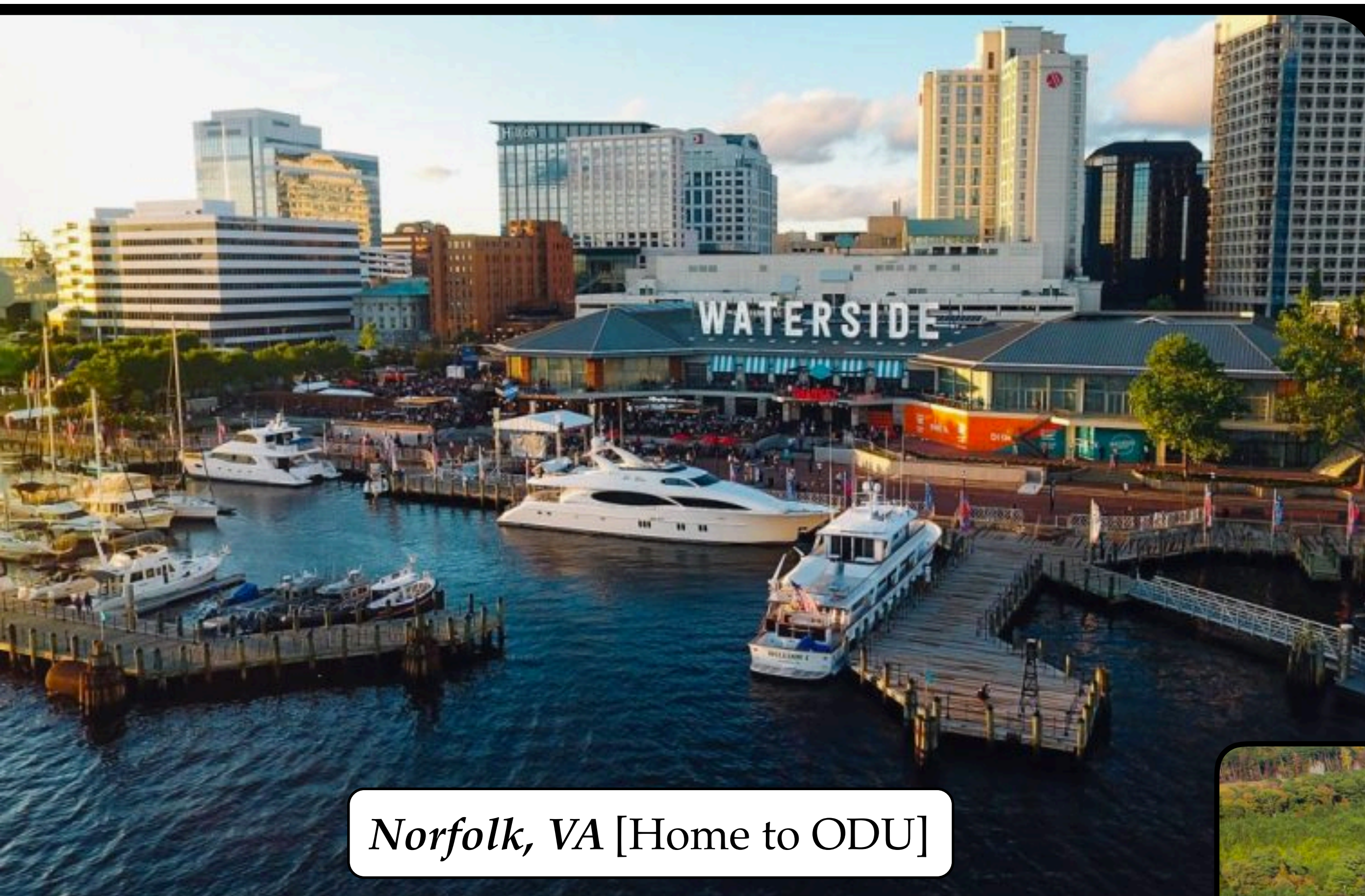
Scattering and resonances in LQCD

Raúl Briceño - http://bit.ly/rbricenoPhD



Norfolk, VA [Home to ODU]

OLD DOMINION UNIVERSITY

Jefferson Lab

hadspec

Elba, 2019

JLab, VA

low energy QCD in the 21st cent.

Amplitude analysis

Experiments

QCD

low energy QCD in the 21st cent.

Amplitude analysis

GOAL:

Get insights to the governing patterns and rules of QCD from emergent phenomena

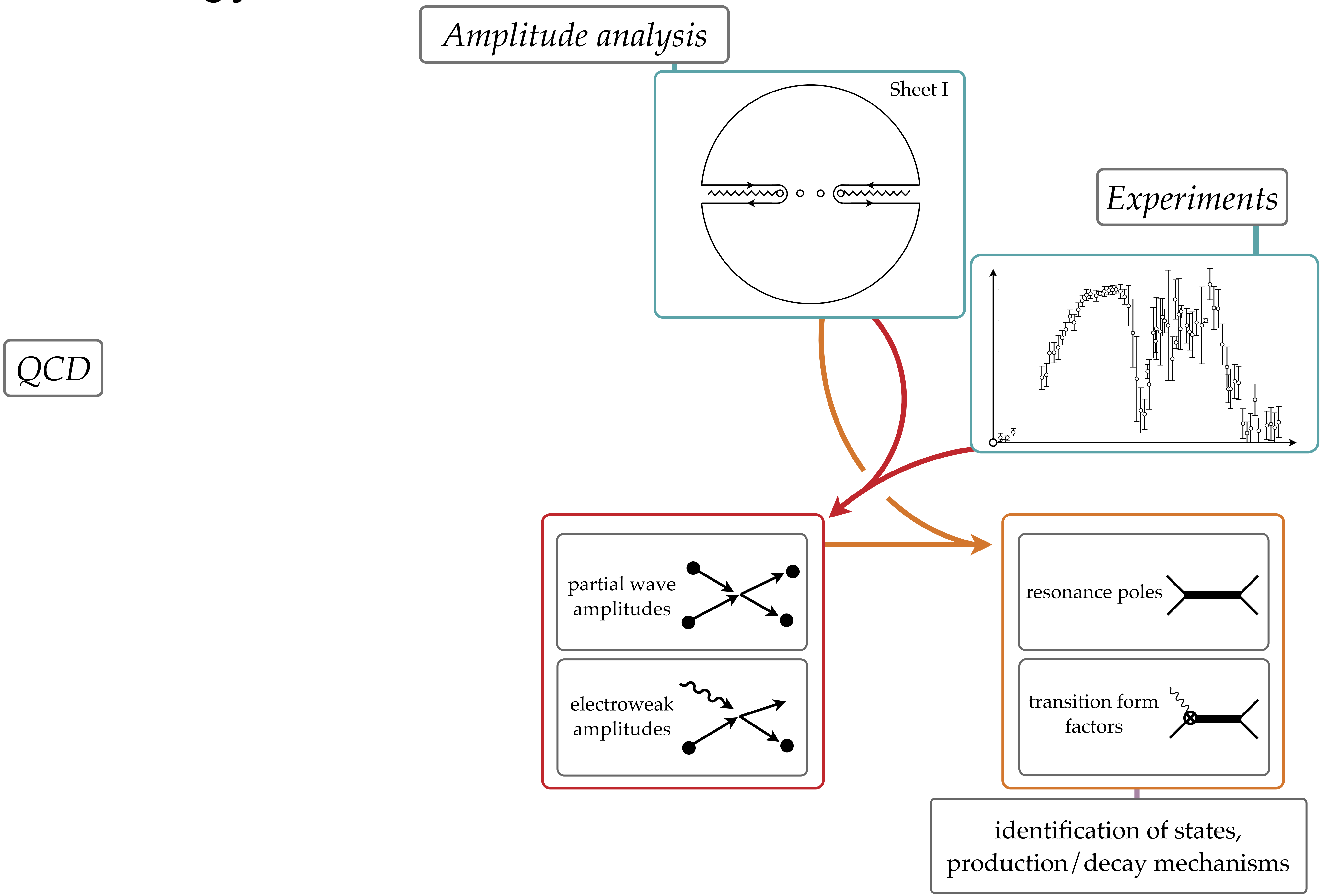
Observables to test our understanding:

- purely hadronic cross sections
- Electroweak production and decay rates
- …

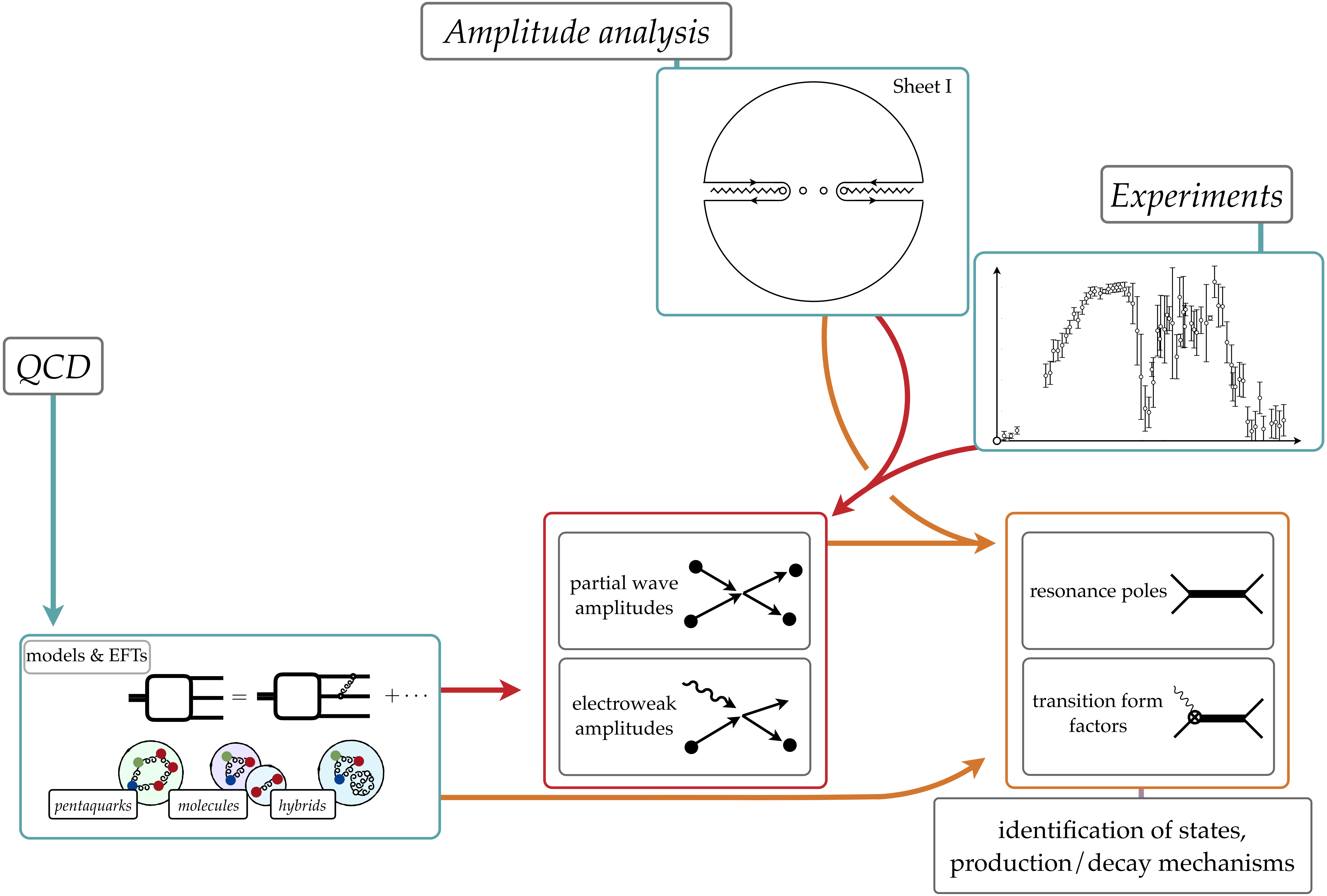
Experiments

QCD

low energy QCD in the 21st cent.



low energy QCD in the 21st cent.



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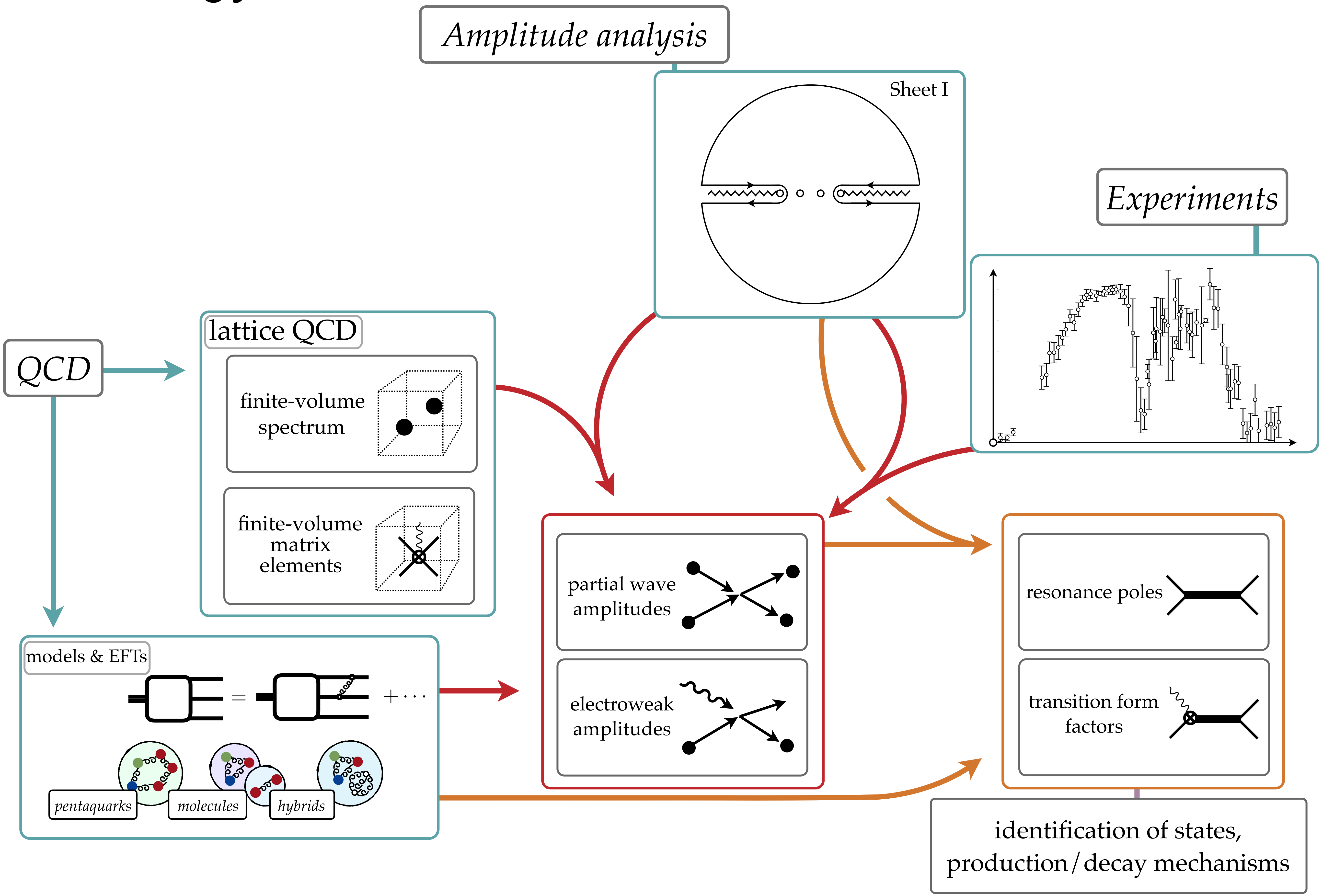
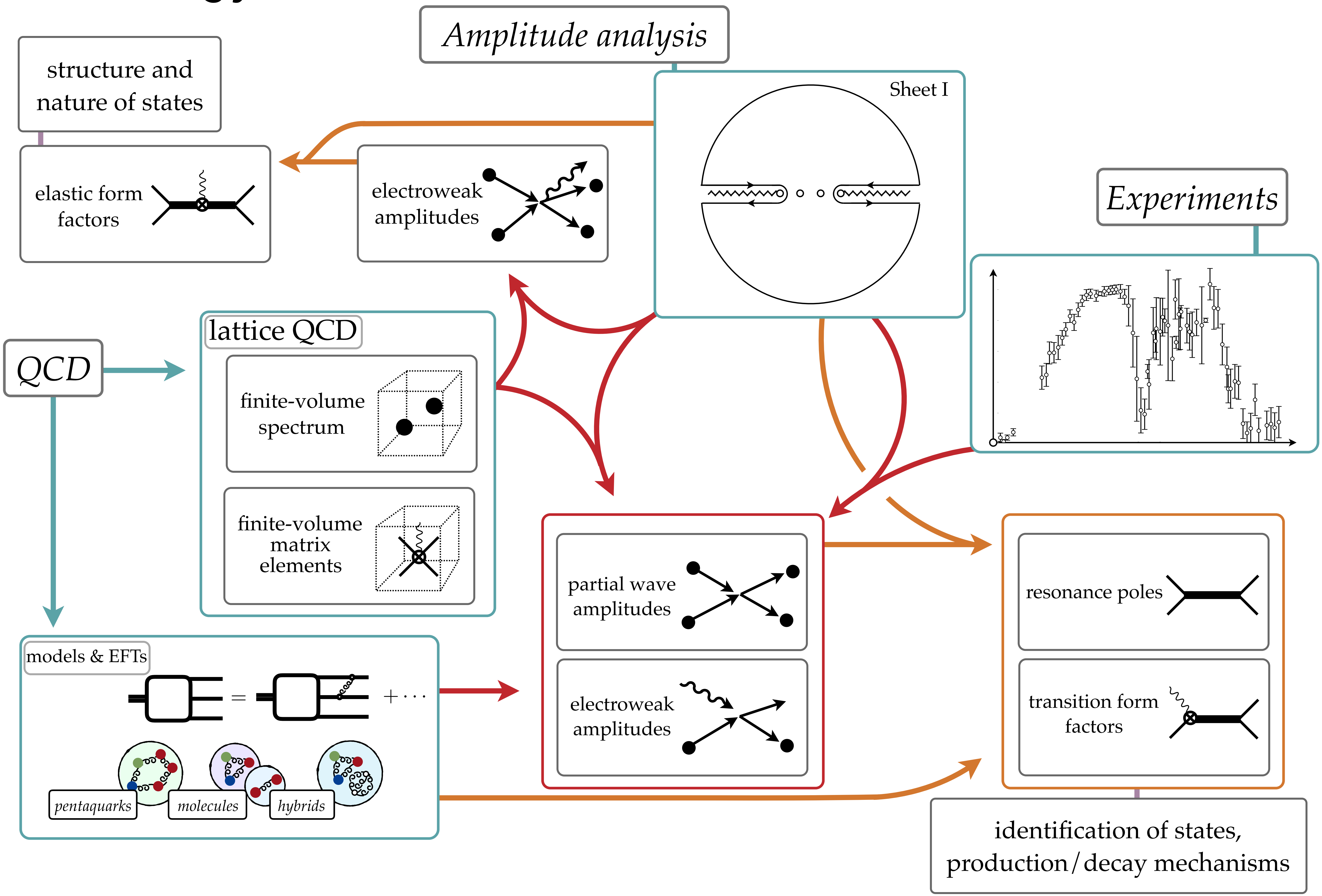


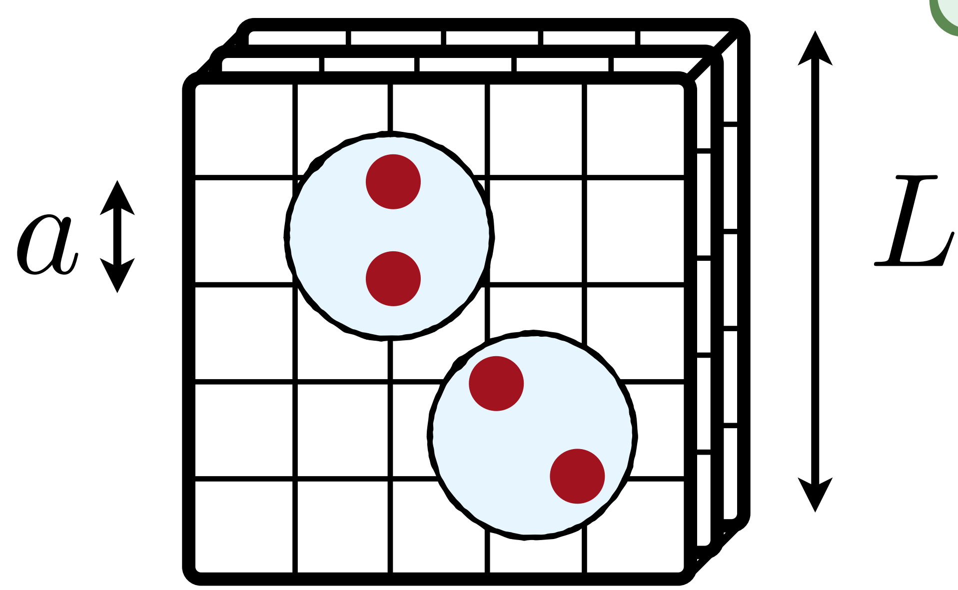
low energy QCD in the 21st cent.



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume

$$D_\mu = \left(\quad \right) \updownarrow (L/a)^3 \times (T/a)$$



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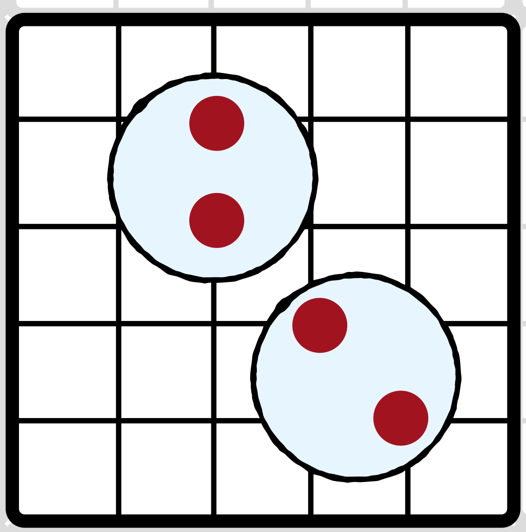
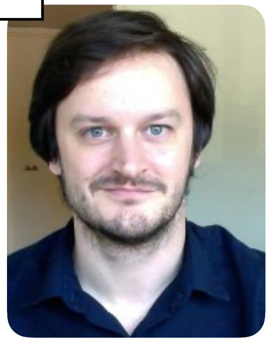


Never free!
No asymptotic states!
No scattering!

Three big ideas

. . . already out of date

Dudek



Young

Scattering processes and resonances from lattice QCD

Raúl A. Briceño, Jozef J. Dudek, and Ross D. Young
Rev. Mod. Phys. **90**, 025001 – Published 18 April 2018

Article | References | Citing Articles (7) | PDF | HTML | Export Citation

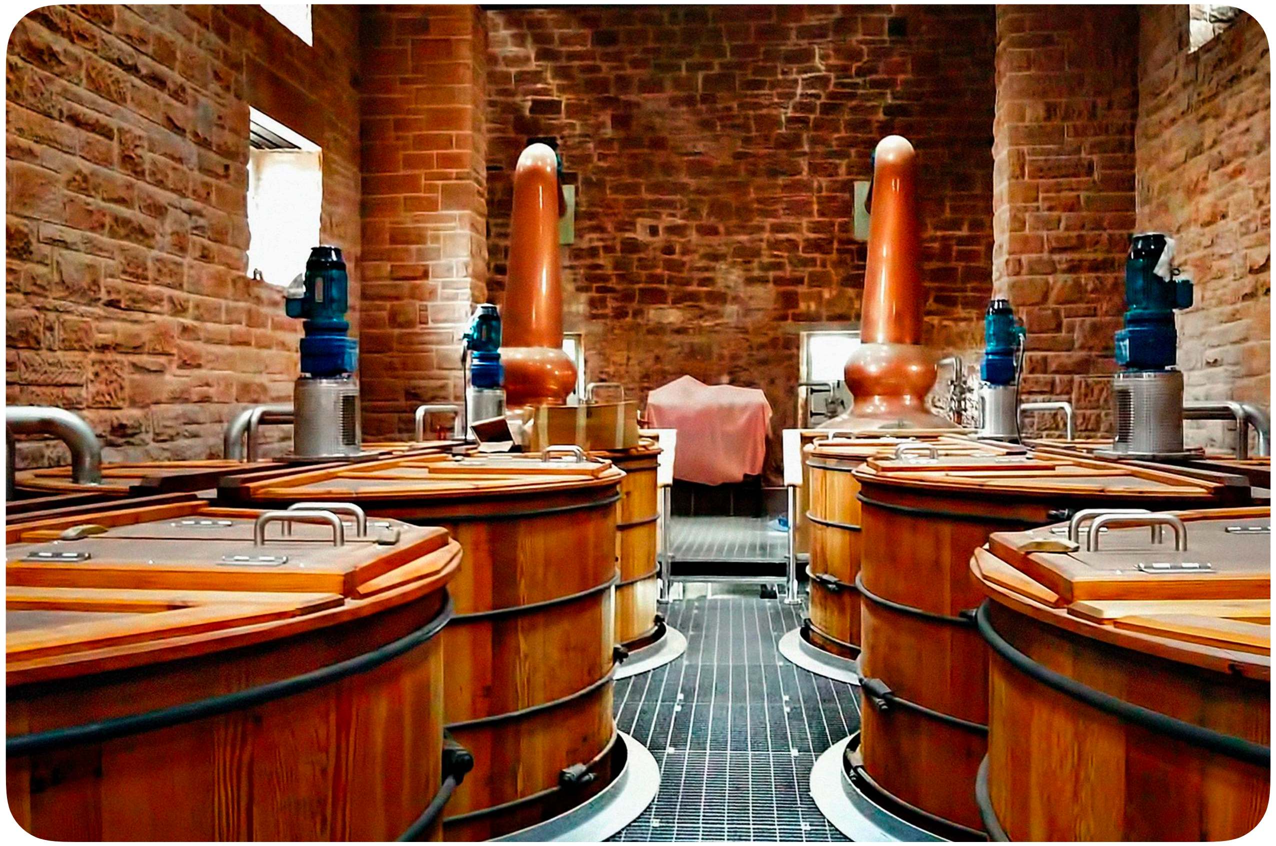
ABSTRACT

The vast majority of hadrons observed in nature are not stable under the strong interaction; rather they are resonances whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy nonperturbative region, and in addition many probes of the limits of the electroweak sector of the standard model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a

Three big ideas

- Distillation [Peardon, *et al.* (Hadron Spectrum, 2009)]
  - use eigenvectors of the Gauge-covariant Laplacian as a way to:
    - smear [distill: boil off the high modes]
    - reduce the size operators / propagators

$$\mathbf{1}_{N_c \times N_x \times N_y \times N_z} \Longrightarrow \sum_k^N V^{(k)}\, V^{(k)\dagger}$$



Three big ideas

- Distillation
- Generalized eigenvalue problem

$$C_{ab}(t) \equiv \langle 0|\mathcal{O}_b(t)\mathcal{O}_a^\dagger(0)|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t},$$

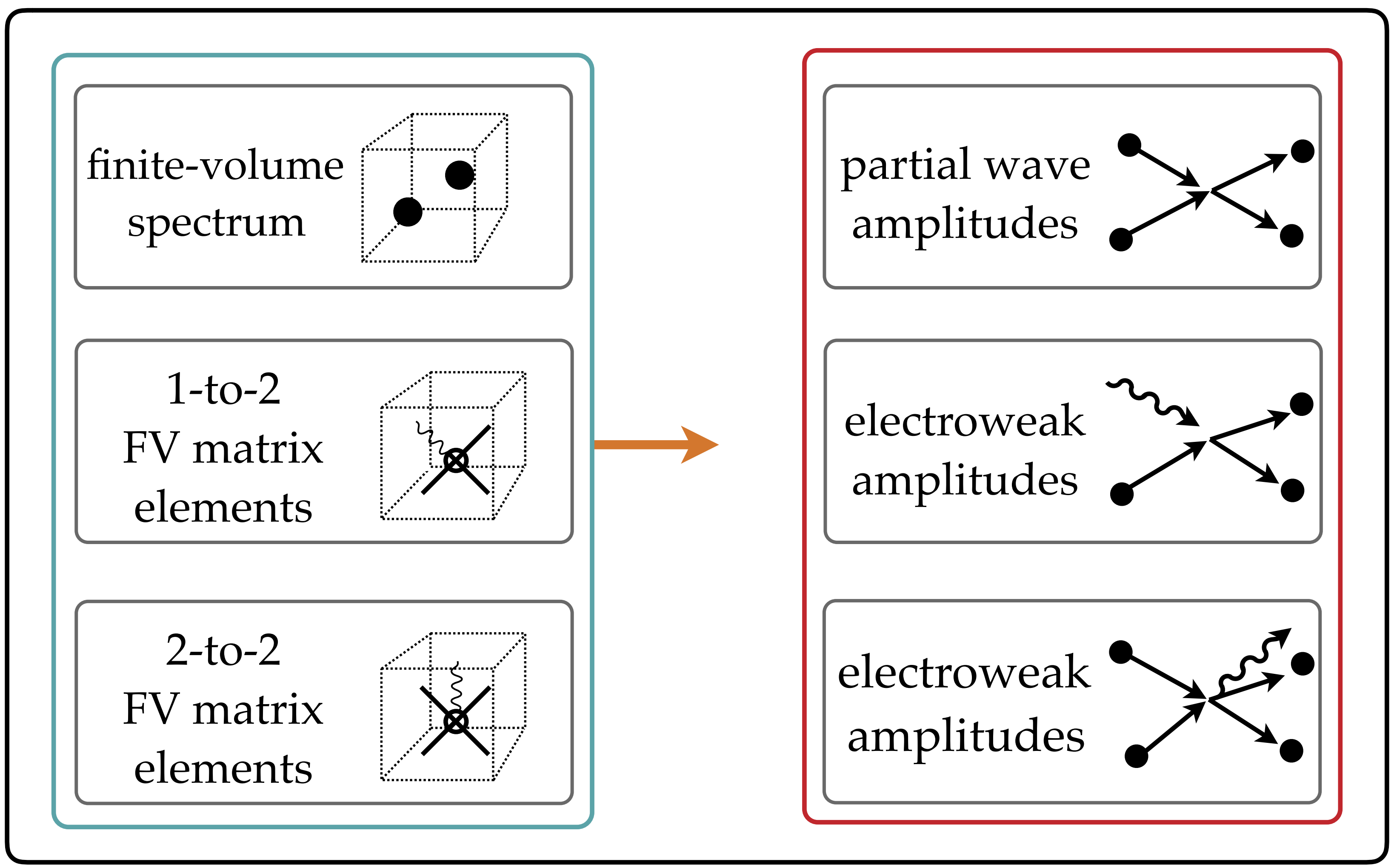
$$C(t)\,\vec{v}^{(n)}(t,t_0) = \lambda_n(t,t_0) C(t_0)\,\vec{v}^{(n)}(t,t_0),$$

$$\lambda_n(t,t_0) = e^{-E_n(t-t_0)} + \cdots$$

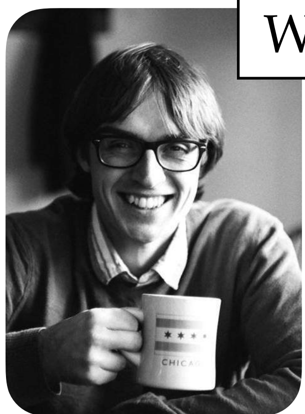
Michael (1985) , Luscher & Wolff (1990), Blossier et al. (2009)

