Perturbations (and boundaries) in Flocking systems

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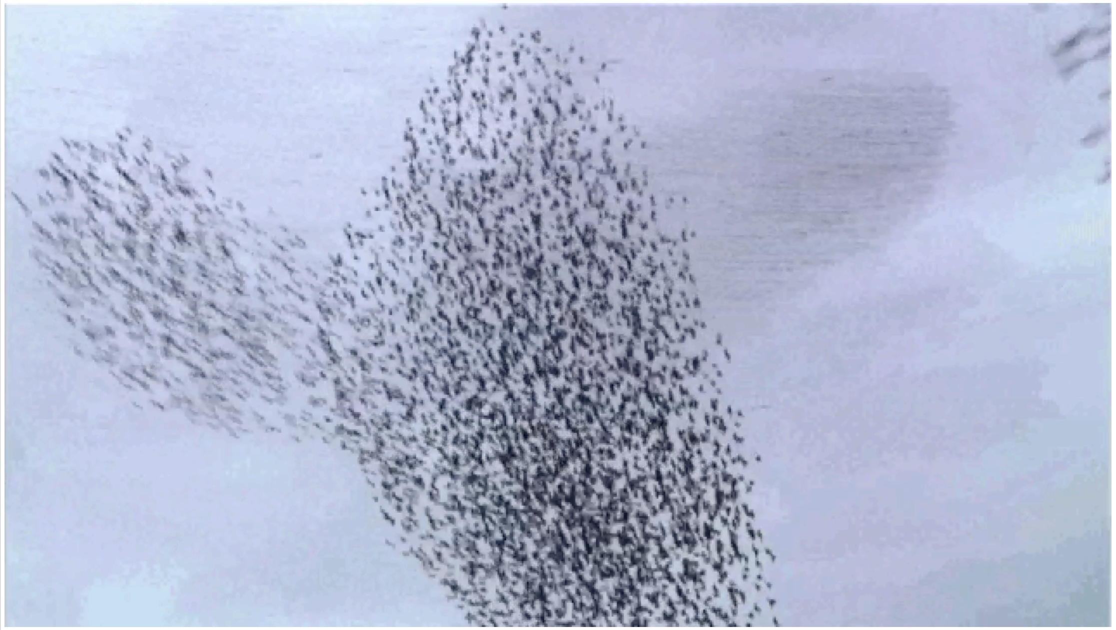

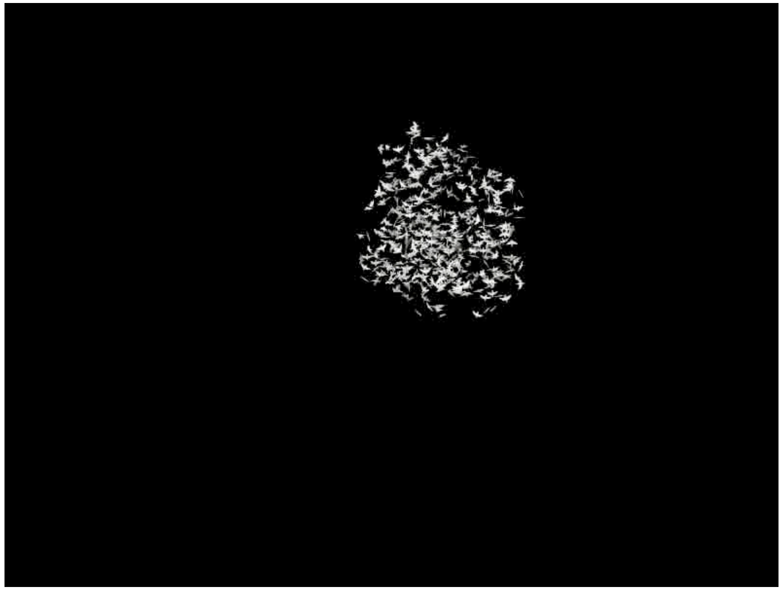

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MARIE CURIE ACTIONS

Introduction: active matter

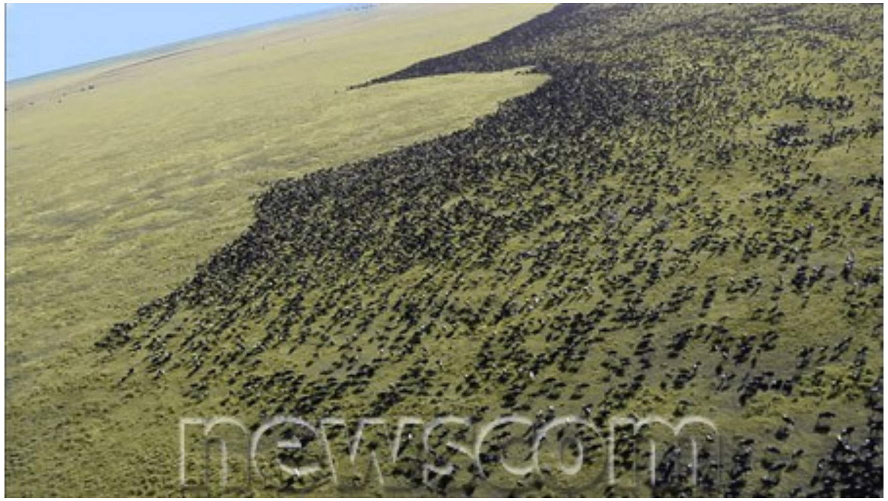
Active particles are able to extract and dissipate energy from their surroundings to produce systematic and coherent motion

- Energy enters and exits the system → out of equilibrium
- Energy is spent to perform actions, typically move (self-propel) in a non-thermal way
- In active systems, energy is injected and dissipated in the bulk, not from the boundaries, in a way that does not explicitely breaks any simmety

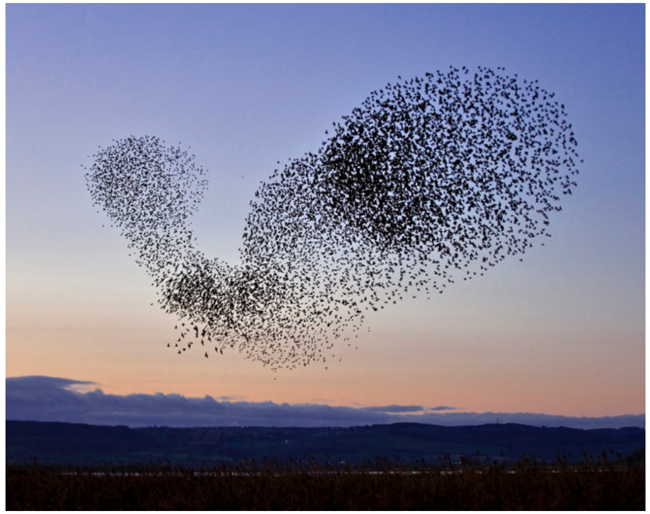
Flocking active matter

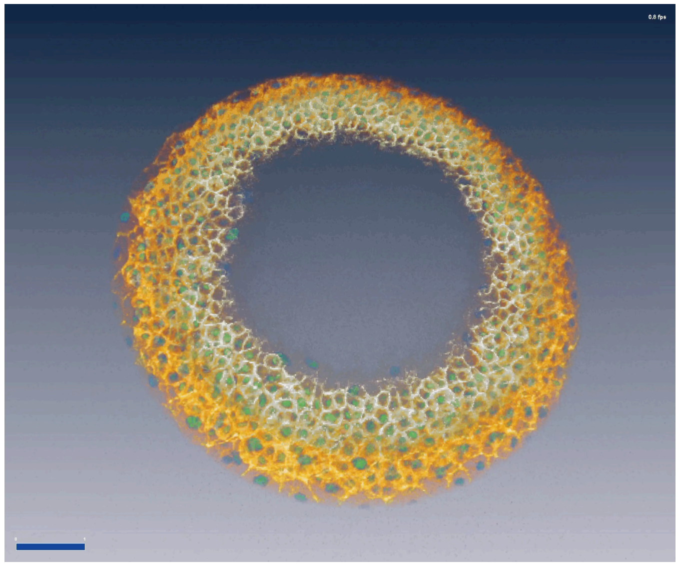
spontaneous symmetry breaking to collective motion

Wildbeasts



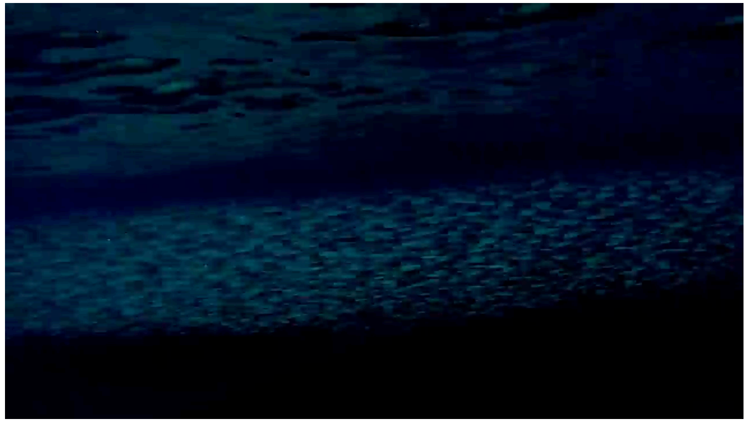
Starlings flocks





Cellular migration

Fish schooling



The Vicsek universality class: a paradigma for collective motion

Which essential ingredients you find in the VM?

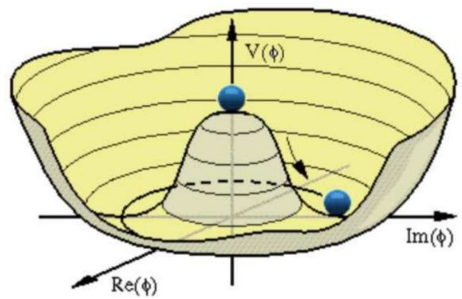
1. Conservation of particles number

2. Dry systems (no hydro interactions)

3. Particles are self propelled, i.e. they move and exchange interacting neighbours

The system is far from equilibrium !!

4. A continuous symmetry can be spontaneously broken (to polar order) by aligning interactions

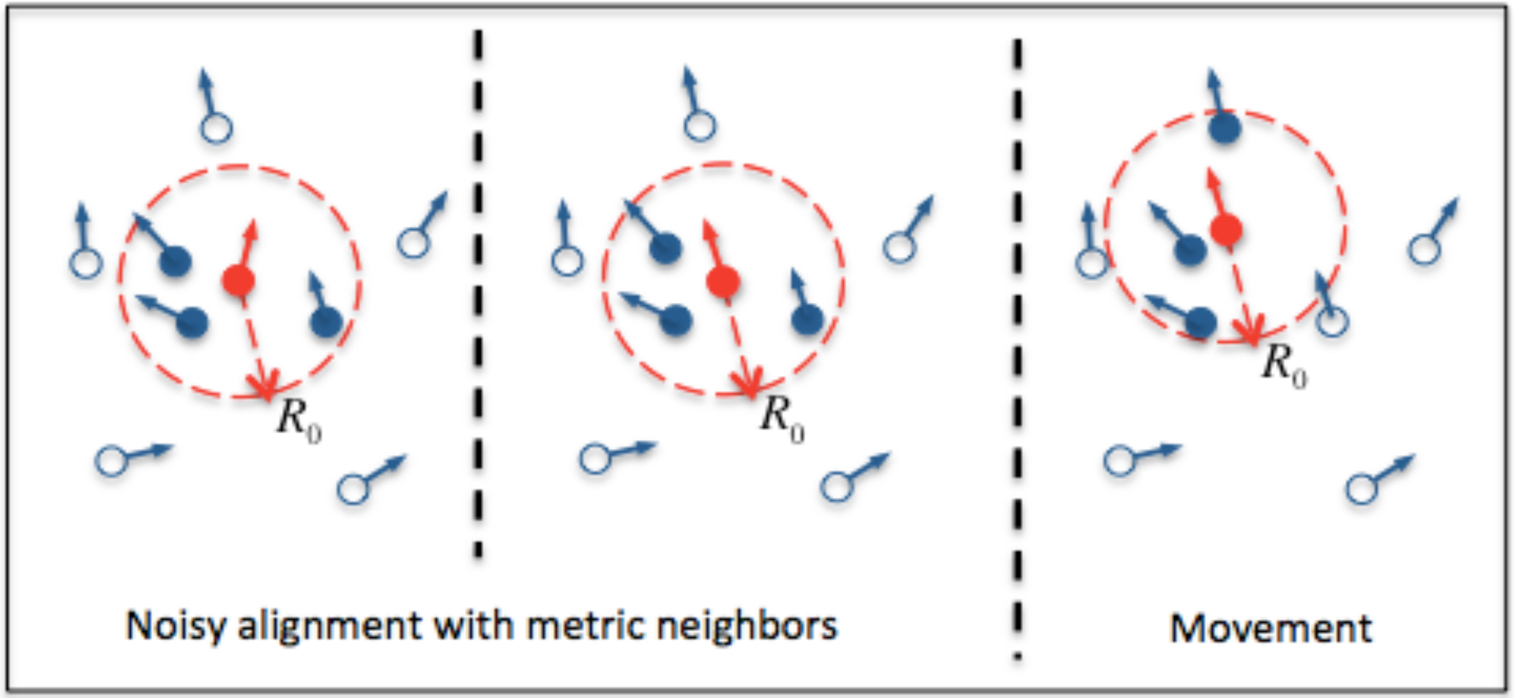


The Vicsek model ("moving XY spins")

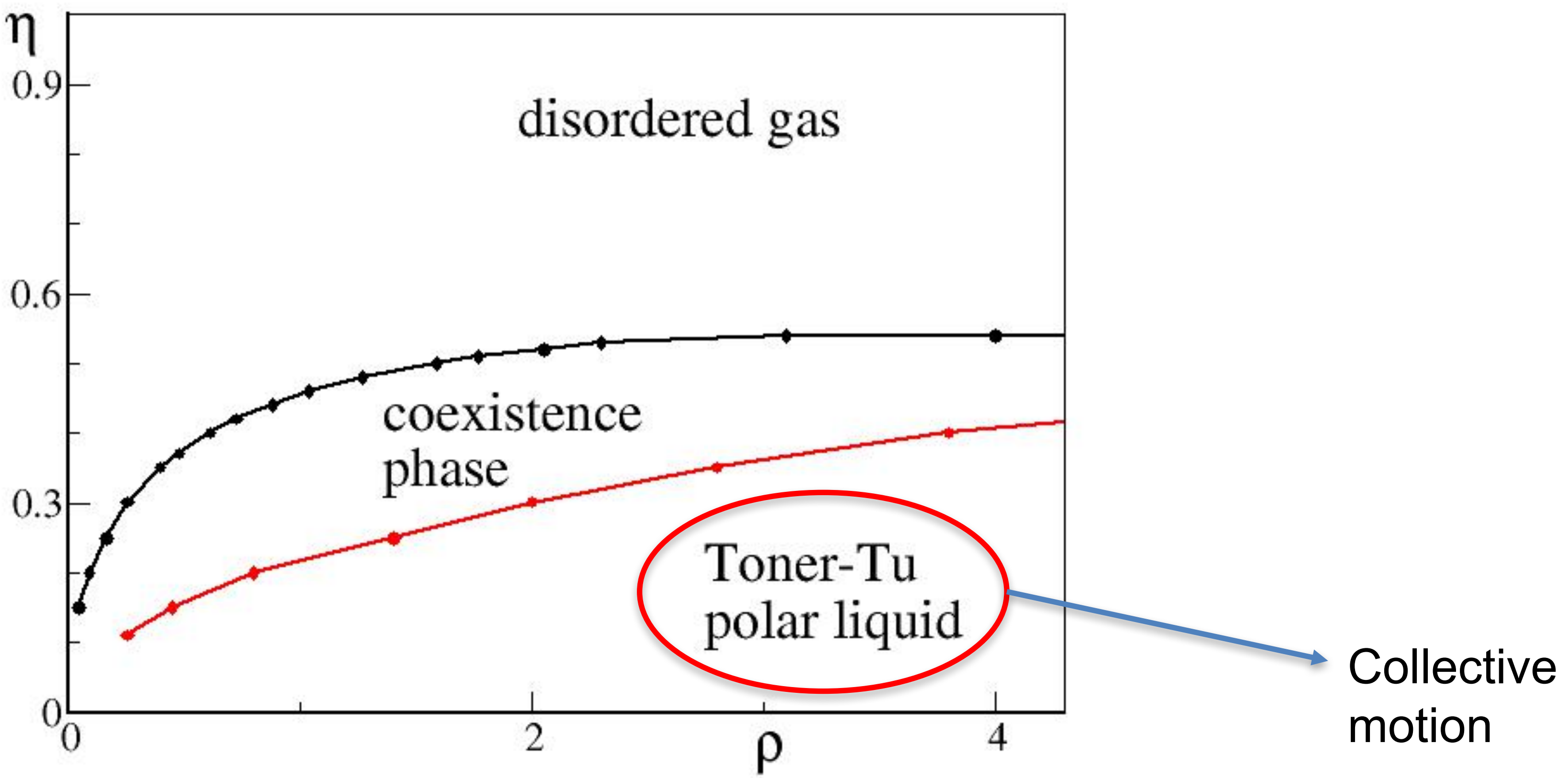
Vicsek et al, PRL (1995)

