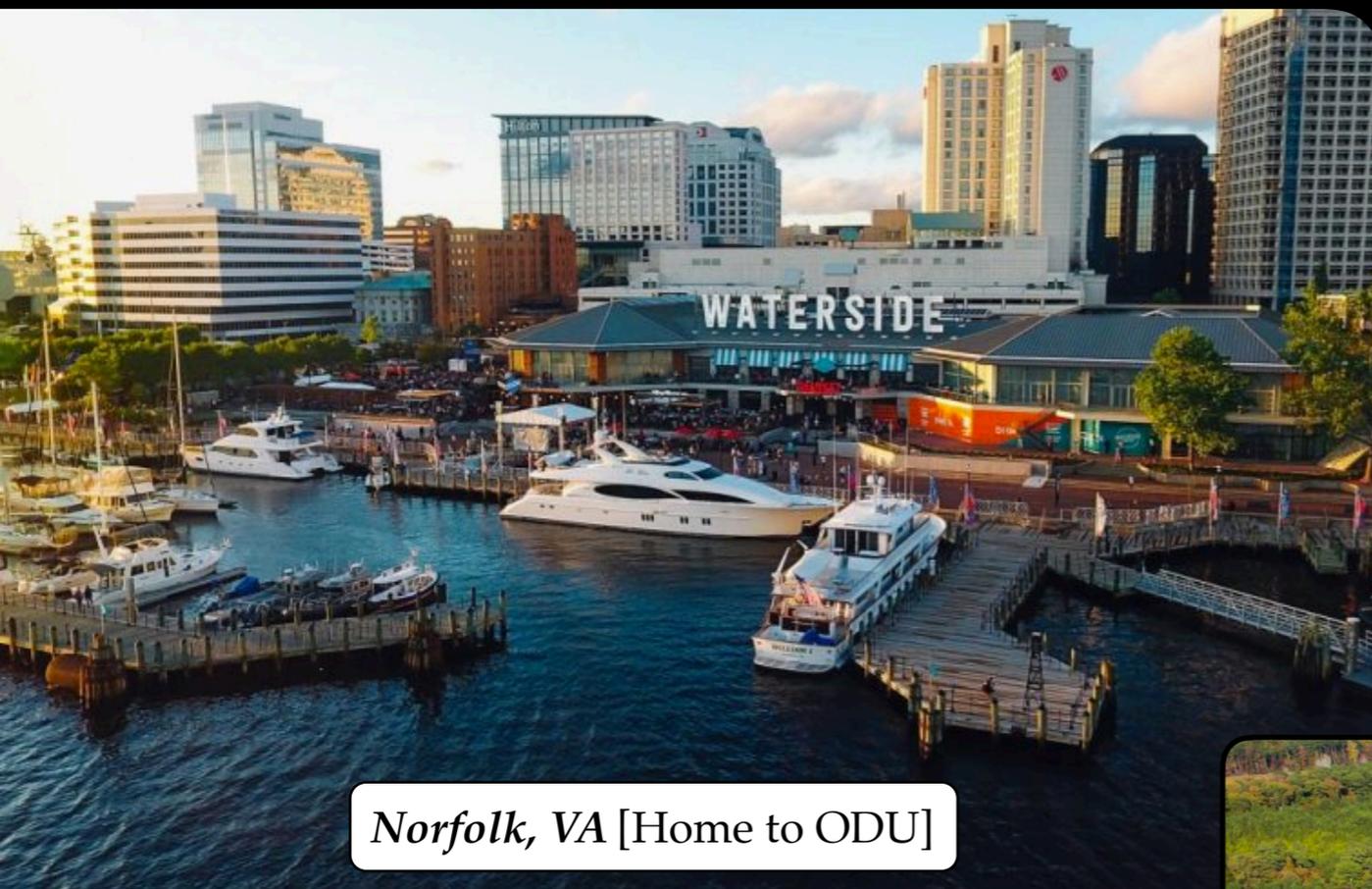


# Scattering and resonances in LQCD

Raúl Briceño - <http://bit.ly/rbricenoPhD>



Norfolk, VA [Home to ODU]



JLab, VA

# low energy QCD in the 21st cent.

*Amplitude analysis*

*Experiments*

*QCD*

# low energy QCD in the 21st cent.

*Amplitude analysis*

GOAL:

*Get insights to the governing patterns and rules of QCD from emergent phenomena*

Observables to test our understanding:

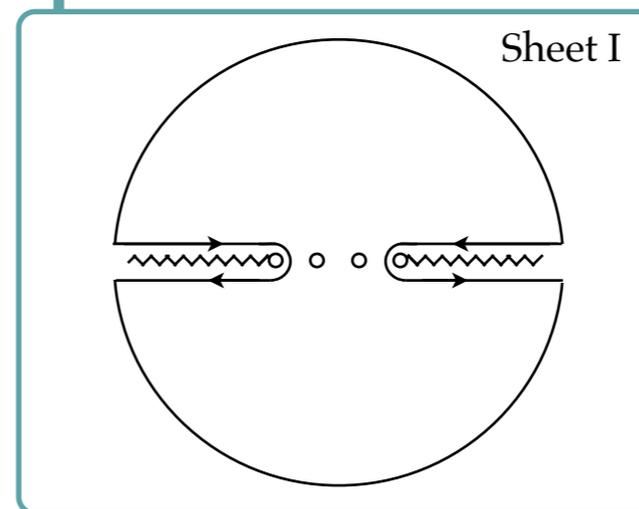
- purely hadronic cross sections
- Electroweak production and decay rates
- ...

*QCD*

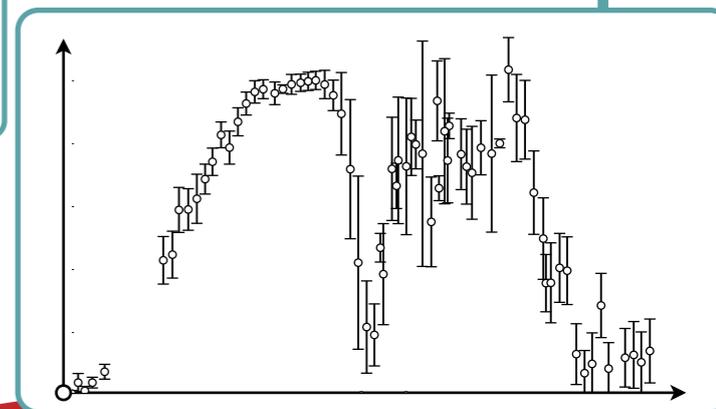
*Experiments*

# low energy QCD in the 21st cent.

*Amplitude analysis*

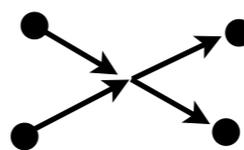


*Experiments*

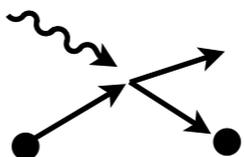


*QCD*

partial wave  
amplitudes



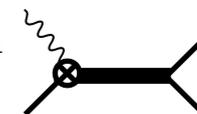
electroweak  
amplitudes



resonance poles



transition form  
factors



identification of states,  
production/decay mechanisms

# low energy QCD in the 21st cent.

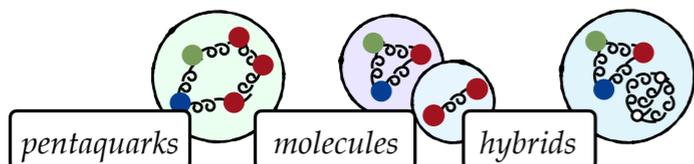
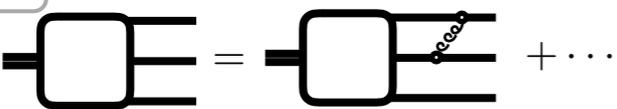
*Amplitude analysis*

Sheet I

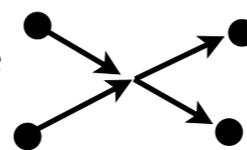
*Experiments*

*QCD*

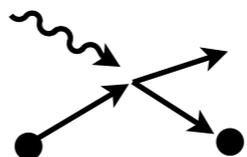
models & EFTs



partial wave  
amplitudes



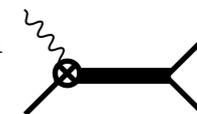
electroweak  
amplitudes



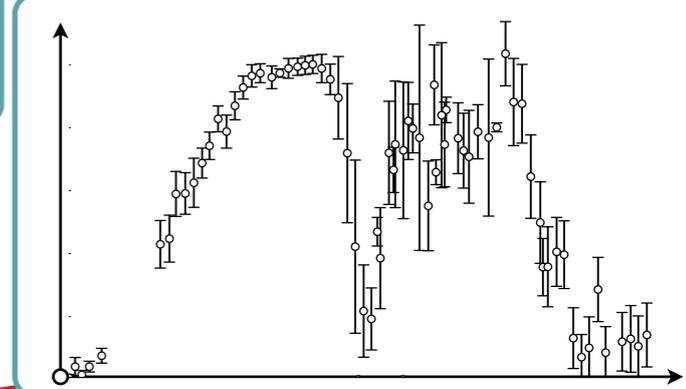
resonance poles



transition form  
factors



identification of states,  
production/decay mechanisms



# low energy QCD in the 21st cent.

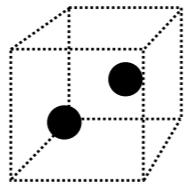
*Amplitude analysis*

*Experiments*

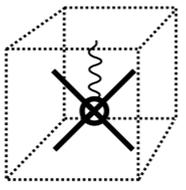
QCD

lattice QCD

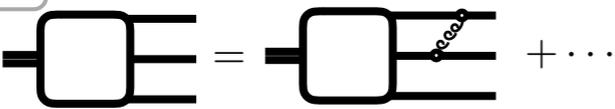
finite-volume spectrum



finite-volume matrix elements



models & EFTs

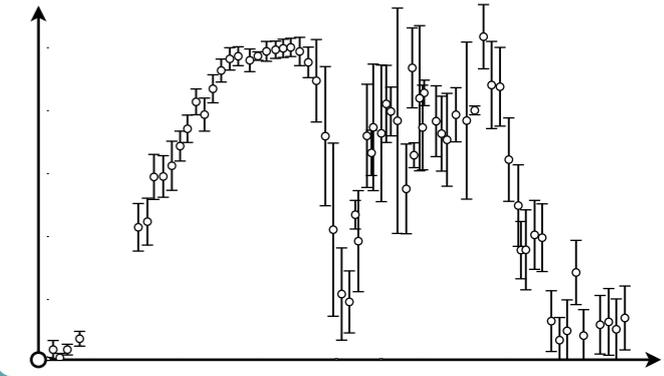
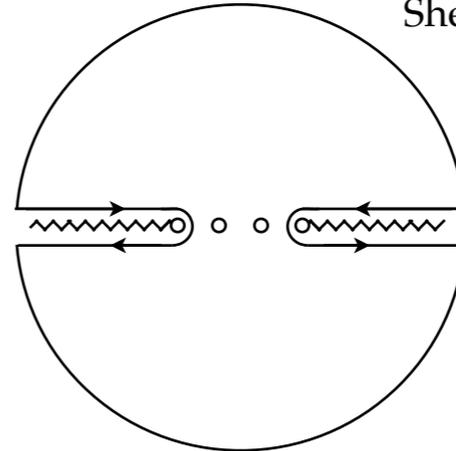


pentaquarks

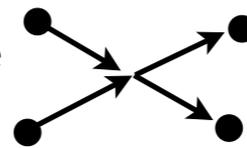
molecules

hybrids

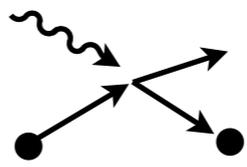
Sheet I



partial wave amplitudes



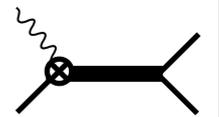
electroweak amplitudes



resonance poles

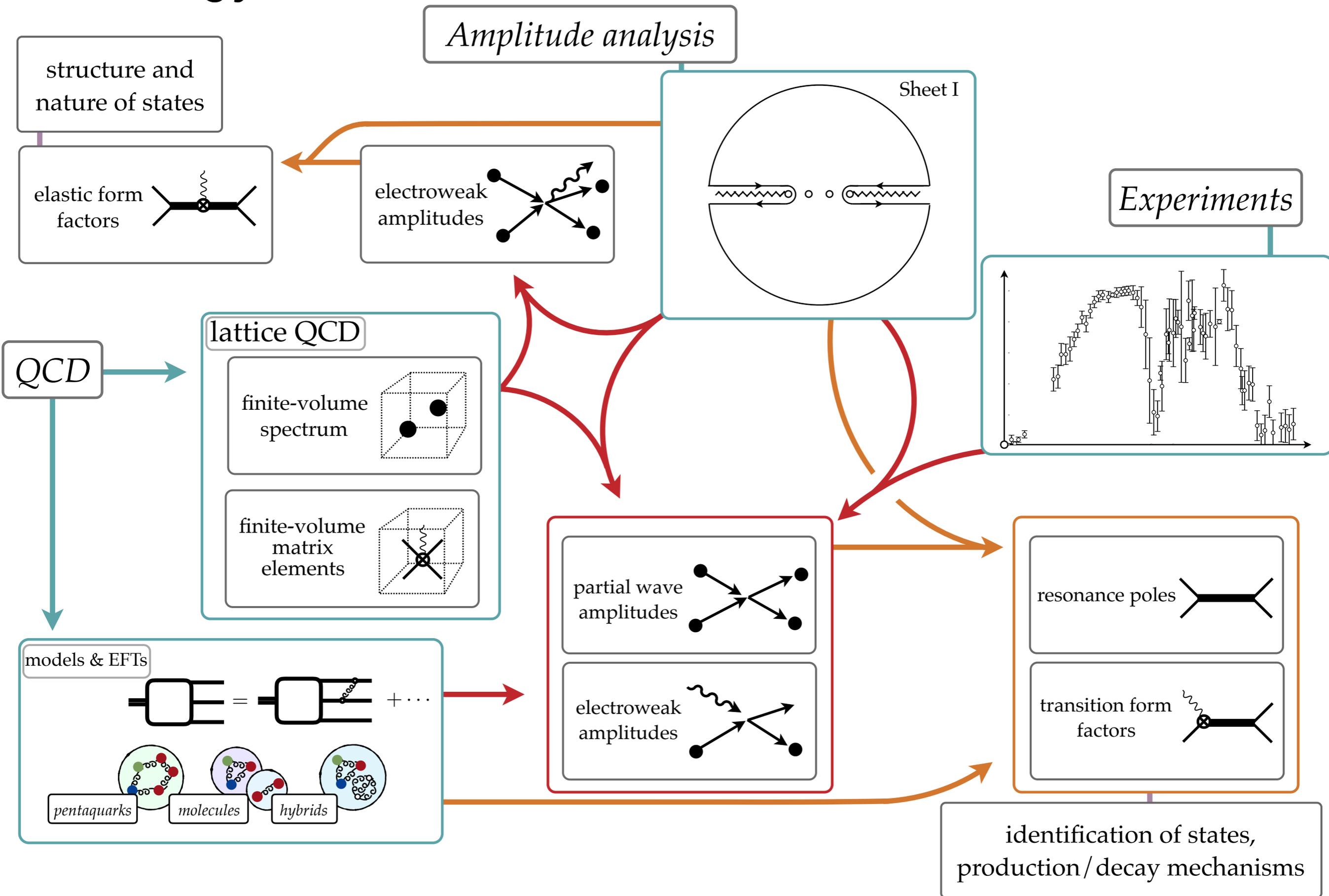


transition form factors



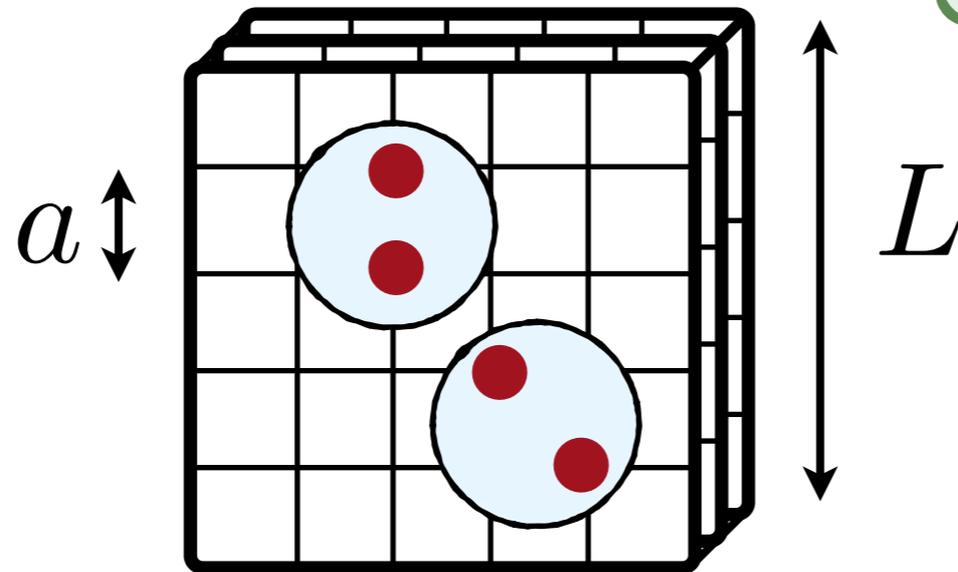
identification of states,  
production/decay mechanisms

# low energy QCD in the 21st cent.



# Lattice QCD

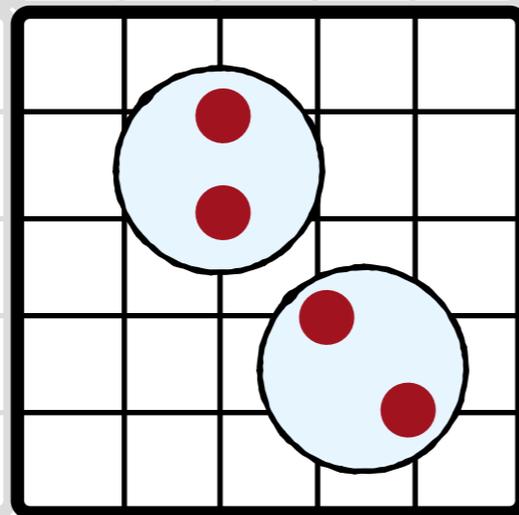
- Wick rotation [Euclidean spacetime]:  $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses:  $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing:  $a \sim 0.03 - 0.15$  fm
- finite volume



$$D_\mu = \left( \right) \updownarrow (L/a)^3 \times (T/a)$$

# Lattice QCD

- Wick rotation [Euclidean spacetime]:  $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses:  $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing:  $a \sim 0.03 - 0.15$  fm
- finite volume



Never free!  
No asymptotic states!  
No scattering!

# Three big ideas

... *already out of date*

Dudek



Young



## Scattering processes and resonances from lattice QCD

Raúl A. Briceño, Jozef J. Dudek, and Ross D. Young  
Rev. Mod. Phys. **90**, 025001 – Published 18 April 2018

Article

References

Citing Articles (7)

PDF

HTML

Export Citation



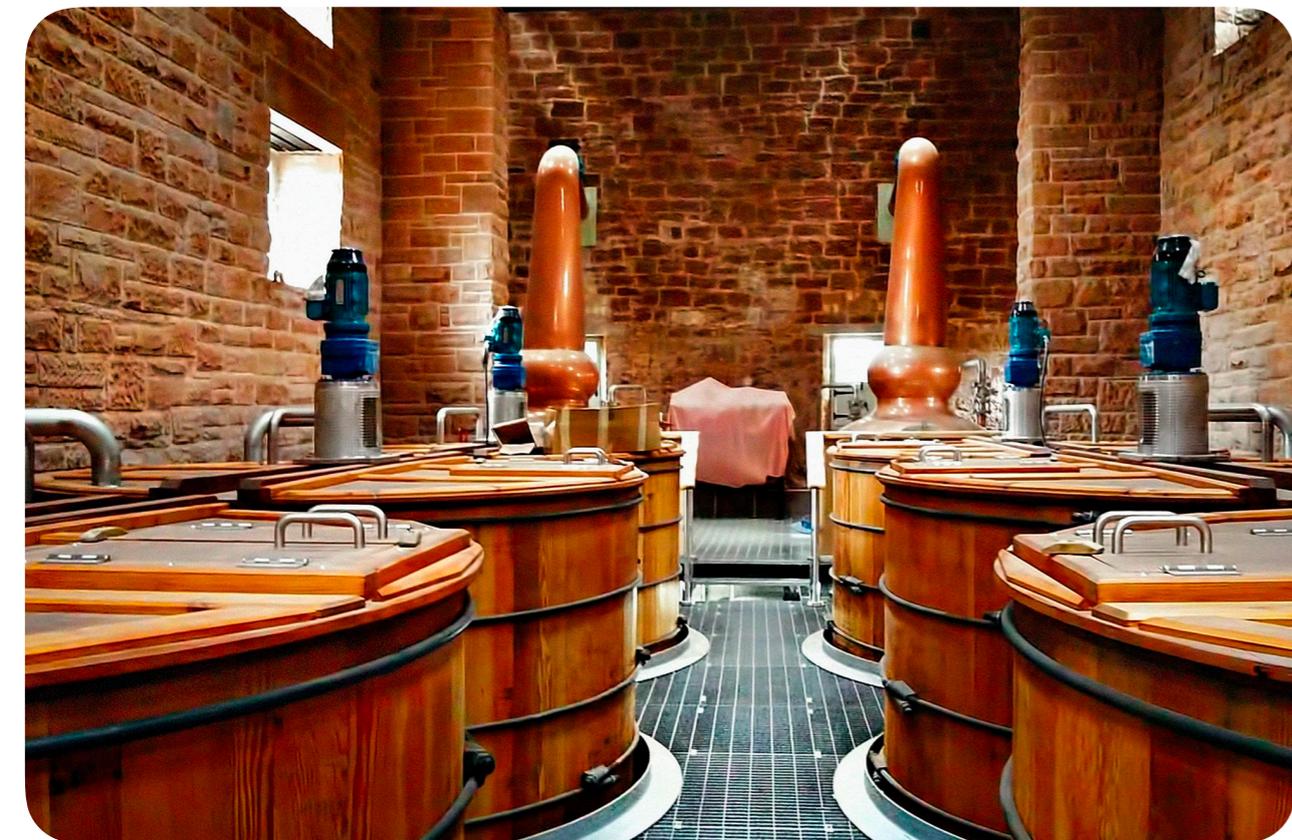
### ABSTRACT

The vast majority of hadrons observed in nature are not stable under the strong interaction; rather they are resonances whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy nonperturbative region, and in addition many probes of the limits of the electroweak sector of the standard model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a

# Three big ideas

- Distillation [Peardon, *et al.* (Hadron Spectrum, 2009)]
- use eigenvectors of the Gauge-covariant Laplacian as a way to:
  - smear [distill: boil off the high modes]
  - reduce the size operators / propagators

$$\mathbf{1}_{N_c \times N_x \times N_y \times N_z} \implies \sum_k^N V^{(k)} V^{(k)\dagger}$$



# Three big ideas

- Distillation
- Generalized eigenvalue problem

$$C_{ab}(t) \equiv \langle 0 | \mathcal{O}_b(t) \mathcal{O}_a^\dagger(0) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t},$$

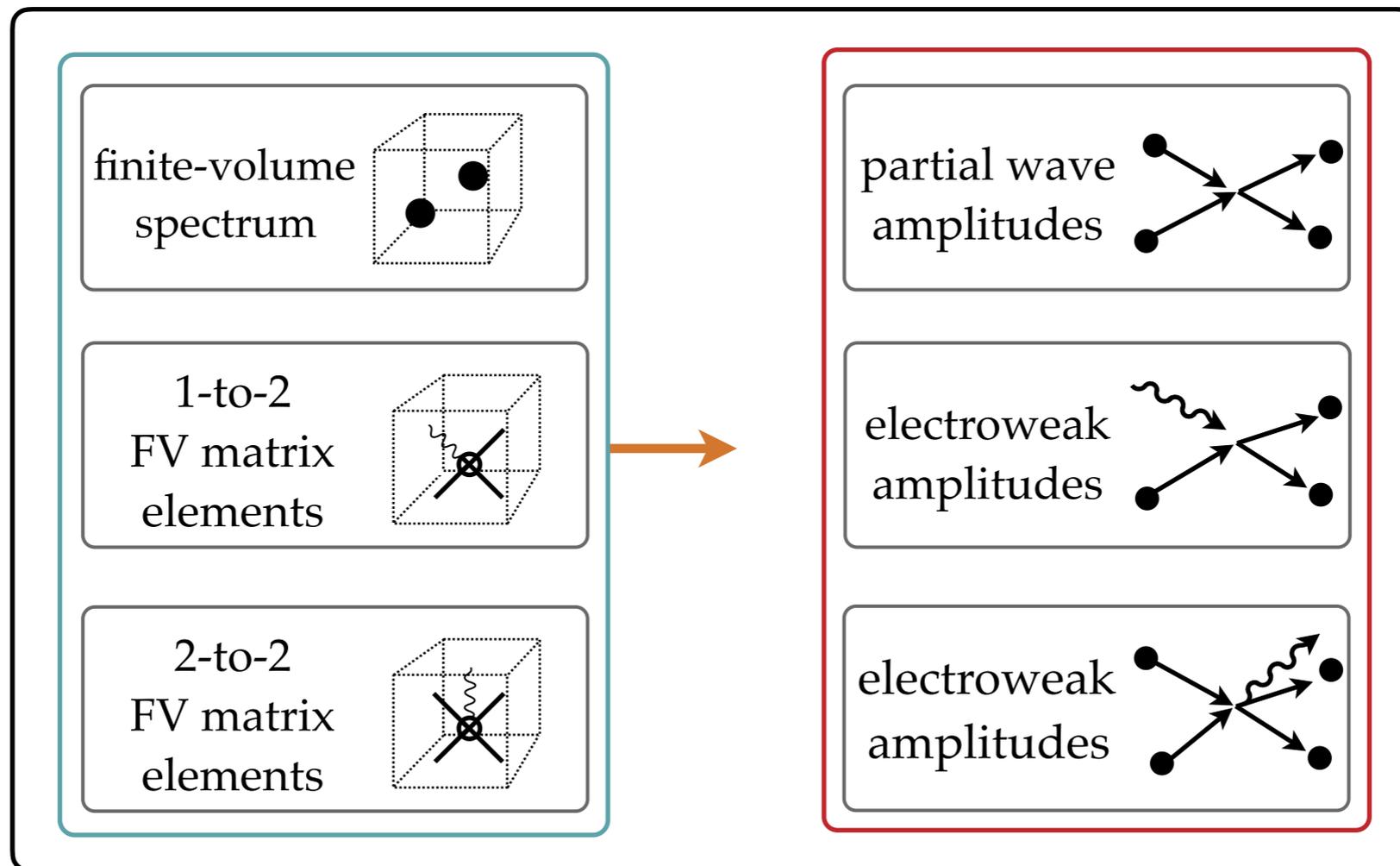
$$C(t) \vec{v}^{(n)}(t, t_0) = \lambda_n(t, t_0) C(t_0) \vec{v}^{(n)}(t, t_0),$$

$$\lambda_n(t, t_0) = e^{-E_n(t-t_0)} + \dots$$

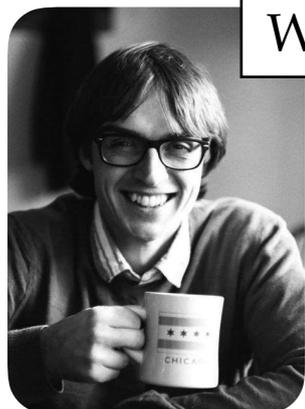
Michael (1985), Luscher & Wolff (1990), Blossier et al. (2009)

# Three big ideas

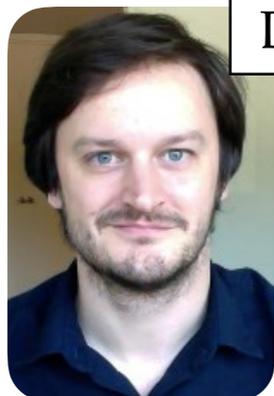
- Distillation
- Generalized eigenvalue problem
- Finite- and infinite-volume mappings
  - no simple connection
  - naive extrapolations do not always make sense



# the light nonets



Wilson



Dudek



Edwards



Thomas



PRL 118, 022002 (2017)

PHYSICAL REVIEW LETTERS

week ending  
13 JANUARY 2017

## Isoscalar $\pi\pi$ Scattering and the $\sigma$ Meson Resonance from QCD

Raul A. Briceño,<sup>1,\*</sup> Jozef J. Dudek,<sup>1,2,†</sup> Robert G. Edwards,<sup>1,‡</sup> and David J. Wilson<sup>3,§</sup>

JLAB-THY-17-2534

## Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the $\sigma, f_0, f_2$ mesons from QCD

Raul A. Briceño,<sup>1,2,\*</sup> Jozef J. Dudek,<sup>1,3,†</sup> Robert G. Edwards,<sup>1,‡</sup> and David J. Wilson<sup>4,§</sup>  
(for the Hadron Spectrum Collaboration)

<sup>1</sup>Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA

<sup>2</sup>Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

<sup>3</sup>Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA

<sup>4</sup>School of Mathematics, Trinity College, Dublin 2, Ireland

PHYSICAL REVIEW LETTERS VOL..XX, 000000 (XXXX)

## Quark-Mass Dependence of Elastic $\pi K$ Scattering from QCD

David J. Wilson,<sup>1,\*</sup> Raúl A. Briceño,<sup>2,3,†</sup> Jozef J. Dudek,<sup>2,4,‡</sup> Robert G. Edwards,<sup>2,§</sup> and Christopher E. Thomas<sup>5,||</sup>

(Hadron Spectrum Collaboration)

