Topology and Center Symmetry in Yang-Mills Theory

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13 December 2019

The XVIII Workshop on Statistical Mechanics and Nonperturbative Field Theory

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Framework

Yang-Mills theory at **LOW-T**:

- **Confined**.
- Strongly coupled $\Rightarrow$ **no** perturbative methods.
- Center symmetry is realized $\rightarrow \langle \mathrm{Tr} P \rangle = 0$.

Yang-Mills theory at **HIGH-T**:

- **Deconfined**.
- Weakly coupled $\Rightarrow$ perturbative/**semiclassical** methods.
- Center symmetry is spontaneously broken $\rightarrow \langle \mathrm{Tr} P \rangle \neq 0$.

How the **properties of the confined phase** and $\langle \mathrm{Tr} P \rangle = 0$ are related?

The Deformed Theory

Consider a **deformed theory** in which center symmetry is recovered even at high-T.

[M. Unsal and L. Yaffe: PRD **78**, (2008) 065035]

Previous lattice study:

- [J.C. Myers and C. Ogilvie: PRD **77**, (2008) 125030].
- [C. Bonati, MC, M. D'Elia: PRD **98**, (2018) 054508].

$$\boxed{\mathrm{S}^{\mathrm{def}} = \mathrm{S}_{\mathrm{W}} + \underbrace{h \sum_{\vec{n}} \left|\mathrm{Tr} P(\vec{n})\right|^2}_{\text{deformation}}}$$

- Gauge **configurations** with $\langle \mathrm{Tr} P \rangle \neq 0$ are **suppressed**.
- The parameter h is chosen in order to restore center symmetry.
- The theory is on $\mathcal{R}^3 \times S^1$ + PBC.

The SU(4) Case

$SU(4) \longrightarrow$ Center Simmetry has two breaking patterns:

$$\boxed{\mathbb{Z}_4 \quad \to \quad \mathrm{Id}} \qquad \boxed{\mathbb{Z}_4 \quad \to \quad \mathbb{Z}_2}$$

The order parameter are

$$\boxed{\langle \mathrm{Tr} P \rangle} \qquad \boxed{\langle \mathrm{Tr} P^2 \rangle}$$

In order to recover the full center symmetry we must consider two deformations:

$$\boxed{\mathrm{S}^{\mathrm{def}} = \mathrm{S_W} + h_1 \sum_{\vec{n}} \left| \mathrm{Tr} P(\vec{n}) \right|^2 + h_2 \sum_{\vec{n}} \left| \mathrm{Tr} P^2(\vec{n}) \right|^2}$$

The Aim of This Work

1. Consider $SU(4)$ **YM** deep in the **deconfined** phase.
2. **Switch** on the **deformations**.
3. **Study** the properties of the **re-confined phase**.

We want to investigate:

- How center symmetry is recovered.
- The phase diagram of the theory.
- The θ dependence of the deformed theory, compared with the one at $T = 0$.

Are the deformed theory and the usual one equivalent?

Summary of Topology

$$\boxed{\mathcal{L}_\theta = \mathcal{L}_{\mathrm{YM}} - i\theta Q(x)} \quad \boxed{F(\theta, T) = -\frac{1}{V_4}\ln \int [dA] \exp\left\{-\int_0^{\frac{1}{T}} dt \int d^3x \mathcal{L}_\theta\right\}}$$

The **free energy** $F(\theta, T)$ can be parametrized as follows:

$$\boxed{F(\theta, T) - F(0, T) = \frac{1}{2}\chi(T)\theta^2 \left(1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right)}$$

and it is easy to see that

$$\boxed{\chi = \frac{\langle Q^2\rangle_{\theta=0}}{V_4}} \qquad \boxed{b_2 = -\frac{1}{12\langle Q^2\rangle_{\theta=0}}\left[\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}\right]}$$

b_2 is a very noisy observable. In order to measure it we used the **imaginary** θ method.
[C. Bonati *et al*: PRD **93**, (2016) 025028].