Three big ideas

- Distillation
- Generalized eigenvalue problem
- Finite- and infinite-volume mappings
  - no simple connection
  - naive extrapolations do not always make sense



the light nonets


Wilson


Dudek


Edwards


Thomas

PRL **118**, 022002 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

JLAB-THY-17-2534

Isoscalar $\pi\pi, K\overline{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]
(for the Hadron Spectrum Collaboration)
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[2] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[3] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*

PHYSICAL REVIEW LETTERS **VOL..XX,** 000000 (XXXX)

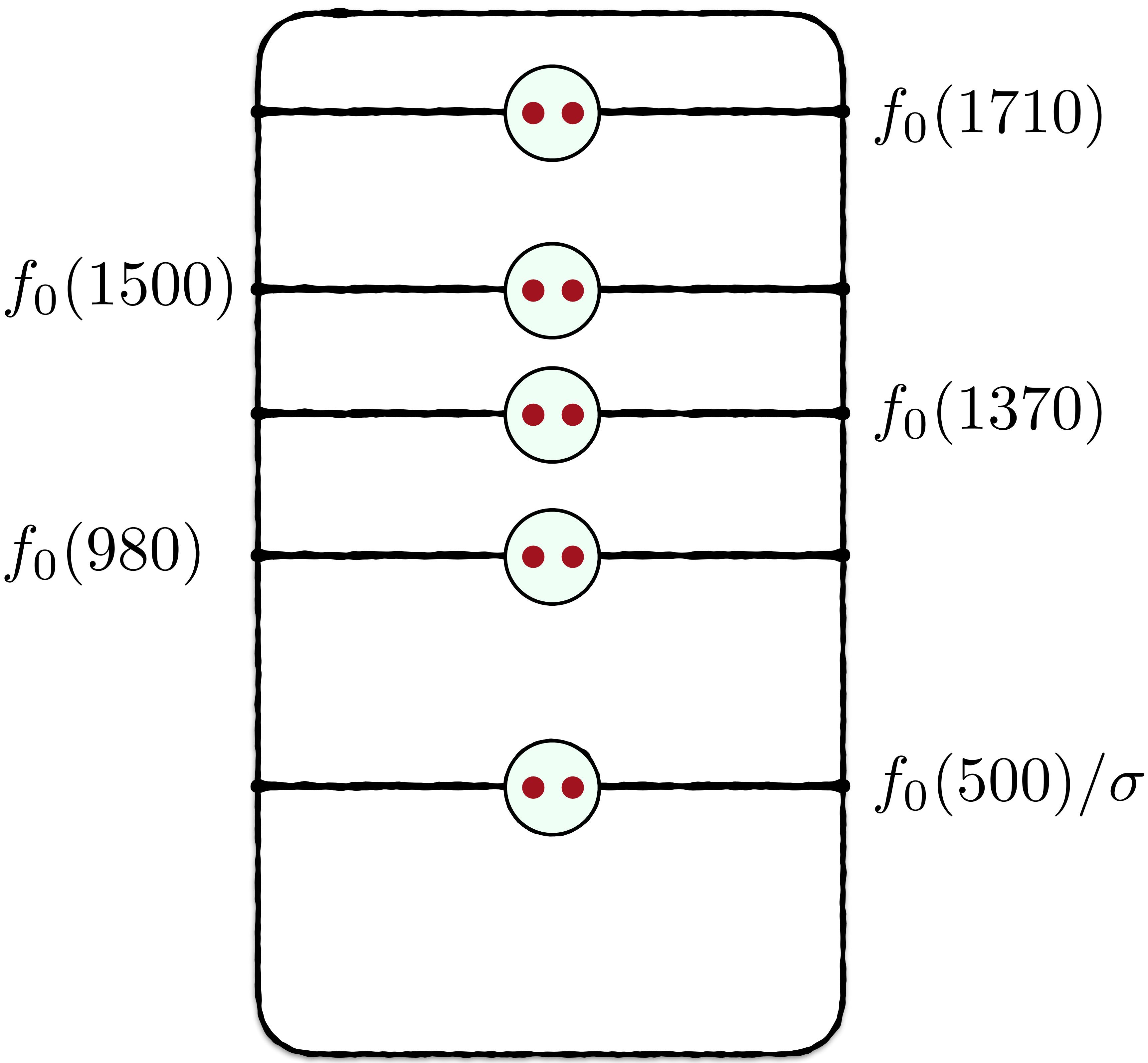
Quark-Mass Dependence of Elastic πK Scattering from QCD

David J. Wilson,[1,*] Raúl A. Briceño,[2,3,†] Jozef J. Dudek,[2,4,‡] Robert G. Edwards,[2,§] and Christopher E. Thomas[5,||]

(Hadron Spectrum Collaboration)

[1] *School of Mathematics, Trinity College, Dublin 2, Ireland*
[2] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[3] *Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA*
[4] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*
[5] *DAMTP, University of Cambridge, Centre for Mathematical Sciences,*

Isoscalar sector



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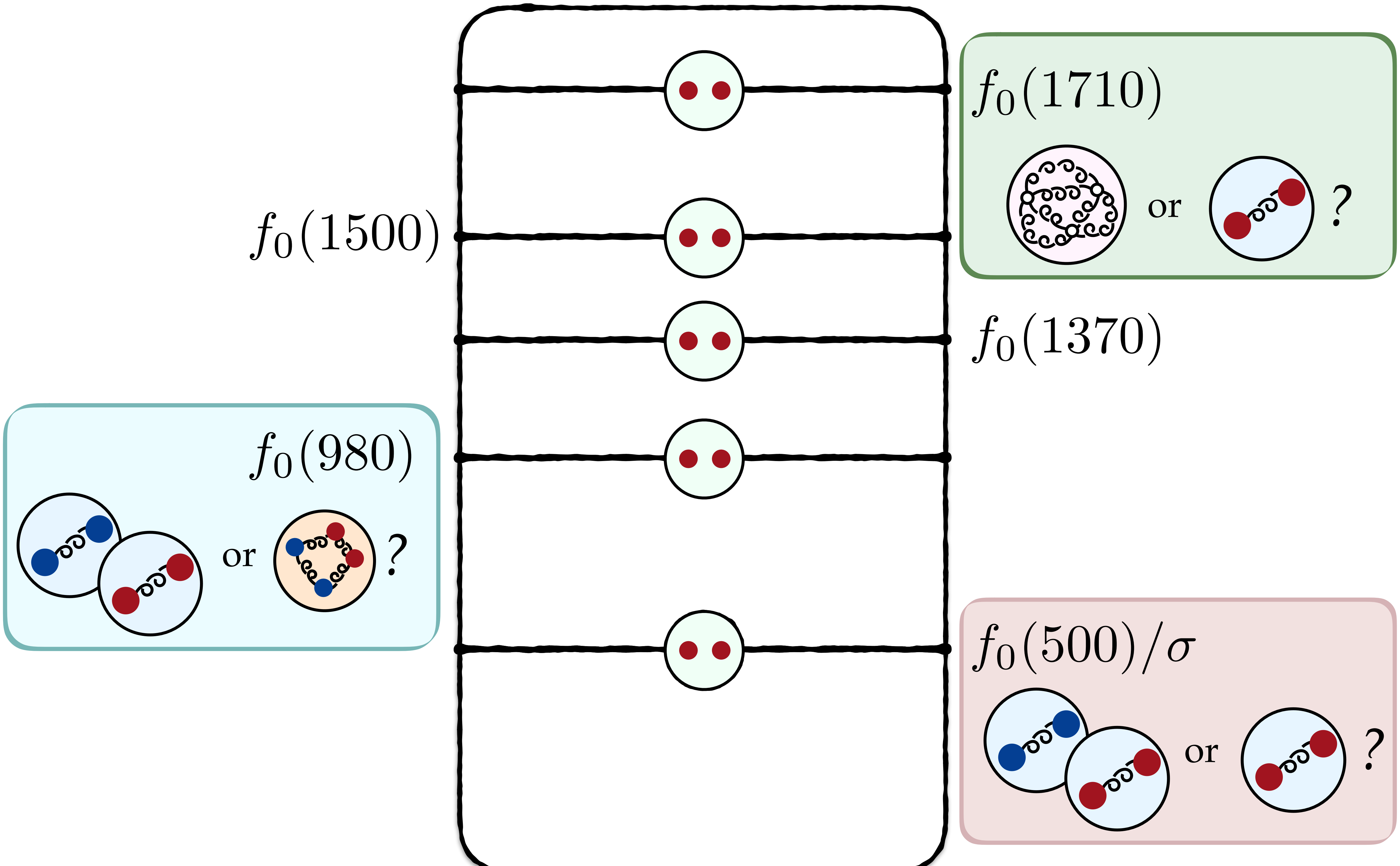
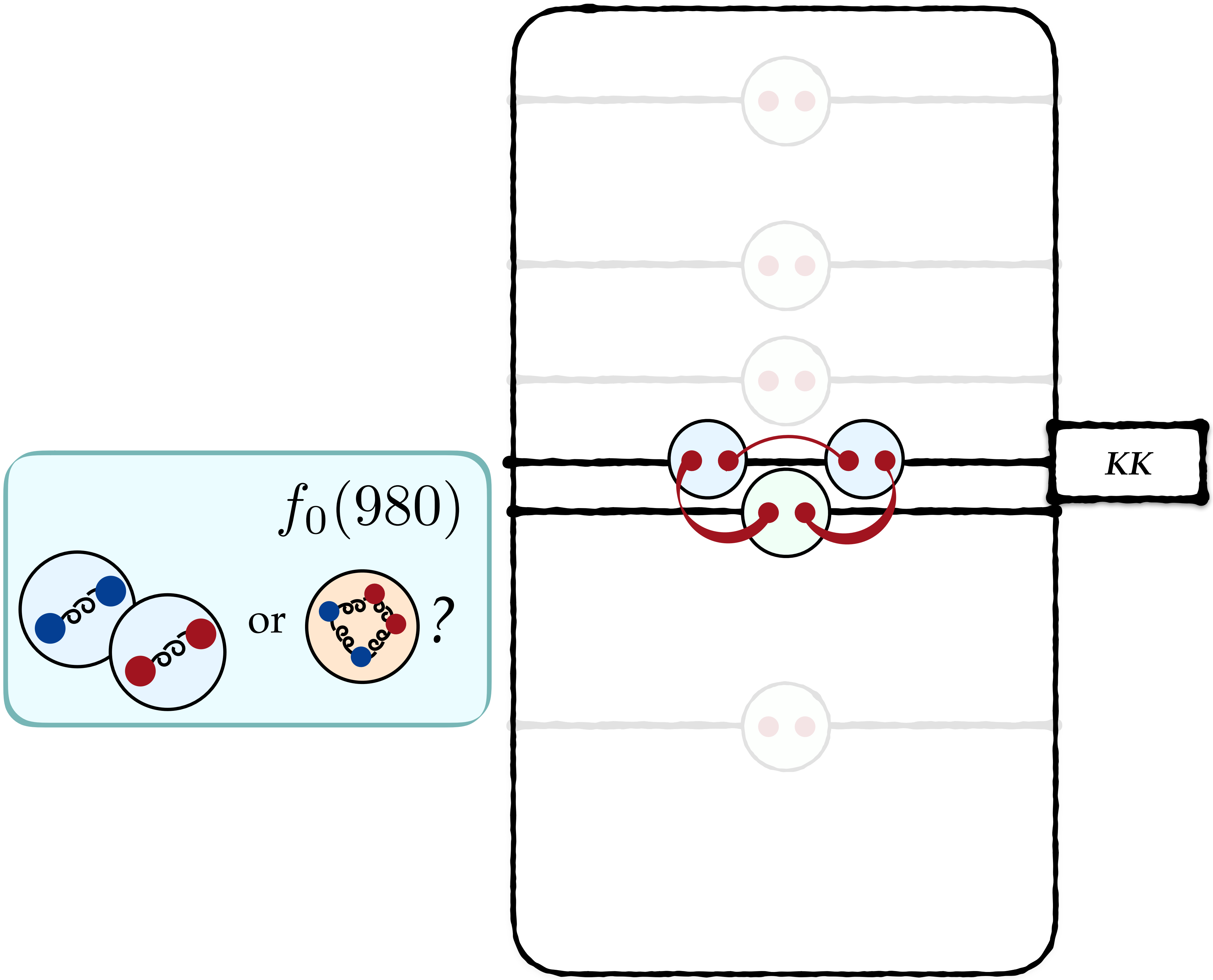
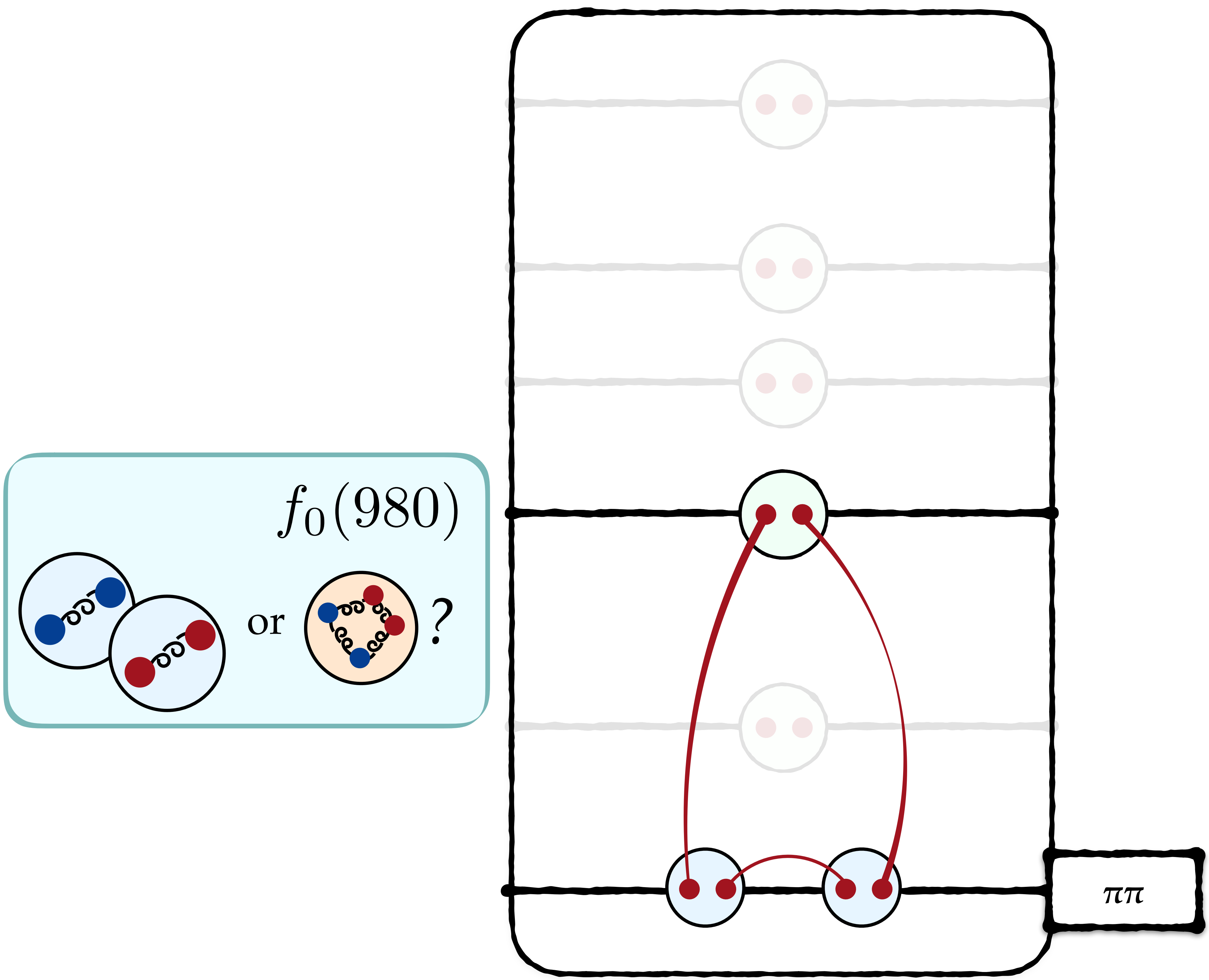


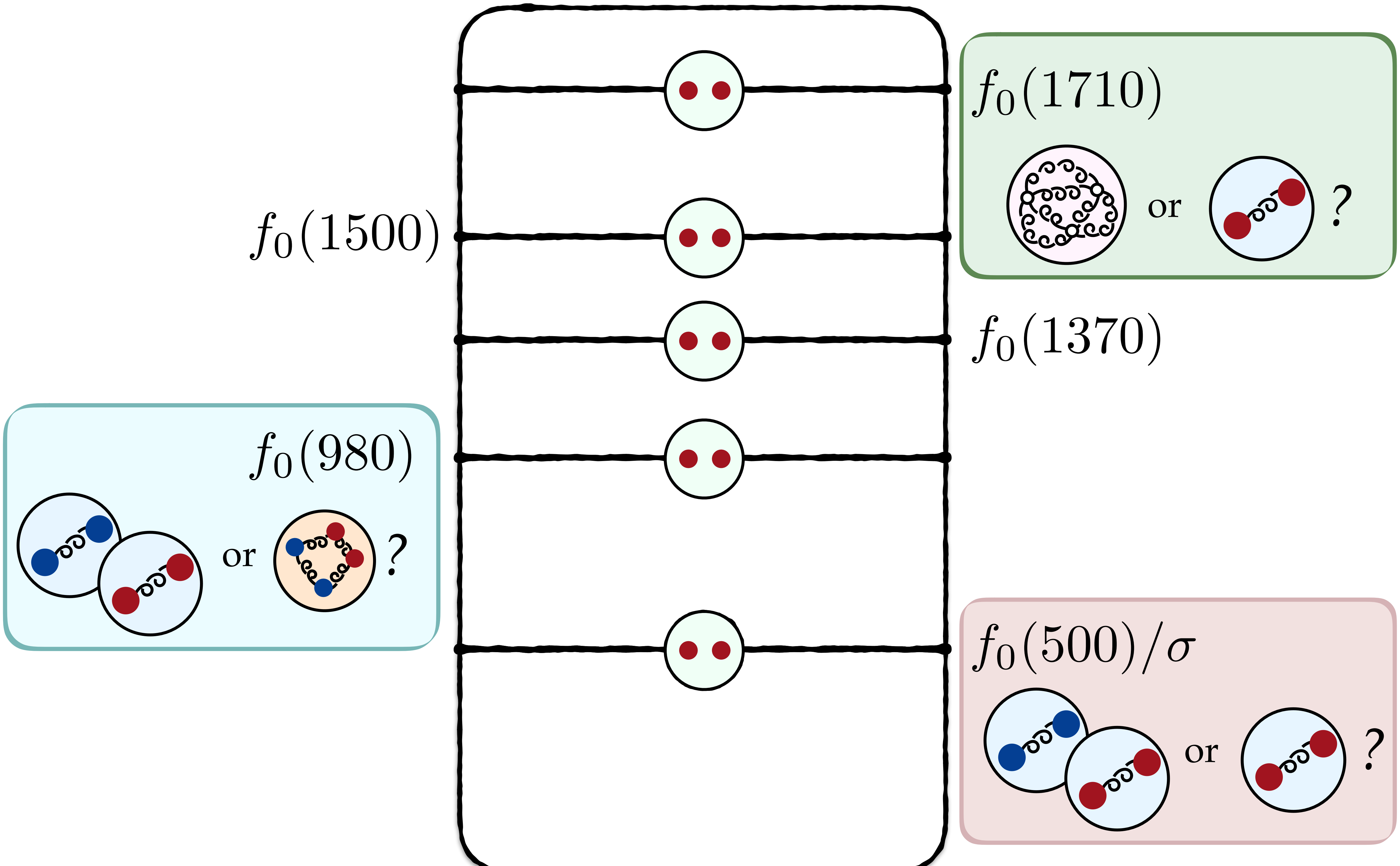
Isoscalar sector



Isoscalar sector

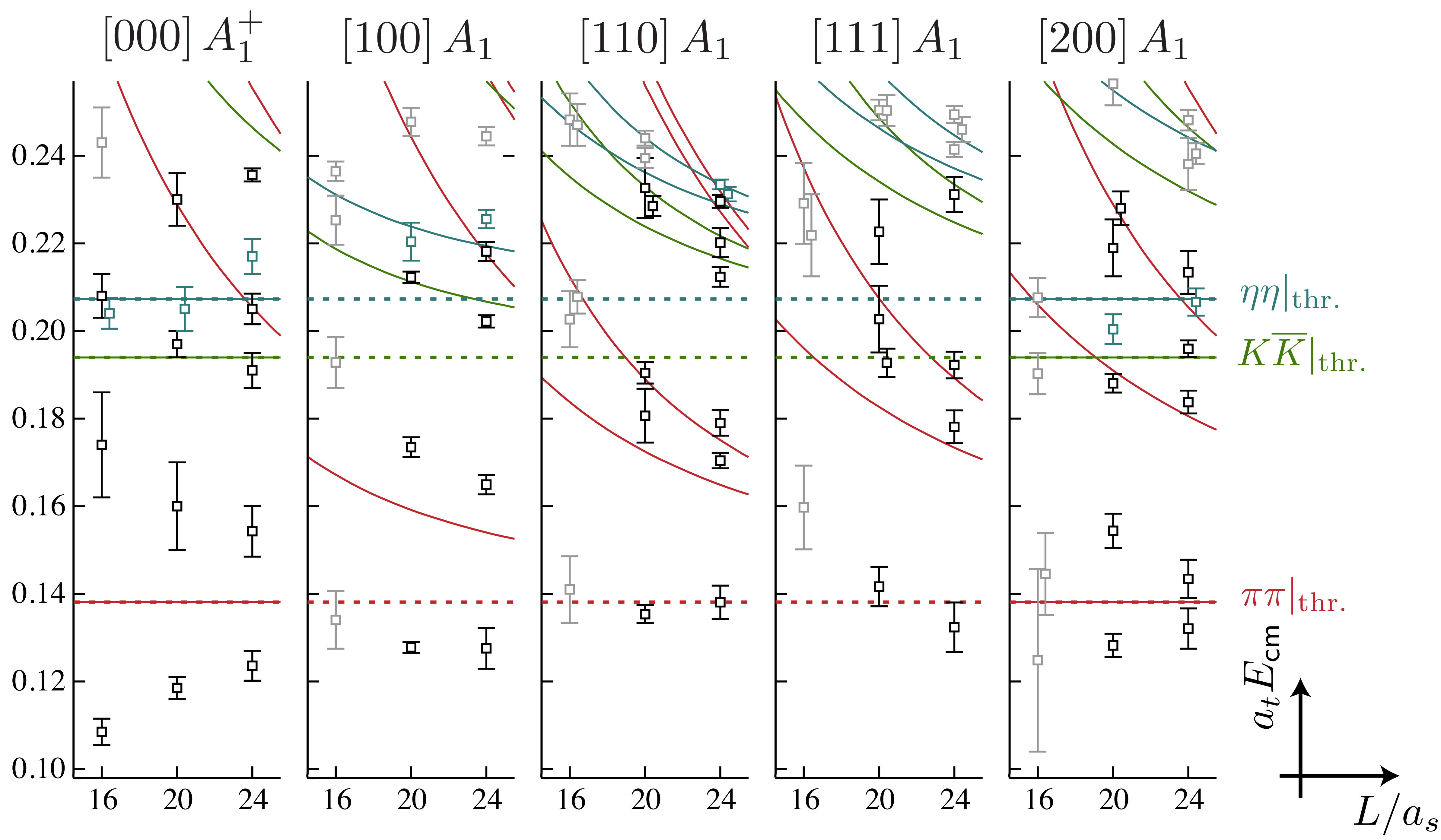


Isoscalar sector



Isoscalar spectra: S-wave dominant

- Multi-meson ops. are crucial
- Spectrum including a large basis: $\{\pi\pi, K\overline{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



m_π=391 MeV

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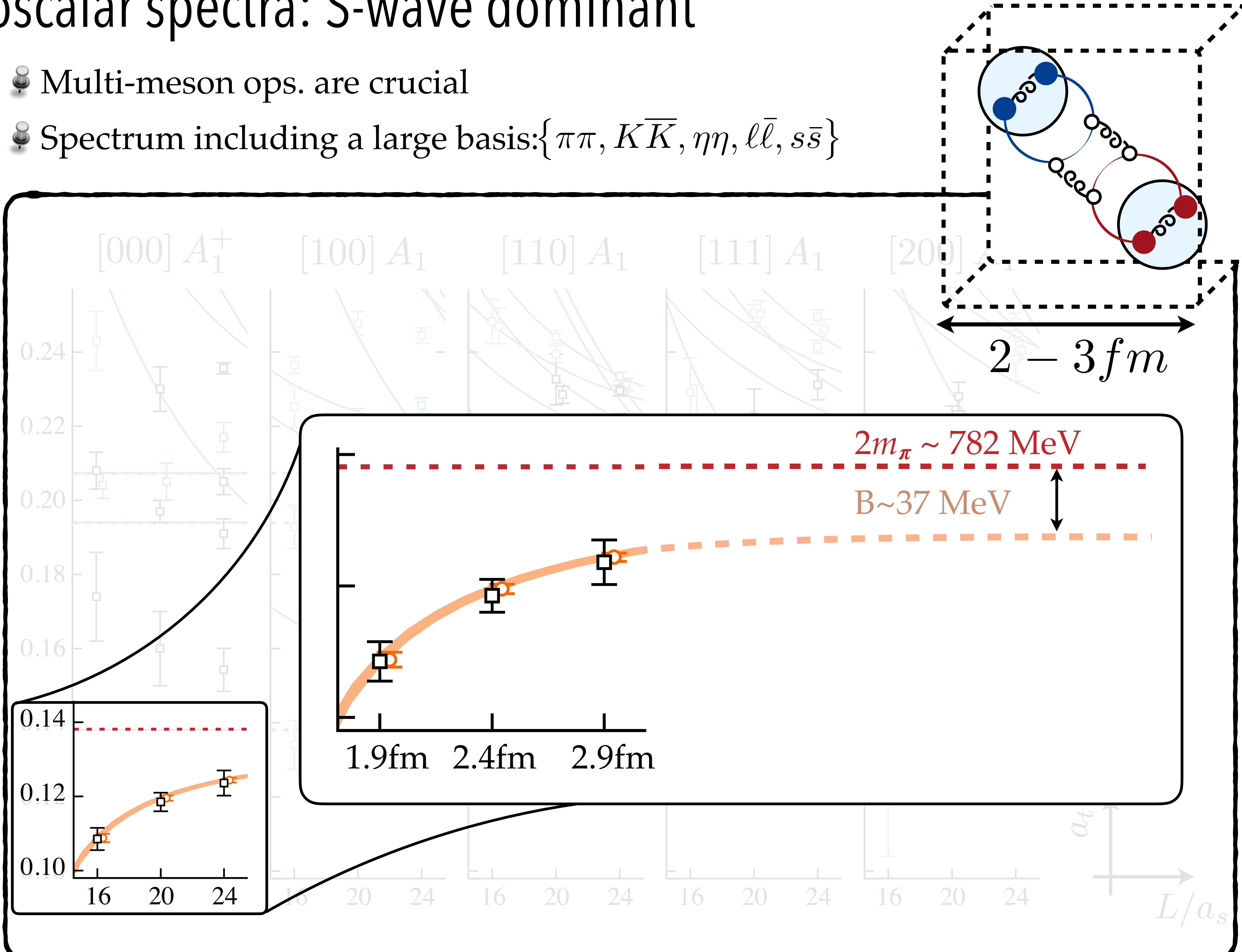

