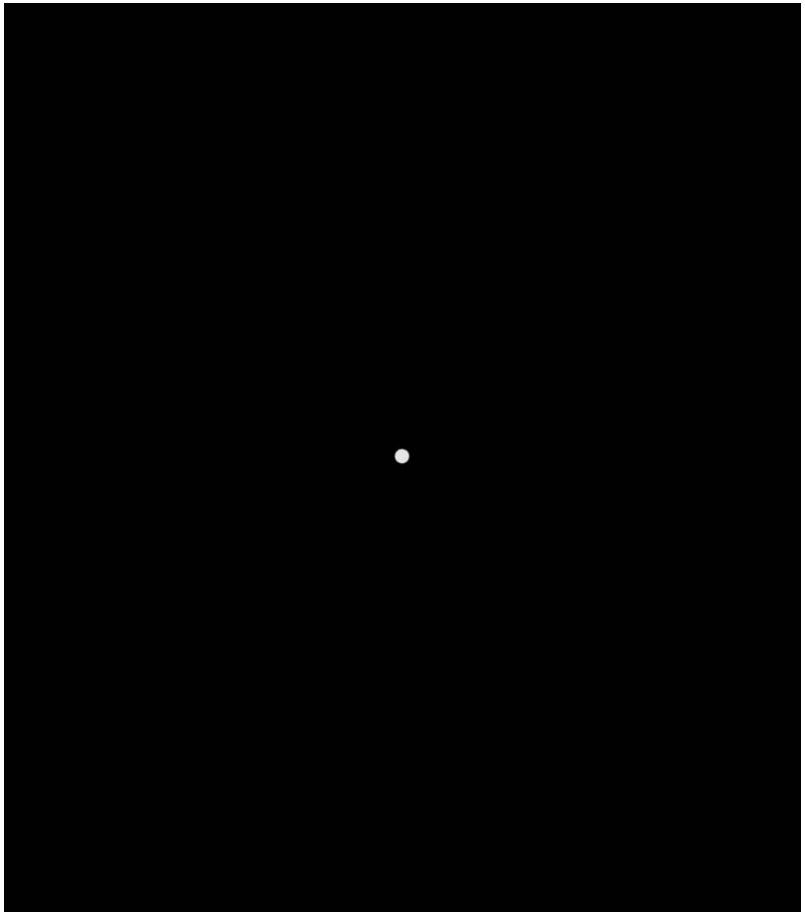


nonperturbative parton distribution function (PDF)

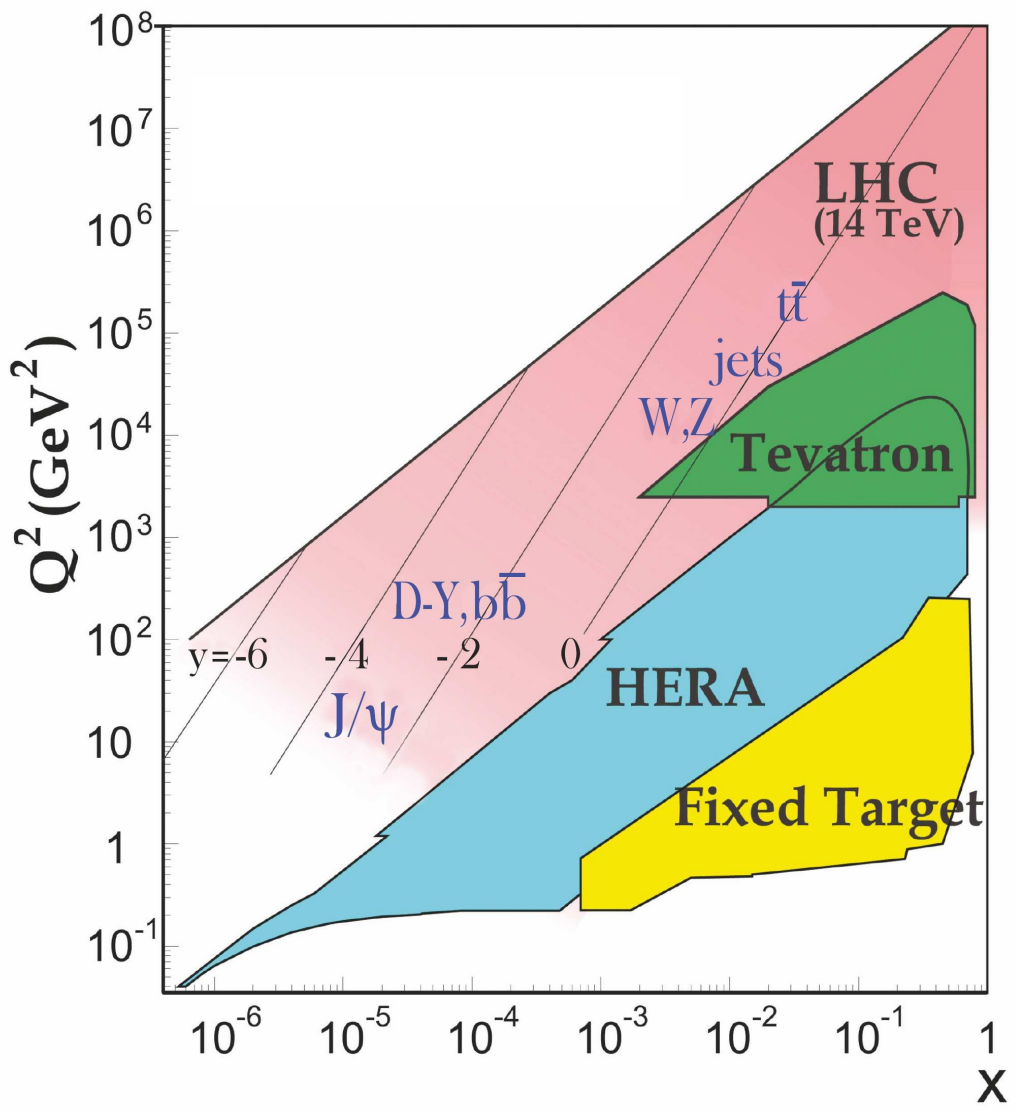
$$x = \frac{Q^2}{2q \cdot P}$$

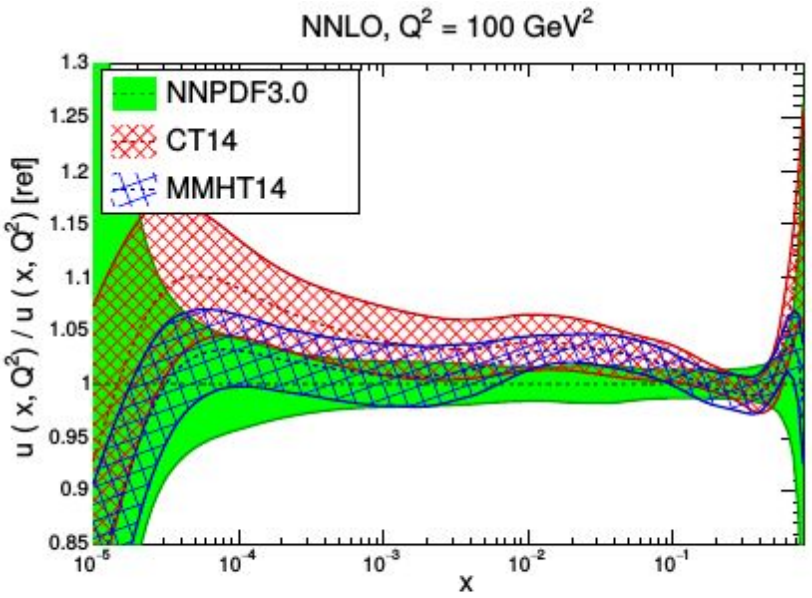
The challenge

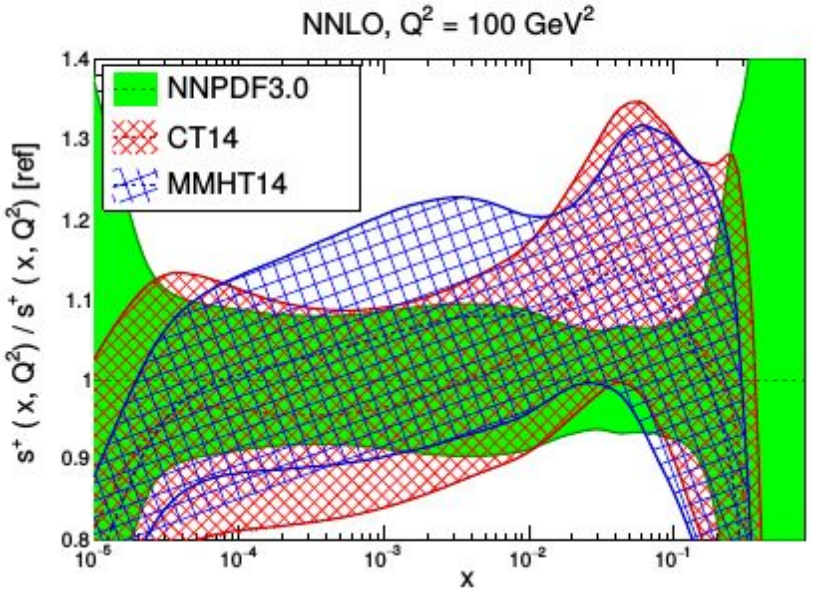




Phenomenological solution

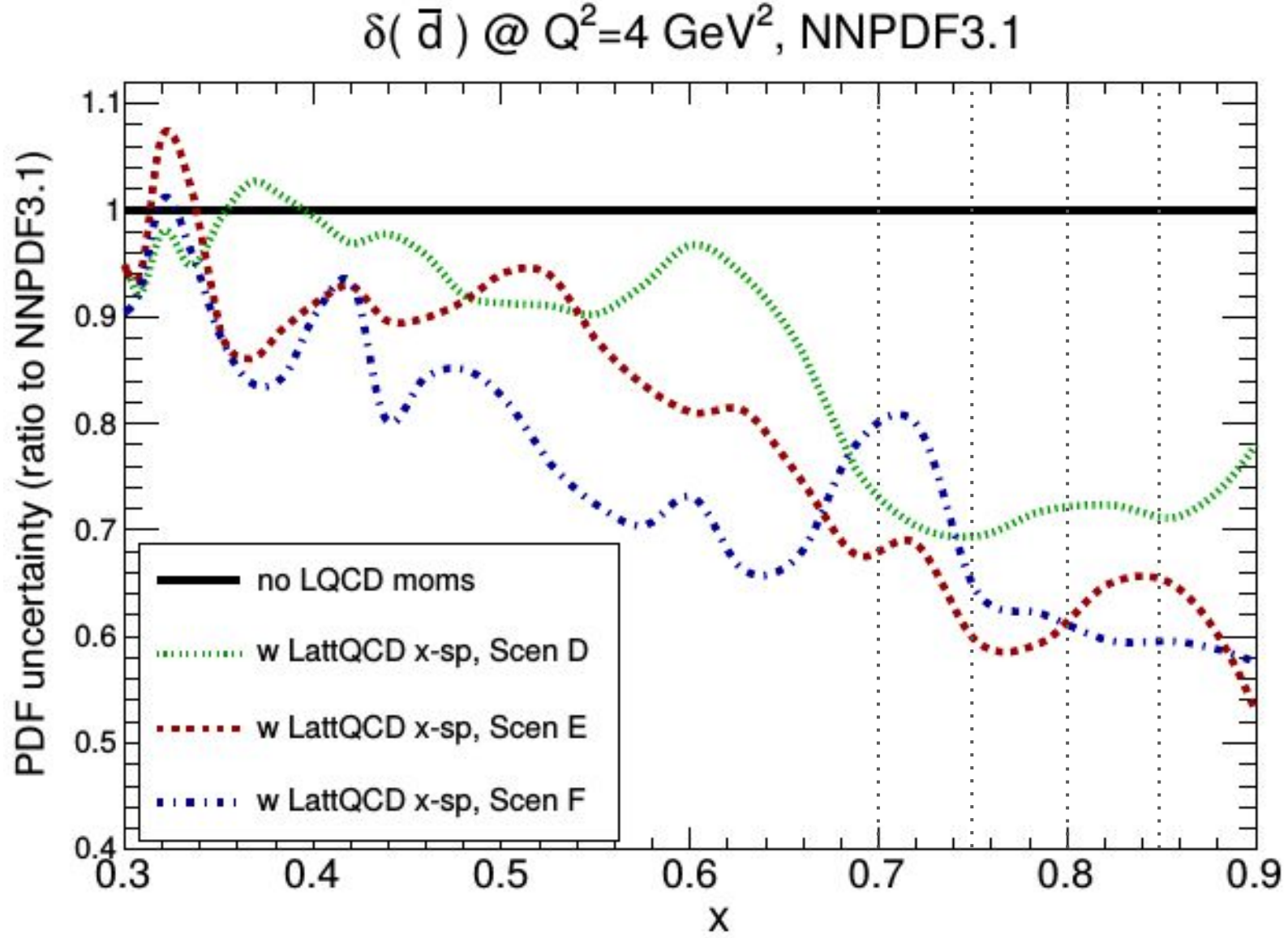






PDG, PRD 98 (2018) 030001

Impact of theory solutions



D: 12%
E: 6%
F: 3%

Lin et al., PPNP 100 (2018) 107

See also Hobbs et al., 1904.00022

Traditional approach

Mellin moments of PDFs

$$a_{j/H}^{(n)}(\mu) = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} \left[f_{j/H}(\xi,\mu) + (-1)^n f_{\bar{j}/H}(\xi,\mu) \right] = \int_{-1}^1 \mathrm{d}\xi\, f_{j/H}(\xi,\mu)$$

related to matrix elements of twist-two operators

$$\left\langle H(P) | \mathcal{O}^{\{\nu_1 \ldots \nu_n\}}(\mu) | H(P) \right\rangle = 2 a_{j/H}^{(n)}(\mu) \left(P^{\nu_1} \cdots P^{\nu_n} - \text{traces} \right)$$

$$\mathcal{O}^{\{\nu_1 \ldots \nu_n\}}(\mu) = Z_{\mathcal{O}}(\mu) \left[i^{n-1} \overline{\psi} \gamma^{\{\nu_1} \overleftrightarrow{D}^{\nu_2} \cdots \overleftrightarrow{D}^{\nu_n\}} \frac{\lambda^a}{2} \psi - \text{traces} \right]$$