Topology and Finite Temperature: MC Results

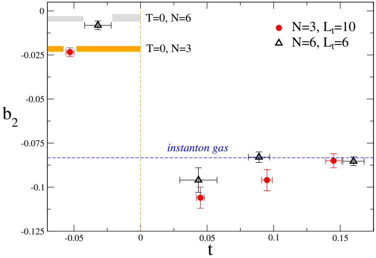
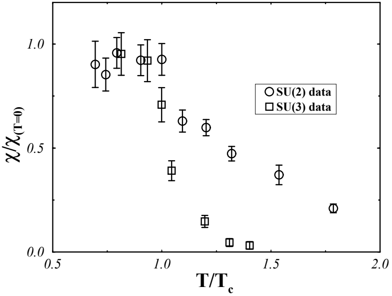


Fig. 3

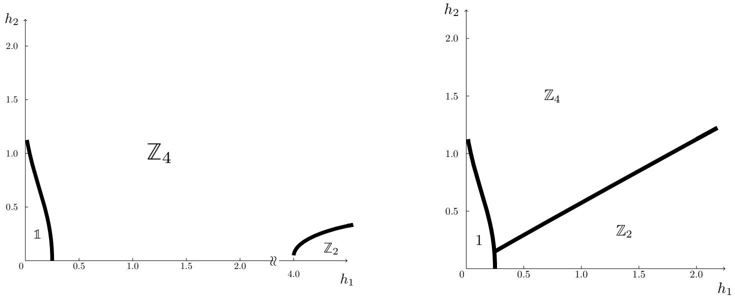


- (sx) [Bonati, D’Elia, Panagopoulos, Vicari: PRL 110 (**25**) 2013]
  (dx) [Allès, D’Elia, Di Giacomo: PLB 412 1997] See also:
  [C. Gattringer, R. Hoffmann and S. Schaefer, PL B535, 358 (2002)]
  [B. Lucini, M. Teper and U. Wenger, Nucl. Phys. B715,461 (2005)]
  [L. Del Debbio, H. Panagopoulos and E. Vicari, JHEP0409, 028 (2004)]
- **Topological properties change drastically from the low-*T* to the high-*T* regime.**

Numerical Results

[arXiv: 1912.02662]