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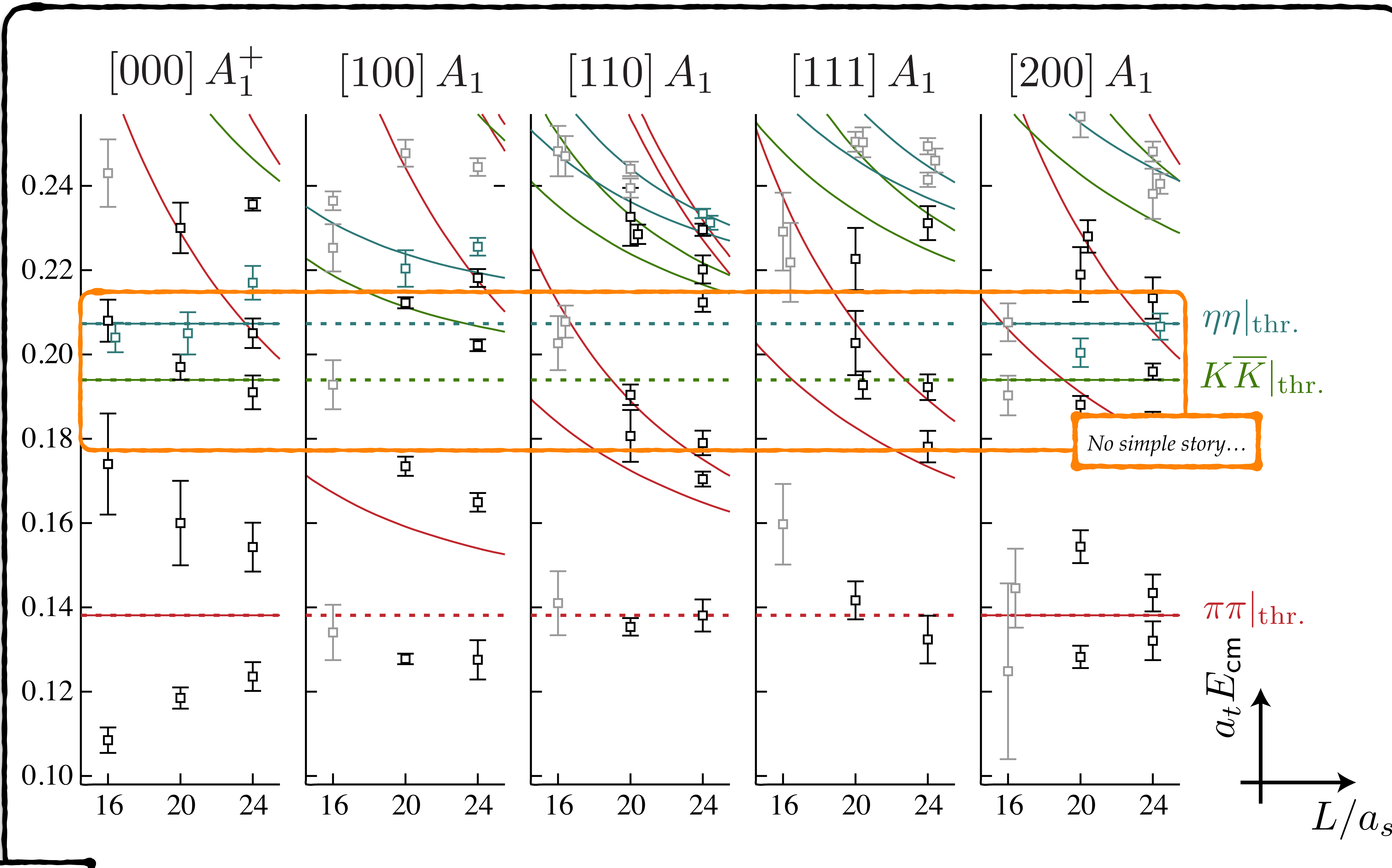
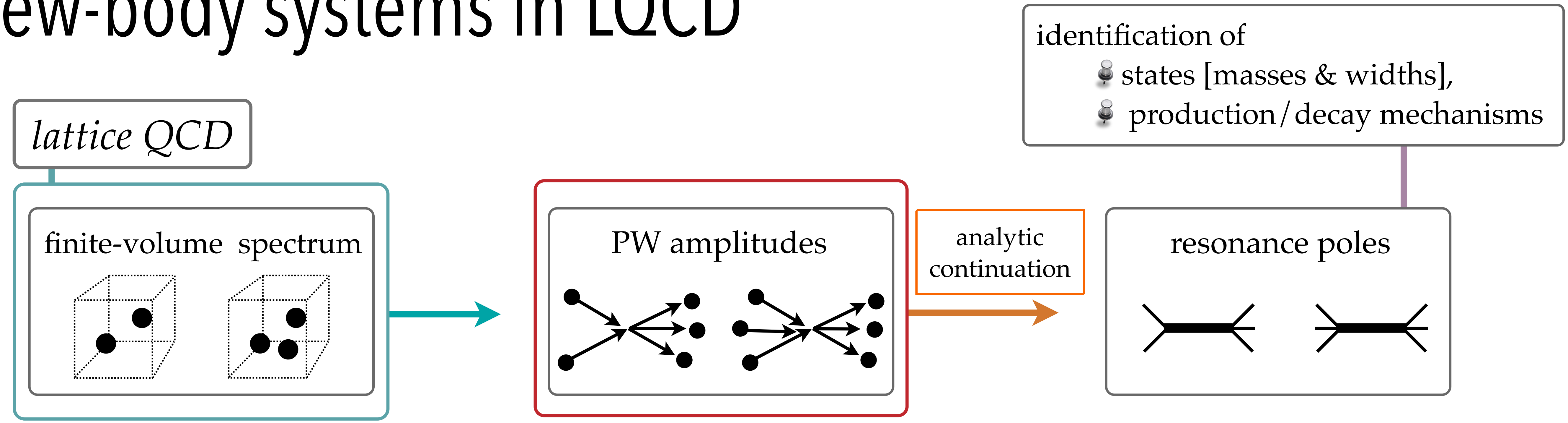


m_π=391 MeV

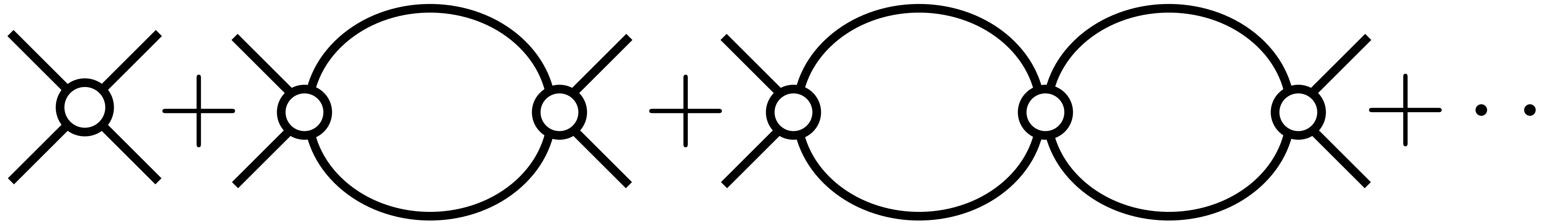
few-body systems in LQCD



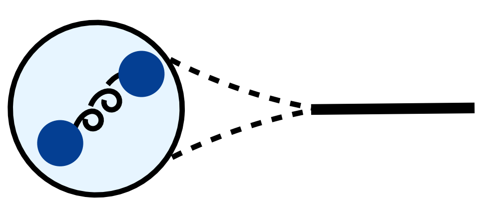
Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} =$$



IR limit of QCD, only interested in hadronic d.o.f.



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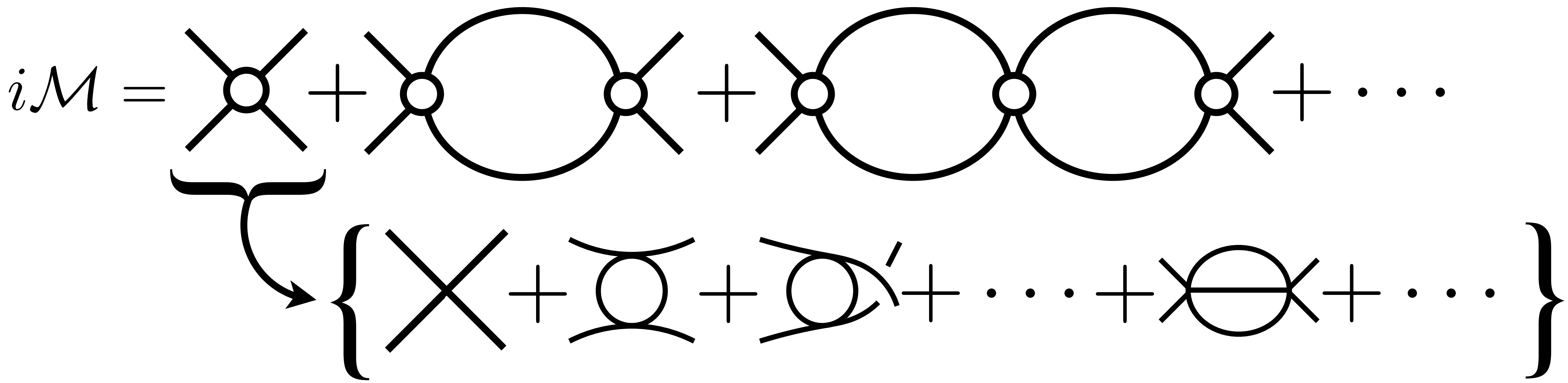

non-perturbative kernel including all diagrams not shown…

"yep, the left hand cut is there"

Two-body scattering

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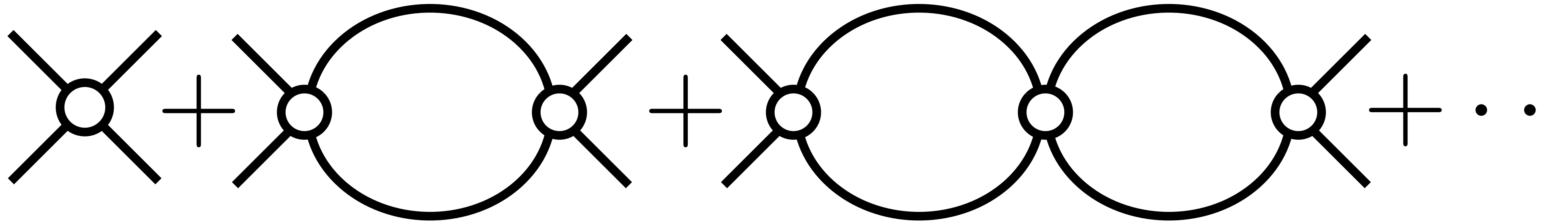
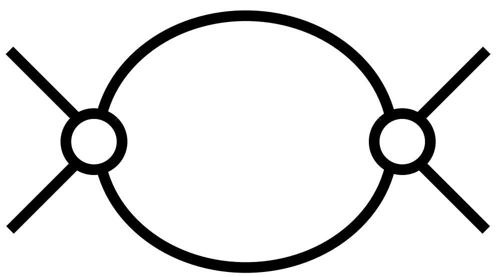
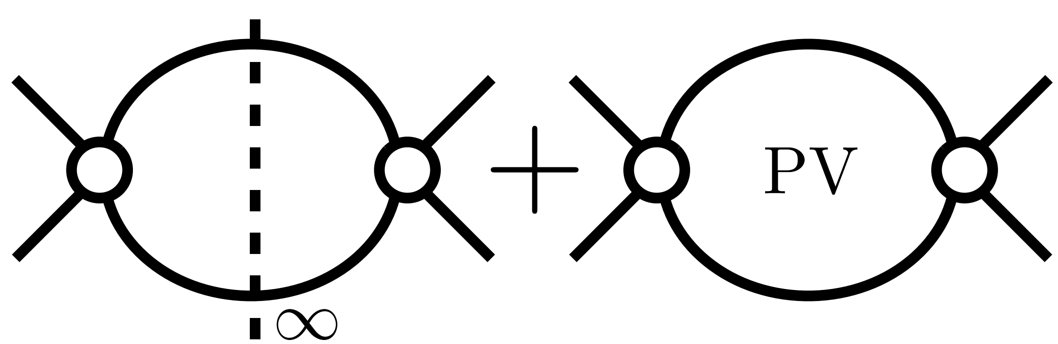
$i\mathcal{M} =$ 



$$= \int \frac{d^4k}{(2\pi)^4} [iB(k,P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P-k)^2 - m^2 + i\epsilon}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k,P)]^2}{(2\omega_k)^2} \pi\delta(E - 2\omega_k) + \text{“PV integral”}$$

$$= [iB_{on}]\rho\,[iB_{on}] + \text{“PV integral”}$$

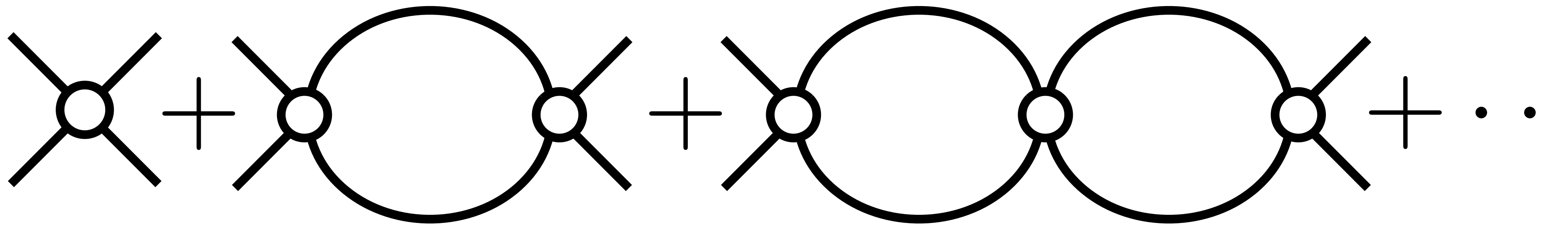
$=$ 

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

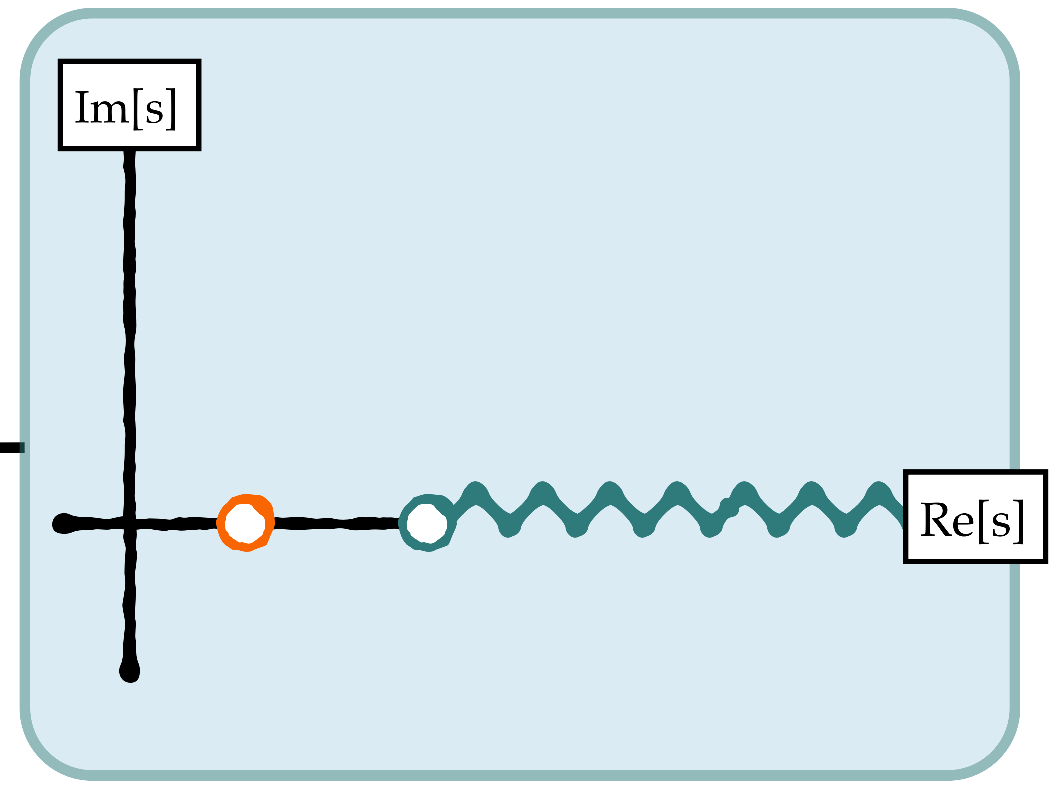
square root singularity.

Two-body scattering

Unitarity using all orders perturbation theory:

$i\mathcal{M} =$ 

2^N sheets for N open channels



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Two-body scattering

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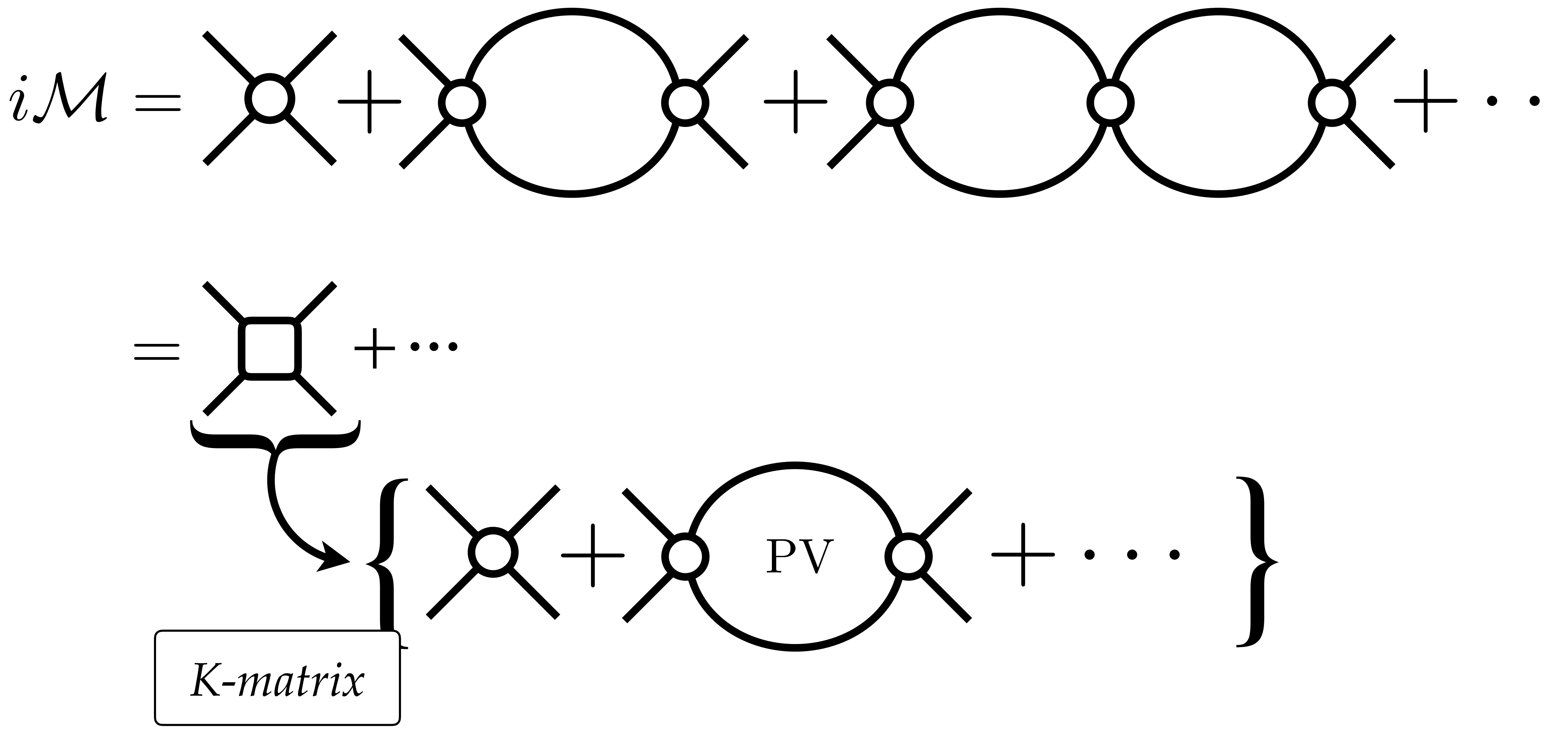
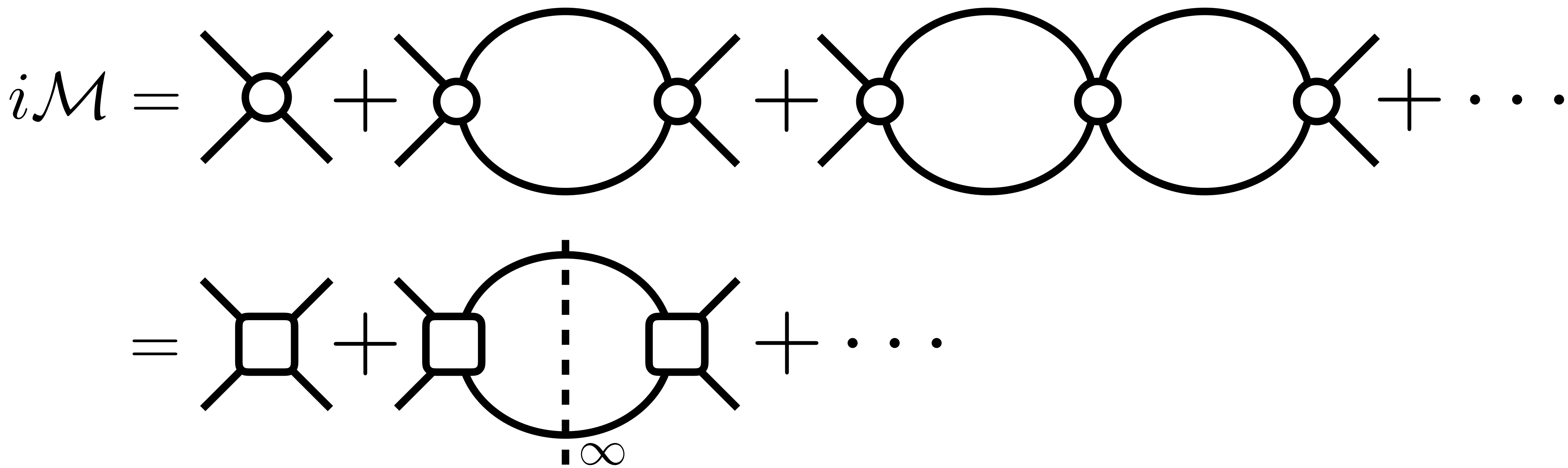


Two-body scattering

Unitarity using all orders perturbation theory:



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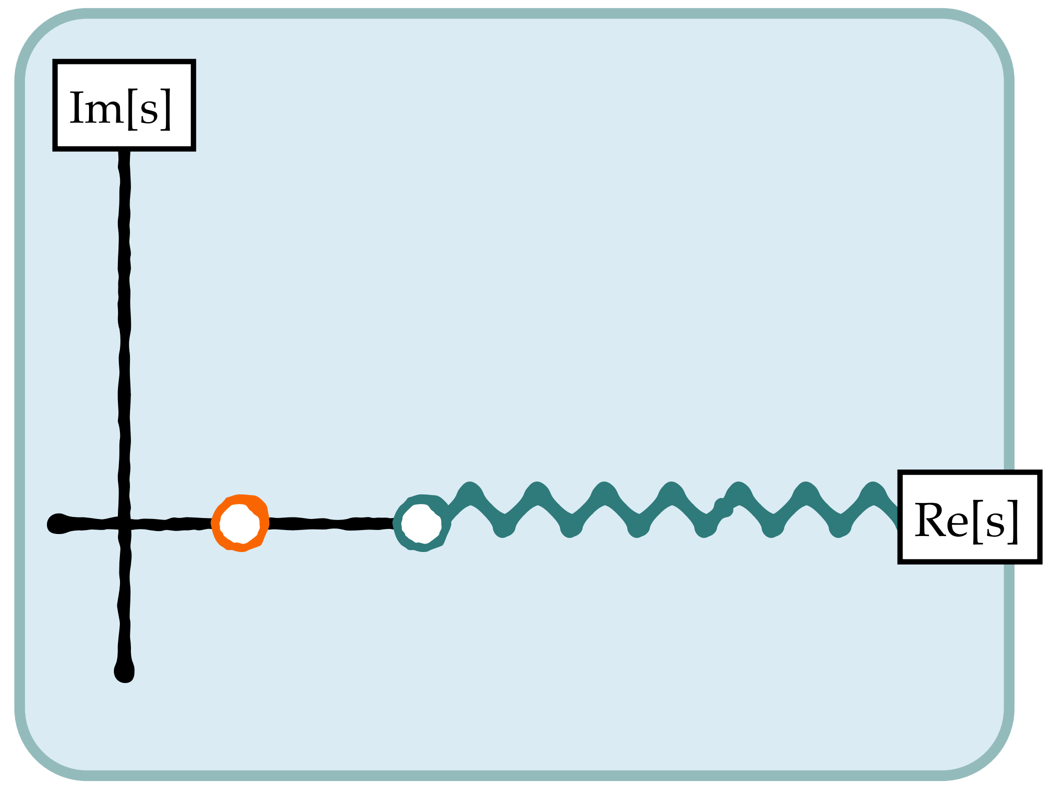

Two-body scattering

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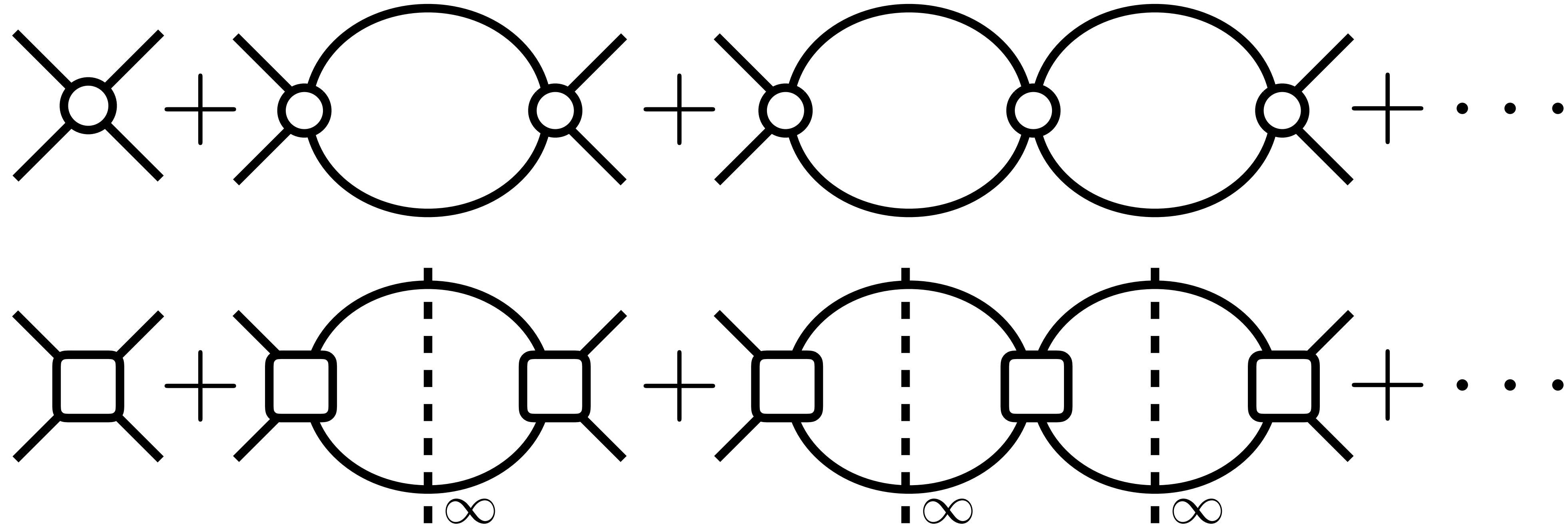
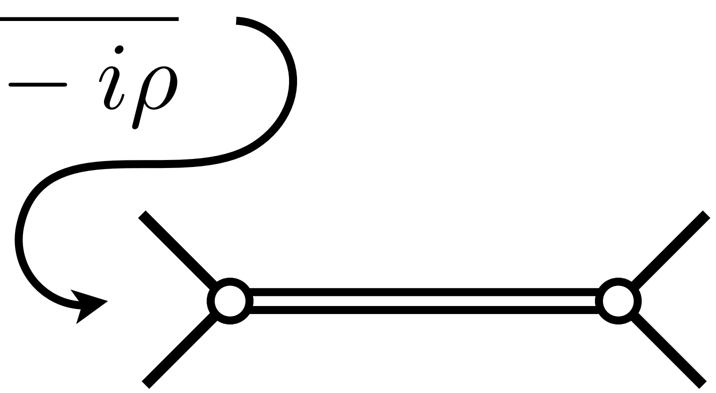
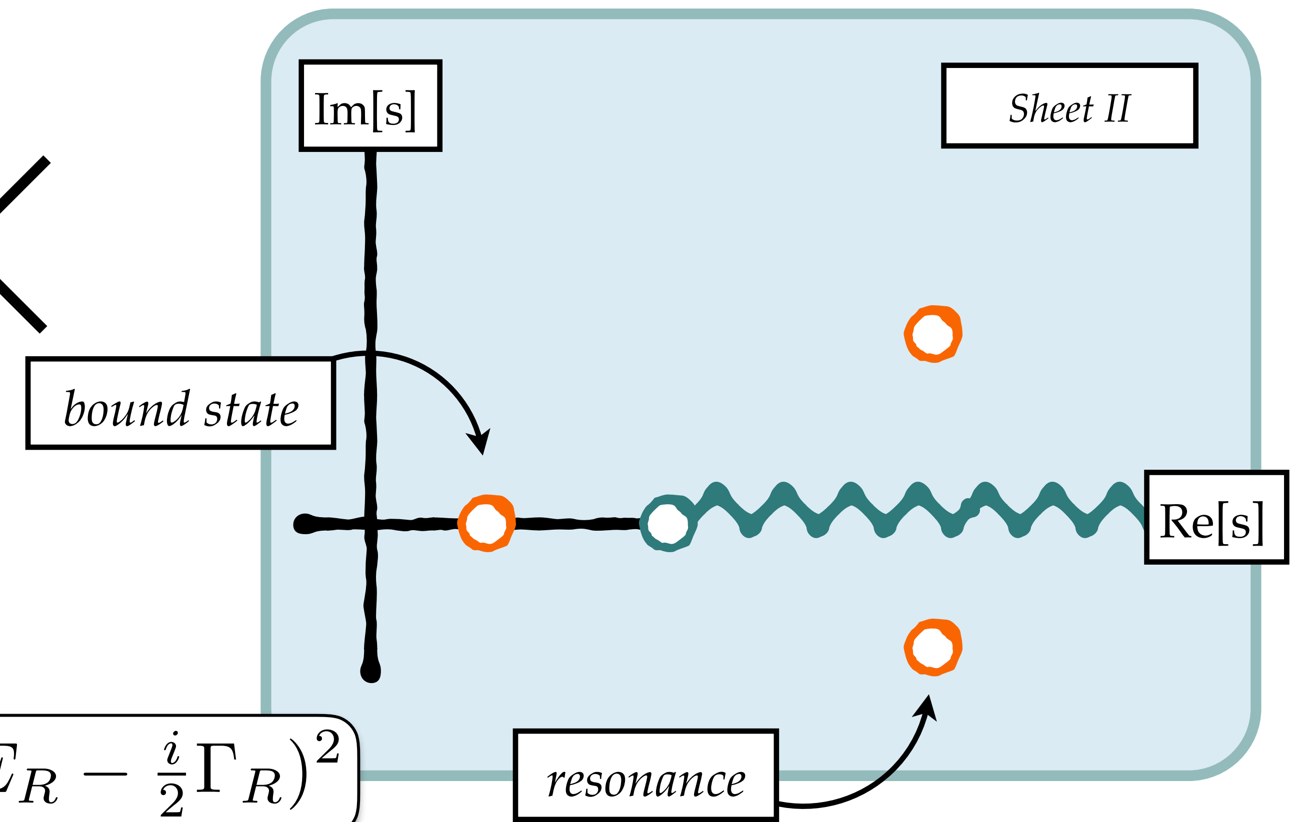