- Off lattice self propelled particles that move with constant speed v_0
- Local *ferromagnetic (or polar)* alignment with local neighbors (inside a metric range R_0.)
- Environmental **white noise**

In d=2 one may write the VM as



Vicsek phase diagram



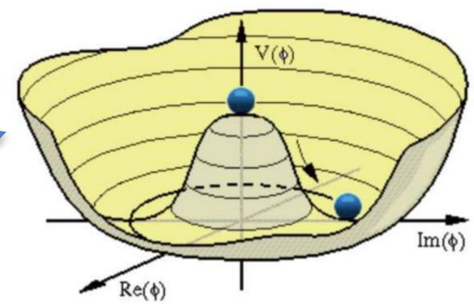
Toner and Tu field theory

Spontaneous symmetry breaking of a continuous symmetry
+ non-equilibrium effects

In the symmetry-broken state, large wavelength velocity fluctuations are easily excited and decay slowly (Nambu-Goldstone modes)

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\partial_t \mathbf{v} + \lambda_1 (\mathbf{v} \cdot \nabla) \mathbf{v} + \lambda_2 (\nabla \cdot \mathbf{v}) \mathbf{v} + \lambda_3 \nabla |\mathbf{v}|^2 = [\alpha - \beta |\mathbf{v}|^2] \mathbf{v}$$
$$- \nabla P + D_0 \nabla^2 \mathbf{v} + D_1 \nabla (\nabla \cdot \mathbf{v}) + D_2 (\mathbf{v} \cdot \nabla)^2 \mathbf{v} + \mathbf{f}.$$



- Long ranged correlations
- Giant number fluctuations

Velocity fluctuations

$\Updownarrow$

Density fluctuations

Toner & Tu Hydrodynamic theory predicts universal long-ranged correlations

E.g.: equal time correlation functions of velocity and density fluctuations

Density structure factor $\delta\rho(\boldsymbol{x}) = \rho(\boldsymbol{x}) - \rho$

$$S(\mathbf{q}, t) \equiv \langle \delta\hat{\rho}(\mathbf{q}, t)\delta\hat{\rho}(-\mathbf{q}, t)\rangle \sim \frac{1}{q^{\sigma}} \quad \text{for } q \to 0$$

Scaling exponents can be determined

By RG under certain conjectures

$$\sigma = \frac{2}{5}(d+1)$$

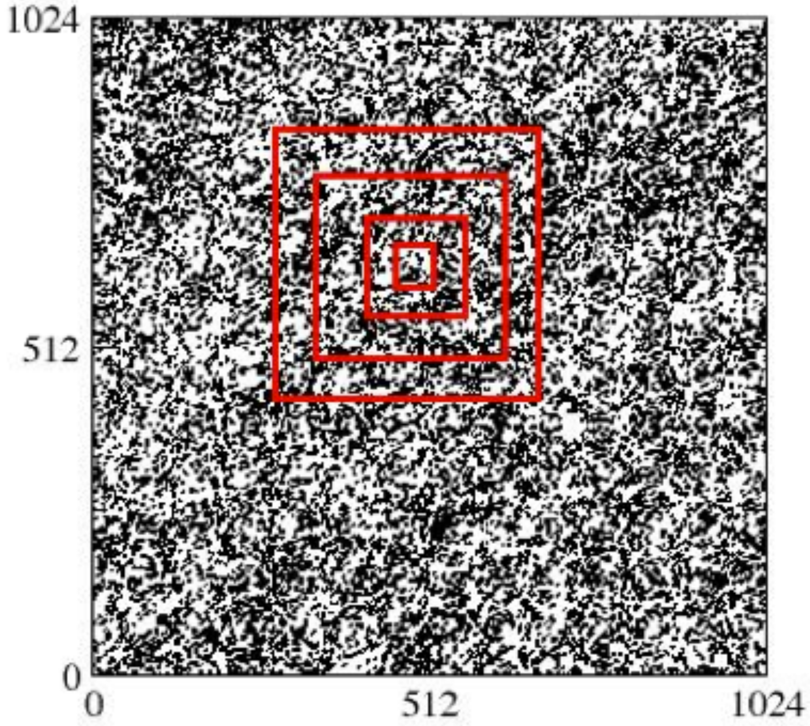
By numerical simulation of microscopic models (HPC)

σ = 1.33(2) $d = 2$

σ σ = 1.75(5) $d = 3$

Giant Number Fluctuations

- Fluctuations in average number of particles are anomalously large:



$$\langle n \rangle = \rho_0 \, \ell^d$$

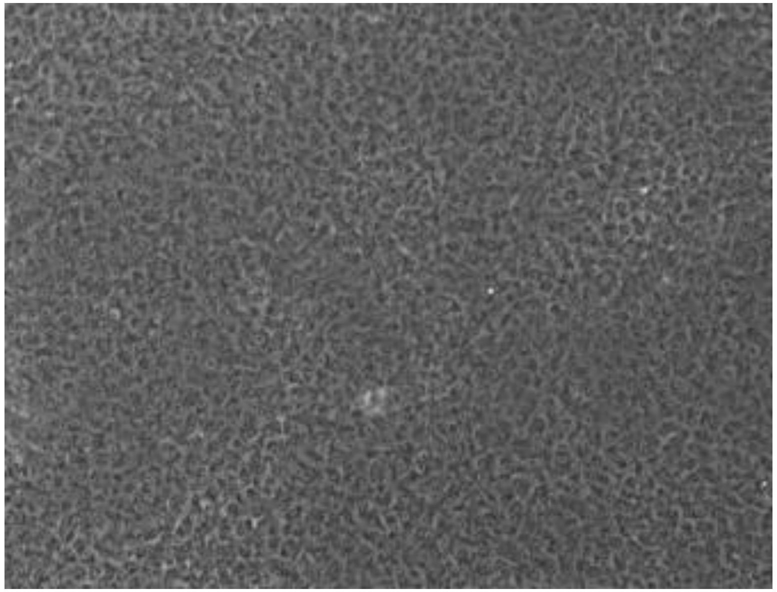
$$\Delta n = \left(\langle (n_t - \langle n \rangle)^2 \rangle\right)^{1/2}$$

$$S(\mathbf{q} \to 0) = \rho_0 \left[\frac{\Delta n^2}{\langle n \rangle} \right]_{n \to \infty}$$

$$S(\mathbf{q} \to 0) \sim \frac{1}{q^\sigma} \sim \ell^\sigma \quad \Rightarrow \quad \Delta n \sim \langle n \rangle^{1/2 + \sigma/(2d)}$$

Experimental validation: qualitative and quantitative

Starling flocks



Human mammary epithelial MCF-10A cells over-expressing RAB5A protein

1. Long range correlations in starlings flocks

2 points (connected) real space correlation function

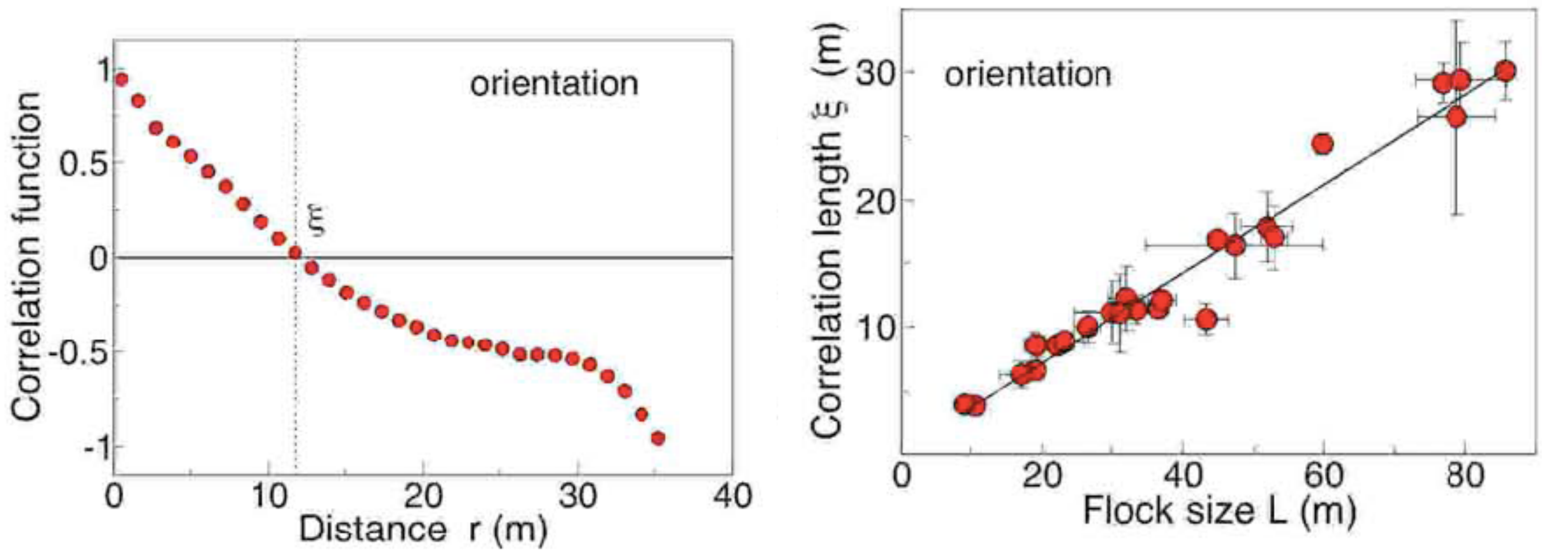
$$C_s(r) = \left\langle \frac{\sum_{ij} \delta \mathbf{s}_i \cdot \delta \mathbf{s}_j \, \delta(r - r_{ij})}{\sum_{ij} \delta(r - r_{ij})} \right\rangle$$

$$\delta \mathbf{s}_i = \mathbf{s}_i - \frac{1}{N} \sum_i \mathbf{s}_i .$$

In finite systems we define a correlation length by

$$C(r = \xi) = 0$$

A



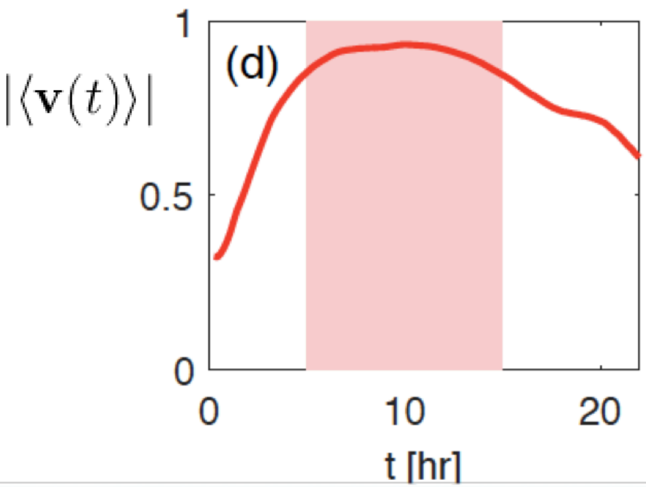
Cavaga et al. PNAS ***107*** *11865 (2010)*

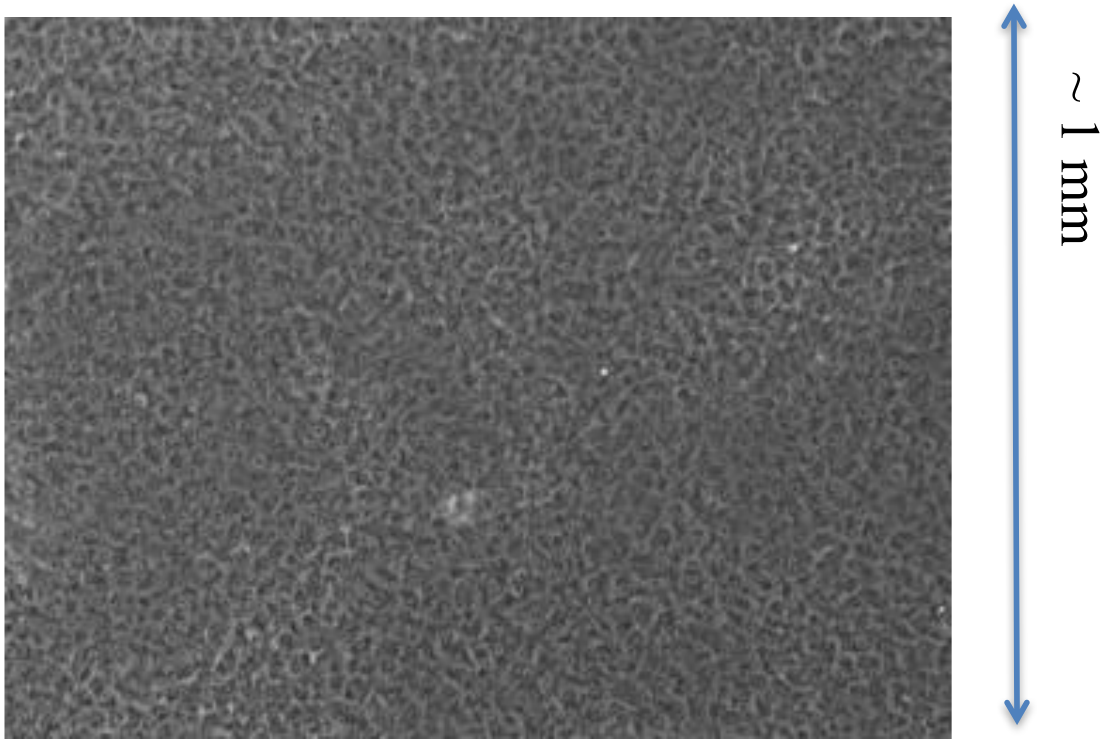
2. In vitro cell migration experiment

Cell tissue **Lab grown human mammary epithelial MCF-10A cells.**

Seeded in well plates and cultured to obtain a large (~ 10^6 cells) hyperconfluent monolayer

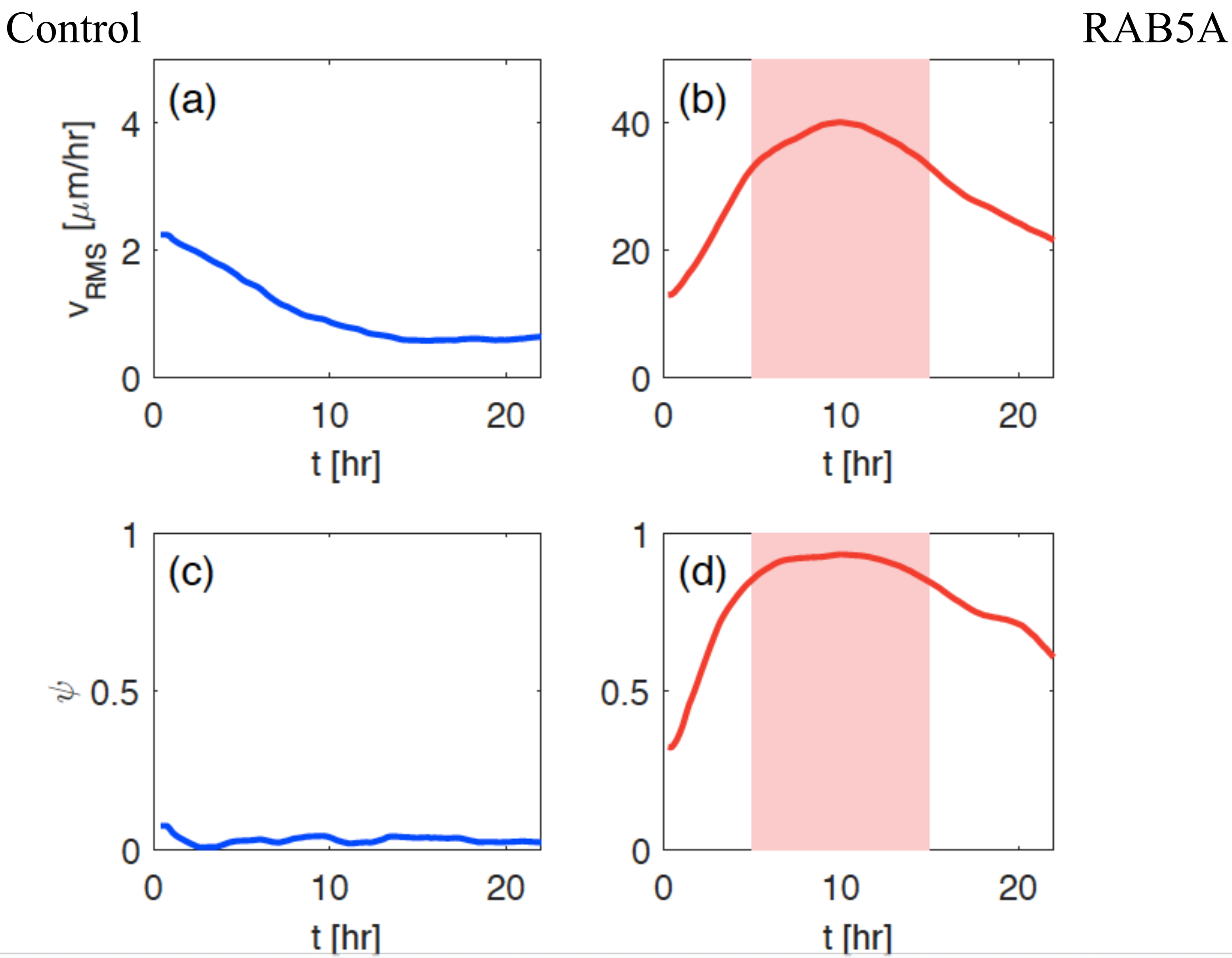
RAB5A expression induces collective motion