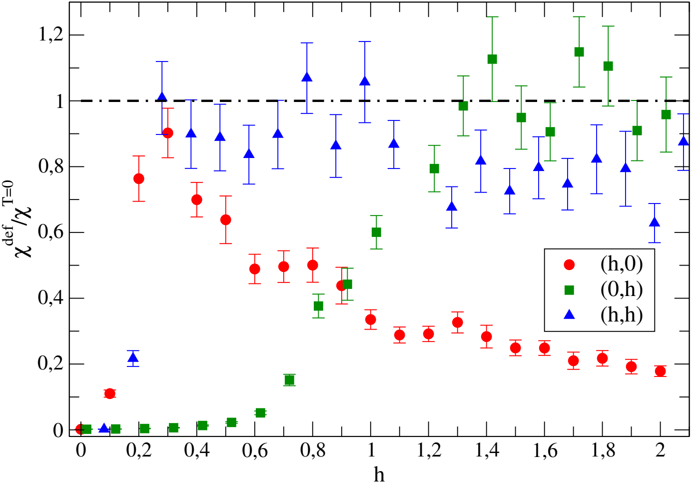
Phase Diagram



$\beta = 11.15$ $\beta = 11.40$

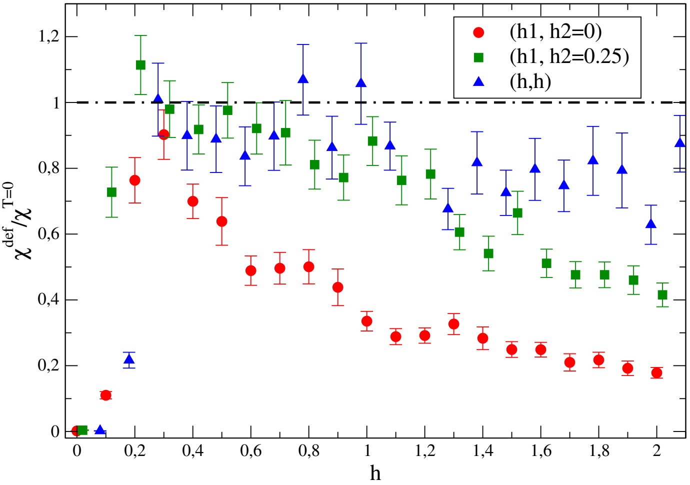
- MC simulations show the breaking to $\mathbb{Z}_2$.
- It is possible to recover center symmetry using only one deformation.
- The $\mathbb{Z}_2$ region becomes larger at higher values of β.

Topological Susceptibility $\beta = 11.40$

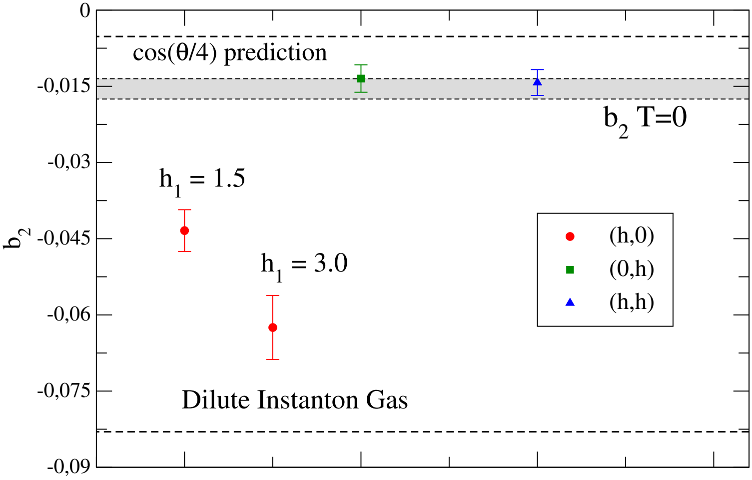


- Both $\langle \mathrm{Tr} P \rangle$ and $\langle \mathrm{Tr} P^2 \rangle$ must be zero to recover the correct $T = 0$ result.

Topological Susceptibility $\beta = 11.40$ part 2



b_2 Coefficient $\beta = 11.40$



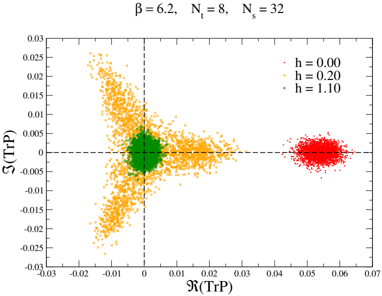
Conclusions

- We study a deformed $SU(N)$ YM theory in which center symmetry is recoverd even at high temperature.
- Once center symmetry is recovered the topological properties of the reconfined phase (χ and b_2) are in agreement with the values obtained at $T = 0$.
- For $SU(N)$ with $N > 3$ we need more than one deformation in order to avoid different breaking patterns of center symmetry.
- In order to obtaint the $T = 0$ values of χ and b_2 in $SU(4)$ center symmetry must not be broken to any subgroup.