$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$



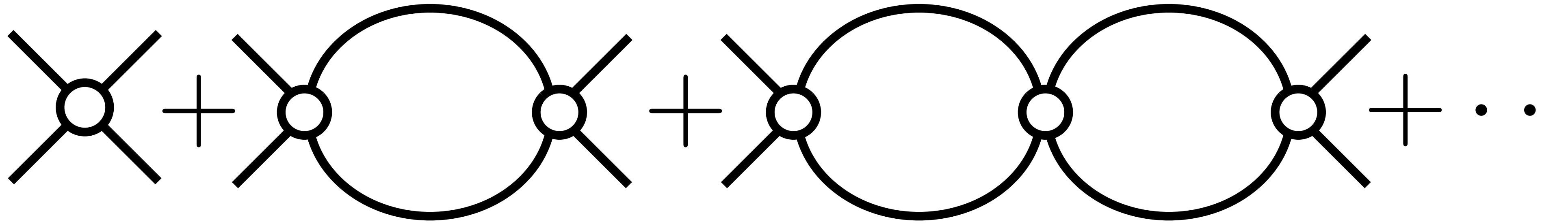


$$s_R = (E_R - \tfrac{i}{2}\Gamma_R)^2$$

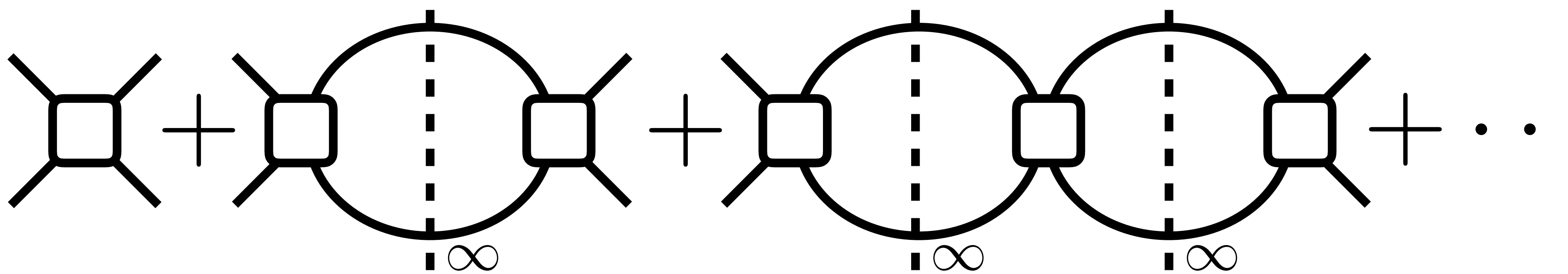
Two-body scattering

Unitarity using all orders perturbation theory:

$i\mathcal{M} =$



$=$



$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$

Equating this to the elastic S matrix…

$$S = e^{2i\delta} = 1 + 2i\rho\mathcal{M}$$

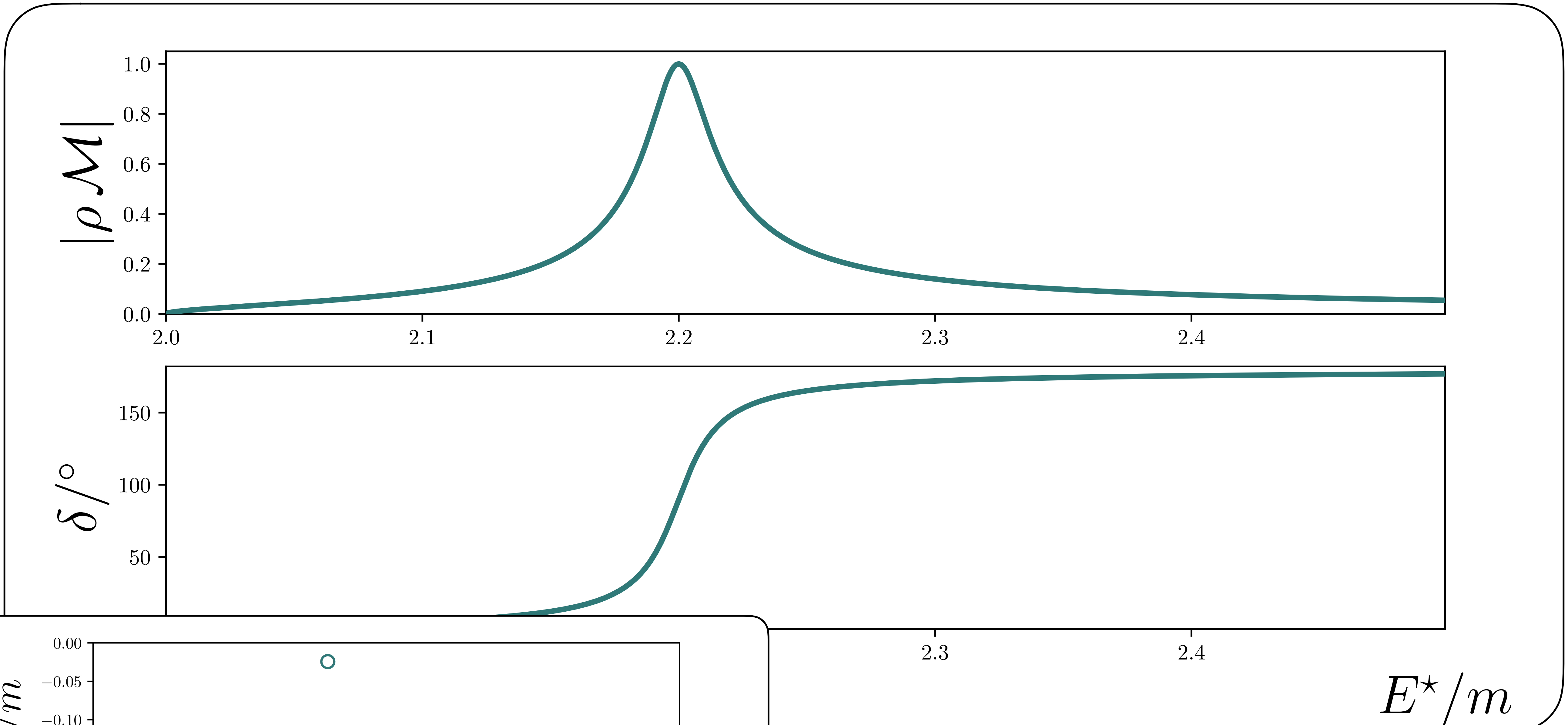
→

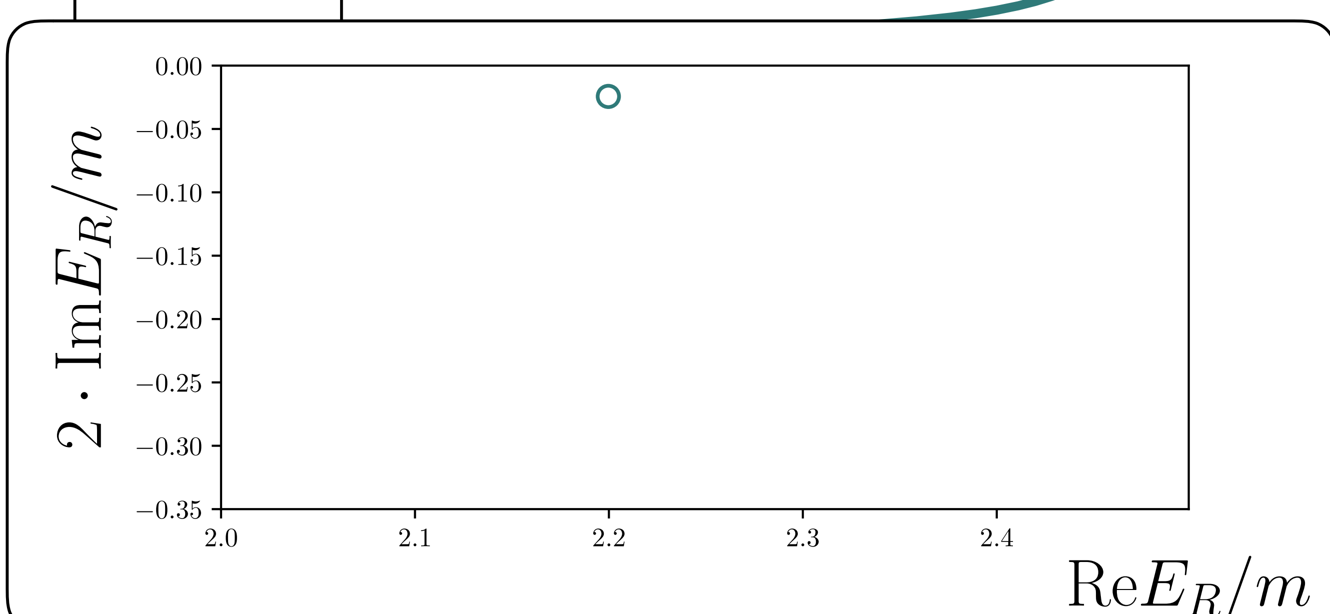
$$\mathcal{K}^{-1} = \rho\cot\delta$$

$$\mathcal{M} = \frac{\sin\delta}{\rho} e^{i\delta}$$

Two-body scattering - resonance

To build some intuition: $\mathcal{M} = \dfrac{\sin\delta}{\rho} e^{i\delta}$





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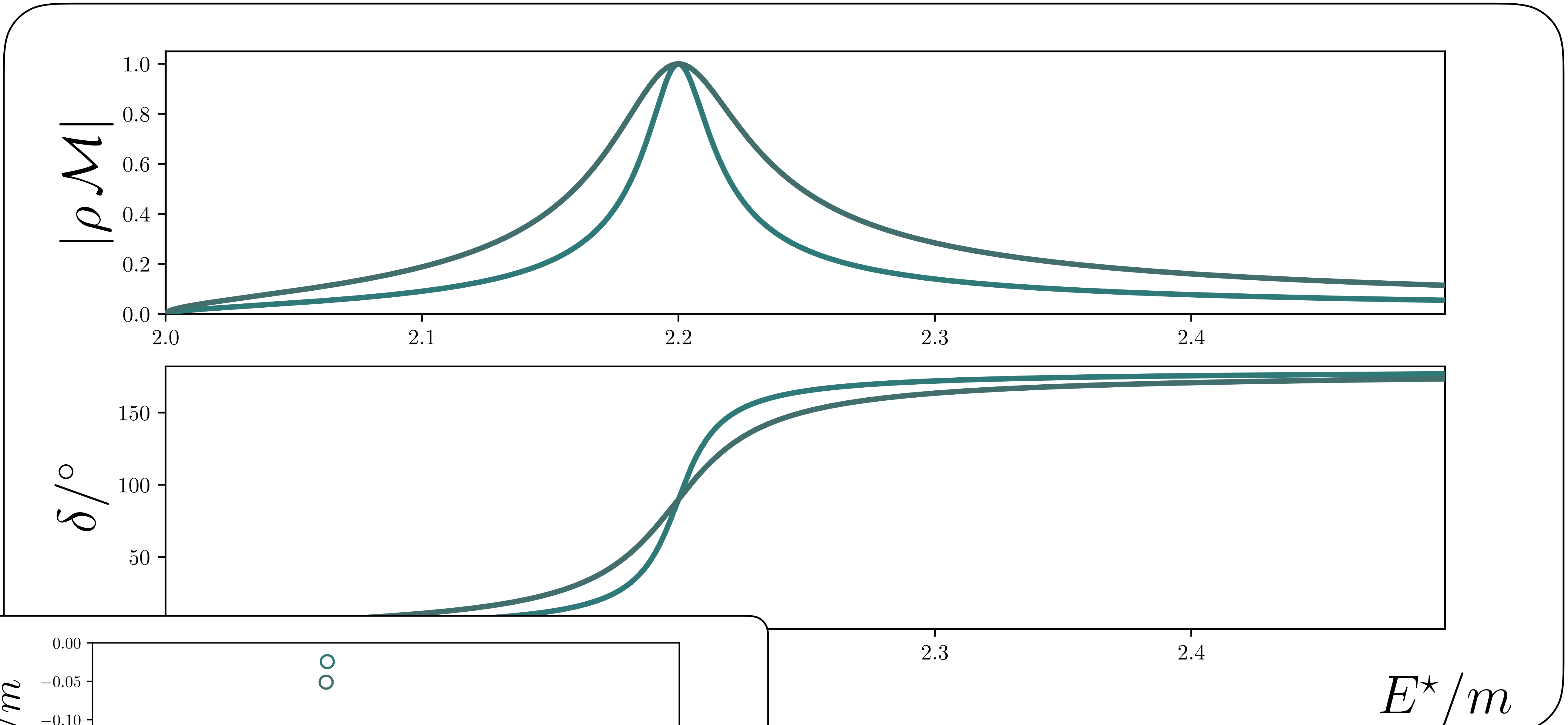

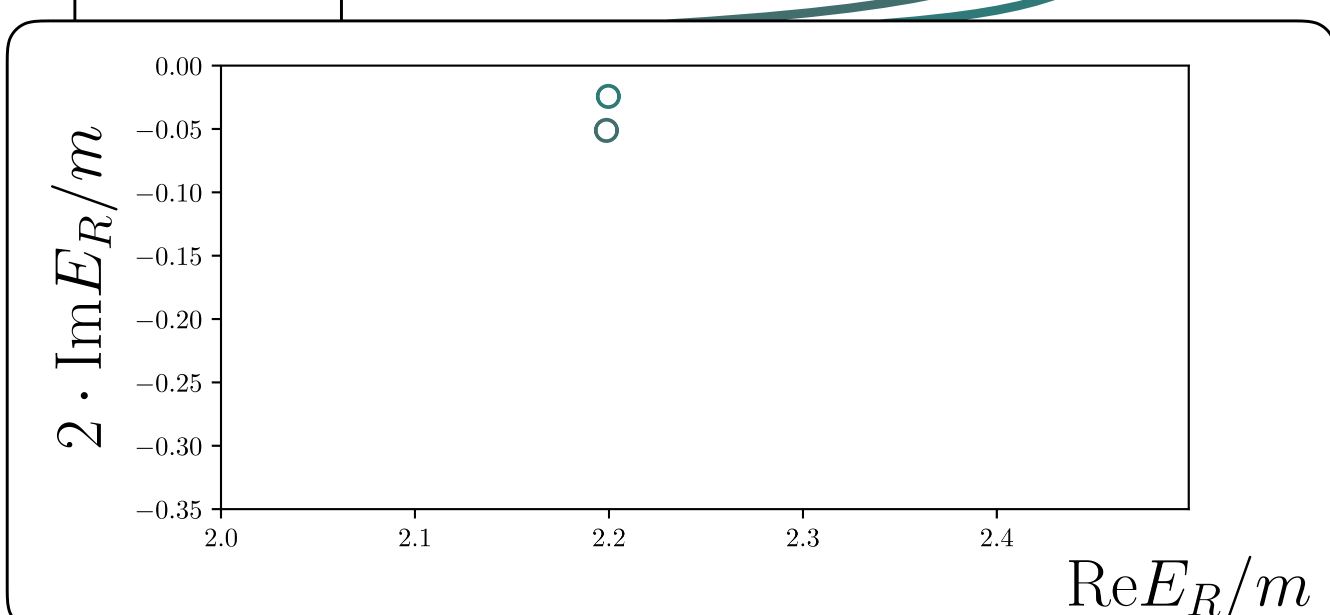

Two-body scattering - resonance

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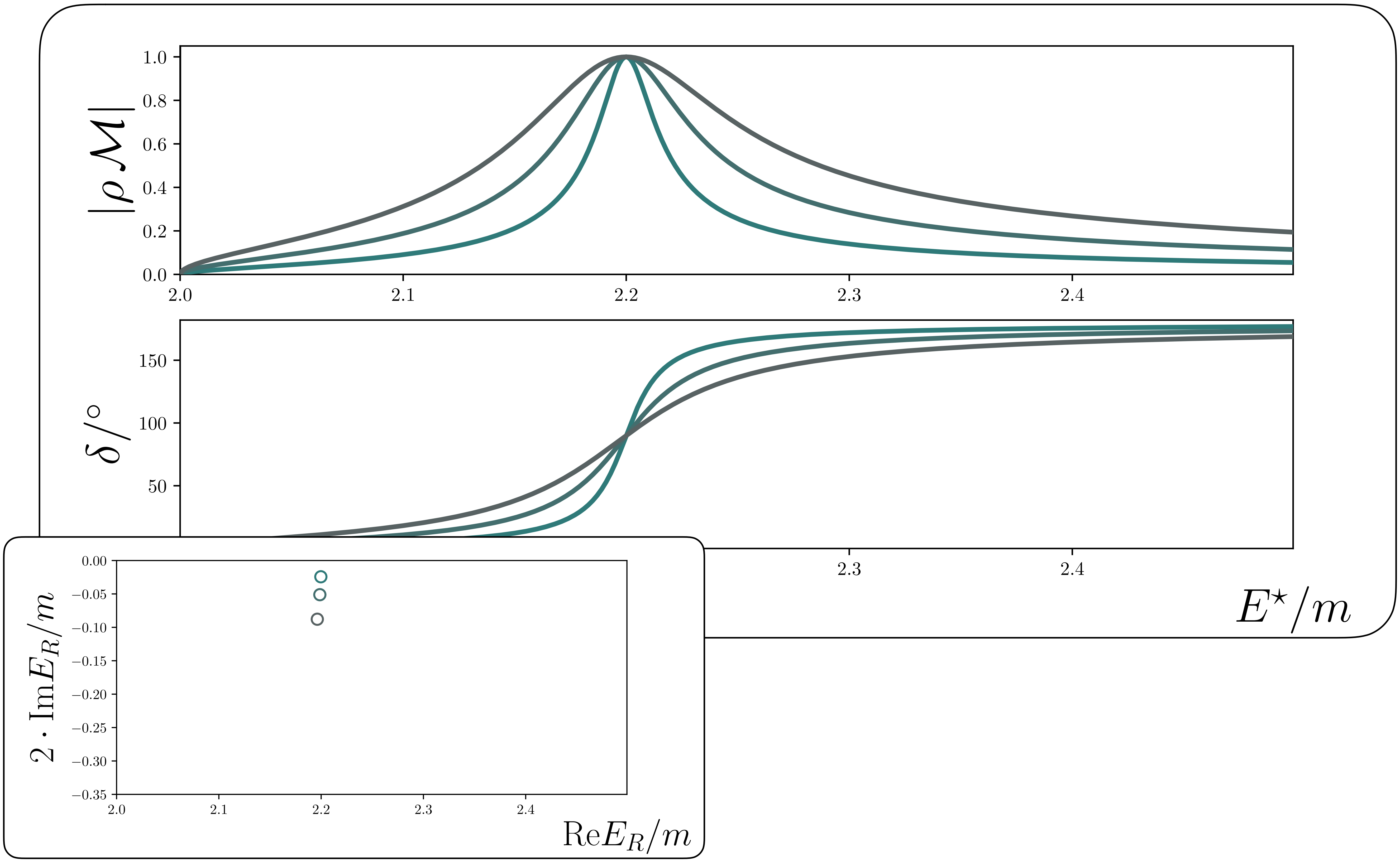
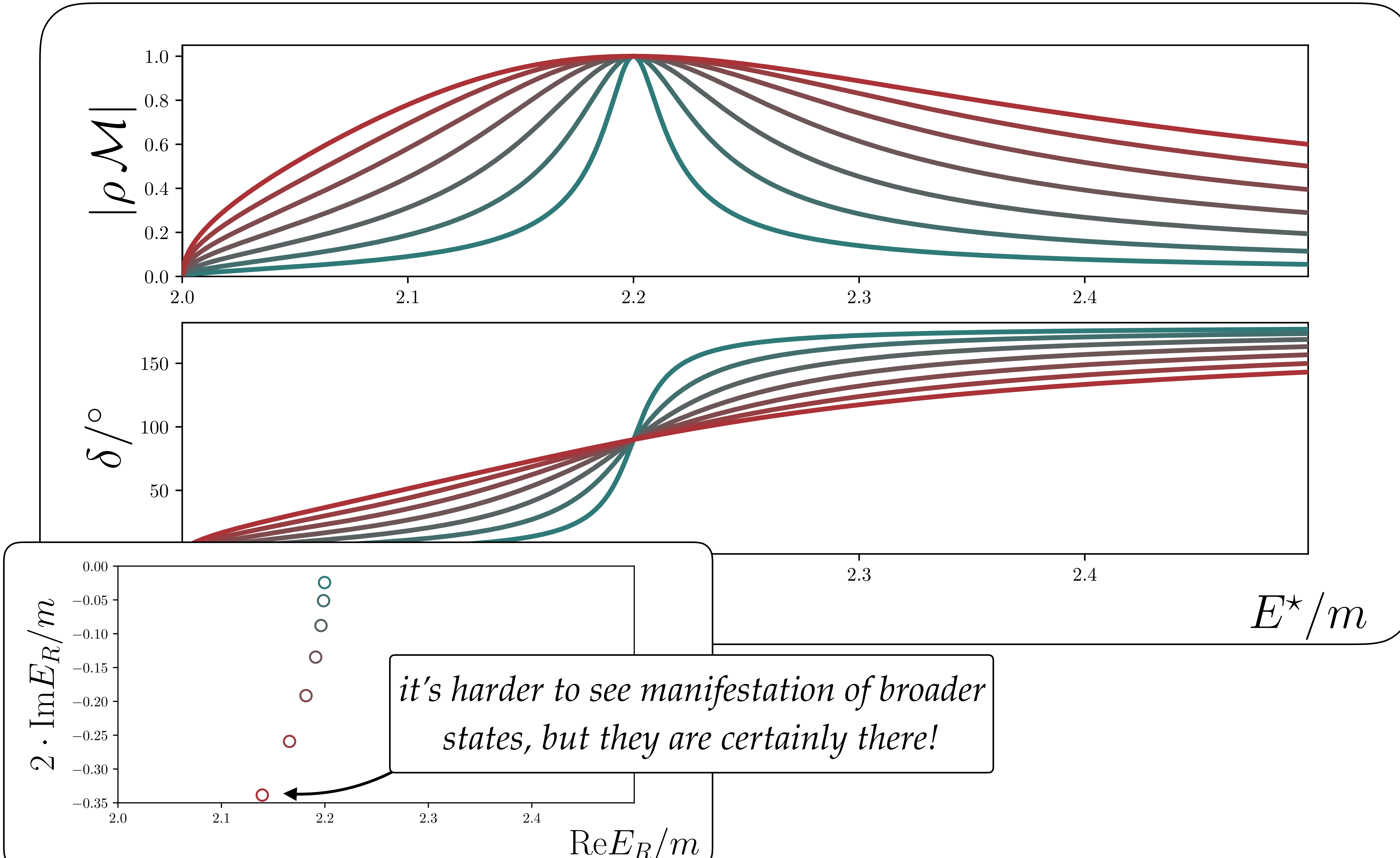


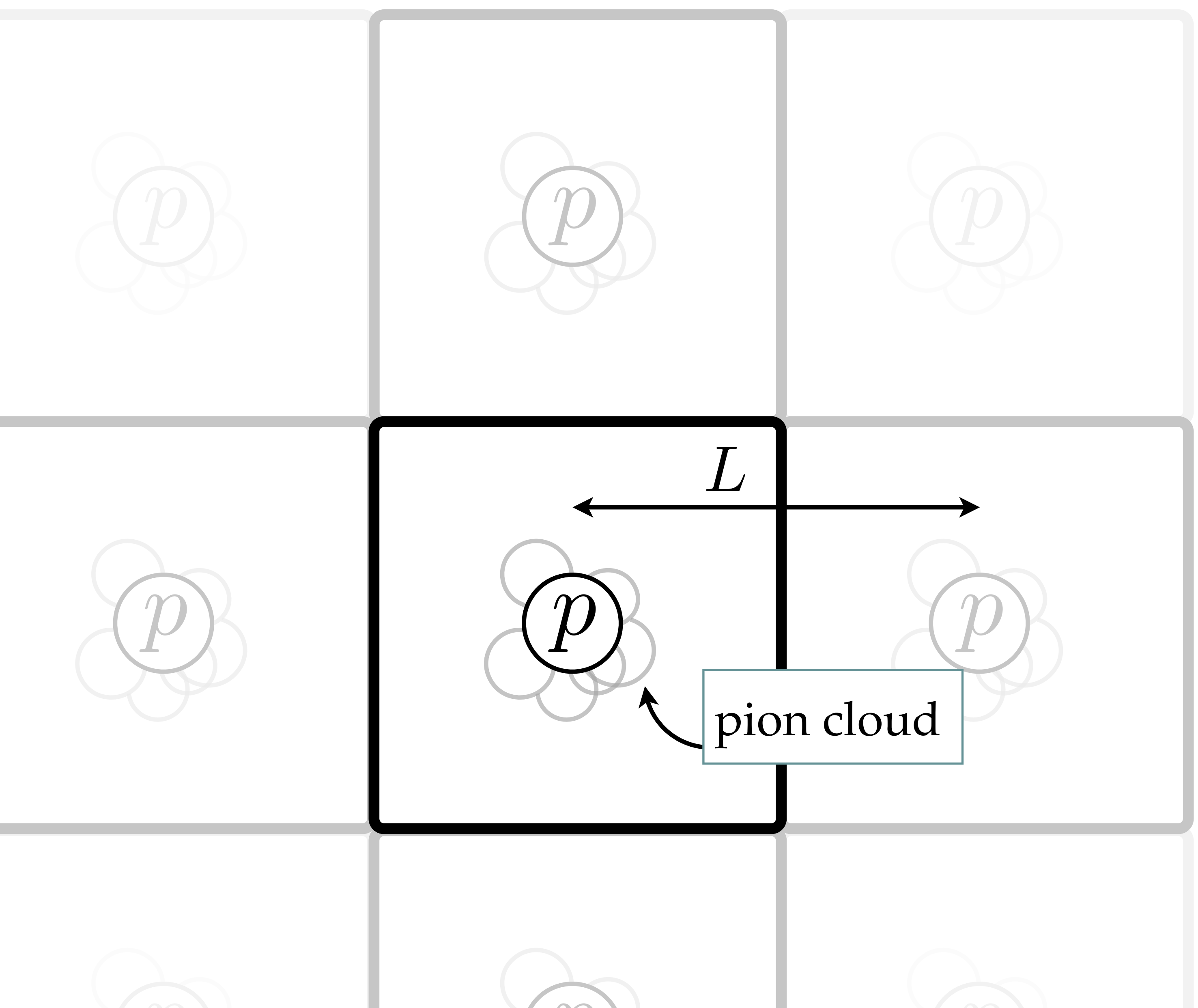
Two-body scattering and resonances

To build some intuition: $\mathcal{M} = \dfrac{\sin\delta}{\rho} e^{i\delta}$



Putting particles in a box

- Finite-volume arise from the interactions with mirror images
- Assuming L >> size of the hadrons ~ $1/m_\pi$
  - This is a purely infrared artifact
  - We can determine these artifact using hadrons are the degrees of freedom
- Note $m_\pi L$ is a natural parameter