Lattice regulator induces power-divergent mixing

$$\overline{\psi} \gamma_4 \gamma_5 \overleftrightarrow{D}_4 \overleftrightarrow{D}_4 \frac{\lambda^a}{2} \psi \sim \frac{1}{a^2} \overline{\psi} \gamma_4 \gamma_5 \frac{\lambda^a}{2} \psi$$

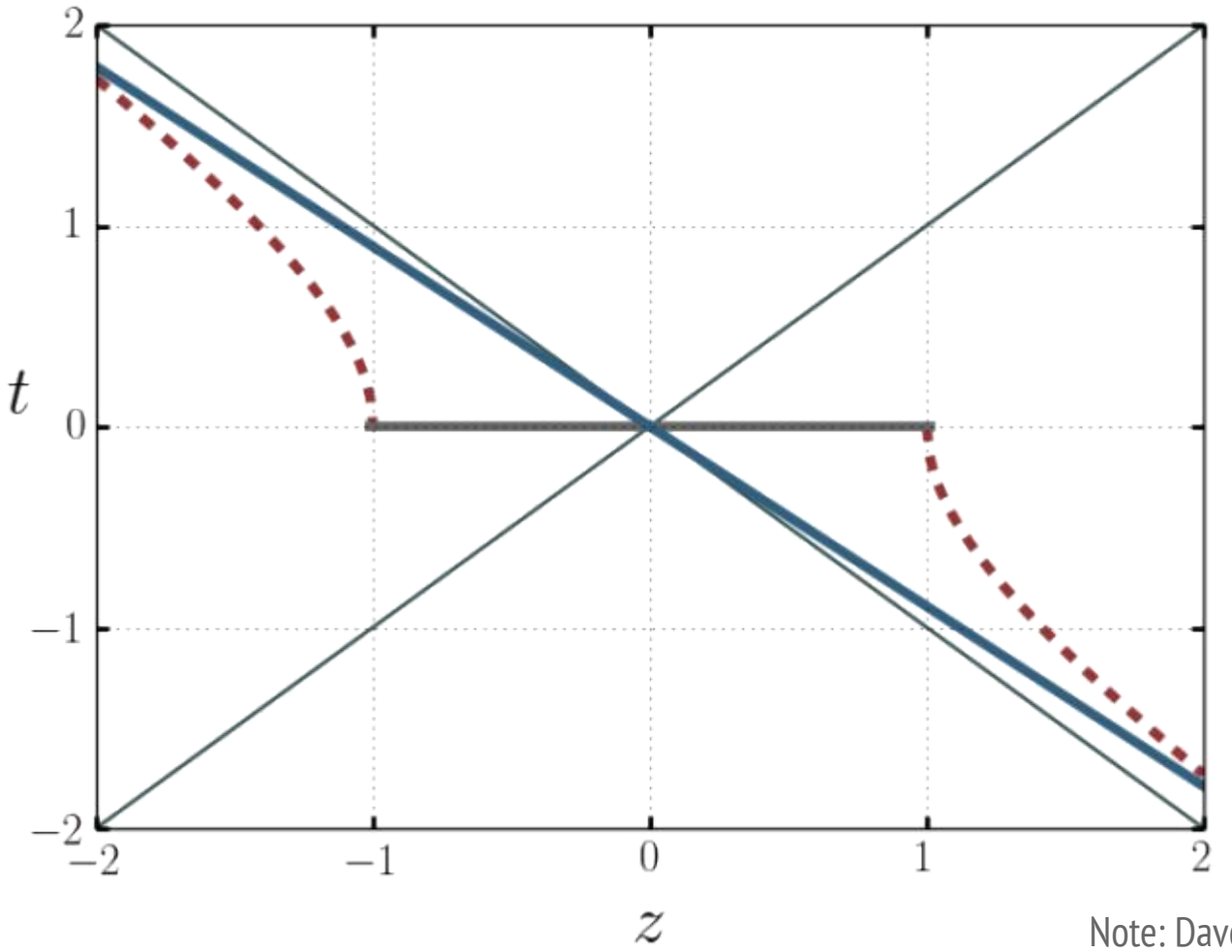
Leads to reconstructions, rather than ab initio calculations

See, e.g. Detmold et al., EPJC 3 (2001) 1

Proposals to calculate higher moments

Davoudi & Savage, PRD 86 (2012) 054505
CJM & Orginos, PRD 91 (2015) 074513

A recent renaissance

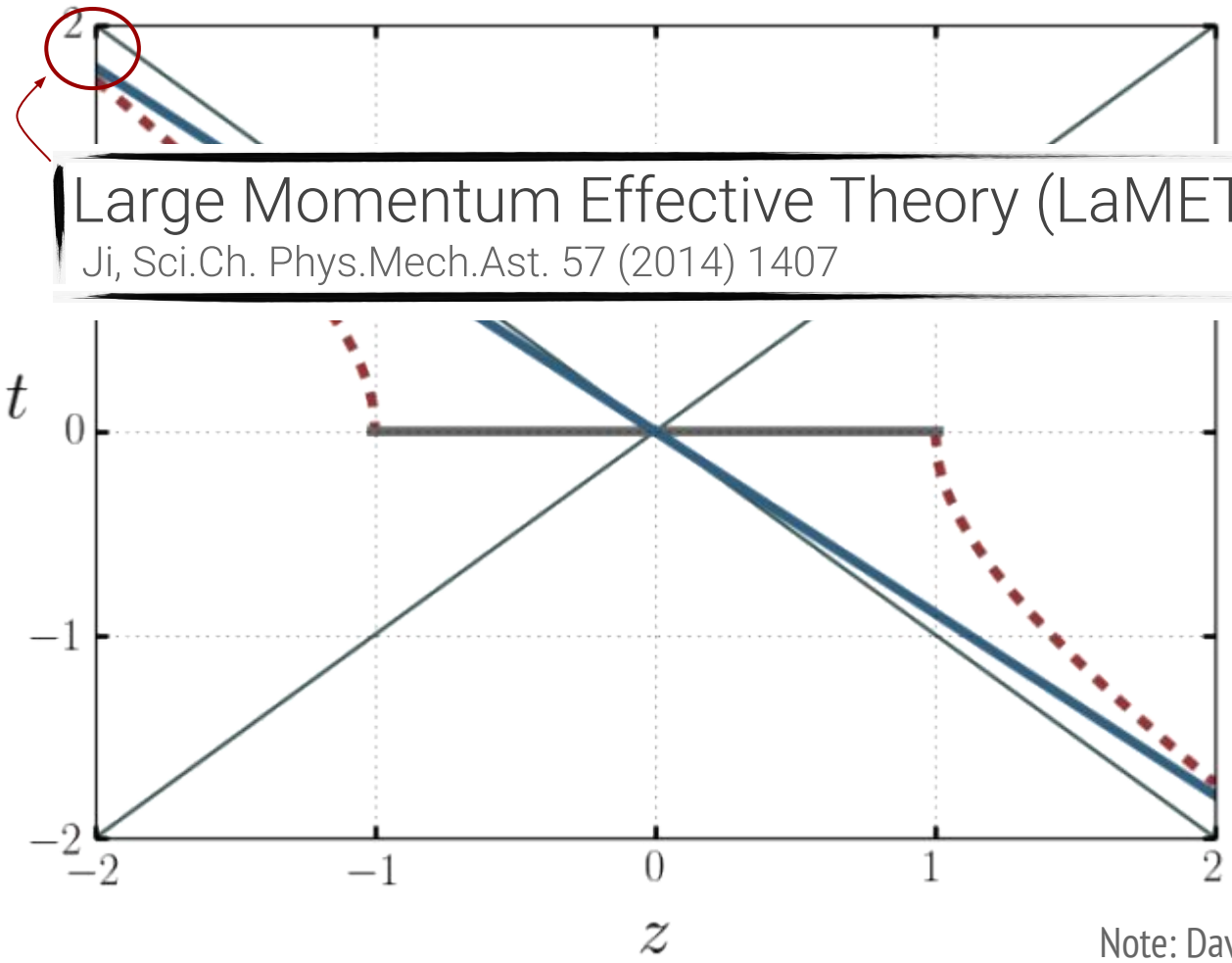


Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025

Note: Davoudi & Savage, PRD 86 (2012) 054505
Musch et al., PRD 83 (2011) 094507
Braun & Müller, EPJC 55 (2008) 349
Detmold & Lin, PRD 73 (2006) 014501
Liu & Dong, PRL 72 (1994) 1790

A recent renaissance

Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025



Note: Davoudi & Savage, PRD 86 (2012) 054505
Musch et al., PRD 83 (2011) 094507
Braun & Müller, EPJC 55 (2008) 349
Detmold & Lin, PRD 73 (2006) 014501
Liu & Dong, PRL 72 (1994) 1790

A plethora of proposals

Quasi and pseudo PDFs	Ji, PRL 110 (2013) 262002 Radyushkin, PRD 96 (2017) 034025
Factorisable matrix elements	Braun & Müller, EPJC 55 (2008) 349 Ma & Qiu, 1404.6860
Fictitious heavy quarks	Detmold & Lin, PRD 73 (2006) 014501
Compton amplitude	Chambers et al., PRL 118 (2017) 242001
Euclidean hadronic tensor	Liu & Dong, PRL 72 (1994) 1790

Lin et al., PPNP 100 (2018) 107
CJM, POS(LATTICE2018) 018
Zhao, IMJPA 33 (2019) 1830033
Cichy & Constantinou, 1811.07248

Ioffe-time distributions

$$h^{(0)}_{j/H}(\zeta = P\cdot n, n^2) = \frac{1}{2P^\mu}\big\langle H(P)|\overline{\psi}(n)W(n,0)\Gamma^\mu_j\psi(0)|H(P)\big\rangle$$

$$W(n(u),0) = \mathcal{P}\exp\Big[-ig_0\int_0^u \mathrm{d}v\frac{\mathrm{d}y^\mu}{\mathrm{d}v}A^a_\mu(y(v))T^a\Big]$$

Radyushkin, PRD 96 (2017) 034025
Musch et al., PRD 83 (2011) 094507

A panoply of distributions

$$f^{(0)}_{j/H}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \left\langle H(P) \left| \overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0) \Gamma_j \psi(0) \right| H(P) \right\rangle$$

PDFs

ξ^- | $n^2 = 0$

$$h^{(0)}_{j/H}(\zeta = P\cdot n, n^2) = \frac{1}{2P^\mu} \left\langle H(P) | \overline{\psi}(n) W(n,0) \Gamma^\mu_j \psi(0) | H(P) \right\rangle$$

P^z | $n^2 = -z^2$ | ζ

quasi PDFs

pseudo PDFs

$$\widetilde{f}^{(0)}_{j/H}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \left\langle H(P) \left| \overline{\psi}(0,z,\mathbf{0}_\mathrm{T}) W(z,0) \Gamma_j \psi(0) \right| H(P) \right\rangle$$

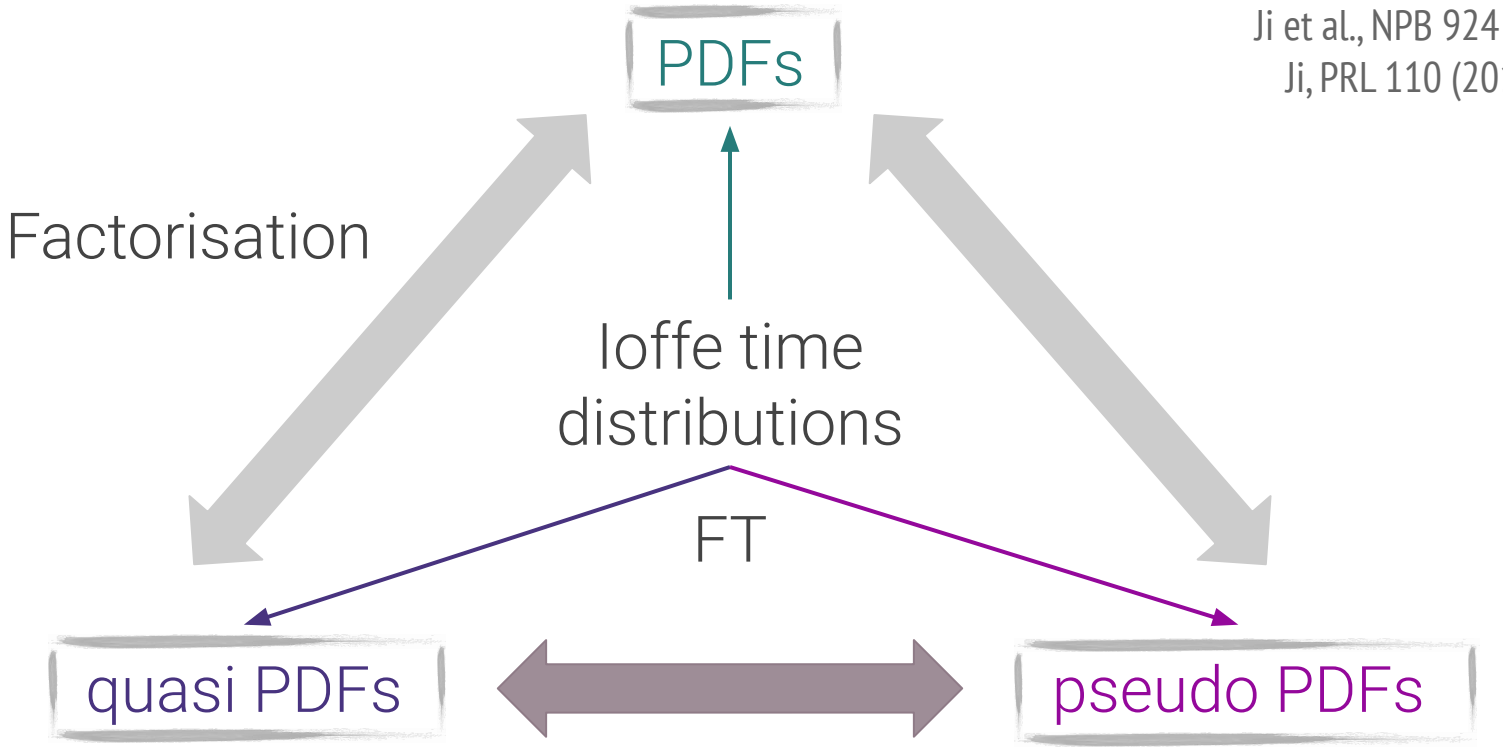
Ji, PRL 110 (2013) 262002

$$\widetilde{p}^{(0)}_{j/H}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\zeta}{4\pi} e^{i\xi\zeta} \left\langle H(P) \left| \overline{\psi}(0,z,\mathbf{0}_\mathrm{T}) W(z,0) \Gamma_j \psi(0) \right| H(P) \right\rangle$$