<sup>1</sup>School of Mathematics, Trinity College, Dublin 2, Ireland

<sup>2</sup>Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

<sup>3</sup>Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA

<sup>4</sup>Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA

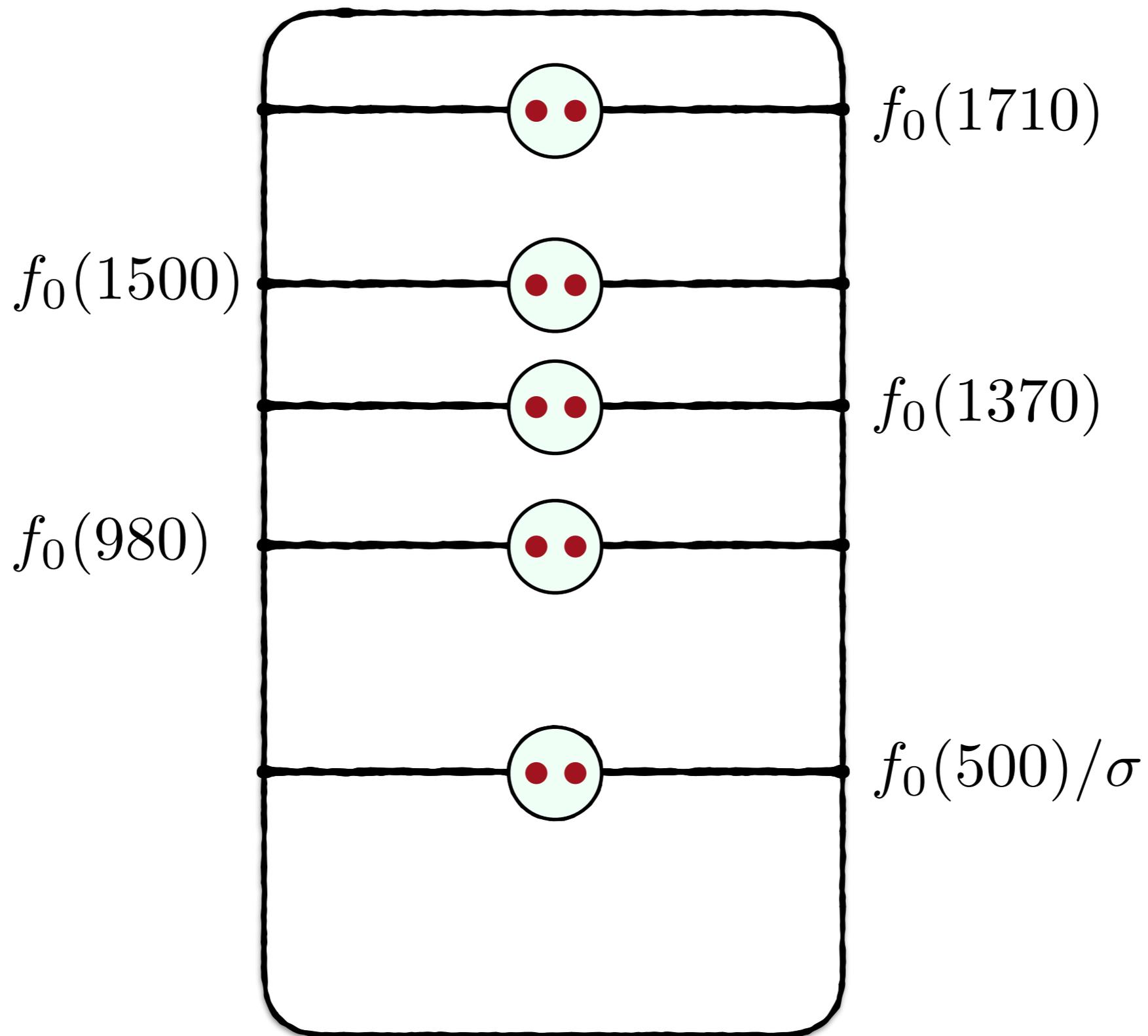
<sup>5</sup>DAMTP, University of Cambridge, Centre for Mathematical Sciences,  
Wilberforce Road, Cambridge CB3 0WA, United Kingdom

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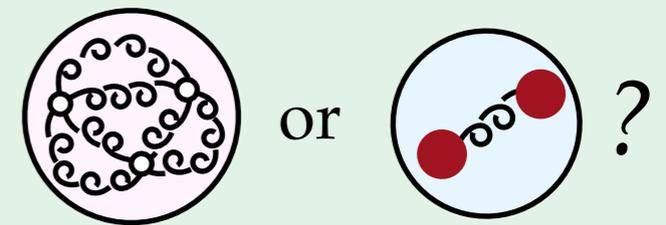
# Isoscalar sector



# Isoscalar sector

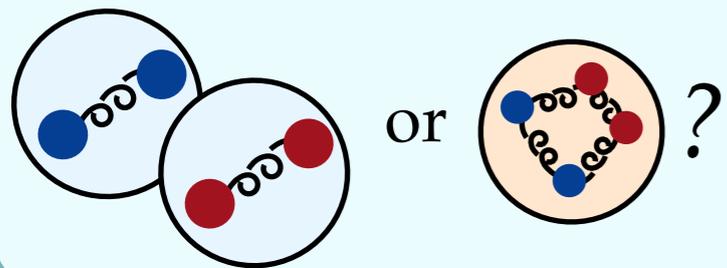
$f_0(1500)$

$f_0(1710)$

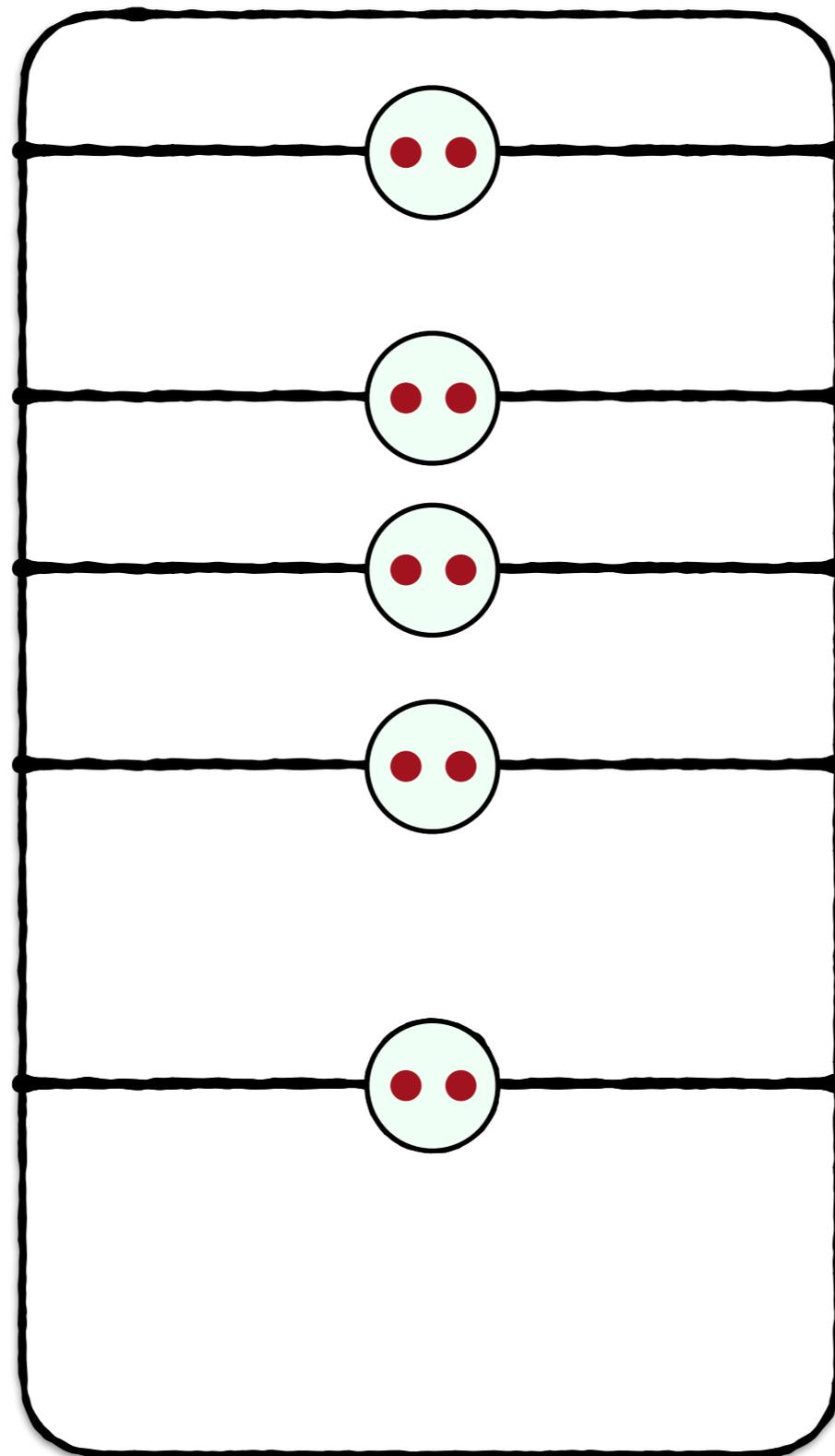
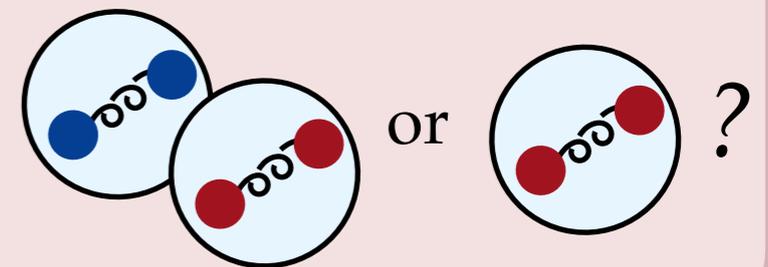


$f_0(1370)$

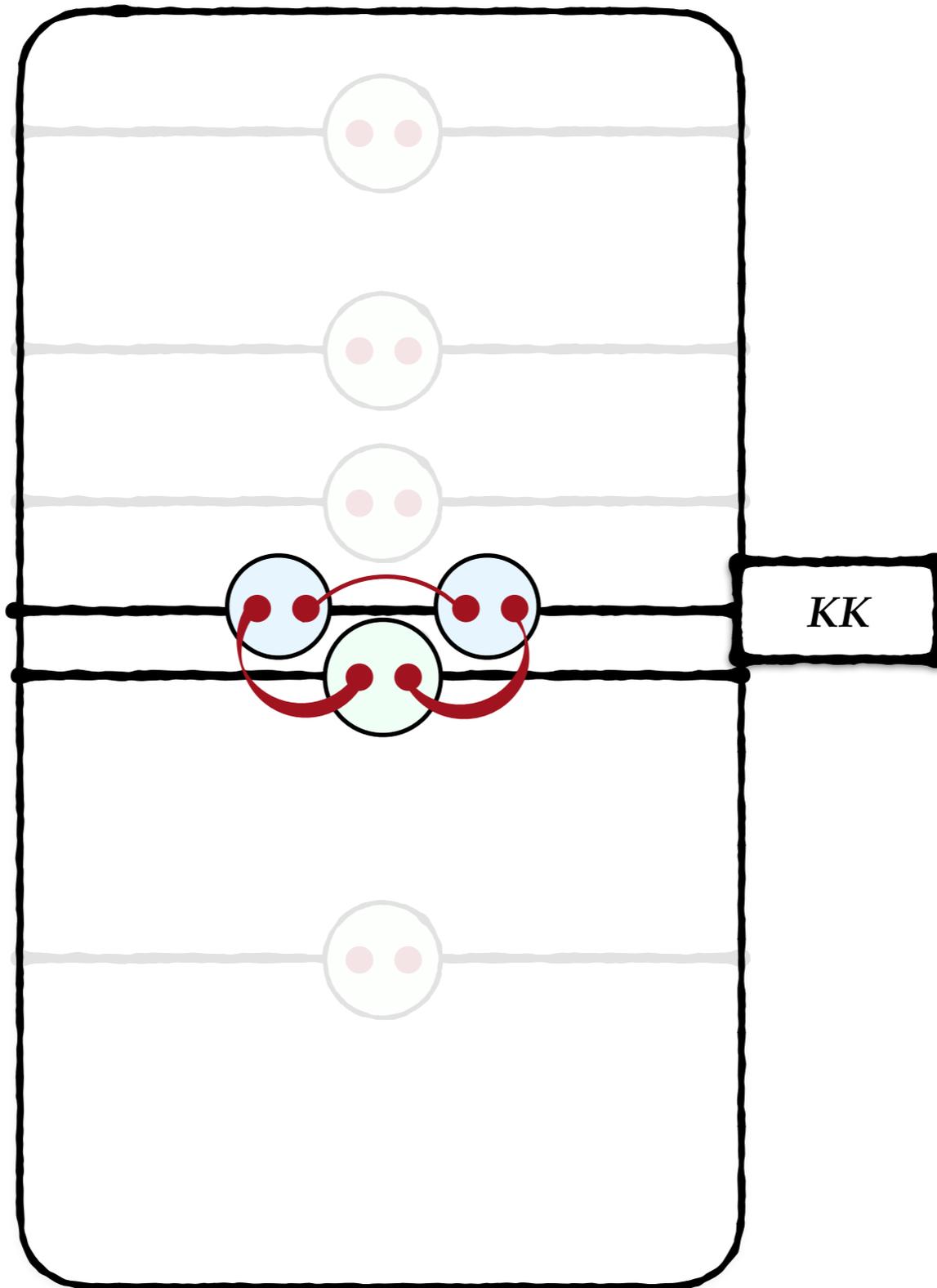
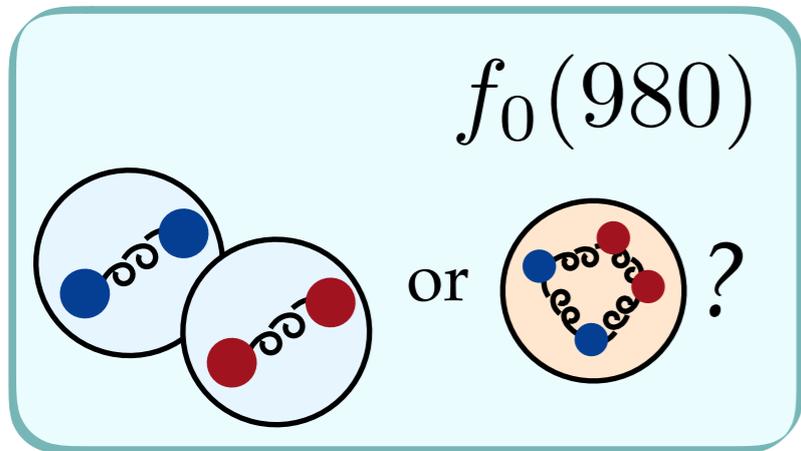
$f_0(980)$



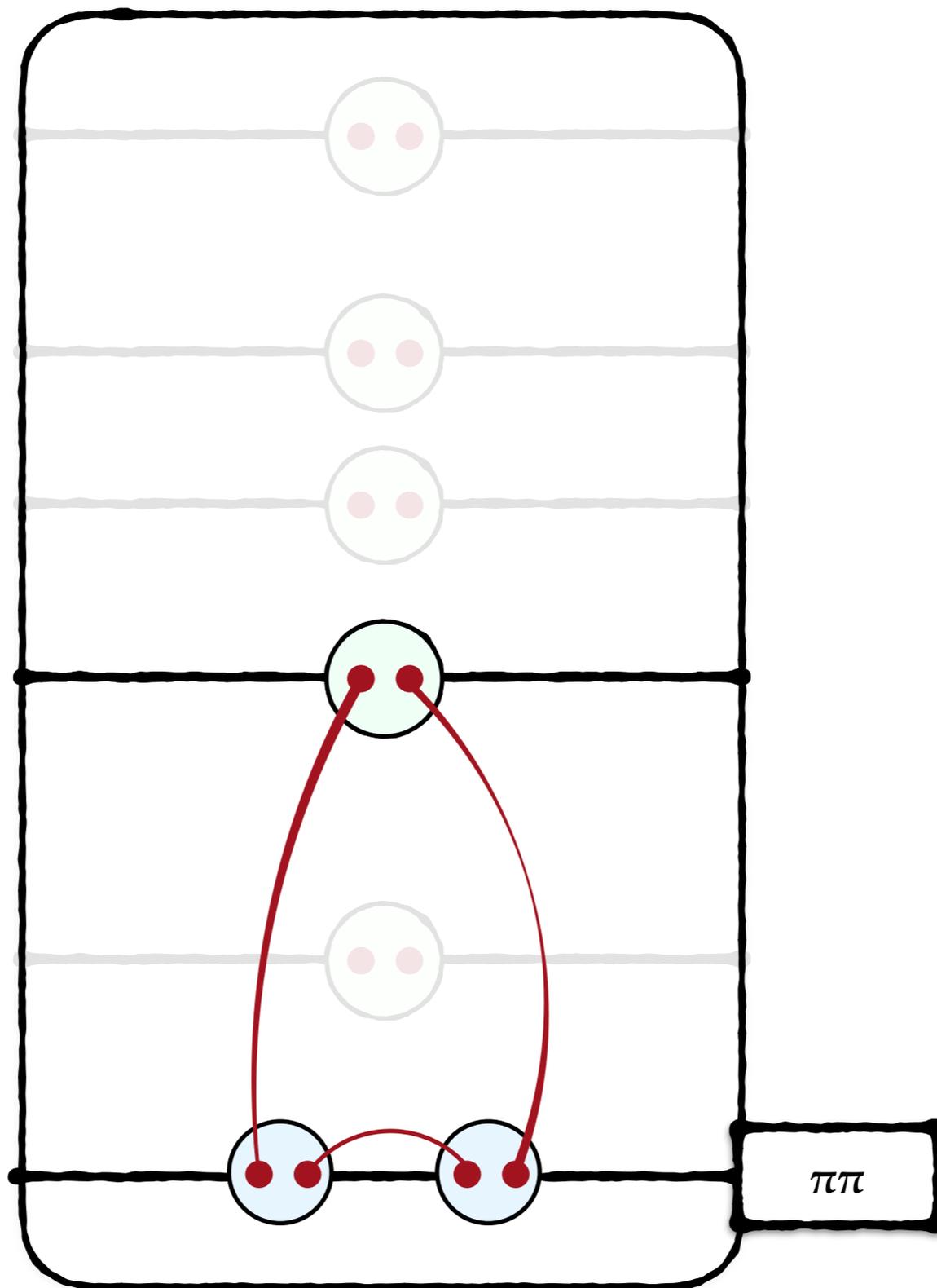
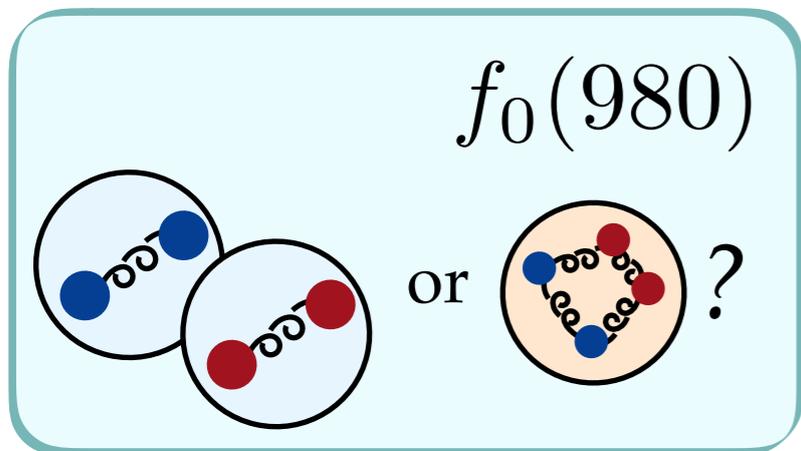
$f_0(500)/\sigma$



# Isoscalar sector



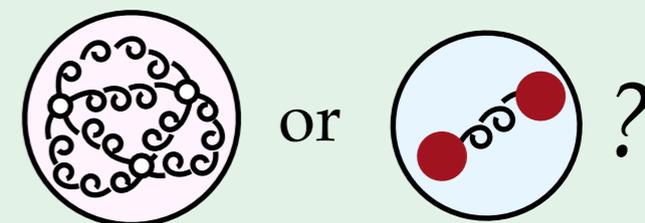
# Isoscalar sector



# Isoscalar sector

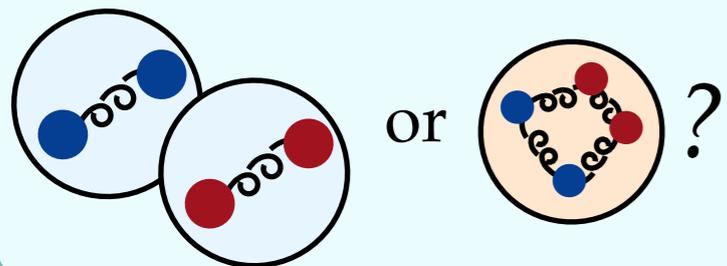
$f_0(1500)$

$f_0(1710)$

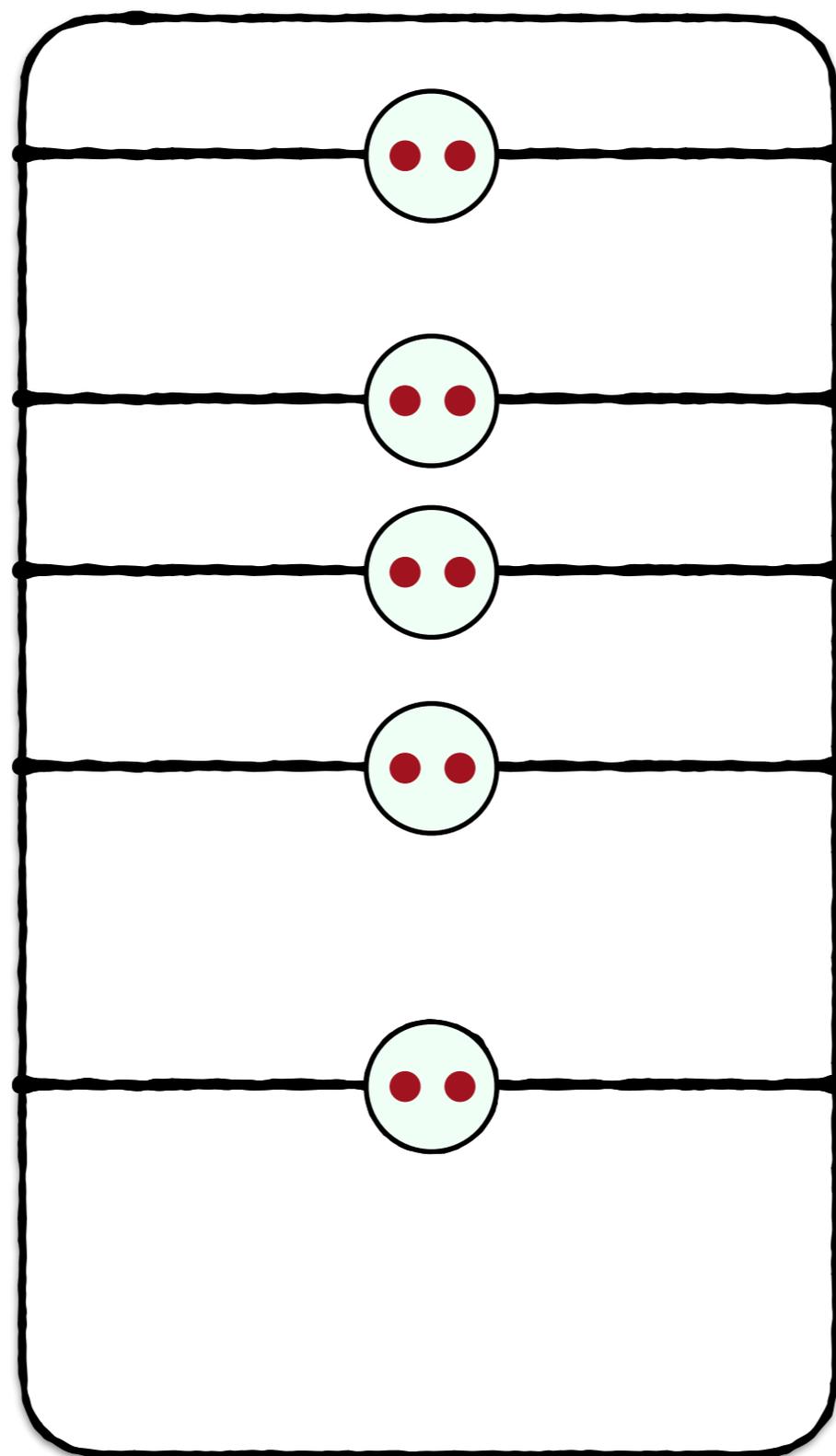
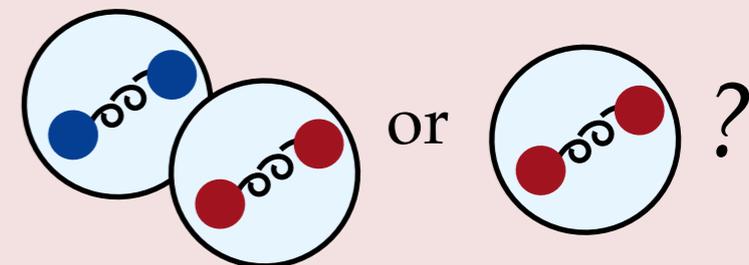


$f_0(1370)$

$f_0(980)$

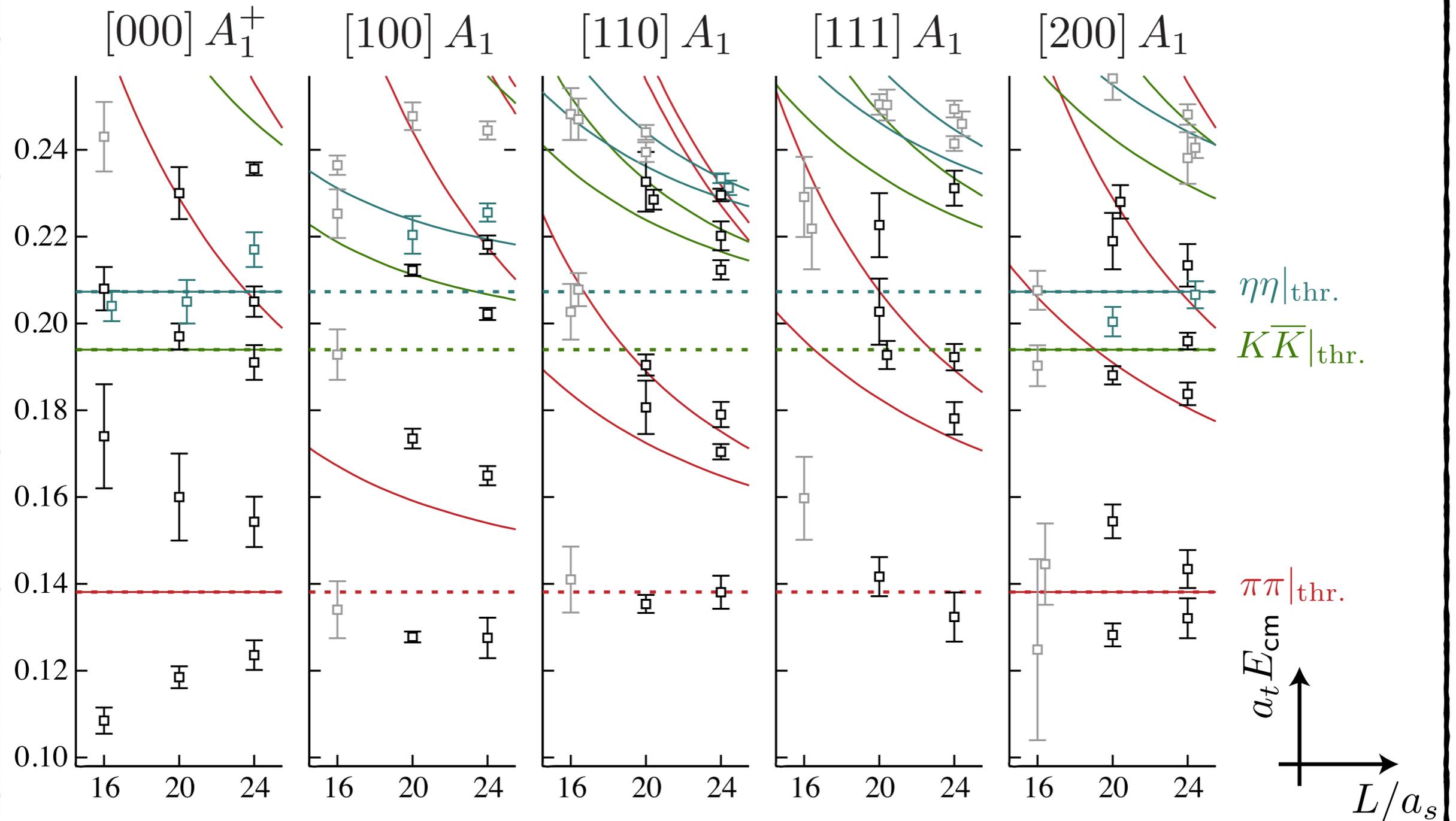


$f_0(500)/\sigma$



# Isoscalar spectra: S-wave dominant

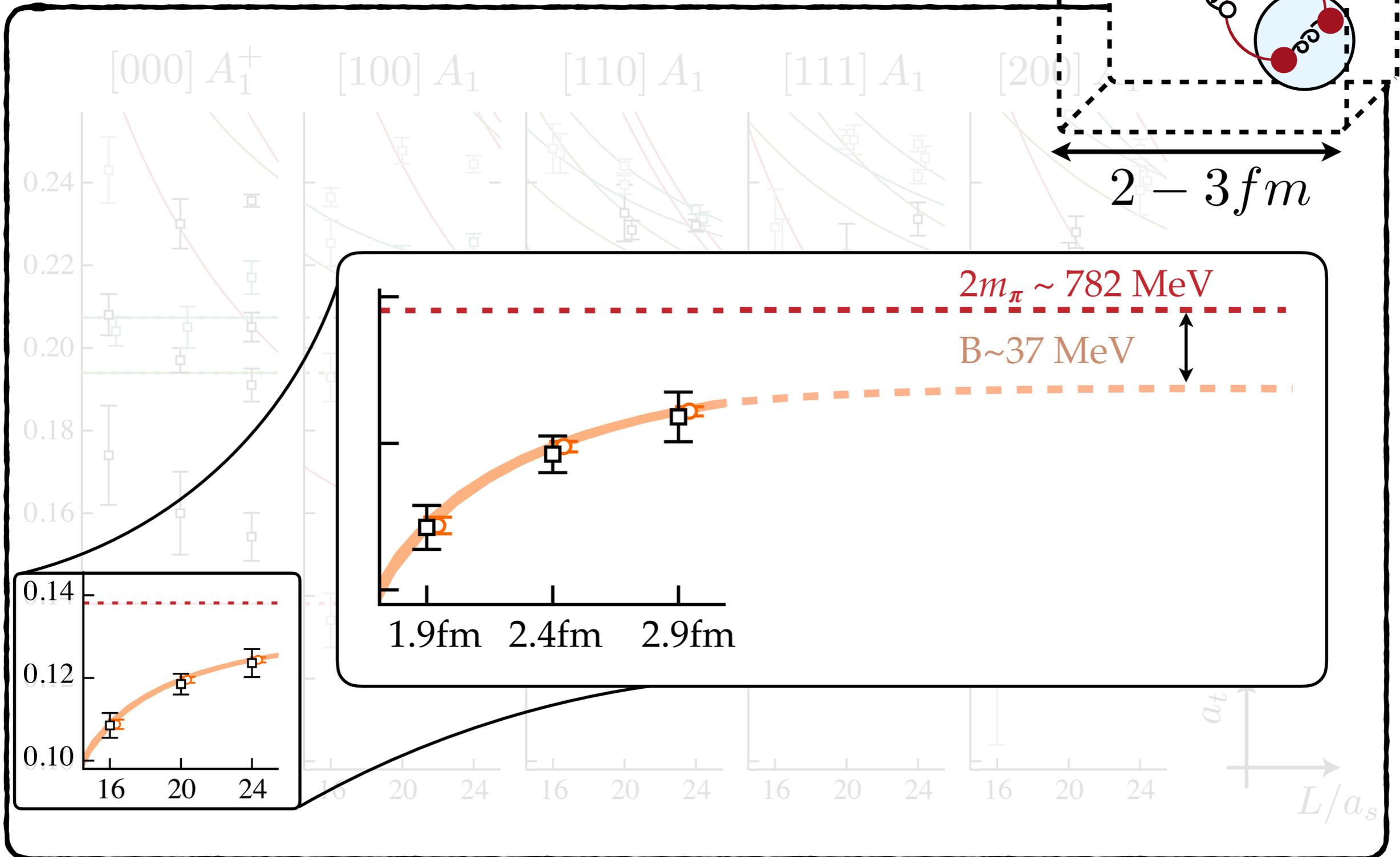
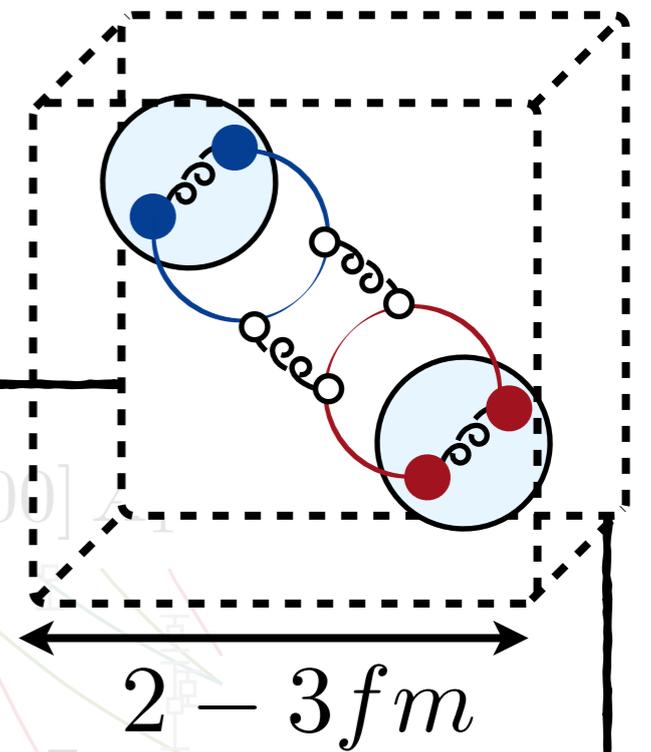
- Multi-meson ops. are crucial
- Spectrum including a large basis:  $\{\pi\pi, K\bar{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