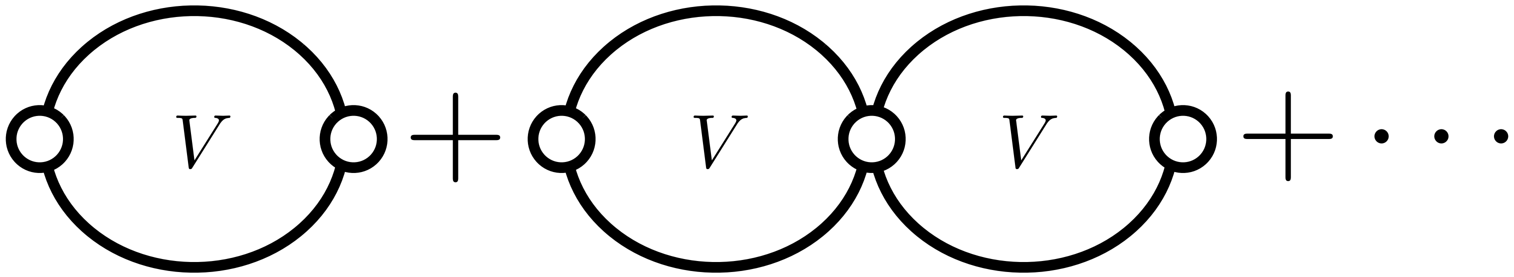


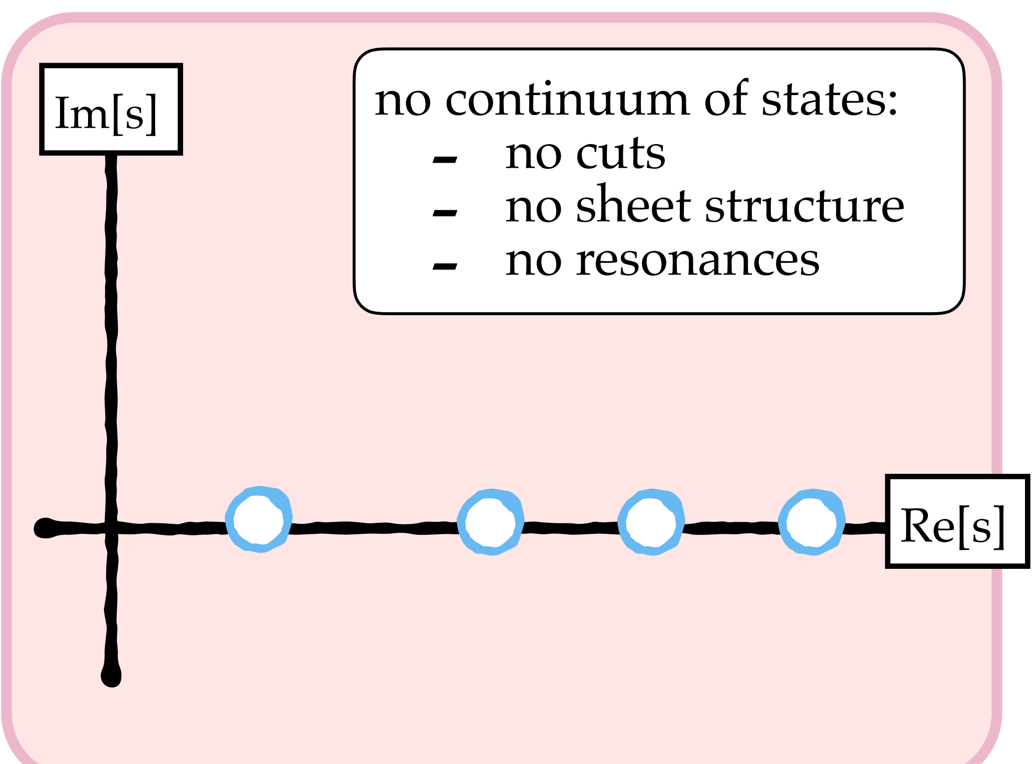
$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

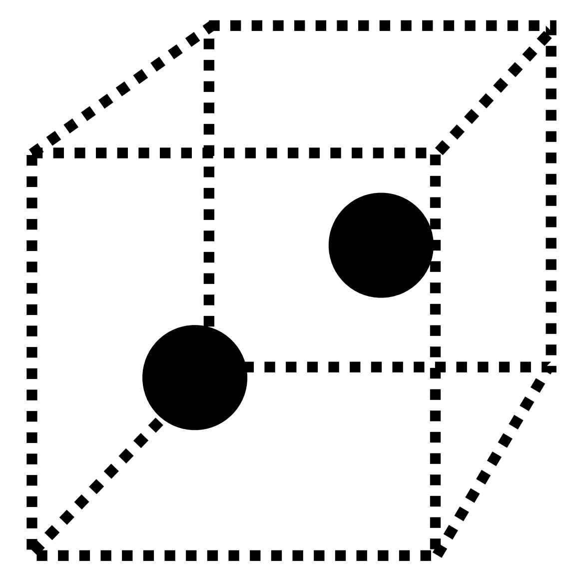
Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) =$$

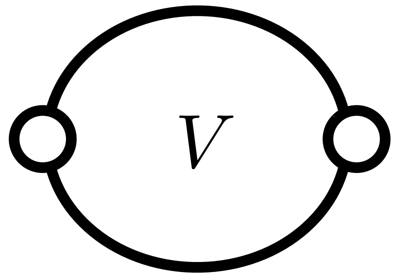


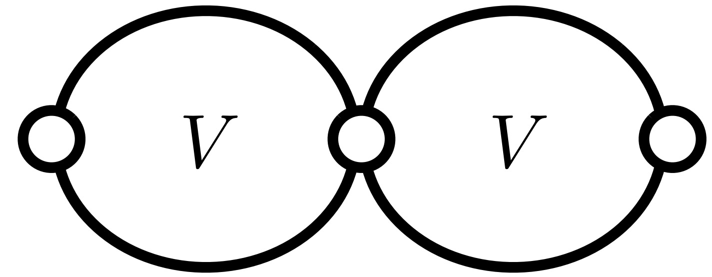




Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

$C_L^{2pt.}(P) =$ 

$+$ 

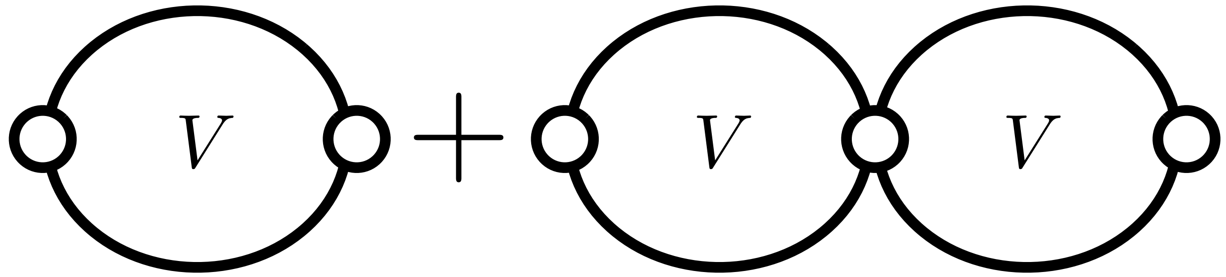
$+ \cdots$

$$= C_\infty(P) + \cdots$$

Two-particle in finite volume

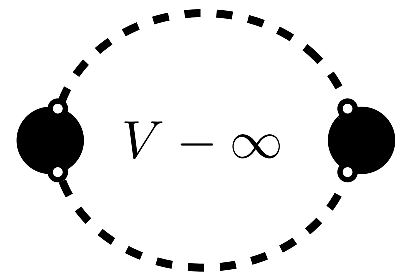
Consider the finite-volume two-particle correlator (E~$2m$):

$$C_L^{2pt.}(P) =$$

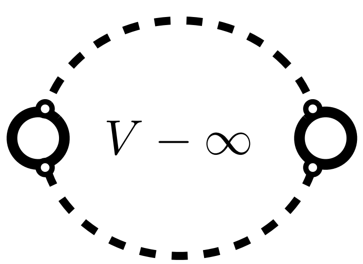


$$+\cdots$$

$$= C_\infty(P) +$$



$$+\cdots$$



$$= (iB)\left(\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3}\right]\frac{1}{(2\omega_k)^2}\,\frac{i}{E-2\omega_k+i\epsilon}\right)(iB)$$

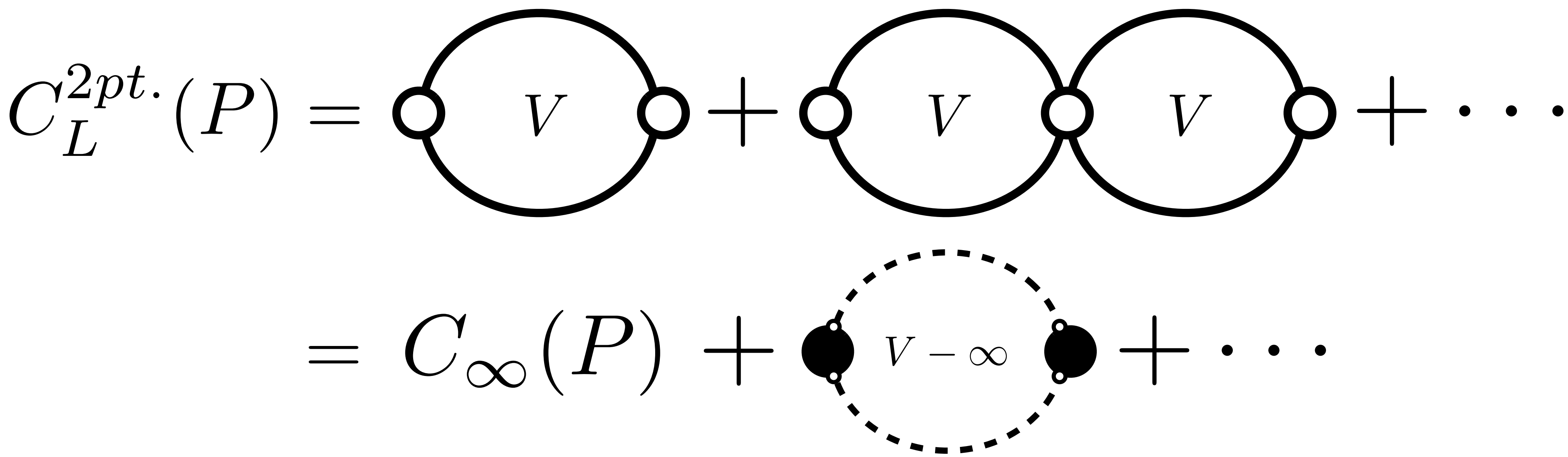
$$\equiv [iB]\, iF\, [iB]$$

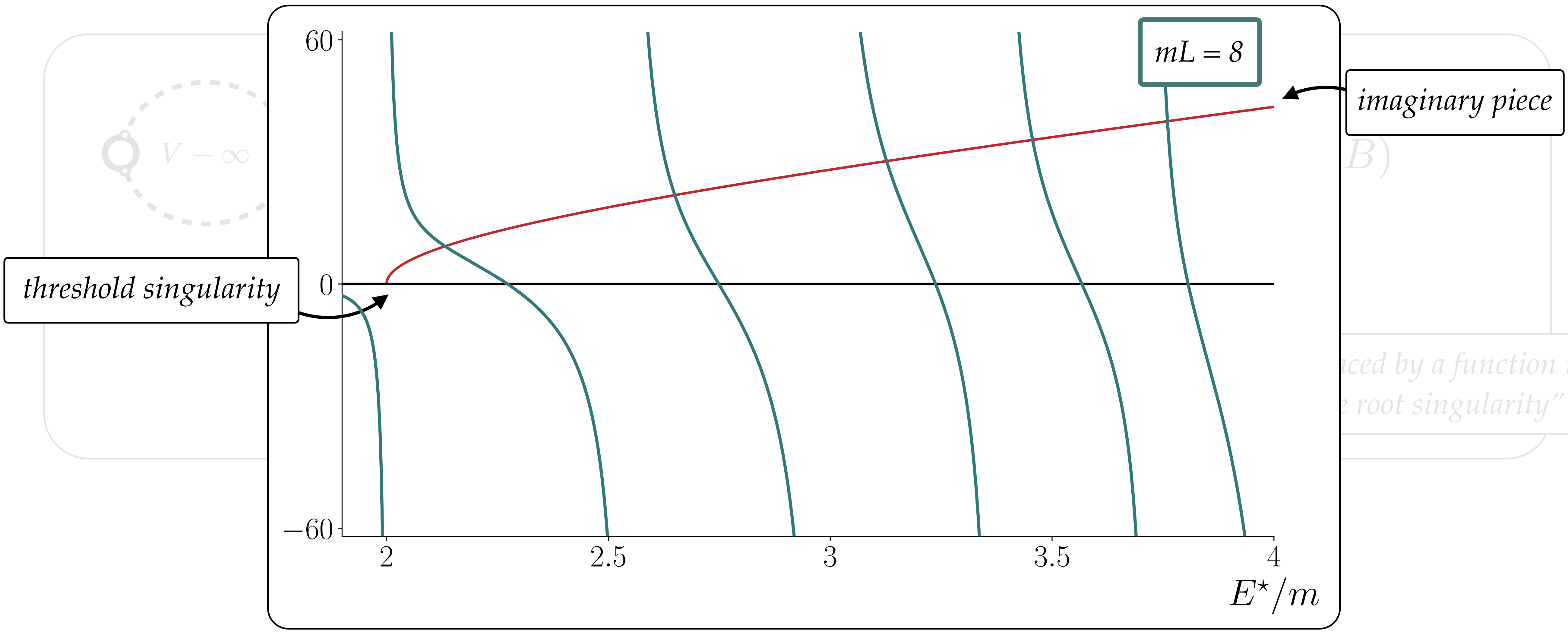
F replaces ρ

a simple square root singularity is replaced by a function that has both simples poles and the square root singularity

Two-particle in finite volume

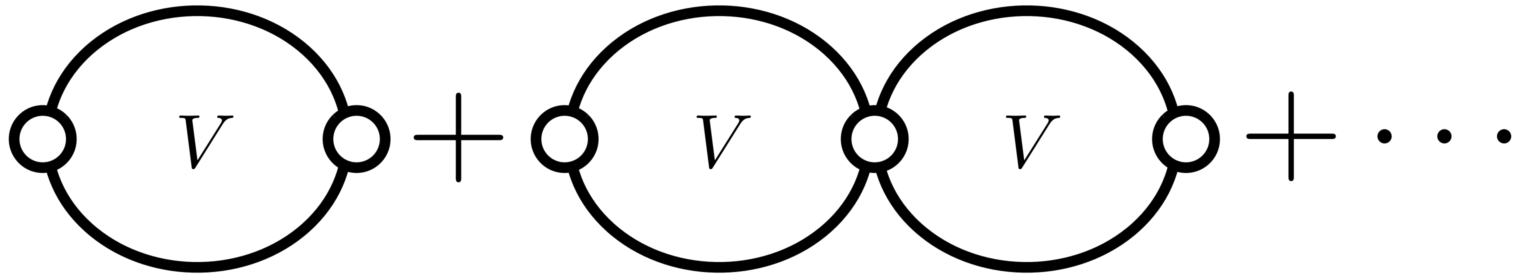
Consider the finite-volume two-particle correlator ($E \sim 2m$):

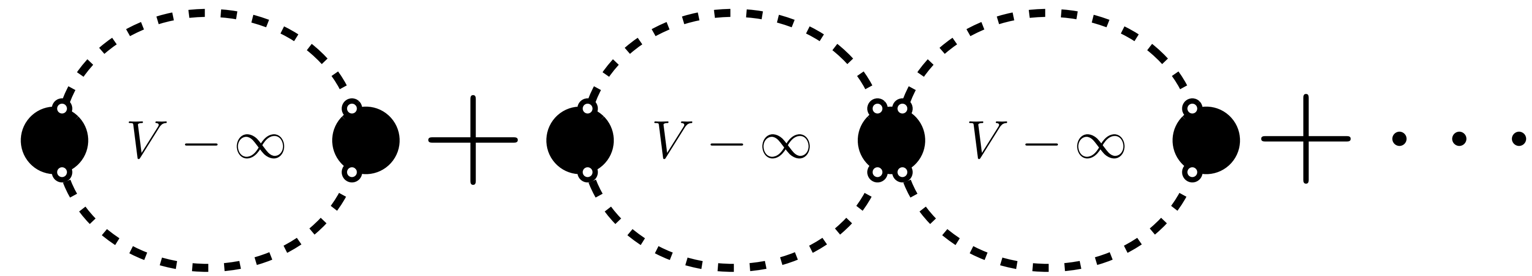




Two-particle in finite volume

Consider the finite-volume two-particle correlator (E~$2m$):

$C_L^{2pt.}(P) =$ 

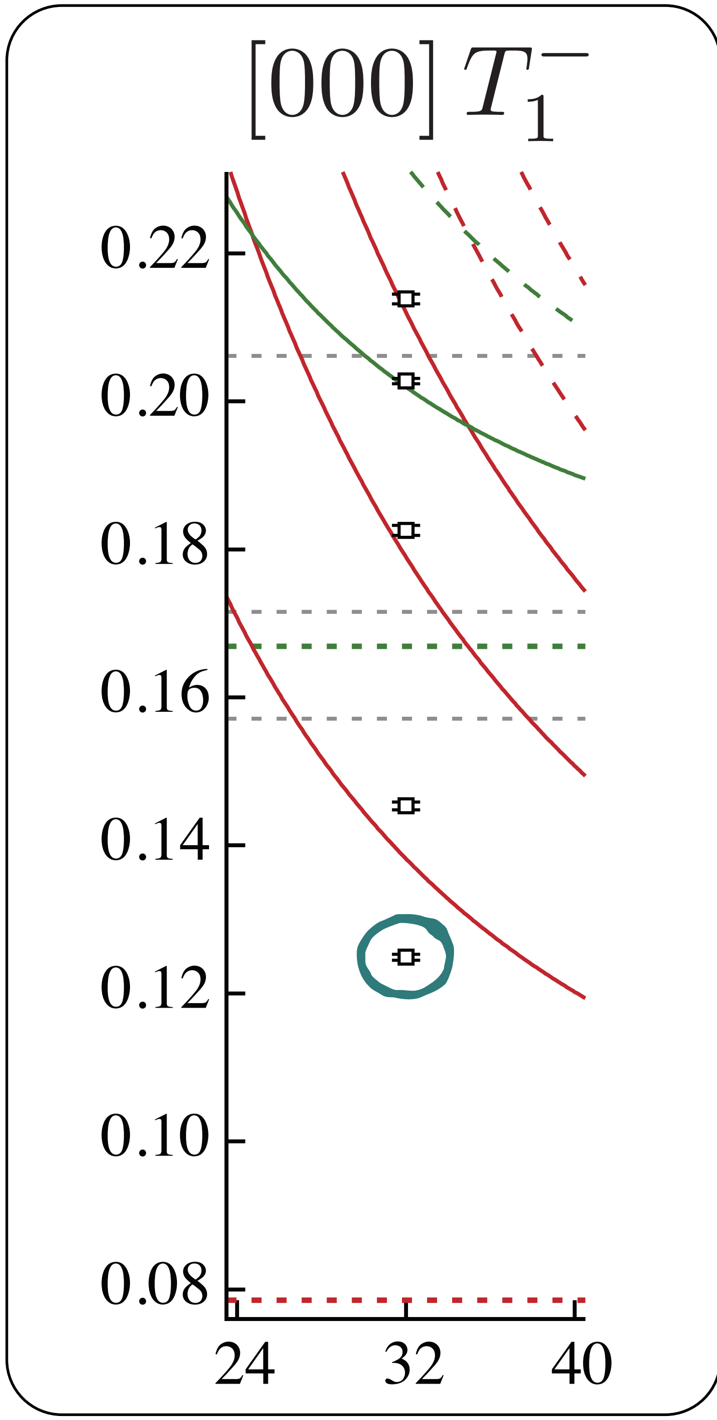
$= C_\infty(P) +$ 

$$= \text{“smooth”} + A\frac{i}{F^{-1}+\mathcal{M}}B^\dagger$$

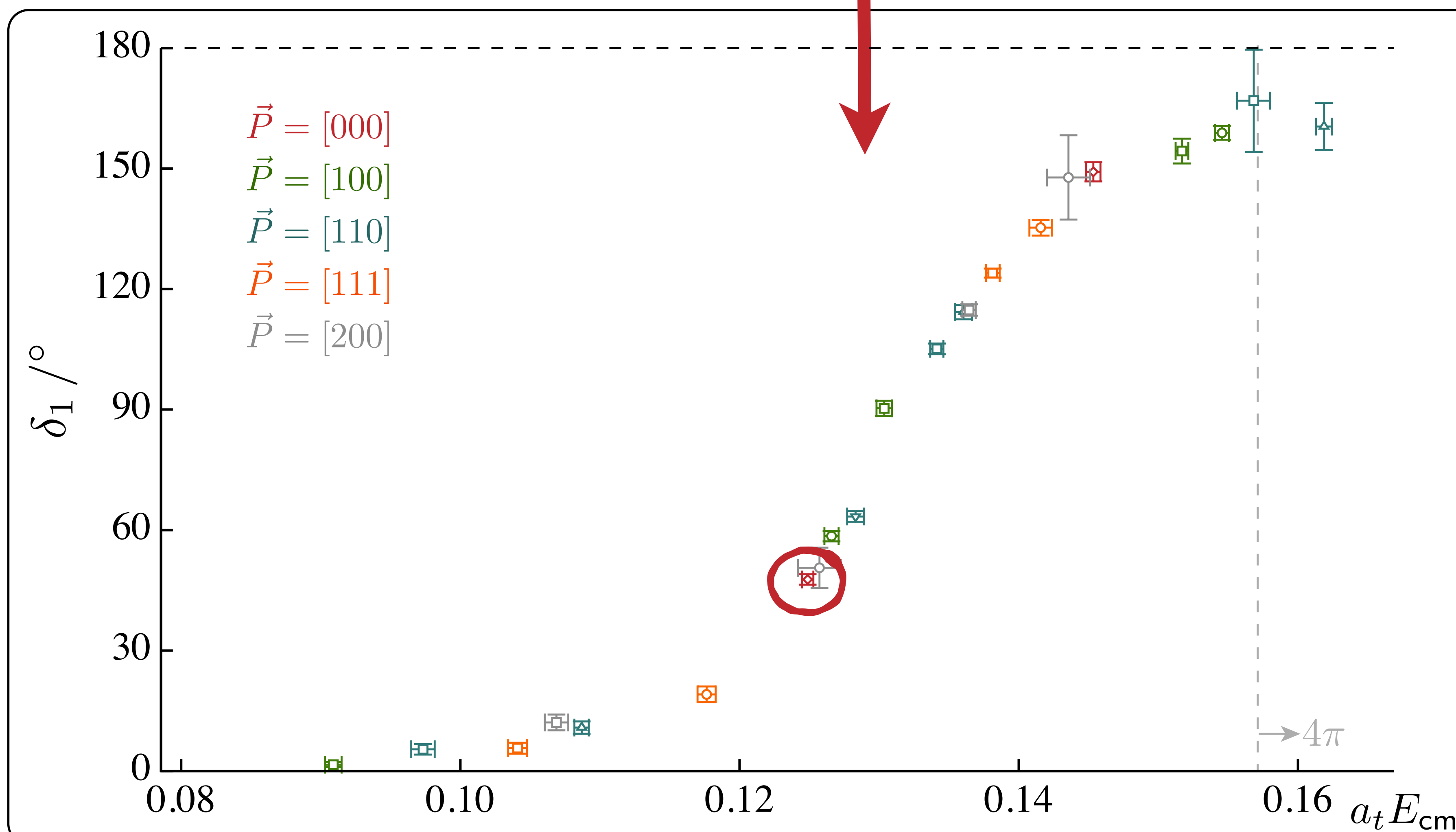
poles satisfy: $\det[F^{-1}(P,L)+\mathcal{M}(P)]=0$

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

$\pi\pi$ Spectrum - (I=1 channel)



$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$

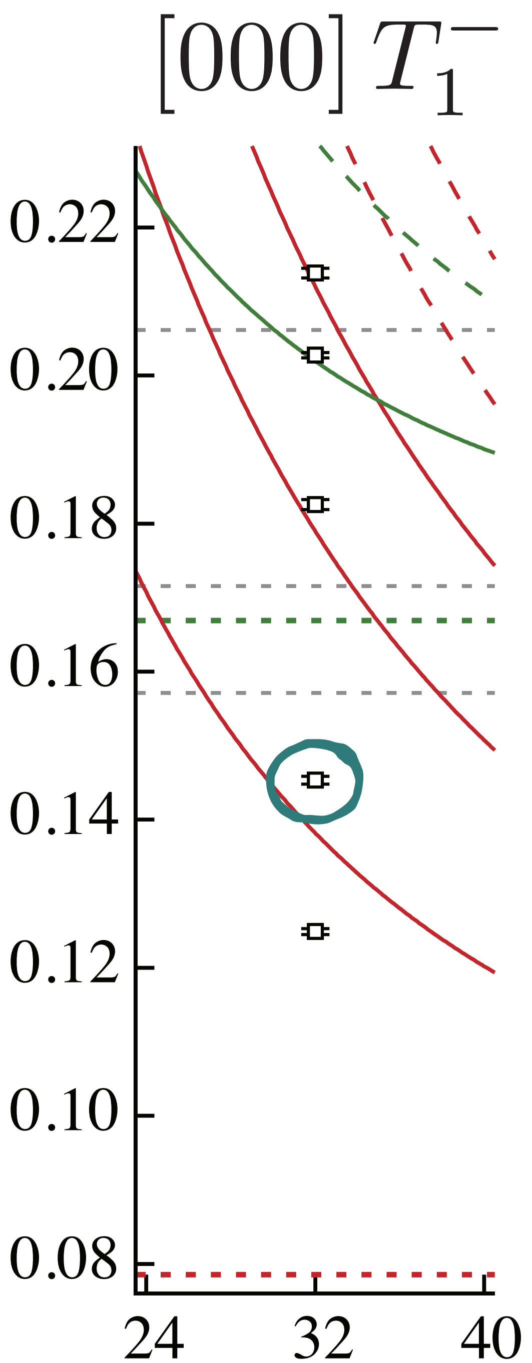


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

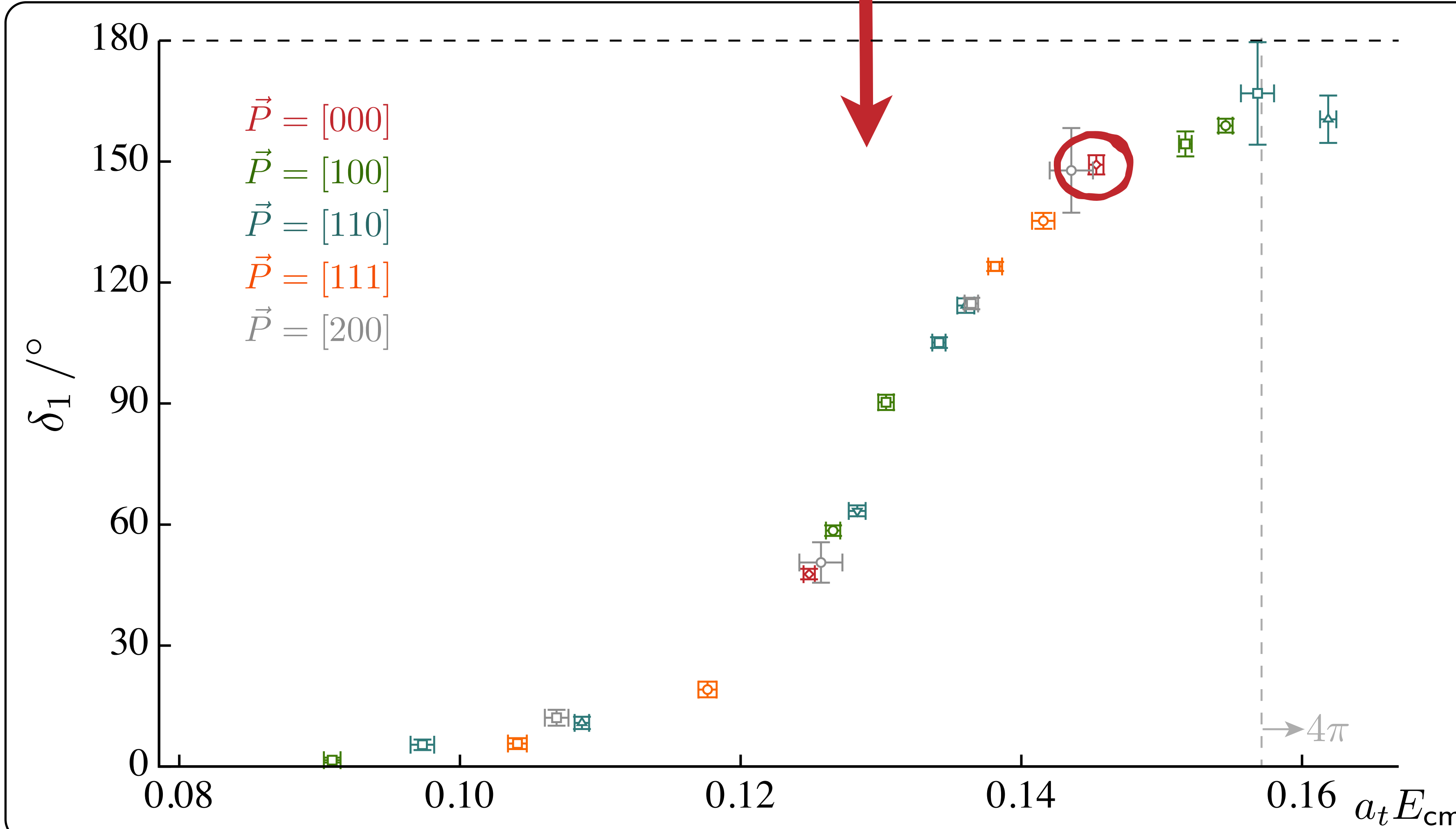
$m_\pi \sim 240$ MeV

Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ Spectrum - (I=1 channel)