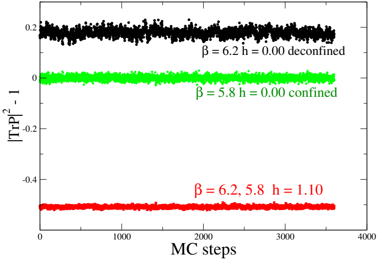
THANK YOU

BACK-UP SLIDES

Restoration of Center Symmetry

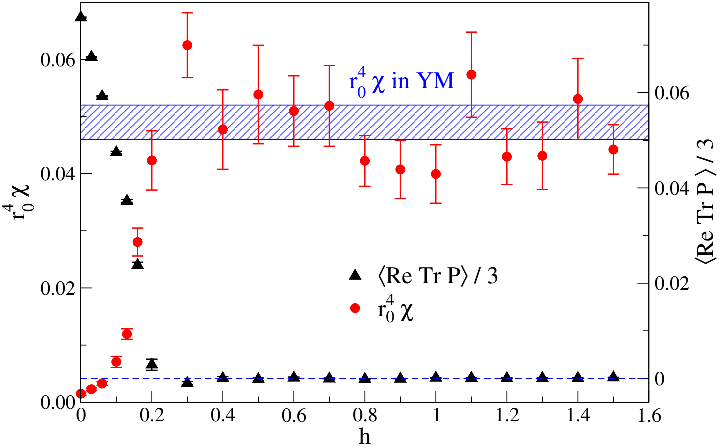


- **Center Symmetry** is **recovered** increasing h.
- The local value of TrP? **Adjoint Polyakov loop**.



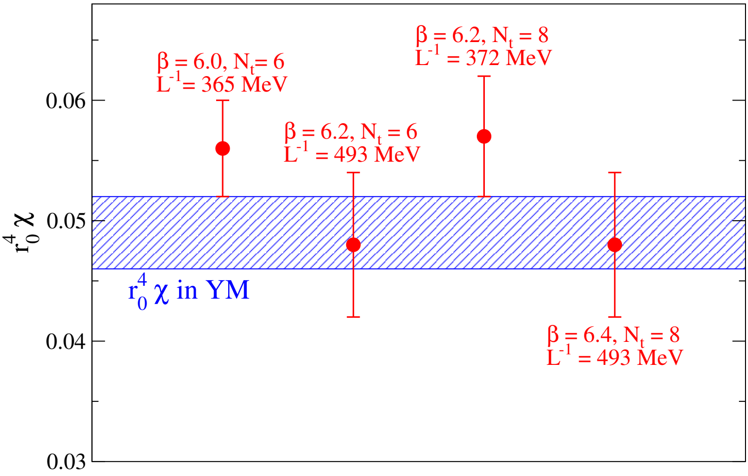
- $P^{\mathrm{adj}} = |\mathrm{Tr}P|^2 - 1$.
- A **negative value** implies that **TrP is close to zero locally**.

$r_0^4\chi$ vs h on $32^3 \times 8$ Lattice, $\beta = 6.4$

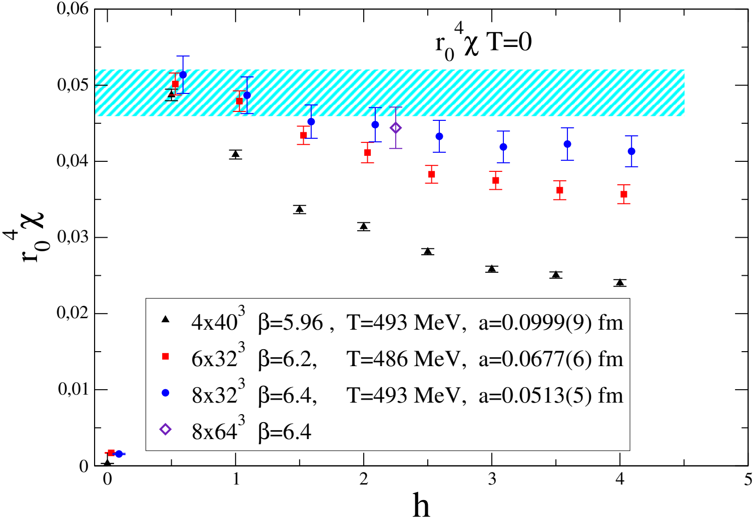


- r_0 is the Sommer parameter, used to fix the scale, and it is approximately 0.5 fm.
- **We assumed that the deformation does not modify the lattice spacing.**
- $r_0^4\chi$ in YM $\rightarrow$ [C. Bonati *et al*: PRD **93**, (2016) 025028].