$m_\pi=391$  MeV

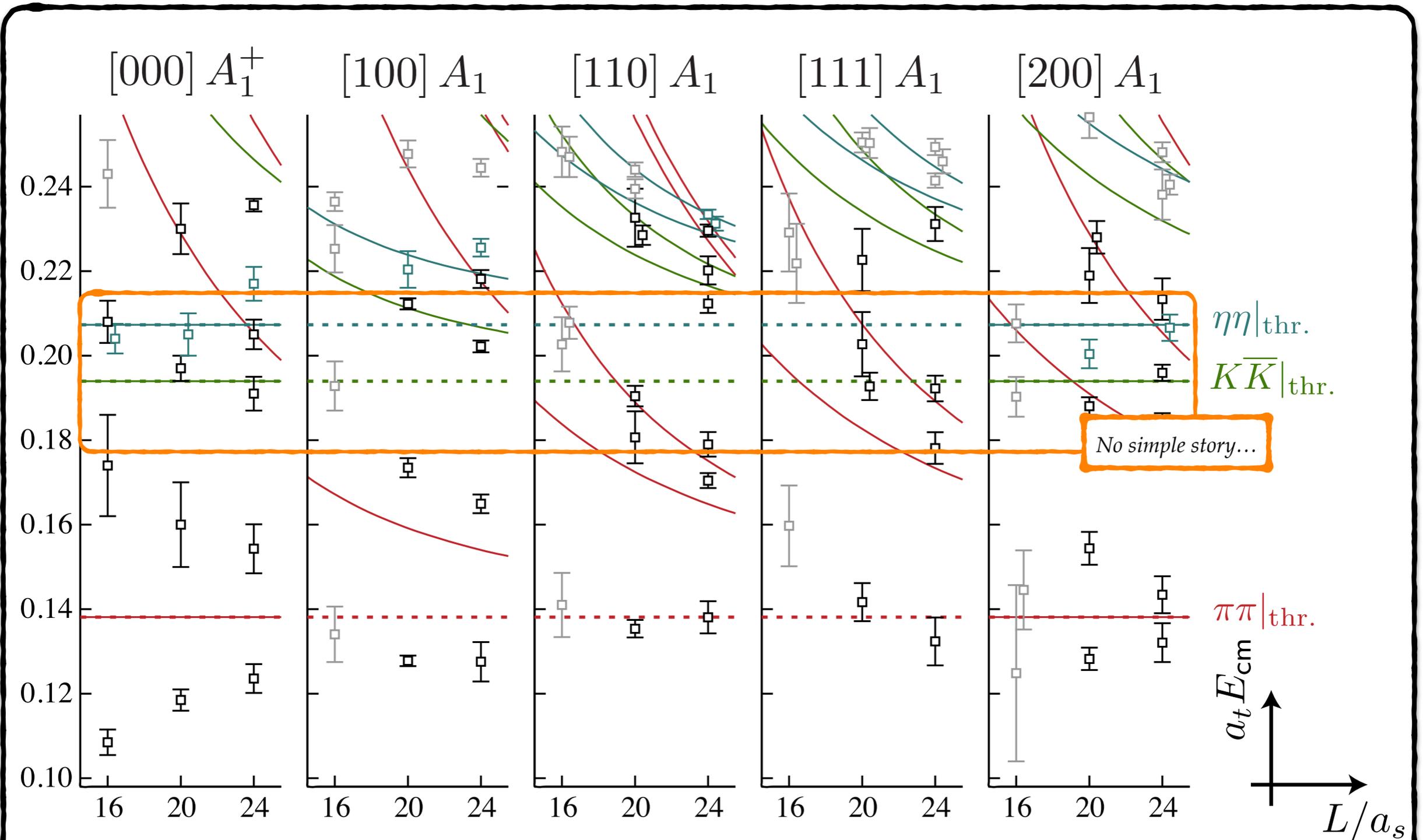
# Isoscalar spectra: S-wave dominant

- Multi-meson ops. are crucial
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# Isoscalar spectra: S-wave dominant

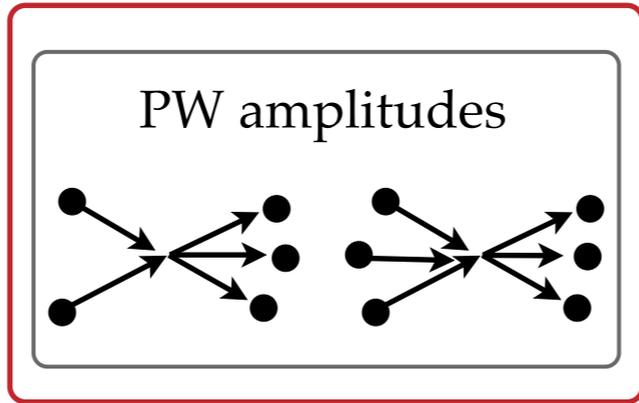
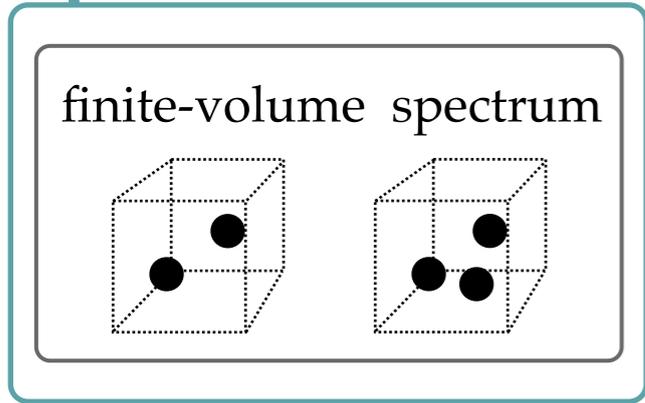
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- Spectrum including a large basis:  $\{\pi\pi, K\bar{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



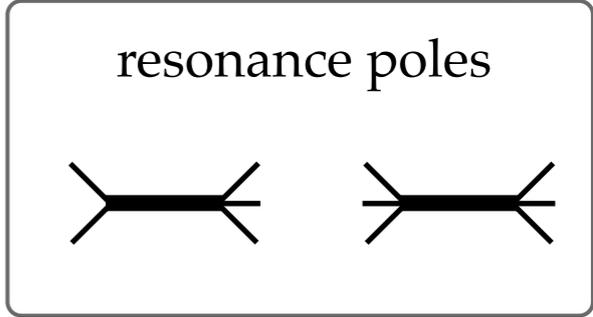
$m_\pi=391$  MeV

# few-body systems in LQCD

*lattice QCD*



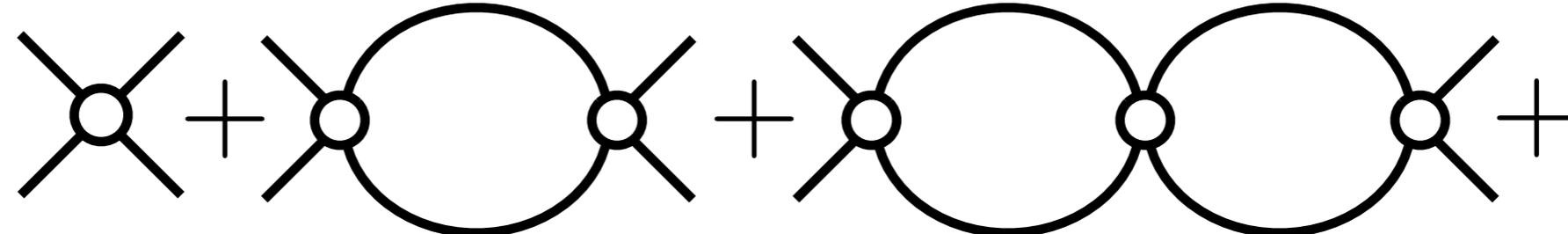
analytic continuation



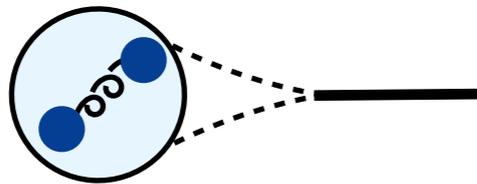
identification of  
• states [masses & widths],  
• production/decay mechanisms

# Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$
The equation shows the expansion of the scattering amplitude  $i\mathcal{M}$  in perturbation theory. The first term is a tree-level diagram with four external lines meeting at a central vertex. The second term is a one-loop diagram consisting of a tree-level vertex connected to a loop, which then connects to another tree-level vertex. The third term is a two-loop diagram with two loops connected in series between the external lines. The series continues with an ellipsis.

*IR limit of QCD, only interested in hadronic d.o.f.*



# Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \underbrace{\text{tree} + \text{one-loop} + \text{two-loop} + \dots}_{\left\{ \text{non-perturbative kernel} \right\}}$$

*non-perturbative kernel including  
all diagrams not shown...*

*“yep, the left hand cut is there”*

# Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

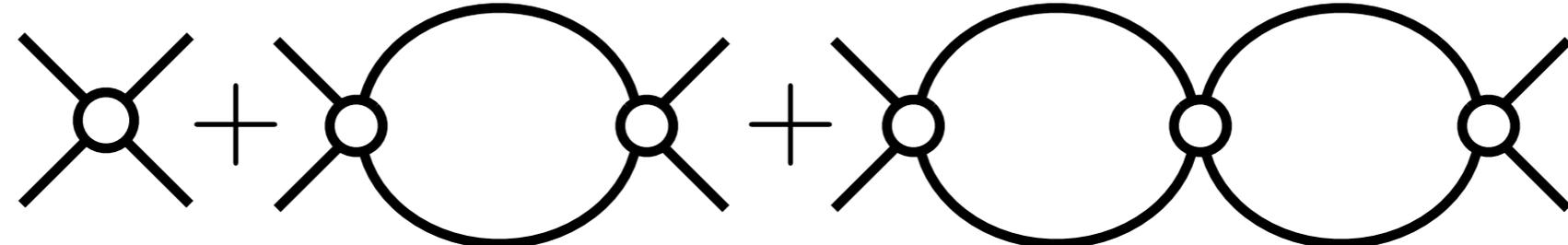
$$\begin{aligned} \text{one-loop} &= \int \frac{d^4 k}{(2\pi)^4} [iB(k, P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P - k)^2 - m^2 + i\epsilon} \\ &= \int \frac{d^3 k}{(2\pi)^3} \frac{[iB(k, P)]^2}{(2\omega_k)^2} \pi \delta(E - 2\omega_k) + \text{“PV integral”} \\ &= [iB_{on}] \rho [iB_{on}] + \text{“PV integral”} \\ &= \text{cut diagram} + \text{PV diagram} \end{aligned}$$

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

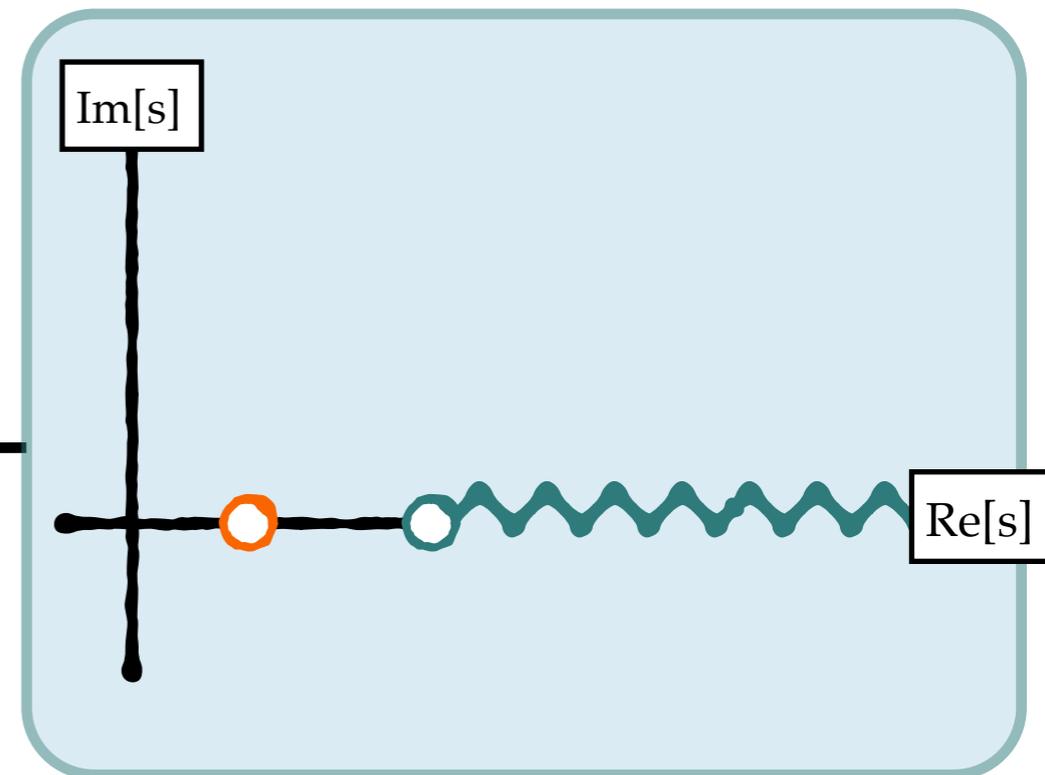
square root singularity.

# Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$


$2^N$  sheets for  $N$  open channels



$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

# Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

$$= \text{square} + \dots$$

$$= \underbrace{\text{square}}_{\text{K-matrix}} \left\{ \text{tree} + \text{one-loop PV} + \dots \right\}$$

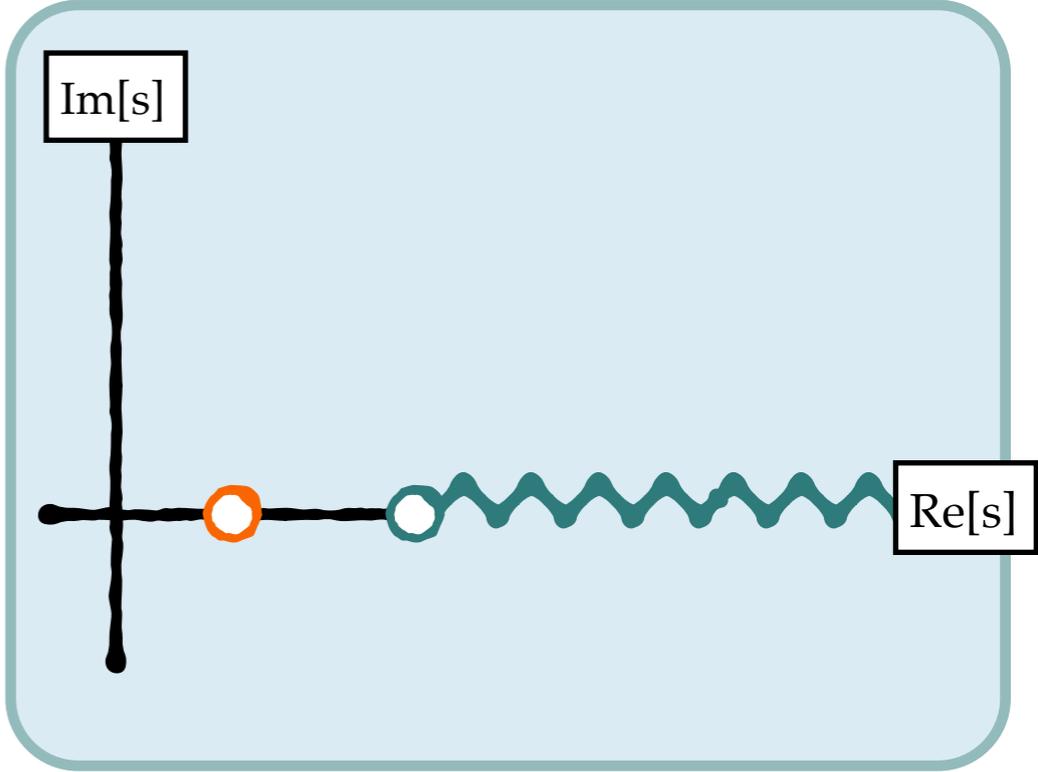
# Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$
$$= \text{tree} + \text{one-loop with cut} + \dots$$

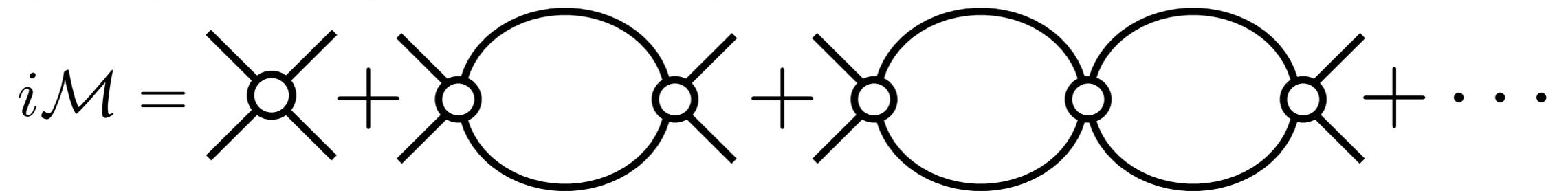
$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

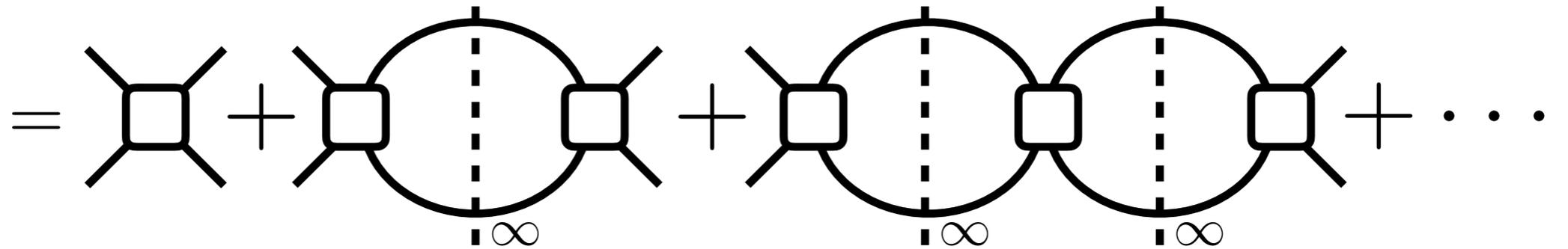
square root singularity.



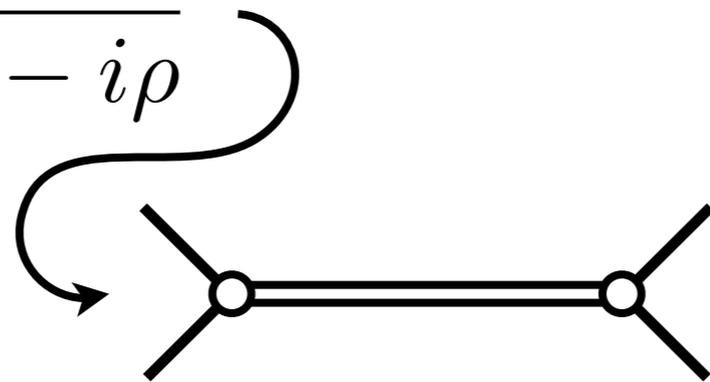
# Two-body scattering

Unitarity using all orders perturbation theory:

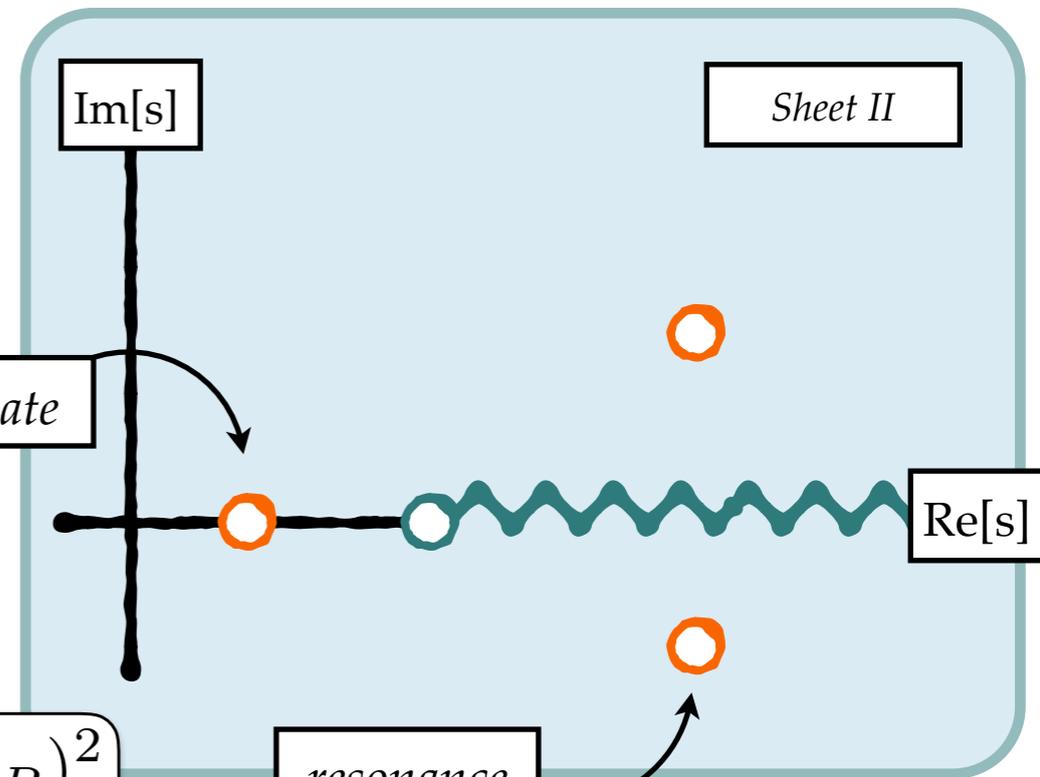
$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$


$$= \text{tree} + \text{one-loop with cut} + \text{two-loop with cut} + \dots$$


$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$



bound state



$$s_R = (E_R - \frac{i}{2}\Gamma_R)^2$$

resonance

# Two-body scattering

Unitarity using all orders perturbation theory:

$$\begin{aligned}
 i\mathcal{M} &= \text{[tree]} + \text{[1-loop]} + \text{[2-loop]} + \dots \\
 &= \text{[tree with cut]} + \text{[1-loop with cut]} + \text{[2-loop with cut]} + \dots \\
 &= \frac{i}{\mathcal{K}^{-1} - i\rho}
 \end{aligned}$$

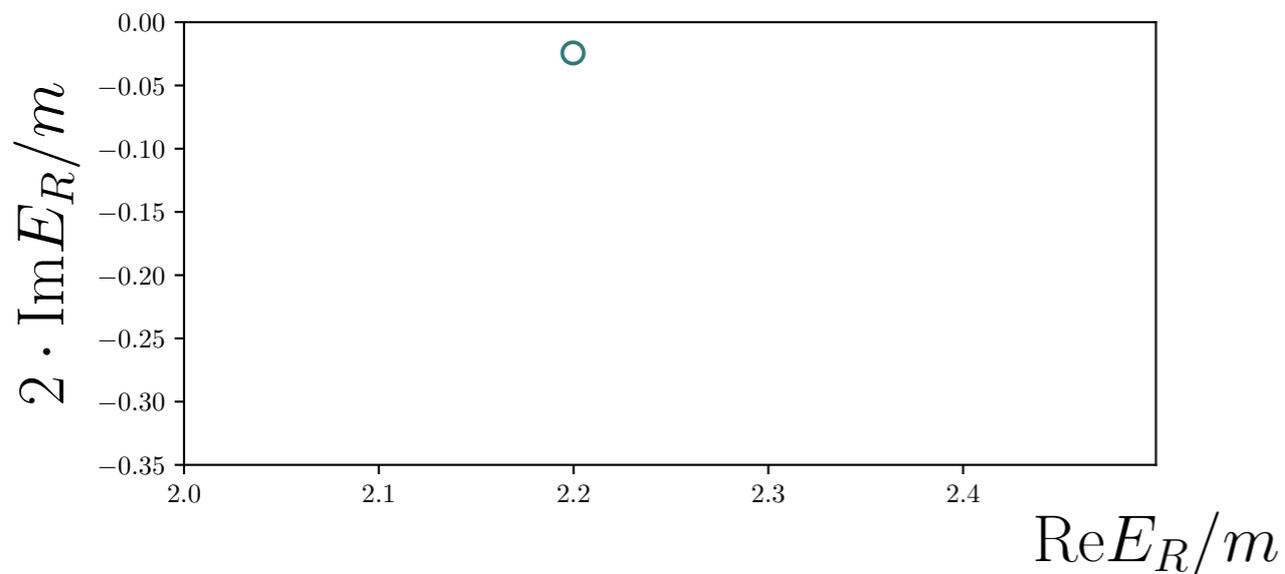
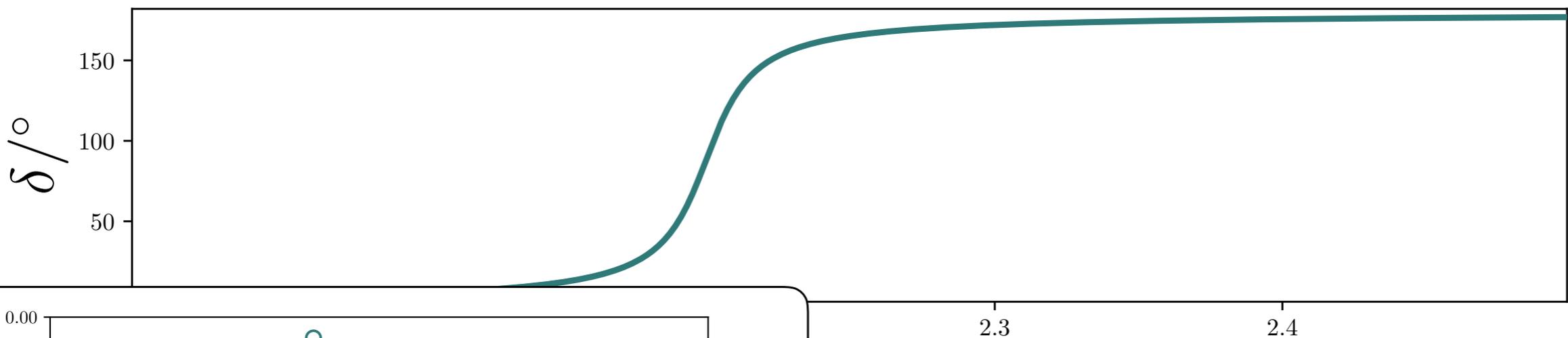
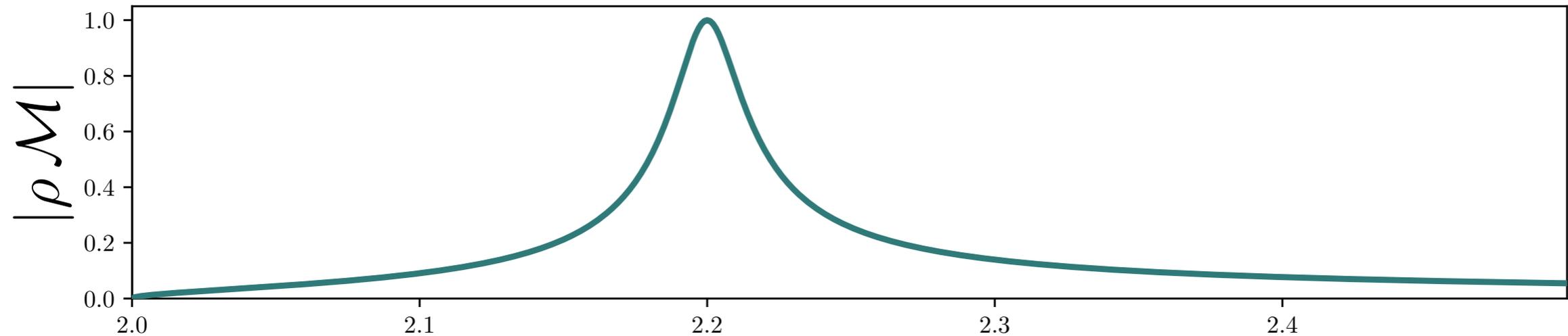
Equating this to the elastic S matrix...