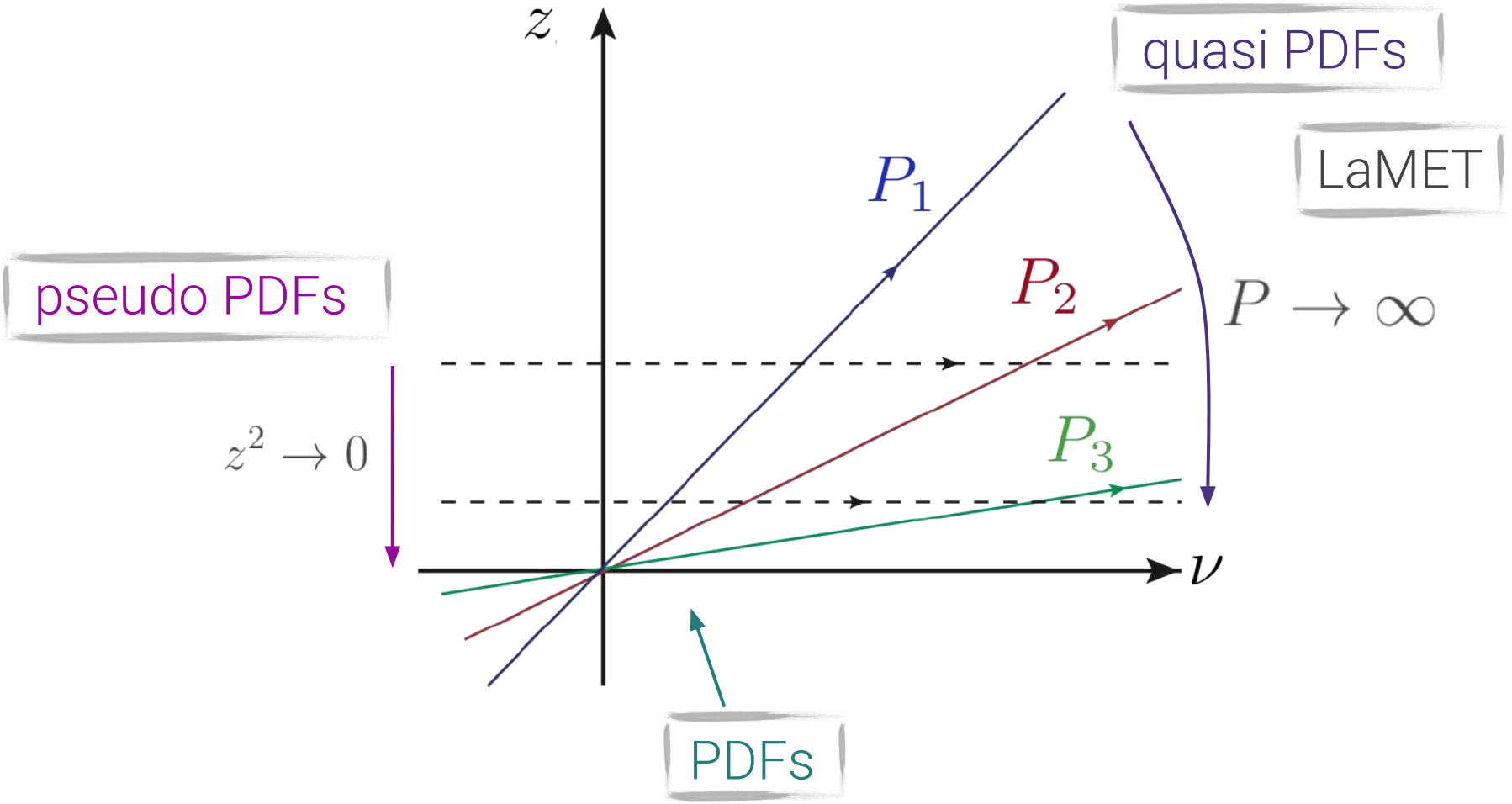
Radyushkin, PRD 96 (2017) 034025

A panoply of distributions

Izubuchi et al., 1801.03917
Zhang, Chen & CJM, PRD 97 (2018) 074508
Radyushkin, PLB 781 (2018) 433
Ji et al., NPB 924 (2017) 326
Ji, PRL 110 (2013) 262002

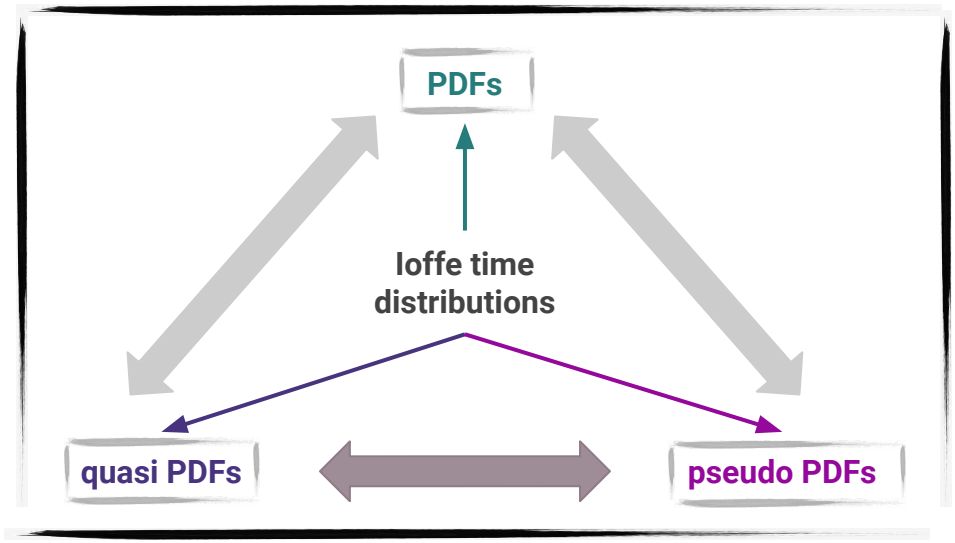


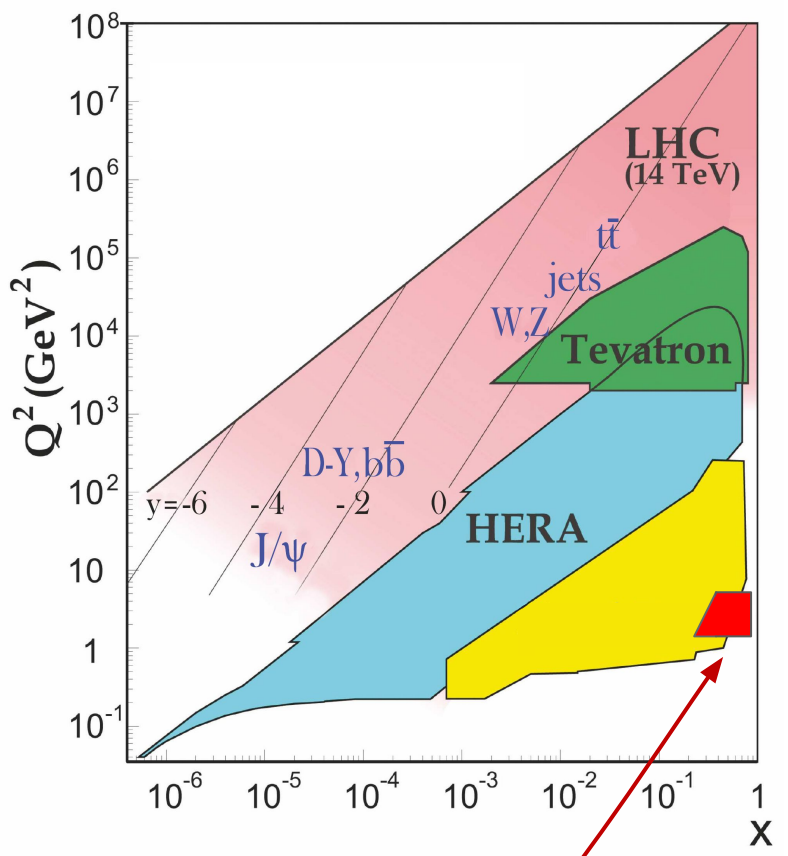
A panoply of distributions



Based on Radyushkin, QCD Evolution 2017

An alternative take





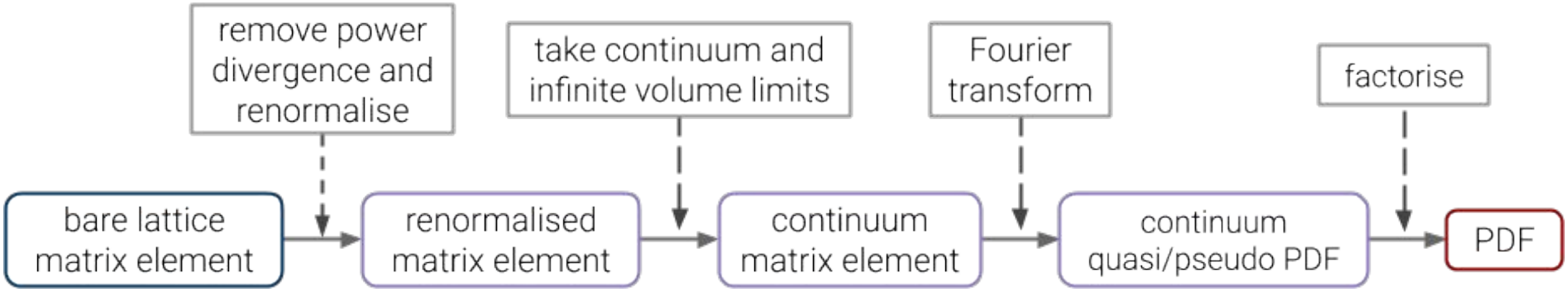
"Lattice ~~cross sections~~" factorisable matrix elements

$$\sigma_{j/H}(\xi, \cdots) \in \left\{ \widetilde{f}^{(0)}_{j/H}(\xi, P^z), \widetilde{p}^{(0)}_{j/H}(\xi, z^2), \cdots \right\}$$

Braun & Müller, EPJC 55 (2008) 349
Ma & Qiu, 1404.6860

In practice

CJM, POS(LATTICE2018) 018



Carlson & Freid, PRD 095 (2017) 094504
Briceño, Hansen & CJM, PRD 96 (2017) 014502

Matrix element extracted from long-time behaviour of Euclidean correlators is identical to that obtained from an LSZ reduction in Minkowski spacetime.