Maliverno C et al Nat. Mater. ***16***, *587 (2017)*

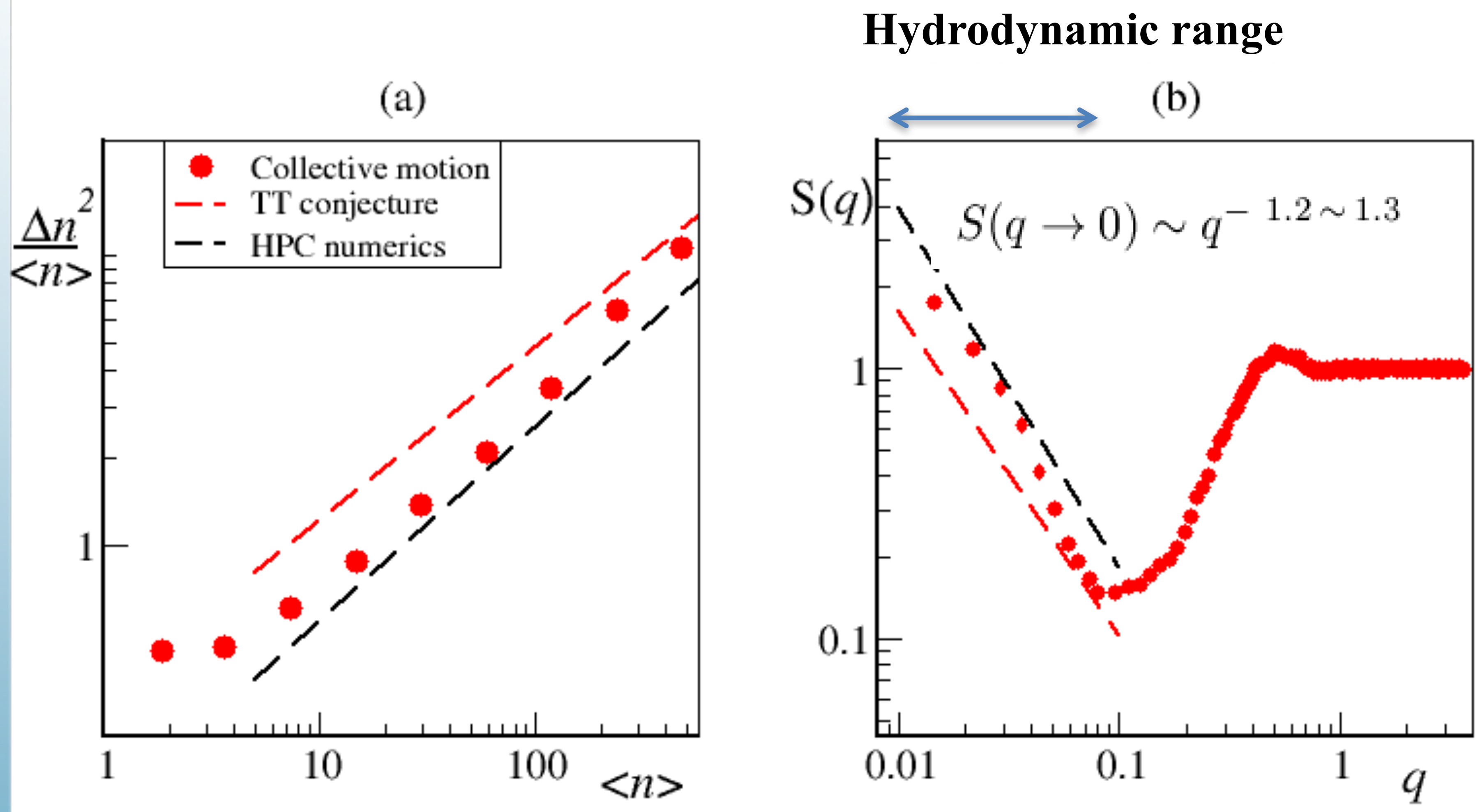
RAB5A promotes a transition to flocking and a reawakening of motility



F. Giavazzi, FG et al. J. Phys D 50 384003 (2017).

RAB5A over-expressed – collective motion

$$S(q) \equiv \langle S(\boldsymbol{q}) \rangle_{|\boldsymbol{q}|=q}$$



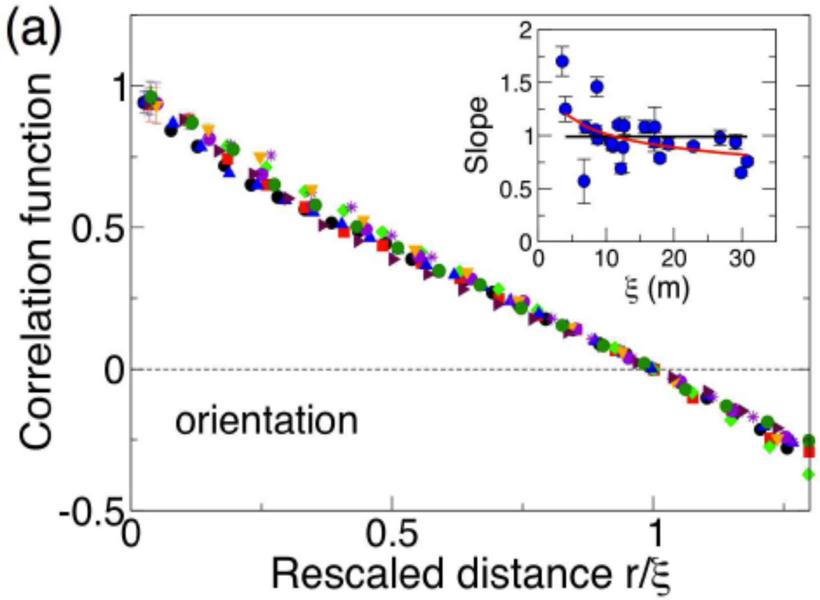
F. Giavazzi, FG et al. J. Phys D 50 384003 (2017).

Beyond Bulk, unperturbed theory

- Flocks are finite

- Flocks are interacting with the rest of the world – external stimuli

1. Problem: observed flocks correlations are surprisingly long ranged



$$C_s(r,L) = r^{-\chi}\, g\left(\frac{r}{\xi}\right)$$

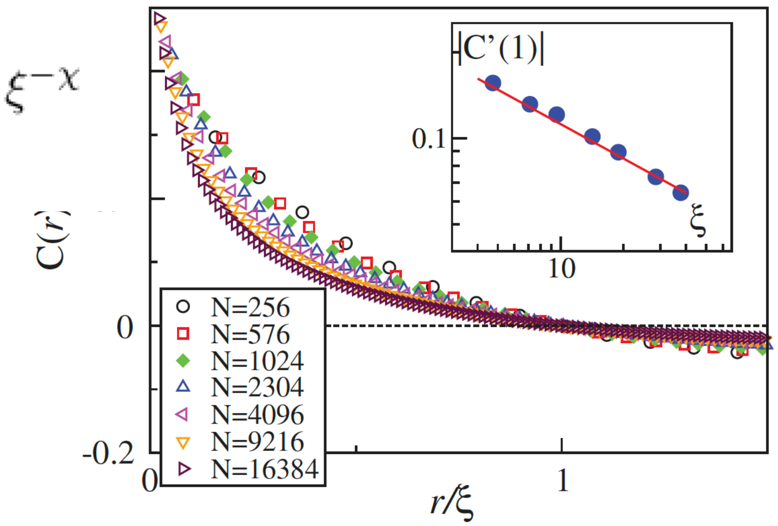
Real Flocks $\chi \sim 0$

$$C(2r, 2L) \sim C(r, L)$$

$$s = -\frac{d}{dy}C(y,L)\bigg|_{y=1} = -\frac{d}{dy}\xi^{-\chi}y^{-\chi}g(y)|_{y=1} \sim \xi^{-\chi}$$

$y = r/\xi$

d=2



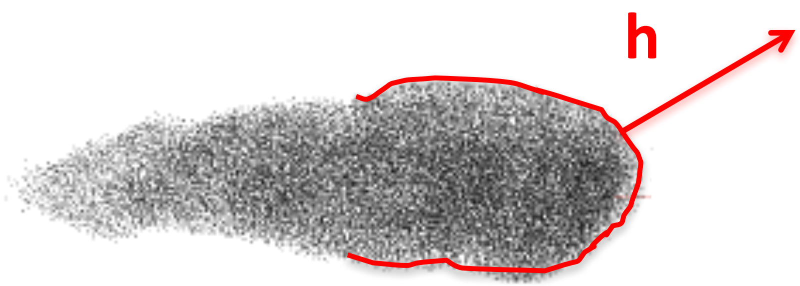
Vicsek flocks (d=3) $\chi \sim 1.2$

(d=2) $\chi \sim 0.6$

Dynamic perturbations localized on the flock boundary

The origin of these anomalous correlations is not in the SPP, out-of-equilibrium nature of flocks…

..but in the interaction – through the boundary – with the external world



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An equilibrium set up:

Heisenberg model with a dynamical boundary magnetic field

$$\mathbf{s}_i^{t+1} = \Theta\left[\Theta[\mathbf{s}_i^t + \sum_{j\in\mathcal{N}_i} \mathbf{s}_j^t + \mathbf{g}(\mathbf{r}_i, \mathbf{h}^t)] + \eta\zeta_i^t\right]$$

$$\Theta[\mathbf{v}] = \mathbf{v}/||\mathbf{v}||$$

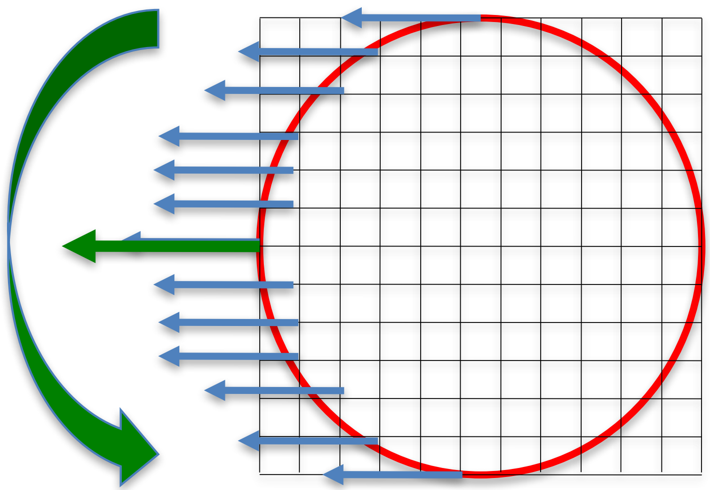
$$\mathbf{g}(\mathbf{r}_i, \mathbf{h}^t) = \mathbf{h}^t \text{ if } \mathbf{r}_i \in \mathcal{B} \text{ and } (\mathbf{h}^t \cdot \mathbf{r}_i) > 0$$

$$\mathbf{g}(\mathbf{r}_i, \mathbf{h}^t) = 0 \text{ otherwise}$$

Spherical domain in a cubic lattice

T<<1 (flocks are very ordered)

Fields only affects part of the boundary



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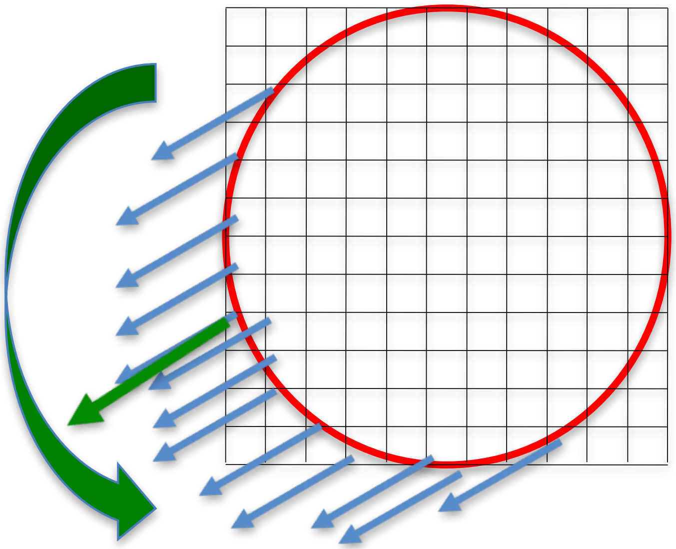


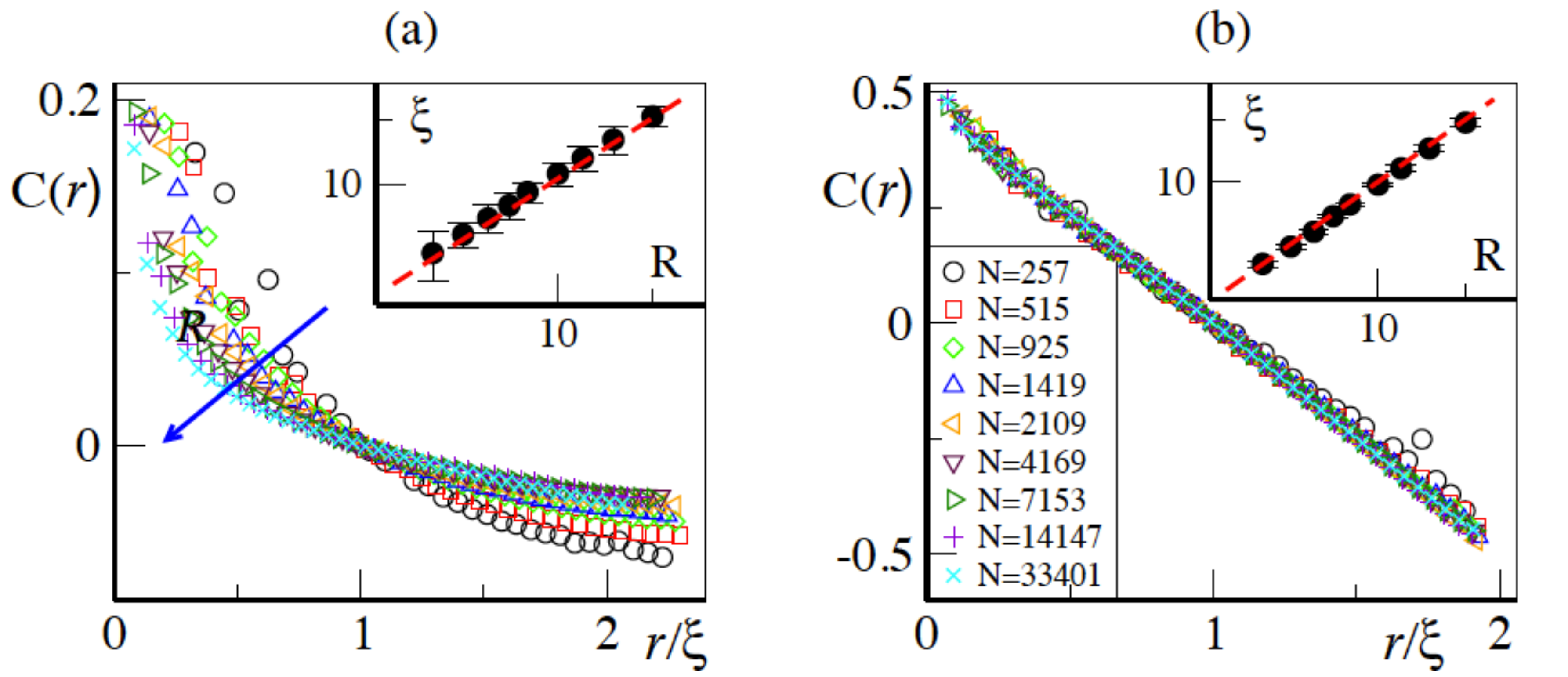
Field $\mathbf{h}^t$ performs a random walk on the spherical surface with typical inversion time

$$\tau_h = R^\alpha$$

$\tau_h \sim R^2$ (Diffusive timescale)