$$S = e^{2i\delta} = 1 + 2i\rho\mathcal{M}$$

$$\begin{aligned}
 \mathcal{K}^{-1} &= \rho \cot \delta \\
 \mathcal{M} &= \frac{\sin \delta}{\rho} e^{i\delta}
 \end{aligned}$$

# Two-body scattering - resonance

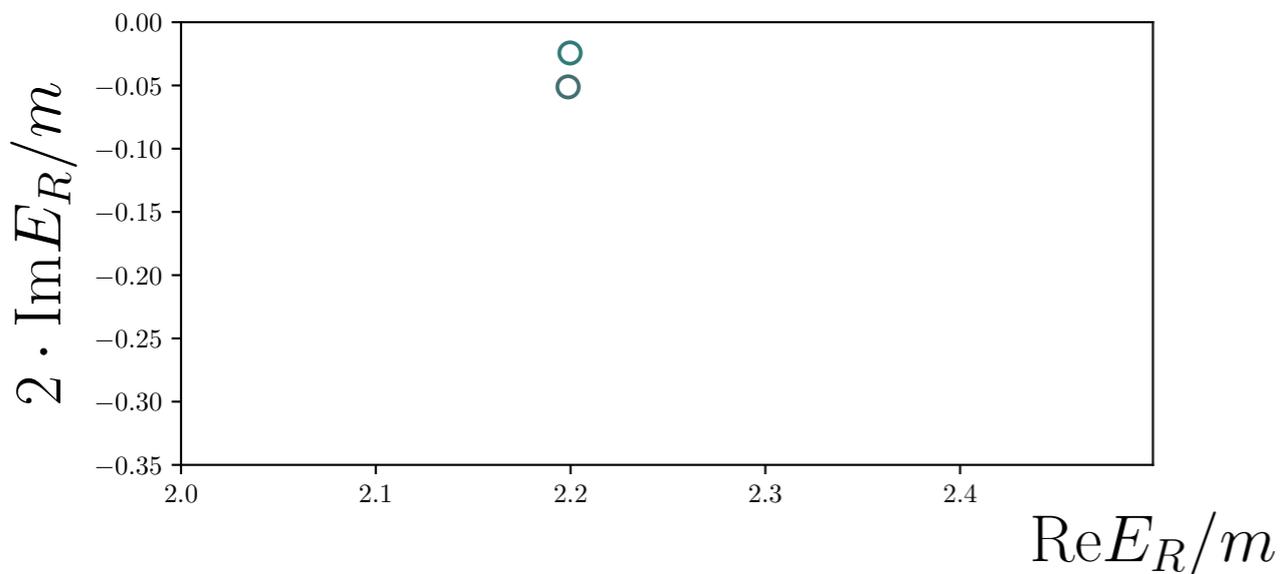
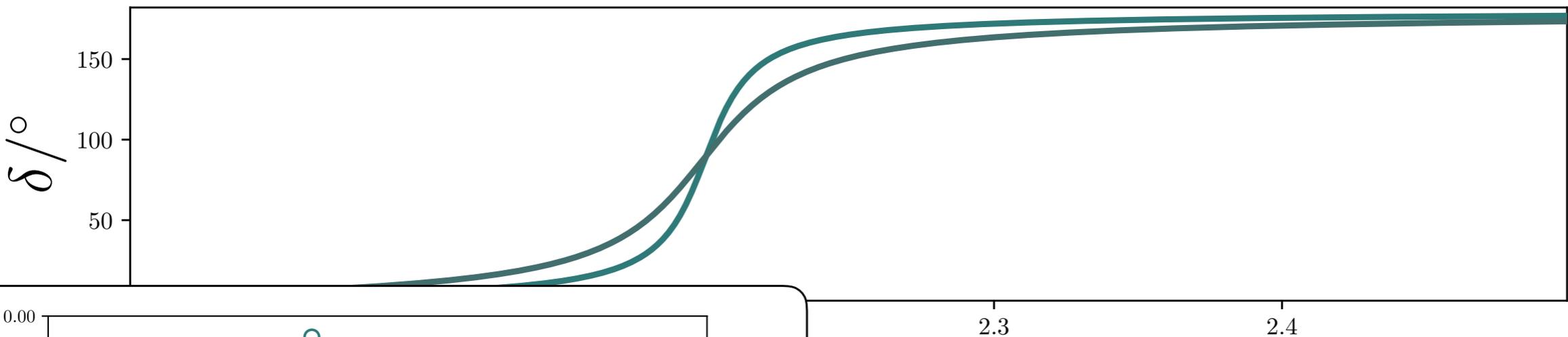
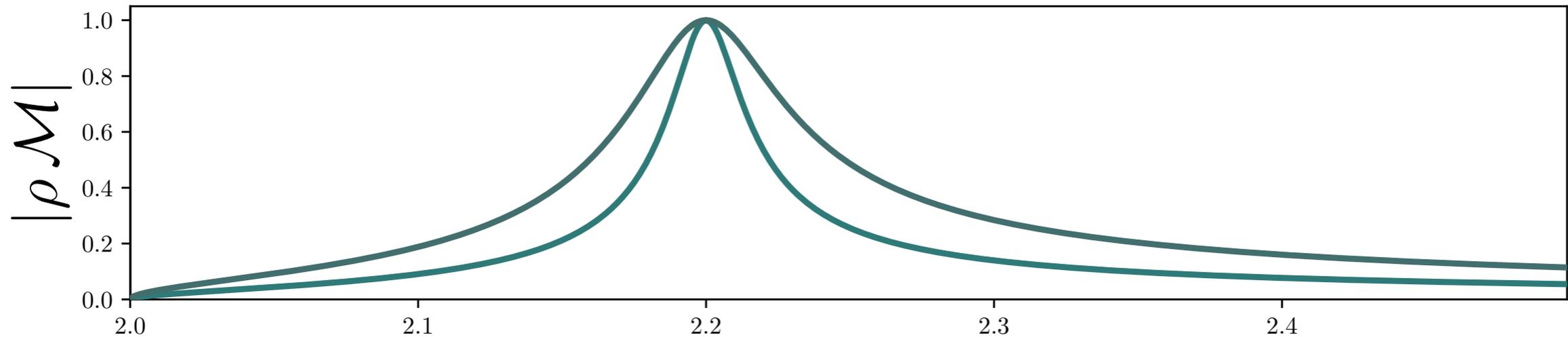
To build some intuition:  $\mathcal{M} = \frac{\sin \delta}{\rho} e^{i\delta}$



$E^*/m$

# Two-body scattering - resonance

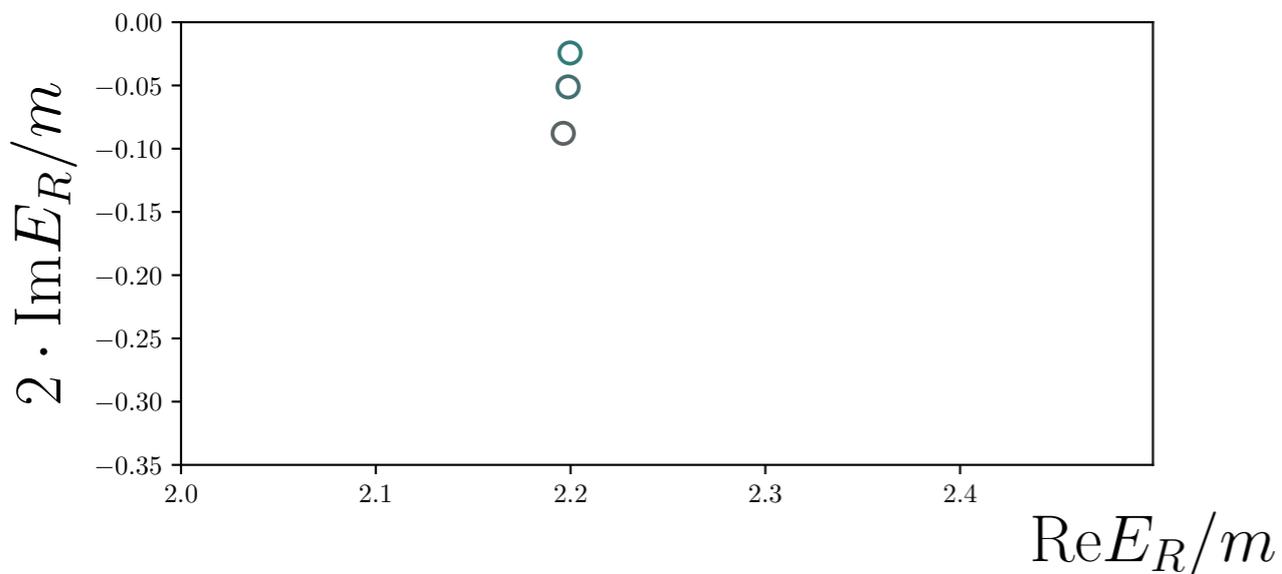
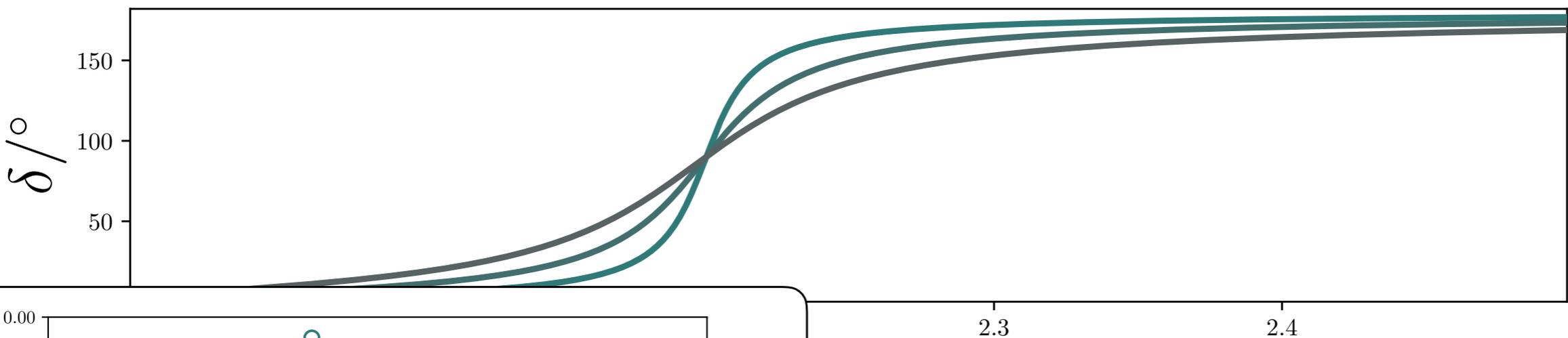
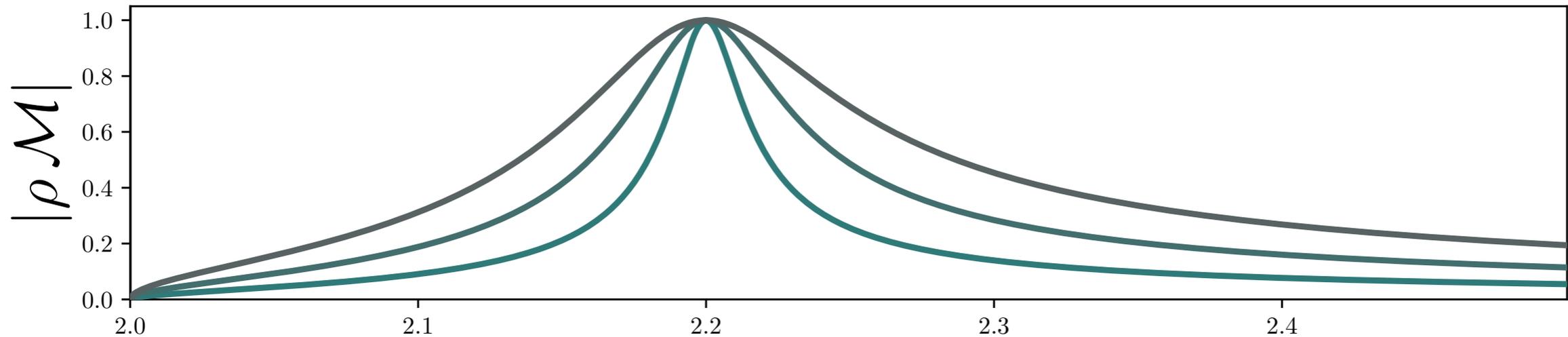
To build some intuition:  $\mathcal{M} = \frac{\sin \delta}{\rho} e^{i\delta}$



$E^*/m$

# Two-body scattering - resonance

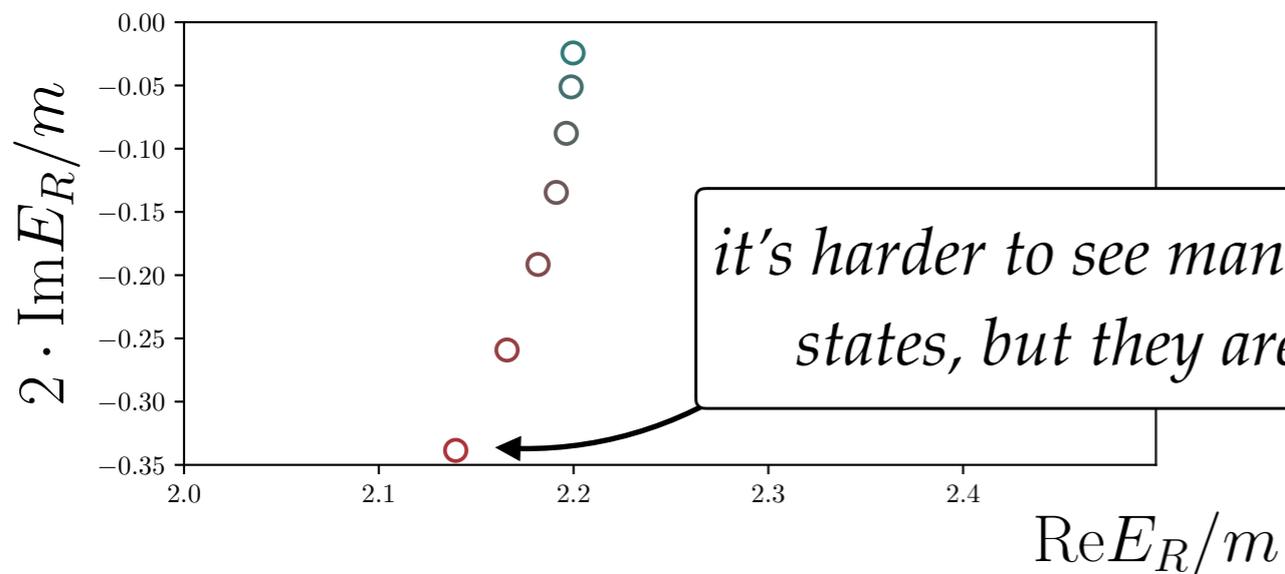
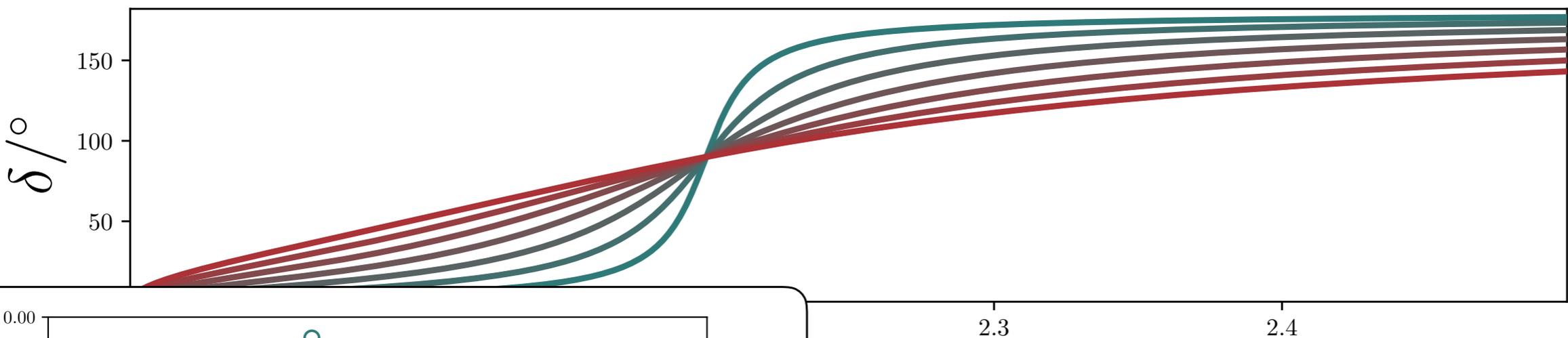
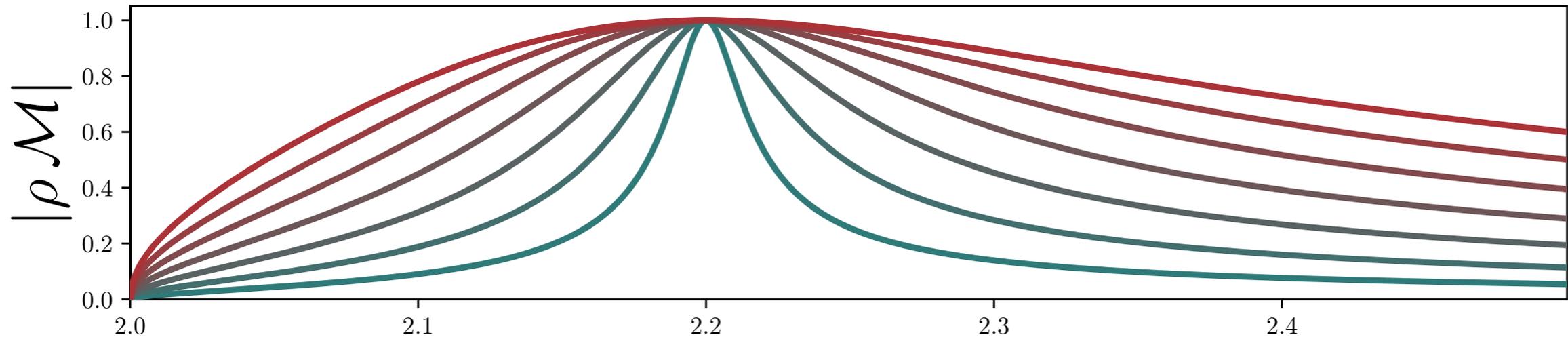
To build some intuition:  $\mathcal{M} = \frac{\sin \delta}{\rho} e^{i\delta}$



$E^*/m$

# Two-body scattering and resonances

To build some intuition:  $\mathcal{M} = \frac{\sin \delta}{\rho} e^{i\delta}$



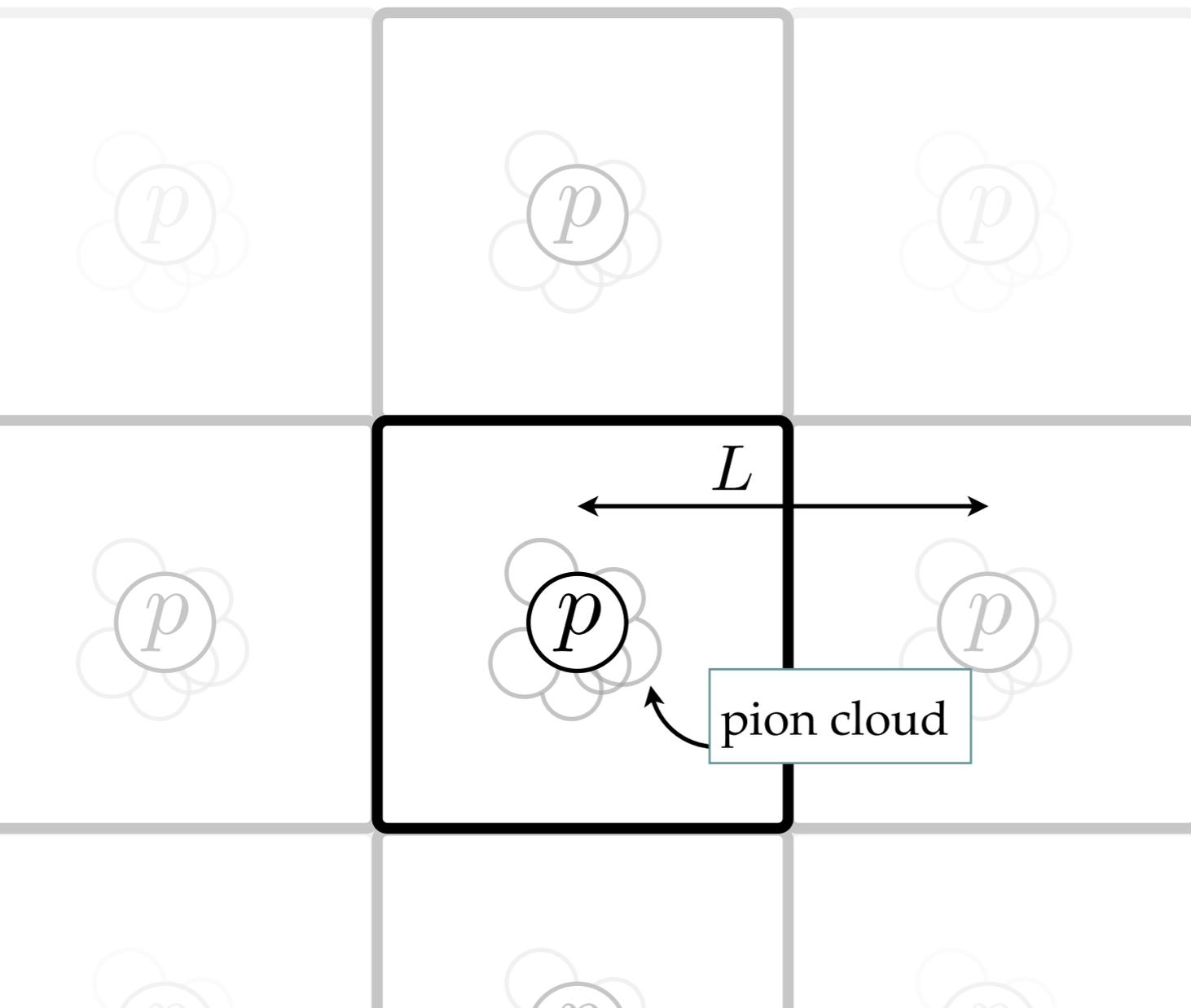
*it's harder to see manifestation of broader states, but they are certainly there!*

$E^*/m$

$\text{Re} E_R / m$

# Putting particles in a box

- Finite-volume arise from the interactions with mirror images
- Assuming  $L \gg$  size of the hadrons  $\sim 1/m_\pi$ 
  - This is a purely infrared artifact
  - We can determine these artifact using hadrons are the degrees of freedom
- Note  $m_\pi L$  is a natural parameter



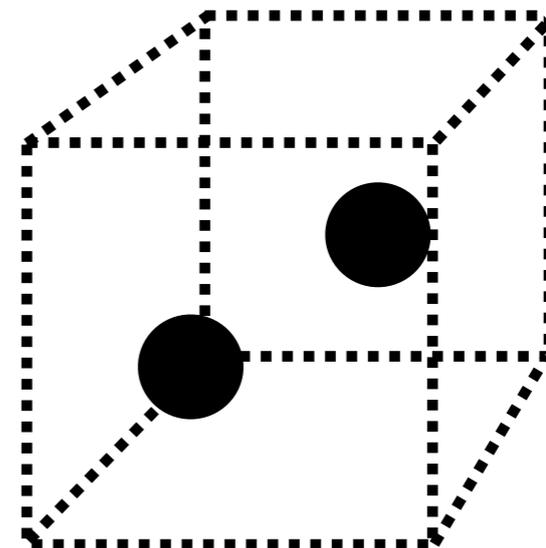
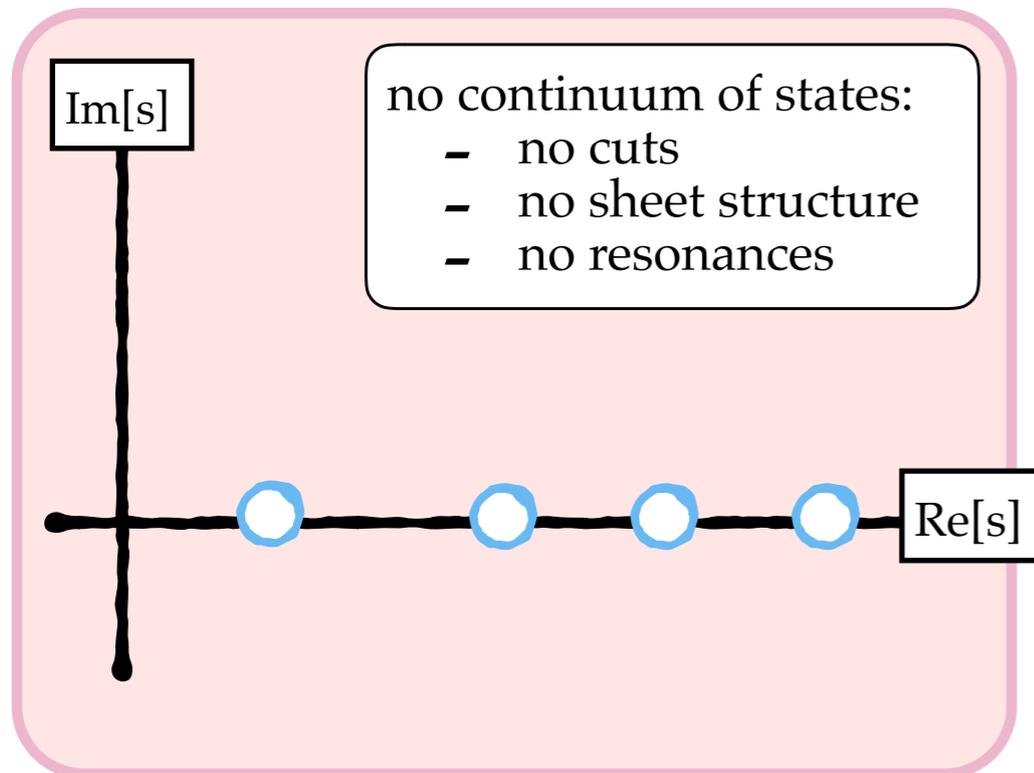
$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

# Two-particle in finite volume

Consider the finite-volume two-particle correlator ( $E \sim 2m$ ):

$$C_L^{2pt.}(P) = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$

The diagram shows a series of terms in a sum. The first term is a circle labeled 'V' with two small white circles on its left and right sides. The second term is two such circles connected at their right and left sides respectively. This pattern continues with an ellipsis.



# Two-particle in finite volume

Consider the finite-volume two-particle correlator ( $E \sim 2m$ ):

$$C_L^{2pt.}(P) = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$
$$= C_\infty(P) + \dots$$

The diagrams are Feynman diagrams for two-particle correlators. The first diagram is a single circle with two external legs, labeled  $V$ . The second diagram is two such circles connected at a vertex, also labeled  $V$ . Ellipses indicate higher-order terms in the expansion.