- Considered scalar effective theory
- Demonstrated results independent of spacetime signature, to all orders in perturbation theory

Operator multiplicatively renormalisable in coordinate space

Ji et al., PRL 120 (2018) 112001
Ishikawa et al., PRD 96 (2017) 094019

Multiple approaches to nonperturbative renormalisation

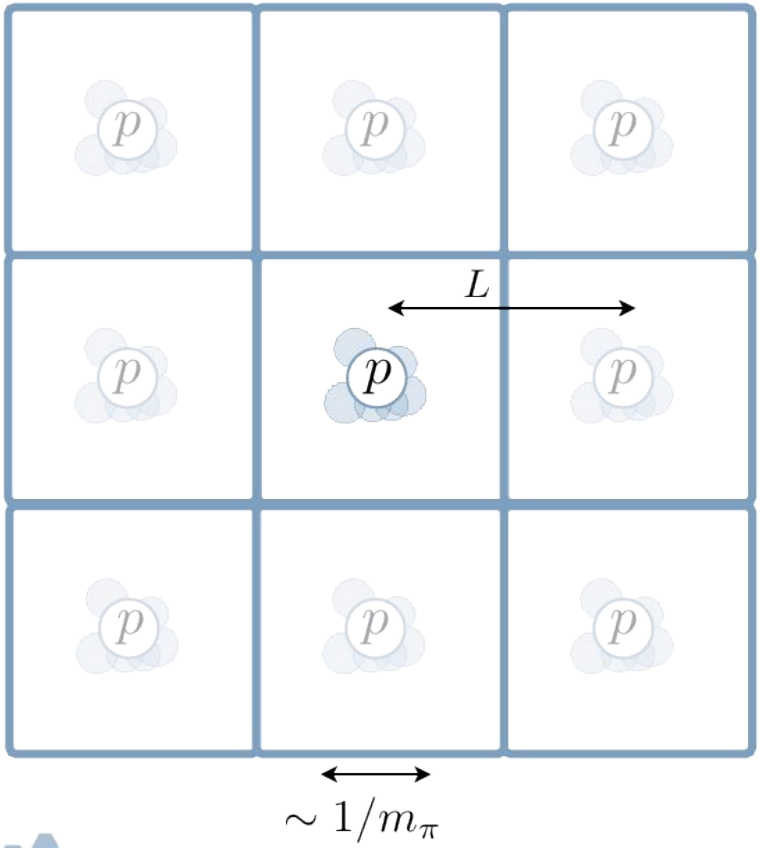
1. Exponential mass counterterm
2. RI/MOM, RI', and RI-xMOM schemes
3. "Reduced" pseudo PDFs
4. Gradient flow

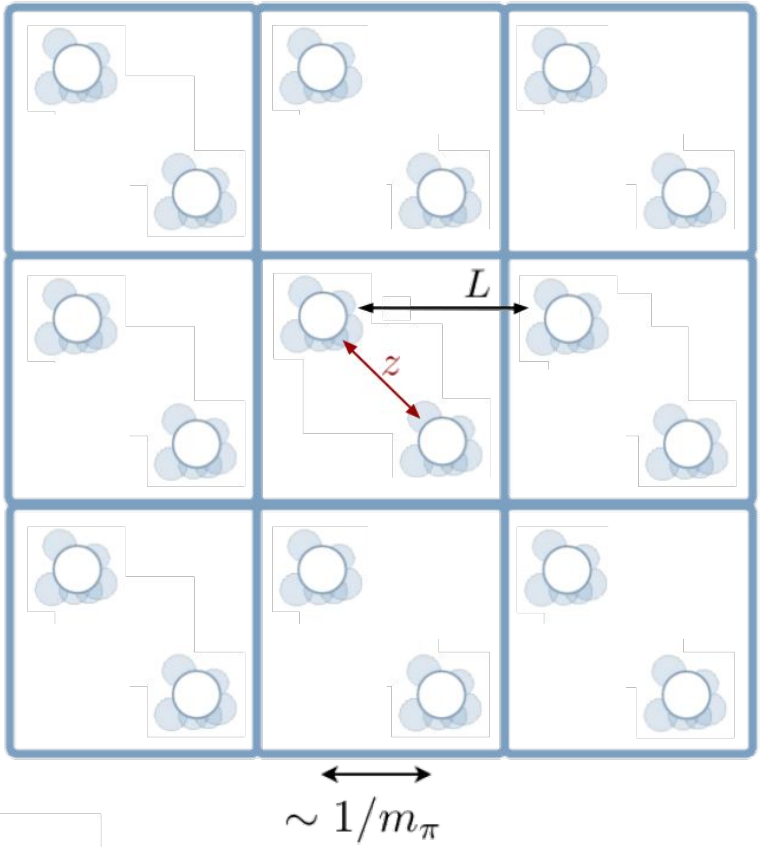
Ishikawa et al., 1609.02018
Chen et al., NPB 12 (2016) 004
Spanoudes & Panagopoulos, 1805.01164
Stewart & Zhao, PRD 97 (2018) 054512
Green et al., 1707.07152
Alexandrou et al., NPB 923 (2017) 394
Radyushkin, PRD 96 (2017) 034025
CJM, PRD 97 (2018) 054507
CJM & Orginos, JHEP 03 (2017) 116

bare lattice matrix element → renormalised matrix element → continuum matrix element → continuum quasi/pseudo PDF → PDF

Discretisation and finite volume effects yet to be understood.

Briceño, Guerrero, Hansen & CJM, PRD 98 (2018) 014511





bare lattice matrix element → renormalised matrix element → continuum matrix element → continuum quasi/pseudo PDF → PDF

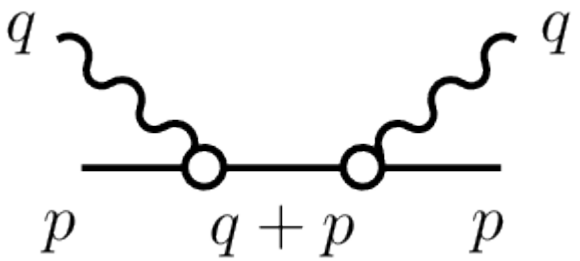
First work to address finite volume effects

- analytic study in a toy effective theory
- two species of scalar field, “pion” and “nucleon”
- spatially-extended current operators

Briceño, Guerrero, Hansen & CJM, PRD 98 (2018) 014511

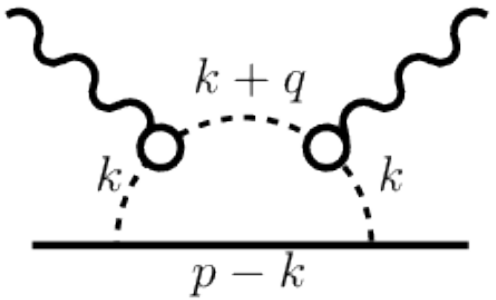
Pion external states

$$\delta\mathcal{M}_{\mathrm{L}}^{\mathrm{LO}} \sim \frac{e^{-m_\pi |L-z|}}{(L-z)^{3/2}}$$



Nucleon external states

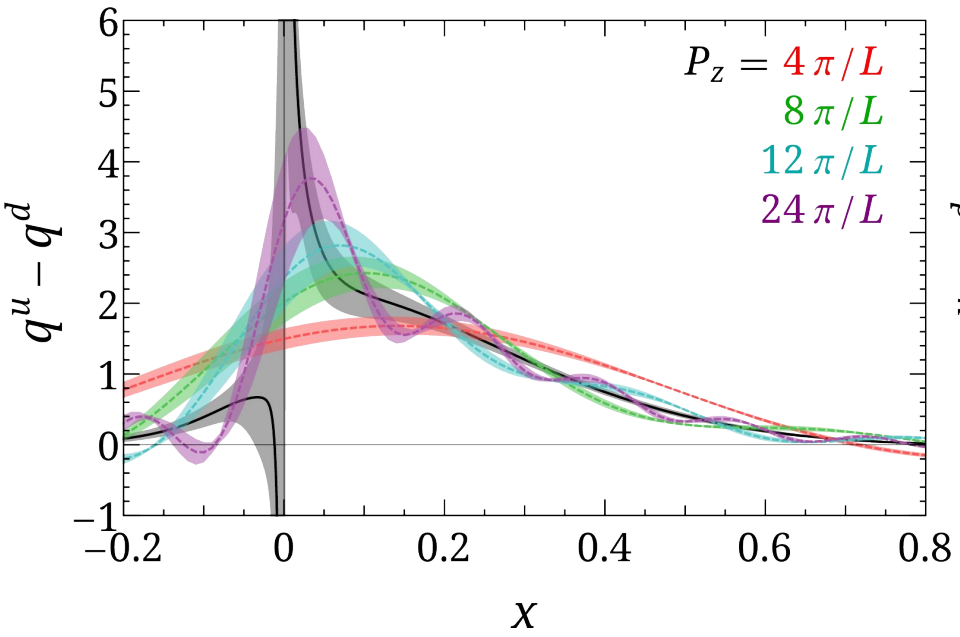
$$\delta\mathcal{M}_{\mathrm{L}} \sim \sqrt{\frac{L-z}{z^3}} e^{-m_\pi L}$$

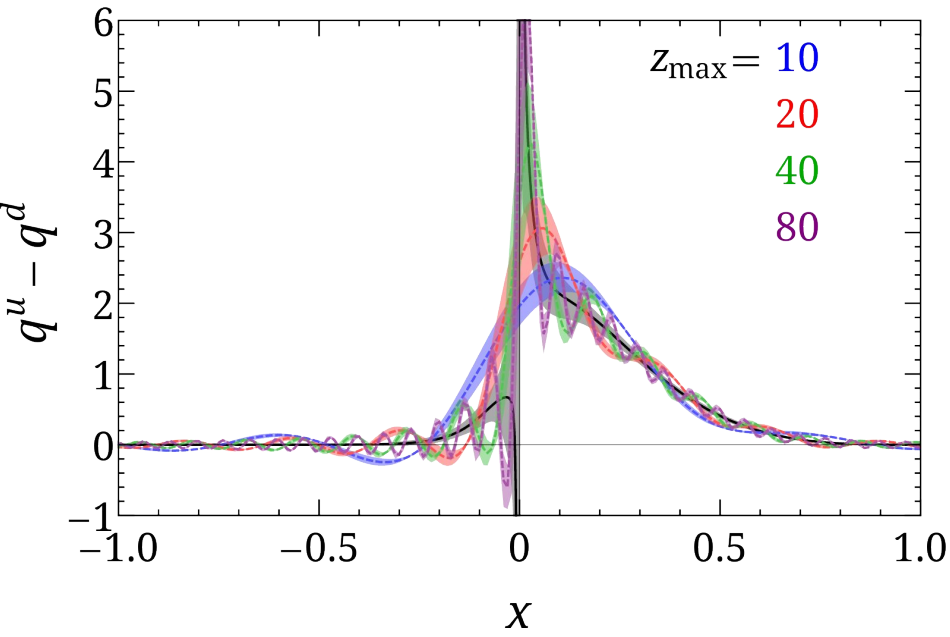


bare lattice matrix element → renormalised matrix element → continuum matrix element → continuum quasi/pseudo PDF → PDF

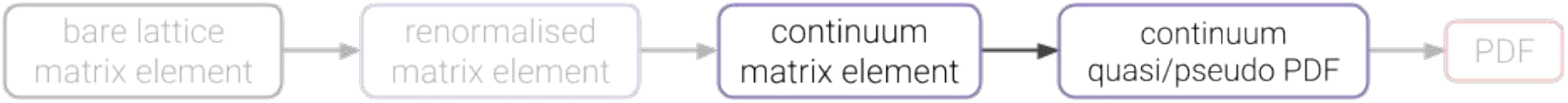
Limited range of spatial separations, and finite number of data points, leads to an inverse problem for the Fourier transform of the lattice matrix element.

Chen et al., 1711.07858





Lin et al., PRD 98 (2018) 054054



Multiple approaches to inverse problem

1. "Model" lattice data in coordinate space
2. Low-pass filter
3. "Derivative" method

Lin et al., PRD 98 (2018) 054054

4. Gaussian smearing

Chen et al., 1711.07858

5. Backus-Gilbert
6. Neural networks
7. Bayesian inference

Karpie et al., JHEP 04 (2019) 057

8. "Model" lattice data in *x*-space

Izubuchi et al., 1905.06349
[See also Sufian et al, PRD 99 (2019) 074507]

Relationship between distributions has been studied extensively

- Factorisation

Izubuchi et al., 1801.03917
Zhang, Chen & CJM, PRD 97 (2018) 074508
Radyushkin, PLB 781 (2018) 433
Ji et al., NPB 924 (2017) 326
Ji, PRL 110 (2013) 262002

- Scheme-matching

Y.-S. Liu et al., 1807.06566
Stewart & Zhao, PRD 97 (2018) 054512
Chen et al., PRD 97 (2018) 014505
Alexandrou et al., NPB 923 (2017) 394
Chen et al., NPB 915 (2017) 1
Xiong et al., 1705.00246
Alexandrou et al., PRD 92 (2015) 014502
Xiong et al., PRD 90 (2014) 014051

- “Target mass”/higher-twist corrections

Radyushkin, PLB 770 (2017) 514
Chen et al., 911 NPB (2016) 246
Bali et al., PRD 93 (2016) 094515
Alexandrou et al., PRD 92 (2015) 01502

Factorisation

Factorisation theorems relate distributions

$$\widetilde{f}_{j/H}(\xi, P^z, \mu_{\rm R}) = \int_{-1}^{1} \frac{{\rm d}y}{|y|} C_j^{(\widetilde{f})}\left(\frac{\xi}{y}, \frac{\mu_{\rm R}}{\mu}, \frac{\mu}{yP^z}\right) f_{j/H}(y,\mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right)$$

$$\widetilde{p}_{j/H}(\xi, z^2, \mu^2_{\rm R}) = \int_{-1}^{1} \frac{{\rm d}y}{|y|} C_j^{(\widetilde{p})}\left(\frac{\xi}{y}, \frac{\mu_{\rm R}}{\mu}, \mu^2 z^2\right) f_{j/H}(y,\mu) + \mathcal{O}\left(M^2 z^2, \Lambda^2_{\rm QCD} z^2\right)$$

Zhang, Chen & CJM, PRD 97 (2018) 074508
Radyushkin, PLB 781 (2018) 433
Ji et al., NPB 924 (2017) 326
Ji, PRL 110 (2013) 262002

Coefficients related by

$$C_j^{(\widetilde{f})}\left(\xi, \frac{\mu_{\rm R}}{\mu}, \frac{\mu}{yP^z}\right) = \int_{-\infty}^{\infty} \frac{{\rm d}\xi}{2\pi} e^{i\xi z} \int_{-1}^{1} {\rm d}\alpha\, e^{-i\alpha z} C_j^{(\widetilde{p})}\left(\alpha, \frac{\mu_{\rm R}}{\mu}, \frac{\mu^2 z^2}{(yP^z)^2}\right)$$

