Topology via Spectral Projectors with Staggered Fermions

Based on:

C. Bonanno[†*], G. Clemente, M. D'Elia and F. Sanfilippo,
Topology via Spectral Projectors with Staggered Fermions,
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Topology, θ-dependence and axion phenomenology

Possible solution to strong-CP problem: pseudo-scalar field (Peccei-Quinn axion) coupled to Q and anomalous $U(1)_{\rm PQ}$ symmetry:

$$Q = \frac{1}{64\pi^2}\int d^4x\, \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x),$$

$$\mathcal{L}_a \supset \left(\frac{a}{f_a} - \theta\right) Q, \qquad a \underset{U(1)_{\rm PQ}}{\longrightarrow} a + \alpha.$$

The coupling with Q relates the axion effective potential to the θ-dependence of the QCD free energy density:

$$f(\theta) = \frac{1}{2}\chi\theta^2\left(1 + \sum_{n=1}^{\infty} b_{2n}\theta^{2n}\right), \qquad V_{\mathit{eff}}(a) = f\left(\theta = \frac{a}{f_a}\right),$$

$$\text{where } \chi = \left.\frac{\langle Q^2\rangle}{V}\right|_{\theta=0}, \qquad b_2 = -\frac{1}{12}\left.\frac{\langle Q^4\rangle - 3\langle Q^2\rangle^2}{\langle Q^2\rangle}\right|_{\theta=0}, \ \dots$$

$$\implies m_a^2 \propto \chi, \qquad \lambda_4 \propto b_2, \ \dots$$

Axion dark matter

The QCD axion, being weakly coupled to the Standard Model, is also a good Dark Matter candidate.
In this context, the behavior of the axion effective potential at high temperatures is extremely relevant to access today axion relic abundance and mass, which are essential inputs for present and future experimental researches.

Being these quantities related to QCD topological observables $(\chi, b_2, \ldots)$, this constitutes a strong motivation to study the topic of θ-dependence in QCD at high-temperatures.

$$\chi = \frac{\langle Q^2 \rangle}{V}\bigg|_{\theta=0}, \qquad b_2 = -\frac{1}{12}\frac{\langle Q^4 \rangle - 3\langle Q^2 \rangle^2}{\langle Q^2 \rangle}\bigg|_{\theta=0}, \ldots$$

$$m_a^2 \propto \chi, \qquad \lambda_4 \propto b_2, \ldots$$

Numerical challenge - 1

In the QCD path-integral, field configurations are weighted with the determinant of the Dirac operator:

$$\det\{\not{D} + m_q\} = \prod_{\lambda\in\mathbb{R}} (i\lambda + m_q)\,.$$

The Index Theorem relates the presence of zero-modes in the spectrum of $\not{D}$ to the topological charge of the gluon field:

$$Q = \mathrm{Index}\{\not{D}\} = \mathrm{Tr}\{\gamma_5\} = n_+ - n_-.$$

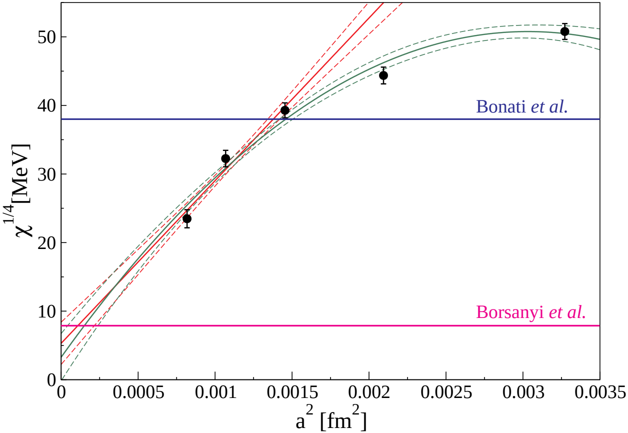
If a configuration has $Q \neq 0$, lowest eigenvalues are $\lambda_{min} = m_q$.