$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

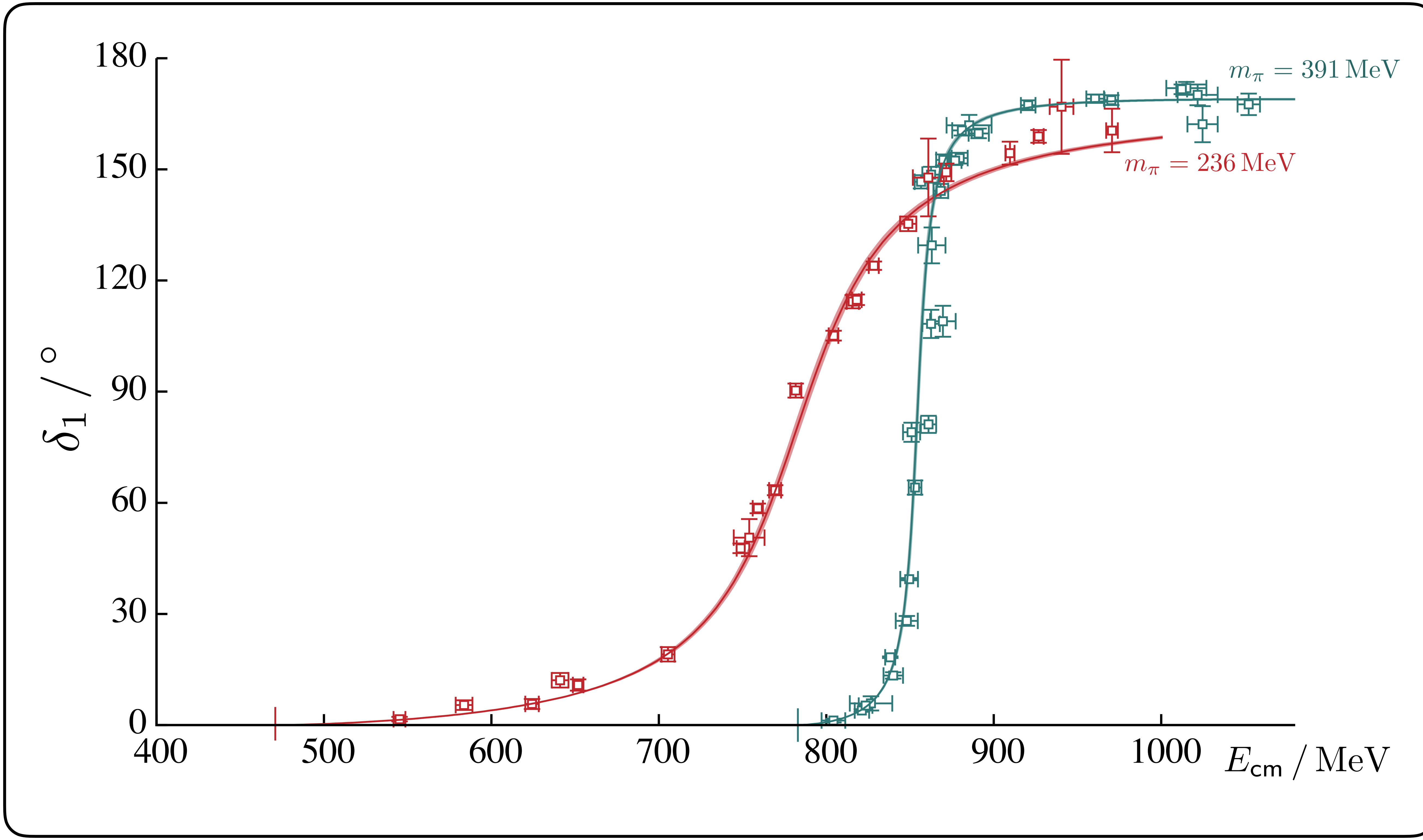


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

$m_\pi \sim 240$ MeV

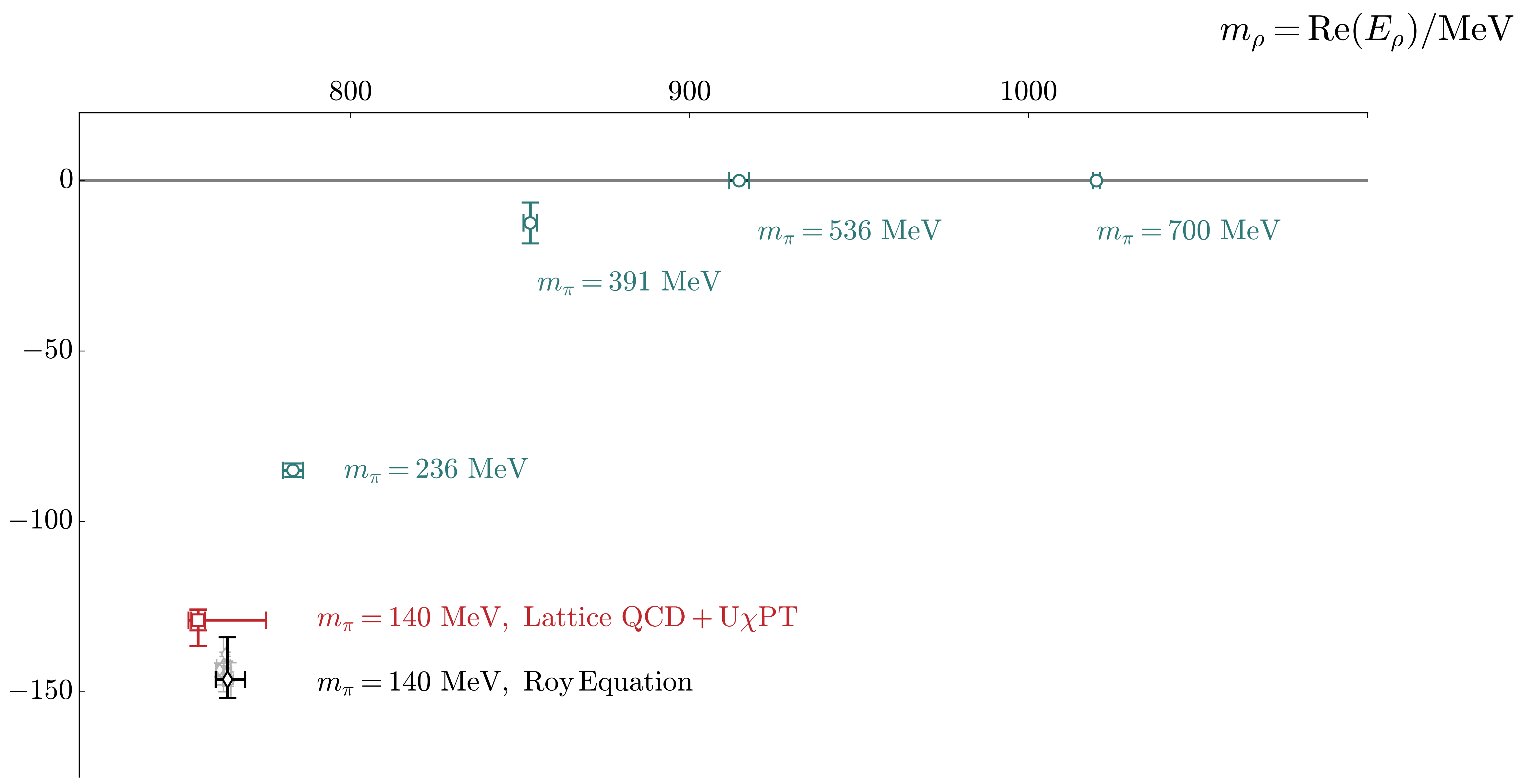
Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering - (I=1 channel)



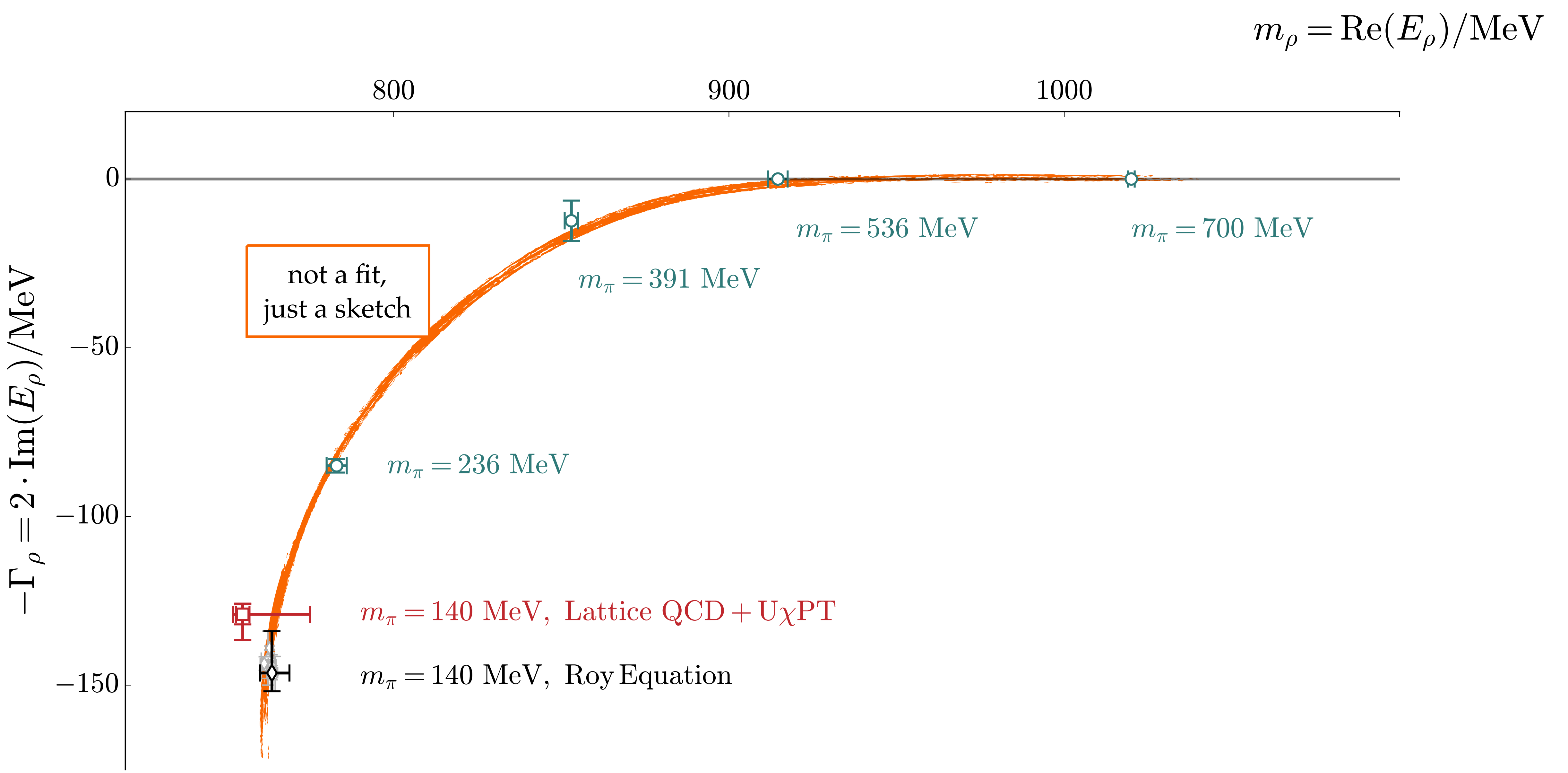
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

The ρ *vs* m_π



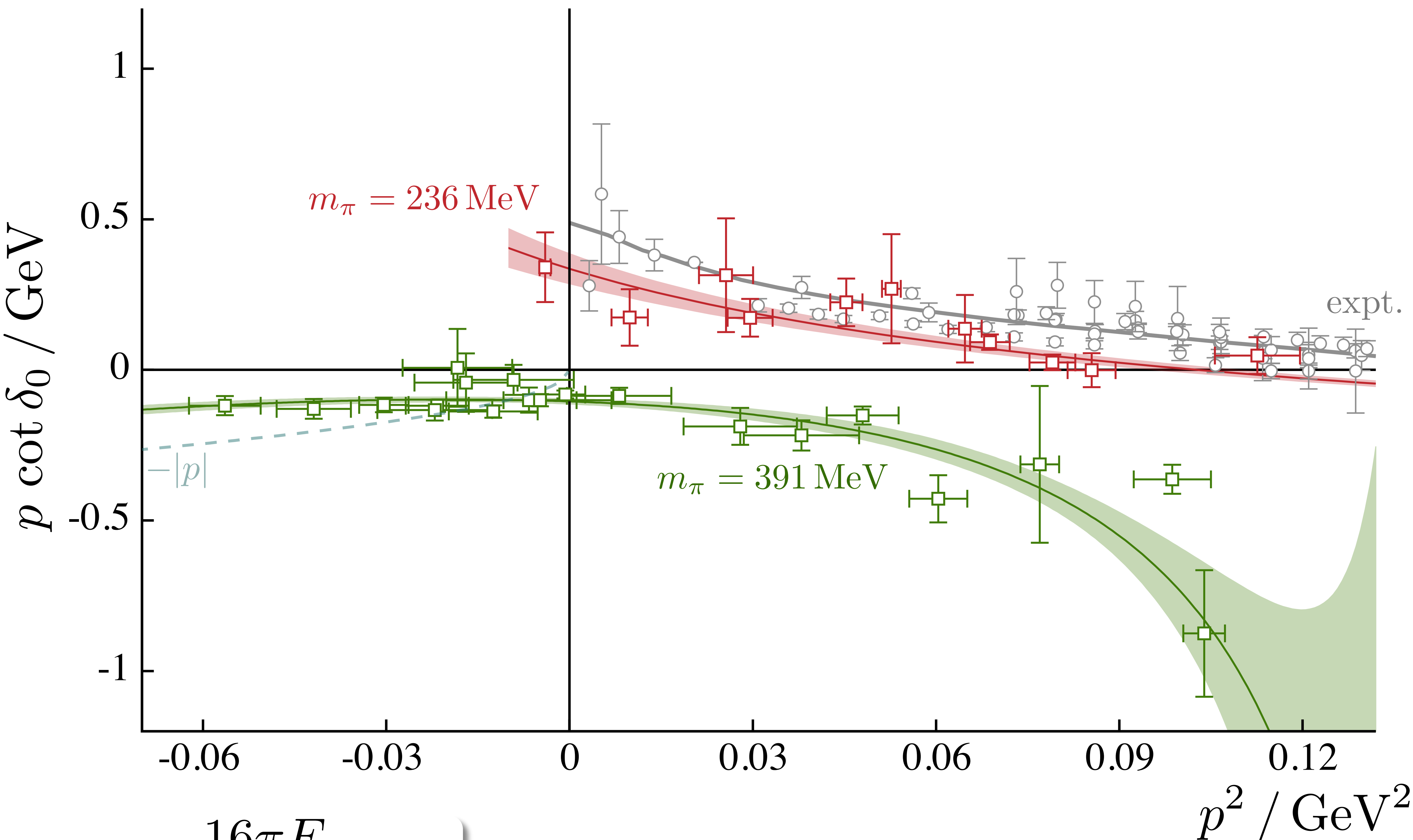
Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

The ρ *vs* m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

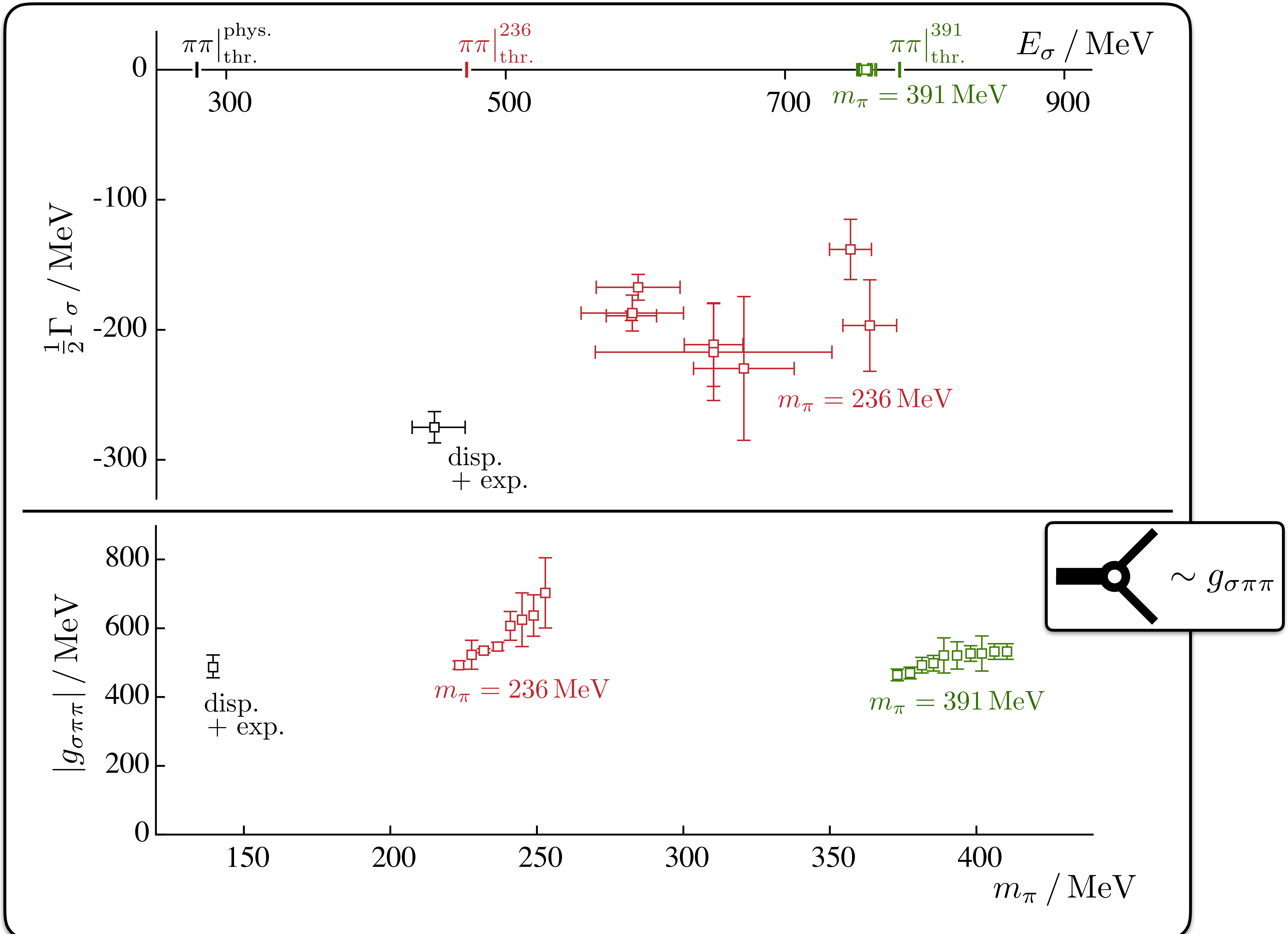
$\pi\pi$ scattering - (I=0 channel)



$$\mathcal{M}_0 = \frac{16\pi E_{\rm cm}}{p \cot \delta_0 - ip}$$

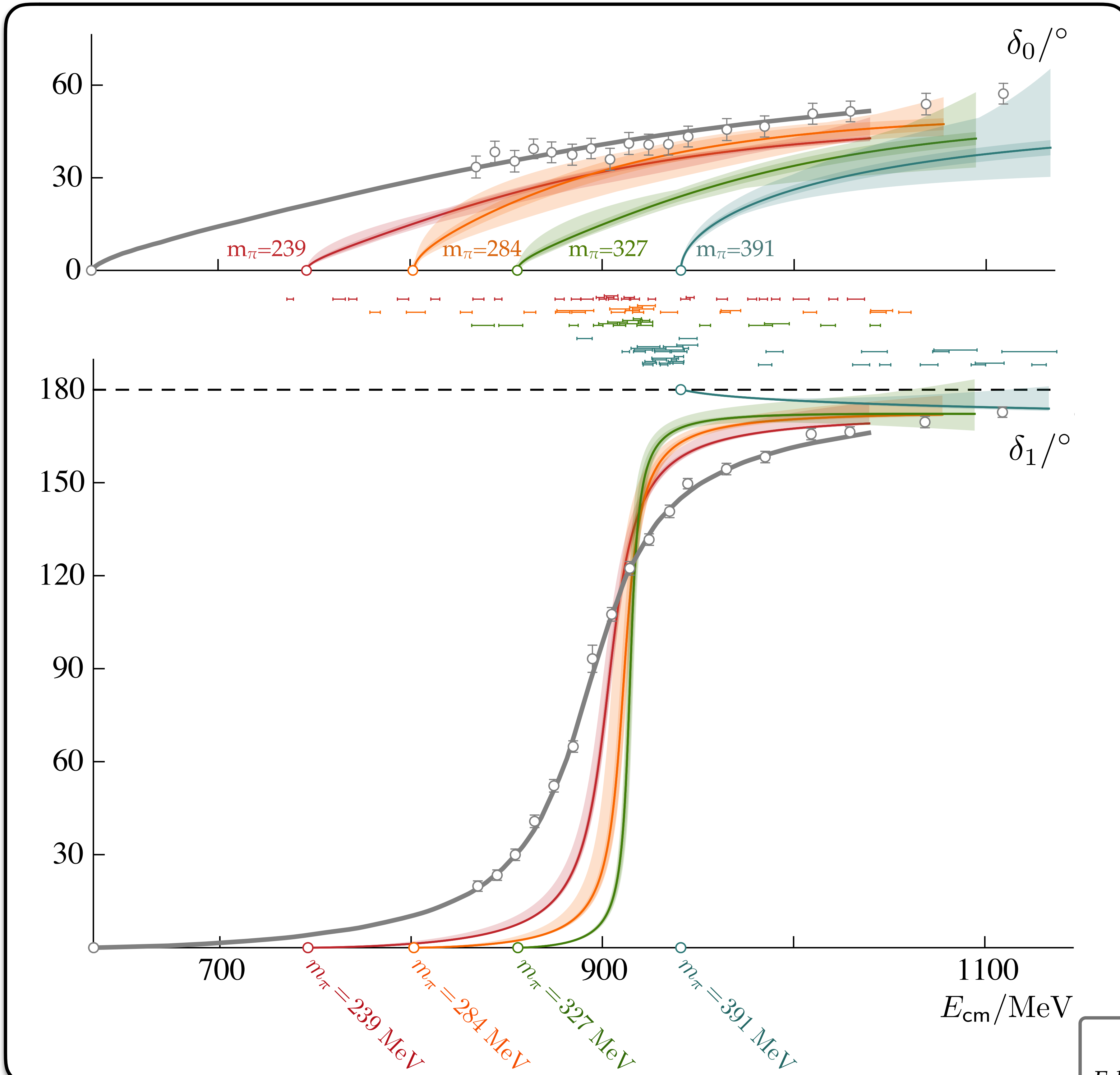
RB, Dudek, Edwards, Wilson - PRL (2017)

The σ vs m_π



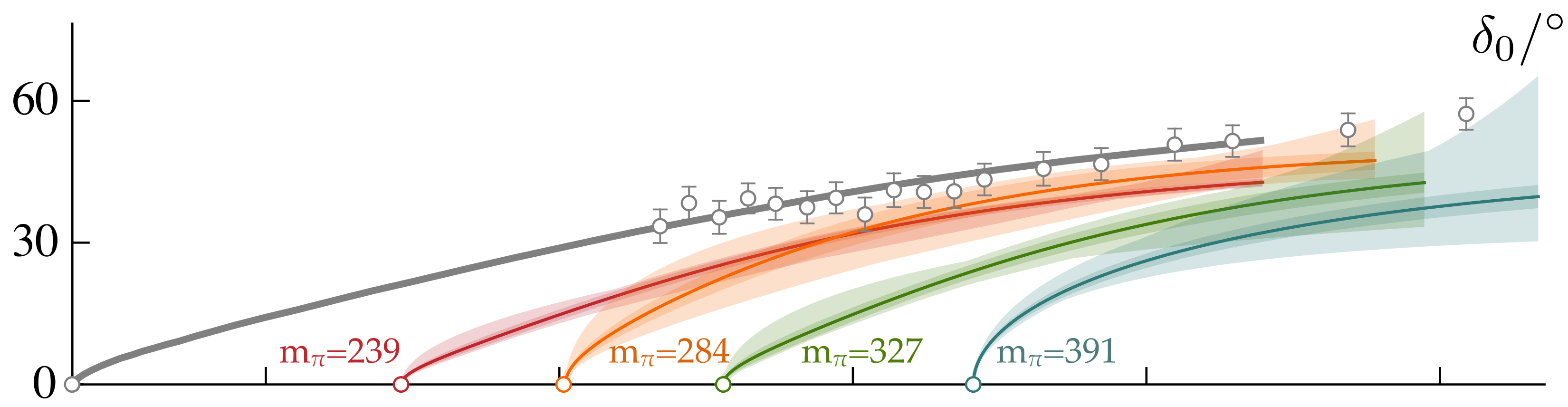
$\pi\pi|^{\text{phys.}}_{\text{thr.}}$
$\pi\pi|^{236}_{\text{thr.}}$
$\pi\pi|^{391}_{\text{thr.}}$
E_σ / MeV
0
300
500
700
900
$m_\pi = 391$ MeV
-100
-200
-300
$\frac{1}{2}\Gamma_\sigma$ / MeV
$m_\pi = 236$ MeV
disp.
+ exp.
800
600
400
200
0
$|g_{\sigma\pi\pi}|$ / MeV
disp.
+ exp.
$m_\pi = 236$ MeV
$m_\pi = 391$ MeV
$\sim g_{\sigma\pi\pi}$
150
200
250
300
350
400
m_π / MeV

πK scattering - (I=1/2 channel)



Wilson, RB, Dudek, Edwards, & Thomas (2019)

πK scattering - (I=1/2 channel)