Weak field

Strong field



$\chi \approx 1$

$\chi \approx 0$

Hints of a theory -- Heisenberg hamiltonian in spin-wave approximation

Velocity correlation functions are expressed as **a superposition of eigenmodes** (plane waves on cubic lattice) (eigenvalues weighted) of the Discrete Laplacian matrix A (closely related to local connectivity)

$$C_{ij}^{\mathrm{eq}} = \langle \boldsymbol{\pi}_i \cdot \boldsymbol{\pi}_j \rangle = \sum_{a>1} w_i^a w_j^a \frac{2}{\beta \lambda_a} \qquad A_{ij} = \delta_{ij} \sum_k n_{ik} - n_{ij}$$

Scale free behavior reflects in a **gapless** eigenspectrum of A

$$\Delta\lambda \sim 1/R^2$$

$$\lambda_1 = 0$$

Rotational symmetry

$$\lambda_2 \sim R^{-2} \to 0$$

Massless, scale free correlations

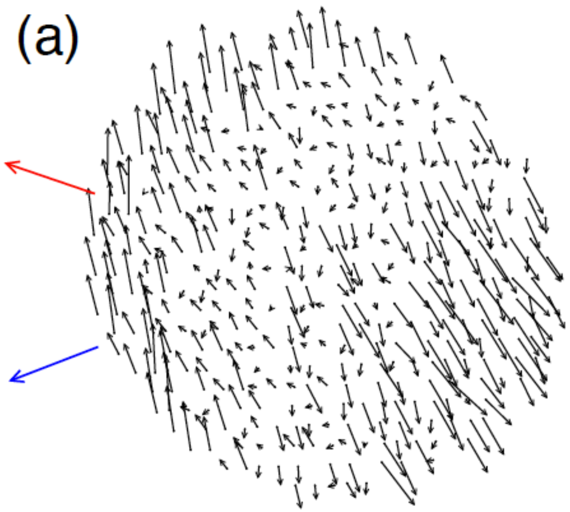
A **localized external field** (e.g. on the **boundary**) does not open a gap and does not create a mass

Velocity fluctuations closely resemble lower order eigenmodes

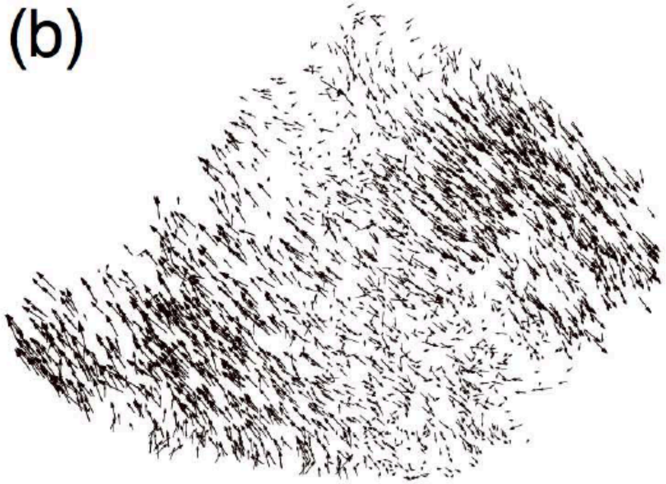
Heisenberg with strong dynamical field

Flocks

(a)



(b)



A

Unperturbed flocking model

Dynamical field added in a Langevin Eq. representation

$$\frac{d\mathbf{s}_i}{dt} = \sum_j n_{ij}\mathbf{s}_j + \mathbf{h}_i - \mu_i \mathbf{s}_i + \boldsymbol{\xi}_i$$

μ is a Lagrange multiplier to enforce $||\mathbf{s}_i|| = 1$

Dynamical field effect

Dynamical correlation function for t >>1

$$C_{ij}(t) = C_{ij}^{\rm eq} + \boxed{2\sum_{a,b} w_i^a w_j^b \int_0^t ds e^{-(\lambda_a+\lambda_b)(t-s)} \mathbf{m}_a^\perp(s) \cdot \mathbf{h}_b^\perp(s)}, \quad = C_{ij}^{EQ} + C_{ij}^{NEQ}$$

$$\mathbf{m}_a^\perp(t) = \mathbf{m}_a^\perp(0) e^{-\lambda_a t} + \int_0^t dt' e^{-\lambda_a(t-t')} \mathbf{h}_a^\perp(t').$$

Decay timescale for the contributions of each eigenmode *a* to the nonequilibrium term

$$\tau_a \sim 1/\lambda_a$$

$$\tau_h >> \tau_a = 1/\lambda_a$$

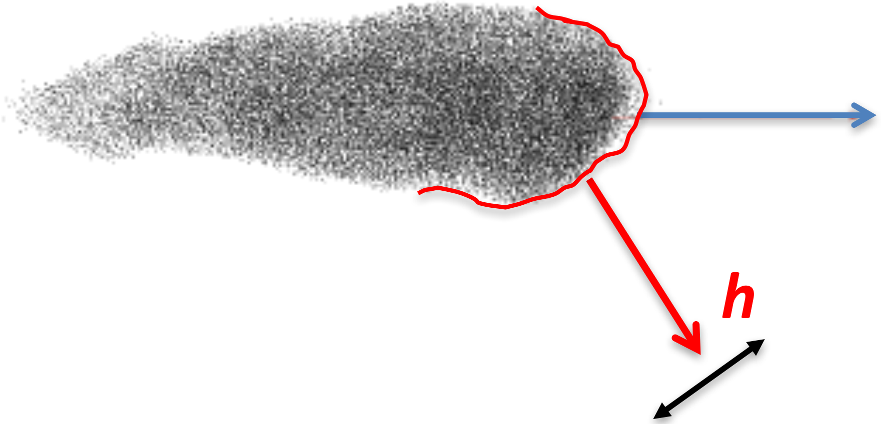
If the field **h** is slow changing, all nonequilibrium contribution decays fast and nothing changes, correlations are similar to equilibrium

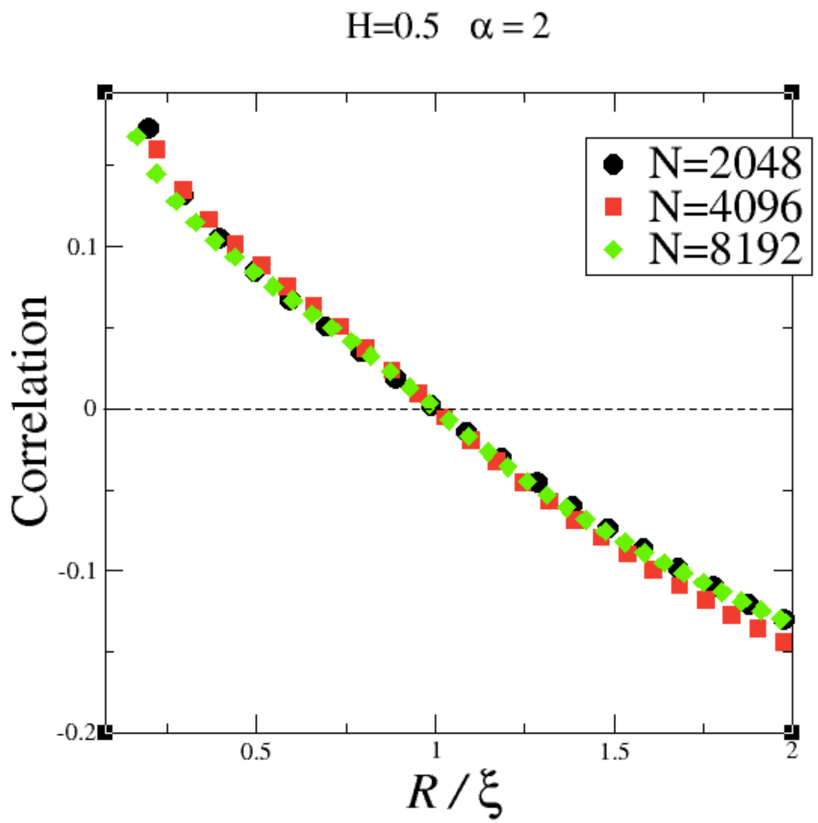
$$\tau_h \leq \tau_a = 1/\lambda_a$$

If the field dynamics is fast enough, the field keeps exciting low modes contributions, which may change the correlations, provided that

Slowest mode timescale $1/\lambda_a \sim R^2$

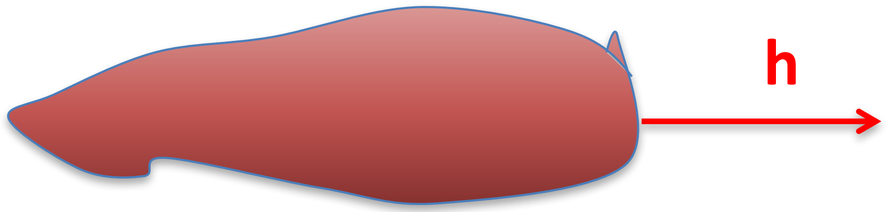
Numerical simulations in flocking models





2. Linear Response theory in Flocking systems

- Linear response in symmetry breaking systems is a classical problem in equilibrium statistical field-theory,
- Response to external threats and biological significance of group response mechanisms.
- Control of biological and synthetic flocks



e.g. Vicsek model in an external field

$$
\begin{aligned}
\mathbf{r}_i^{t+1} &= \mathbf{r}_i^t + v_m \mathbf{v}_i^t \\
\mathbf{v}_i^{t+1} &= (\mathcal{R}_\eta \circ \vartheta)\left(\sum_{j \in S_i} \mathbf{v}_j^t + \mathbf{h} \right)
\end{aligned}
$$

Global field induces symmetry breaking

Asymptotic response of the **order parameter** $\mathbf{\Phi}$ when a constant infinitesimal field is applied in the bulk

$$\Phi = \frac{1}{N}\left|\sum_{i=1}^{N}\mathbf{v}_i\right|$$

$$\chi_{\alpha\beta} = \frac{\partial \phi_\alpha}{\partial h_\beta}$$ Tensor susceptibility

Transversal susceptibility

$$\chi_\perp = \frac{\Phi(0)}{h} \sim \frac{1}{h}$$

Longitudinal susceptibility

$$\chi_{//} = \frac{\Phi(h)-\Phi(0)}{h} \xrightarrow{h\to 0} ?$$

At equilibrium also the longitudinal component diverges

$$\chi_\perp^{eq} \sim \frac{1}{h}$$

$$\chi_\parallel^{eq} \sim \frac{1}{\sqrt{h}}$$ (d=3)

TT phenomenological hydrodynamics with external field

$$\partial_t \rho + \nabla \cdot (\mathbf{v}\rho) = 0$$ (continuity eq.)

$$\partial_t \mathbf{v} + \boldsymbol{\Lambda}\left[\nabla \mathbf{v}\mathbf{v}\right] = U(\rho, |\mathbf{v}|)\mathbf{v} + \mathbf{D}\left[\nabla\nabla\mathbf{v}\right] + \mathbf{F_P} + \mathbf{f} + \mathbf{h}$$

External field: fix a direction in space and drives the motion accordingly

Can be derived either by:

1. **Phenomenological hydrodynamics**
2. **Direct coarse-graining: e.g. Kinetic approaches (Boltzmann-Ginzburg-Landau approach)**

J. Toner, Y. Tu, Phys Rev Lett 75 4326 (1995); , Phys Rev E 58 4828 (1998).

TT phenomenological hydrodynamics with external field

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advective

$$\mathbf{\Lambda}\left[\nabla \mathbf{v}\mathbf{v}\right] \equiv \lambda_1(\mathbf{v}\cdot\nabla)\mathbf{v} + \lambda_2(\nabla\cdot\mathbf{v})\mathbf{v} + \lambda_3\nabla(|\mathbf{v}|^2)$$

Some kind of material derivative (time + convective derivatives), but with extra terms since Galileian invariance is broken

TT phenomenological hydrodynamics with external field

$$\partial_t \rho + \nabla \cdot (\mathbf{v}\rho) = 0$$

$$\partial_t \mathbf{v} + \mathbf{\Lambda}\left[\nabla \mathbf{v}\mathbf{v}\right] = U(\rho, |\mathbf{v}|)\mathbf{v} + \mathbf{D}\left[\nabla\nabla\mathbf{v}\right] + \mathbf{F_P} + \mathbf{f} + \mathbf{h}$$

Diffusive, viscous terms

$$\mathbf{D}\left[\nabla\nabla\mathbf{v}\right] \equiv D_1 \nabla(\nabla \cdot \mathbf{v}) + D_2(\mathbf{v} \cdot \nabla)^2 \mathbf{v} + D_3 \nabla^2 \mathbf{v};$$

TT phenomenological hydrodynamics with external field

$$\partial_t \rho + \nabla \cdot (\mathbf{v}\rho) = 0$$

$$\partial_t \mathbf{v} + \mathbf{\Lambda}\,[\nabla \mathbf{v}\mathbf{v}] = U(\rho, |\mathbf{v}|)\mathbf{v} + \mathbf{D}\,[\nabla\nabla \mathbf{v}] + \mathbf{F_P} + \mathbf{f} + \mathbf{h}$$

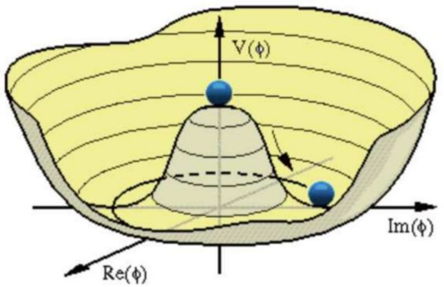
Spontaneous symmetry breaking force

pressure

$$\mathbf{F_P} \equiv -\nabla P_1 - \mathbf{v}\,(\mathbf{v} \cdot \nabla P_2)$$

$$P_i(\rho) = P_i^0 + \sum_{n=1}^{\infty} \sigma_{i,n}(|\vec{v}|)\delta\rho^n$$

$$U = \mu(\rho) - \beta(\rho)|\mathbf{v}|^2 = -\frac{\partial V}{\partial |\mathbf{v}|}$$



$$v_0(0) = \sqrt{\mu/\beta},$$

TT phenomenological hydrodynamics with external field

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$$\partial_t \mathbf{v} + \mathbf{\Lambda}\left[\nabla \mathbf{v}\mathbf{v}\right] = U(\rho, |\mathbf{v}|)\mathbf{v} + \mathbf{D}\left[\nabla\nabla\mathbf{v}\right] + \mathbf{F_P} + \mathbf{f} + \mathbf{h}$$

$$< f_i(\mathbf{r}, t) f_j(\mathbf{r}', t') >= \Delta \delta_{ij} \delta^d(\mathbf{r} - \mathbf{r}')\delta(t - t')$$

order parameter $\Phi(h) \equiv |\langle \mathbf{v}(\mathbf{r}, t)\rangle|$

No fluctuations (mean field):

$$\rho(\mathbf{r},t)=\rho_0,\ \mathbf{v}(\mathbf{r},t)=\mathbf{v}_0(\mathbf{h}).$$

$$U(v_0(h),\rho_0)\approx -\frac{h}{v_0}\neq 0$$

$$\Phi(h)\equiv|\langle\mathbf{v}(\mathbf{r},t)\rangle| = v_0(0)+O(h)$$

At mean field level, response is linear in *h*

Consider fluctuations:

$$\rho(\mathbf{r},t) = \rho_0 + \delta\rho(h;\mathbf{r},t)$$

Slow modes

$$\mathbf{v}(\mathbf{r},t) = \mathbf{v}_0(h) + \delta v_{\parallel}(h;\mathbf{r},t)\,\mathbf{e}_{\parallel} + \mathbf{v}_{\perp}(h;\mathbf{r},t)$$

$$\langle \delta\rho \rangle = \langle \mathbf{v}_{\perp} \rangle = \mathbf{0}$$

$$\Phi(h) = v_0(h) + \langle \delta v_{\parallel} \rangle = v_0(0) + \langle \delta v_{\parallel} \rangle + O(h)$$