Izubuchi et al., PRD 98 (2018) 056004

View as "lattice ~~cross sections~~" factorisable matrix elements

$$\sigma_{j/H}\left(\zeta, z^2, P^2\right) = \int_{-1}^{1} \frac{{\rm d}y}{y} K_j^{(\sigma)}\left(y\zeta, z^2, y^2P^2, \mu^2\right) f_{j/H}(y,\mu) + \mathcal{O}(\Lambda^2_{\rm QCD} z^2)$$

Ma & Qiu, 1404.6860
Ma & Qiu, PRL 120 (2018) 022003

Higher twist effects

Corrections in factorisation relations:

$$\widetilde{f}_{j/H}(\xi, P^z, \mu_{\mathrm{R}}) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} C_j^{(\widetilde{f})}\left(\frac{\xi}{y}, \frac{\mu_{\mathrm{R}}}{\mu}, \frac{\mu}{yP^z}\right) f_{j/H}(y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}\right)$$

- "target mass corrections"
- higher twist effects

Target mass corrections worked out to all orders in perturbation theory

Chen et al., NPB 911 (2016) 246

Higher twist effects a large source of systematic uncertainty

Quasi and pseudo PDFs expected to have different *x*-dependent effects

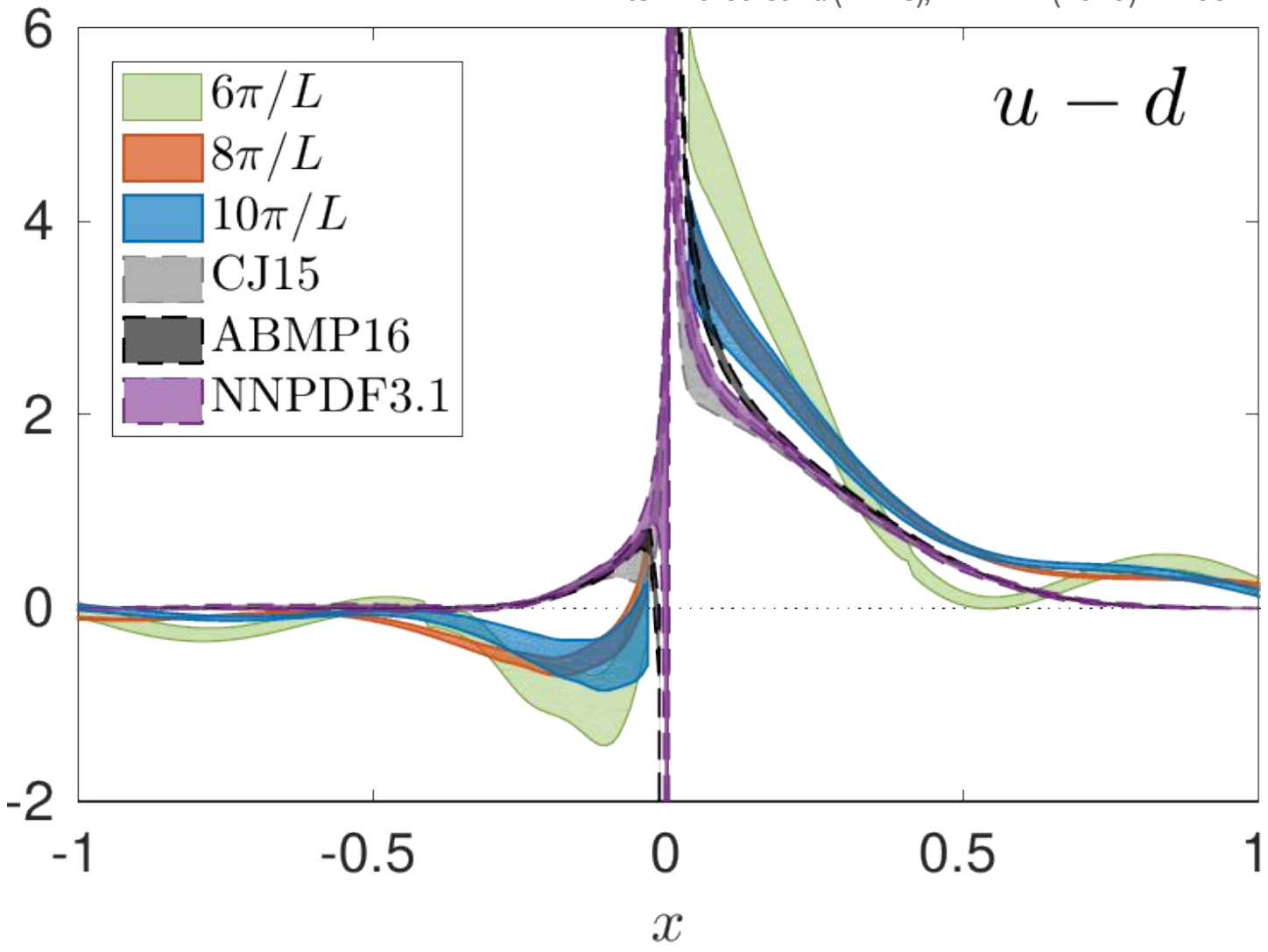
$$\widetilde{f}_{j/H}(\xi, P^z, \mu) = f_{j/H}(\xi, \mu) + \mathcal{O}\left(\alpha_S, \frac{\Lambda^2_{\mathrm{QCD}}}{(xP^z)^2(1-x)}\right)$$

Braun et al., PRD 99 (2019) 014013

$$\widetilde{p}_{j/H}(\xi, z^2, \mu) = f_{j/H}(\xi, \mu) + \mathcal{O}\left(\alpha_S, z^2\Lambda^2_{\mathrm{QCD}}(1-x)\right)$$

bare lattice matrix element → renormalised matrix element → continuum matrix element → continuum quasi/pseudo PDF → PDF

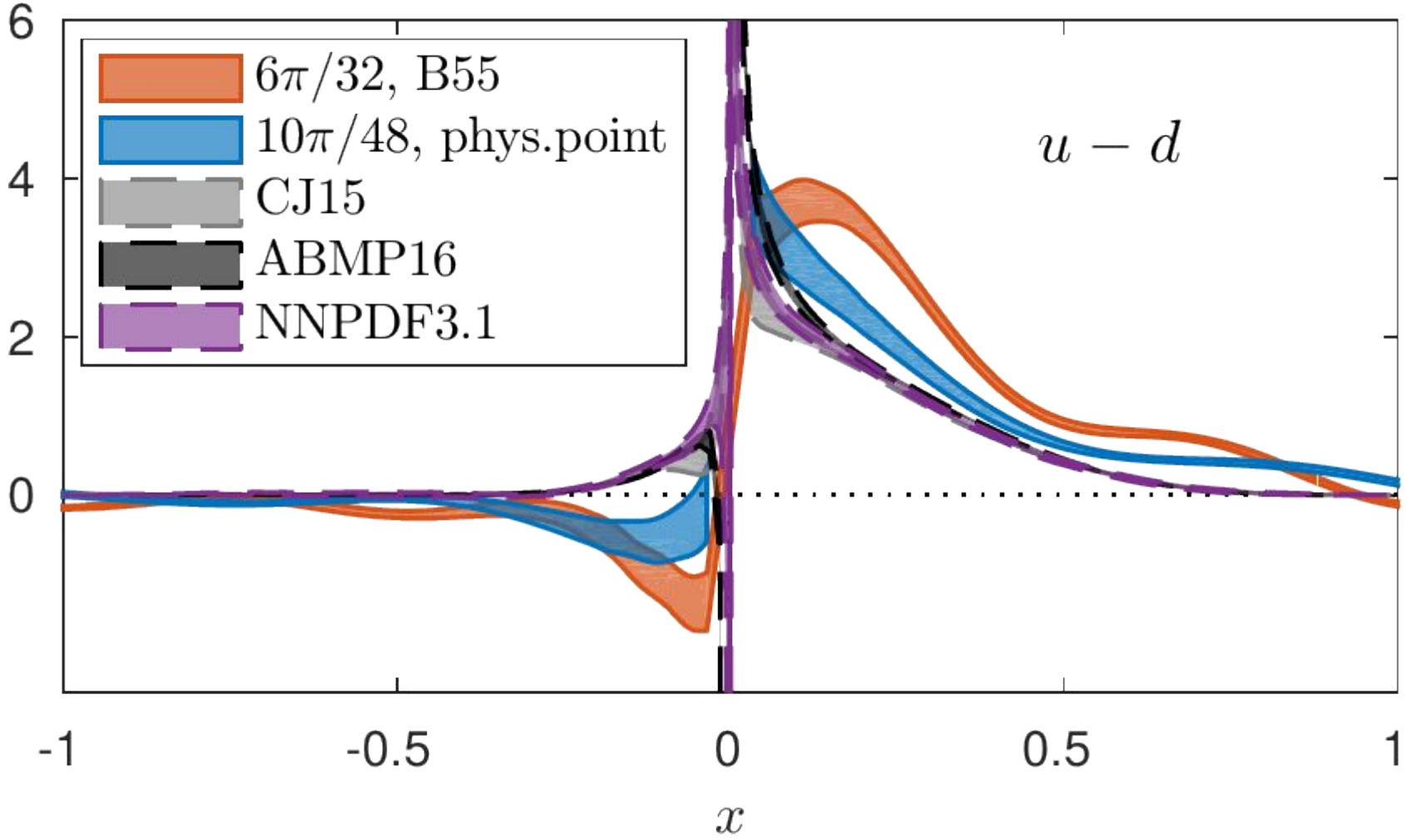
Alexandrou et al. (ETMC), PRL 121 (2018) 112001



See also: Chen et al (LP^3). 1803.04393

bare lattice matrix element → renormalised matrix element → continuum matrix element → continuum quasi/pseudo PDF → PDF

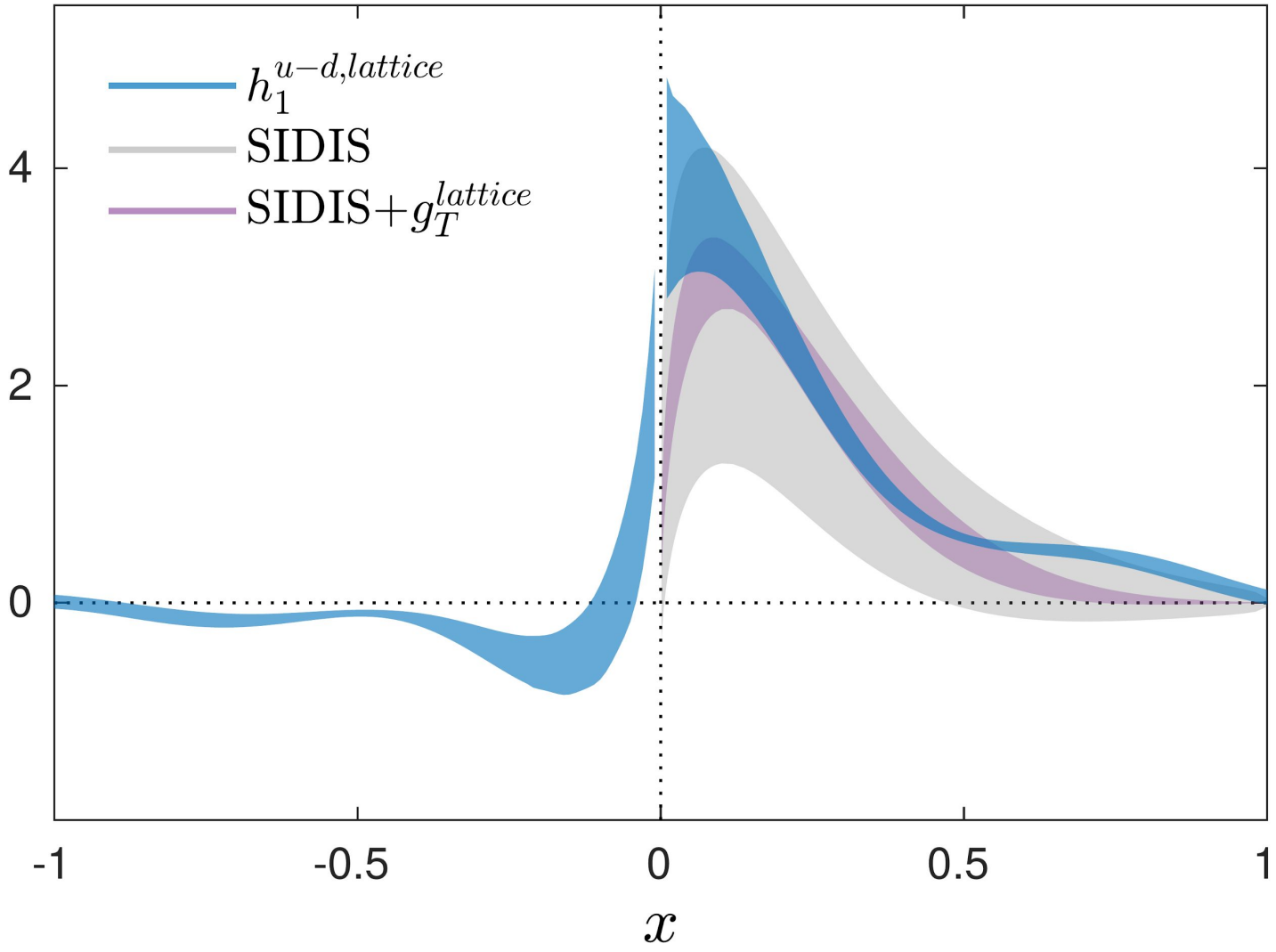
Alexandrou et al. (ETMC), PRL 121 (2018) 112001