On the lattice, however, some fermionic discretizations (e.g., staggered) do not have exact zero-modes. $\implies$ The determinant fails to efficiently suppress non-zero charge configurations!

$$\lambda_{min} = m_q \longrightarrow m_q + i\lambda_0, \qquad \lambda_0 \underset{a\to 0}{\to} 0.$$

Numerical challenge - 2

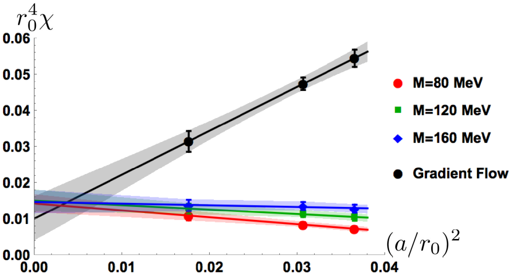
This results in large discretization corrections $\Longrightarrow$ Continuum extrapolation not under control (Bonati et al., 2018):



In (Borsanyi et al., 2016) a continuum estimation of χ at high-T has been obtained by reweighting configurations with the continuum lowest eigenvalues by hand.

Fermionic topological charge

Another possible solution, which does not require any ad hoc assumption, could be to switch, through the index theorem, to fermionic definitions of Q. Using the same "bad" operator to weight configurations and to count eigenmodes to measure Q may introduce smaller lattice artifacts.



Idea supported by results at $T = 0$ (Alexandrou et al., 2017): twisted mass Wilson fermions employed for the MC evolution and for the measure of χ through spectral projectors $\longrightarrow$ improved scaling of χ towards the continuum!

Goal: extension of spectral projectors to staggered fermions in view of an application to ongoing high-T QCD simulations with staggered fermions.

Spectral projectors with staggered fermions

In the continuum, only zero-modes contribute to Q. This is not true on the lattice, due to the absence of exact zero-modes:

$$Q = \mathrm{Tr}\{\gamma_5\} \longrightarrow \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M\},$$

$$\mathbb{P}_M = \textstyle\sum_{|\lambda_k| \leq M} u_k u_k^\dagger, \qquad \not{D}_{stag} u_k = i\lambda_k u_k.$$

Since 1 staggered fermion corresponds to $N_f = 4$ quark flavors in the continuum limit, taste degeneration has to be considered to avoid mode over-counting:

$$Q_{0stag} = \frac{1}{N_f} \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M\}.$$

Lattice charge gets a renormalization $Z_Q = \frac{Z_P}{Z_S}$, which can be derived from Ward identities for the flavor-singlet axial current:

$$Q_{stag} = \frac{Z_P}{Z_S} Q_{0stag}, \qquad \left(\frac{Z_P}{Z_S}\right)^2 = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M \Gamma_5 \mathbb{P}_M\}\rangle}.$$

Choice of the cut-off mass M

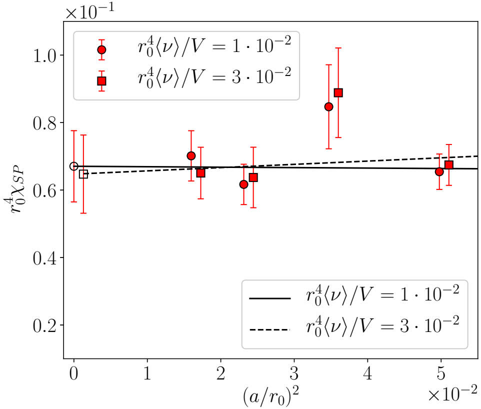
The cut-off mass M is irrelevant in the continuum limit but a prescription to keep its renormalized value $M_R = M/Z_S$ constant as $a \to 0$ is needed to guarantee $O(a^2)$ corrections:

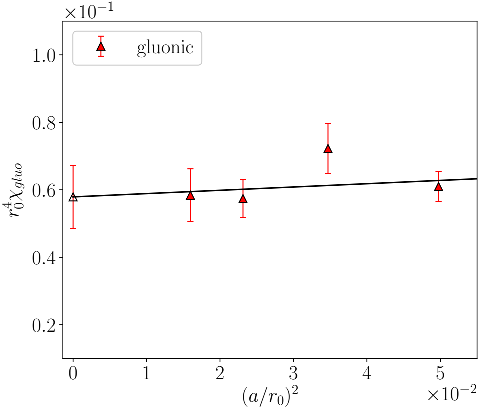
$$\chi_{SP}(a, M_R) = \chi + c(M_R)a^2 + O(a^4).$$