# Two-particle in finite volume

Consider the finite-volume two-particle correlator ( $E \sim 2m$ ):

$$\begin{aligned}
 C_L^{2pt.}(P) &= \text{[Diagram: circle with two external lines and label } V \text{]} + \text{[Diagram: two circles connected by a line, each with two external lines and label } V \text{]} + \dots \\
 &= C_\infty(P) + \text{[Diagram: dashed circle with two external lines and label } V - \infty \text{]} + \dots
 \end{aligned}$$

$$\begin{aligned}
 \text{[Diagram: dashed circle with two external lines and label } V - \infty \text{]} &= (iB) \left( \left[ \frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} \right) (iB) \\
 &\equiv [iB] iF [iB]
 \end{aligned}$$

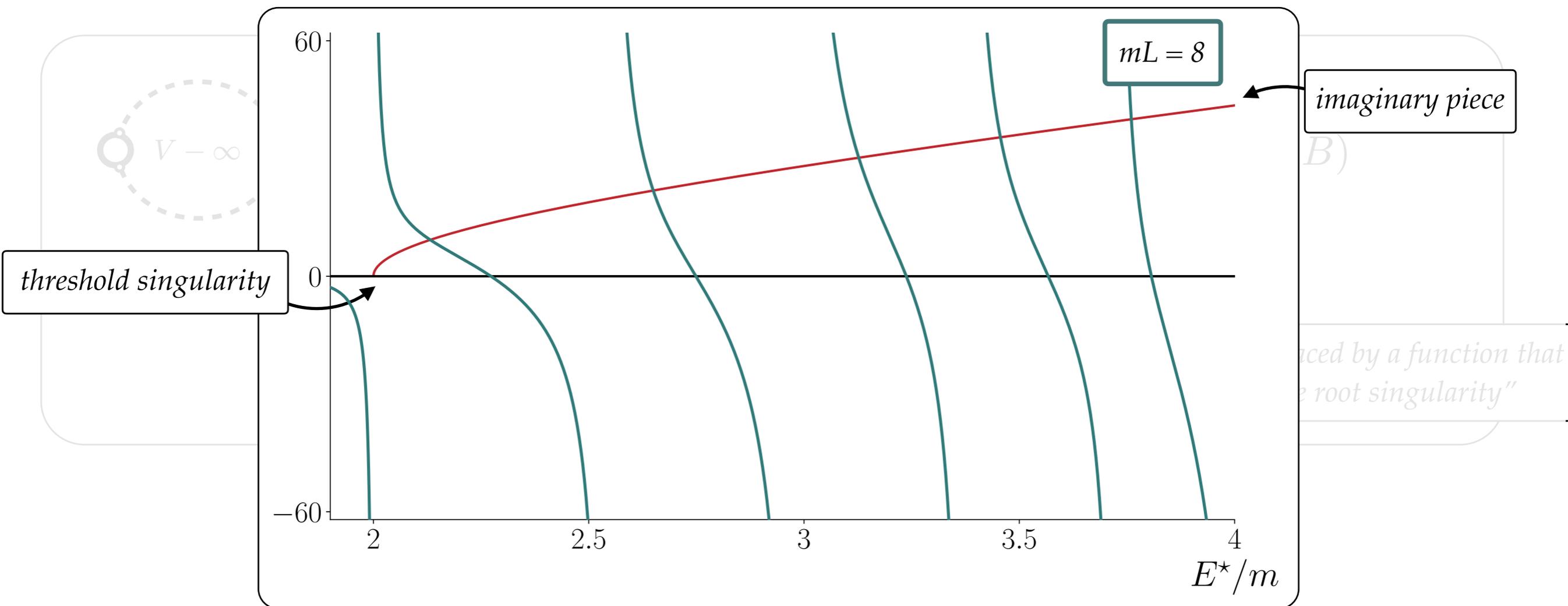
*F replaces  $\rho$*

*a simple square root singularity is replaced by a function that has both simple poles and the square root singularity*

# Two-particle in finite volume

Consider the finite-volume two-particle correlator ( $E \sim 2m$ ):

$$\begin{aligned}
 C_L^{2pt.}(P) &= \text{[Diagram: circle with two external lines and vertex } V] + \text{[Diagram: two circles with two external lines and vertices } V] + \dots \\
 &= C_\infty(P) + \text{[Diagram: dashed circle with two external lines and vertices } V - \infty] + \dots
 \end{aligned}$$



# Two-particle in finite volume

Consider the finite-volume two-particle correlator ( $E \sim 2m$ ):

$$\begin{aligned} C_L^{2pt.}(P) &= \text{[diagram: two circles labeled } V \text{ connected by a line]} + \text{[diagram: two circles labeled } V \text{ connected by a line, with a second circle attached to the middle]} + \dots \\ &= C_\infty(P) + \text{[diagram: dashed circle labeled } V - \infty \text{ with two black dots]} + \text{[diagram: two dashed circles labeled } V - \infty \text{ with two black dots]} + \dots \\ &= \text{“smooth”} + A \frac{i}{F^{-1} + \mathcal{M}} B^\dagger \end{aligned}$$

poles satisfy:  $\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$

• Lüscher (1986, 1991)

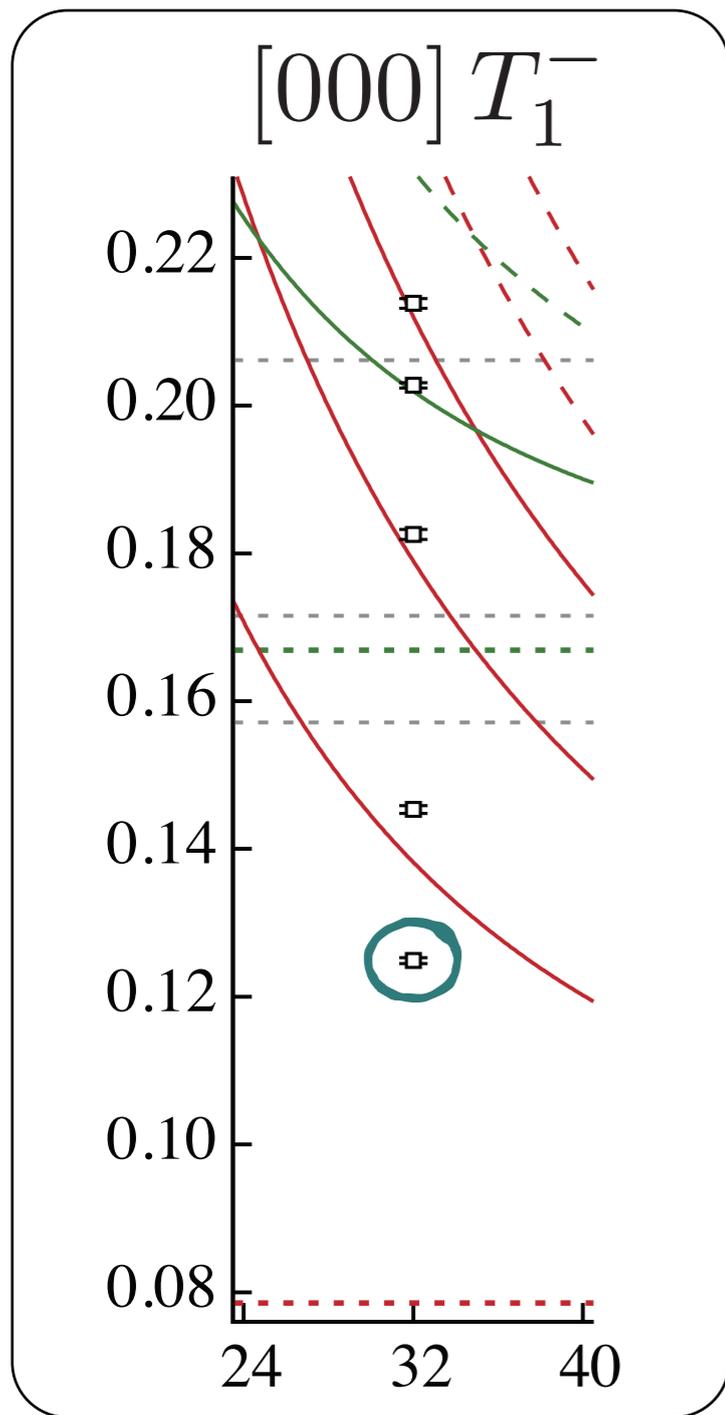
• Rummukainen & Gottlieb (1995)

• Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005)

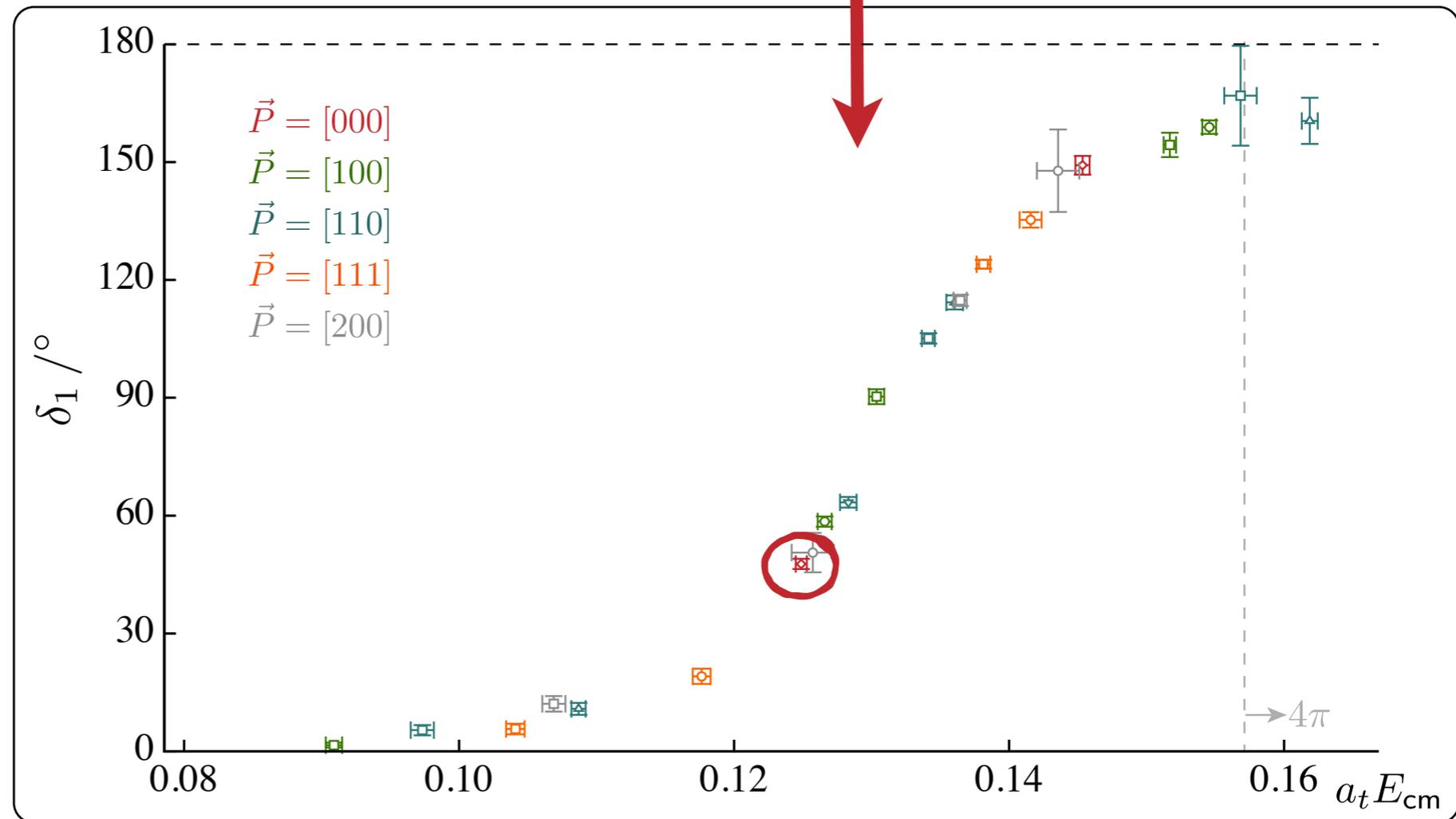
• Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)

• RB (2014)

# $\pi\pi$ Spectrum - ( $l=1$ channel)



$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$

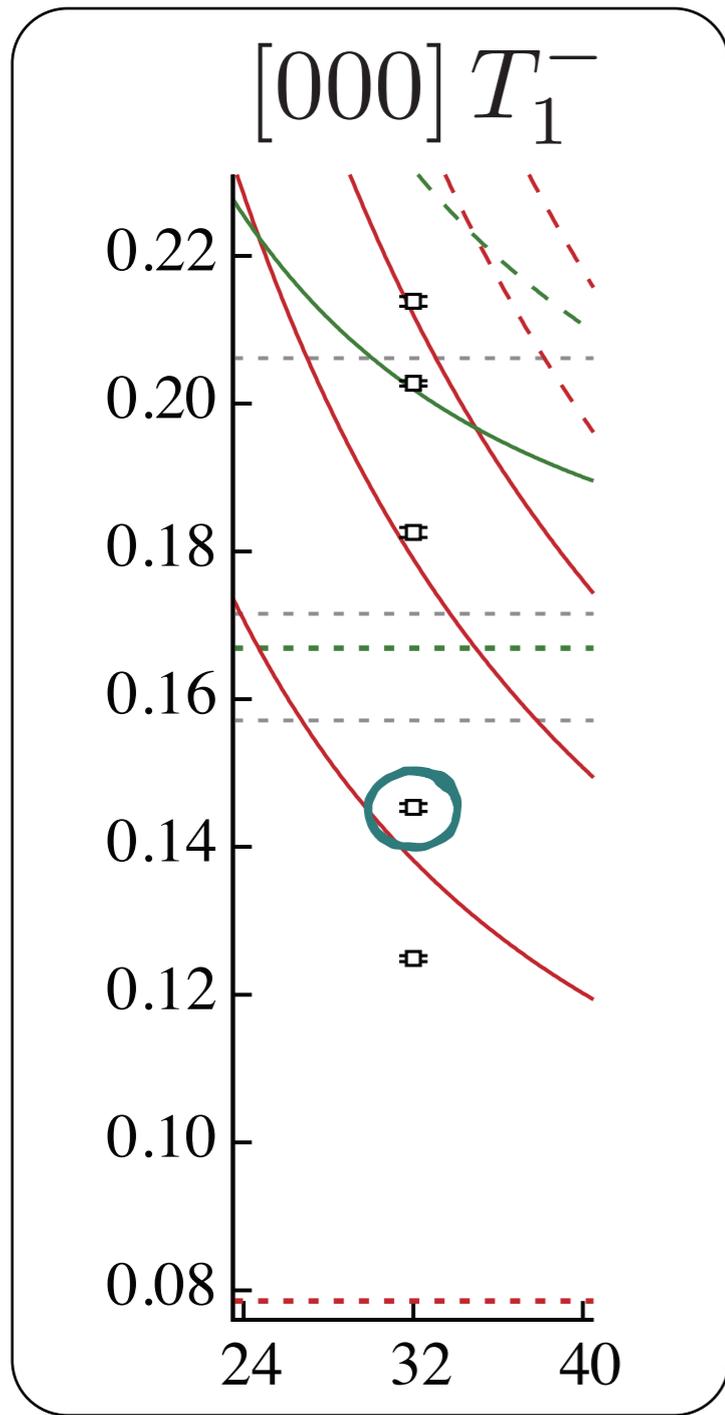


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

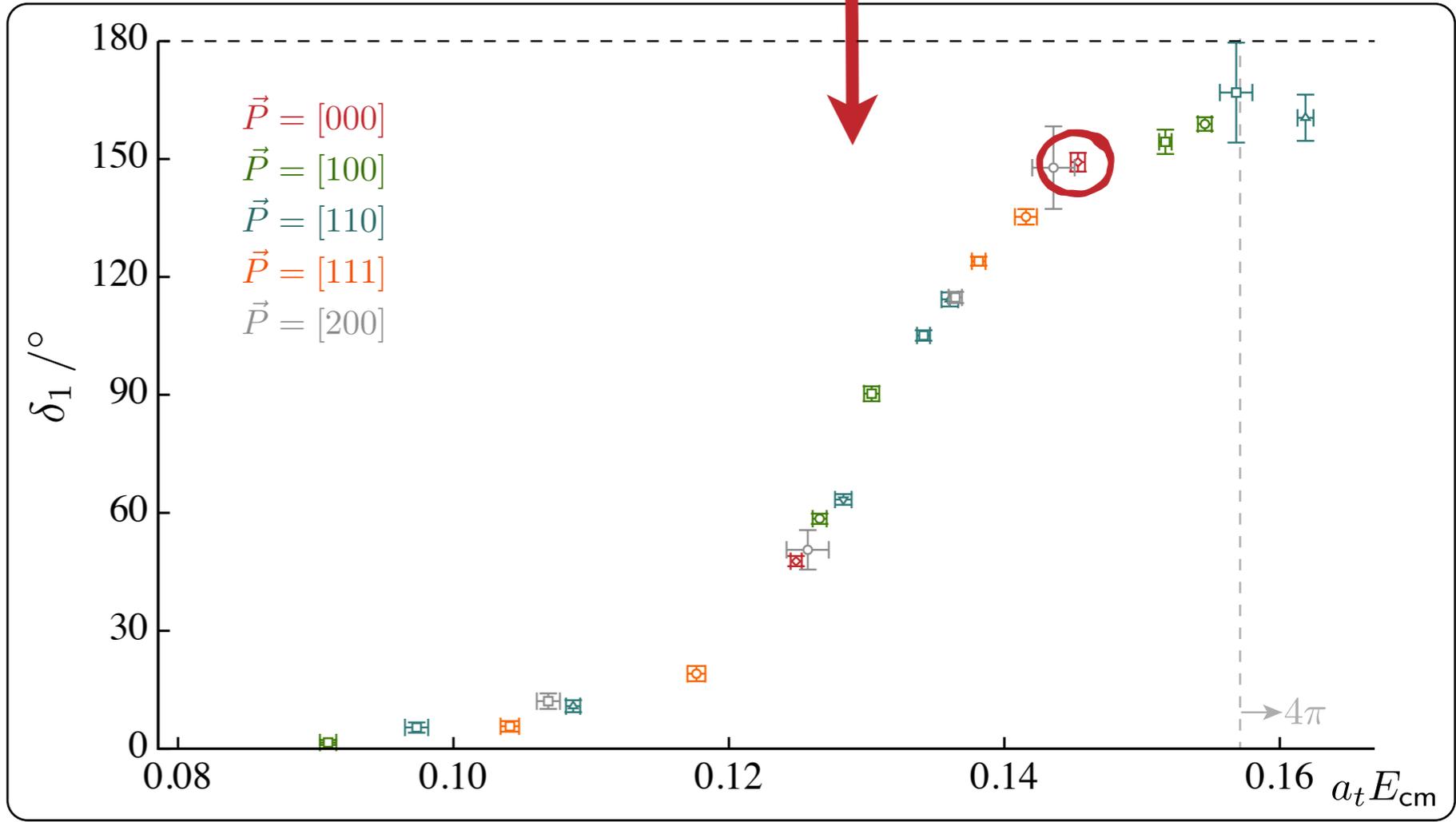
$$m_\pi \sim 240 \text{ MeV}$$

Wilson, RB, Dudek, Edwards & Thomas (2015)

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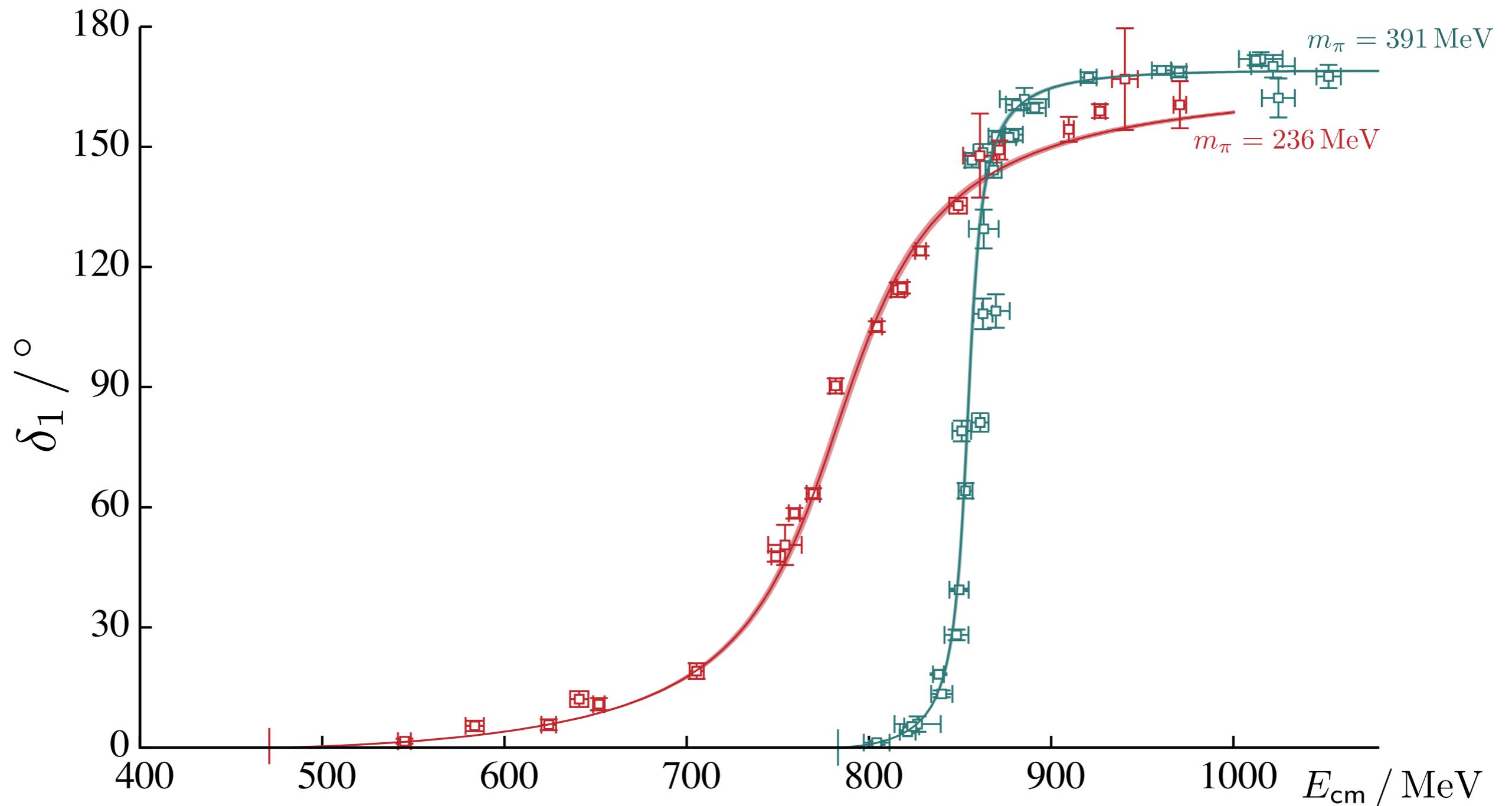


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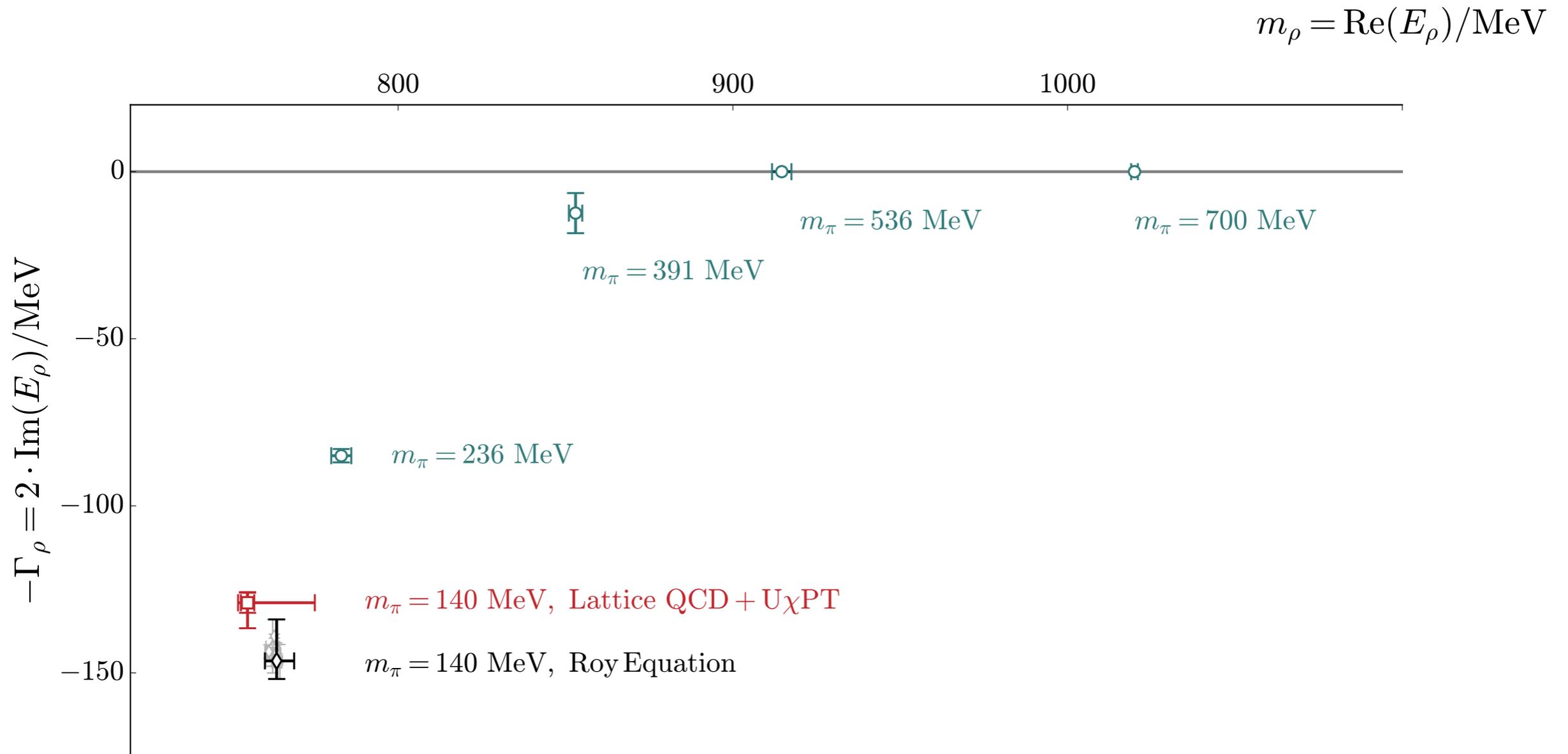
# $\pi\pi$ scattering - ( $l=1$ channel)



Dudek, Edwards & Thomas (2012)

Wilson, RB, Dudek, Edwards & Thomas (2015)

# The $\rho$ vs $m_\pi$



Lin *et al.* (2009)

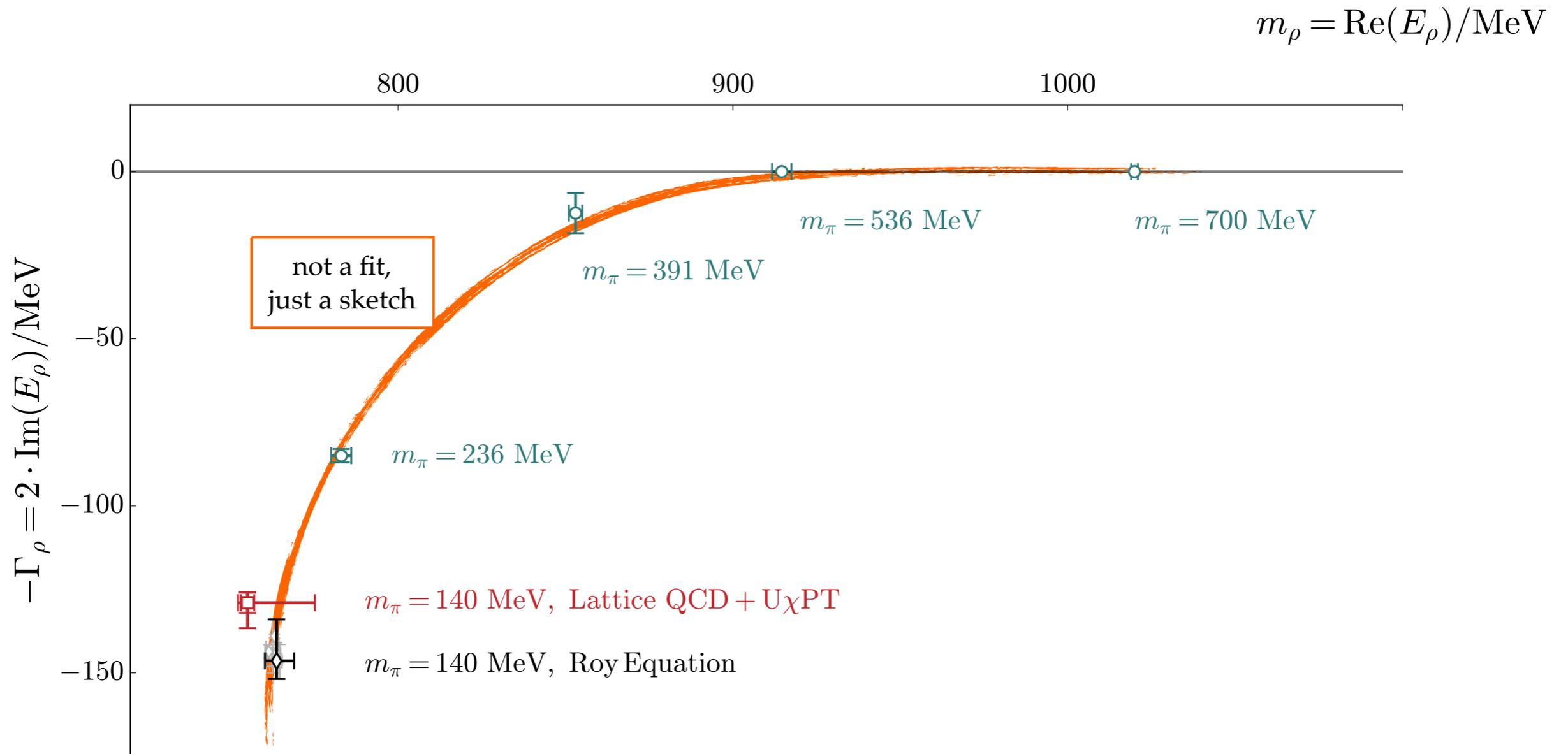
Dudek, Edwards, Guo & Thomas (2013)

Dudek, Edwards & Thomas (2012)

Wilson, RB, Dudek, Edwards & Thomas (2015)

Bolton, RB & Wilson (2015)