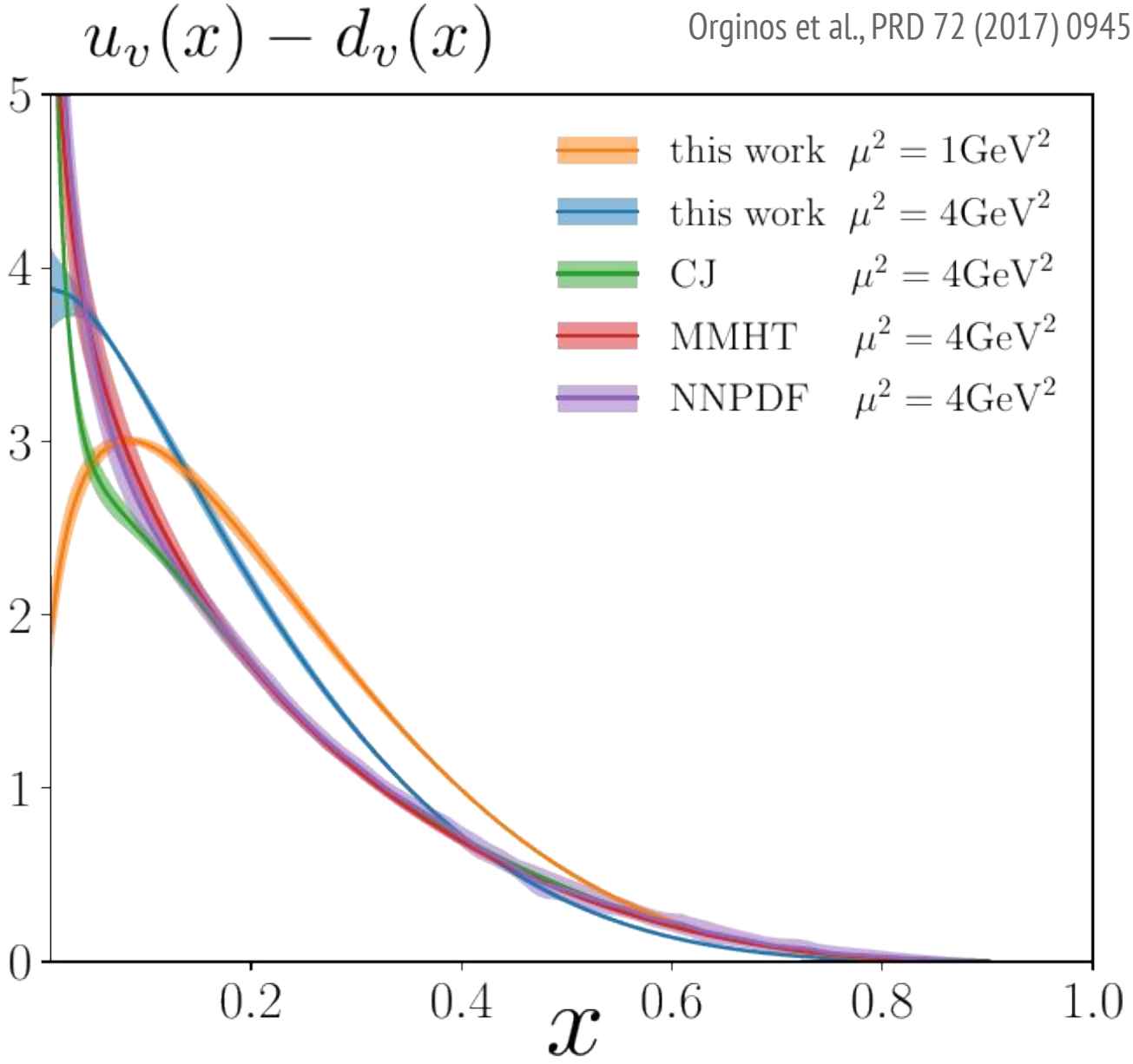
See also: Chen et al (LP^3). 1803.04393

bare lattice matrix element → renormalised matrix element → continuum matrix element → continuum quasi/pseudo PDF → PDF

Alexandrou et al. (ETMC), PRD 98 (2018) 091503

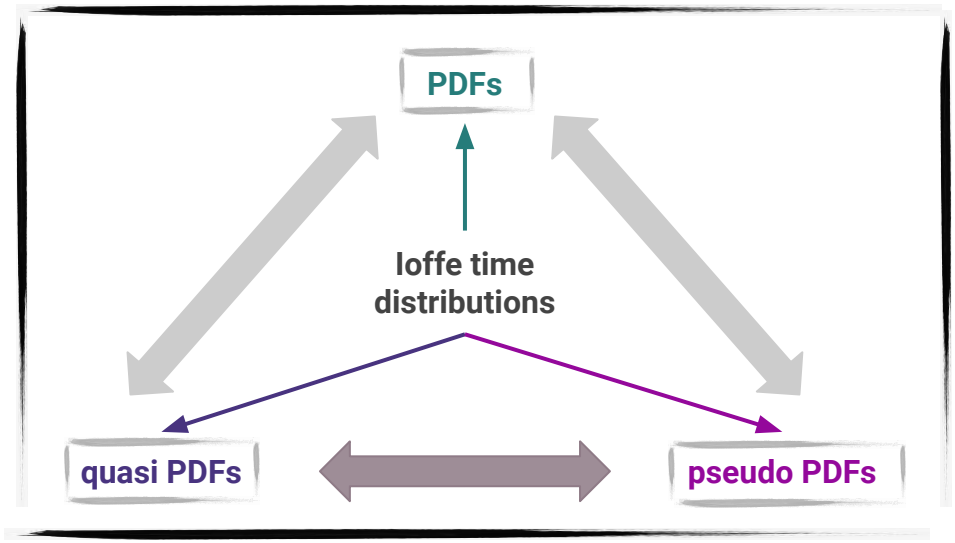


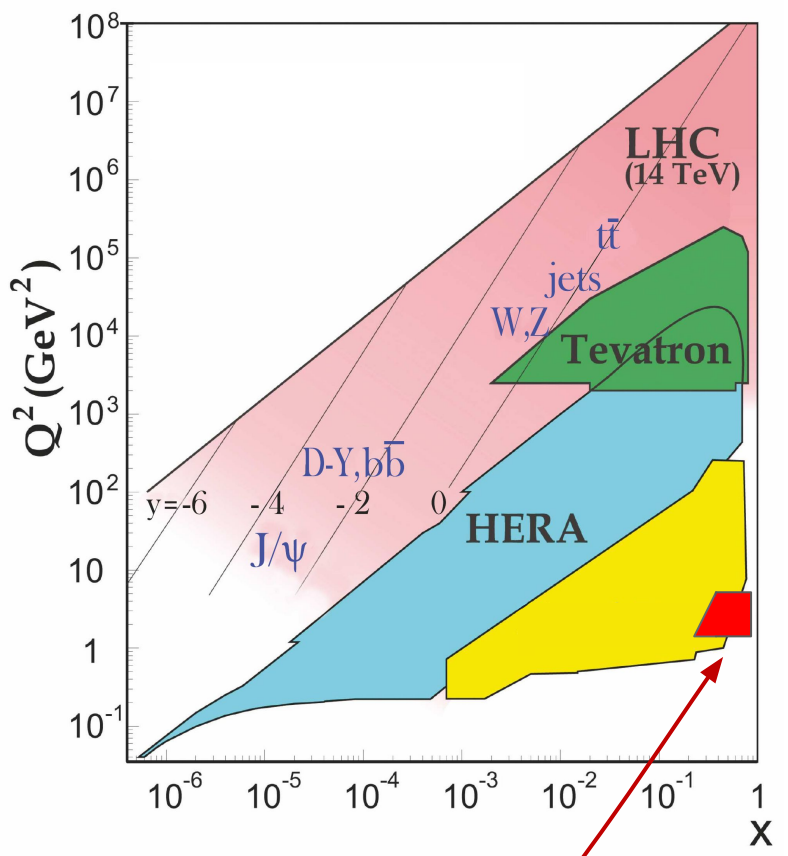
See also: Liu et al (LP^3). 1810.05043



Orginos et al., PRD 72 (2017) 094503

A panoply of distributions: reminder





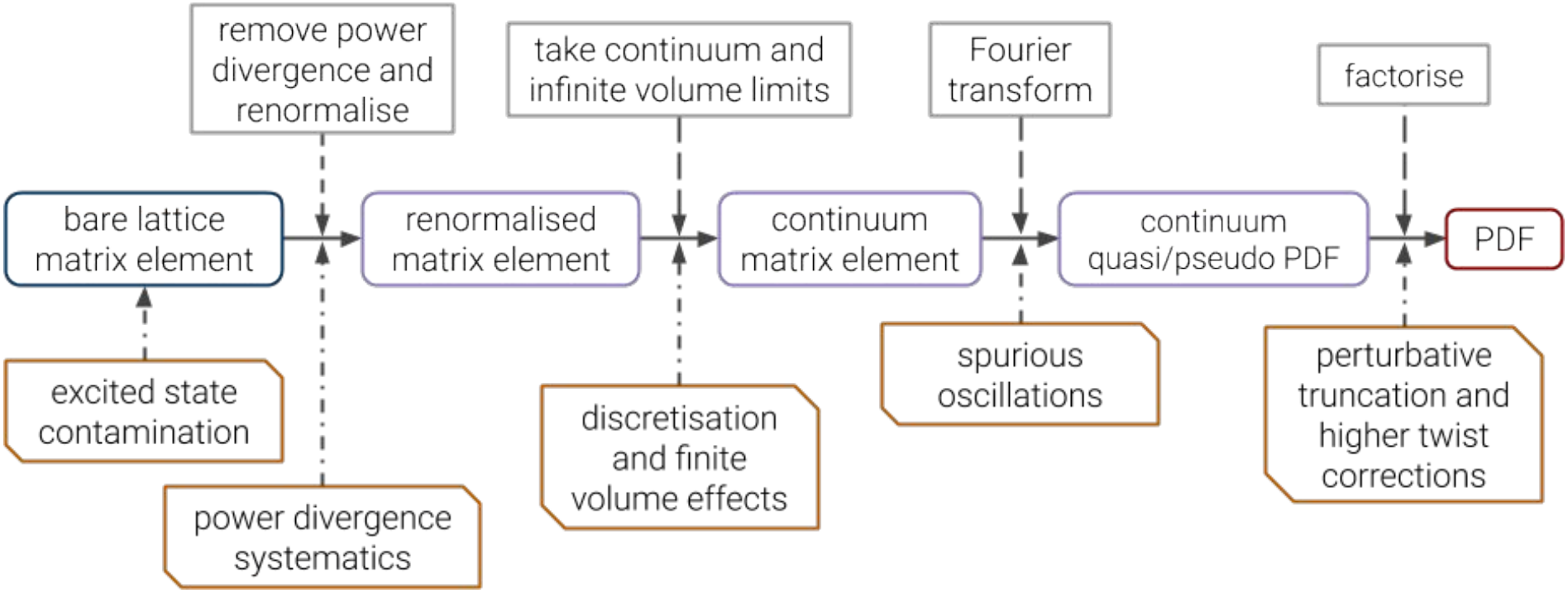
"Lattice ~~cross sections~~" factorisable matrix elements

$$\sigma_{j/H}(\xi, \cdots) \in \left\{ \widetilde{f}^{(0)}_{j/H}(\xi, P^z), \widetilde{p}^{(0)}_{j/H}(\xi, z^2), \cdots \right\}$$

Braun & Müller, EPJC 55 (2008) 349
Ma & Qiu, 1404.6860

In practice: a reminder

CJM, POS(LATTICE2018) 018



A plethora of proposals

Quasi and pseudo PDFs

Factorisable matrix elements

Fictitious heavy quarks

Compton amplitude

Euclidean hadronic tensor

Lin et al., PPNP 100 (2018) 107
CJM, POS(LATTICE2018) 018
Zhao, IMJPA 33 (2019) 1830033
Cichy & Constantinou, 1811.07248

Factorisable matrix elements

Position-space correlation functions
- match directly to perturbative QCD
- encode all necessary information
- avoid challenges of nonlocal renormalisation

Braun & Müller, EPJC 55 (2008) 349
Bali et al., EPJC 78 (2018) 217

Reinterpreted as “factorisable matrix elements”

Ma & Qiu, PRL 120 (2018) 022003
Ma & Qiu, 1404.6860

Factorisation carried out in coordinate or momentum representation

Generally require
- small spatial separations for factorisation
- large Ioffe time to distinguish pion DA shapes