Longitudinal fluctuations affect response

Longitudinal fluctuations are enslaved to slow modes:

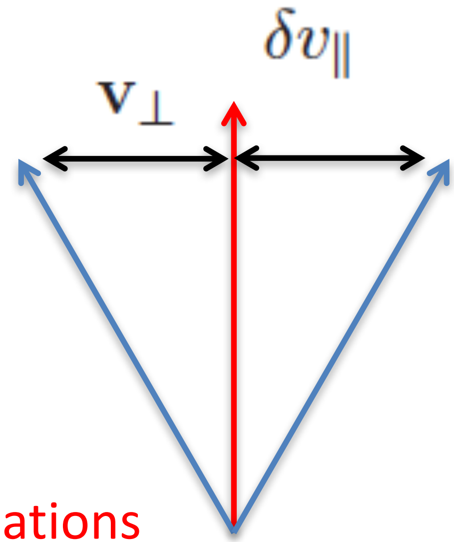
J. Toner, Phys. Rev. E 86 , 031918 (2012).

$$\partial_t \delta\rho = [\partial_t \delta\rho]_{h=0}$$
$$\partial_t \mathbf{v}_\perp = [\partial_t \mathbf{v}_\perp]_{h=0} - h_v \mathbf{v}_\perp$$

$$h_v = h/v_0$$

$$\delta v_\parallel \approx -\frac{|\mathbf{v}_\perp(h)|^2}{2v_0(0)} + \mu_1 \delta\rho + (\mu_2 \partial_t + \mu_3 \partial_\parallel)\,\delta\rho + \mu_4 \nabla_\perp \cdot \mathbf{v}_\perp + O(h)\,.$$

Spatial average



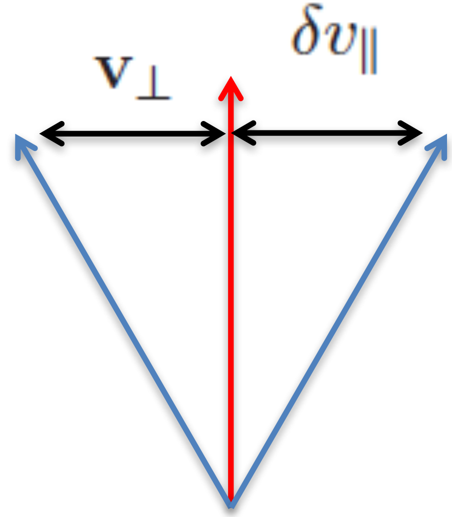
$$\langle \delta v_\parallel \rangle \approx -\frac{\langle |\mathbf{v}_\perp(h)|^2 \rangle}{2v_0(0)} + O(h)$$

Link between longitudinal and transversal fluctuations

AKA: Principle of conservation of modulus in equilibrium

A. Z. Patashinskii and V. L. Pokrovskii, Zh. Eksp. Teor. Fiz. 64 , 1445 (1973).

Order parameter depends on transversal fluctuations



$$\Phi(h) \approx v_0(0) - \frac{\langle|\mathbf{v}_\perp(h)|^2\rangle}{2v_0(0)} + O(h)$$

Linear response is given by the correlation function *C*…

$$\delta\Phi(h) \equiv \Phi(h) - \Phi(0) \approx \frac{\langle|\mathbf{v}_\perp(0)|^2\rangle - \langle|\mathbf{v}_\perp(h)|^2\rangle}{2v_0(0)} + O(h).$$

$$\langle|\mathbf{v}_\perp(0)|^2\rangle - \langle|\mathbf{v}_\perp(h)|^2\rangle \equiv C\left(L_\perp, L_\parallel, \{\mu_i^0\}, h_v\right)$$

… whose scaling can be determined by DRG techniques

$$C\left(L_\perp, L_\parallel, \{\mu_i^0\}, h_v\right) = b^{2\alpha} C\left(b^{-1}L_\perp, b^{-\zeta}L_\parallel, \{\mu_i^b\}, b^z h_v\right)$$

$$h_v \equiv \frac{h}{v_0(0)} \qquad ; \; b = \left(\frac{h_v}{h_v^*}\right)^{-1/z}$$

$$\delta\Phi = h^{1-\nu} f\left(L h^{\frac{1}{z}}\right) \propto \begin{cases} h^{1-\nu}, & h \gg L^{-z} \\ h L^{\gamma}, & h \ll L^{-z} \end{cases}$$

$\nu = 1 + 2\alpha/z$

$\gamma = \nu z$

HPC: $\alpha(d=2) \approx -0.3$ $\quad z(d=2) \approx 1.33$

$\alpha(d=3) \approx -0.6$ $\quad z(d=3) \approx 1.75$

RG conjecture $\quad \nu = \dfrac{4-d}{d+1}, \quad z = \dfrac{2(d+1)}{5}, \quad \gamma = \dfrac{2(4-d)}{5}$

Diverging longitudinal susceptibility in the thermodynamic limit

$$\chi_{//} = \frac{\delta\Phi(h)}{h} \sim h^{-\nu}$$

Early works overlooked this, e.g.

A. Czirok, H. E. Stanley, and T. Vicsek, J. Phys. A 30 , 1375 (1997).

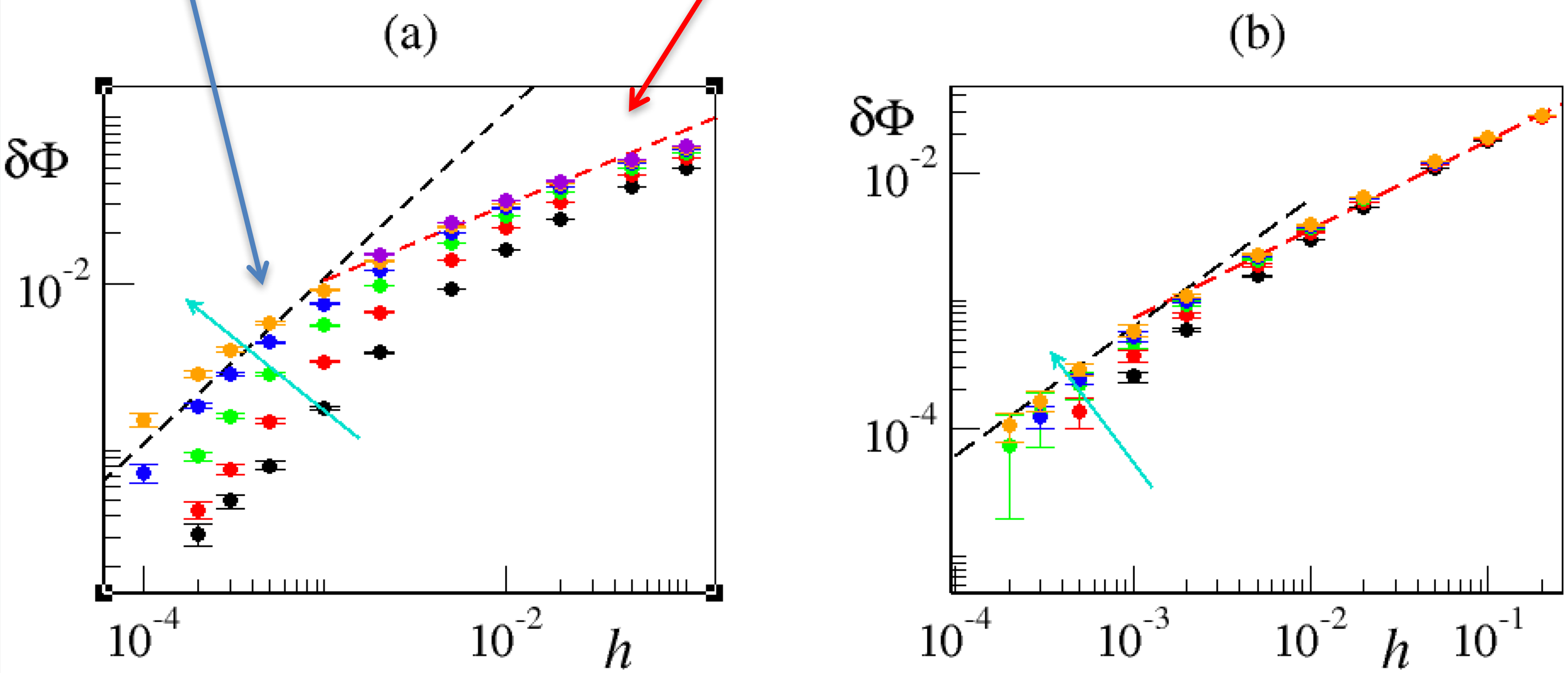
At and above the upper critical dimension d_c=4 the susceptibility is finite

•*N. Kyriakopoulos, FG, J. Toner, New Journal of Physics, 18, 073039 (2016).*

Numerical simulatios for response – VM + field

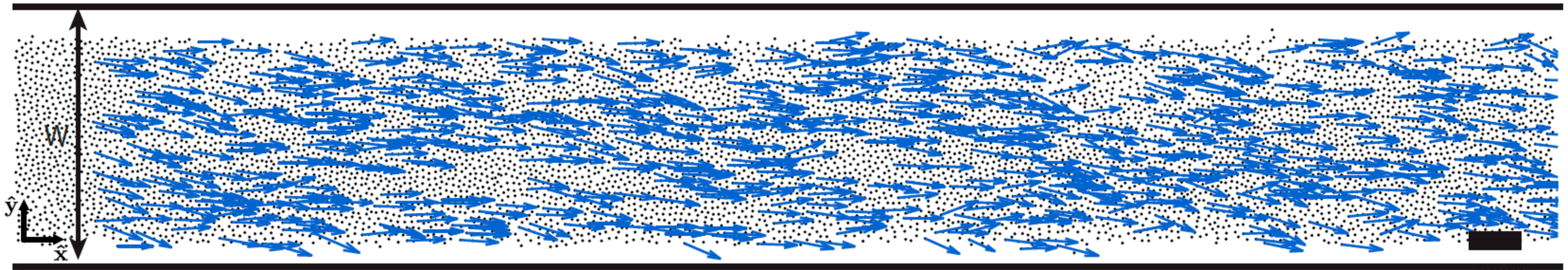
Linear, finite size regime scaling

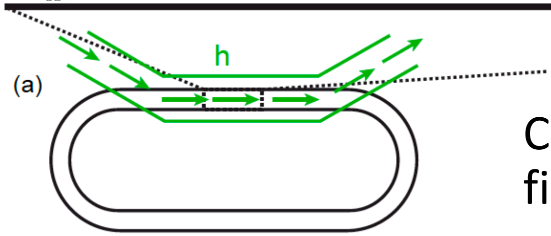
Large size regime scaling $h \gg L^{-z}$



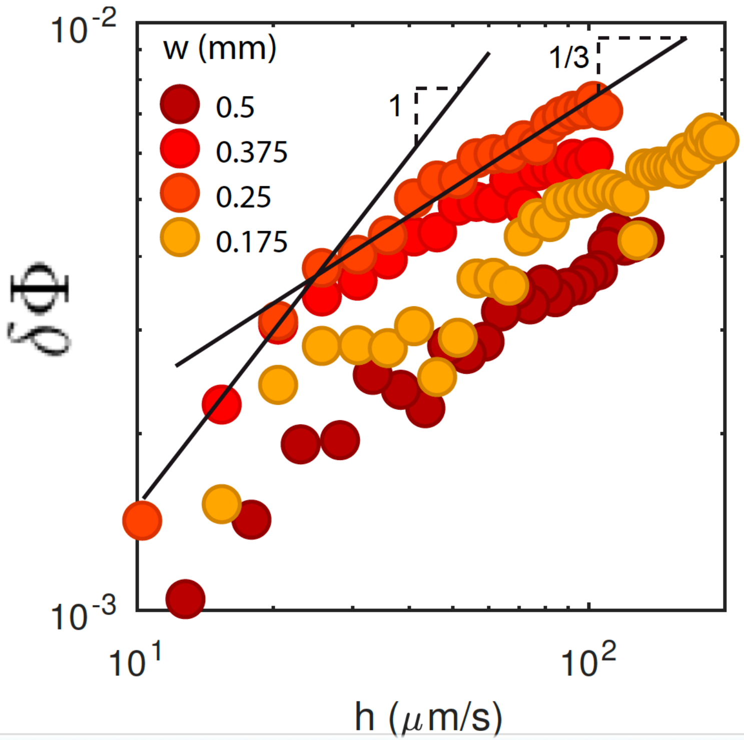
d=2 *d*=3

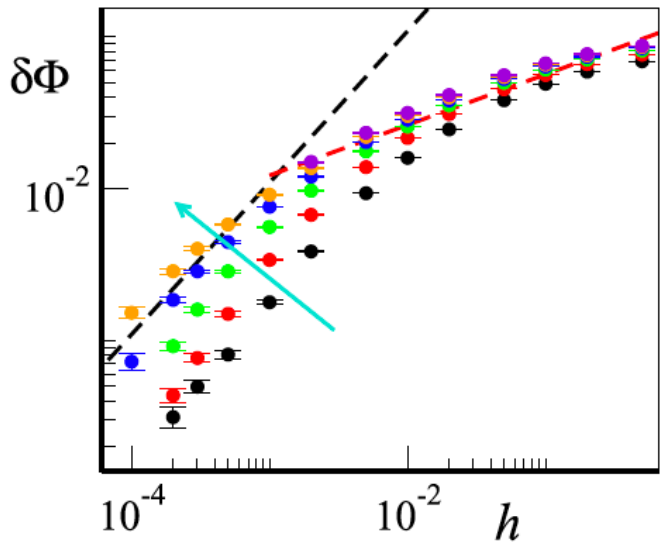
Experiment: Longitudinal response and susceptibility





Colloidal Quincke rollers subjected to a flow field (in green) of speed *h*





Theoretical predictions are expected to hold as long as the external field affects a **finite** fraction of particles.