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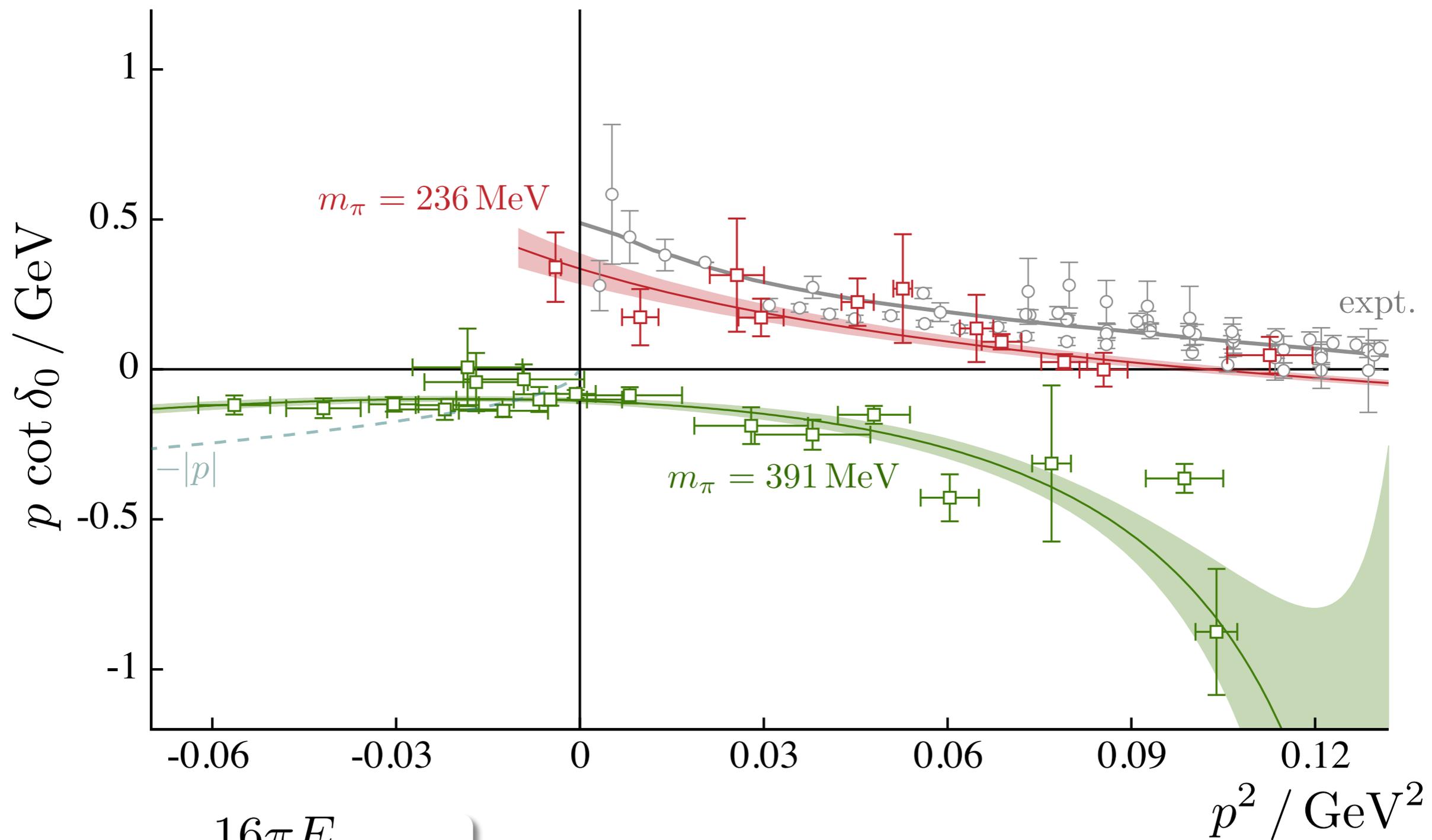
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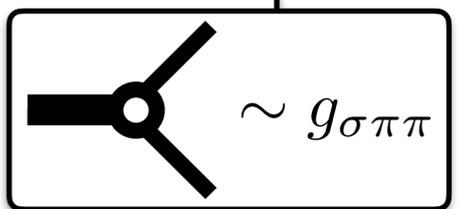
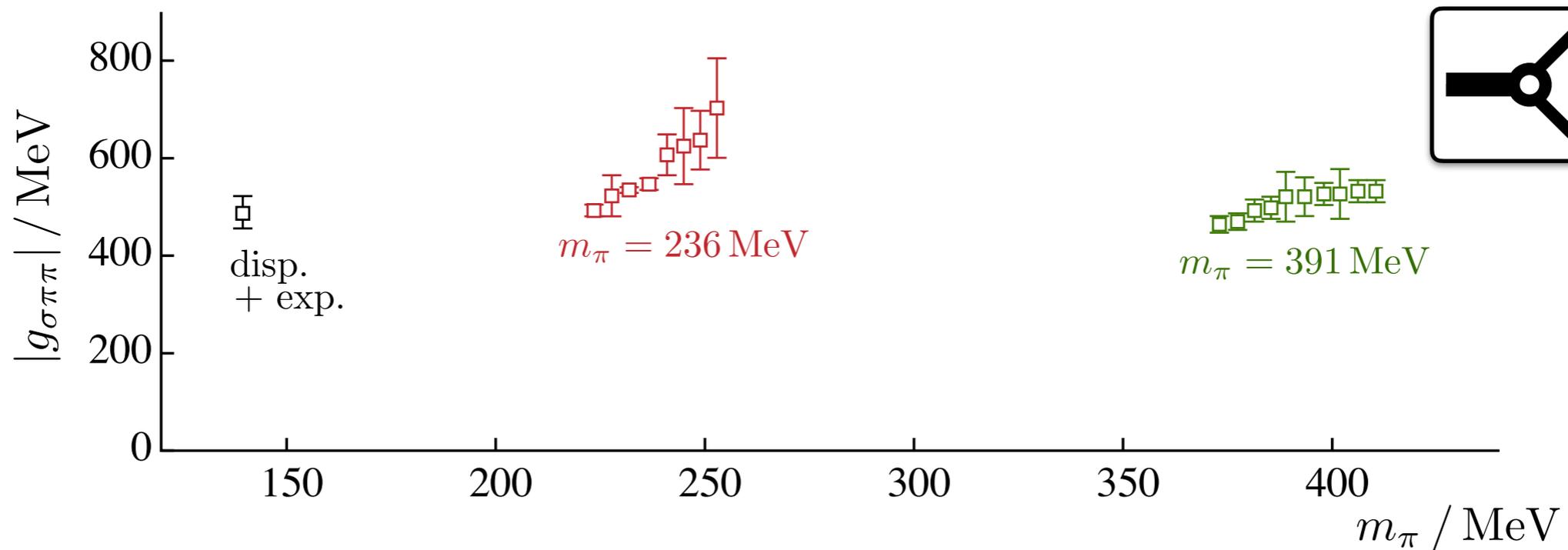
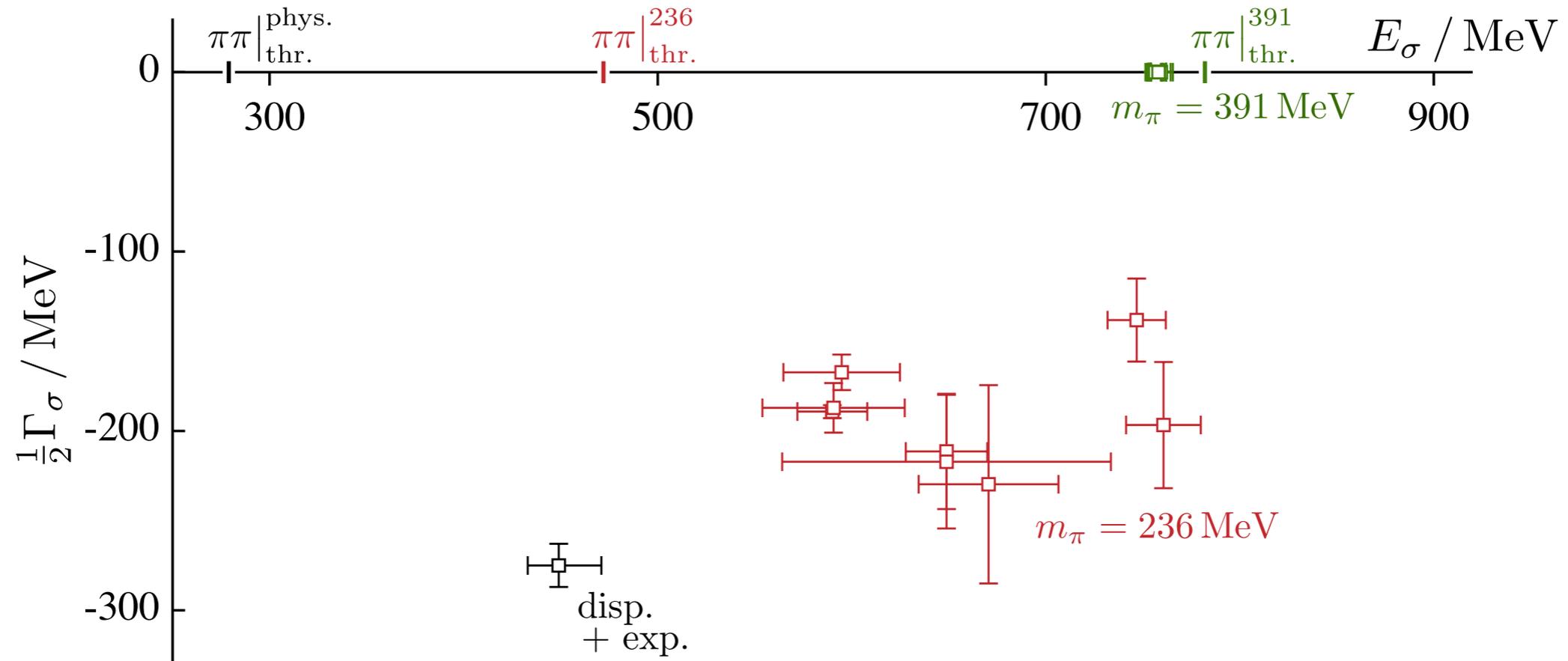
Bolton, RB & Wilson (2015)

# $\pi\pi$ scattering - ( $l=0$ channel)

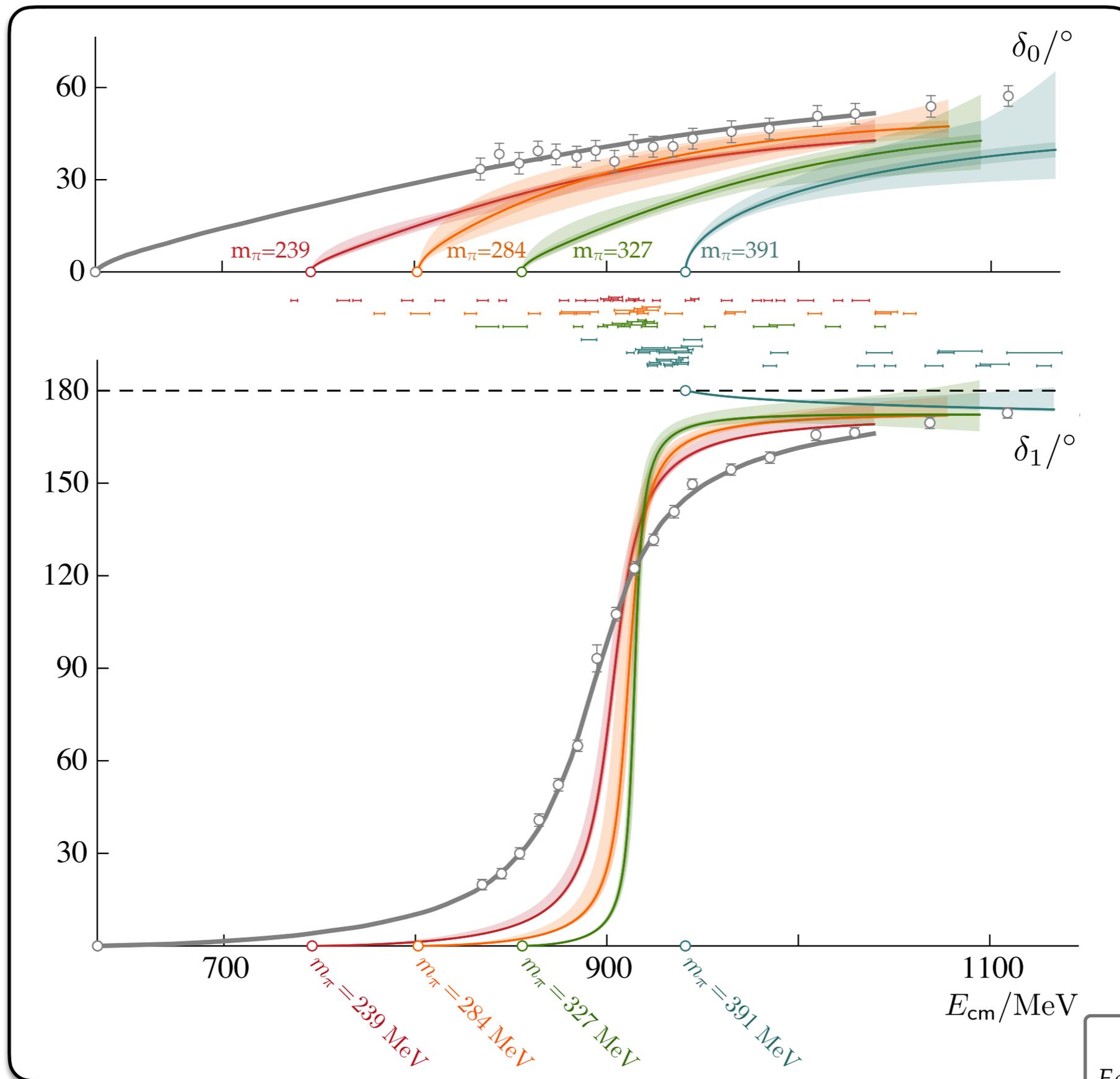


$$\mathcal{M}_0 = \frac{16\pi E_{\text{cm}}}{p \cot \delta_0 - ip}$$

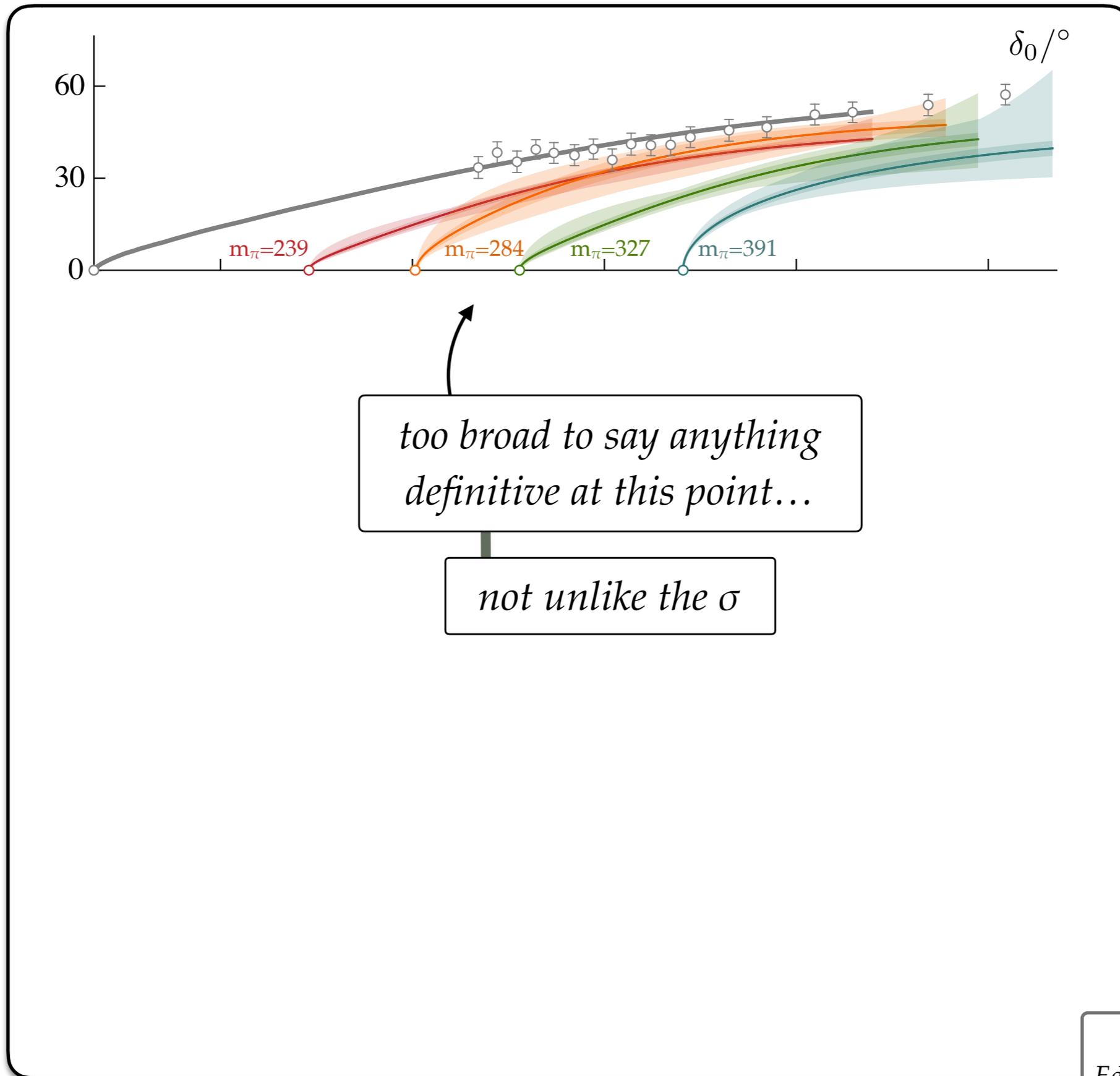
# The $\sigma$ vs $m_\pi$



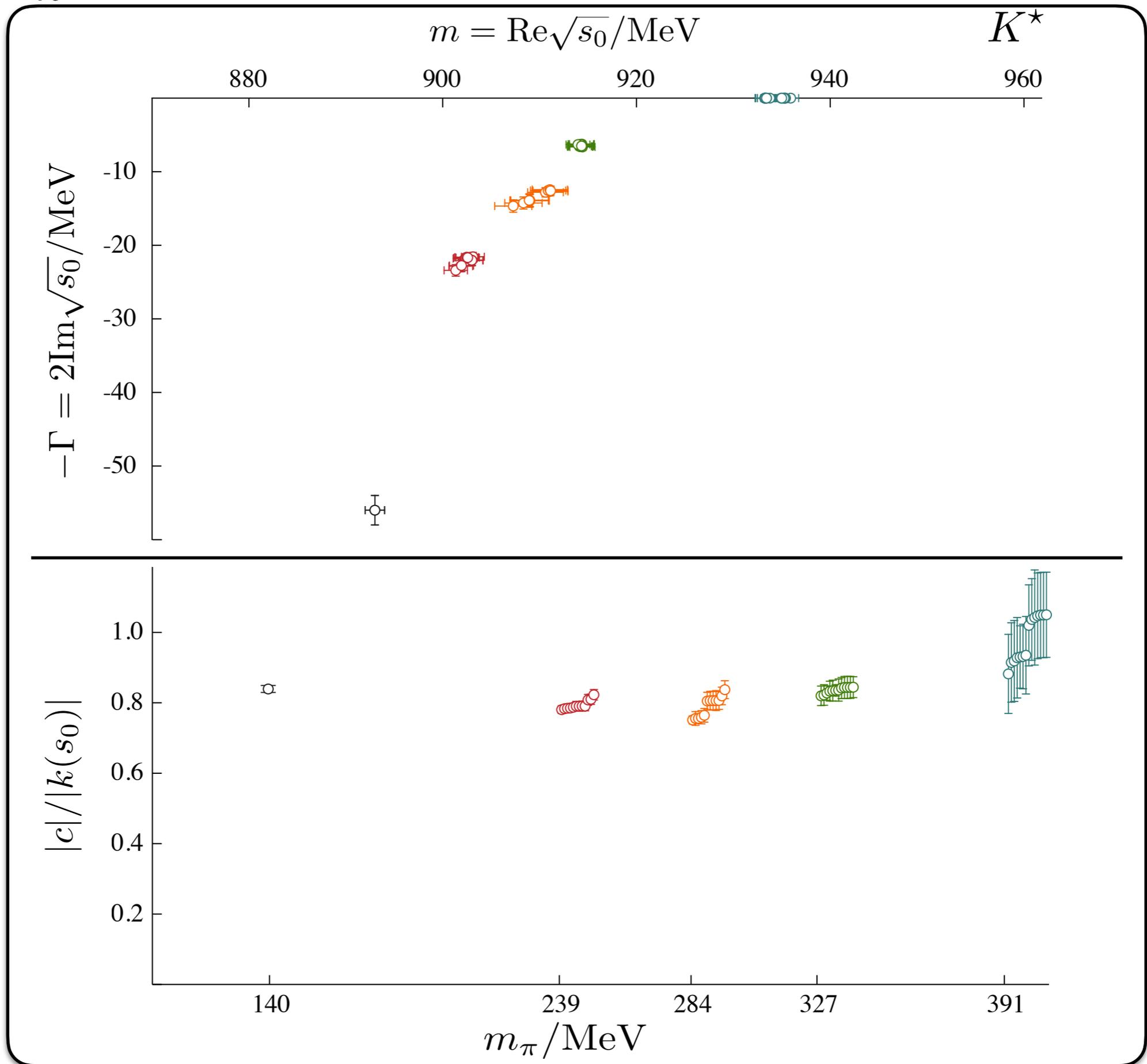
# $\pi K$ scattering - ( $l=1/2$ channel)



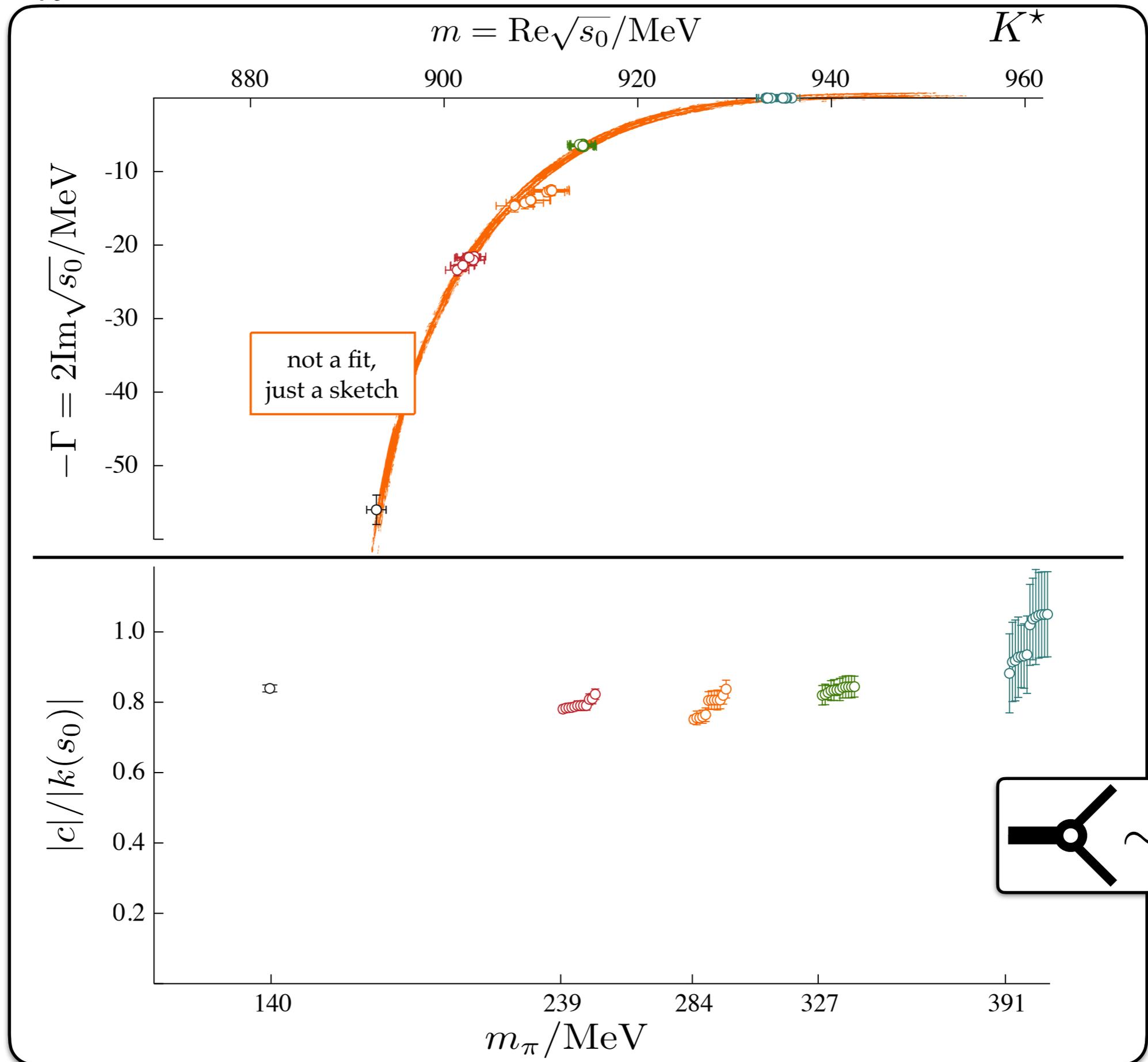
# $\pi K$ scattering - ( $l=1/2$ channel)



# The $K^*$ vs $m_\pi$



# The $K^*$ vs $m_\pi$

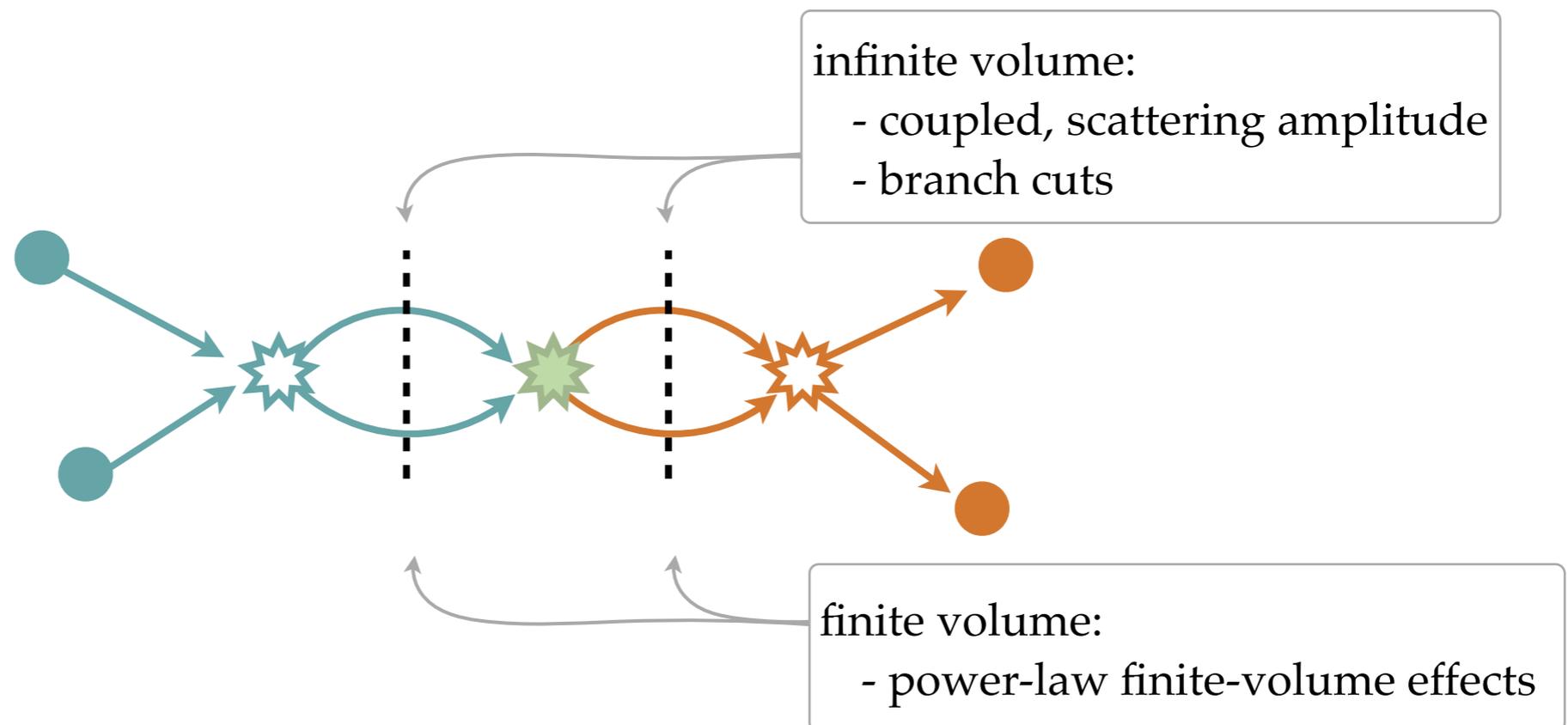


# Multi-channel systems - the cutting edge!

📌 Above  $2m_K$ , there is not a one-to-one correspondence

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\bar{K}} \\ \mathcal{M}_{\pi\pi,K\bar{K}} & F_{K\bar{K}}^{-1} + \mathcal{M}_{K\bar{K},K\bar{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),  
Hansen & Sharpe / RB & Davoudi (2012)