$r_0^4\chi$ on Different Lattices ($N_s = 32$)

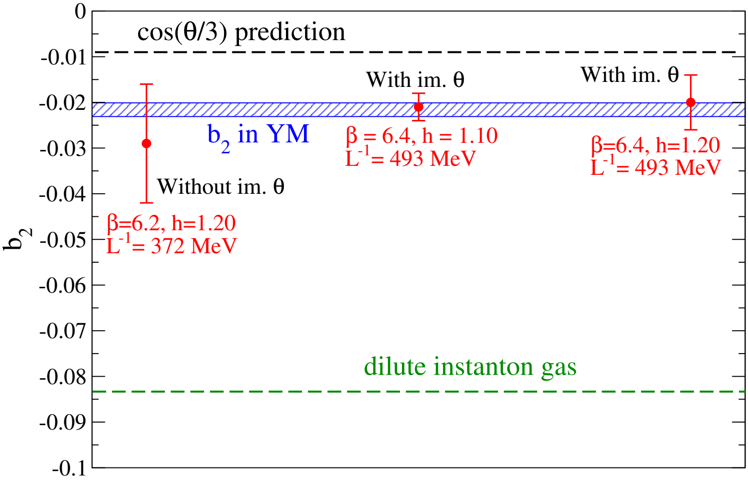


Beware of N_t



$r_0^4\chi$ T=0
$r_0^4\chi$
h
0,06
0,05
0,04
0,03
0,02
0,01
0
0
1
2
3
4
5
$4x40^3$ β=5.96 , T=493 MeV, a=0.0999(9) fm
$6x32^3$ β=6.2, T=486 MeV, a=0.0677(6) fm
$8x32^3$ β=6.4, T=493 MeV, a=0.0513(5) fm
$8x64^3$ β=6.4

b_2 on $N_t = 8$ $N_s = 32$



- b_2 **is dimensionless** $\Rightarrow$ **we do not need to fix** a.

Discretisation of The Topological Charge

- In our simulations we will use the discretisation of the topological charge with definite parity

$$q_L(x) = -\frac{1}{2^9\pi^2} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \varepsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left[\Pi_{\mu\nu}\Pi_{\rho\sigma}\right]$$

- In the continuum limit $q_L(x)$ must be corrected by a renormalization factor Z introduced by the lattice discretisation

$$q_L(x) \;\rightarrow\; a^4 Z q(x) + O(a^6)$$

- We remove UV fluctuation using the Cooling procedure.

Dilute Instanton Gas Approximation (DIGA)

We can describe our system as a gas of weakly interacting objects called (anti-) instantons which carry a topological charge equal to (minus) one and a finite action.

The free energy of this system is given by

$$\boxed{F(\theta) \approx \chi\,(1 - \cos\theta) \quad \rightarrow \quad b_2 = -\frac{1}{12}}$$

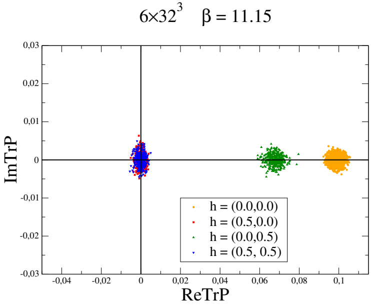
Lattice Spacing and the Deformation on $SU(3)$

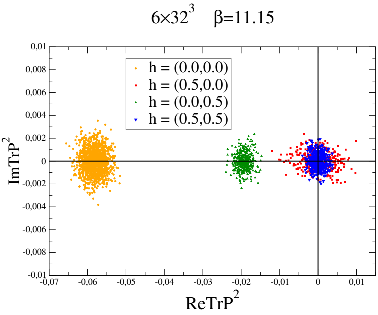
β	h	t_0/a^2
5.96	0.0	2.7854(62)
5.96	1.0	2.8087(69)
5.96	2.0	2.8063(74)

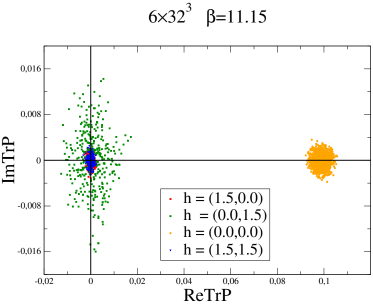
β	h	t_0/a^2
6.17	0.0	5.489(14)
6.17	1.0	5.530(16)
6.17	2.0	5.498(16)