•*A. Morin, D. Bartolo,* Phys. Rev. X **8**, 021037 *(2018)*

What about correlation functions ?

$$L_c(h) \sim h^{-1/z}$$

Intrinsic, field dependent length scale

$$r \sim L_c(h)$$

Exponential cut off ~ L_c in real space correlations

$$q \ll \Lambda(h) \sim \frac{1}{L_c(h)}$$

Small q divergence is suppressed in Fourier

$$S_\rho(q) \sim \frac{1}{\langle \bar{b} \rangle q^z + h}$$

At leading order in q and h

$z = \frac{6}{5} \ (d = 2)$ $\quad$ $z = \frac{8}{5} \ (d = 3)$ $\quad$ $z = 2 \ (d \geq 4)$

[Probably, under some RG conjecture]

TT equations for fluctuations

$$\rho(\mathbf{r},t)=\rho_0+\delta\rho(h;\mathbf{r},t)$$

Slow modes

$$\mathbf{v}(\mathbf{r},t)=\mathbf{v}_0(h)+\delta v_\parallel(h;\mathbf{r},t)\,\mathbf{e}_\parallel+\mathbf{v}_\perp(h;\mathbf{r},t)$$

$$\partial_t\delta\rho=-\rho_0\nabla_\perp\cdot\mathbf{v}_\perp-w_1\nabla_\perp\cdot(\mathbf{v}_\perp\delta\rho)-v_2\partial_\parallel\delta\rho+D_{\rho_\parallel}\partial_\parallel^2\delta\rho+D_{\rho_\perp}\nabla_\perp^2\delta\rho$$
$$+D_{\rho v}\partial_\parallel(\nabla_\perp\cdot\mathbf{v}_\perp)+\rho_0\mu_2\partial_t\partial_\parallel\delta\rho-\mu_1\partial_\parallel(\delta\rho^2)+\mu_5\partial_\parallel(|\mathbf{v}_\perp|^2),$$

$$\partial_t\mathbf{v}_\perp=-\lambda_1^0 v_0\partial_\parallel\mathbf{v}_\perp-\lambda_1^0(\mathbf{v}_\perp\cdot\nabla_\perp)\mathbf{v}_\perp-g_1\delta\rho\partial_\parallel\mathbf{v}_\perp-g_2\mathbf{v}_\perp\partial_\parallel\delta\rho-\frac{c_0^2}{\rho_0}\nabla_\perp\delta\rho$$
$$-g_3\nabla_\perp(\delta\rho^2)+D_B\nabla_\perp(\nabla_\perp\cdot\mathbf{v}_\perp)+D_3\nabla_\perp^2\mathbf{v}_\perp+D_\parallel\partial_\parallel^2\mathbf{v}_\perp+g_t\partial_t\nabla_\perp\delta\rho$$
$$+g_\parallel\partial_\parallel\nabla_\perp\delta\rho+\mathbf{f}_\perp-h_v\mathbf{v}_\perp,$$

$$h_v\equiv\frac{h}{v_0(0)}$$

Solve linearized dynamics in Fourier

(both space and time)

$$\delta\hat{\rho}(\mathbf{q},\omega) = \int dt \int d^d r\, \delta\rho(\mathbf{r},t) e^{-i(\omega t - \mathbf{r}\cdot\mathbf{q})}$$

$$\hat{\mathbf{v}_\perp}(\mathbf{q},\omega) = \int dt \int d^d r\, \mathbf{v}_\perp(\mathbf{r},t) e^{-i(\omega t - \mathbf{r}\cdot\mathbf{q})}$$

Retain leading orders in (small) q and h

$$S_\rho(\mathbf{q},\omega) = \langle \delta\hat{\rho}(\mathbf{q},\omega)\delta\hat{\rho}(-\mathbf{q},-\omega)\rangle$$

Integrate in ω to get equal time structure factor

$$S_\rho(\mathbf{q}) = \langle \delta\hat{\rho}(\mathbf{q})\delta\hat{\rho}(-\mathbf{q})\rangle \approx \frac{\Delta\rho_0^2 v_0}{2c_0^2}\,\frac{1}{b(\theta_q)q^2 + h}$$

Linear theory SF

Nonlinear effects can be accounted for by RG arguments $\quad q^2 \to q^z$

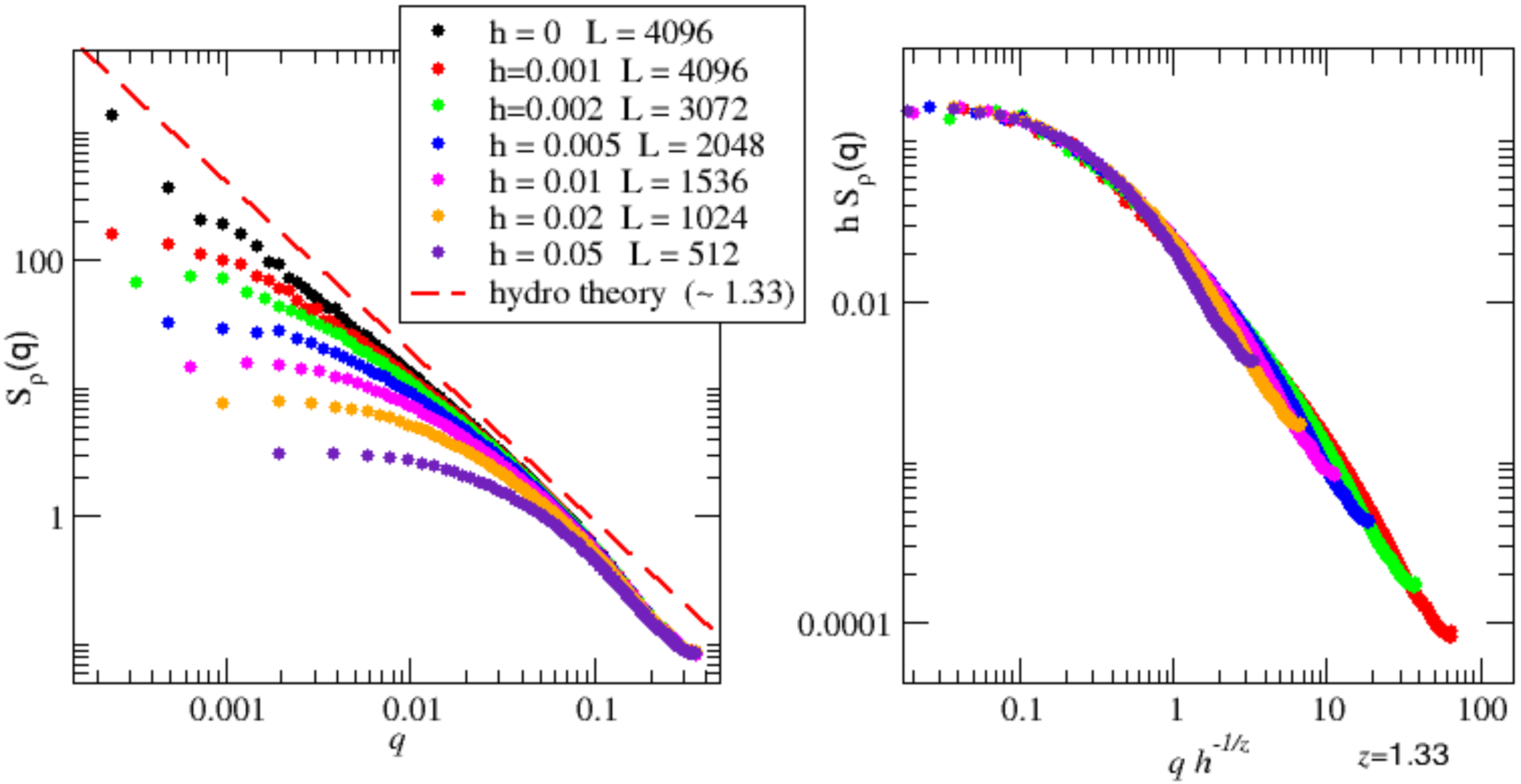
Average over spatial directions, to get the « isotropic » structure factor

$$S_\rho(q) \sim \frac{1}{\langle \bar{b}\rangle q^z + h}$$

Numerical results (Vicsek model, d=2)

$$S_\rho(q) \sim \frac{1}{\langle \bar{b} \rangle q^z + h}$$

Density Structure factor



Driven, not spontaneous collective motion

(how to tell the difference with short timeseries)

A simple (universal) recipe for discriminating spontaneous from driven collective motion

1. Hope your moving system is large enough

2. Compute the structure factor

$$S_\rho(\mathbf{q}) = \frac{1}{N}\left\langle \left| \sum_{j=1}^{N} e^{-i\mathbf{q}\cdot\mathbf{r}_j} \right|^2 \right\rangle$$

3. Check low q behavior.

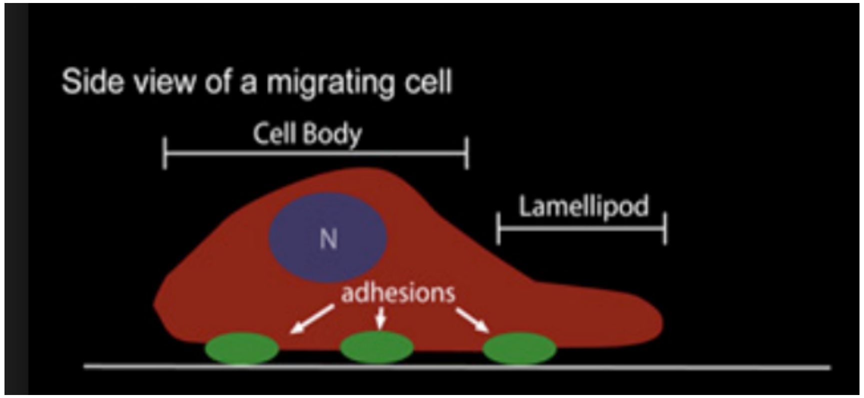
Diverging (spontaneous) or *constant* (driven)

$$S_\rho(q \ll 1) \qquad q > \frac{1}{L}$$

4. Criteria may be sufficient but not necessary for establishing directed collective motion

Perspectives: a controlled experiment

Cellular tissue migrating on a **micrograted substrate**



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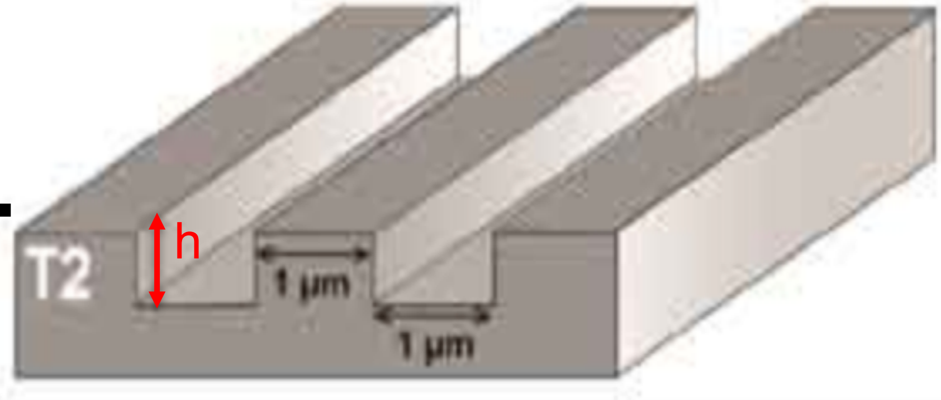

Perspectives: into the wild?

Thank you for your attention!

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(former) Aberdeen group:

PhD students:

N. Kyriakopoulos *(former)*

M. Carlu

M. Faggian

C. Zancok

Post-doc

S. Ngo *(former):*

Collaborations:

H. Chate (Paris, Fr)

J. Toner (Oregon, US)

A. Cavagna (Rome)

I. Giardina (Rome)

R. Cerbino (Milan)

F. Giavazzi (Milan)

G. Scita (Milan)

Financial support:

EPSRC Pioneering research and skills

SUPA

MARIE CURIE ACTIONS

COSMOS

Driven, not spontaneous collective motion

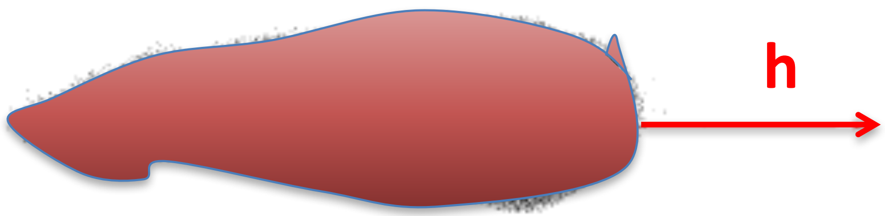
Particle still interact but also orient with an external field

e.g. Vicsek model in an external field

Global field induces symmetry breaking

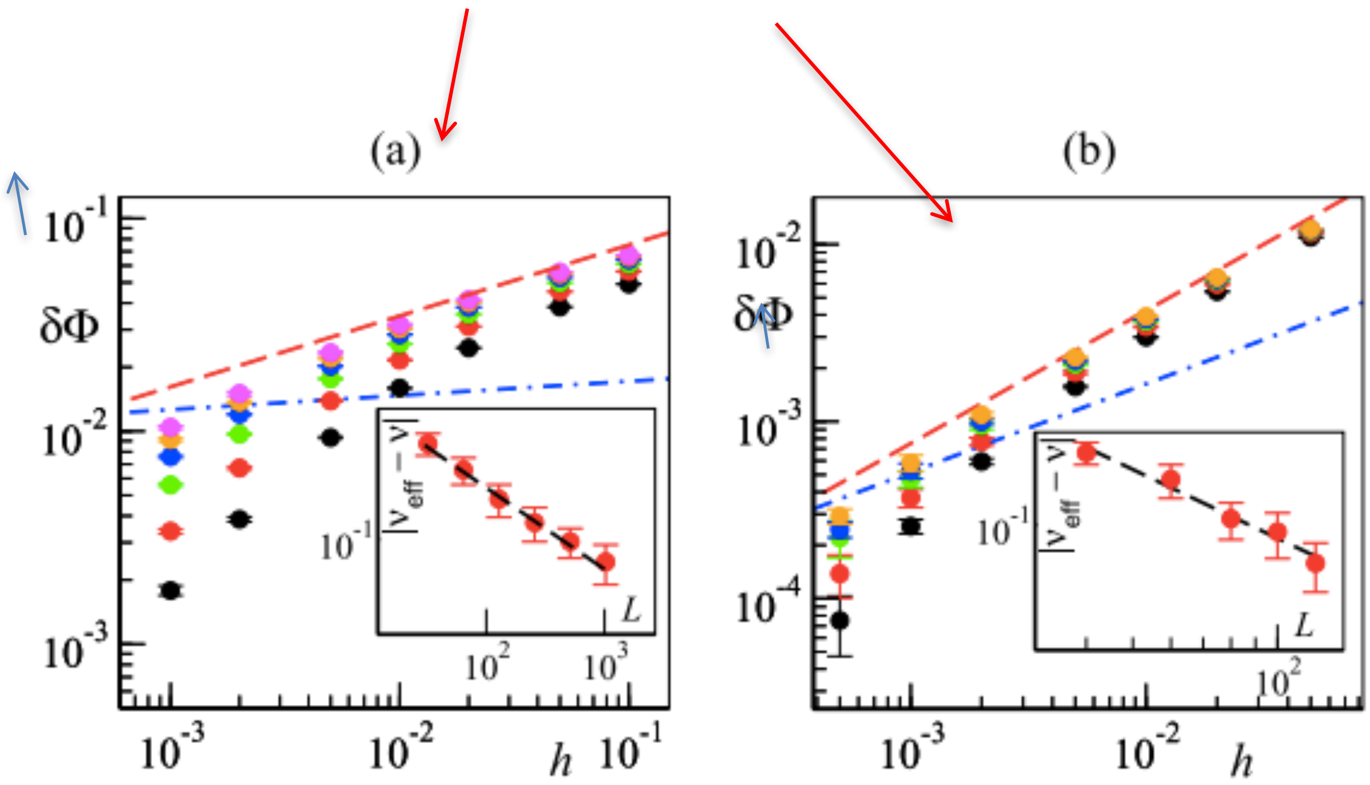
$$\mathbf{r}_i^{t+1} = \mathbf{r}_i^t + v_m \mathbf{v}_i^t$$

$$\mathbf{v}_i^{t+1} = (\mathcal{R}_\eta \circ \vartheta)\left(\sum_{j \in S_i} \mathbf{v}_j^t + \mathbf{h}\right)$$



Large system size (thermodynamic behavior)