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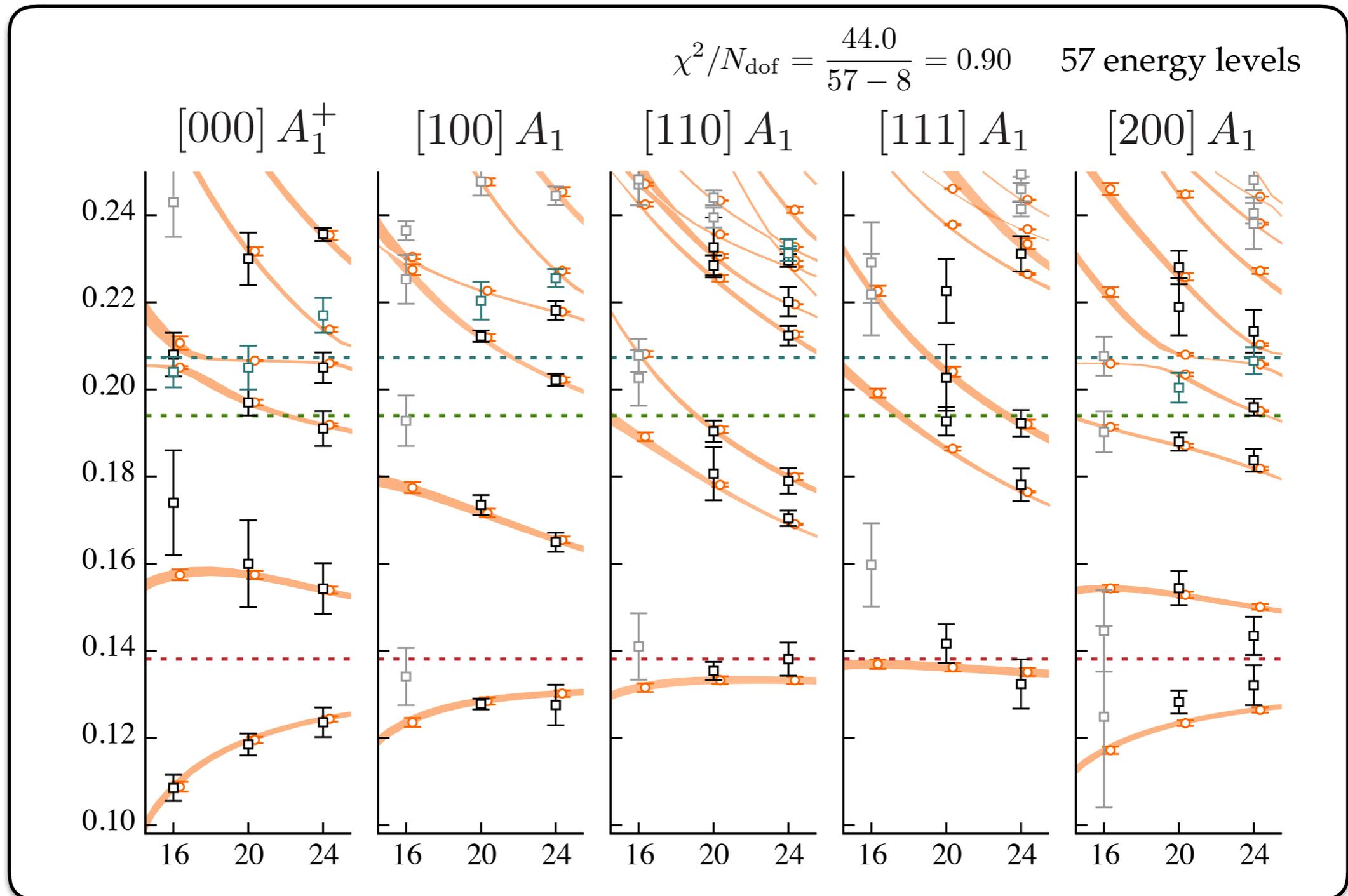
Feng, Li, & Liu (2004),  
Hansen & Sharpe / RB & Davoudi (2012)

- In general, must constrain  $(1/2) [N^2 + N]$  functions of energy
- Need that many energy levels at the same energy
- Alternatively, parametrize scattering amplitude and do a global fit

# Coupled-channels analysis

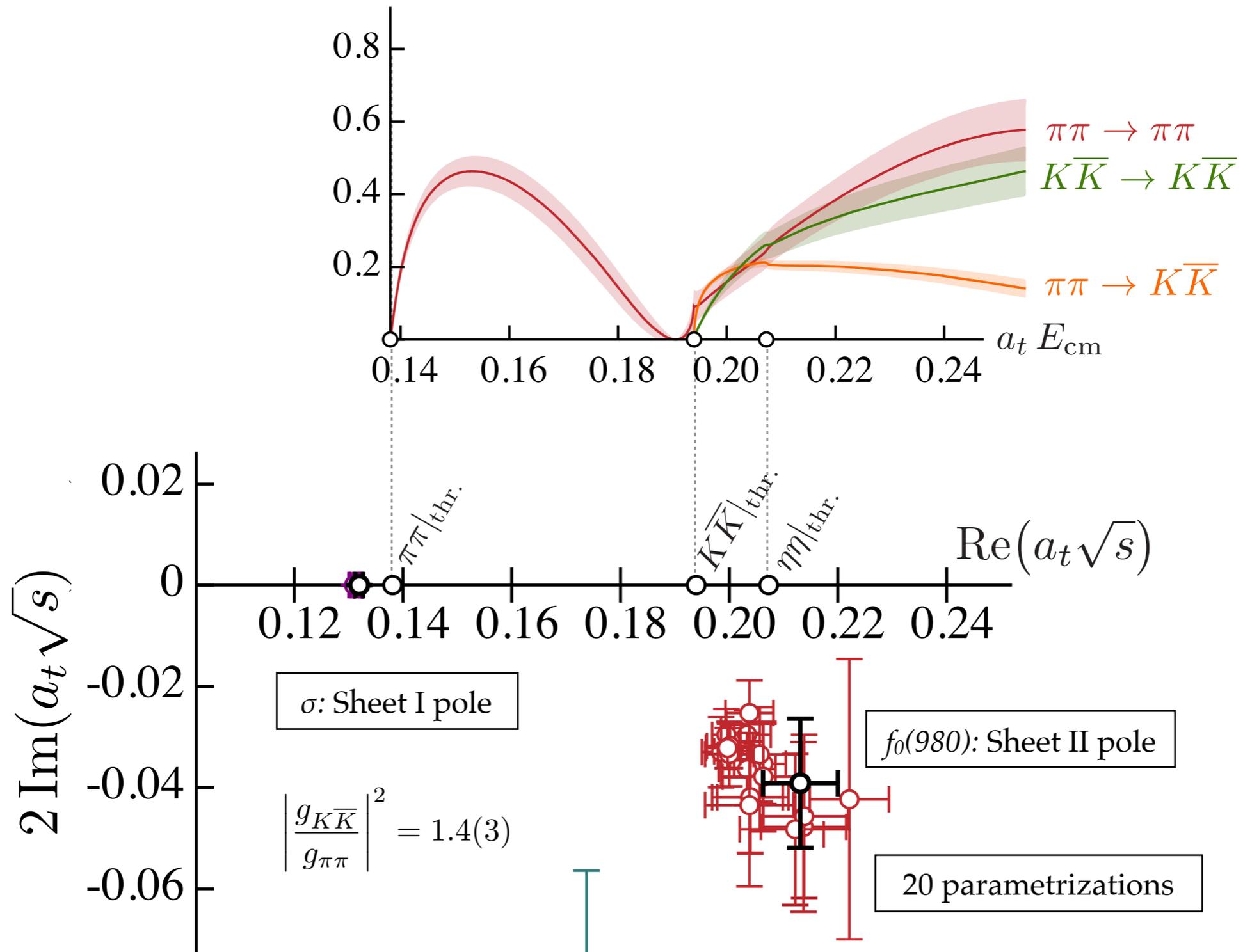
📌 S-wave above  $2m_\pi$ ,  $2m_K$ , and  $2m_\eta$

📌 Ansatz  $\mathbf{K}^{-1}(s) = \begin{pmatrix} a + bs & c + ds & e \\ c + ds & f & g \\ e & g & h \end{pmatrix}$



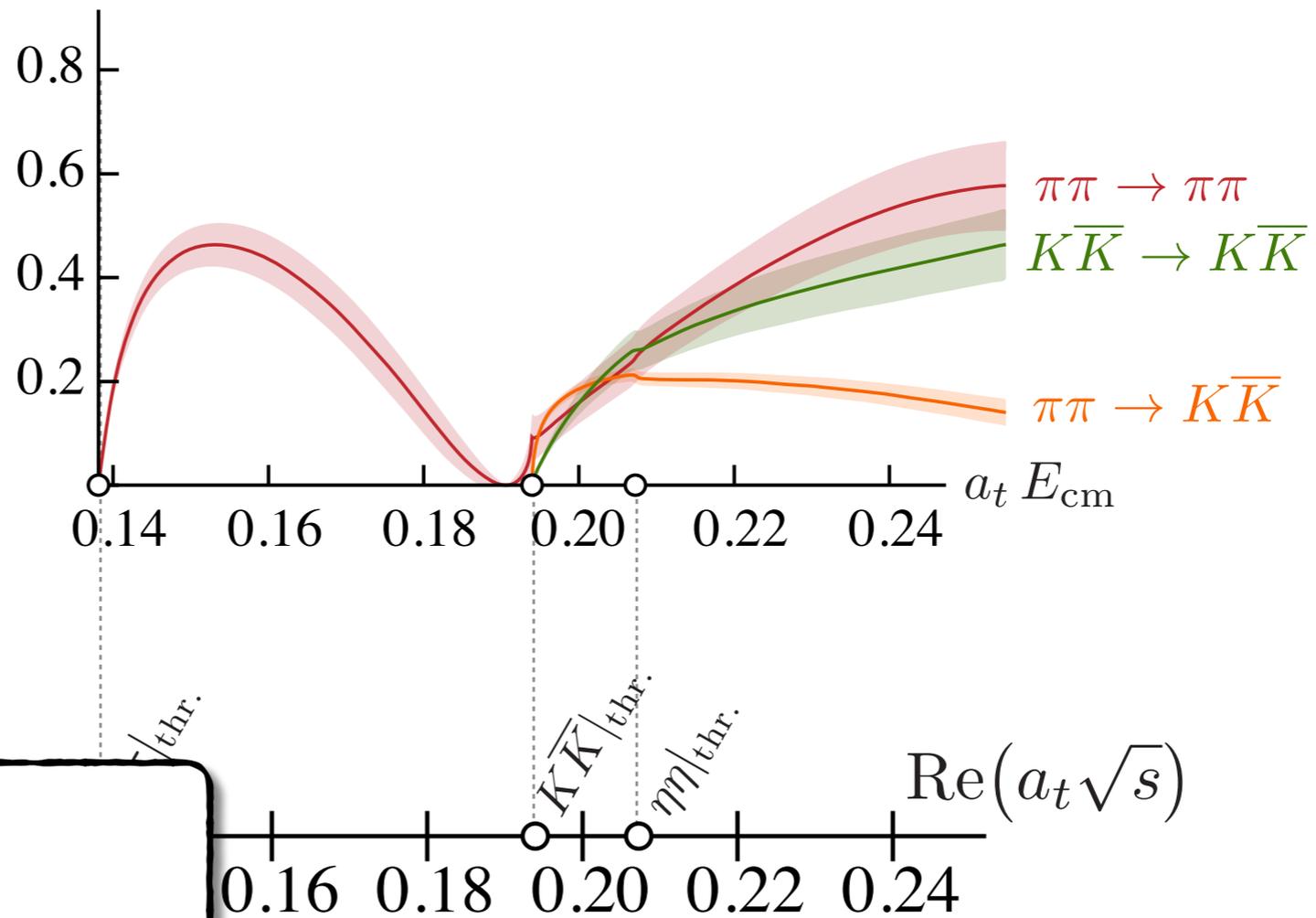
# Scalar poles: $\sigma$ and $f_0(980)$

• Near poles:  $\mathcal{M} \sim \frac{g^2}{s_0 - s}$

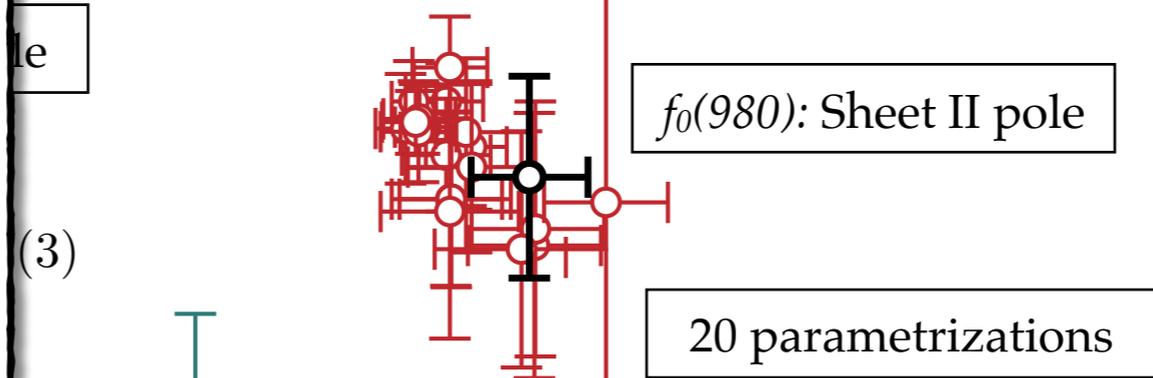
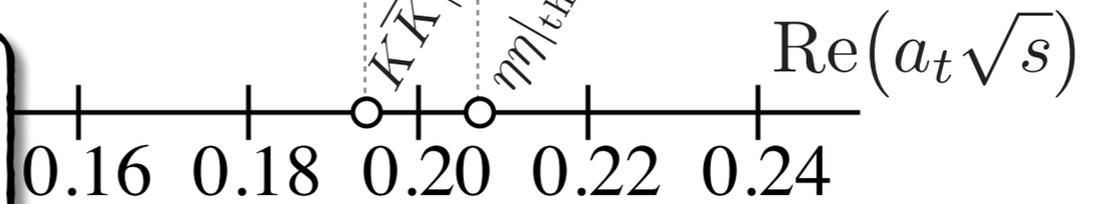
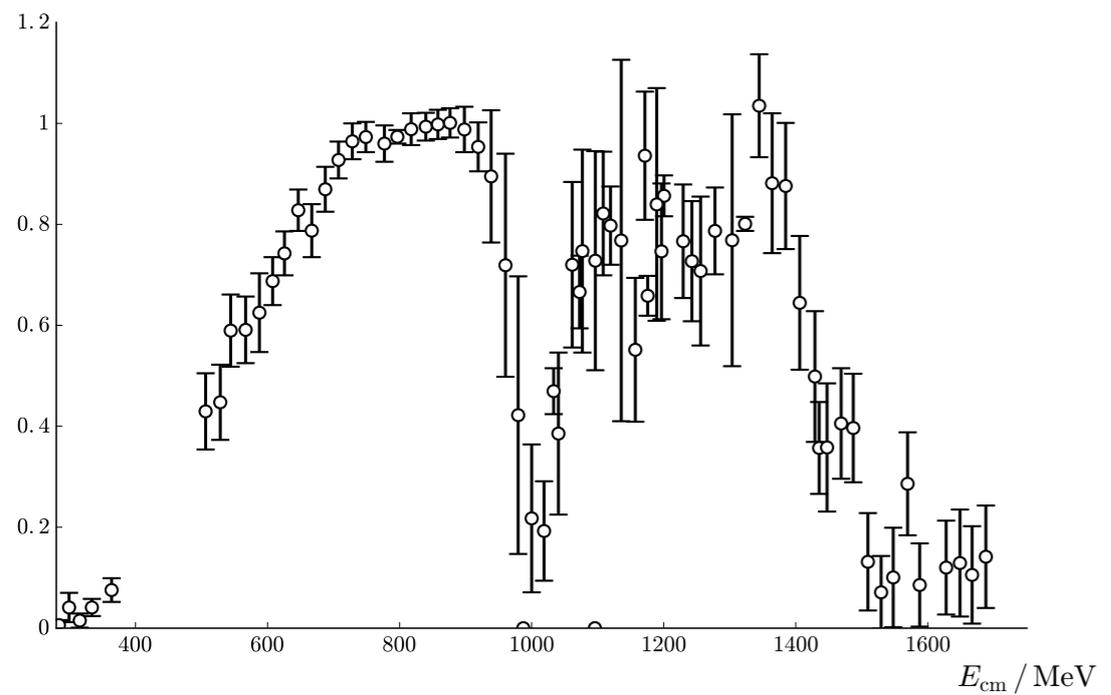


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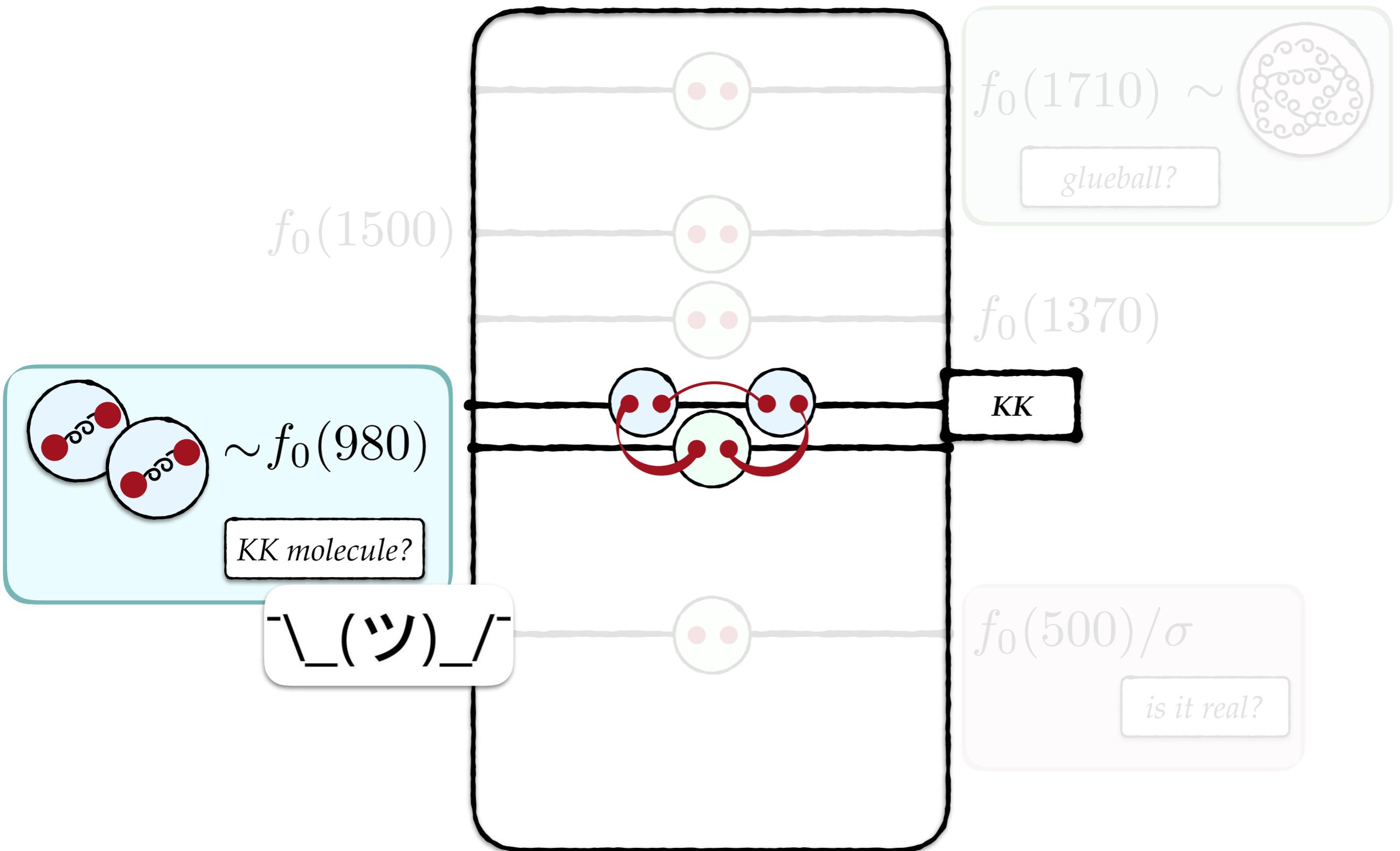
0.02



(3)

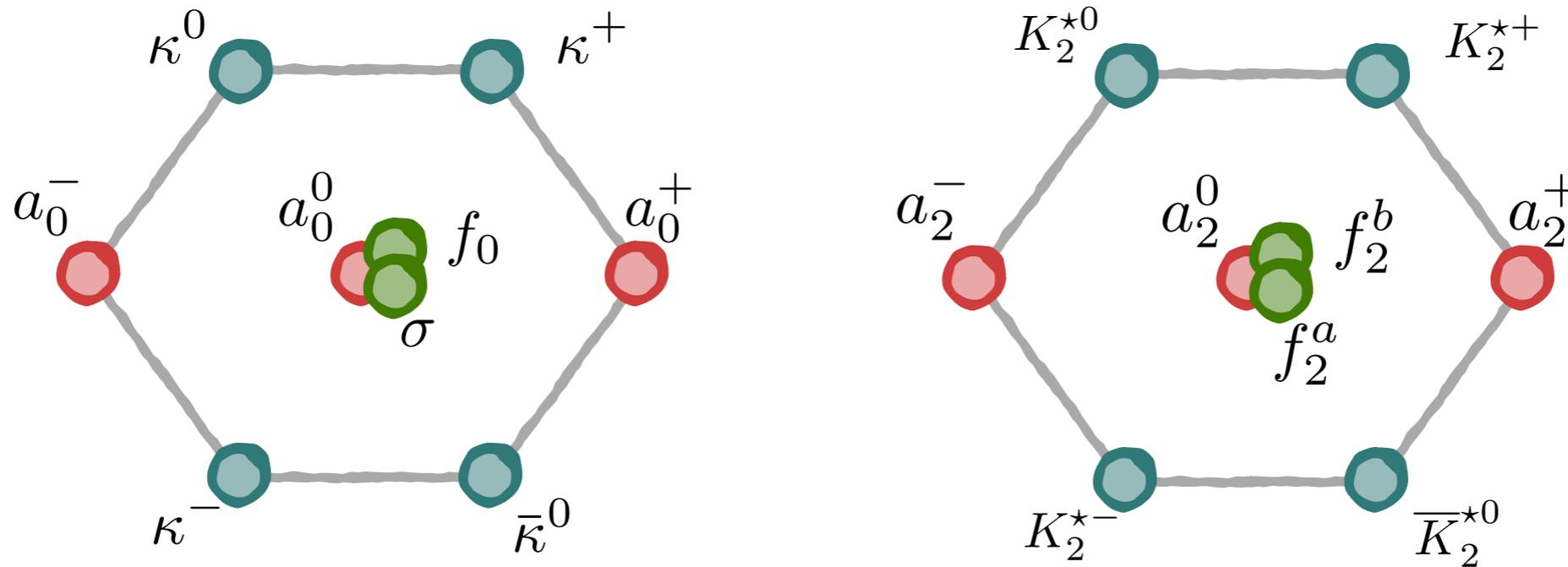


# The isoscalar, scalar sector



# Tensor and scalar nonets

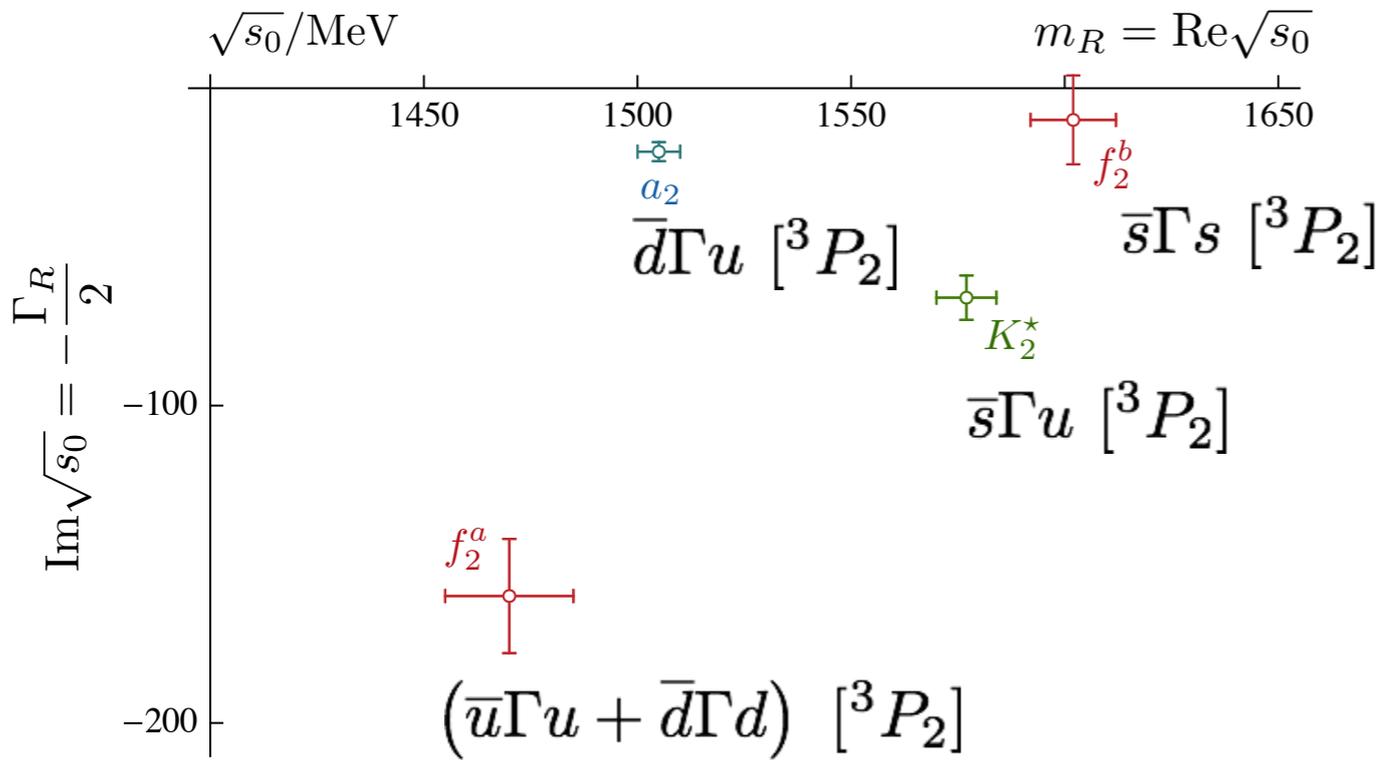
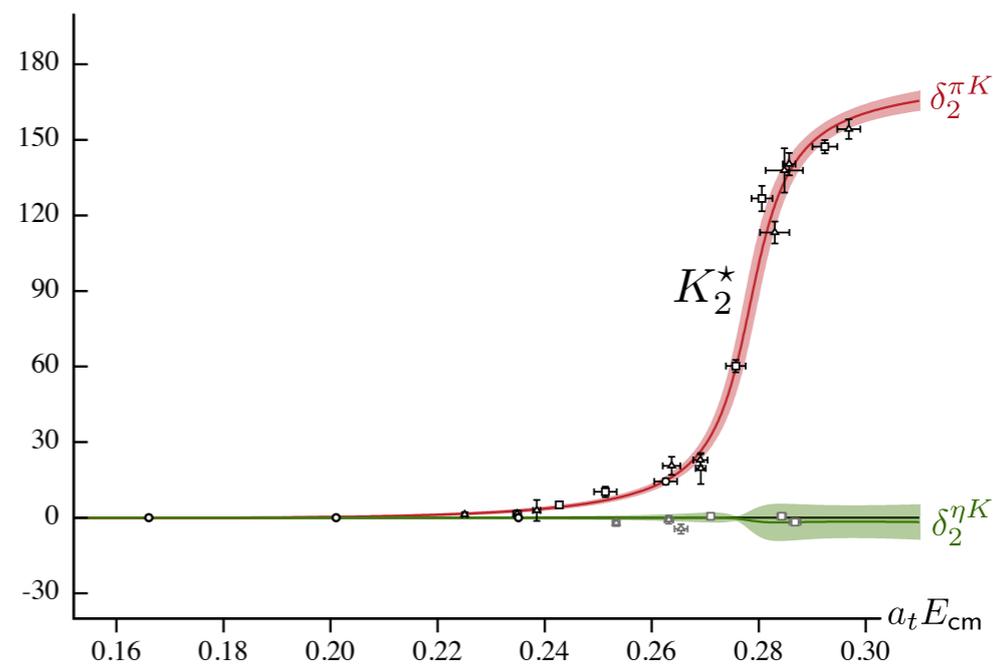
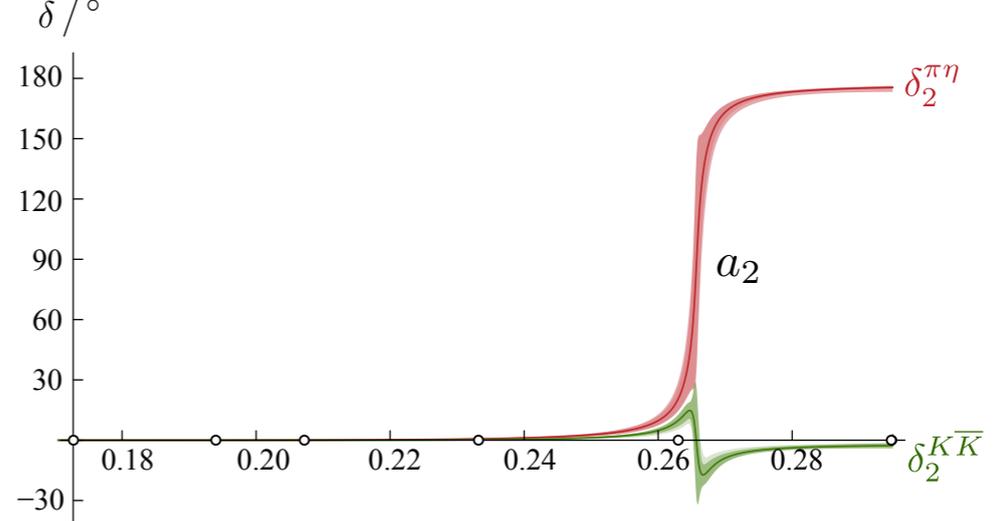
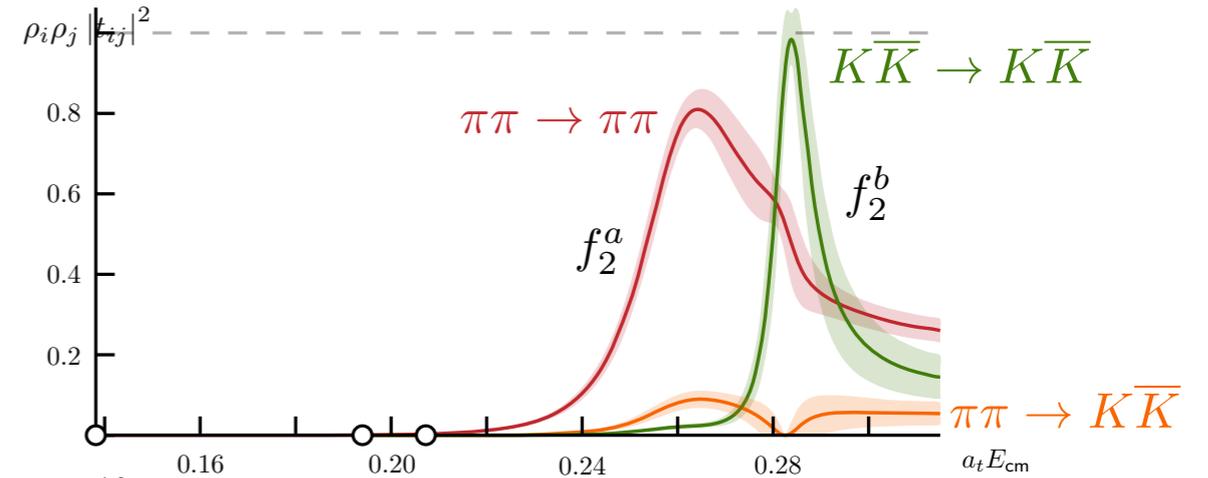
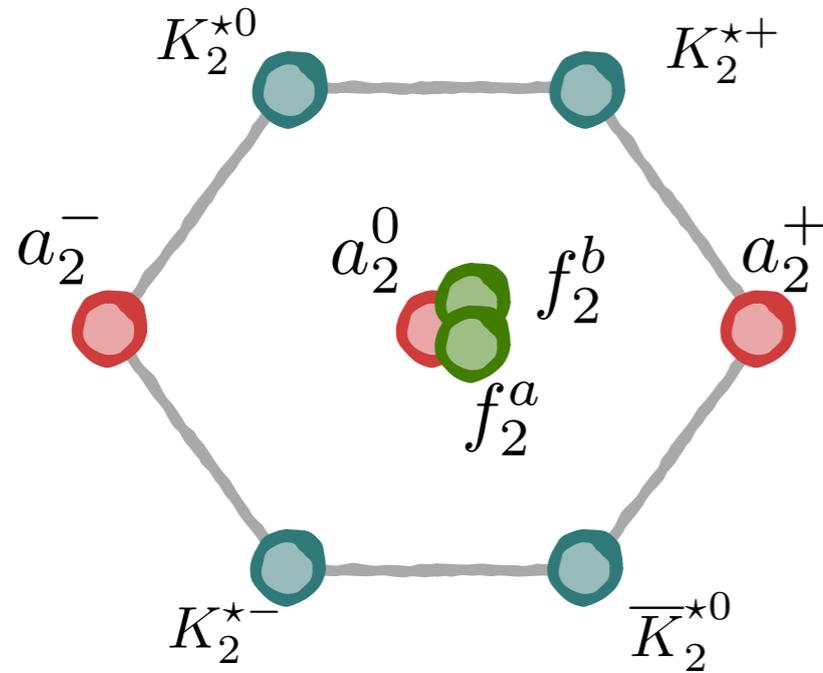
📌 First complete determination of the scalar and tensor nonets from LQCD :