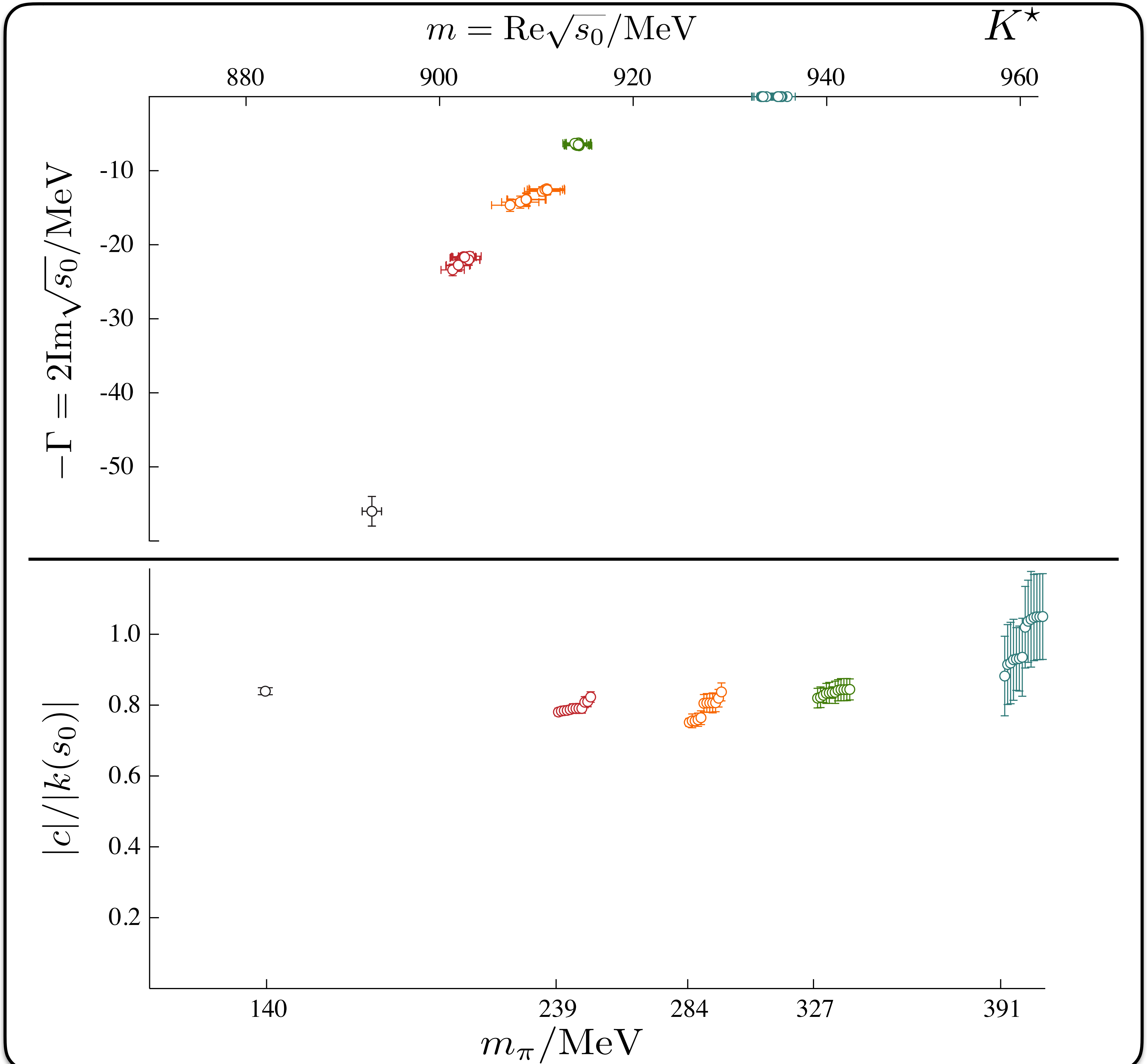


too broad to say anything definitive at this point…

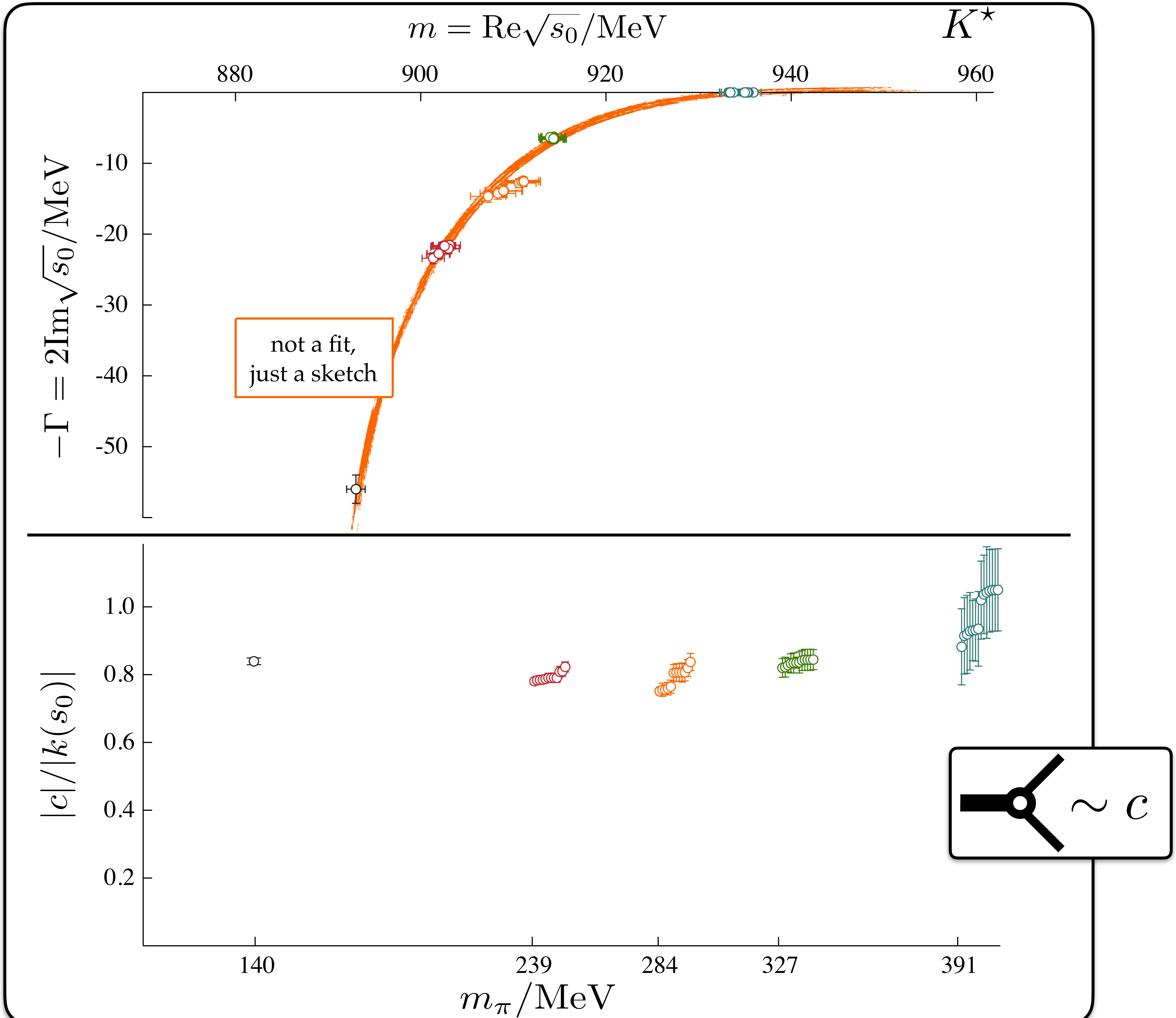
not unlike the σ

Wilson, RB, Dudek, Edwards, & Thomas (2019)

The K^* vs m_π


$m = \mathrm{Re}\sqrt{s_0}/\mathrm{MeV}$
$K^\star$
880
900
920
940
960
$-\Gamma = 2\mathrm{Im}\sqrt{s_0}/\mathrm{MeV}$
-10
-20
-30
-40
-50
$|c|/|k(s_0)|$
1.0
0.8
0.6
0.4
0.2
140
239
284
327
391
m_π/MeV

The K^* vs m_π



$m = \mathrm{Re}\sqrt{s_0}$/MeV
$K^\star$
880
900
920
940
960
$-\Gamma = 2\mathrm{Im}\sqrt{s_0}$/MeV
-10
-20
-30
-40
-50
not a fit,
just a sketch
$|c|/|k(s_0)|$
1.0
0.8
0.6
0.4
0.2
140
239
284
327
391
m_π/MeV
$\sim c$

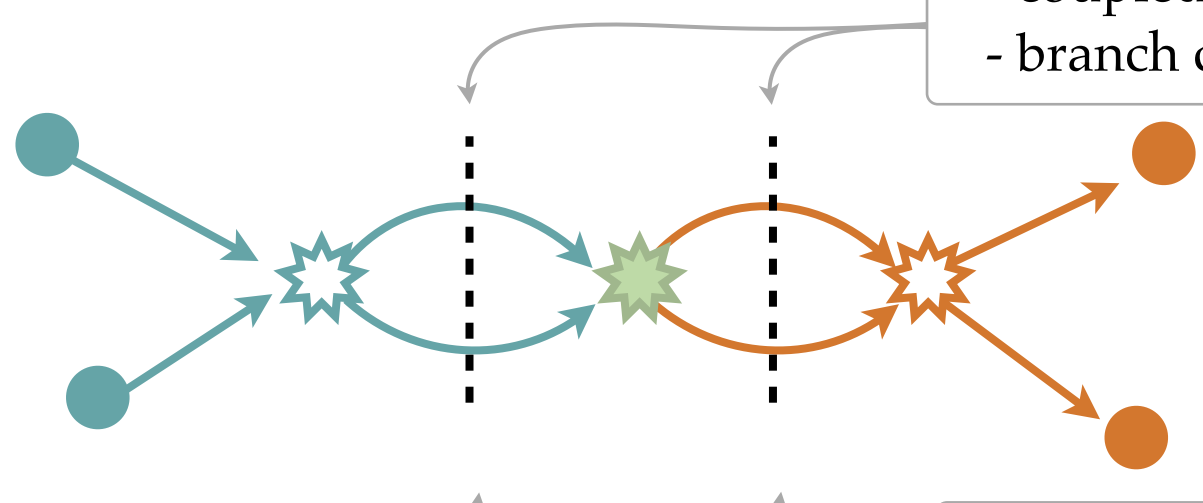
Multi-channel systems - the cutting edge!

- Above $2m_K$, there is not a one-to-one correspondence

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)

infinite volume:
- coupled, scattering amplitude
- branch cuts



finite volume:
- power-law finite-volume effects

Multi-channel systems - the cutting edge!

- Above $2m_K$, there is not a one-to-one correspondence

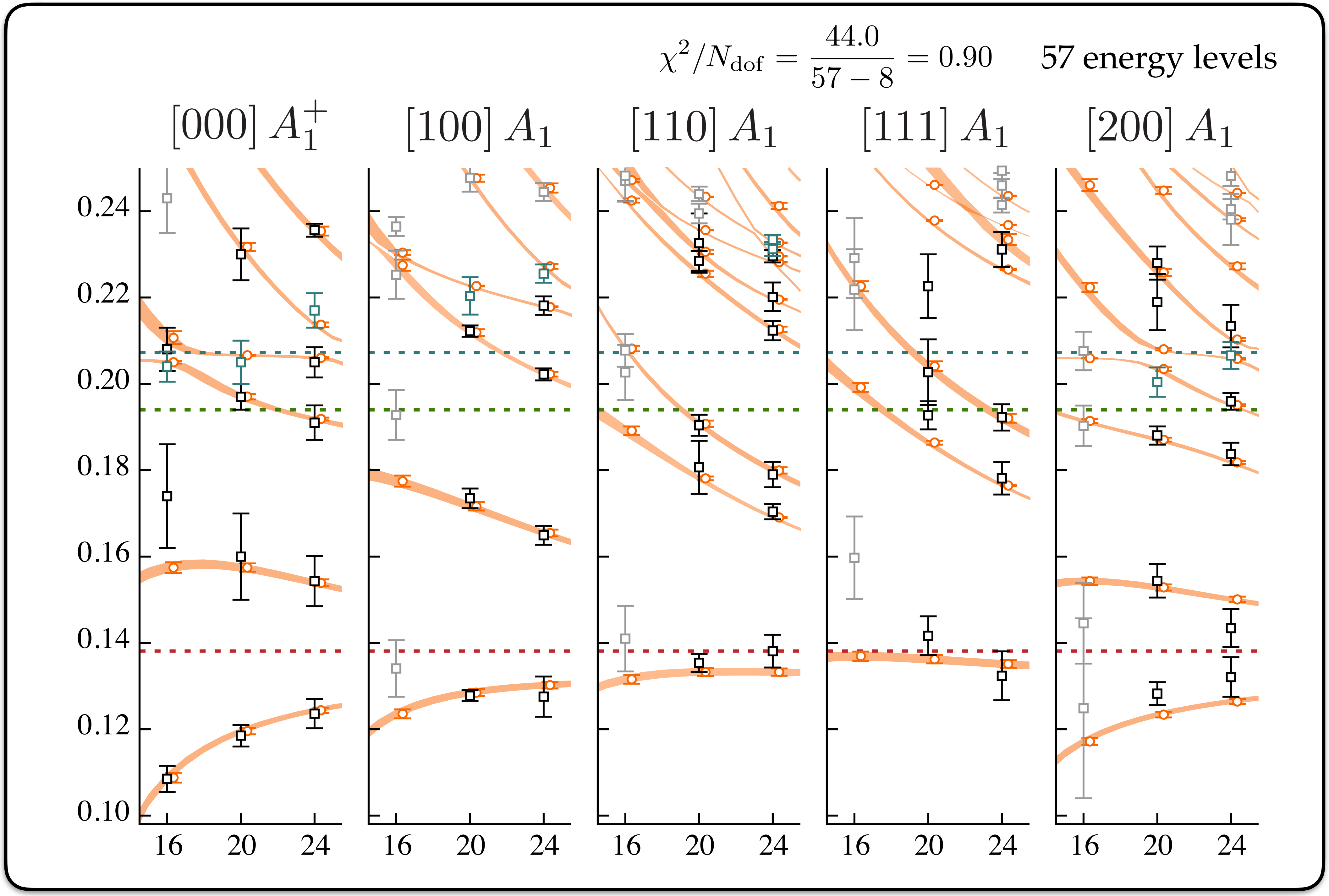
$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)

- In general, must constrain $(1/2)\,[N^2 + N]$ functions of energy
- Need that many energy levels at the same energy
- Alternatively, parametrize scattering amplitude and do a global fit

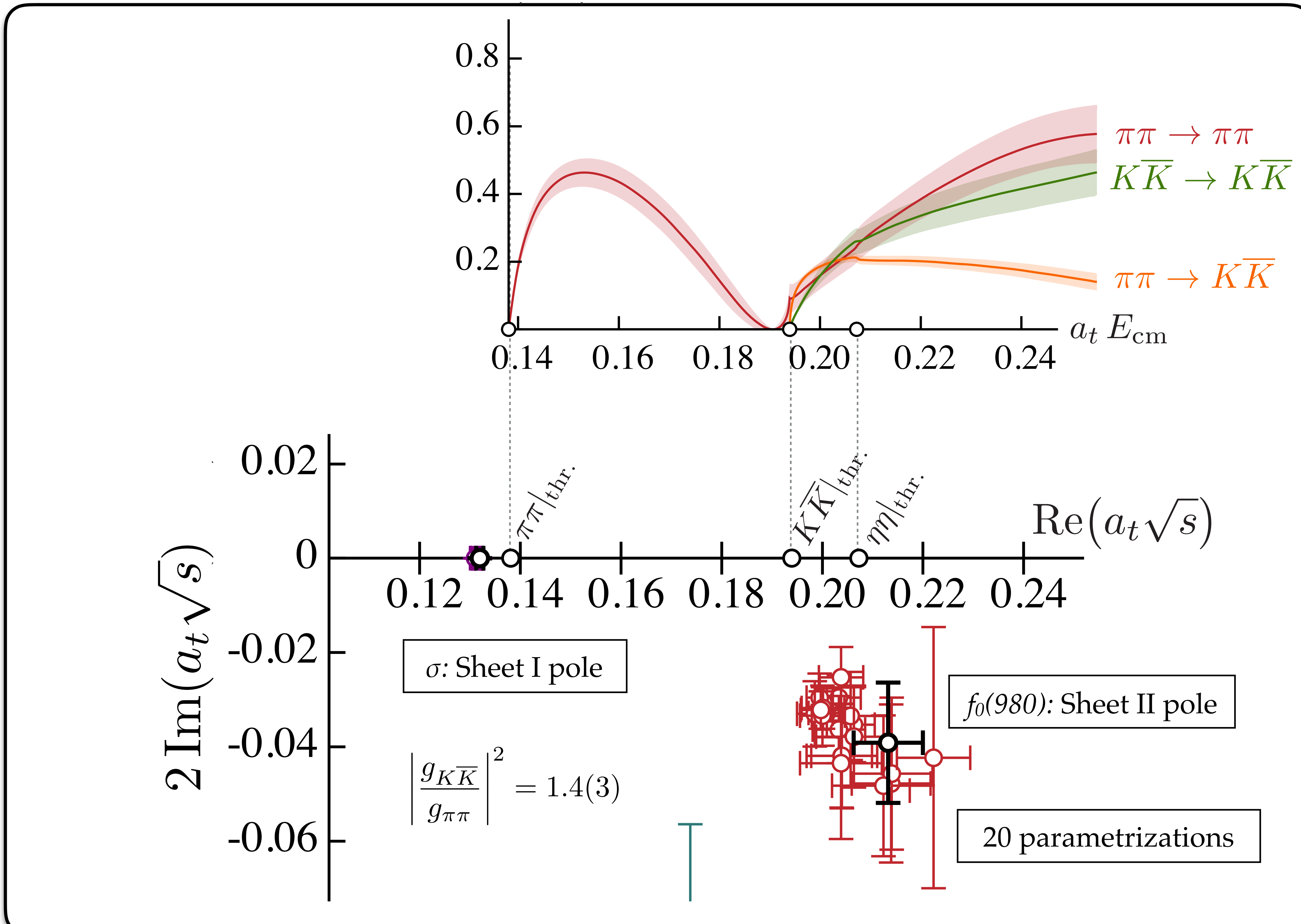
Coupled-channels analysis

- S-wave above $2m_\pi$, $2m_K$, and $2m_\eta$
- Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$



Scalar poles: σ and $f_0(980)$

- Near poles: $\mathcal{M} \sim \dfrac{g^2}{s_0 - s}$



Scalar poles: σ and $f_0(980)$

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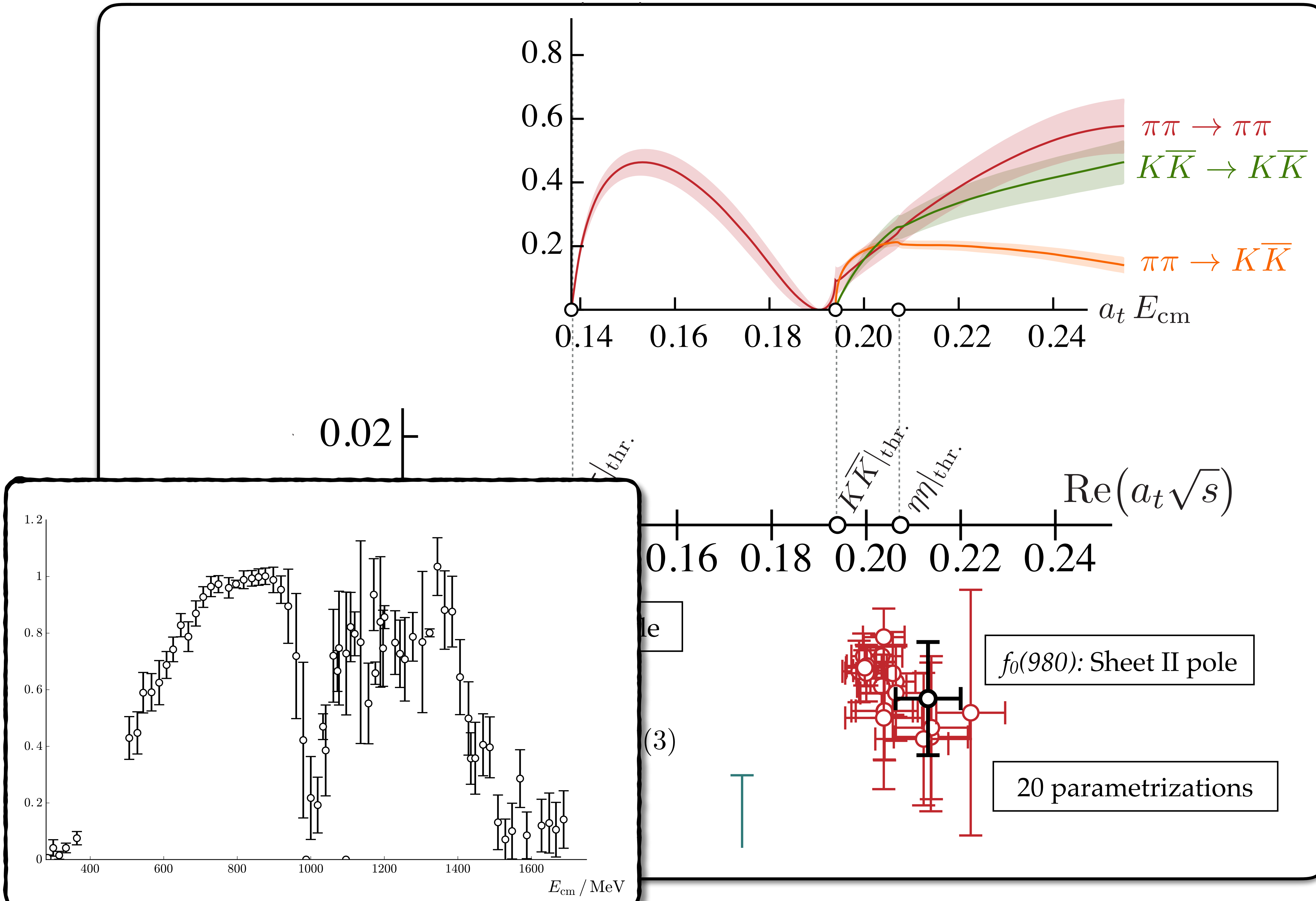
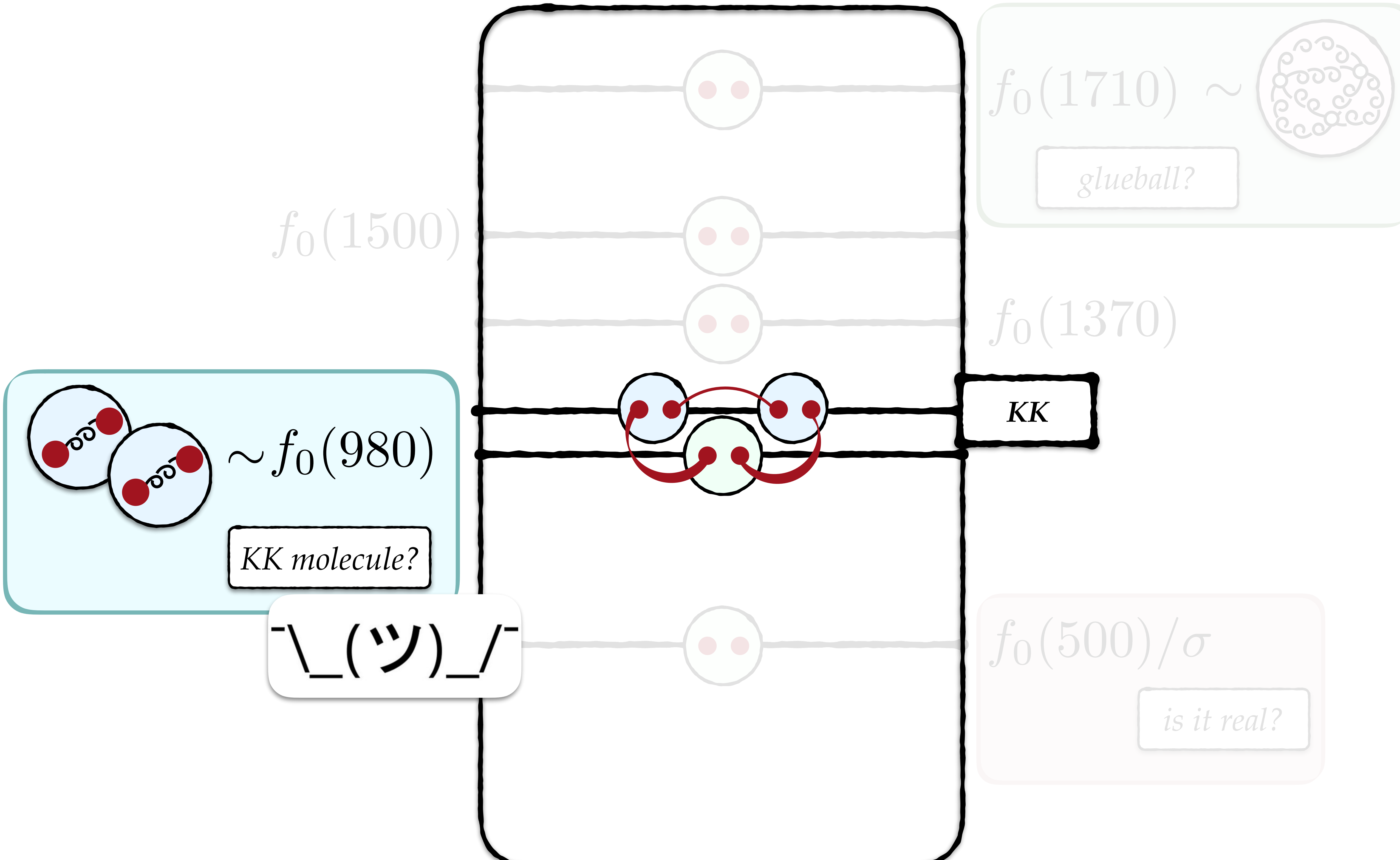
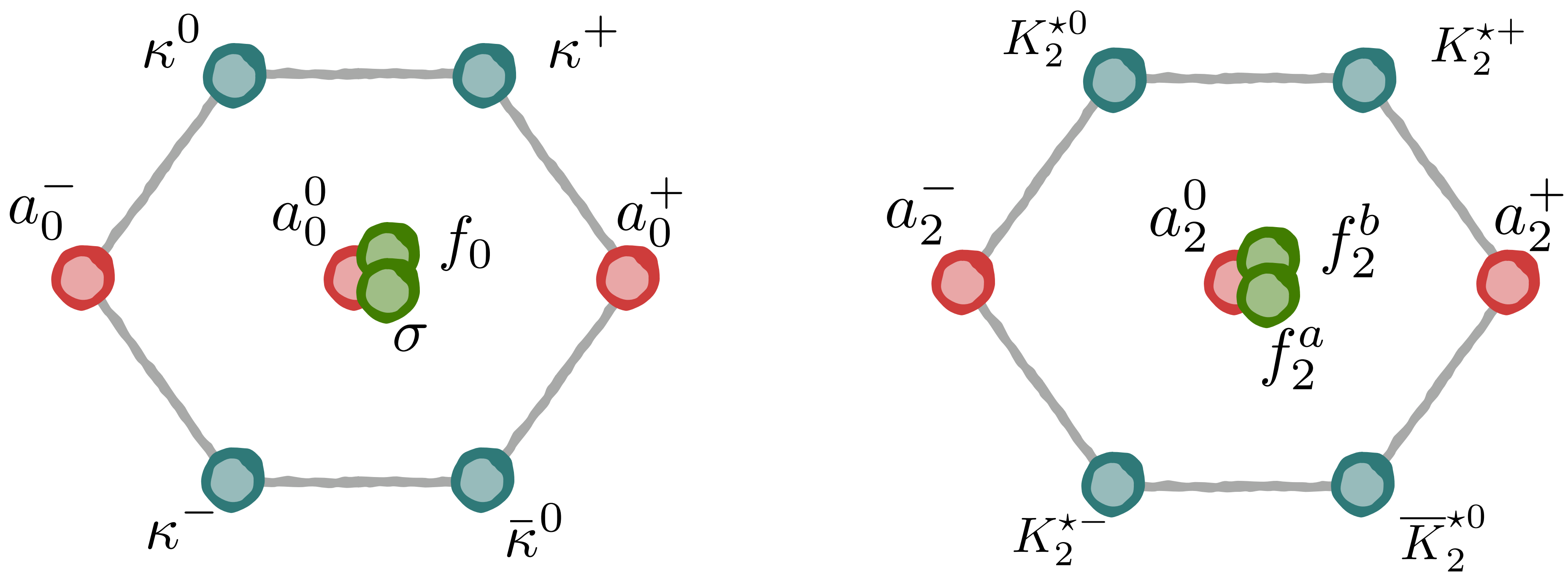


The isoscalar, scalar sector



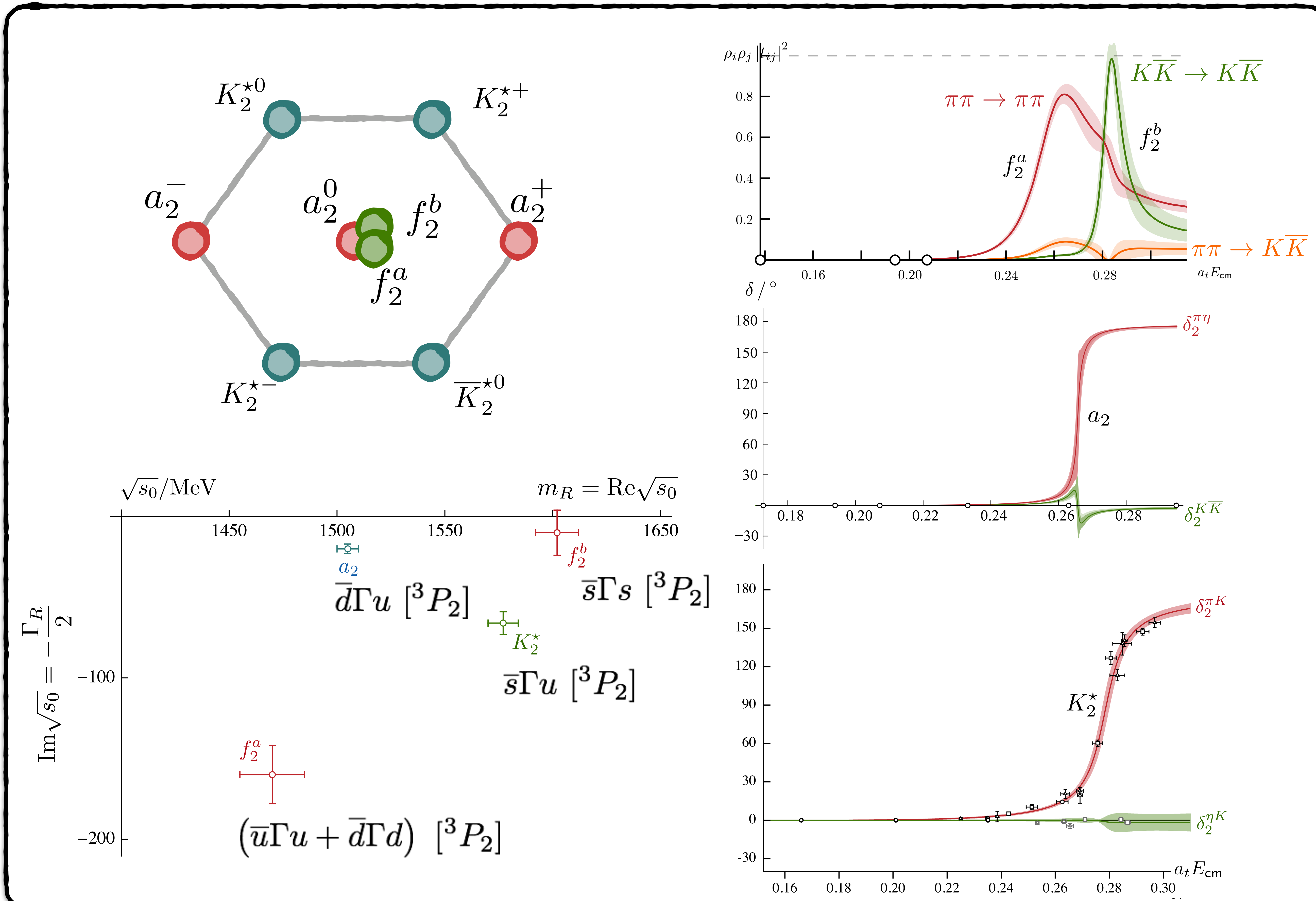
Tensor and scalar nonets

- First complete determination of the scalar and tensor nonets from LQCD :