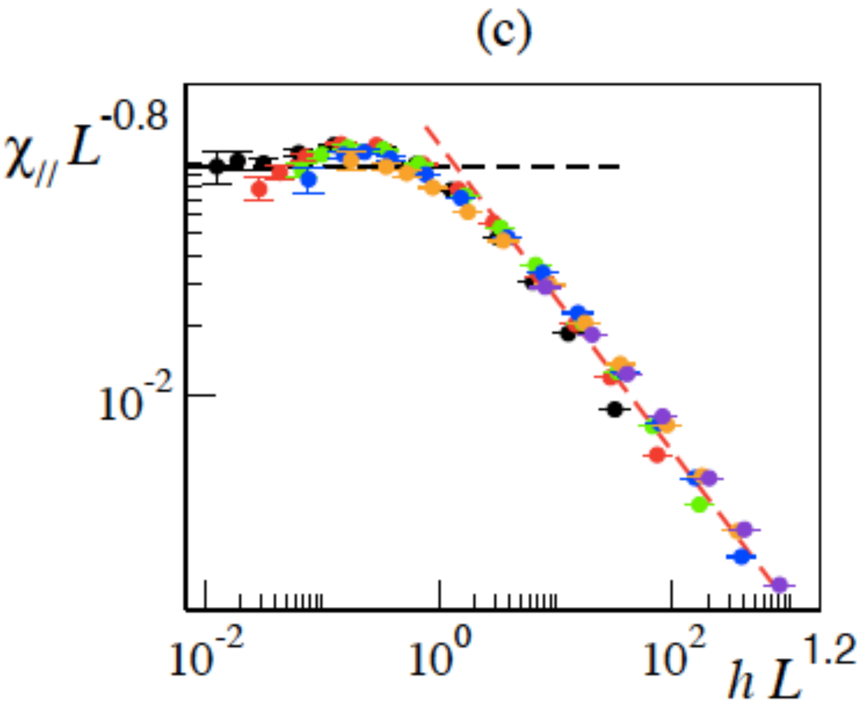
$$1-\nu$$



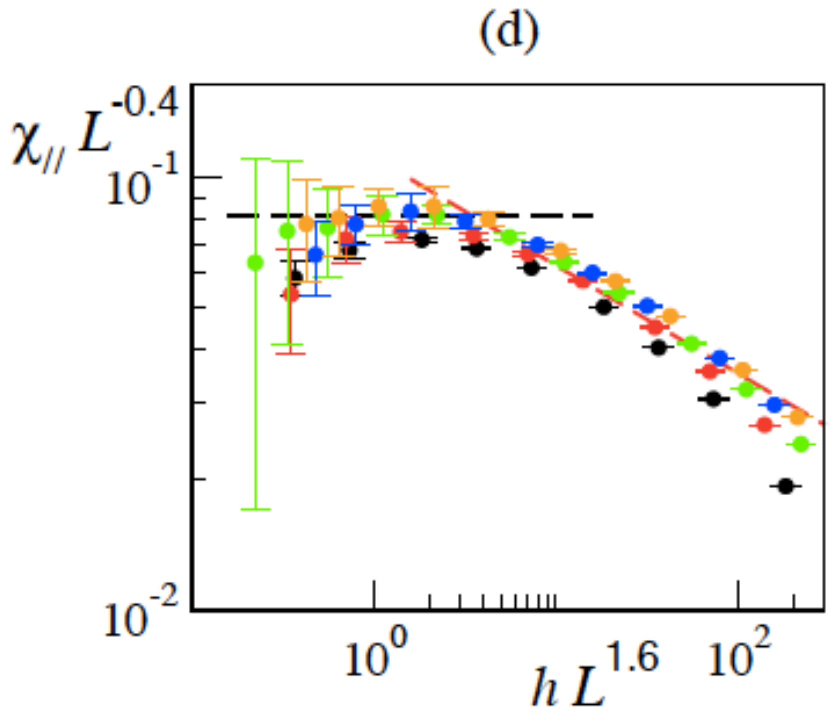
d=2

d=3

Data collapse tests all 3 response exponents



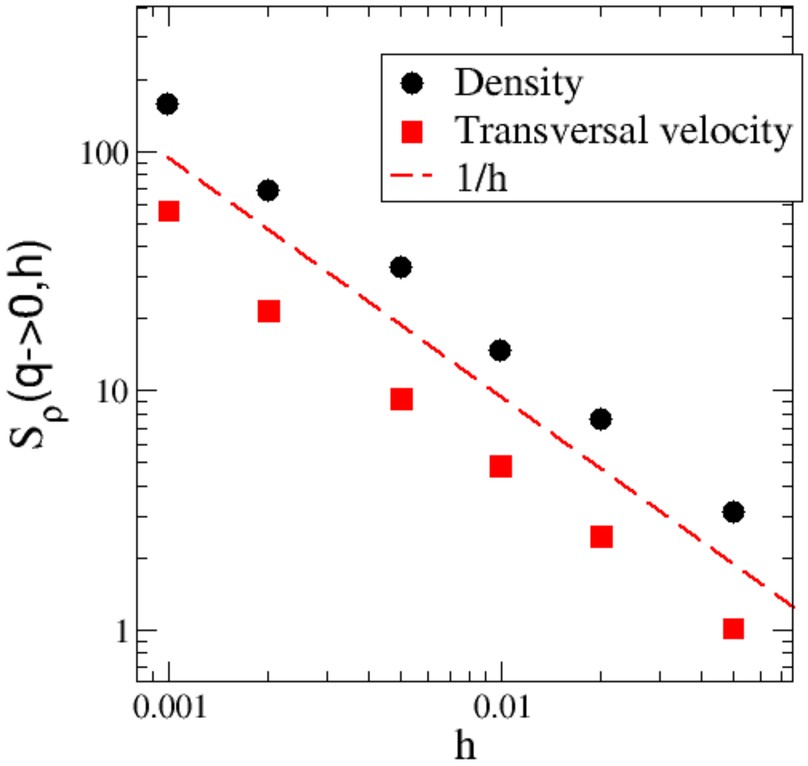
d=2



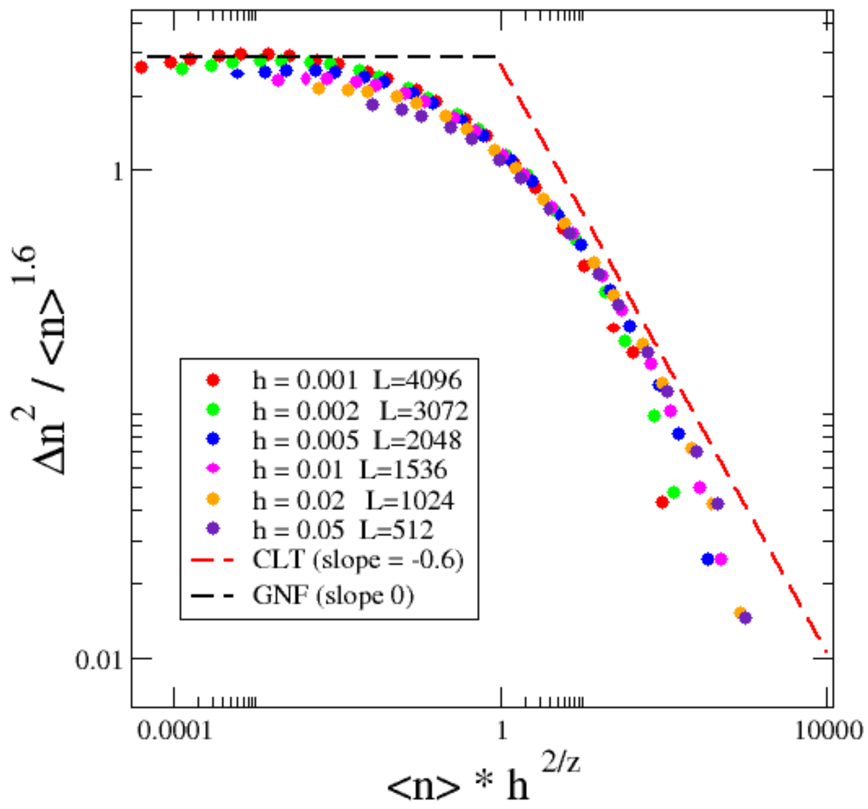
d=3

Numerical results (Vicsek model, d=2)

$$S_\rho(q) \sim \frac{1}{\langle \bar{b} \rangle q^z + h}$$



Giant number fluctuations



GNF cutted off at $n_c(h) \sim L_c(h)^2 \sim h^{-2/z}$

Flocking -- Conclusions …

- Active matter: Fundametal class of non-equilibrium system. Biologically inspired
- Some reasonable theoretical understanding especially for low density, dry systems. Hydrodynamic behavior based on symmetry and conservation laws
- Relevant experiments exist (animal groups, motility assays, driven granular matter, cellular tissues, etc.)

… & Perspectives

- Biological relevance: can active matter help explain biologically relevant problems
- Synthetic active matter: swarming nanoparticles (medical applications), biomimetic materials, funtionalized colloids

- Response to perturbations (linear and finite regimes) and control
- Genericity of mesoscale behavior in high density active matter/flocks
- Thermodynamic approaches
- Finite systems, boundary effects, etc.
- Long range hydrodynamic interactions in active suspensions

Adding a cohesive interaction

$$\mathbf{s}_i^{t+1} = \Theta\left[\alpha \sum_{k \in V_i} \mathbf{s}_k^t + \beta \sum_{k \in V_i} f_{ik} \mathbf{e}_{ik}^t + \eta\, m_i^t\, \vec{\xi}_i^{\,t}\right]$$

Orientation + attraction-repulsion + noise

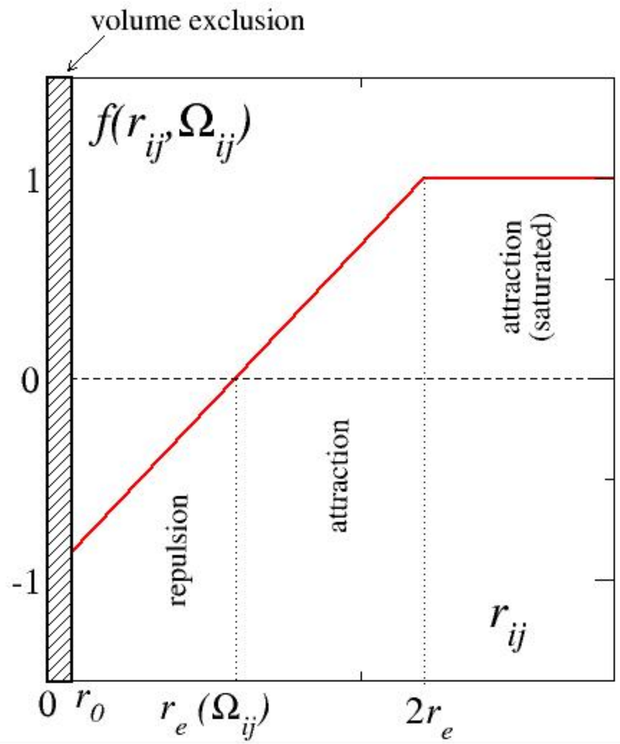
$$\left|\vec{\xi}_i^{\,t}\right| = 1 \qquad \left\langle \vec{\xi}_i^{\,t} \cdot \vec{\xi}_j^{\,t'} \right\rangle \sim \delta_{ij}\delta_{tt'}$$

V_j are Voronoi neigbours

$$\Theta[\mathbf{v}] = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

$$\mathbf{r}_i^{t+1} = \mathbf{r}_i^t + v_0\, \mathbf{s}_i^{t+1}$$

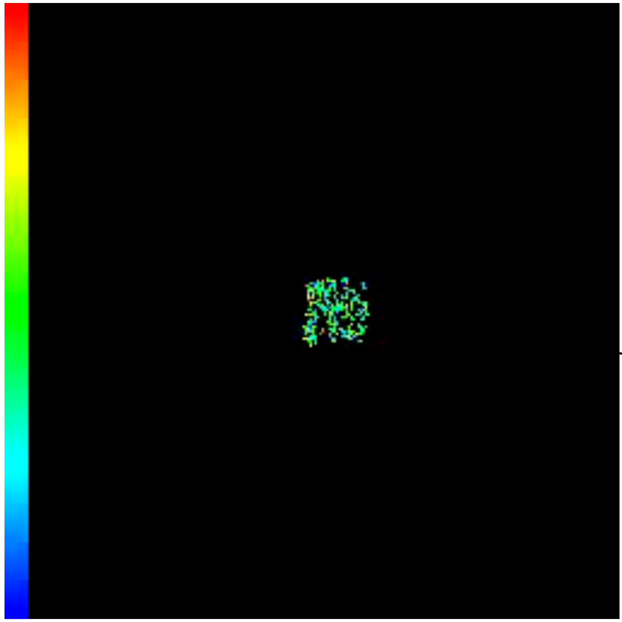
$$f_{jk} = \begin{cases} -\infty & \text{if} \quad r_{jk} < r_0 \\ \dfrac{r_{jk} - r_e}{r_e} & \text{if} \quad r_0 \le r_{jk} < 2r_e \\ 1 & \text{if} \quad 2r_e \le r_{jk} \end{cases}$$

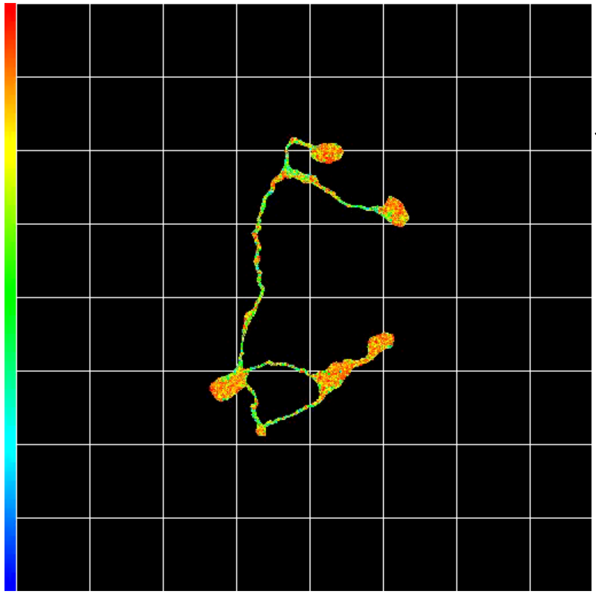


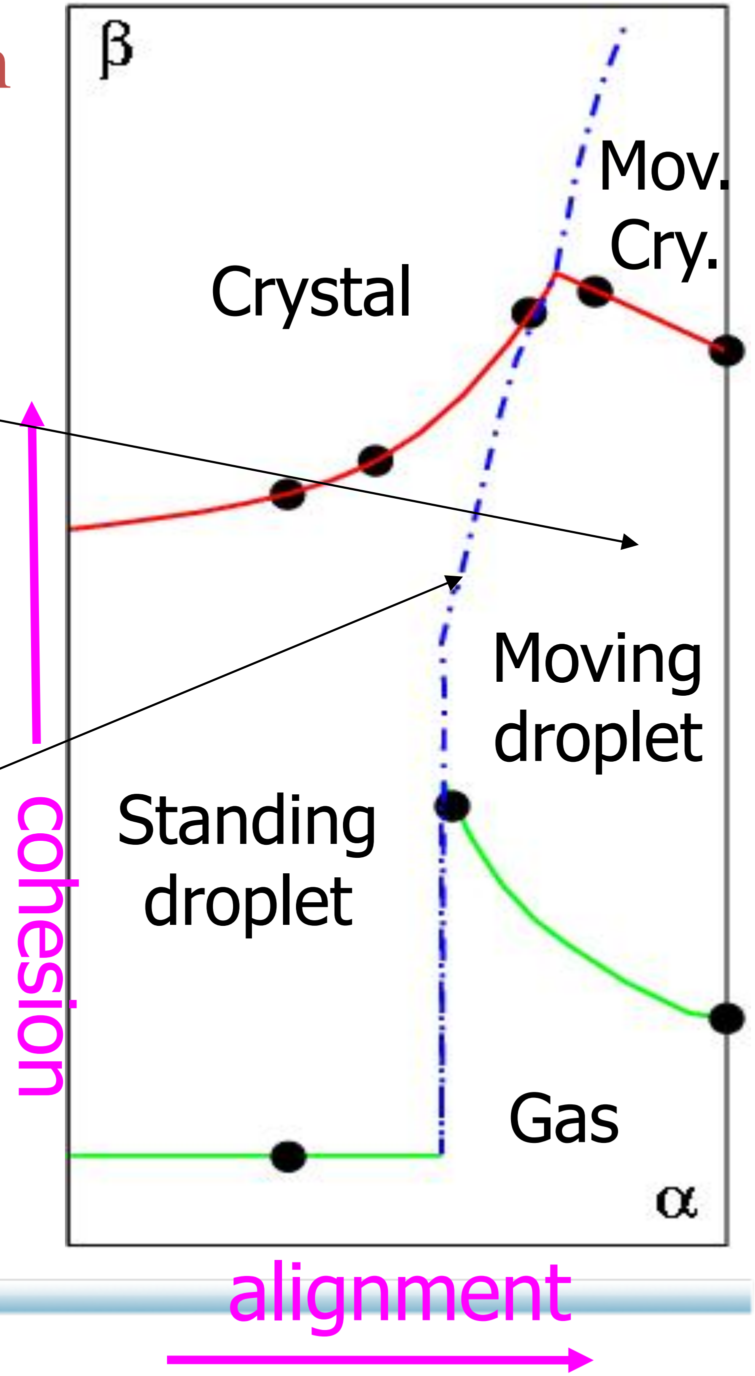
Some field study evidence:
Tien JH, Levin SA, Rubenstein DI (2004)