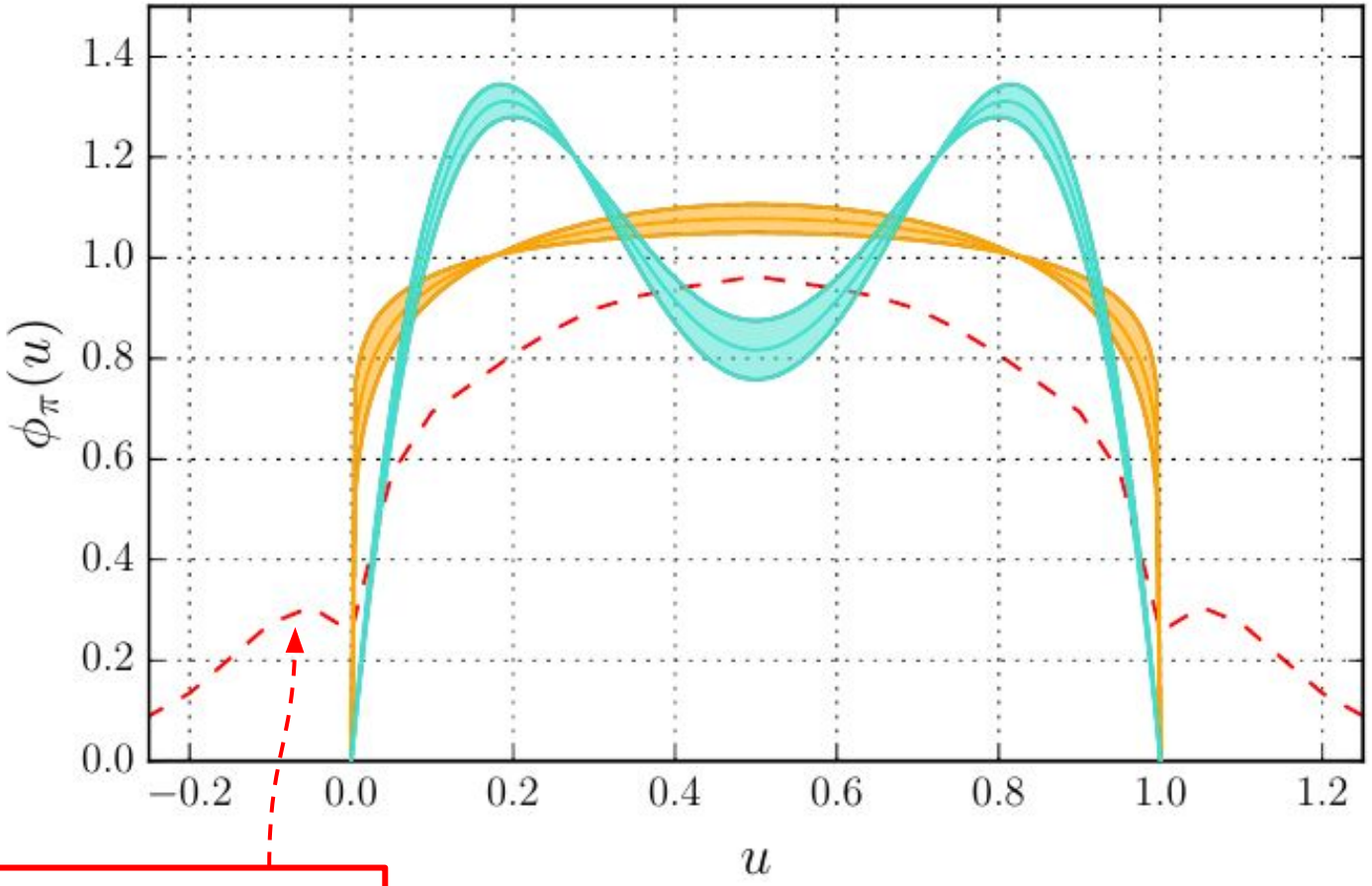
Using different current combinations can access higher twist effects

Factorisable matrix elements

$$\Phi_\pi(\zeta = P\cdot z) = \frac{2(\pi z^2)^2}{f_\pi \zeta}\langle \pi^0(P)|[\bar{u}q](z/2)[\bar{q}\gamma_5 u](-z/2)|0\rangle$$

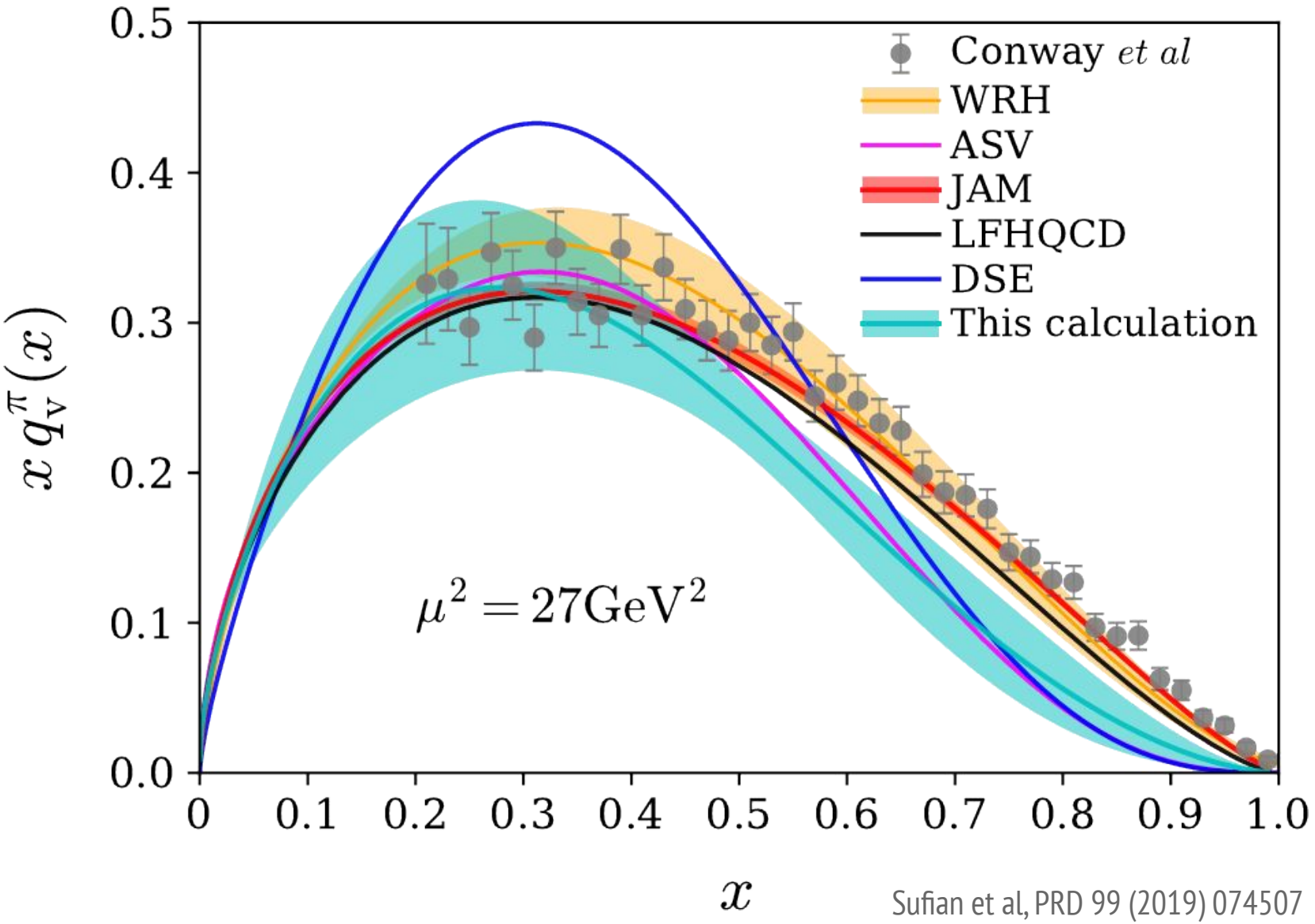
$$\Phi_\pi(\zeta) = \int_0^1 \mathrm{d}x\, e^{i(x-1/2)\zeta}\phi_\pi(x)$$

Bali et al., EPJ C78 (2018) 217
Bali et al, PRD 98 (2018) 094507



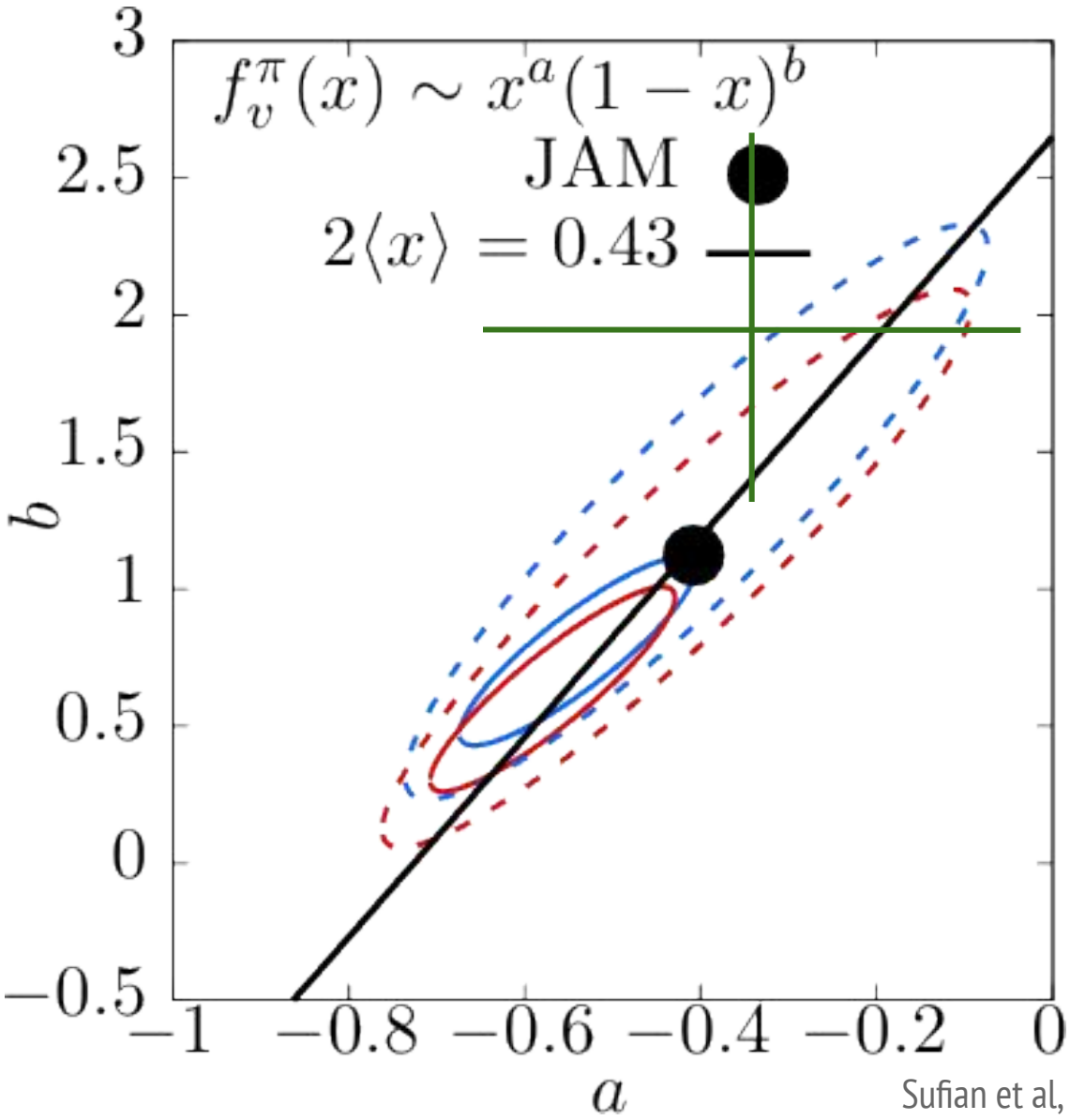
Chen et al (LP^3), 1712.10025

Factorisable matrix elements



Sufian et al, PRD 99 (2019) 074507

Pion PDF comparison



Sufian et al, PRD 99 (2019) 074507
superimposed on Izubuchi et al., 1905.06349

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Fictitious heavy quarks

Calculate Mellin moments of pion DA from

Detmold & Lin, PRD 73 (2006) 014501

$$U^A_{[\mu\nu]}(P,q) = \int \mathrm{d}^4x \, \langle O | T\{ A^{(\Psi,\psi)}_{[\mu}(x) A^{(\Psi,\psi)}_{\nu]}(0)\} | \pi^+(P) \rangle$$

via flavour-changing axial current

$$A^{(\Psi,\psi)}_\mu(x) = \overline{\Psi}\gamma_\mu\gamma_5\psi + \overline{q}\gamma_\mu\gamma_5\Psi$$