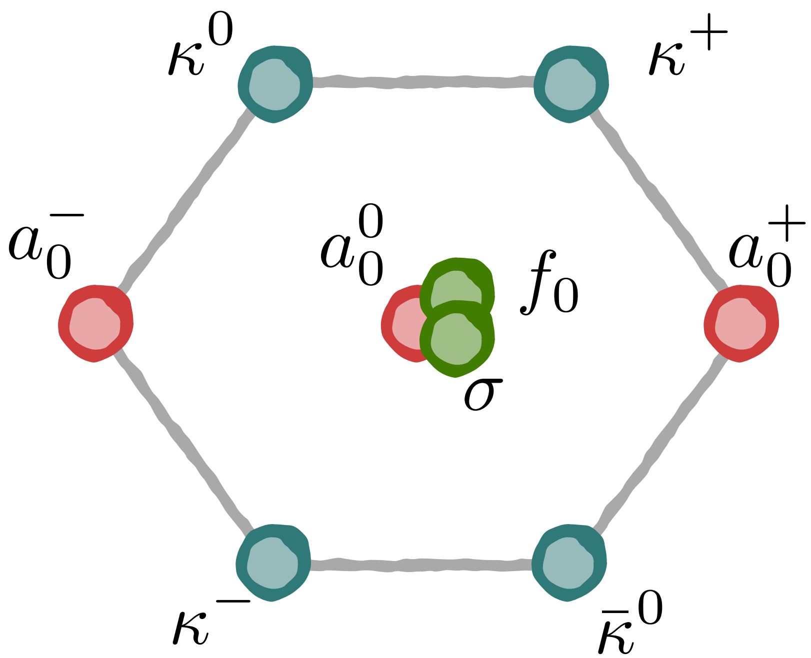


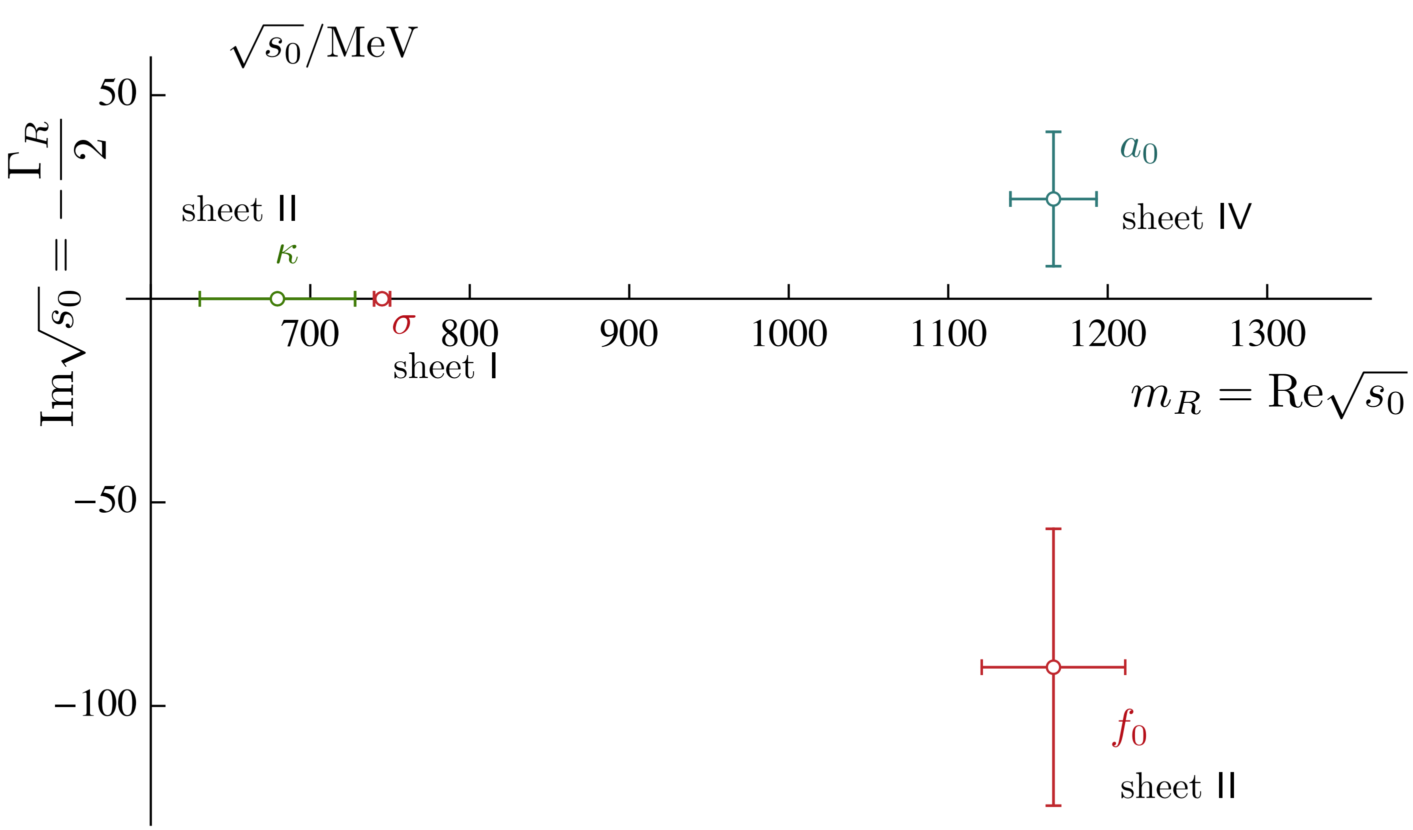
ππ, KK, ηη:	RB, Dudek, Edwards - PRL (2017)
	RB, Dudek, Edwards - PRD (2017)
Kπ, Kη:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
πη, KK:	Dudek, Edwards, Wilson - PRD (2016)

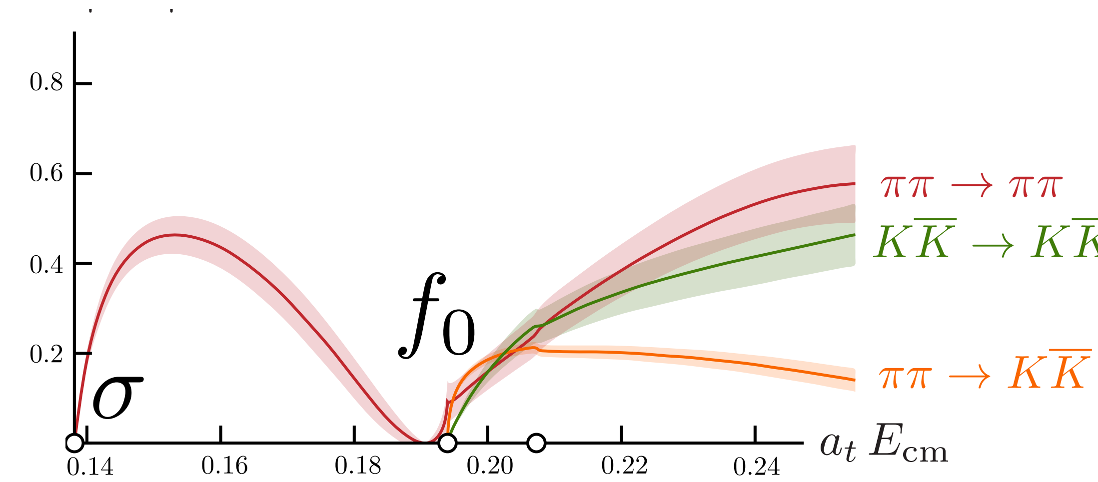
Tensor nonet

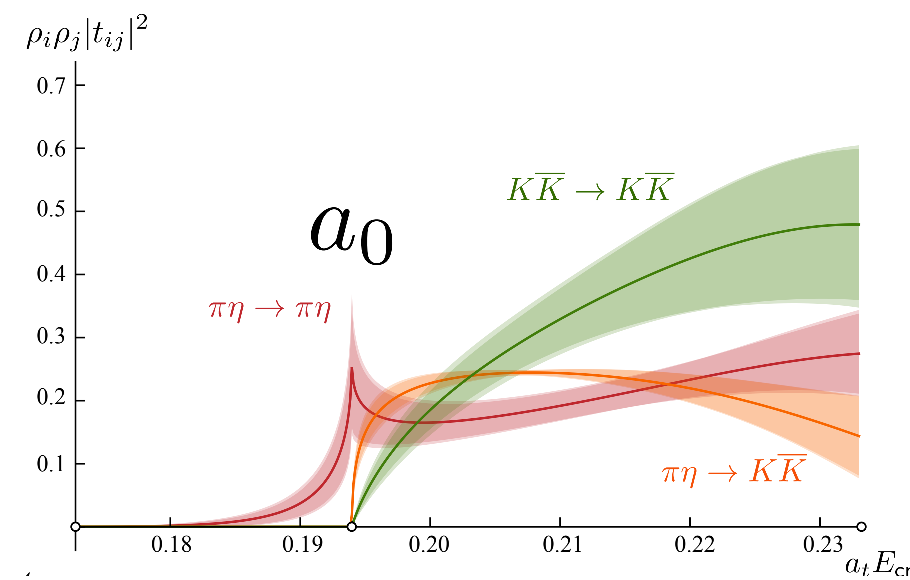


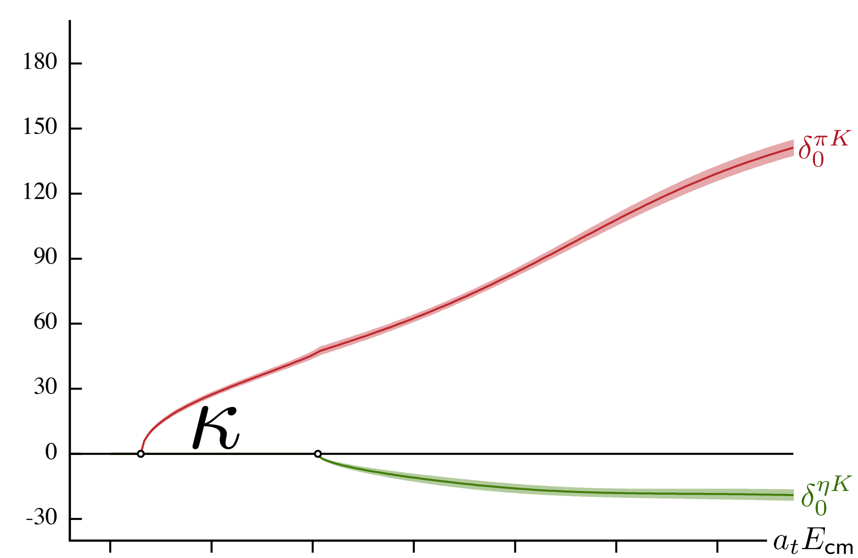
Scalar nonet









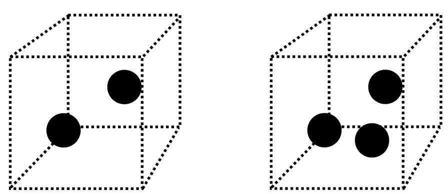


few-body systems in LQCD

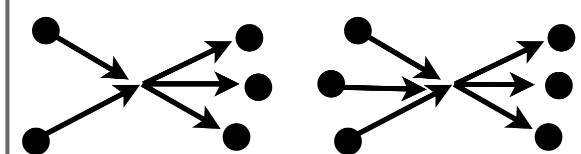
identification of
- states [masses & widths],
- production / decay mechanisms

lattice QCD

finite-volume spectrum

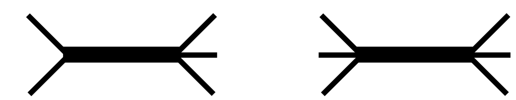


PW amplitudes



analytic continuation

resonance poles



these tools have been applied to study unitarity and the finite-volume spectrum in the 3-body sector

inside the box

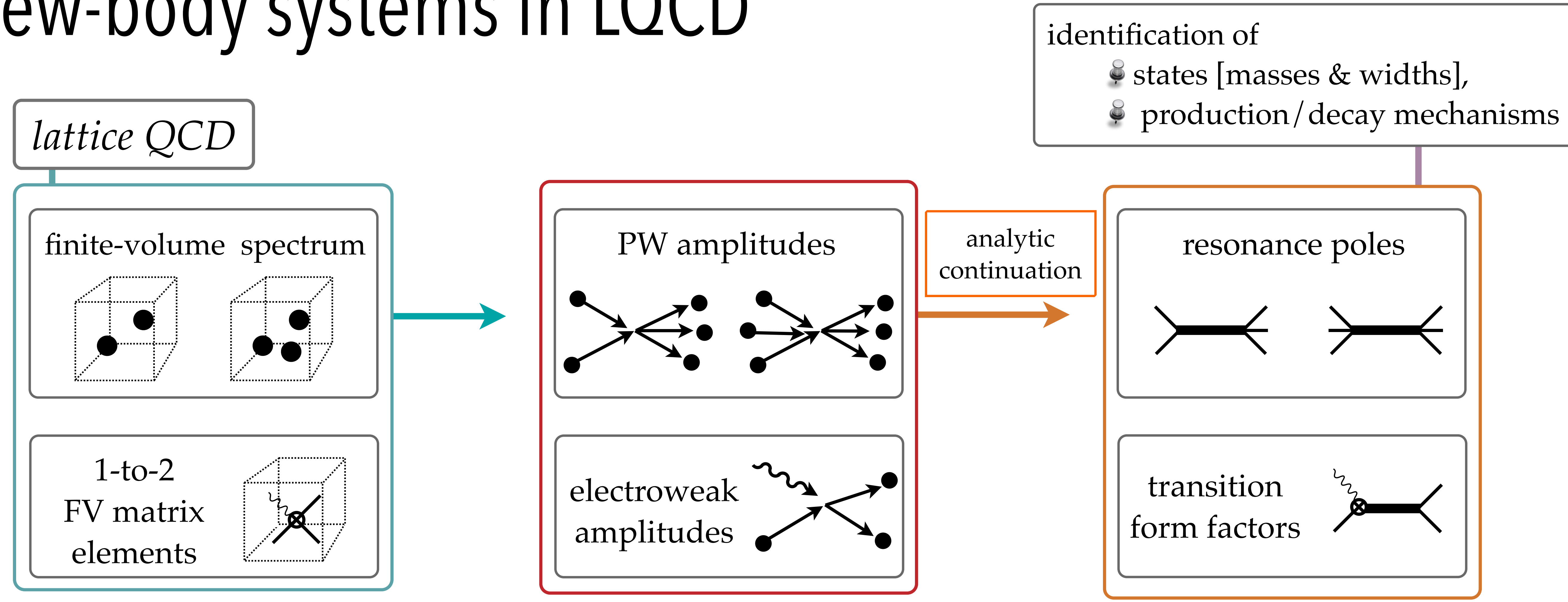
Sharpe

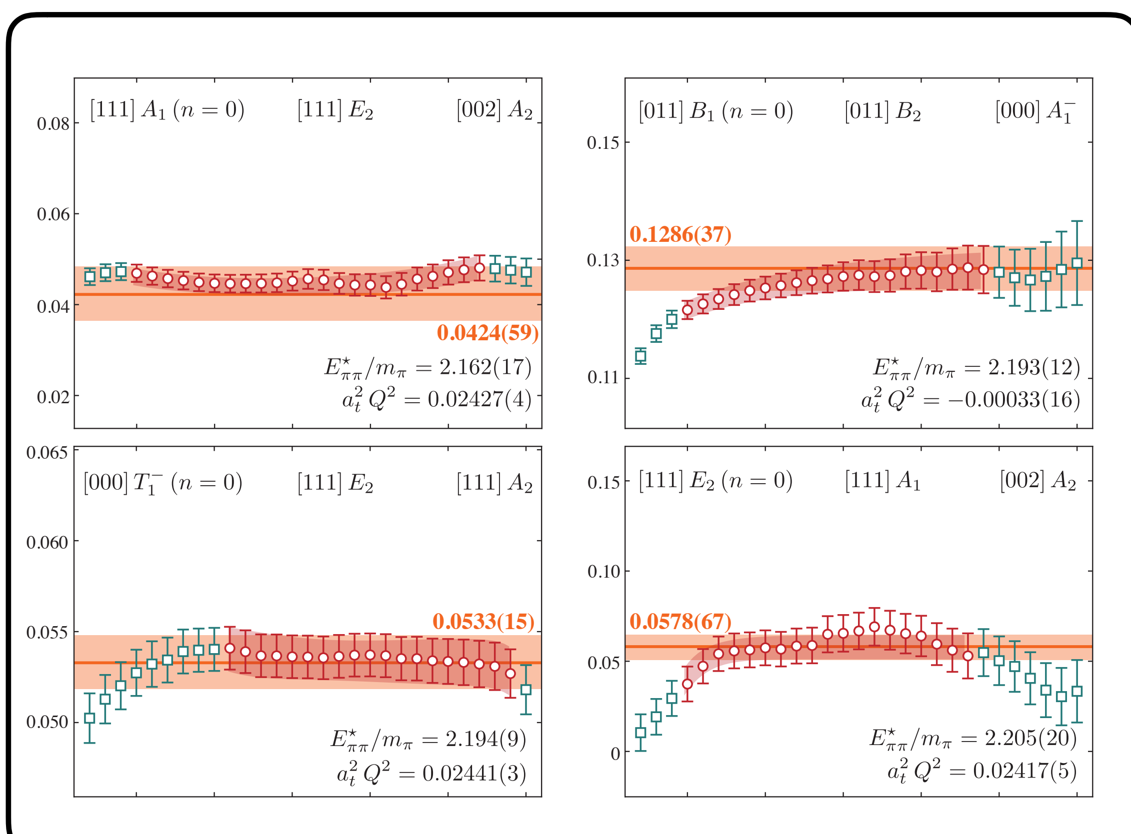
Hansen

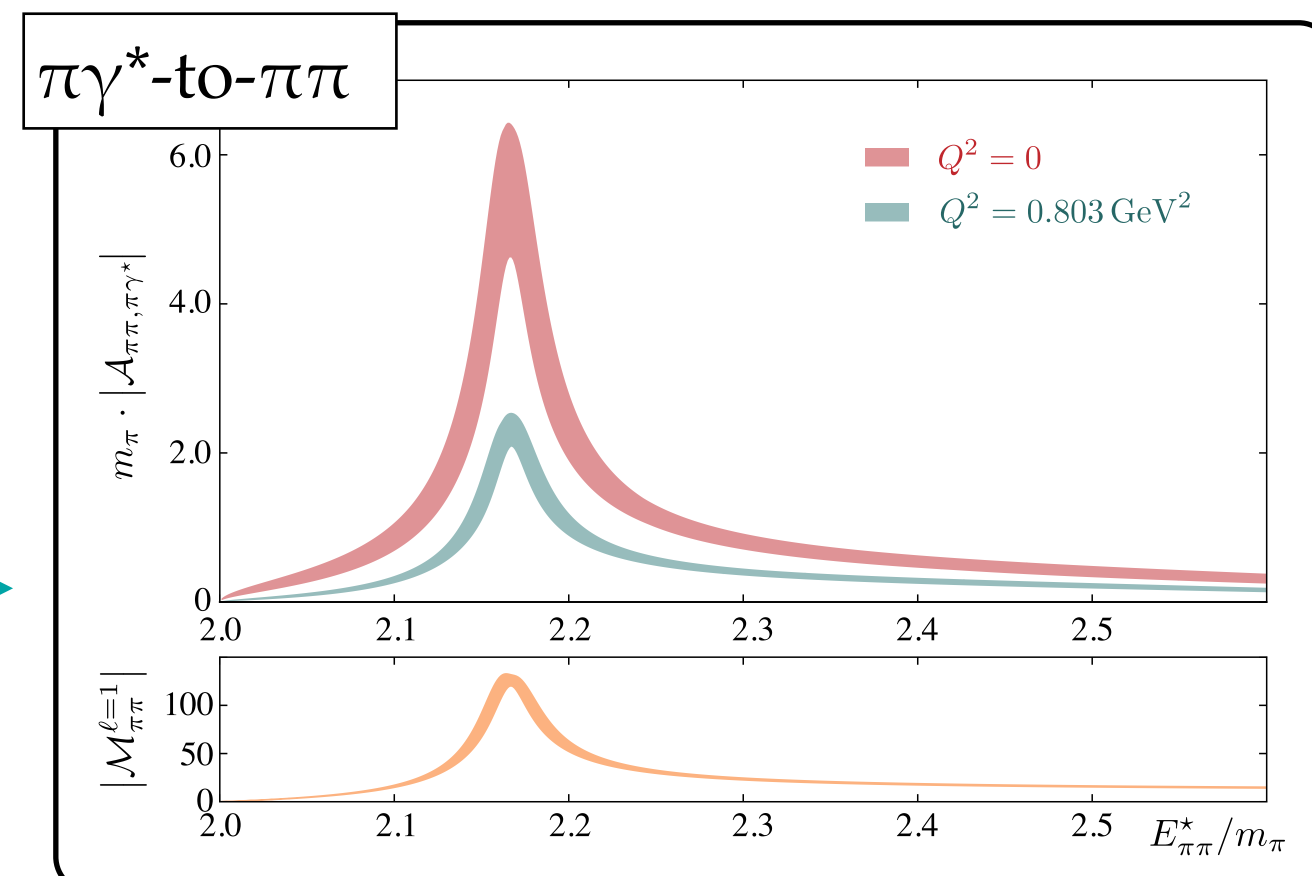
outside the box

Szczepaniak

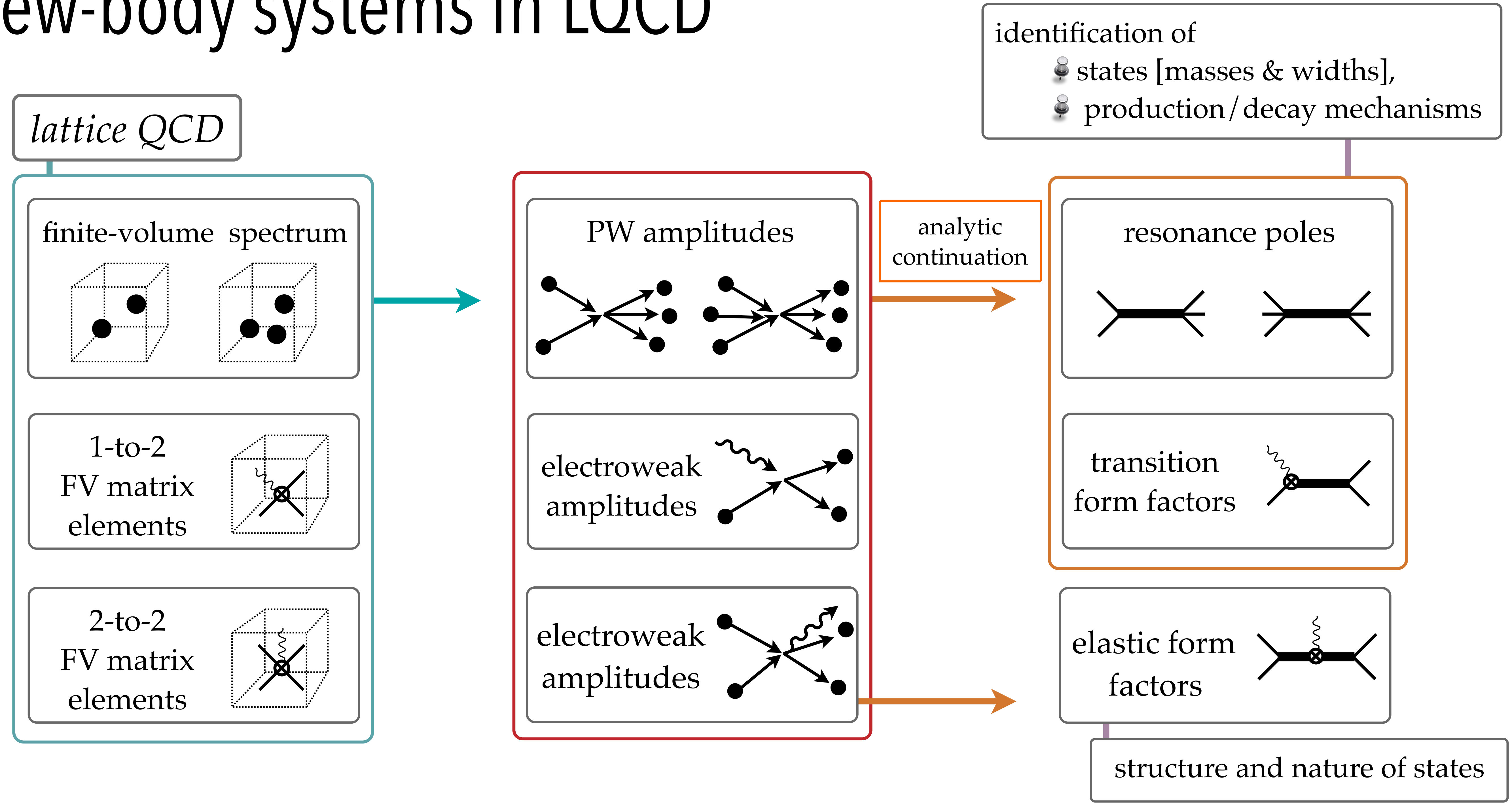
few-body systems in LQCD





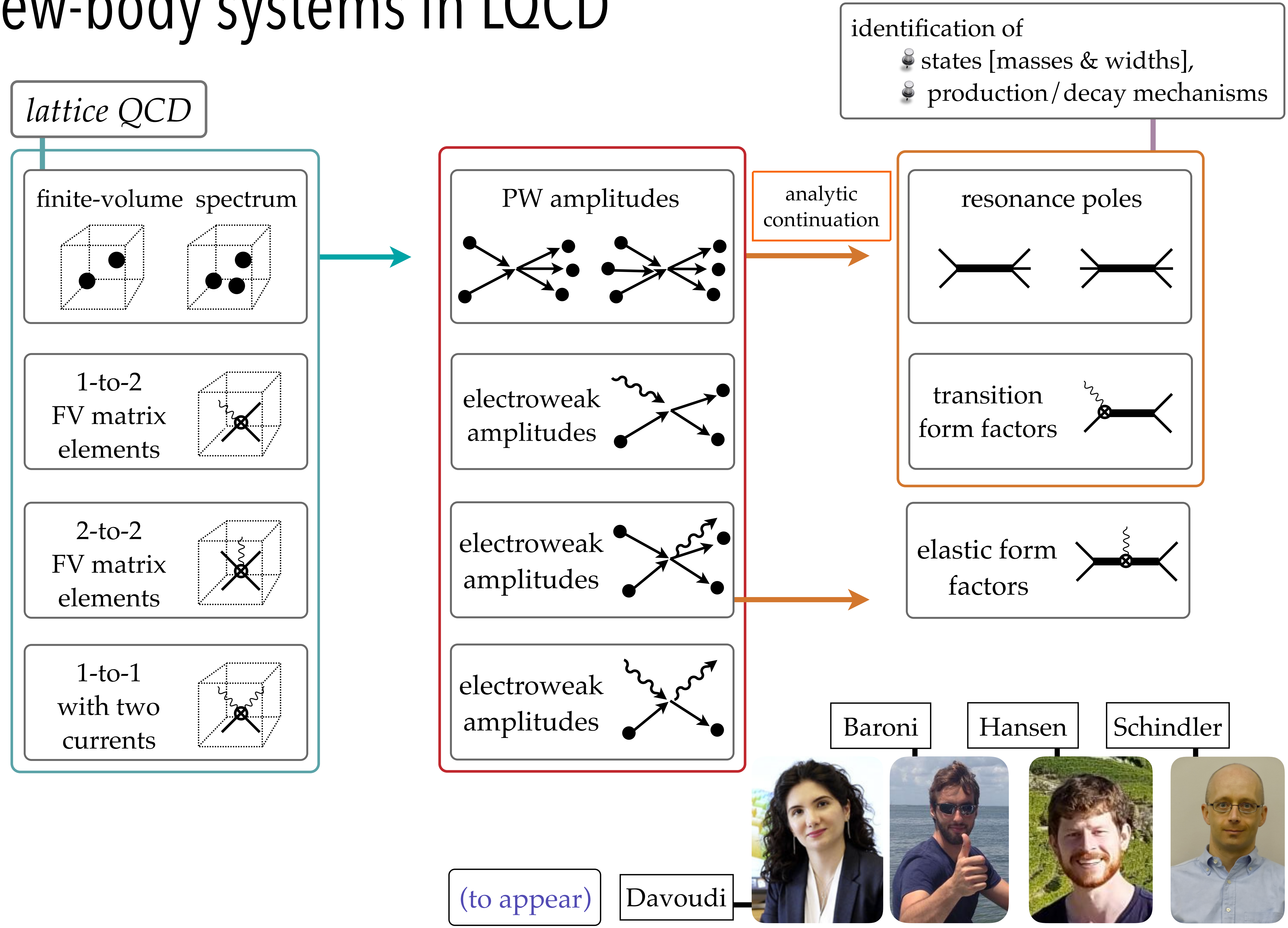


few-body systems in LQCD



- RB & Hansen (2015)
- Baroni, RB, Hansen, Ortega (2018)

few-body systems in LQCD



the future is nuclear

These techniques are being tested and implemented for *A=0* systems first, but they are necessary and will be applied for light nuclear systems…