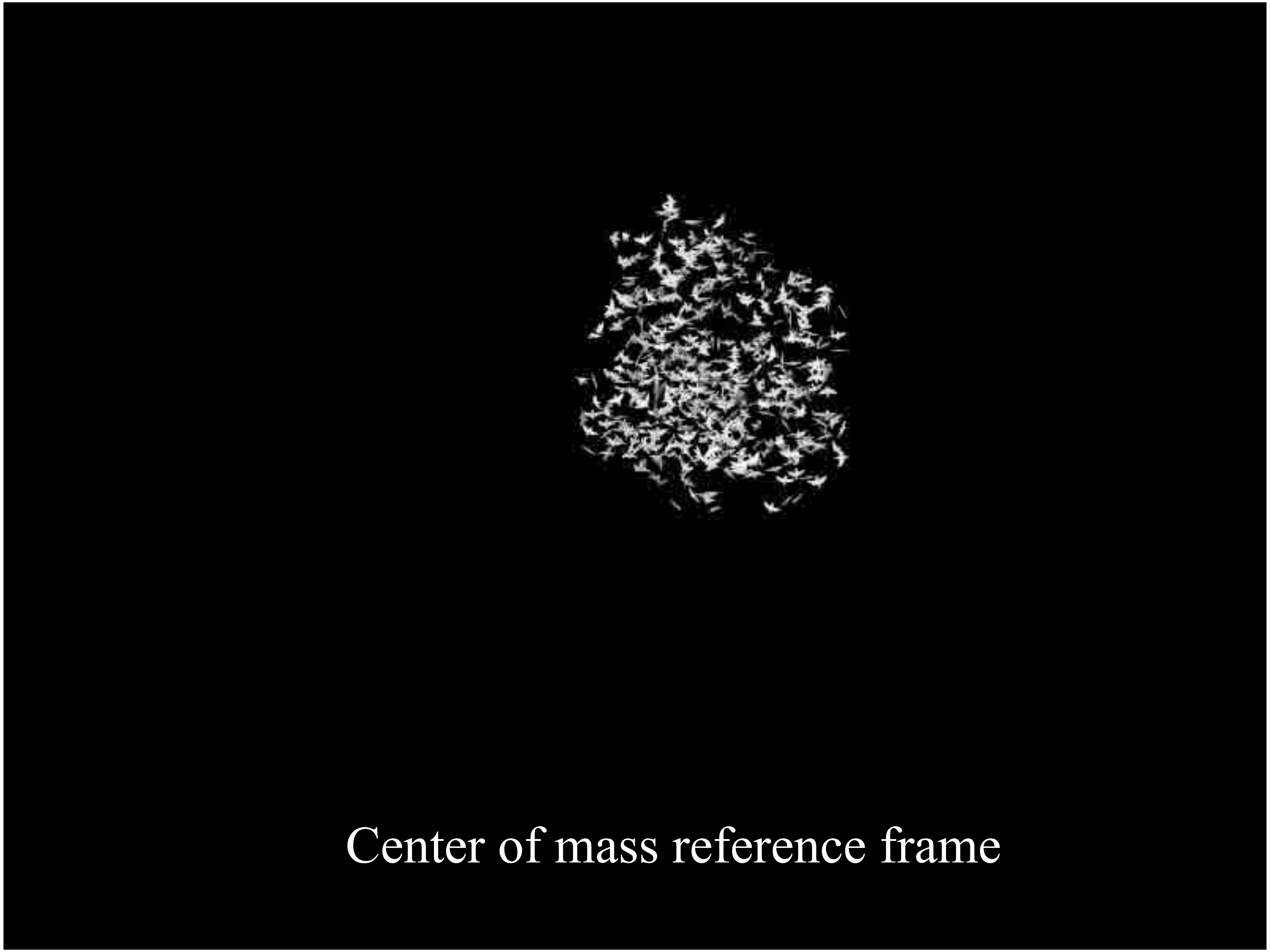
G. Grégoire, H. Chaté & Y. Tu *Physica D* **181,** 157 (2003)

Phase diagram

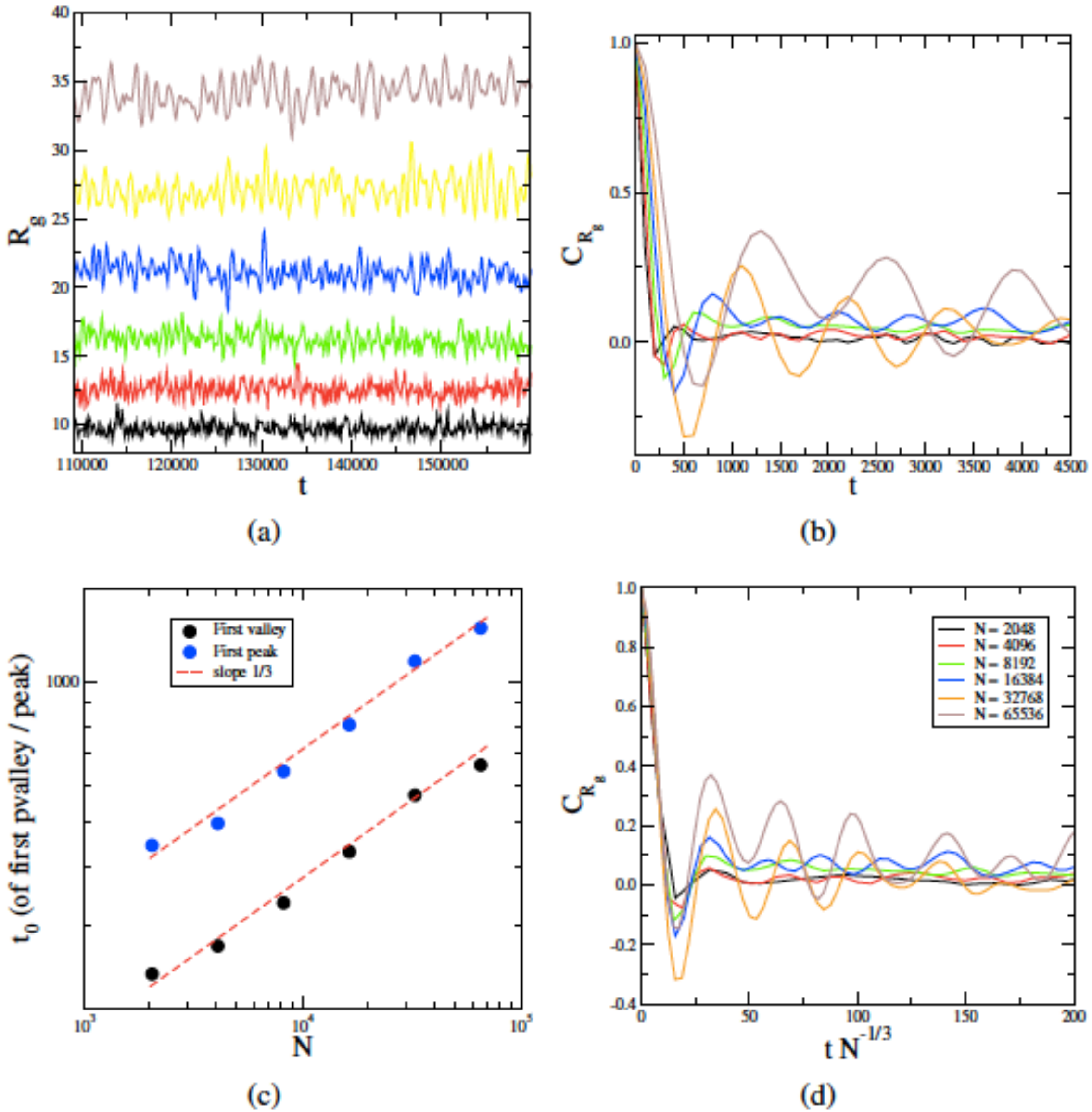







Center of mass reference frame

Border fluctuations in d=3



Border fluctuations in d=3

Equilibrium droplet fluctuation frequency

$$\omega_{eq}^2 \sim \frac{\sigma_S}{N}$$

Flocks fluctuation frequency

$$\omega_f^2 \sim \frac{\sigma_S}{N^{2/3}}$$

Faster fluctuations in active flocks

Conclusions

First **experimental measure of GNF and structure** in biological active matter showing long range polar order

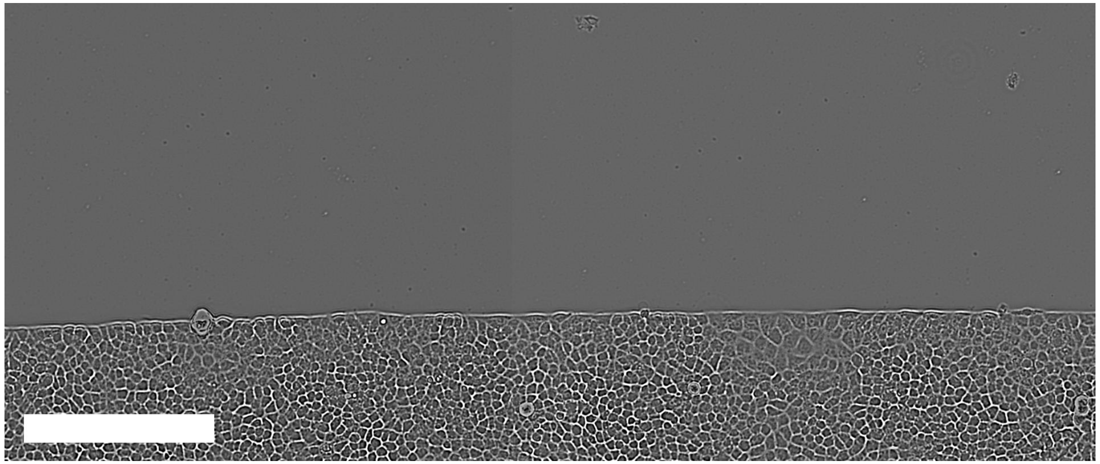
A **simple mechanical model** of soft self-propelled disks **reproduces fairly well a wide range of scale**, at the **local**, **mesoscopic** and **hydrodynamic** range.

At the experimental level, the flocking transition is accompanied by **local fluidization**. In simulations, this can be achieved by a large increase of self-propulsion speed. This suggests that an (indirect ?) effect of **RAB5A** expression is to **reduce the mechanical feedbacks (contact inhibition of locomotion) that suppress cellular motility in the disordered control**

Perspectives

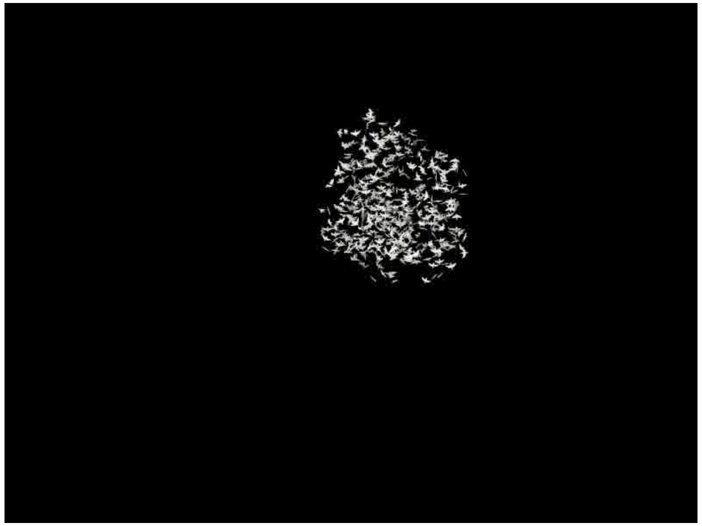
1. Use larger FOVs, measure velocity fluctuations,
2. Interaction of local stresses and elastic modes w. velocity fluctuations

3. Boundary instability/unjamming induced by activity



Wound healing in-vitro experiments
L. Sepulveda et al. Plos Comp. Bio (2013)

finite flock model



Hydrodynamic theory predicts universal long-ranged properties

Due to symmetry breaking (i.e.: there is a preferential directions) correlations are anisotropic

Velocity (connected) correlations

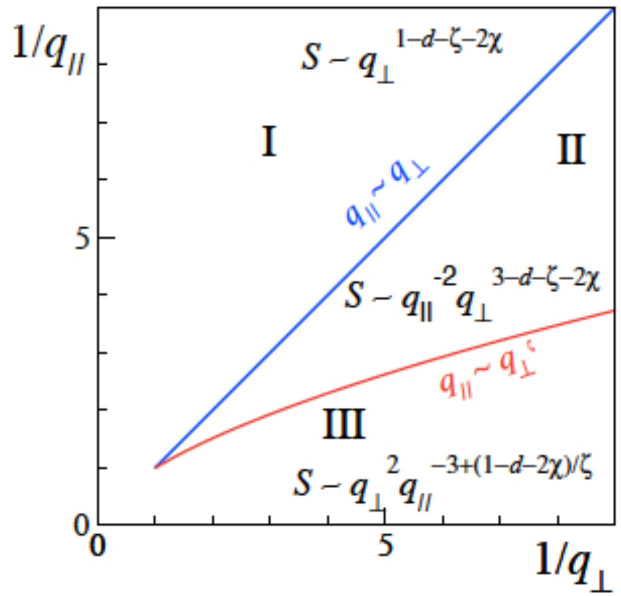
$$S(\mathbf{q}) \sim \begin{cases} q_\perp^{1-d-\zeta-2\chi}, & \\ q_\parallel^{-2} q_\perp^{3-d-\zeta-2\chi}, & q_\perp^\zeta \gg q_\parallel \gg q_\perp \\ q_\parallel^{-3+(1-d-2\chi)/\zeta} q_\perp^2, & q_\perp^\zeta \ll q_\parallel \end{cases} \qquad C_v(\vec{R}) = \langle \delta\vec{v}(\vec{r}+\vec{R},t) \cdot \delta\vec{v}(\vec{r},t) \rangle = |R_\perp|^{2\chi} f_v$$

$$C_v(\vec{R}) = \langle \delta\vec{v}(\vec{r}+\vec{R},t) \cdot \delta\vec{v}(\vec{r},t) \rangle = |R_\perp|^{2\chi} f_v \left(\frac{|R_\parallel|/l_0}{(|R_\perp|/l_0)^\xi} \right)$$

Toner & Tu Hydrodynamic theory predicts universal long-ranged correlations

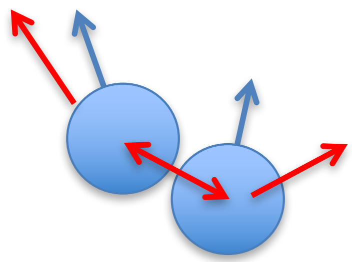
Anysotropic structure

$$S(\mathbf{q}) \sim \begin{cases} q_\perp^{1-d-\zeta-2\chi}, & q_\parallel \ll q_\perp \\ q_\parallel^{-2} q_\perp^{3-d-\zeta-2\chi}, & q_\perp^\zeta \gg q_\parallel \gg q_\perp \\ q_\parallel^{-3+(1-d-2\chi)/\zeta} q_\perp^2, & q_\perp^\zeta \ll q \end{cases}$$



A better model at short scales: Collisional Vicsek model (CVM)

$$\dot{\mathbf{r}}_i = v_0 \hat{\mathbf{n}}(\theta_i) + \beta \sum_{j}^{N_0} \mathbf{F}_{ij}$$



$$\dot{\theta}_i = \frac{1}{\tau}(\theta_i - \psi_i) + \xi_i$$

$$\mathbf{v_i} \equiv \dot{\mathbf{r}}_i = v_i \left(\cos\psi_i, \sin\psi_i\right),$$

$$\mathbf{F}_{ij} = \begin{cases} 0 & \text{if } r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j| \geq (\sigma_i + \sigma_j) \\ \left[r_{ij} - (\sigma_i + \sigma_j)\right] \hat{\mathbf{r}}_{ij} & \text{if } r_{ij} < (\sigma_i + \sigma_j) \end{cases}$$

$$\langle \xi_i \rangle = 0$$
$$\langle \xi_i(t) \xi_j(t') \rangle = \eta^2 \delta_{ij} \delta(t - t').$$

Rescale time and space

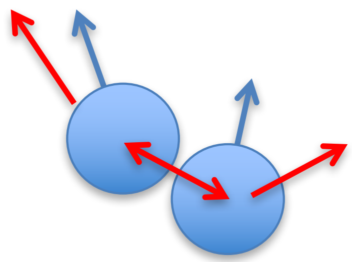
$$\beta = \langle \sigma \rangle = 1$$

Szabo B, Szollosi GJ, Gonci B, Juranyi Z, Selmeczi D and Vicsek T 2006 Phys. Rev. E 74(6) 061908

Henkes S, Fily Y and Marchetti M C 2011 Phys. Rev. E 84(4) 040301

A better model at short scales: Collisional Vicsek model (CVM)

$$\dot{\mathbf{r}}_i = v_0 \hat{\mathbf{n}}(\theta_i) + \beta \sum_j^{N_0} \mathbf{F}_{ij}$$



$$\dot{\theta}_i = \frac{1}{\tau}(\theta_i - \psi_i) + \xi_i$$

$$\mathbf{v_i} \equiv \dot{\mathbf{r}}_i = v_i\,(\cos\psi_i, \sin\psi_i).$$

Realignment timescale τ

Self-propulsion speed v_0

Packing fraction $\phi = \rho\pi\langle\sigma_i^2\rangle = 1.2$

Noise $\eta = 0.45.$

Polydispersivity = 20%

Szabo B, Szollosi GJ, Gonci B, Juranyi Z, Selmeczi D and Vicsek T 2006 Phys. Rev. E 74(6) 061908

Henkes S, Fily Y and Marchetti M C 2011 Phys. Rev. E 84(4) 040301