To avoid the direct computation of Z_S for each lattice spacing, we devised the following strategies:

- full QCD: since a line of constant physics is known, to fix M_R it is sufficient to tune M to keep M/m_q constant;
- quenched case: $\langle \nu(M) \rangle / V$, the number density of eigenmodes found below M, is proportional to M_R and independent on a; thus, to fix M_R it is sufficient to tune M to keep $\langle \nu \rangle / V$ (in physical units) constant.

Topological susceptibility in the quenched case - $T = 0$



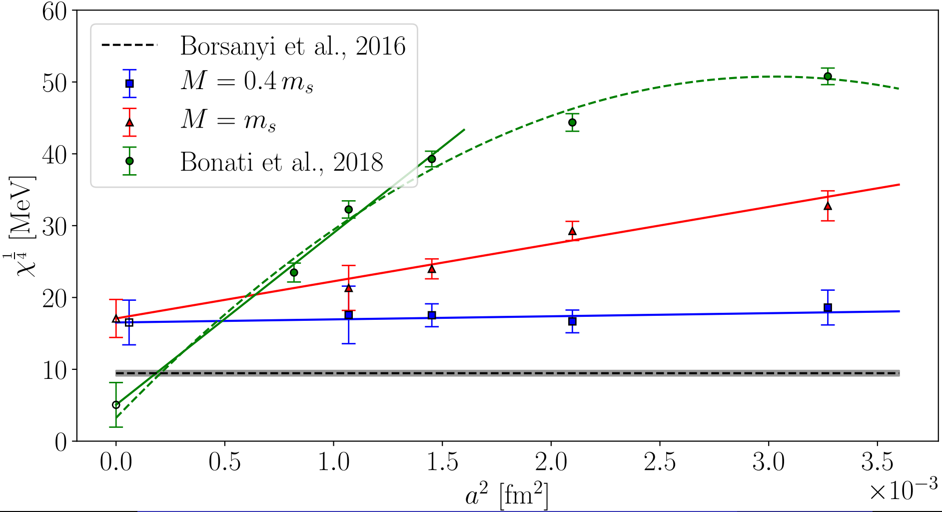


(CB, Clemente, D'Elia, Sanfilippo, 2019)	Stag. spectral proj. $r_0^4 \langle\nu\rangle / V = 10^{-2}$	Stag. spectral proj. $r_0^4 \langle\nu\rangle / V = 3 \cdot 10^{-2}$	Gluonic definition
$r_0^4 \chi_{\mathrm{YM}} \cdot 10^3$	67(11)	65(12)	58(9)

Other works	Overlap fermions + index theorem (Del Debbio et al., 2005)	Wilson spectral projectors (Lüscer & Palombi, 2010)
$r_0^4 \chi \cdot 10^3$	59(3)	67(3)

Full QCD with $N_f = 2 + 1$ rooted stout staggered fermions - $T = 430$ MeV - PRELIMINARY RESULTS

Preliminary results for the spectral measure of χ taken from the Master Thesis of Francesco D'Angelo (Pisa U.).



Summary of the talk:

- derivation of the spectral projectors expression for the staggered fermionic topological charge,
- discussion of two different methods to fix the eigenmodes cut-off M in physical units,
- results for $T = 0$, quenched case: spectral and gluonic determinations perfectly agree within errors,
- preliminary results for $T = 430$ MeV, full QCD: the staggered spectral measure, as it has been shown for Wilson fermions, has sensibly smaller lattice artifacts compared to the standard gluonic determination.

Future outlooks:

- study the zero-temperature case in full QCD with staggered spectral projectors,
- extend the full QCD high-T study to higher temperatures and refine present results.

Thank you for your attention!

Spectral projectors determination of b_2 in the quenched case - $T = 338$ MeV