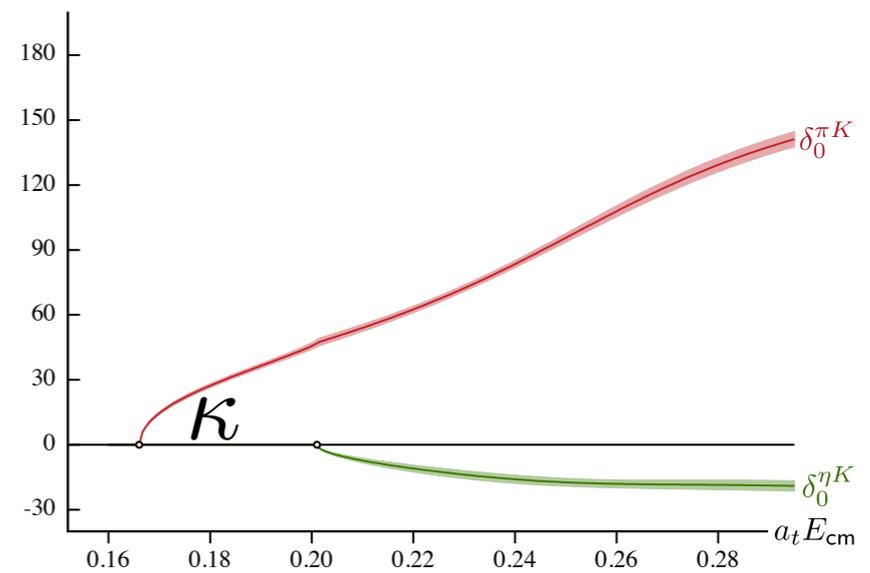
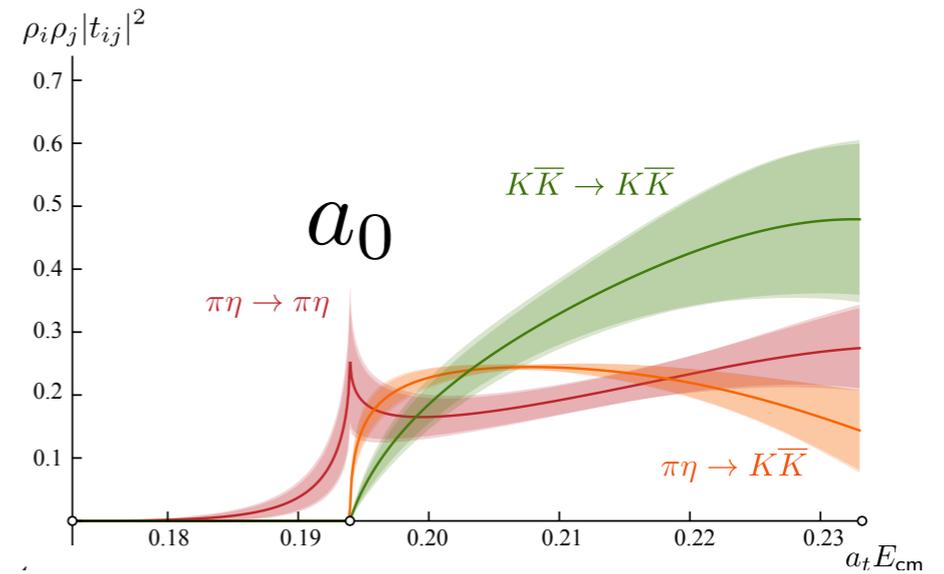
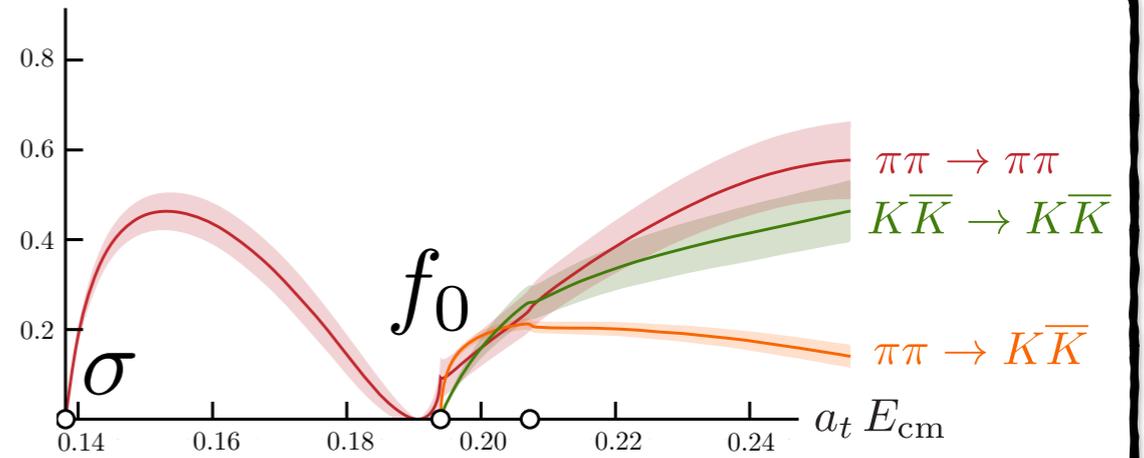
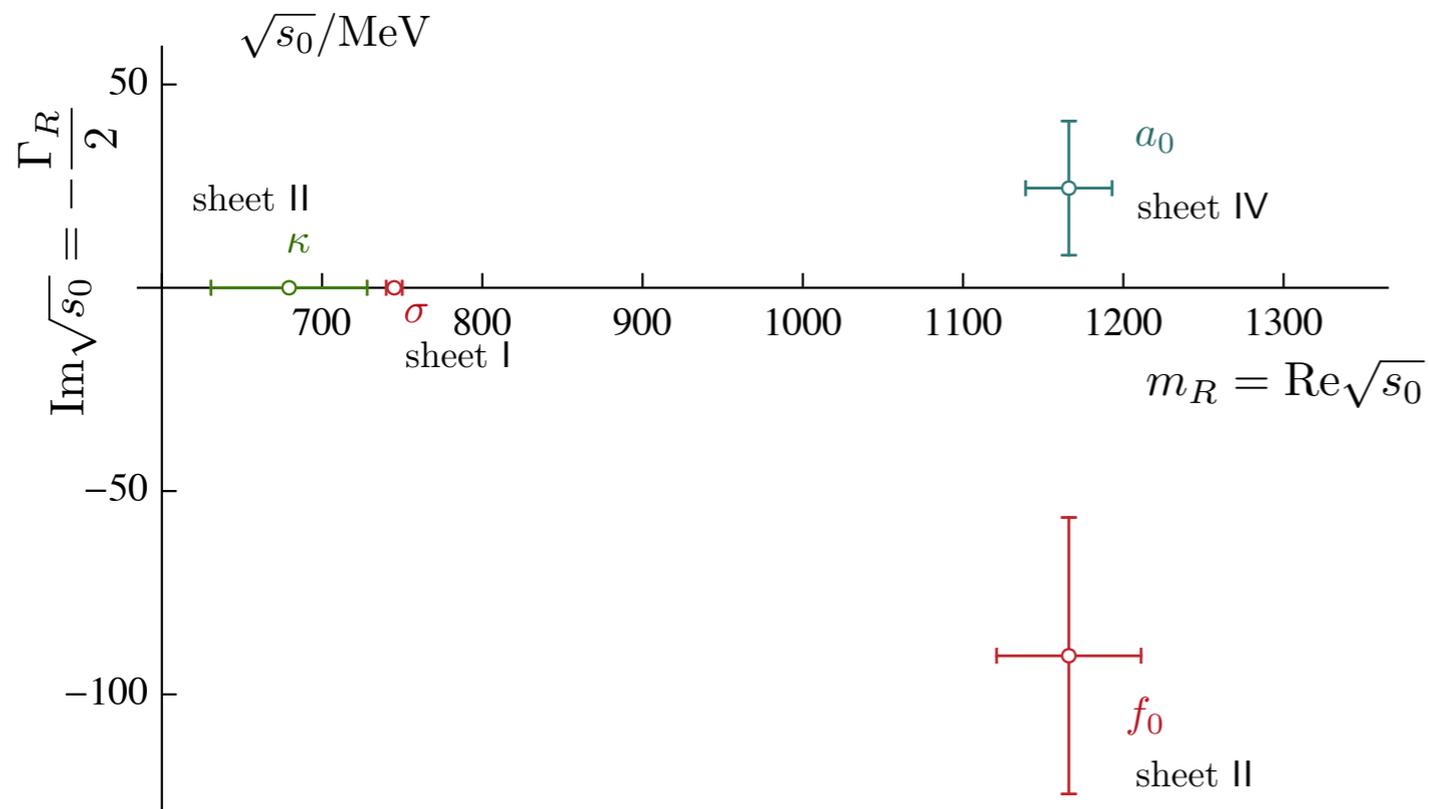
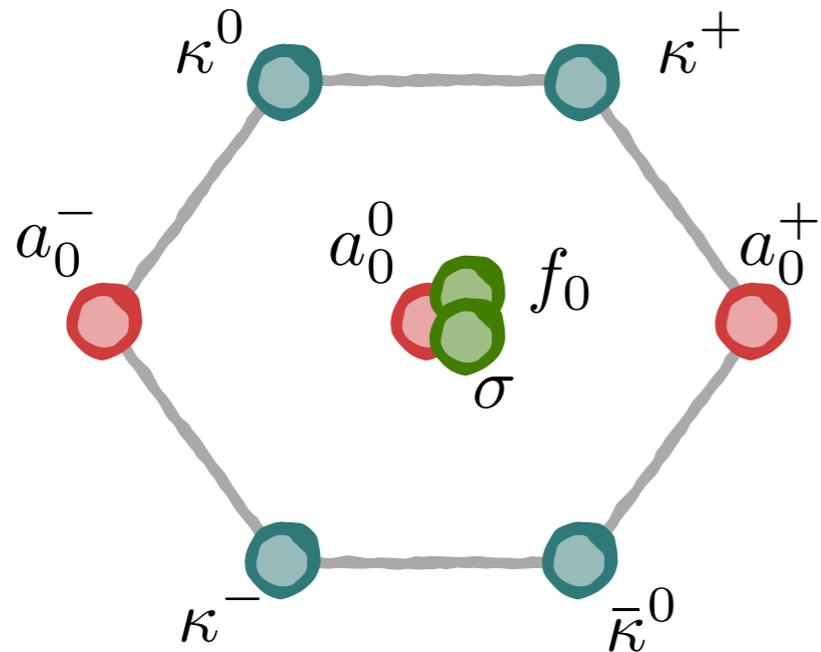
$\pi\pi, KK, \eta\eta$ :	RB, Dudek, Edwards - PRL (2017) RB, Dudek, Edwards - PRD (2017)
$K\pi, K\eta$ :	Dudek, Edwards, Thomas, Wilson - PRL (2015) Wilson, Dudek, Edwards, Thomas - PRD (2015)
$\pi\eta, KK$ :	Dudek, Edwards, Wilson - PRD (2016)

had spec

# Tensor nonet

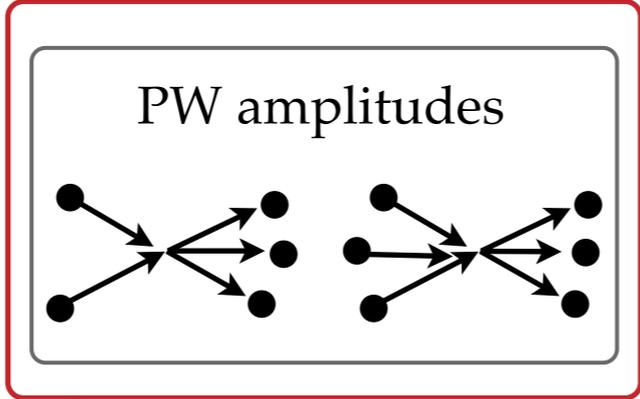
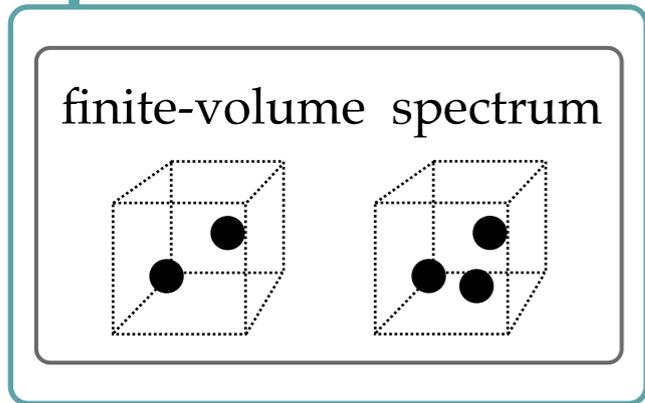


# Scalar nonet

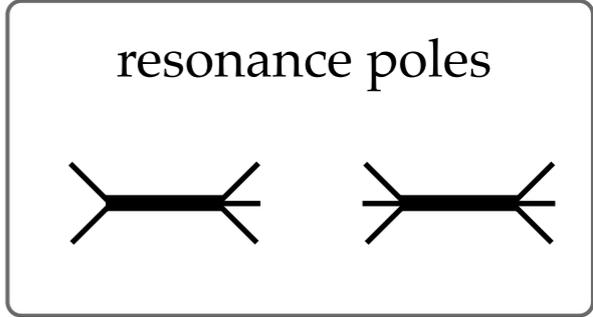


# few-body systems in LQCD

*lattice QCD*



analytic continuation



identification of  
• states [masses & widths],  
• production/decay mechanisms

*these tools have been applied to study unitarity and the finite-volume spectrum in the 3-body sector*

inside the box



Sharpe



Hansen

outside the box

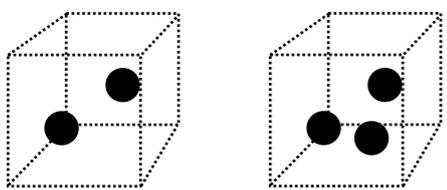


Szczepaniak

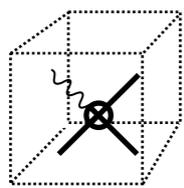
# few-body systems in LQCD

*lattice QCD*

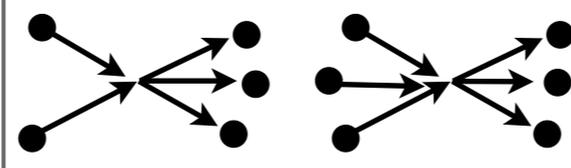
finite-volume spectrum



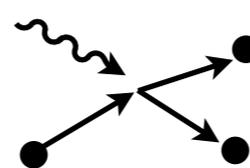
1-to-2  
FV matrix  
elements



PW amplitudes



electroweak  
amplitudes

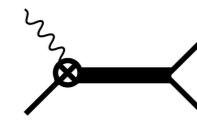


analytic  
continuation

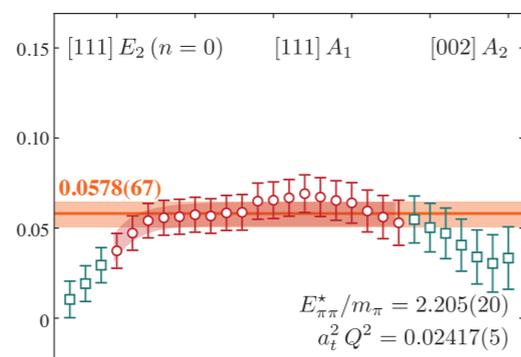
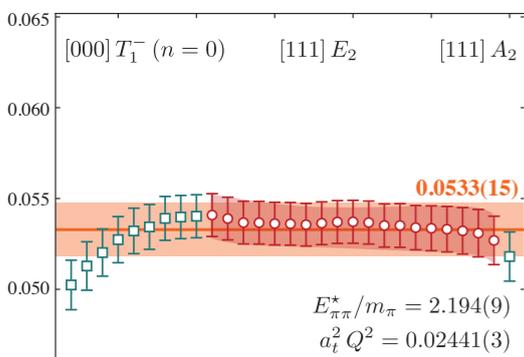
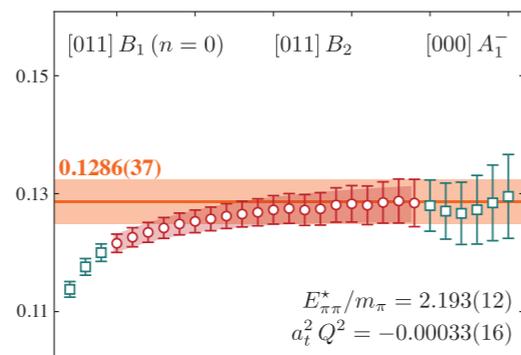
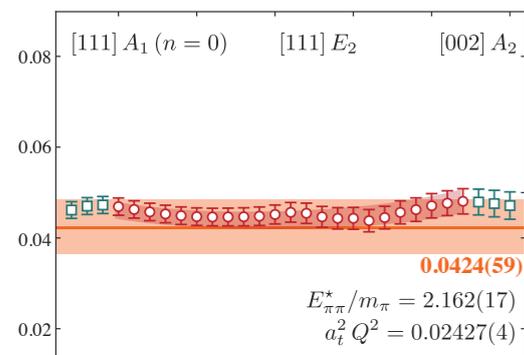
resonance poles



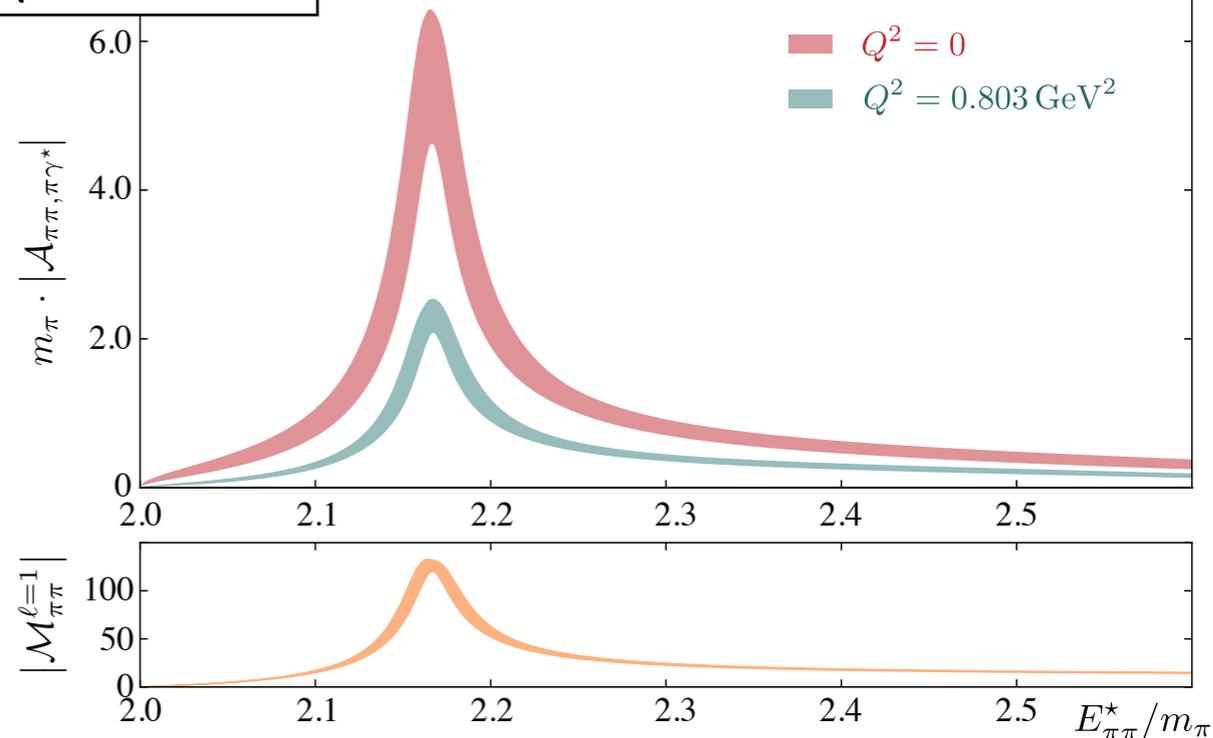
transition  
form factors



identification of  
 states [masses & widths],  
 production/decay mechanisms

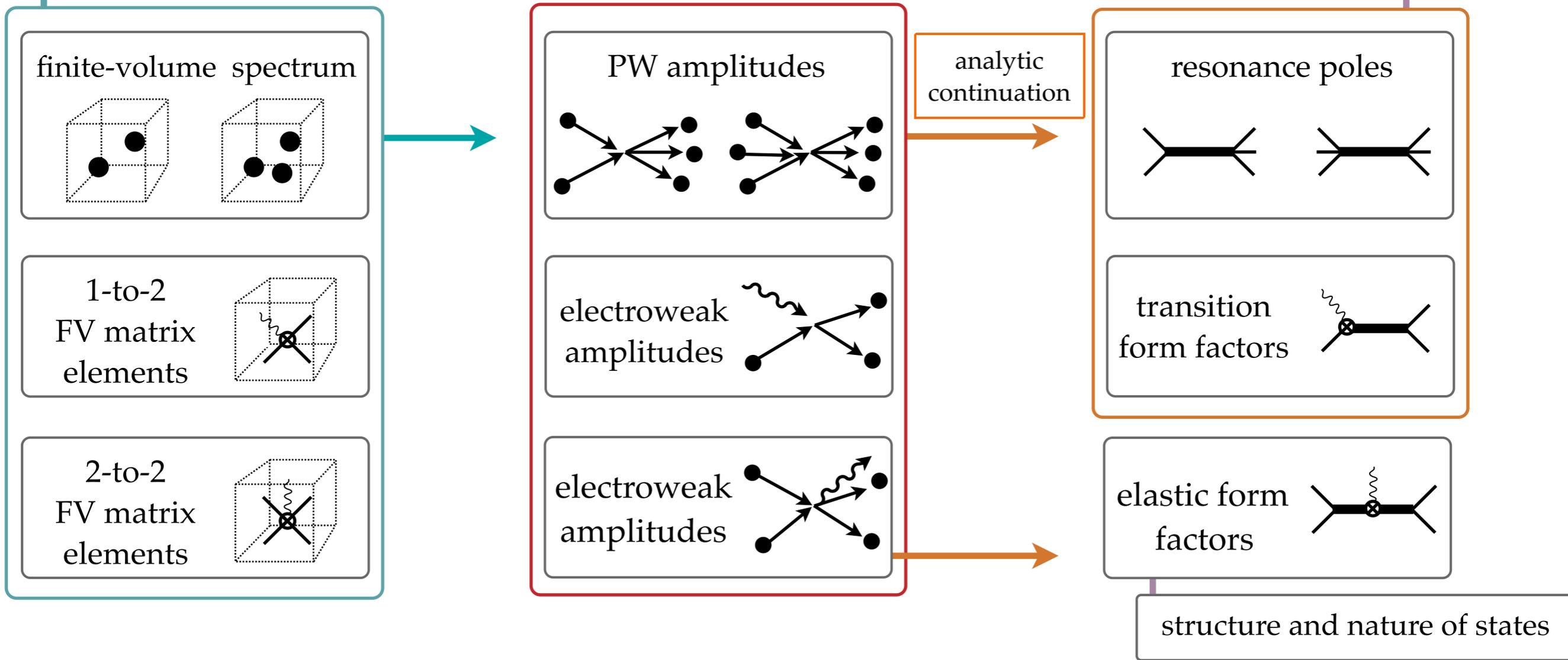


$\pi\gamma^*$ -to- $\pi\pi$



# few-body systems in LQCD

*lattice QCD*



Baroni      Hansen      Ortega

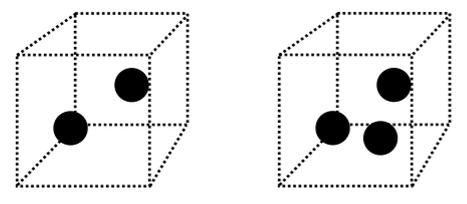


- RB & Hansen (2015)
- Baroni, RB, Hansen, Ortega (2018)

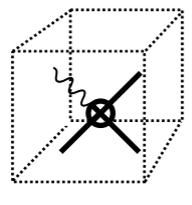
# few-body systems in LQCD

*lattice QCD*

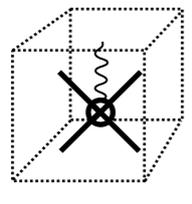
finite-volume spectrum



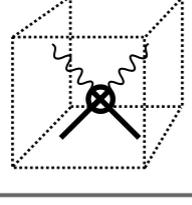
1-to-2 FV matrix elements



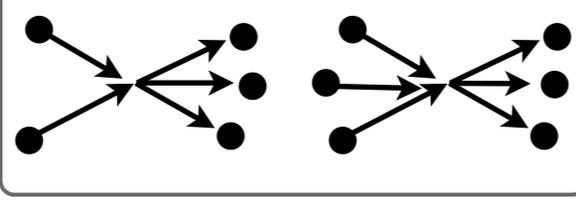
2-to-2 FV matrix elements



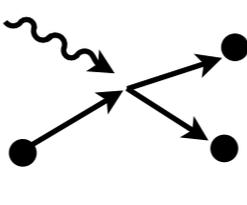
1-to-1 with two currents



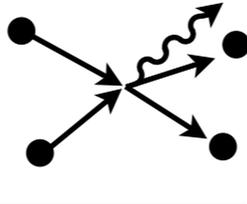
PW amplitudes



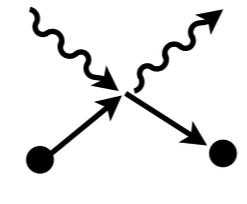
electroweak amplitudes



electroweak amplitudes

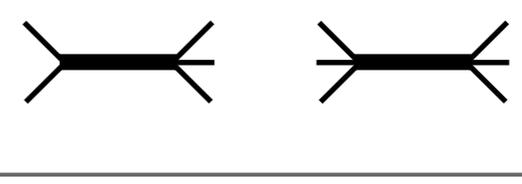


electroweak amplitudes

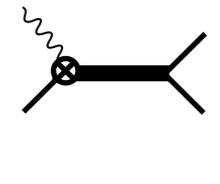


analytic continuation

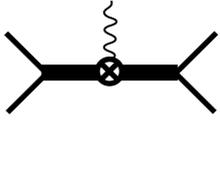
resonance poles



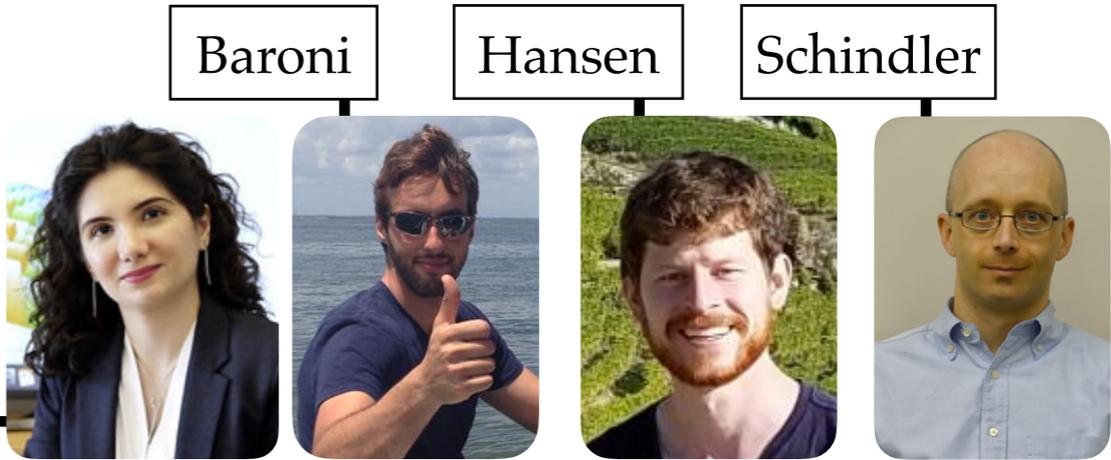
transition form factors



elastic form factors



identification of  
• states [masses & widths],  
• production/decay mechanisms



(to appear)

Davoudi

# the future is nuclear

These techniques are being tested and implemented for  $A=0$  systems first, but they are necessary and will be applied for light nuclear systems...