in “unphysical” region

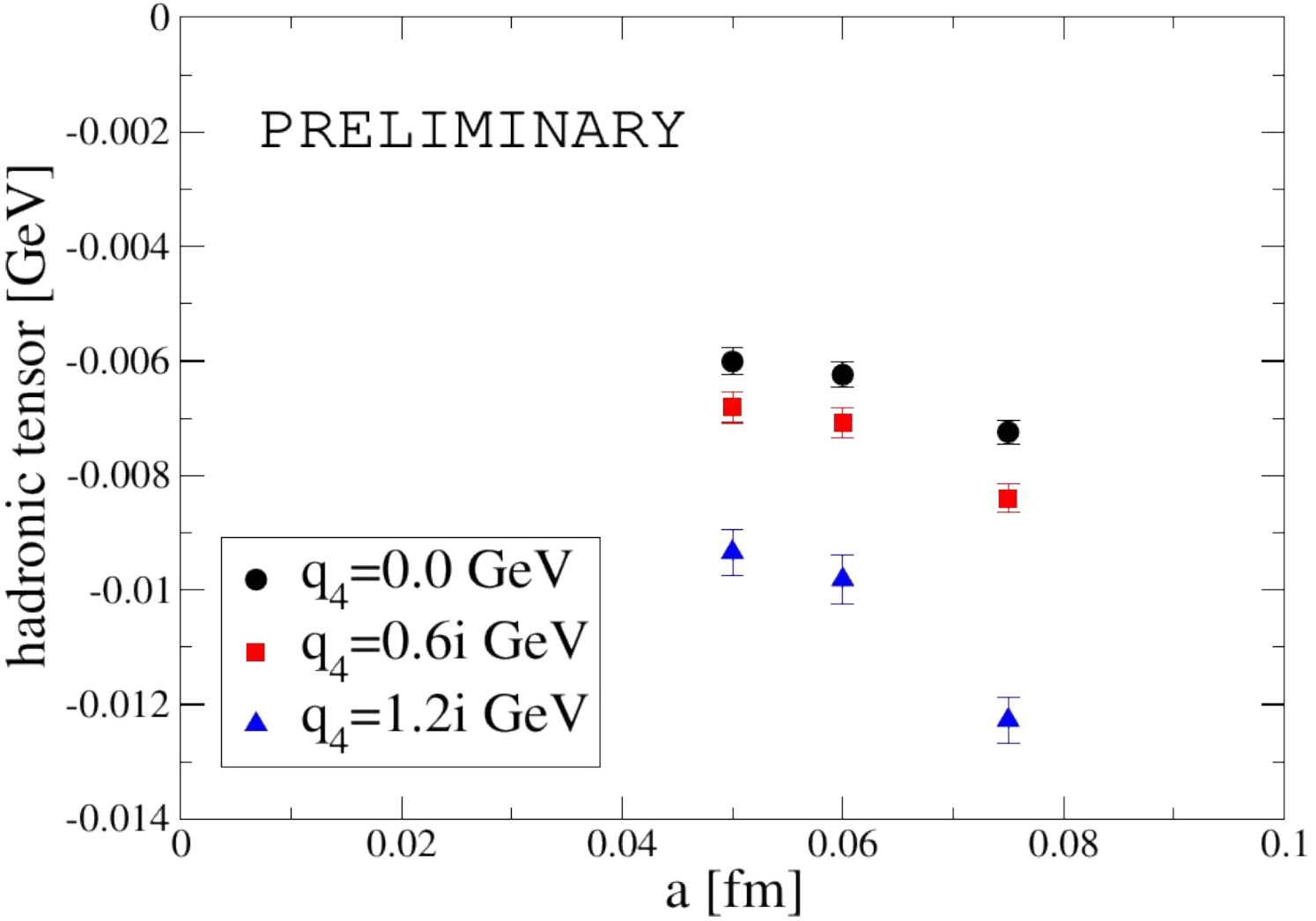
$$(P_M + q_M)^2 < (m_\Psi + \Lambda_{\mathrm{QCD}})^2$$

Requires Fourier transform of lattice matrix element

$$U^A_{[\mu\nu]}(P,q) = \int_{\tau_{\min}}^{\tau_{\max}} \mathrm{d}\tau \, e^{iq_4\tau} \int \mathrm{d}^3\mathbf{x} \, \langle O | T\{ A^{(\Psi,\psi)}_\mu(\mathbf{x},\tau) A^{(\Psi,\psi)}_\mu(0)\} | \pi(P) \rangle$$

And very fine lattice spacing

Fictitious heavy quarks



Detmold et al., PoS(LATTICE2018), 1810.12194

Compton amplitude

Calculate via a Feynman-Hellman method

- avoids renormalisation and mixing issues
- can disentangle higher-twist contributions

Chambers et al., PRL 118 (2017) 242001

$$T^{(H)}_{\mu\nu}(P,q) = \int \mathrm{d}^4x\, e^{iq\cdot x} \langle H(P)|\mathcal{T}\{J_\mu(x)J_\nu(0)\}|H(P)\rangle$$

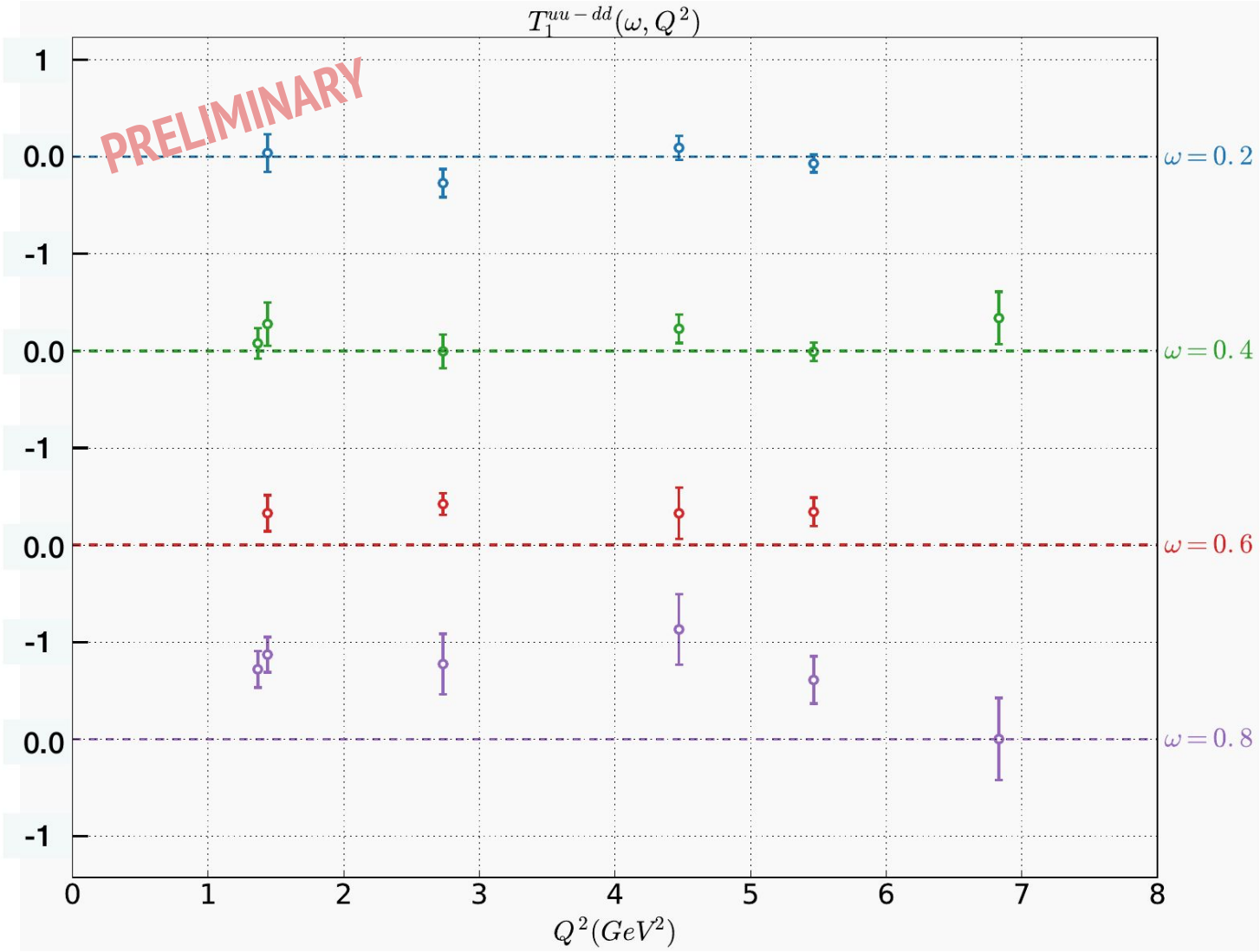
$$T^{(H)}_{33}(P,q) = \sum_{n=2,4,\ldots}^{\infty} 4\left(\frac{2P\cdot q}{q^2}\right)^n \int_0^1 \mathrm{d}x\, x^{n-1} F_1^{(H)}(x,q^2)$$

$$T^{(H)}_{33}(P,q) = 4\left(\frac{2P\cdot q}{q^2}\right) \int_0^1 \mathrm{d}x \frac{2P\cdot q/q^2x}{1-(2P\cdot q/q^2x)^2} F_1^{(H)}(x,q^2)$$

Challenges

- must reconstruct inverse Mellin transform

Compton amplitude



K. Somfleth, Lattice 2018

$$\omega = \frac{2P \cdot q}{q^2}$$

Euclidean hadronic tensor

Formulate hadronic tensor in Euclidean path-integral formalism

- renormalisation straightforward
- frame invariant

Liu & Dong, PRL 72 (1994) 1790
Liu, PRD 62 (2000) 074501
Liu, PoS(LATTICE 2015) 115

$$W_{\mu\nu}^{(H)}(\mathbf{q},\tau) = \frac{1}{4\pi}\sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}}\langle H(P)|J_\mu(\mathbf{x},\tau)J_\nu(0,0)|H(P)\rangle$$

$$W_{\mu\nu}^{(H)}(q^2, q\cdot P) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\tau\, e^{q\cdot P\tau} W_{\mu\nu}(\mathbf{q},\tau)$$

Challenges:

- requires calculation of four-point function
- difficult inverse problem to convert signature

Present implementation does not address interplay with finite volume

Hansen, Meyer & Robaina, PRD 96 (2017) 094513

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} Distribution functions

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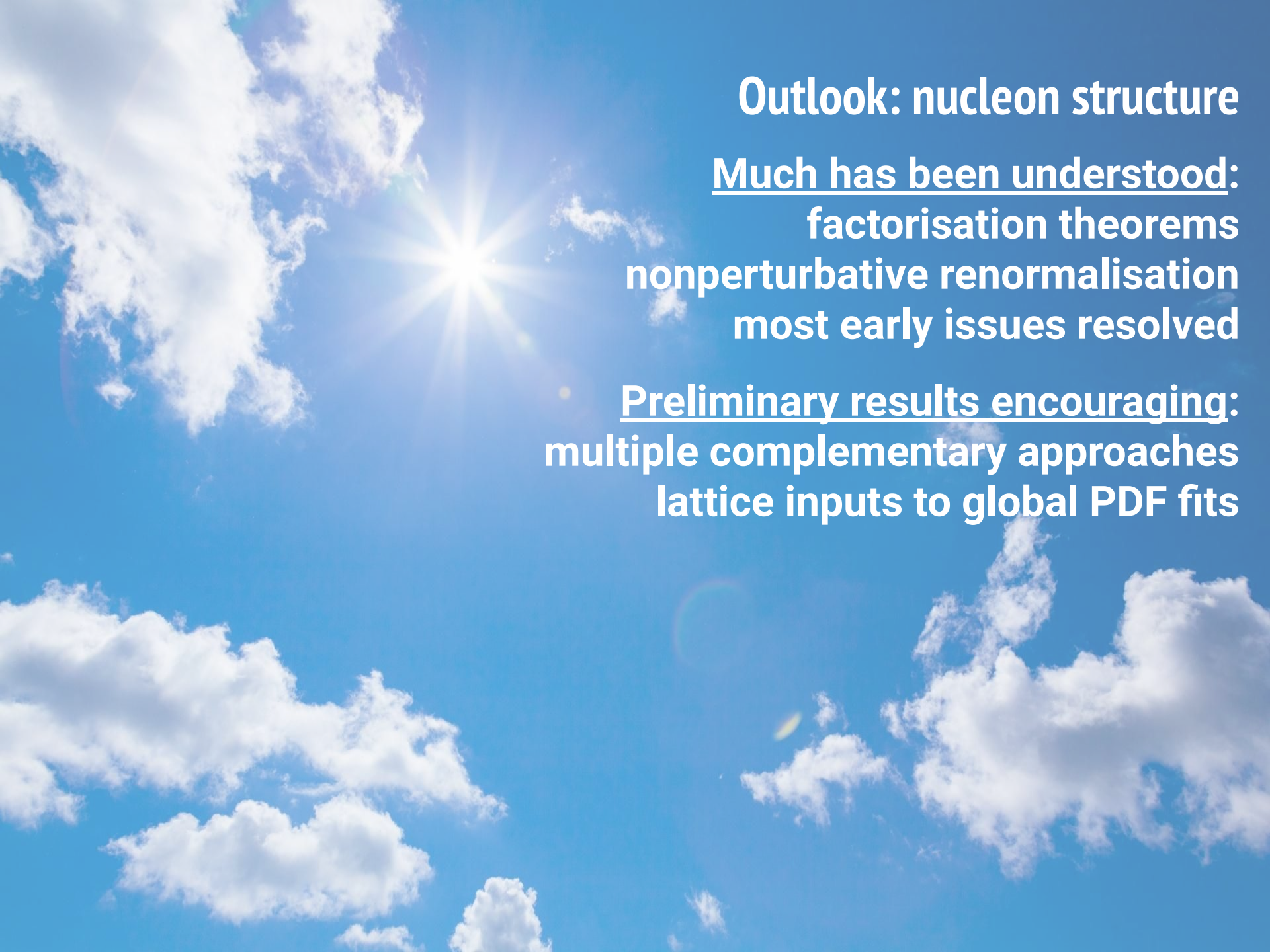
} Structure functions

Lin et al., PPNP 100 (2018) 107
CJM, POS(LATTICE2018) 018
Zhao, IMJPA 33 (2019) 1830033
Cichy & Constantinou, 1811.07248

Outlook: nucleon structure



Outlook: nucleon structure

Much has been understood:
factorisation theorems
nonperturbative renormalisation
most early issues resolved

Preliminary results encouraging:
multiple complementary approaches
lattice inputs to global PDF fits

Outlook: nucleon structure

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factorisation theorems
nonperturbative renormalisation
most early issues resolved

Preliminary results encouraging:
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lattice inputs to global PDF fits

Any unknown unresolved theoretical issues?

Systematic uncertainties unexplored
excited state effects?
enhanced discretisation or finite volume effects?
need larger momenta
inverse problem is hard

Looking forward?

Controlling systematics is now key:

- physical pion masses, multiple lattice spacings and volumes
- larger momenta
- two-loop perturbation theory

Perennial inverse problem

- direct comparison of PDFs will go out of fashion

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Controlling systematics is now key:

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Perennial inverse problem

- direct comparison of PDFs will go out of fashion

Eventually move to PDFs in light nuclei

Other light-cone quantities?

- can we formulate a true effective theory?

Other strongly-interacting theories?


Thank you
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