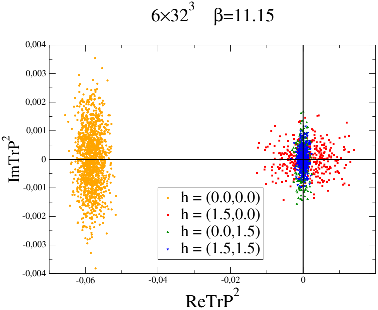
- To test the independence of the lattice spacing on h we determined the scale t_0 defined by gradeient flow. See [M. Luscher: JHEP **1403**, 092 (2014)].
- $\beta = 5.96 \quad \rightarrow \quad 24^4$ lattices.
  $\beta = 6.17 \quad \rightarrow \quad 32^4$ lattices.
- Data coincides with those at $h = 0$ up to less than 1%.

Scatter Plots $SU(4)$ $\beta = 11.15$

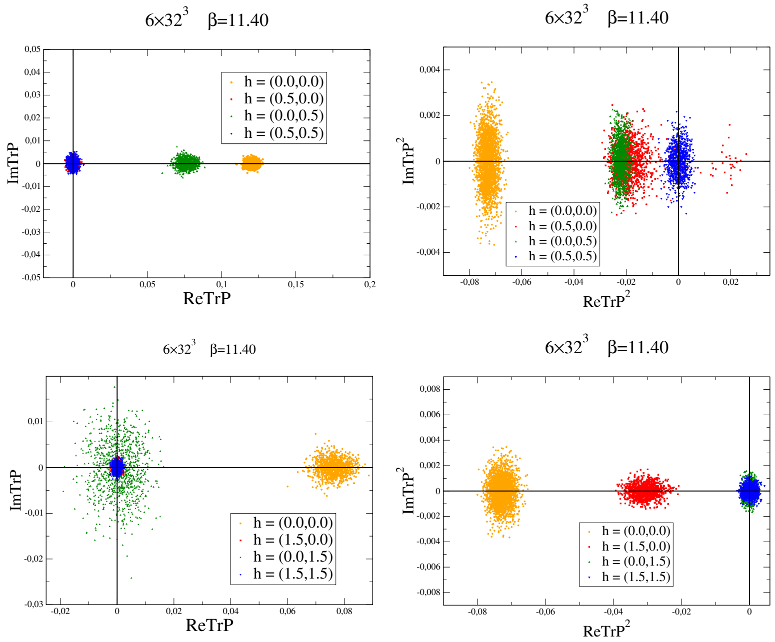








Scatter Plots $SU(4)$ $\beta = 11.40$



Imaginary Theta Method

We add to the Lagrangian an imaginary θ term

$$\boxed{S^{\mathrm{def},i\theta} = S_W + h\sum_{\vec{n}} \left|\mathrm{Tr}P(\vec{n})\right|^2 - \theta_L Q_L}$$

where Q_L is the clover discretisation of Q.
We perform simulations using different values of θ_L and we obtain χ, b_2 and Z with a combined fit of the first four cumulants

$$\frac{\langle Q\rangle}{V_4} = \chi Z\theta_L \left(1 - 2b_2Z^2\theta_L^2 + 3b_4Z^4\theta_L^4 + \cdots\right)$$

$$\frac{\langle Q^2\rangle_c}{V_4} = \chi \left(1 - 6b_2Z^2\theta_L^2 + 15b_4Z^4\theta_L^4 + \cdots\right)$$

$$\frac{\langle Q^3\rangle_c}{V_4} = \chi \left(-12b_2Z\theta_L + 60b_4Z^3\theta_L^3 + \cdots\right)$$

$$\frac{\langle Q^4\rangle_c}{V_4} = \chi \left(-12b_2 + 180b_4Z^2\theta_L^2 + \cdots\right)$$